

## AP Statistics Chapter 2 – Describing Location in a Distribution

### 2.1: Measures of Relative Standing and Density Curves

#### Density Curve

A **density curve** is a curve that

- is always on or above the horizontal axis, and
- has area exactly 1 underneath it.

A density curve describes the overall pattern of a distribution. The area under the curve and above any range of values is the proportion of all observations that fall in the range.

#### Example

The density curve below left is a rectangle. The area underneath the curve is  $4 \cdot 0.25 = 1$ .

The figure on the right represents the proportion of data between 2 and 3 ( $1 \cdot 0.25 = 0.25$ ).



#### Median and Mean of a Density Curve

- The **median** of a density curve is the **equal-areas point**, the point that divides the area under the curve in half.
- The **mean** of a density curve is the **balance point**, at which the curve would balance if made of solid material.
- The median and mean are the same for a symmetric density curve. They both lie at the center of the curve. The mean of a skewed curve is pulled away from the median in the direction of the long tail.

#### Normal Distributions

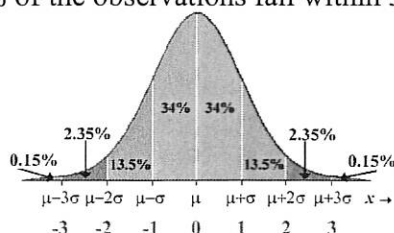
A normal distribution is a curve that is

- mound-shaped and symmetric
- based on a continuous variable
- adheres to the 68-95-99.7 Rule

#### The 68-95-99.7 Rule

In the normal distribution with mean  $\mu$  and standard deviation  $\sigma$ :

- 68% of the observations fall within  $1\sigma$  of the mean  $\mu$ .
- 95% of the observations fall within  $2\sigma$  of the mean  $\mu$ .
- 99.7% of the observations fall within  $3\sigma$  of the mean  $\mu$ .



## 2.2: Normal Distributions

### Standardizing and z-Scores

If  $x$  is an observation from a distribution that has mean  $\mu$  and standard deviation  $\sigma$ , the **standardized value of  $x$**  is

$$z = \frac{x - \mu}{\sigma}$$

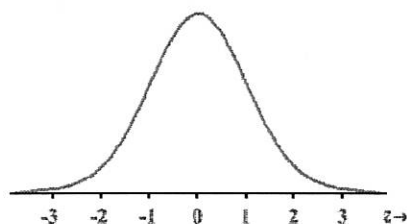
A standardized value is often called a **z-score**.

### Standard Normal Distribution

- The standard normal distribution is the normal distribution  $N(0, 1)$  with mean 0 and standard deviation 1.
- If a variable  $x$  has any normal distribution  $N(\mu, \sigma)$  with mean  $\mu$  and standard deviation  $\sigma$ , then the standardized variable

$$z = \frac{x - \mu}{\sigma}$$

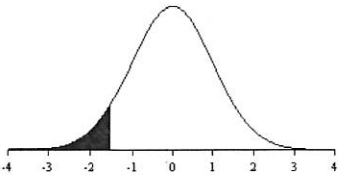
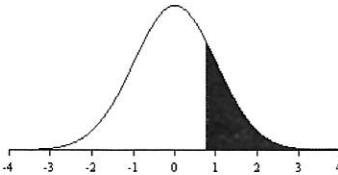
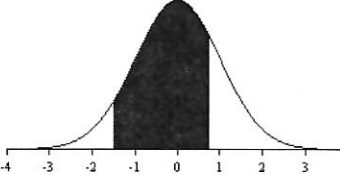
has the standard normal distribution (see diagram below).



### The Standard Normal Table

**Table A** is a table of areas under the standard normal curve. The table entry for each value  $z$  is the area under the curve to the left of  $z$ .

### Standard Normal Calculations

Area to the left of $z$ ( $Z < z$ )	Area to the right of $z$ ( $Z > z$ )	Area between $z_1$ and $z_2$
		
Area = Table Entry	Area = $1 - \text{Table Entry}$	Area = difference between Table Entries for $z_1$ and $z_2$

### Inverse Normal Calculations

Working backwards from the area, we find  $z$ , then  $x$ . The value of  $z$  is found using Table A *in reverse*. The value of  $x$  is found, from  $z$ , using the formula below

$$x = \mu + z \cdot \sigma$$

## AP Statistics - Chapter 2 MC Study Guide

### Multiple Choice

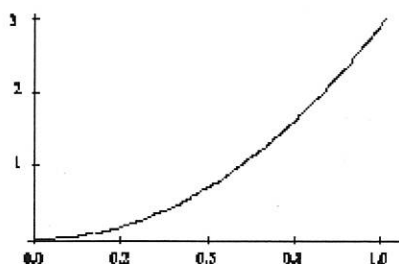
Identify the choice that best completes the statement or answers the question.

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1. The heights of American men aged 18 to 24 are approximately normally distributed with mean 68 inches and standard deviation 2.5 inches. Only about 5% of young men have heights outside the range
- 65.5 inches to 70.5 inches
  - 63 inches to 73 inches
  - 60.5 inches to 75.5 inches
  - 58 inches to 78 inches
  - none of the above

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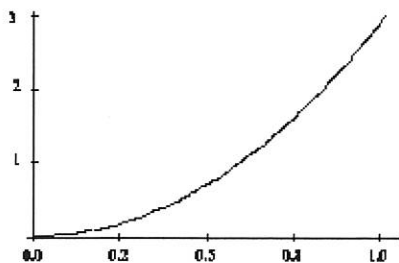
2. For the density curve shown below, which statement is true?



- The density curve is symmetric.
- The density curve is skewed right.
- The area under the curve between 0 and 1 is 1.
- The density curve is normal.
- None of the above is correct.

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3. For the density curve shown below, which statement is true?



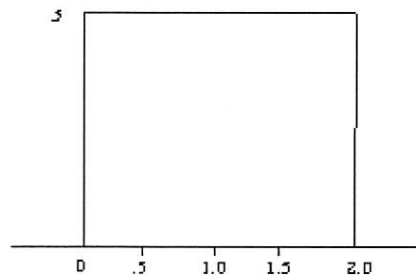
- The mean and median are equal.
- The mean is greater than the median.
- The mean is less than the median.
- The mean could be either greater than or less than the median.
- None of the above is correct.

- ☐ 4. The area under the standard normal curve corresponding to  $-0.3 < Z < 1.6$  is
- a. 0.3273
  - b. 0.4713
  - c. 0.5631
  - d. 0.9542
  - e. None of the above

- ☐ 5. Suppose that sixteen-ounce bags of chocolate chips cookies are produced with an actual mean weight of 16.1 ounces and a standard deviation of 0.1 ounce. The percentage of bags that will contain between 16.0 and 16.1 ounces is
- a. 10
  - b. 16
  - c. 34
  - d. 68
  - e. none of the above

- ☐ 6. A company produces packets of soap powder labeled "Giant Size 32 Ounces." The actual weight of soap powder in a box has a normal distribution with a mean of 33 oz. and a standard deviation of 0.8 oz. What proportion of packets are underweight (i.e., weigh less than 32 oz.)?
- a. 0.159
  - b. 0.212
  - c. 0.106
  - d. 0.841
  - e. 0.115

- ☐ 7. For the density curve below, what percent of the observations lie between 0.5 and 1.2?

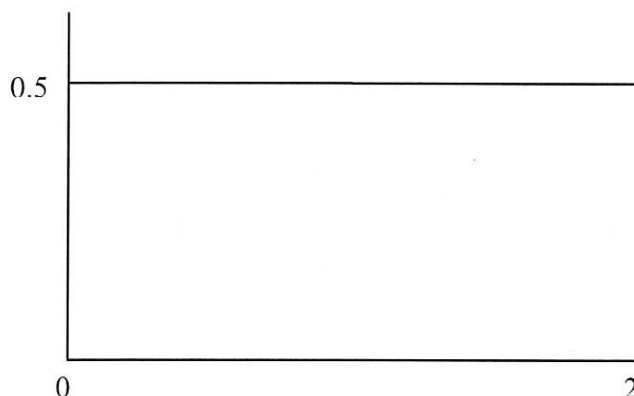


- a. 25%
- b. 35%
- c. 50%
- d. 68%
- e. 70%

- ▼ 8. If the heights of 99.7% of American men are between 5'0" and 7'0", what is your estimate of the standard deviation of the height of American men?
- a. 1"
  - b. 3"
  - c. 4"
  - d. 6"
  - e. 12"
- ▼ 9. Suppose that the distribution of math SAT scores from your state this year is normally distributed with mean 480 and standard deviation 100 for males, and mean 440 and standard deviation 120 for females. If someone who scores 780 or higher on math SAT can be considered a genius, what is the proportion of geniuses among the male SAT takers?
- a. 26%
  - b. 13%
  - c. 3%
  - d. 1.3%
  - e. 0.13%
- ▼ 10. The average yearly snowfall in Chillyville is normally distributed with a mean of 55 inches. If the snowfall in Chillyville exceeds 60 inches in 15% of the years, what is the standard deviation?
- a. 4.81 inches
  - b. 5.18 inches
  - c. 6.04 inches
  - d. 8.93 inches
  - e. The standard deviation cannot be computed from the given information.

## AP Statistics Chapter 2 Practice Test

1. The density curve below is uniform from  $(0, 0.5)$  to  $(2, 0.5)$ .



- a) Verify that the area below this density curve is 1.
  - b) What proportion of observations in this density curve are less than 0.75?
  - c) What proportion of observations in this density curve are greater than 1.5?
2. In a study of elite distance runners, the mean weight was reported to be 63.1 kilograms (kg), with a standard deviation of 4.8 kg.

- a) Assuming that the distribution of weights is normal, sketch the density curve of the weight distribution, with the horizontal axis marked in kilograms.

**Using your sketch and the 68-95-99.7 rule, answer the following:**

- b) What range represents the middle 68% of runner's weights?
  - c) What percentage of runner's weights are less than 58.3 kg?
  - d) What percentage of runners weighs between 63.1 kg and 72.7 kg?
3. A lunch stand in the business district has a mean daily gross income of \$520 with a standard deviation of \$50. Assume that the daily gross income is normally distributed.
- a) In what proportion of days is the gross income less than \$400?
  - b) In what proportion of days is the gross income more than \$600?
  - c) In what proportion of days is the gross income between \$400 and \$600?
  - d) What does the gross income need to be for a day to be in the top 10% of all days?