SNU Fourth Industrial Revolution Academy

Basic Math for Big Data

Homework 4

Due: August 3, 10:00 AM

Reminders

- T.A.: Chiwan Park (chiwanpark@snu.ac.kr)
- The points of this homework add up to 100.
- This has to be done individually like all the homeworks.
- Please answer clearly; illegible handwriting may get no points.
- Whenever you are making an assumption, please state it clearly.
- If you have a question about assignments, please upload your question in FIRA portal.

Submissions

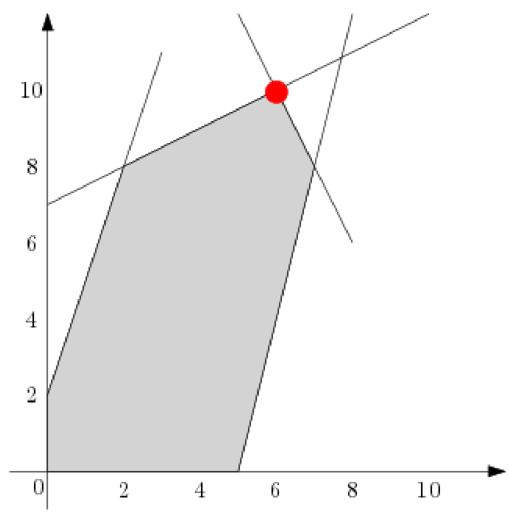
- You can submit your homework in the class or via email (only PDFs are accepted).
- Do not submit the homework in a photography form.

Question 1 [15 points]

Answer the following questions to solve the following linear programming problem containing two variables, x and y.

$$\begin{array}{ll} \text{maximize} & 2x + 3y \\ \text{subject to} & -3x + y \leq 2 \\ & 4x + 2y \leq 44 \\ & 4x - y \leq 20 \\ & -x + 2y \leq 14 \\ & 0 \leq x, y \end{array}$$

(a) Draw graphs of the linear constraints into an xy-plane and shade the feasible region. [10 points]



(b) Find the optimal solution of the problem and denote the solution in the graph of (a). [5 points]

The optimal solution $x = 6$ and $y = 10$ is colored in red. The value of objective function is 42.

Question 2 [15 points]

Consider a bakery which sells two types of bread, chocolate cream bread and baguette for 10 and 4 dollars, respectively. The chocolate cream bread consumes 100 grams of flour and 10 grams of chocolate, while the baguette consumes only 50 grams of flour. The bakery has 3 kilograms of flour and 100 grams of chocolate. Answer the following questions.

(a) Formulate an appropriate linear programming problem to maximize revenue. [10 points]

Let x be the number of chocolate cream bread and y be the number of baguette. Then, we have the following linear programming problem:

$$\begin{array}{ll} \text{maximize} & 10x + 4y \\ \text{subject to} & 100x + 50y \leq 3000 \\ & 10x \leq 100 \\ & x \geq 0, y \geq 0 \end{array}$$

(b) Find the optimal solution for the problem in (a). [5 points]

The optimal solution for the problem is $x = 10$ and $y = 40$. The revenue with the optimal solution is 260.

Question 3 [30 points]

A bronze-medal manufacturing company can produce three types of the medal, A, B, and C which sell for 18, 29, and 25 dollars and consume 0.5, 0.2, and 0.75 kilograms of copper and 0.2, 0.4 and 0.2 grams of tin, respectively. The company has 0.6 kilograms of tin and 1500 kilograms of copper, and are required to produce at least 1000 medals of A, 200 medals of B, and 400 medals of C. Answer the following questions.

(a) Formulate an appropriate linear programming problem to maximize revenue. [15 points]

Let a, b, and c be the number of medals A, B, and C, respectively. Then, we have the following linear programming problem:

$$\begin{array}{ll} \text{maximize} & 18a + 29b + 25c \\ \text{subject to} & 0.5a + 0.2b + 0.75c \leq 1500 \\ & 0.2a + 0.4b + 0.2c \leq 600 \\ & a \geq 1000 \\ & b \geq 200 \\ & c \geq 400 \end{array}$$

(b) Find the optimal solution for the problem in (a) using the linear programming solver in R. Note that you should submit your R script via email. [15 points]

By the linear programming solver in R, we get $a = 1000$, $b = 384$, $c = 1230$. The revenue is \$59886.

Question 4 [40 points]

Table 1 shows the needs of couriers to provide delivery services for the following three months. The cost of a courier is $8000 per month. At the beginning of the first month, the delivery company has 20 staffs, but the number of staffs can be adjusted each month.

Table 1. Needs of couriers for the following three months

Month	1	2	3
Needed couriers	30	60	55

Couriers can be hired and fired at the beginning of each month. Newly hired couriers can start working at the same month, and fired couriers stop working the same day they are fired. The cost of firing a courier is $10000, and the hiring cost of a courier is $5000. If it is convenient, the company can have a staff of couriers larger than the actual needs. Answer the following questions.

(a) Formulate an appropriate linear programming problem to minimize costs. [25 points]

We define two symbols h_i, f_i, and $s_i$which mean the number of couriers hired at the beginning of month i, the number of couriers fired at the beginning of month i, and the number of staffs during month i, respectively. Then, $\mathrm{s_i} = h_i - f_i + s_{i-1}$ for each month i and the initial number of staffs $\mathrm{s_0} = 20$.

From the definition of s_i, we get s_1, s_2, and s_3 as follows:

$$\begin{aligned} s_1 &= h_1 - f_1 + 20 \\ s_2 &= h_2 - f_2 + h_1 - f_1 + 20 \\ s_3 &= h_3 - f_3 + h_2 - f_2 + h_1 - f_1 + 20 \end{aligned}$$

Applying these variables, we obtain the cost equation as follows:

$$\begin{aligned} Cost &= 5000 \cdot \sum h_i + 10000 \cdot \sum f_i + 8000 \cdot \sum s_i \\ &= 29000 \cdot h_1 + 21000 \cdot h_2 + 13000 \cdot h_3 - 10000 \cdot f_1 - 6000 \cdot f_2 + 2000 f_3 \\ &\qquad + 480000 \end{aligned}$$

We derive the linear programming problem by combining the equations above.

$$
\begin{array}{ll}
\text{minimize} & Cost \\
\text{subject to} & h_1 - f_1 \geq 10 \\
& h_2 - f_2 + h_1 - f_2 \geq 40 \\
& h_3 - f_3 + h_2 - f_2 + h_1 - f_1 \geq 35 \\
& h_1, h_2, h_3, f_1, f_2, f_3 \geq 0
\end{array}
$$

(b) Find the optimal solution for the problem in (a) using the linear programming solver in R. Note that you should submit your R script via email. [15 points]

By the linear programming solver in R, we obtain $h_1 = 10, h_2 = 30, h_3 = f_1 = f_2 = f_3 = 0$. The cost is $1400000.