AP Statistics Chapter 5 – Probability: What are the Chances?

5.1: Randomness, Probability and Simulation

Probability

The **probability** of any outcome of a chance process is a number between 0 and 1 that describes the proportion of times the outcome would occur in a very long series of repetitions.

Simulation

The imitation of chance behavior, based on a model that accurately reflects the situation, is called a **simulation**.

Performing of a Simulation – The 4-Step Process

1. **State**: Ask a question of interest about some chance process.
2. **Plan**: Describe how to use a chance device to imitate one repetition of the process. Tell what you will record at the end of each repetition.
3. **Do**: Perform many repetitions of the simulation.
4. **Conclude**: Use the results of your simulation to answer the question of interest.

5.2: Probability Rules

Sample Space

The **sample space *S*** of a chance process is the set of all possible outcomes.

Probability Models

Descriptions of chance behavior contain two parts:

A **probability model** is a description of some chance process that consists of two parts:

- a sample space S and
- a probability for each outcome.

For example: When a fair 6-sided die is rolled, the Sample Space is S = {1, 2, 3, ,4,5, 6}. The probability for a fair die would include the probabilities of these outcomes, which are all the same.

Outcome	**1**	**2**	**3**	**4**	**5**	**6**
Probability	1/6	1/6	1/6	1/6	1/6	1/6

Event

An **event** is any collection of outcomes from some chance process. That is, an event is a subset of the sample space. Events are usually designated by capital letters, like *A, B, C*, and so on.

For example: For the probability model above we might define event A = die roll is odd. The elements of the sample space S that fits this event are {1, 3, 5}. The probability of the event A, written as P(A) is the 3/6 or ½. So we would write P(A) = 0.5, in decimal form.

The Basic Rules of Probability

- For any event A, $0 \le P(A) \le 1$.
- If S is the sample space in a probability model, $P(S) = 1$.
- In the case of equally likely outcomes,

$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

- **Complement rule:** $P(A^C) = 1 - P(A)$
- **Addition rule for mutually exclusive events:** If A and B are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B)$. Also be familiar with the notation: $\boldsymbol{P(A \cup B)}$.

Mutually Exclusive Events

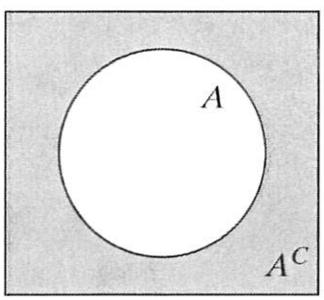
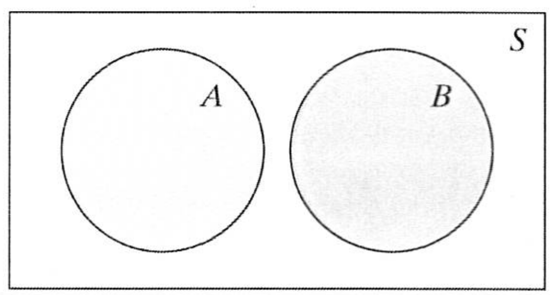
Two events A and B are **mutually exclusive** (or **disjoint**) if they have no outcomes in common and so can never occur together—that is, if $P(A \text{ and } B\ ) = 0$. Alternate notation: $\boldsymbol{P(A \cap B)}$.

For example: Using a deck of playing cards and drawing a card at random, the events A = card is a King, and B = card is a Queen are mutually exclusive because a single card cannot be both a King and a Queen. Thus we can calculate the probability of A or B as the sum of their individual probabilities - P(A or B) = P(A) + P(B).

General Addition Rule

If A and B are any two events resulting from some chance process, then
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Venn Diagrams and Probability

<table>
<tr>
<td>The complement A^c contains exactly the outcomes that are not in A.
</td>
<td>The events A and B are mutually exclusive (disjoint) because they do not overlap. That is, they have no outcomes in common.
</td>
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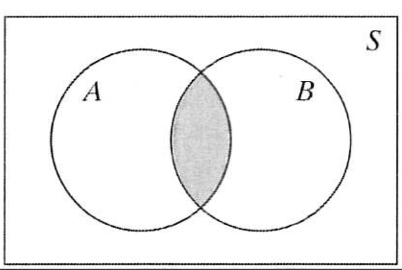
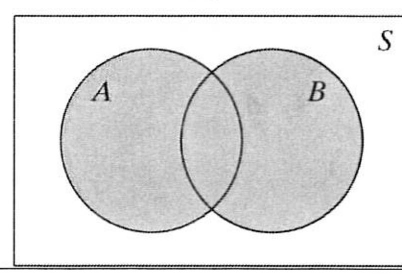
<tr>
<td>The intersection of events A and B (A ∩ B) is the set of all outcomes in both events A and B.
A ∩ B
</td>
<td>The union of events A and B (A ∪ B) is the set of all outcomes in either event A or B.
A ∪ B
</td>
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5.3: Conditional Probability and Independence

Conditional Probability

The probability that one event happens given that another event is already known to have happened is called a **conditional probability**.

Suppose we know that event A has happened. Then the probability that event B happens given that event A has happened is denoted by $P(B \mid A)$. The symbol "" is read as "given that," so we read $P(B \mid A)$ as the probability that B occurs given that A has already occurred.

Calculating Conditional Probability

To find the conditional probability $P(A \mid B)$, use the formula

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

The conditional probability $P(B \mid A)$ is given by

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)}$$

The General Multiplication Rule

The probability that events A and B both occur can be found using the general multiplication rule

$$P(A \cap B) = P(A) \bullet P(B \mid A),$$

where $P(B \mid A)$ is the conditional probability that event B occurs given that event A has already occurred.

Conditional Probability and Independence

Two events A and B are **independent** if the occurrence of one event does not change the probability that the other event will happen. In other words, events A and B are independent if $P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$.

The Multiplication Rule for Independent Events

If A and B are independent events, then the probability that A and B both occur is

$$P(A \cap B) = P(A) \bullet P(B)$$

Chapter 5 - Probability

Multiple Choice

Identify the choice that best completes the statement or answers the question.

1. An assignment of probability must obey which of the following?
   a. The probability of any event must be a number between 0 and 1, inclusive.
   b. The sum of all the probabilities of all outcomes in the sample space must be exactly 1.
   c. The probability of an event is the sum of the outcomes in the sample space which make up the event.
   d. All of the above.
   e. A and B only.

2. Students at University X must be in one of the class ranks—freshman, sophomore, junior, or senior. At University X, 35% of the students are freshmen and 30% are sophomores. If a student is selected at random, the probability that her or she is either a junior or a senior is
   a. 30%.
   b. 35%.
   c. 65%.
   d. 70%.

In a particular game, a fair die is tossed. If the number of spots showing is either four or five, you win $1. If the number of spots showing is six, you win $4. And if the number of spots showing is one, two, or three, you win nothing. You are going to play the game twice.

3. The probability that you win $4 both times is
   a. 1/6
   b. 1/3
   c. 1/36
   d. 1/4
   e. 1/12

4. The probability that you win money at least once in the two games is
   a. .75
   b. .50
   c. .25
   d. .125
   e. .1

An event A will occur with probability 0.5. An event B will occur with probability 0.6. The probability that both A and B will occur is 0.1.

5. The conditional probability of A given B is
   a. 0.5.
   b. 0.3.
   c. 0.2.
   d. 1/6.
   e. cannot be determined from the information given.

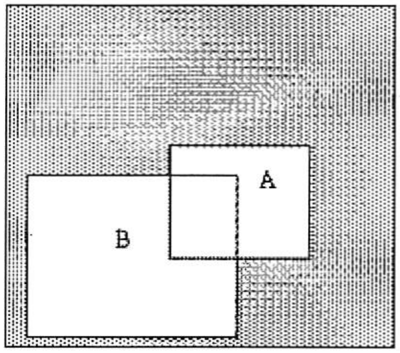
6. Experience has shown that a certain lie detector will show a positive reading (indicates a lie) 10% of the time when a person is telling the truth and 95% of the time when a person is lying. Suppose that a random sample of 5 suspects is subjected to a lie detector test regarding a recent one-person crime. Then the probability of observing no positive reading if all suspects plead innocent and are telling the truth is
   a. 0.409
   b. 0.735
   c. 0.00001
   d. 0.590
   e. 0.99999

7. If you buy one ticket in the Provincial Lottery, then the probability that you will win a prize is 0.11. If you buy one ticket each month for five months, what is the probability that you will win at least one prize?
   a. 0.55
   b. 0.50
   c. 0.44
   d. 0.45
   e. 0.56

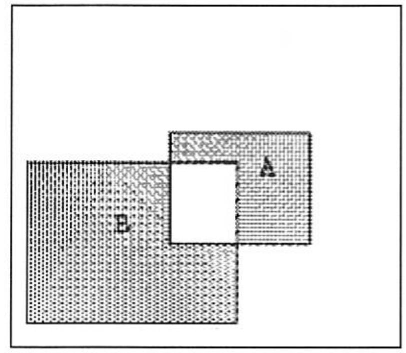
8. Suppose that A and B are two independent events with $P(\text{A}) = .2$ and $P(\text{B}) = .4$. $P(A \cap B^c)$ is
   a. 0.08.
   b. 0.12.
   c. 0.52.
   d. 0.60.

9. A plumbing contractor puts in bids in on two large jobs. Let the event that the contractor wins the first contract be A and the event that the contractor wins the second contract be B. Which of the Venn diagrams has shaded the event that the contractor wins exactly one of the contracts?

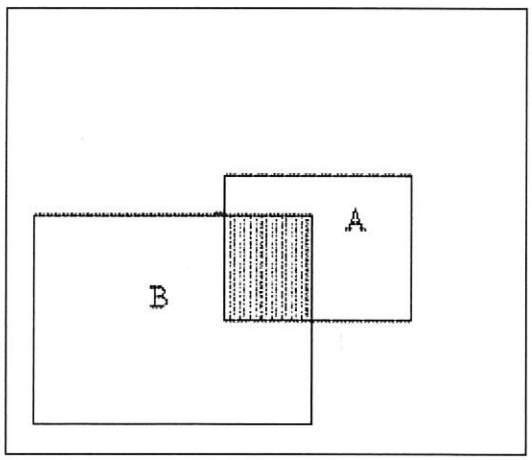
a.



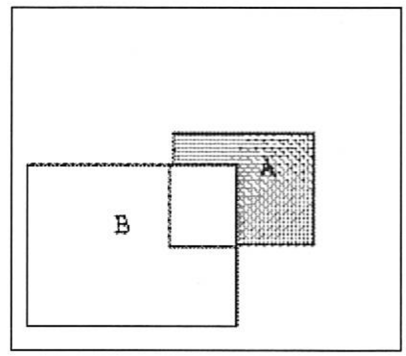
c.



b.



d.



10. A die is loaded so that the number 6 comes up three times as often as any other number. What, then, is the probability of rolling a 6?

a. .125
b. .250
c. .375
d. .500
e. None of the above.

AP Statistics: Chapter 5 Practice Free Response exam

1. Below is a two-way table that describes responses of 120 subjects to a survey in which they were asked, "Do you exercise for at least 30 minutes four or more times per week?" and "What kind of vehicle do you drive?"

		Car type			
		Sedan	SUV	Truck	Total
Exercise?	Yes	25	15	12	52
	No	20	24	24	68
	Total	45	39	36	120

Suppose one person from this sample is randomly selected.

(a) What is the probability that the person selected drives an SUV?

(b) What is the probability that the person selected drives either a sedan or a truck?

(c) What is the probability that the person is a non-exerciser that drives a sedan?

(d) What is the probability that the person is a non-exerciser or drives a sedan?

(e) What is the probability that the person selected drives a truck given that they exercise?

2. There are 35 students in Ms. Ortiz's Calculus class. One day, 24 students turned in their homework and 14 turned in test corrections. Eight of these students turned in both homework and test corrections. Suppose we randomly select a student from the class.

(a) Complete a Venn diagram below so that it describes the chance process involved here. Let H = the event "turned in homework" and C = the event "turned in corrections."

(b) Using your Venn diagram, determine the probability that a student did not turn in homework or their test corrections.

(c) Using your Venn diagram, determine the probability that a student turned in homework given that they turned in test corrections.

3. You have been handed a bag containing four $1 bills and two $5 bills. You will reach in the bag (blindly) and select two bills at random.

(a) Sketch a complete tree diagram to represent the outcomes of this random selection.

Use your tree diagram to answer the following:

(b) What is the probability that you end up with $6?

(c) What is the probability that the two selected bills are the same dollar amount?