Reconstruction of Video Using SVD with Delays

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Abstract

Singular value decomposition (SVD) is a matrix factorization that can be used to compress or filter data. We apply this idea to reconstruct video acquired with noise. Our goal is to interpret the correlation of the time frames of the data and cancel the noise using delays of time frames.

General Aims

The general aims of our research project can be summarised in the following points:

1. Understanding the general concept of **Singular Value Decomposition (SVD)**
2. Developing better method to remove noise from noisy video data using delays.
3. Apply the method to more complex video data.

Singular Value Decomposition

For every $m \times n$ matrix A, there are orthonormal sets $\{u_1, ..., u_m\}$ and $\{v_1, ..., v_n\}$, together with nonnegative numbers $s_1 \leq ... \leq s_n \leq 0$, satisfying

$$Av_i = s_i u_i (1 \leq i \leq n)$$

v_i is called the **right singular vector** of the matrix A, the u_i is the **left singular vector** of A, and the s_i is the **singular value** of A. Then,

$$A = USV^T$$

where U is an $m \times m$ matrix, whose columns are the left singular vectors u_i, V is an $n \times n$ matrix, whose columns are the right singular vectors v_i, and S is a diagonal $m \times n$ matrix, whose diagonal entries are the singular values s_i.

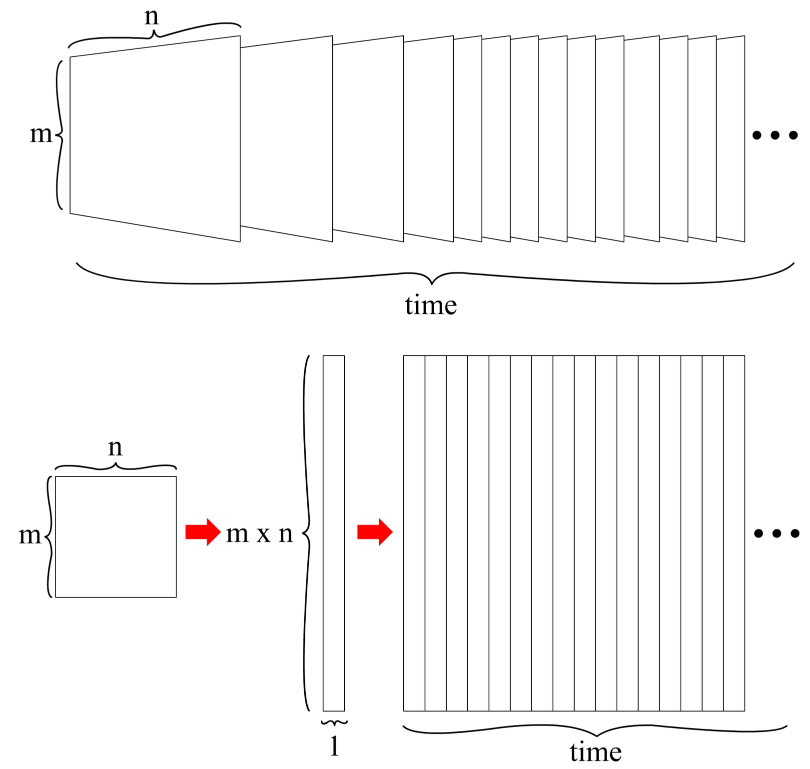
Low Rank Approximation Property

$$A = USV^T = s_1 u_1 v_1^T + s_2 u_2 v_2^T + ... + s_r u_r v_r^T$$

$s_i u_i v_i^T$ $(1 \leq i \leq r)$ will represent the i th mode of matrix A.

Applying SVD to Video Data

Video data can be expressed as 3D matrix **m × n × time**. In order to apply SVD to video data, we need to convert 3D into 2D matrix.



1. Video data

$= m \times n \times time$

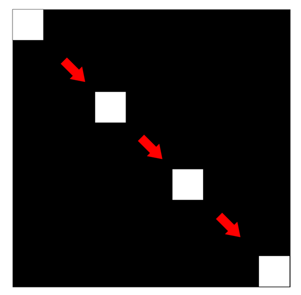
2. One frame

$= m \times n$
$= (m * n) \times 1$

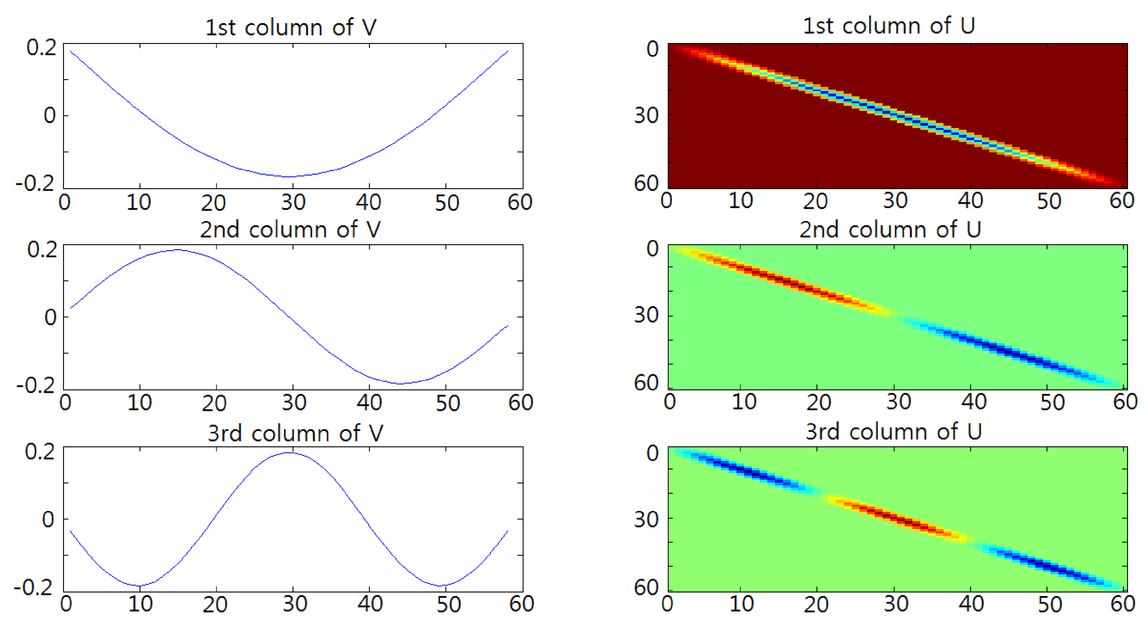
3. Reshaping

$= m \times n \times time$
$= (m * n) \times time$

Example of SVD with Video



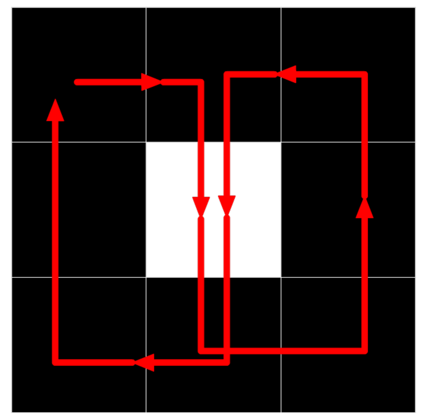
We applied SVD to the simple video that 3×3 box moving in 64×64, in a diagonal direction.
As a result, we obtained matrices of U and V. Each column of U and row of V represent a different mode. For example, i th column and row of U and V are i th mode of the video data. The graph of i th mode of U gives us i th Eigen frame. The graph of V tells us the time frequency of each mode of U. The frequency increases as mode increases.



SVD to Video Data

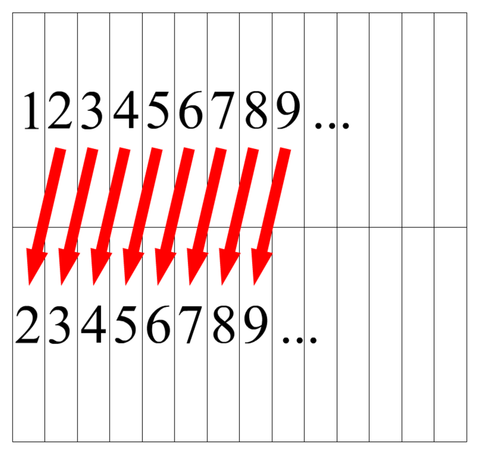
Adding Noise to Video data

In real life situation, most of the video data contain noises. In order to apply SVD to a realistic situation, noise values are added to video data, using random numbers generated by Matlab.



The video data we used is a one unit white box moving in 3×3 matrix making shape of infinity symbol ∞. We found that SVD didn't work very well when noise was added to the video data. The interesting thing about this infinity shape video is that when a white box is at the middle, SVD is not able to predict which way the box would move, left or right.

SVD with Delays



One delay of a time frame

1. The figure shows the example of one delay of a time frame.

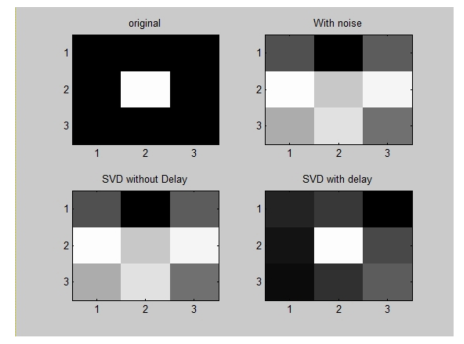
2. The second time frame is copied and pasted below the first time frame.

3. $i+1$ th time frame is copied and pasted below i th time frame.

4. $1 \leq i \leq$ (total time frames - the number of delays)

5. If $i{+}1$ through $i{+}k$ th time frames are copied and pasted below i th time frame, then it is k number of delays of time frames.

Results



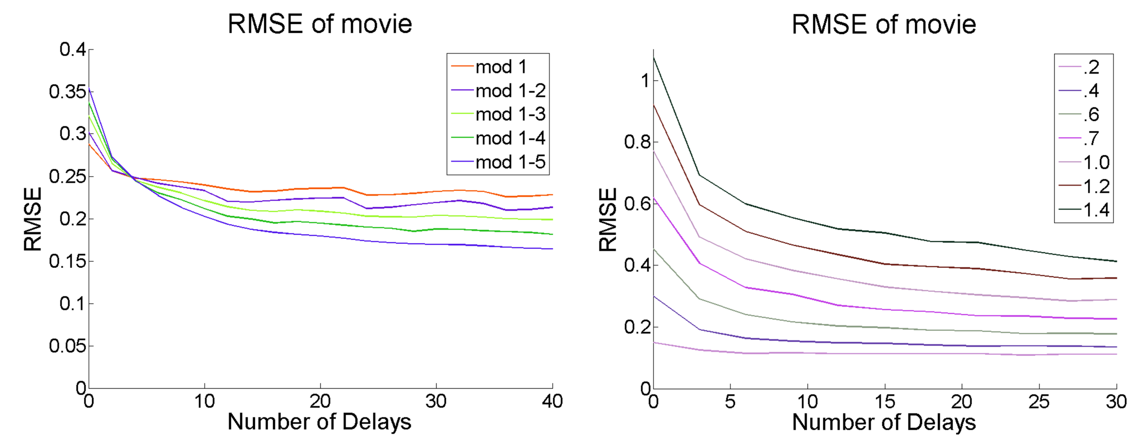
Top left: The original video data

Top right: The video data with noise.

Bottom left: Reconstructed video after SVD is applied without delays

Bottom right: With delays.

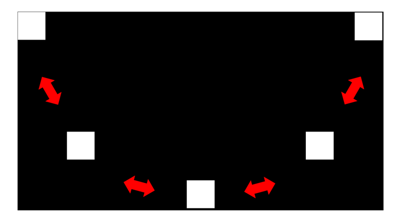
All four images are captured at the same time.



Two graphs show changes of RMSE with an increase in number of delays with different mods and different noise values.

- RMSE decreases as delays increase.
- The method works better with a video with more noises.

More complex video data is analyzed and the result is shown on the right.

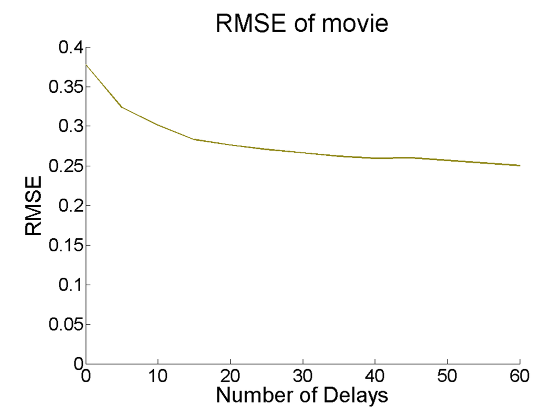


The video is a pendulum motion.

$40 \times 80 \times 300$ matrix.

The value of noises = 1

Same result: Delays improve SVD



References

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[1] Sauer, Timothy. Numerical Analysis 2nd Edition. Pearson, 2011. Print.