SNU Fourth Industrial Revolution Academy

Basic Math for Big Data

Homework 1

Due: July 6, 10:00 AM

Reminders

- T.A.: Chiwan Park (chiwanpark@snu.ac.kr)

- The points of this homework add up to 100.

- This has to be done individually like all the homeworks.

- Please answer clearly; illegible handwriting may get no points.

- Whenever you are making an assumption, please state it clearly.

- If you have a question about assignments, please upload your question in FIRA portal.

Submissions

- You can submit your homework in the class or via email (only PDFs are accepted).

- Do not submit the homework in a photography form.

Question 1 [12 points]

Let p, q, and r be the propositions:

- p: You get an A on the final exam.

- q: You do every exercise in our textbook.

- r: You get an A in this class.

Write the following propositions using p, q, and r and logical connectives including negations.

a) You get an A in this class, but you do not do every exercise in our textbook.

b) You get an A on the final, you do every exercise in our textbook, and you get an A in this class.

c) To get an A in this class, it is necessary for you to get an A on the final.

d) You get an A on the final, but you do not do every exercise in our textbook; nevertheless, you get an A in this class.

e) Getting an A on the final and doing every exercise in our book is sufficient for getting an A in this class.

f) You will get an A in this class if and only if you either do every exercise in our textbook or you get an A on the final.

Question 2 [16 points]

Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements "x is a duck", "x is one of my poultry", "x is an officer", and "x is willing to waltz", respectively. Express each of the following statements using quantifiers; logical connectives; $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.

a) No ducks are willing to waltz.

b) No officers ever decline to waltz.

c) All my poultry are ducks.

d) My poultry are not officers.

Question 3 [22 points]

Prove that given a real number x there exist unique numbers n and ϵ such that $x = n - \epsilon$ where n is an integer, and $0 \leq \epsilon < 1$.

Question 4 [25 points]

The symmetric difference of sets A and B, denoted by $A \oplus B$, is the set containing those elements in either A or B, but not in both A and B. Answer the following questions.

a) Find the symmetric difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$. [4 points]

b) Show that $A \oplus B = (A \cup B) - (A \cap B)$. [7 points]

c) Show that $A \oplus B = (A - B) \cup (B - A)$. [7 points]

d) Show that $(A \oplus B) \oplus B = A$. [7 points]

Question 5 [10 points]

A palindrome is a string whose reversal is identical to the string. How many bit strings of length n are palindromes?

Question 6 [15 points]

Let $n_1, n_2, \ldots n_t$ be positive integers. Show that if $n_1 + n_2 + \cdots + n_t - t + 1$ objects are placed into t boxes, then for some i where $0 < i \leq t$, the i-th box contains at least n_i objects.