ECON 8402. International Macro

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This problem set is due on **Tuesday November 9**, during class.

Everyone should write their own individual answers, but you are encouraged to work in groups.

1 Static Trade Model

International trade provides a nice illustration of how we can use the equivalence of Pareto optima and competitive equilibria to characterize equilibria in multi-agent economies. Consider a static two-country economy with two goods, apples a and bananas b, and two agents, one representing each country. Country 1 is endowed with y_1 apples and country 2 is endowed with y_2 bananas. Each country has the same constant elasticity utility function:

$$U(a, b) = \left[(1-\omega) a^{\rho} + \omega b^{\rho}\right]^{\frac{1}{\rho}}$$

with parameters $0 < \omega < 1$ and $\rho < 1$ The elasticity of substitution is $\sigma = 1/(1-\rho)$

(a) State and solve an optimum problem corresponding to an arbitrary Pareto optimal allocation. What is the implicit relative price q of apples to bananas?

(b) Use the solution to (a) to find the competitive equilibrium associated with the initial endowments.

(c) Suppose we wanted to choose parameters to match an observed import share: say, the ratio of imports at market prices to the endowment at market prices for country 1. Given a value of ρ, show how we might use the observed import share to choose a value for ω.

(d) Comment briefly on how your solution changes if countries have different preferences — for example, if ω differs between them.

2 Complete markets, SOE, investment and the trade balance

There is a Small Open Economy (SOE) with a representative agent that can buy and sell assets contingent on states of the world. The assets are bought or sold as well by risk neutral foreigners who required an expected interest rate of R between periods.

Assume that the representative agent in the SOE has log utility over consumption and maximizes the discounted sum of expected utility using a discount factor of β.

Suppose that the representative agent owns an endowment $y(s^t)$ where s is the state of the world. Let $\pi(s^t)$ denote the probability of the history s^t.

(a) Suppose an Arrow Debreu market opens at the beginning of time. At what prices, $q(s^t)$, can the representative consumer in the SOE buy and sell consumption at state s^t from the initial state $t = 0$? Hint: Relate them to probabilities and R.

(b) Write down the time 0 problem of the representative agent in the small open economy given the prices of consumption. What can you tell about the nature of the solution? How does it relates to β and R? How does uncertainty affects the solution?

(c) Define the trade balance in the small open economy as output (endowment) minus consumption: $TB = Y - C$. What's the correlation of the trade balance with the endowment process in the SOE? In the data this correlation is counter-cyclical. Do we have a problem?

(d) Suppose that the representative agent in the SOE, owns instead of an endowment, a production technology of the form:

$$y(s^t) = z(s^t)f(k(s^{t-1}))$$

where k represents the capital stock in the previous period. Capital is accumulated by the following equation,

$$k(s^t) = i(s^t) + (1 - \delta)k(s^{t-1})$$

where δ is the depreciation rate and i is investment.

Assume that at any state one unit of investment can be created with one unit of consumption, and vice-versa (the price of investment in any state s^t is equal to the price of consumption).

(e) Write down the new time 0 budget constraint of the representative agent. Write down the time 0 problem of the representative agent given the consumption prices. Show that there is a separation of the investment decision and the consumption problem (meaning: the solution to the investment decision does not depends on the level of consumption attained by the representative agent).

(f) What is the first order condition for investment?

(g) Suppose that the shocks faced by the small open economy are i.i.d.: $\pi(s^t) = \pi(s_t)$. Is investment moving with the business cycle? What do we need for investment to be procyclical?

(h) Finally, define the trade balance as $TB = Y - C - I$. What's the correlation of the TB with output when the shocks are i.i.d.? What's the correlation between savings and investment?

(i) Describe what could happen if the shocks where not i.i.d.? Could we ever obtain a counter-cyclical trade balance? What about the correlation between savings and investment?

3 Risk Sharing with no Assets – An Example of Cole-Obsfeld

There is only one period, two countries (1 and 2) and **two goods** (A and B). The representative agent in each country has a utility function given by:

$$u(c^A, c^B) = \frac{(c^A c^B)^{\frac{1-\rho}{2}}}{1-\rho}$$

where $\rho > 0$ and where c^A represents the consumption of good A and similarly for c^B.

There is a state of world indexed by $s \in S$. In state $s \in S$, which occurs with probability $\pi(s) > 0$, country 1 is endowed with $e^A(s)$ units of good A while country 2 is endowed with $e^B(s)$ units of good B (Note country 1 has no endowment of good B, and country 2 has no endowment of good A).

(a) Solve for Pareto optimal allocations.

(b) Suppose that there are no financial markets but countries can trade goods after the state of the world is realized. Show that the competitive equilibrium allocation that results without financial assets is Pareto efficient ex-ante (that is, before the state of the world is known).

(c) Given your answer to part (b), what happens to the relative price of good A in states where $e^A(s)/e^B(s)$ is low? Explain in words how risk sharing is achieved.

(d) Why is (b) an important result to consider when thinking about risk sharing across countries?

4 A (Numerical) Recursive Pareto Problem

There are two economies, A and B. Every period, there are two possible realizations of the state $S = \{s_1, s_2\}$. The total endowment of the single good in each state is such that $Y(s_1) = 1$ and $Y(s_2) = 1/2$. In all periods, the probabilities of each state are $\pi(s_1) = \pi(s_2) = 1/2$, which are independent of history (i.i.d.).

The utility function of the representative agent in each country is given by

$$\sum_t \sum_{s^t} \beta^t \pi(s^t) u(c^i(s^t))$$

Let v denote the utility value promised to the households in country A, before the realization of uncertainty this period. Let $P(v)$ denote the maximum utility that can be attained by households in country B, given that a value of v has been promised to the households in country A. Then, we can write the Pareto problem recursively as follows:

$$P(v) = \max_{c(s), w(s)} \sum_{s \in S} \pi(s) \{u(Y(s) - c(s)) + \beta P(w(s))\}$$

subject to:

$$v \leq \sum \pi(s) \{u(c(s)) + \beta w(s)\}$$

Be sure that you understand why is this a Pareto problem. The following questions will ask you to go to the computer and solve the above problem using value function iteration. In what follows suppose that $u(c) = c^{1/2}$, $\beta = 0.8$.

(a) **A grid for** v. Note that the maximum that can be promised to the households in country A is $\overline{V} = \sum_{s \in S} \pi(s) u(Y(s))/(1 - \beta)$ and the minimum value is $\underline{V} = \sum_{s \in S} \pi(s) u(0)/(1 - \beta)$. Create a grid for v in the computer. Don't make it too big (maybe just 20 points). Let's call that grid *vgrid*.

(b) **Iterating the value function.** For any initial value function guess, $P_i(v)$, iterate the value function. That is, for every v in *vgrid* solve numerically the maximization problem

$$P_{i+1}(v) = \max_{c(s),w(s)\in[\underline{V},\overline{V}]} \sum_{s\in S} \pi(s) \left\{u(Y(s) - c(s)) + \beta P_i(w(s))\right\}$$

subject to:

$$v \leq \sum \pi(s) \left\{u(c(s)) + \beta w(s)\right\}$$

and keep iterating until P_{i+1} is sufficiently close to P_i.

There are many ways you could do this. A simple (albeit brute-force) way is to try all possible combinations of $c(s_1), w(s_1), w(s_2)$ and solve for the $c(s_2)$ that will make the "promise keeping" constraint, i.e. $v \leq \sum \pi(s) \left\{u(c(s)) + \beta w(s)\right\}$, hold with equality.

Here are the steps. Construct a grid for $c(s_1)$ from 0 to $Y(s_1)$ (don't make it too big, say 20 points). Loop over all possibles v in *vgrid*. For every value of v, find numerically the combination of $c(s_1)$ in the consumption grid, and $w(s_1)$ and $w(s_2)$ in *vgrid* that, once you obtained $c(s_2)$ from above, maximizes the objective given your guess for P. Iterate until convergence.

You are free to improve on this algorithm if you want.

(c) **A check.** Consider now the sequence problem, and let λ denote the multiplier on the households from country A. Compute the solution to the planning problem analytically for any λ. Check numerically that as you move λ, you trace out the value function found in step (b).