

$$\begin{aligned}
(x - A_1)^2 + (y - B_1)^2 + (z - C_1)^2 &= [c(t_1 - d)]^2 \\
(x - A_2)^2 + (y - B_2)^2 + (z - C_2)^2 &= [c(t_2 - d)]^2 \\
(x - A_3)^2 + (y - B_3)^2 + (z - C_3)^2 &= [c(t_3 - d)]^2 \\
(x - A_4)^2 + (y - B_4)^2 + (z - C_4)^2 &= [c(t_4 - d)]^2
\end{aligned}$$

$$f(x + \Delta x) \cong f(x) + \Delta x f'(x) + \frac{\Delta x^2}{2} f''(x) + \dots$$

$$f(x + \Delta x) = f(x) + \Delta x f'(x)$$

$$\text{Assume } f(x + \Delta x) = 0, \text{ then}$$

$$f(x) + \Delta f'(x) = 0$$

$$f(x) = -\Delta x f'(x)$$

$$\Delta x = \frac{-f(x)}{f'(x)}$$

$$x_{new} = x + \Delta x = x - \frac{f(x)}{f'(x)}$$

$$\overrightarrow{F}(\overrightarrow{x}) = \overrightarrow{0}$$

$$\overrightarrow{F}(\overrightarrow{x} + \Delta \overrightarrow{x}) = \overrightarrow{F}(\overrightarrow{x}) + DF(\overrightarrow{x})\Delta \overrightarrow{x}$$

$$(1) \quad DF(\overrightarrow{x}) = \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \cdots & \frac{\partial F_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_n}{\partial x_1} & \frac{\partial F_n}{\partial x_2} & \cdots & \frac{\partial F_n}{\partial x_n} \end{pmatrix}$$

$$\overrightarrow{x}_{new} = \overrightarrow{x}_{old} + \Delta \overrightarrow{x} = \overrightarrow{x}_{old} - DF(\overrightarrow{x})^{-1} \cdot F(\overrightarrow{x})$$

$$A_i = \rho \cos \phi_i \cos \theta_i$$

$$B_i = \rho \cos \phi_i \sin \theta_i$$

$$C_i = \rho \sin \theta_i$$

satellite ranges $R_i = \sqrt{A_i^2 + B_i^2 + (C_i - 6370)^2}$ and travel times $t_i = d + R_i/c$

$$\text{error magnification factor} = \frac{\|(\Delta x, \Delta y, \Delta z)\|_\infty}{c \|\Delta t_1, \dots, \Delta t_m\|_\infty}$$

$$(2) \quad -DF(\vec{x})\Delta\vec{x} = F(\vec{x})$$

$$(3) \quad -DF^T(\vec{x})DF(\vec{x})\Delta\vec{x} = DF^T(\vec{x})F(\vec{x})$$

$$(4) \quad -(DF^T(\vec{x})DF(\vec{x}))^{-1}DF^T(\vec{x})DF(\vec{x})\Delta\vec{x} = (DF^T(\vec{x})DF(\vec{x}))^{-1}DF^T(\vec{x})F(\vec{x})$$

$$(5) \quad \Delta\vec{x} = -(DF^T(\vec{x})DF(\vec{x}))^{-1}DF^T(\vec{x})F(\vec{x})$$