Lab Report 3 - Digital Filters

ECE 3101L Signals and Systems

Section 3

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Prelab/Background Theory

Filtering

Filtering is the removal or attenuation of undesired frequency components from an input signal. Filter specifications are usually given in the frequency domain but filters can be realized in the time domain using formulas such as the difference equation. Techniques such as z-transforms and the discrete-time Fourier Transform (DTFT) are used to transfer the signal between domains to assure a correct implementation with the required specs.

Difference Equation

In general, a linear and time invariant digital filter can be specified by its difference equation. The difference equation is given in the form:

$$y(n) = \sum_{i=0}^{N-1} b_i x(n - i) - \sum_{k=1}^{M} a_k y(n - k)$$

where b_i and a_k are filter defining coefficients, $x(n - i)$ are the inputs, and $y(n - k)$ are the previous outputs. N is the number of zeros of the filter and M is the number of poles in the filter.

From the definition of the z-transform, a delay of k samples in the time domain is equivalent to multiplication by z^{-k} in the z domain. z is defined as $e^{j\theta}$. In the discrete frequency domain the difference equation becomes,

$$Y(z) = \sum_{i=0}^{N-1} b_i z^{-i} X(z) - \sum_{k=1}^{M} a_k z^{-k} Y(z).$$

With some rearranging we can solve for the transfer function of the filters

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{i=0}^{N-1} b_i z^{-i}}{1 + \sum_{k=1}^{M} a_k z^{-k}}$$

We see that both the numerator and the denominator of the transfer functions are polynomials of degree N and M respectively. There will be N values of z where the numerator is equal to 0 and M values of z where the numerator is equal to 0.

Filter Design Process

To design our filters, the frequencies for each pole or zero are specified.Then the value of z at these frequencies are determined. The polynomial or polynomial coefficients whose roots are the z values are derived. One polynomial is derived from the zeros (numerator) and the other from the poles (denominator). This can be done using the MATLAB function 'poly'. The transfer function is then simply the ratio of the zero polynomial and the pole polynomial.

Usually, the max gain of the filter is desired to be 1. The transfer function is multiplied by the constant b_0. This constant is found by finding the max gain of the transfer function and taking its inverse. Setting b_0 to this value will cause the product of the max gain and b_0 to be 1 as they are inverses. Finding the max gain of the transfer function is done by first finding the frequency for which the maximum value occurs. This frequency is the frequency which is the furthest from a zero or closest to pole. The frequency's corresponding z value is plugged into the transfer function to find the max gain.

FIR Filter

In the case where $a_k = 0$ for all k, the filter is considered an FIR filter or Finite Impulse Response Filter. This means that the second sum on the right side of the time domain difference equation is equal to 0 and the current input is no longer affected by previous outputs. In this case, the filter is said to have no poles, only zeros. FIR filters have linear phase responses.

$$y(n) = \sum_{i=0}^{N-1} b_i x(n - i)$$

IIR Filters

When $a_k \neq 0$, the filter is considered an IIR filter or an Infinite Impulse Response filter. This type of filter has zeros as well as poles. The phase response of IIR filters are non linear.

$$y(n) = \sum_{i=0}^{N-1} b_i x(n - i) - \sum_{k=1}^{M} a_k y(n - k)$$

Filter Response

Consider the general digital transfer function

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{i=0}^{N-1} b_i z^{-i}}{1+\sum_{k=1}^{M} a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{N-1} z^{-(N-1)}}{(1+a_0) + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_M z^{-M}}$$

$$H(z) = \frac{(1-q_1 z^{-1})(1-q_2 z^{-1})\ldots(1-q_{N-1} z^{-1})}{(1-p_1 z^{-1})(1-p_2 z^{-1})\ldots(1-p_M z^{-1})}$$

Here, z is a complex representation of frequency usually written in polar form, $Ae^{j\theta}$. The magnitude of z is related to stability behavior of a waveform while the angle of z corresponds to waveform frequency.

Considering stable frequencies ($|z| = 1$), as the angle of z goes from 0 to π, the corresponding waveform frequency range is 0 Hz to Nyquist frequency. Analyzing $H(z)$ as z is swept through this range will give us the filter magnitude and phase response.

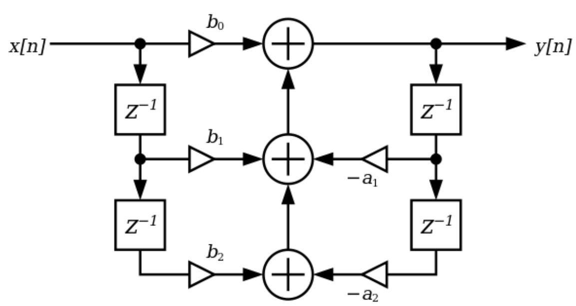
$$H(z) = \frac{(1-q_1 z^{-1})(1-q_2 z^{-1})\ldots(1-q_{N-1} z^{-1})}{(1-p_1 z^{-1})(1-p_2 z^{-1})\ldots(1-p_M z^{-1})}$$

When z is equal to a zero, the magnitude of $H(z)$ will become 0. When z is equal to a pole or near a pole, the magnitude of $H(z)$ will become undefined or very large. With this in mind, poles and zeros could be strategically placed to create specified filters.

Filter Implementation

Filters could be implemented directly from the difference equation in which case it is known as Direct Form I. Delayed inputs and outputs are stored in memory, multiplied by the filter's polynomial coefficients and then summed in accordance to the difference equation. This method requires 2N delays for a filter of order N.

$$y(n) = \sum_{i=0}^{N-1} b_i x(n - i) - \sum_{k=1}^{M} a_k y(n - k)$$



Direct Form II makes use of an intermediate signal called $w(n)/W(z)$. The following equations show how $W(z)$ relates to the delayed inputs and outputs.

The transfer function can be split into two factors containing the new intermediate signal $W(z)$:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + ... + b_{N-1} z^{-(N-1)}}{1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_M z^{-M}}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_M z^{-M}} \qquad \frac{Y(z)}{W(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + ... + b_{N-1} z^{-(N-1)}}{1}$$

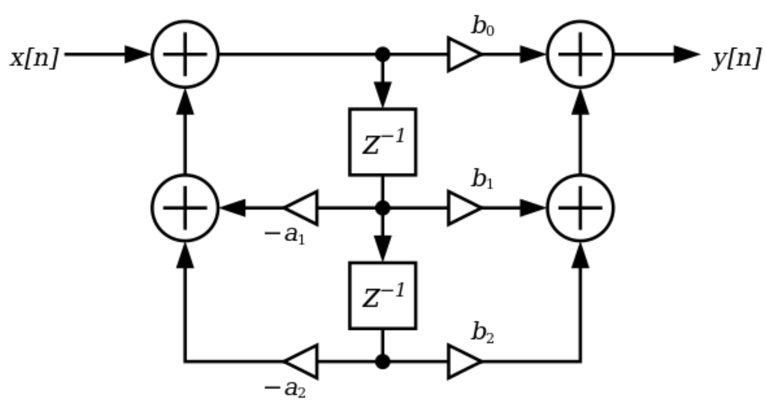
Each factor equation is cross multiplied and returned to the discrete time domain

$$W(z)(1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_M z^{-M}) = X(z)$$
$$w(n) + a_1 w(n-1) + ... + a_M w(n-M) = x(n)$$
$$w(n) = x(n) - a_1 w(n-1) - ... - a_M w(n-M)$$

$$W(z)(b_0 + b_1 z^{-1} + b_2 z^{-2} + ... + b_{N-1} z^{-(N-1)}) = Y(z)$$
$$y(n) = b_0 w(n) + b_1 w(n-1) + ... + b_{N-1} w(n-(N-1))$$

The implementation of these two derived equations requires only N delays to store the previous values of $w(n)$ and is mathematically equivalent to Direct Form I.

$$w(n) = x(n) - a_1 w(n-1) - ... - a_M w(n-M)$$
$$y(n) = b_0 w(n) + b_1 w(n-1) + ... + b_{N-1} w(n-(N-1))$$



Prelab 8 Results

Given Digital Filter Polynomial Coefficients:

Filter 1 Numerator: $[1,\ 1]$

Filter 1 Denominator: $[1,\ \ -.95]$

Filter 2 Numerator: $[1,\ \ -1]$

Filter 2 Denominator: $[1,\ .95]$

Filter 3 Numerator: $[1,\ 0,\ \ -1]$

Filter 3 Denominator: $[1,\ 0,\ .9025]$

Filter 4 Numerator: $[1,\ 0,\ 1]$

Filter 4 Denominator: $[1\ 0\ \ -.9025]$

Polynomial Roots:

Solving for the polynomial roots using MATLAB ***roots*** function:

Filter 1 Zeros: $-1 = 1\,e^{j\pi}$

Filter 1 Poles: $.95 = .95\,e^{j0}$

With a zero at $\theta = \pi$ (maximum frequency) and a pole at $\theta = 0$ (minimum frequency), the filter is a low pass filter

Filter 2 Zeros: $1 = 1\,e^{j0}$

Filter 2 Poles: $-.95 = .95\,e^{j\pi}$

With a zero at $\theta = 0$ (minimum frequency) and a pole at $\theta = 0$ (maximum frequency), the filter is a high pass filter

Filter 3 Zeros: $[1, -1] = [1\,e^{j0},\ 1\,e^{j\pi}]$

Filter 3 Poles: $[.95i,\ \ -.95i] = [.95\,e^{j\frac{\pi}{2}},\ \ .95\,e^{j\frac{-\pi}{2}}]$

With zeros at $\theta = 0$, π (extrema frequencies) and a pole at $\theta = \frac{\pi}{2}$ (center frequency), the filter is a bandpass filter

Filter 4 Zeros: $[i, -i] = [1\,e^{j\frac{\pi}{2}},\ \ 1\,e^{j\frac{-\pi}{2}}]$

Filter 4 Poles: $[.95,\ \ -.95] = [.95\,e^{j0},\ \ .95\,e^{j\pi}]$

With a zero at $\theta = \frac{\pi}{2}$ (center frequency) and poles at $\theta = 0$, π (extrema frequencies), the filter is a bandstop filter.

Transfer Functions and Filter Gain

Filter 1

$$TF = b_0 \frac{1+z^{-1}}{1-.95z^{-1}}$$

$$maximum\ gain\ frequency = 0$$

$$z\ at\ maximum\ gain = e^{j(0)} = 1$$

$$b_0 = \frac{1-.95(1)^{-1}}{1+(1)^{-1}} =.025$$

Filter 2

$$TF = b_0 \frac{1-z^{-1}}{1+.95z^{-1}}$$

$$maximum\ gain\ frequency = \pi$$

$$z\ at\ maximum\ gain = e^{j(\pi)} = \ -1$$

$$b_0 = \frac{1+.95(-1)^{-1}}{1-(-1)^{-1}} =.025$$

Filter 3

$$TF = b_0 \frac{1-z^{-2}}{1+.9025z^{-2}}$$

$$maximum\ gain\ frequency = \frac{\pi}{2}$$

$$z\ at\ maximum\ gain = e^{j(\frac{\pi}{2})} = i$$

$$b_0 = \frac{1+.9025(i)^{-2}}{1-(i)^{-2}} =.0488$$

Filter 4

$$TF = b_0 \frac{1+z^{-2}}{1-.9025z^{-2}}$$

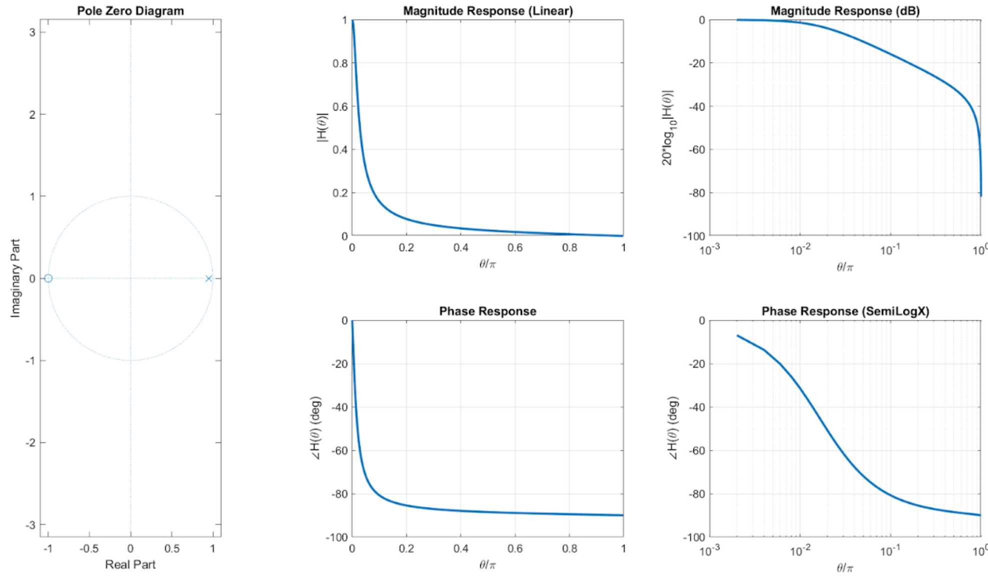
$$maximum\ gain\ frequency = 0$$

$$z\ at\ maximum\ gain = e^{j(0)} = 1$$

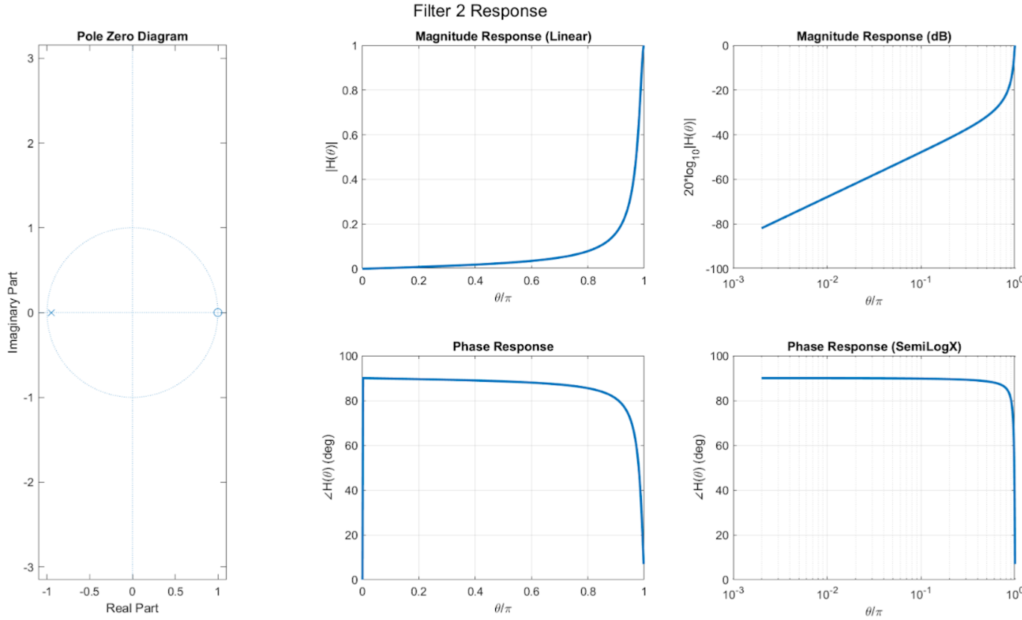
$$b_0 = \frac{1-.9025(1)^{-2}}{1+(1)^{-2}} =.0488$$

<u>Filter Behaviors:</u>

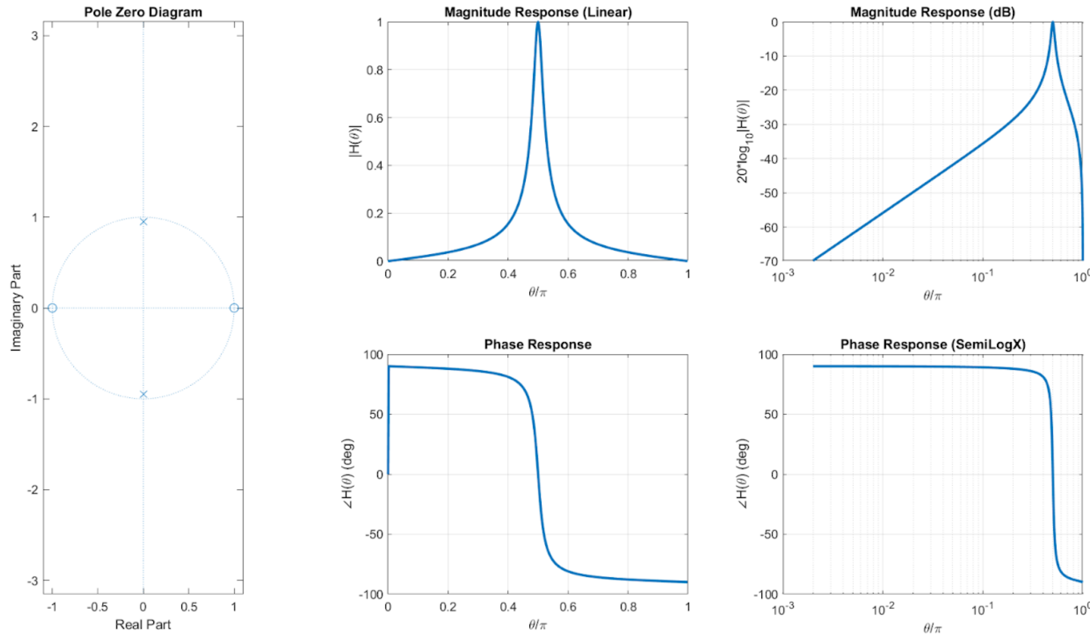
Filter 1 Response



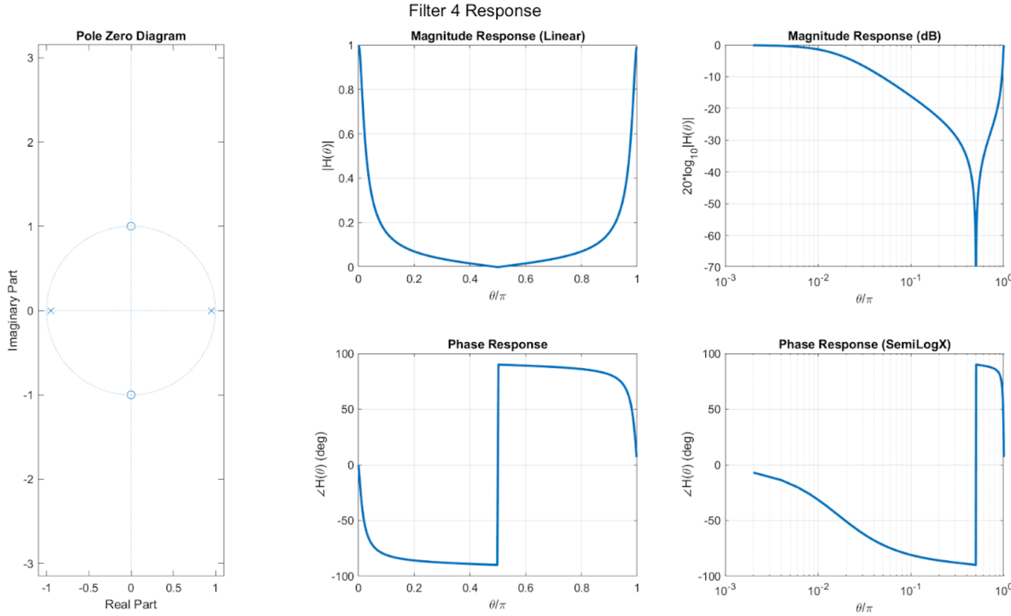
Filter 2 Response



Filter 3 Response



Filter 4 Response



Prelab 9 Results

Given Filter Roots:

The 4 given filters in this prelab have the following zeros:

Fitler 5 Zeros: $-1,\ i,\ -i,\ \frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}$

Fitler 6 Zeros: $1,\ i,\ -i,\ \frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}$

Fitler 7 Zeros: $1,\ -1,\ \frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}$

Fitler 8 Zeros: $i,\ -i,\ \frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}$

These filters have no poles or can be considered to have poles at 0. These filters are thus FIR filters.

Fitler 5 Zeros: $e^{j\frac{\pi}{4}},\ e^{j\frac{\pi}{2}},\ e^{j\frac{3\pi}{4}}, e^{j\pi},\ e^{j\frac{-\pi}{4}},\ e^{j\frac{-\pi}{2}},\ e^{j\frac{-3\pi}{4}}$

Zeros with θ at evenly spaced $\frac{\pi}{4}$ intervals except for $\theta = 0$ (minimum frequency). Maximum gain occurs at minimum frequency. This filter is a lowpass.

Fitler 6 Zeros: $e^{j0},\ e^{j\frac{\pi}{4}},\ e^{j\frac{\pi}{2}},\ e^{j\frac{3\pi}{4}},\ e^{j\frac{-\pi}{4}},\ e^{j\frac{-\pi}{2}},\ e^{j\frac{-3\pi}{4}}$

Zeros with θ at evenly spaced $\frac{\pi}{4}$ intervals except for $\theta = \pi$ (maximum frequency). Maximum gain occurs at maximum frequency. This filter is a highpass.

Fitler 7 Zeros: $e^{j0}, e^{j\frac{\pi}{4}},\ e^{j\frac{3\pi}{4}}, e^{j\pi},\ e^{j\frac{-\pi}{4}},\ e^{j\frac{-3\pi}{4}}$

Zeros with θ at evenly spaced $\frac{\pi}{4}$ intervals except for $\theta = \frac{\pi}{2}$ (center frequency). Maximum gain occurs at center frequency. This filter is a band pass.

Fitler 8 Zeros: $e^{j\frac{\pi}{4}}, e^{j\frac{\pi}{2}}\ e^{j\frac{3\pi}{4}},\ e^{j\frac{-\pi}{4}},\ e^{j\frac{-\pi}{2}},\ e^{j\frac{-3\pi}{4}}$

Zeros with θ at evenly spaced $\frac{\pi}{4}$ intervals except for $\theta = 0$, π (extrema frequencies). Maximum gain occurs at extrema frequencies. This filter is a band stop.

Transfer Function and Filter Gain

Filter 5

$$TF = b_0 \frac{(1-(-1)z^{-1})(1-(i)z^{-1})(1-(-i)z^{-1})(1-(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}})z^{-1})(1-(-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}})z^{-1})(1-(\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}})z^{-1})(1-(-\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}})z^{-1})}{1}$$

$$TF = b_0(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6} + z^{-7})$$

$$maximum\ gain\ frequency = 0$$

$$z\ at\ maximum\ gain = e^{j(0)} = 1$$

$$b_0 = \frac{1}{\left|(1+(1)^{-1}+(1)^{-2}+(1)^{-3}+(1)^{-4}+(1)^{-5}+(1)^{-6}+(1)^{-7})\right|} =.125$$

Filter 6

$$TF = b_0 \frac{(1-(1)z^{-1})(1-(i)z^{-1})(1-(-i)z^{-1})(1-(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}})z^{-1})(1-(-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}})z^{-1})(1-(\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}})z^{-1})(1-(-\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}})z^{-1})}{1}$$

$$TF = b_0(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} - z^{-5} + z^{-6} - z^{-7})$$

$$maximum\ gain\ frequency = \pi$$

$$z\ at\ maximum\ gain = e^{j(\pi)} =- 1$$

$$b_0 = \frac{1}{\left|1-(-1)^{-1}+(-1)^{-2}-(-1)^{-3}+(-1)^{-4}-(-1)^{-5}+(-1)^{-6}-(-1)^{-7})\right|} =.125$$

Filter 7

$$TF = b_0 \frac{(1-(1)z^{-1})(1-(-1)z^{-1})(1-(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}})z^{-1})(1-(-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}})z^{-1})(1-(\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}})z^{-1})(1-(-\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}})z^{-1})}{1}$$

$$TF = b_0(1 - z^{-2} + z^{-4} - z^{-6})$$

$$maximum\ gain\ frequency = \pi/2$$

$$z\ at\ maximum\ gain = e^{j(\pi/2)} = i$$

$$b_0 = \frac{1}{\left|(1-(-i)^{-2}+(-i)^{-4}-(-i)^{-6})\right|} =.25$$

Filter 8

$$TF = b_0 \frac{(1-(i)z^{-1})(1-(-i)z^{-1})(1-(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}})z^{-1})(1-(-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}})z^{-1})(1-(\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}})z^{-1})(1-(-\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}})z^{-1})}{1}$$

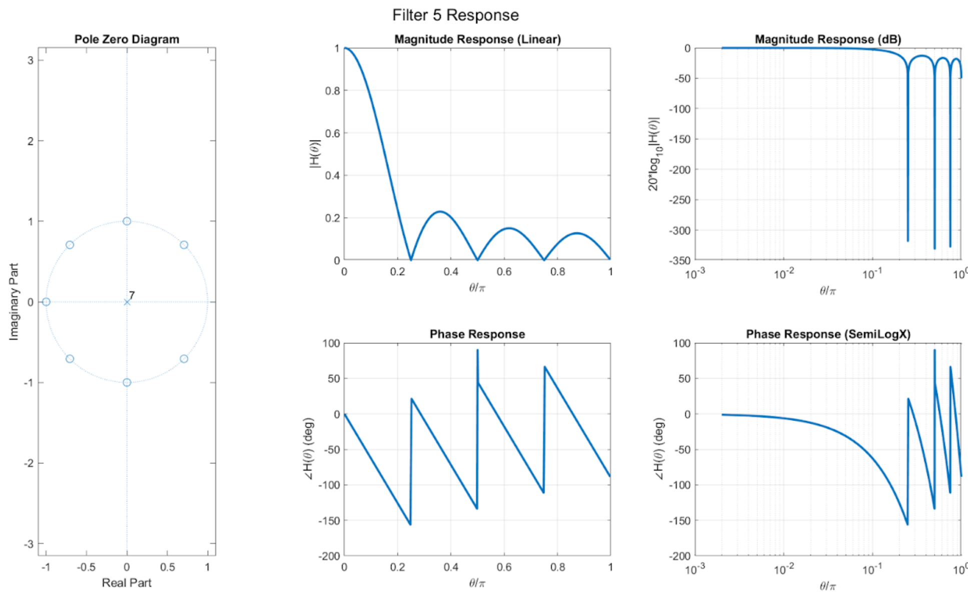
$$TF = b_0(1 + z^{-2} + z^{-4} + z^{-6})$$

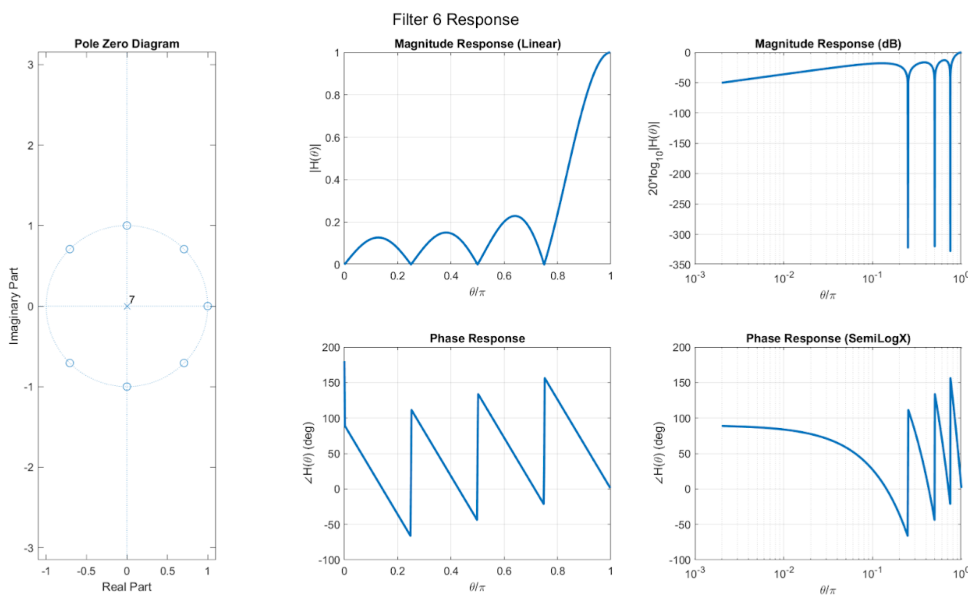
$$maximum\ gain\ frequency = 0$$

$$z\ at\ maximum\ gain = e^{j(0)} = 1$$

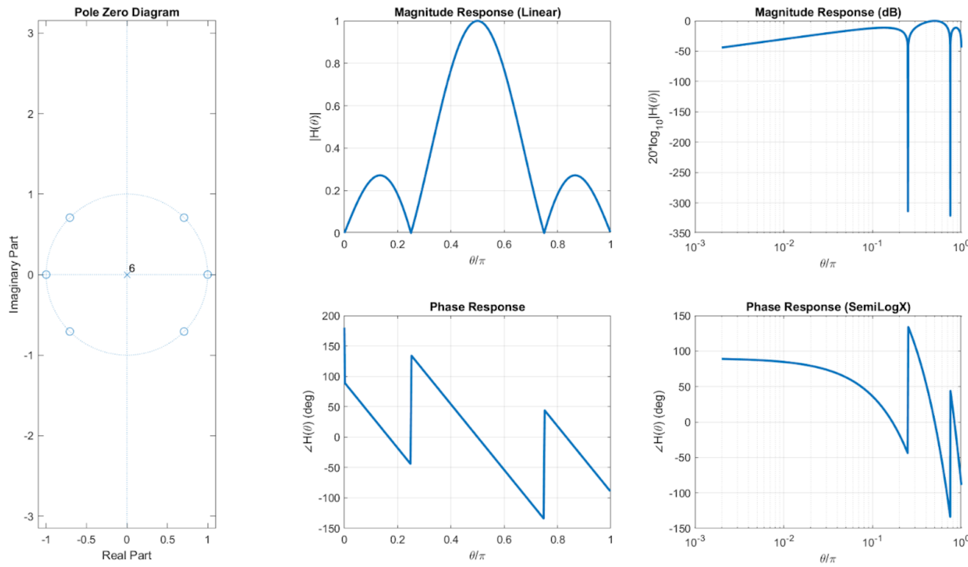
$$b_0 = \frac{1}{\left|(1+(1)^{-2}+(1)^{-4}+(1)^{-6})\right|} =.25$$

Filter Behaviors:

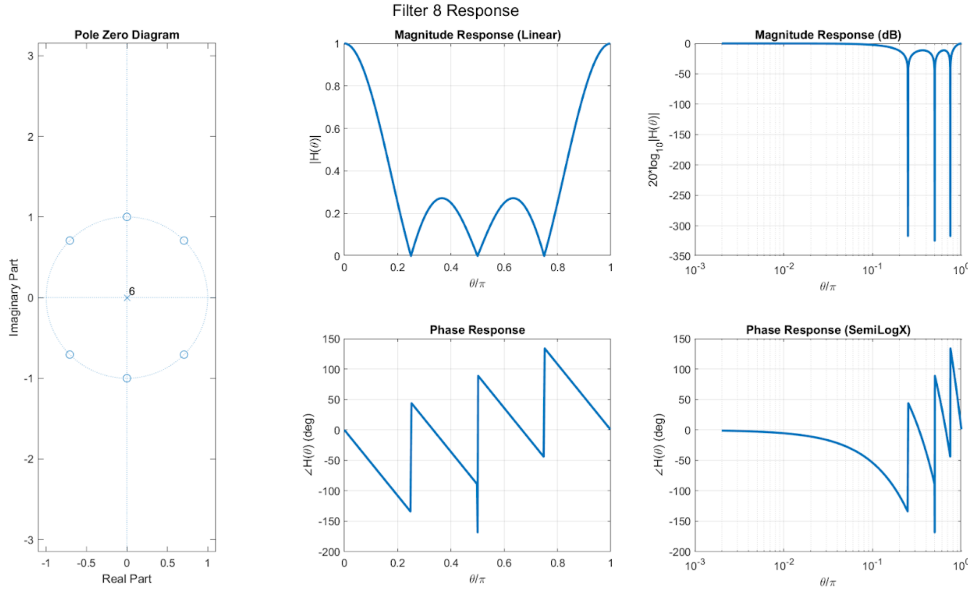




Filter 7 Response



Filter 8 Response



Prelab 10 Results

Given Filter Roots:

Filter 9 Zero: $e^{i\frac{\pi}{5}}$ Filter 10 Zero: $e^{i\frac{\pi}{4}}$

Filter 11 Zero: $e^{i\frac{\pi}{3}}$ Filter 12 Zero: $e^{i\frac{2\pi}{3}}$ Filter 13 Zero: $e^{i\pi}$

Filters 9-11 have max gain at frequency π. Filters 12 and 13 have max gain at frequency 0. These are the frequencies furthest from their respective zeros.

Filter 14 Pole: $.98e^{i\frac{\pi}{5}}$ Filter 15 Pole: $.98e^{i\frac{\pi}{3}}$ Filter 16 Pole: $.98e^{i\frac{2\pi}{3}}$

Filter 17 Pole: $.75e^{i\frac{\pi}{5}}$ Filter 18 Pole: $.75e^{i\frac{\pi}{3}}$ Filter 19 Pole: $.75e^{i\frac{2\pi}{3}}$

Filters 14-19 have max gain at frequencies corresponding to the angle of their poles.

Transfer Function and Filter Gain:

Filter 9: $TF = b_0\frac{(1-e^{i\frac{\pi}{5}}z^{-1})(1-e^{-i\frac{\pi}{5}}z^{-1})}{1} = \frac{(1-1.6180z^{-1}+z^{-2})}{1}$ $b_0 = \frac{1}{(1-1.6180(-1)^{-1}+(-1)^{-2})} =.2763$

Filter 10: $TF = b_0\frac{(1-e^{i\frac{\pi}{4}}z^{-1})(1-e^{-i\frac{\pi}{4}}z^{-1})}{1} = \frac{(1-1.4142z^{-1}+z^{-2})}{1}$ $b_0 = \frac{1}{(1-1.4142(-1)^{-1}+(-1)^{-2})} =.2929$

Filter 11: $TF = b_0\frac{(1-e^{i\frac{\pi}{3}}z^{-1})(1-e^{-i\frac{\pi}{3}}z^{-1})}{1} = \frac{(1-z^{-1}+z^{-2})}{1}$ $b_0 = \frac{1}{(1-(-1)^{-1}+(-1)^{-2})} =.3333$

Filter 12: $TF = b_0\frac{(1-e^{i\frac{2\pi}{3}}z^{-1})(1-e^{-i\frac{2\pi}{3}}z^{-1})}{1} = \frac{(1+z^{-1}+z^{-2})}{1}$ $b_0 = \frac{1}{(1+(1)^{-1}+(1)^{-2})} =.3333$

Filter 13: $TF = b_0\frac{(1-e^{i\pi}z^{-1})(1-e^{-i\pi}z^{-1})}{1} = \frac{(1+2z^{-1}+z^{-2})}{1}$ $b_0 = \frac{1}{(1+2(1)^{-1}+(1)^{-2})} =.25$

Filter 14: $TF = b_0\frac{1}{(1-.98e^{i\frac{\pi}{5}}z^{-1})(1-.98e^{-i\frac{\pi}{5}}z^{-1})} = \frac{1}{(1-1.5857z^{-1}+.9604z^{-2})}$ $b_0 = \frac{(1-1.5857(e^{i\frac{\pi}{5}})^{-1}+.9604(e^{i\frac{\pi}{5}})^{-2})}{1} =.0233$

Filter 15: $TF = b_0\frac{1}{(1-.98e^{i\frac{\pi}{3}}z^{-1})(1-.98e^{-i\frac{\pi}{3}}z^{-1})} = \frac{1}{(1-.98z^{-1}+.9604z^{-2})}$ $b_0 = \frac{(1-.98(e^{i\frac{\pi}{3}})^{-1}+.9604(e^{i\frac{\pi}{3}})^{-2})}{1} =.0345$

Filter 16: $TF = b_0\frac{1}{(1-.98e^{i\frac{2\pi}{3}}z^{-1})(1-.98e^{-i\frac{2\pi}{3}}z^{-1})} = \frac{1}{(1+.98z^{-1}+.9604z^{-2})}$ $b_0 = \frac{(1+.98(e^{i\frac{2\pi}{3}})^{-1}+.9604(e^{i\frac{2\pi}{3}})^{-2})}{1} =.0345$Filter

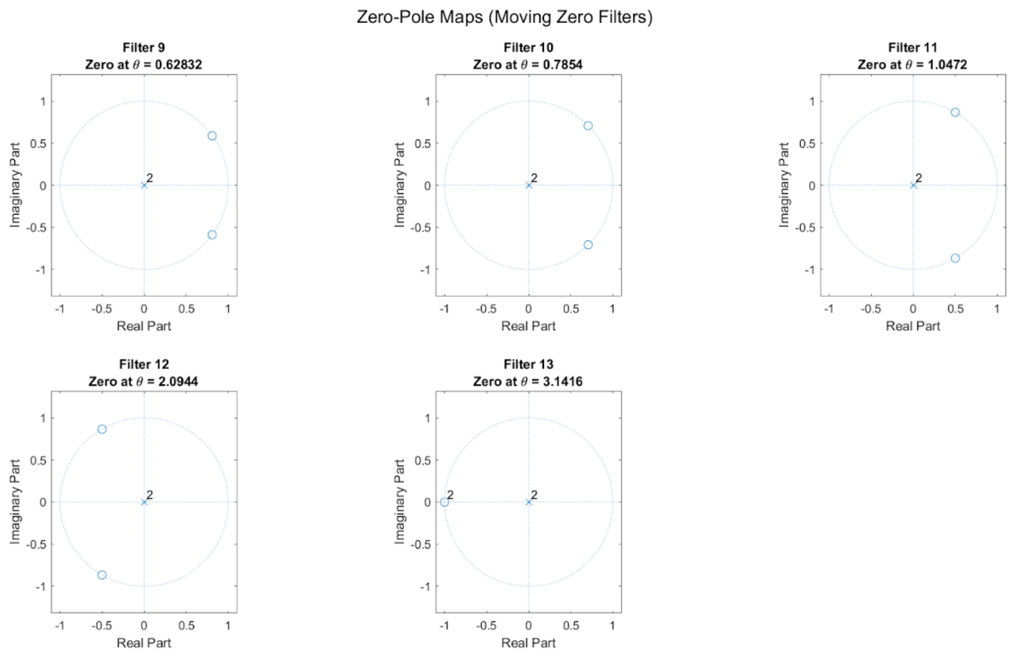
17: $TF = b_0\frac{1}{(1-.75e^{i\frac{\pi}{5}}z^{-1})(1-.75e^{-i\frac{\pi}{5}}z^{-1})} = \frac{1}{(1-1.2135z^{-1}+.5625z^{-2})}$ $b_0 = \frac{(1-1.2135(e^{i\frac{\pi}{5}})^{-1}+.5625(e^{i\frac{\pi}{5}})^{-2})}{1} =.2572$

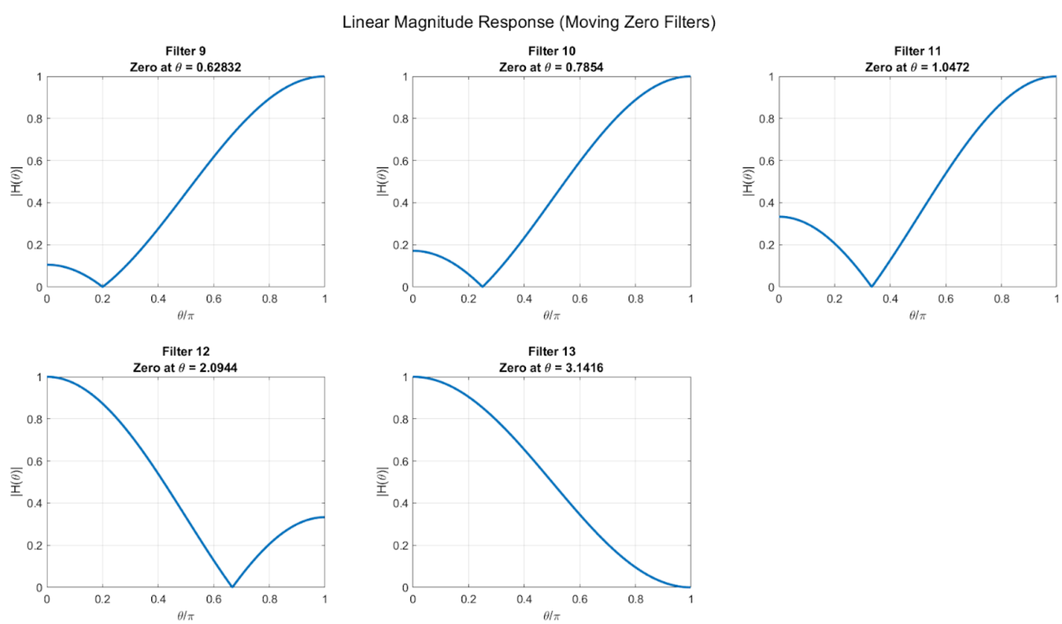
Filter 18: $TF = b_0\frac{1}{(1-.75e^{i\frac{\pi}{3}}z^{-1})(1-.75e^{-i\frac{\pi}{3}}z^{-1})} = \frac{1}{(1-.75z^{-1}+.5625z^{-2})}$ $b_0 = \frac{(1-.75(e^{i\frac{\pi}{3}})^{-1}+.5625(e^{i\frac{\pi}{3}})^{-2})}{1} =.3789$

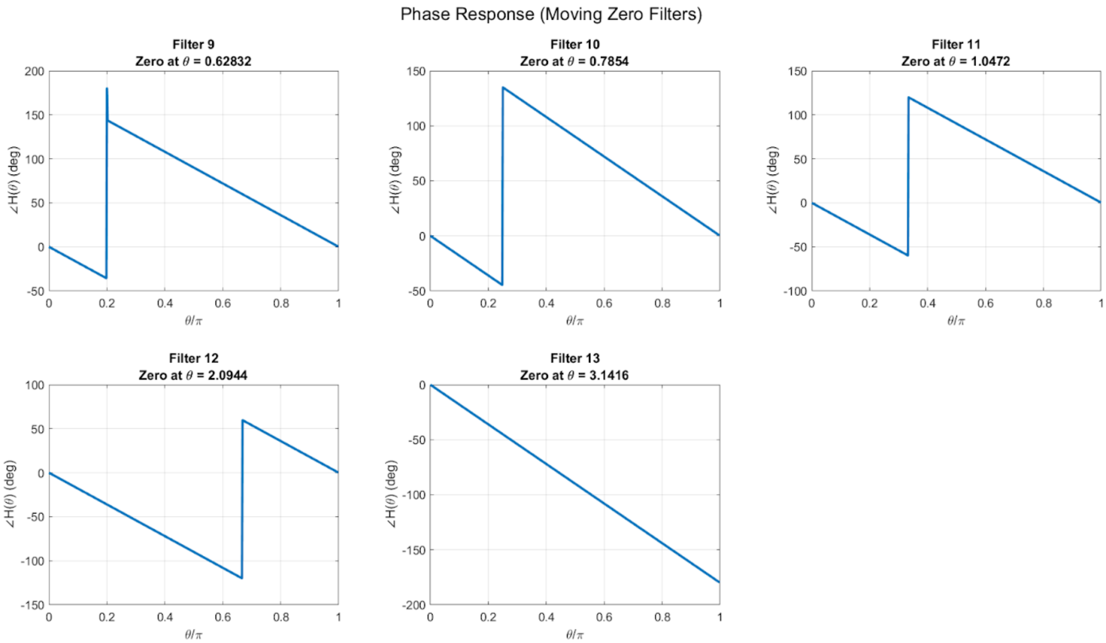
Filter 19: $TF = b_0\frac{1}{(1+.75e^{i\frac{2\pi}{3}}z^{-1})(1-.75e^{-i\frac{2\pi}{3}}z^{-1})} = \frac{1}{(1+75z^{-1}+.5625z^{-2})}$ $b_0 = \frac{(1+.75(e^{i\frac{2\pi}{3}})^{-1}+.5625(e^{i\frac{2\pi}{3}})^{-2})}{1} =.3789$

Filter Behaviors

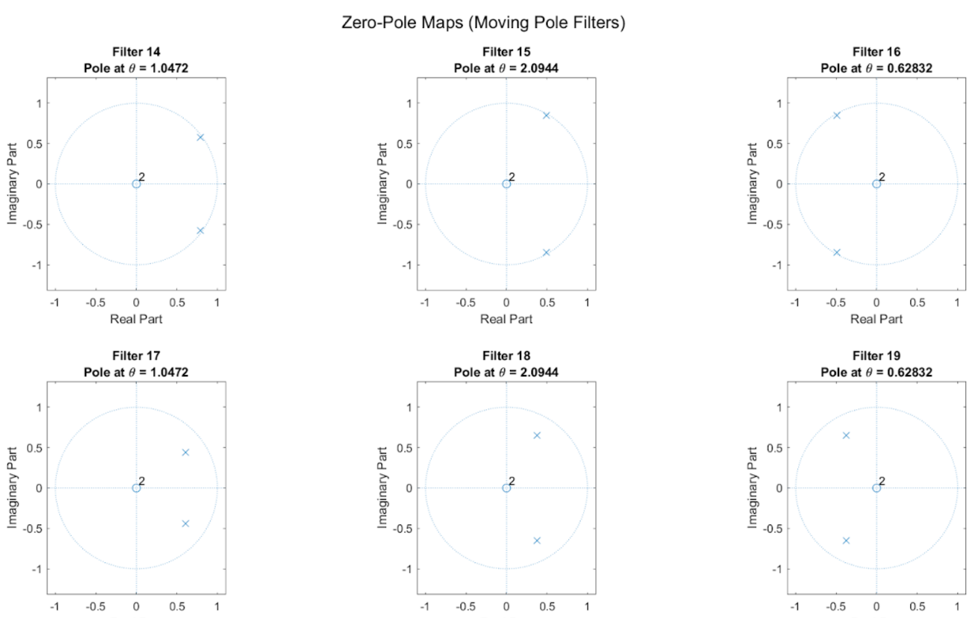
Filters 9-13

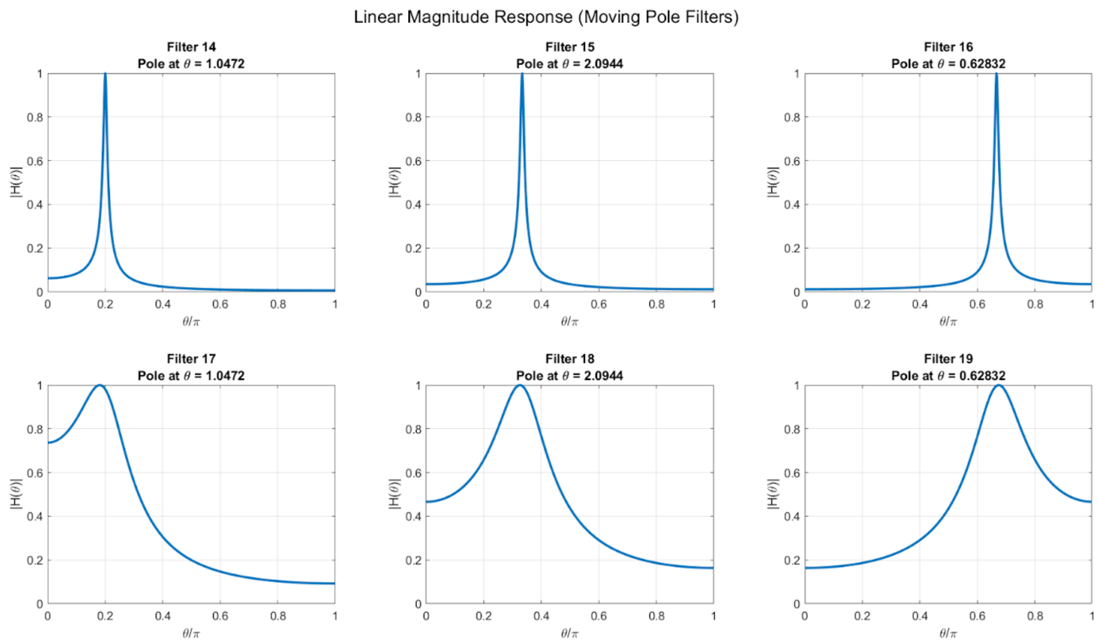


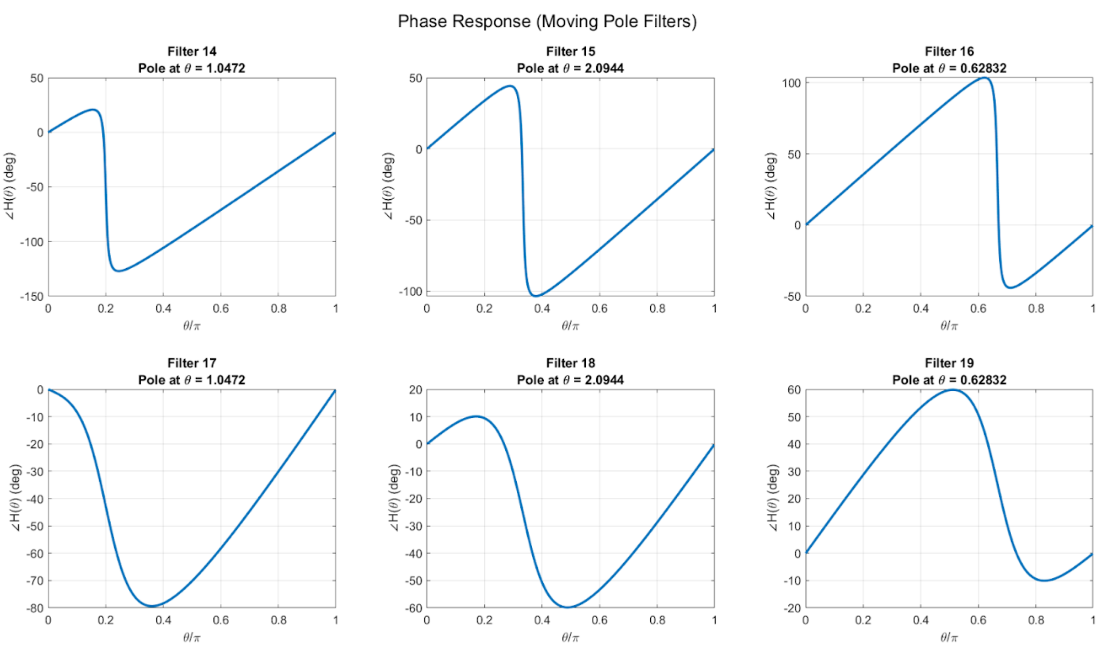




Filters 14-19




Linear Magnitude Response (Moving Pole Filters)
Filter 14
Pole at θ = 1.0472
Filter 15
Pole at θ = 2.0944
Filter 16
Pole at θ = 0.62832
Filter 17
Pole at θ = 1.0472
Filter 18
Pole at θ = 2.0944
Filter 19
Pole at θ = 0.62832
|H(θ)|
θ/π


Phase Response (Moving Pole Filters)
Filter 14
Pole at θ = 1.0472
Filter 15
Pole at θ = 2.0944
Filter 16
Pole at θ = 0.62832
Filter 17
Pole at θ = 1.0472
Filter 18
Pole at θ = 2.0944
Filter 19
Pole at θ = 0.62832
∠H(θ) (deg)
θ/π

Lab 8-10

Lab Procedures

The 19 given filters were applied to a pure sinusoid of fundamental 30 Hz using direct form I code implementation. The sampling rate of the signal was altered to view the effects of the filter at different perceived frequencies. The filter's output amplitude and relative phase shift were calculated and compared to the filter's ideal magnitude and phase response.

The filters were also applied to 5 different audio files including a randomly generated white noise signal. The filtered and unfiltered signals were plotted as well as their FFT spectra. Specific plots were selected to highlight key concepts.

Lab Results

Pure Sinusoids

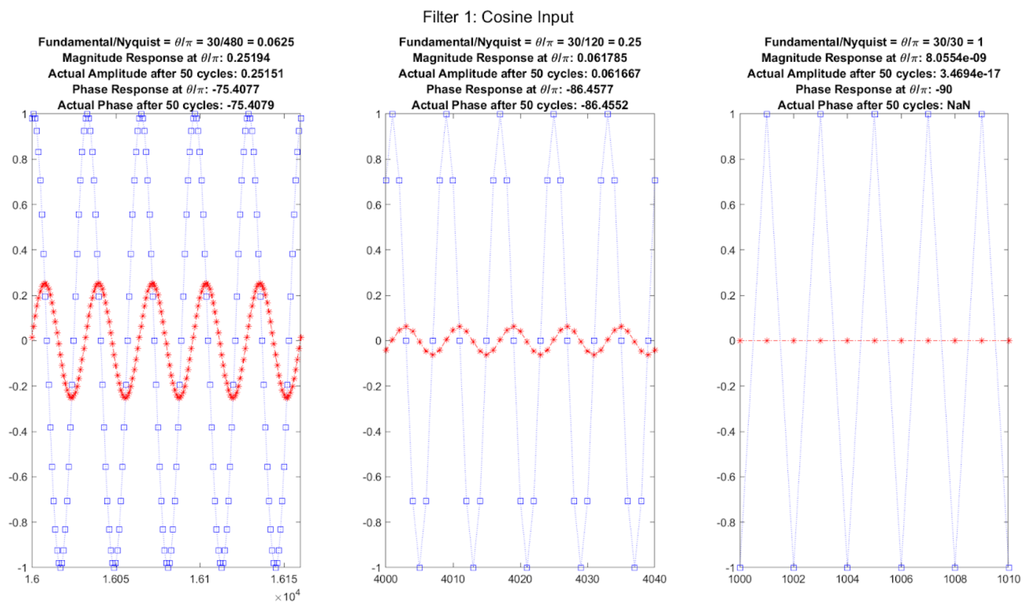


Figure 1

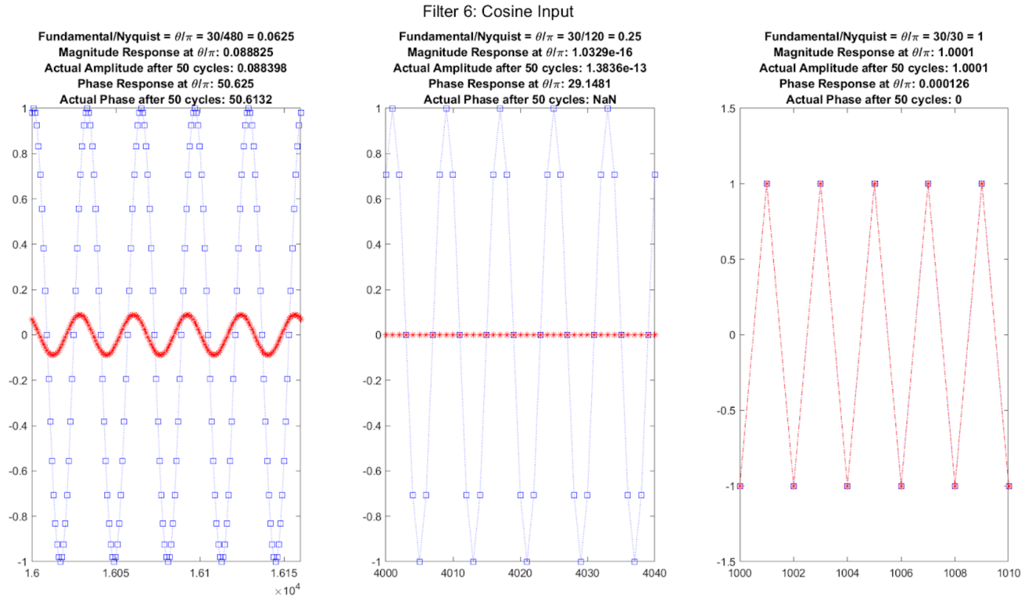


Figure 2

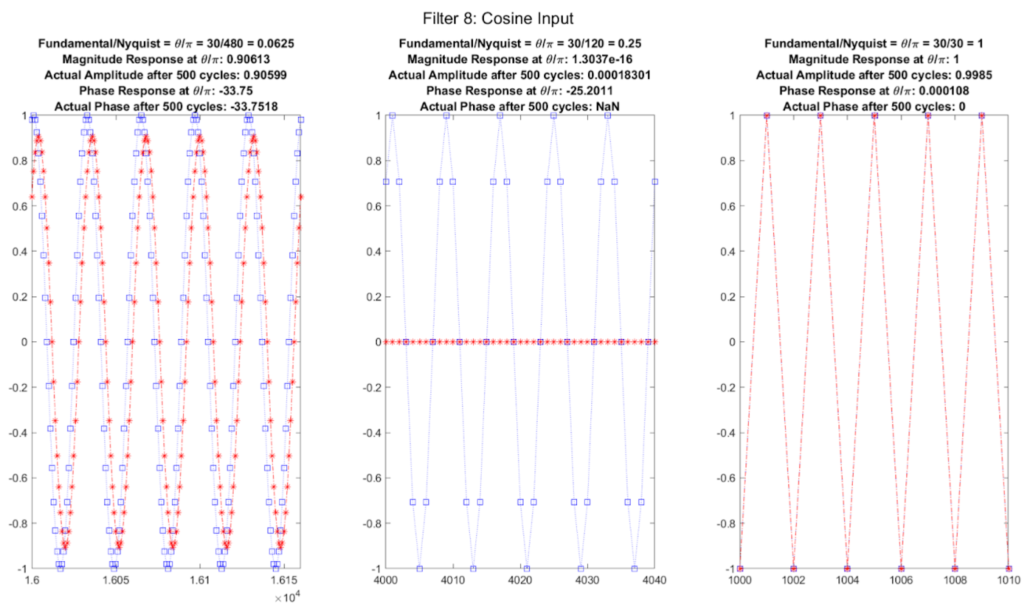


Figure 3

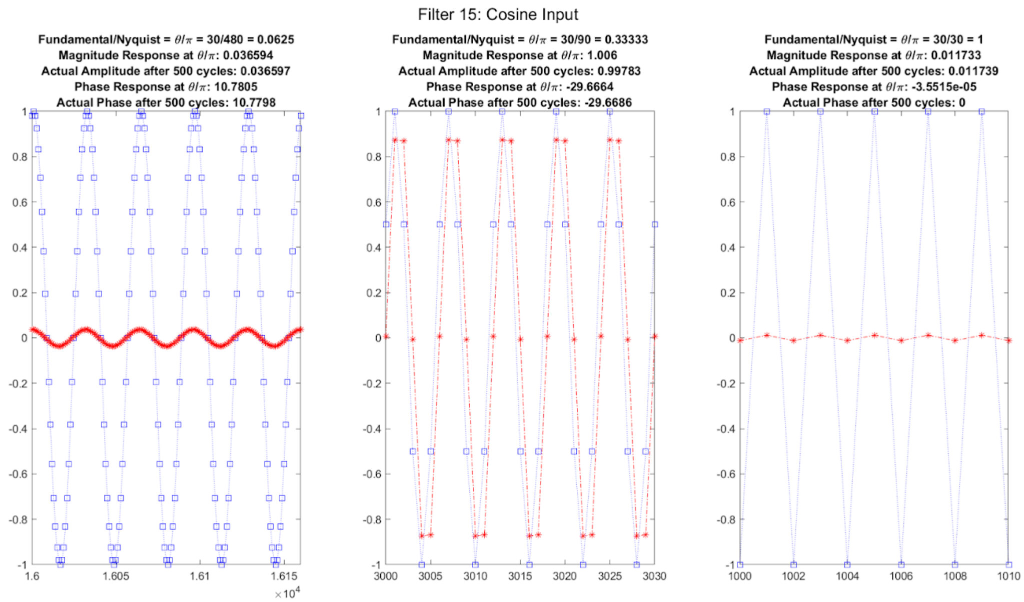


Figure 4

The figures above show an input signal with increasing perceived frequency from left to right. Although the fundamental frequency of the signal stays the same, decreasing sampling frequency increases the perceived frequency of the input.

Filter 1 (Figure 1) shows a basic low pass filter which allows the low frequency to pass but attenuates the signal completely when at Nyquist. Filter 6 (Figure 2) shows high pass filter with a zero at $\theta = \frac{\pi}{4}$. The output signal is attenuated at low perceived frequencies, is removed completely at $\theta = \frac{\pi}{4}$, and remains unaffected at Nyquist. For Filter 15, at $\frac{\theta}{\pi} = .3333$, the maximum value of the output is around .87. The measured amplitude however is noted as .997. This is because the method of measuring magnitude (FFT) could assess implied maximums. The reason the maximums are not sampled is because the number of samples per cycle is not a power of two for $\frac{\theta}{\pi} = .3333$. For this filter the lower and higher frequencies attenuate quickly due to the near 1 pole at magnitude.

At frequencies where the theoretical magnitude output is 0, the actual amplitude is smaller by multiple factors of 10, but this is due to the difference in method precision. Also, measuring the phase of the output when its magnitude is so small results in unreliable results.

Lab 8 - Audio Files

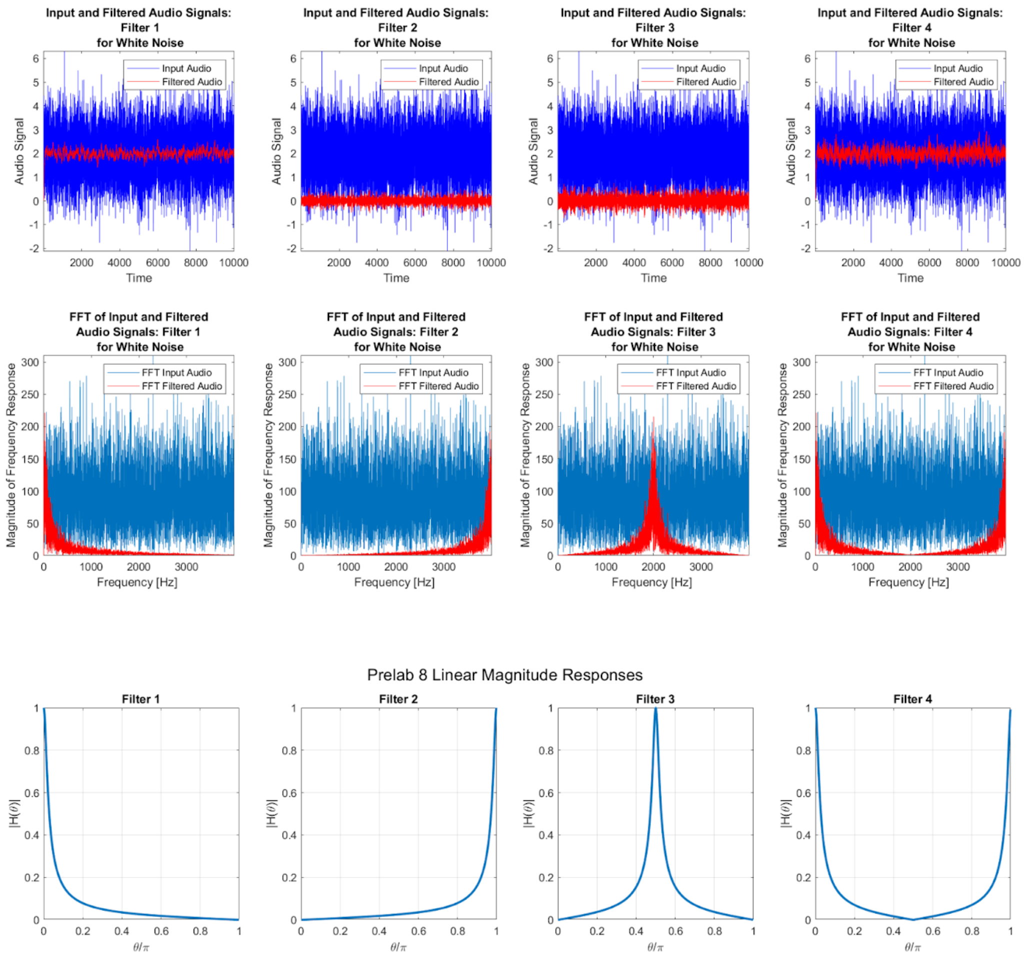


Figure 5

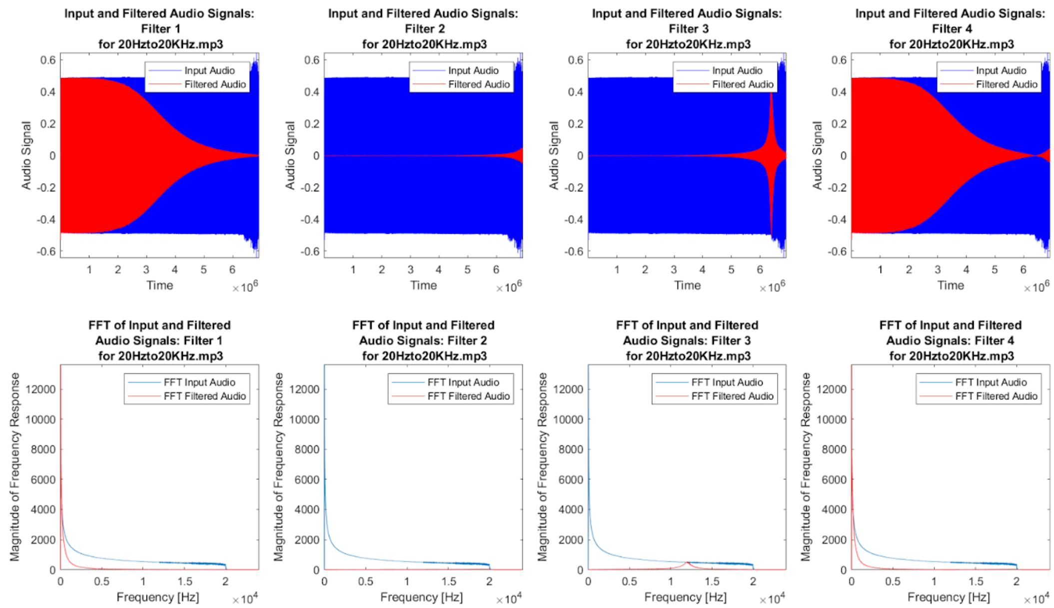


Figure 6

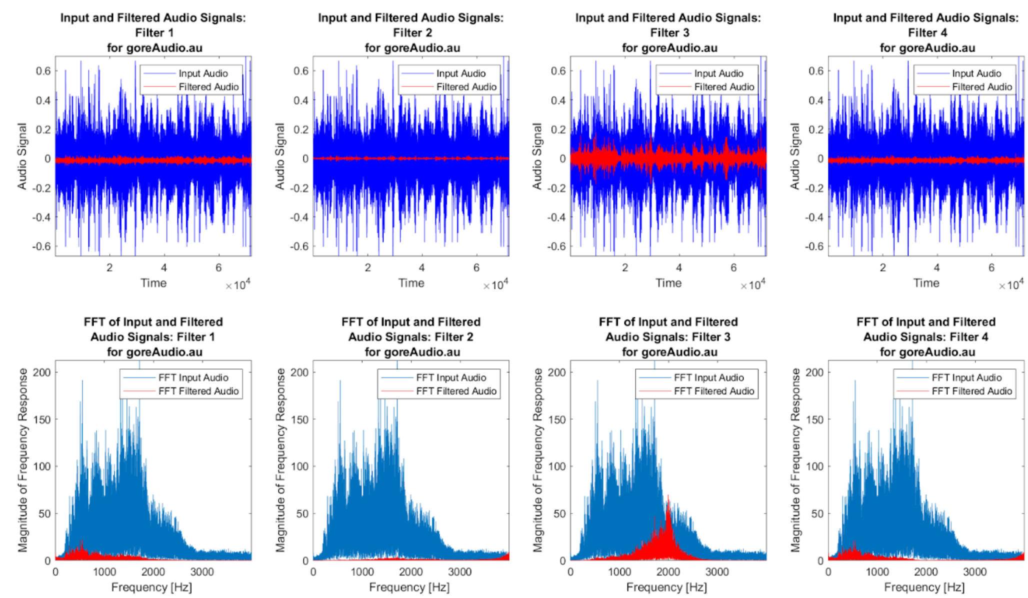


Figure 7

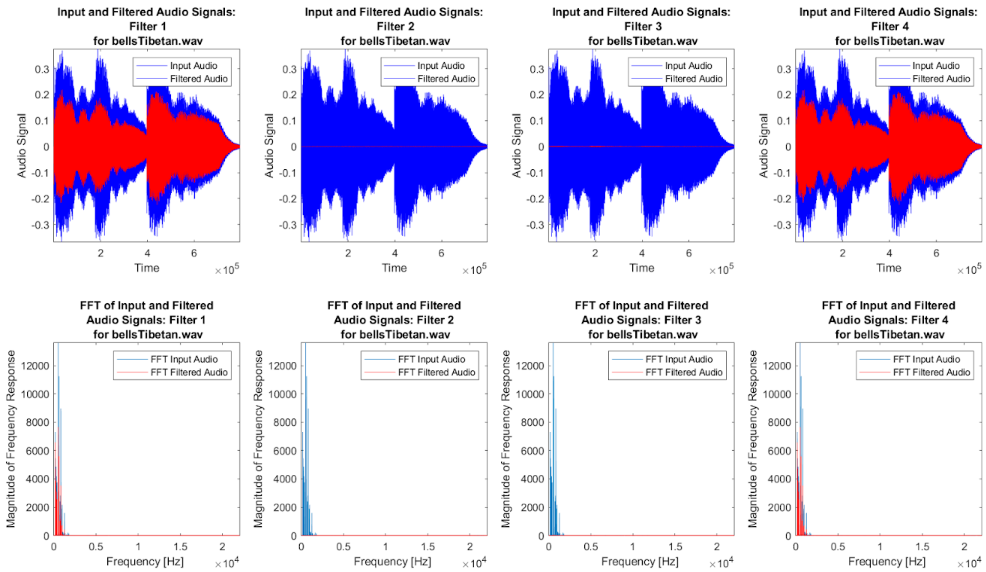


Figure 8

To ensure correct implementation of the filters in MATLAB, the filters were applied to a white noise signal. In white noise, the magnitude of individual frequency components are normally distributed. This causes the magnitudes of neighboring frequency components to have approximately equal averages. When the filters are applied to the white noise signal, the filter's magnitude response is recreated (Figure 4).

The 20Hz to 20KHz sweep is a good signal to test as it has singular frequency content that varies over time. The effects of the filter could be easily visualized in the time domain as well. For these particular filters, the upper cutoff frequency for the highpass and bandstop are too large to capture significant content. Decreasing sampling rate could also help provide more high frequency content with these filters.

These simple Lab 8 filters with steep cutoff slopes are very selective, as seen with the Al Gore audio (Figure 6). Even though the audio has low-mid frequency content, the low pass filter (Filter 1) still attenuates the majority of the signal due to lack of low bass content. Considering that higher frequencies are less distinctive, the high pass filter (Filter 2) might also have too large of a cutoff frequency to preserve a desired 'high frequency range'.

For signals with only low content like the Tibetan Bell file, the lowpass and bandstop filter have almost identical effects. The highpass and bandpass also have similar effects as they both attenuate the signal near completely.

Lab 9 - Audio Files

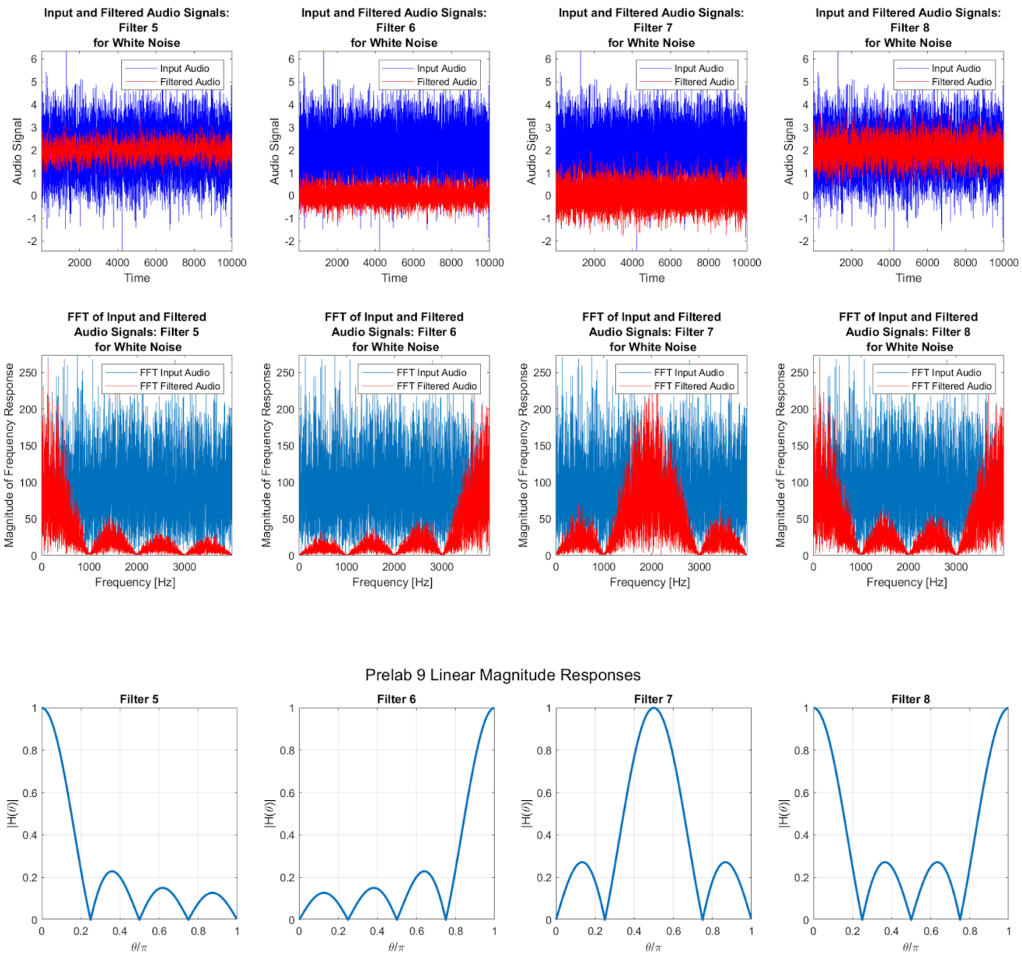


Figure 9

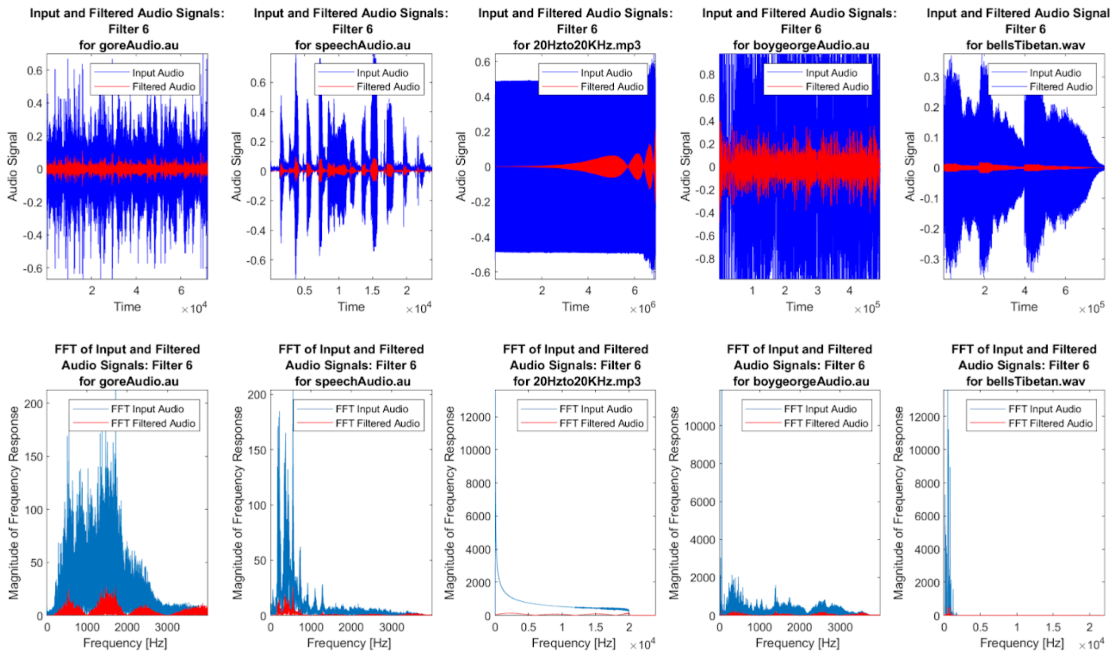


Figure 10

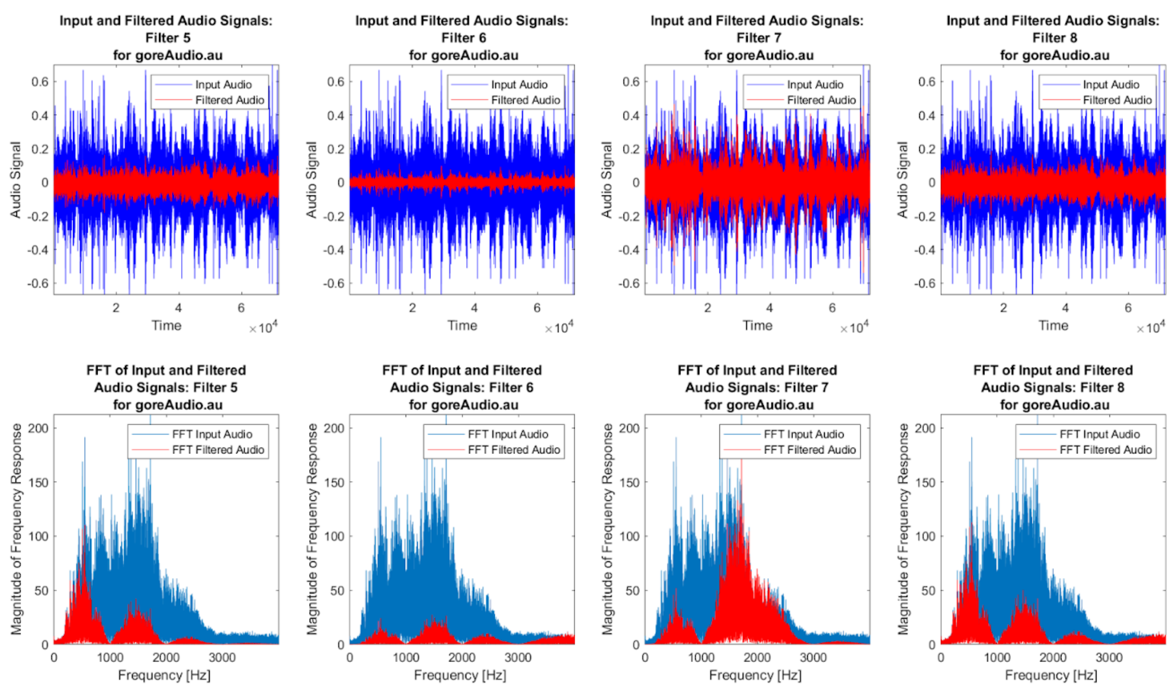


Figure 11

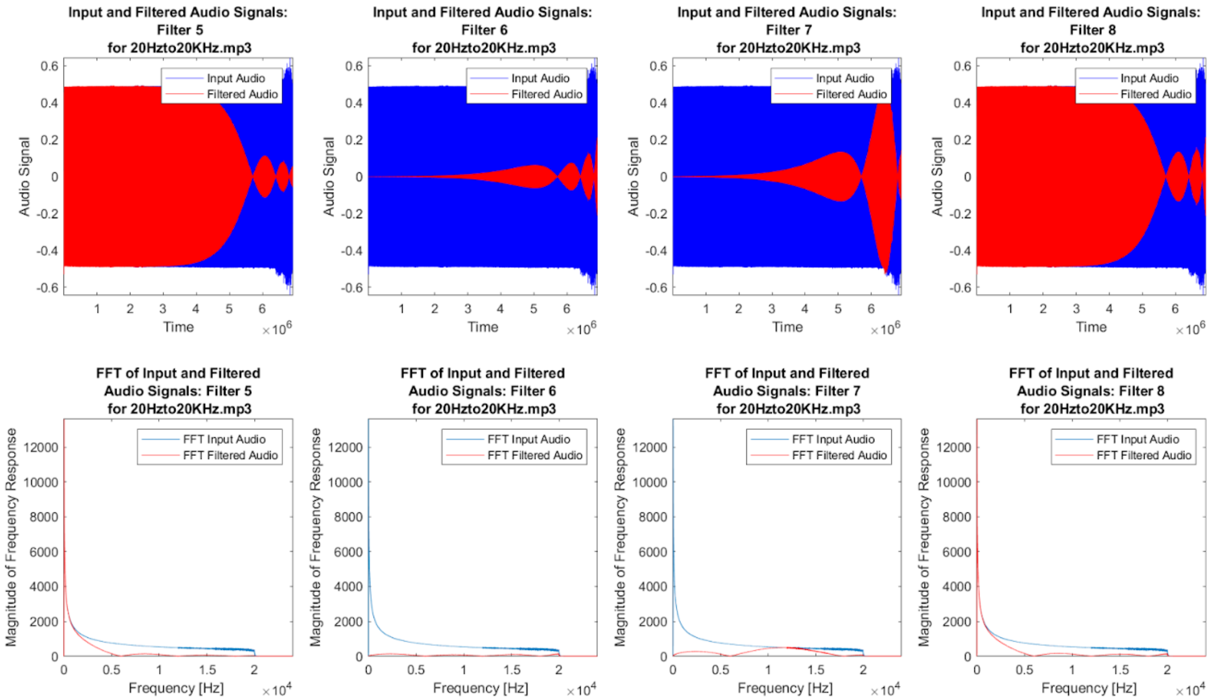


Figure 12

Similar to the Lab 8 filters, the Lab 9 filters' magnitude response are recreated in the filtered white noise FFT. As these filters do not continuously decay, they allow more frequency content from their stopbands to pass. More of the signal is preserved as can be seen in the larger amplitude of the filtered white noise compared to Lab 8.

Even with the added ripples, the highpass and bandpass filters attenuate much more of the signal than the lowpass and bandstop. In general, speech and music audio will have most of their content in the low to mid frequency range, and the attenuation behavior of the filters is expected (Figure 10). The other two audio signals used (20to20kHz and TibetanBells) also contain predominantly low frequency content. An explanation could be that the signals were sampled at a rate chosen to allow even higher non-essential frequencies to be recognized, for the highest fidelity.

The Lab 9 filters attenuate the signal less overall. Since the peaks of the filter magnitude response are caused by lack of zeros rather than presence of poles, they become wider and allow a larger range of frequencies to pass (Figure 11).

Figure 12 shows the presence of the filters' ripples in the filtered 20to20kHz.mp3 time domain plots.

Lab 10 Part 1 - Audio Files

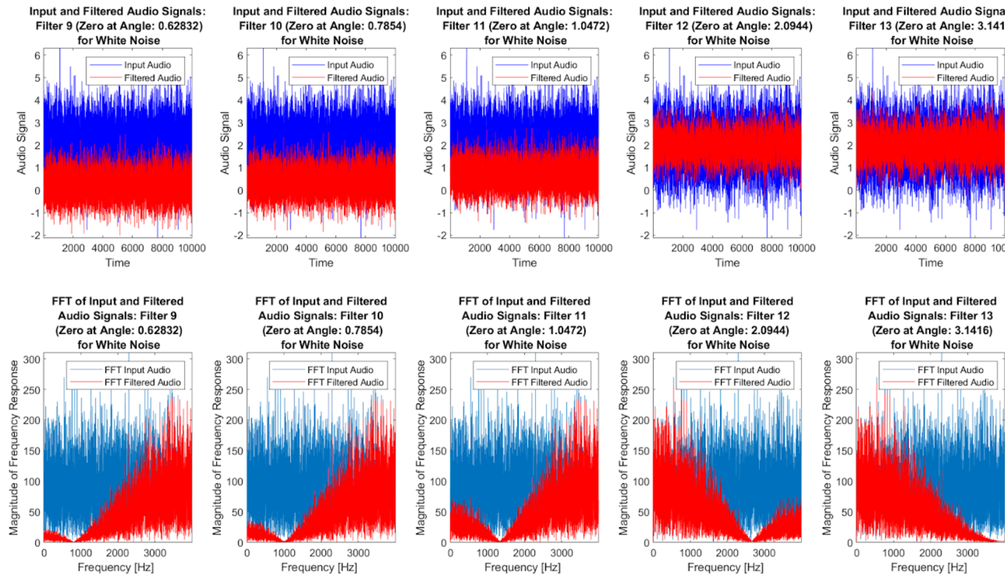


Figure 13

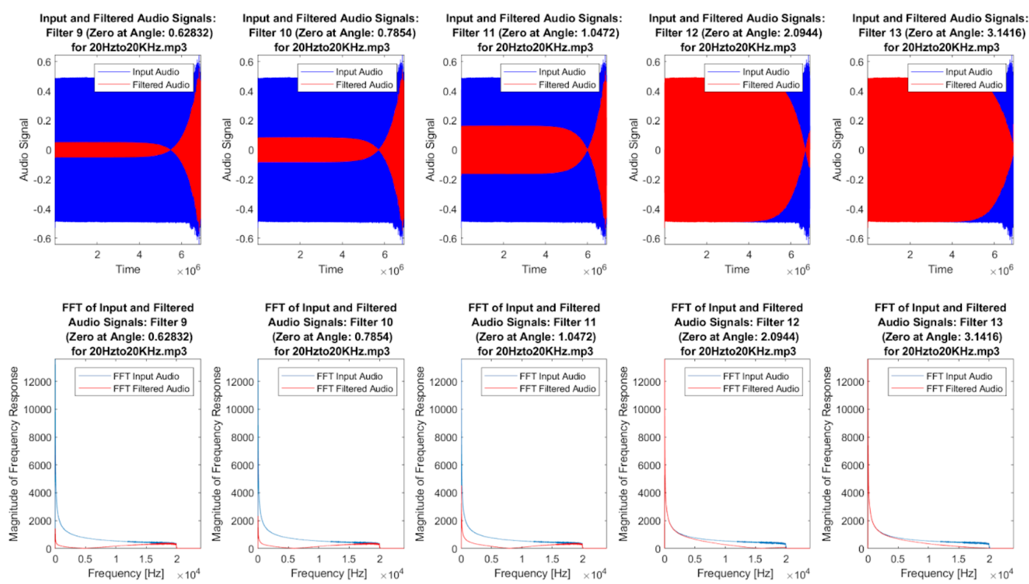


Figure 14

Lab 10 Part 2 - Audio Files

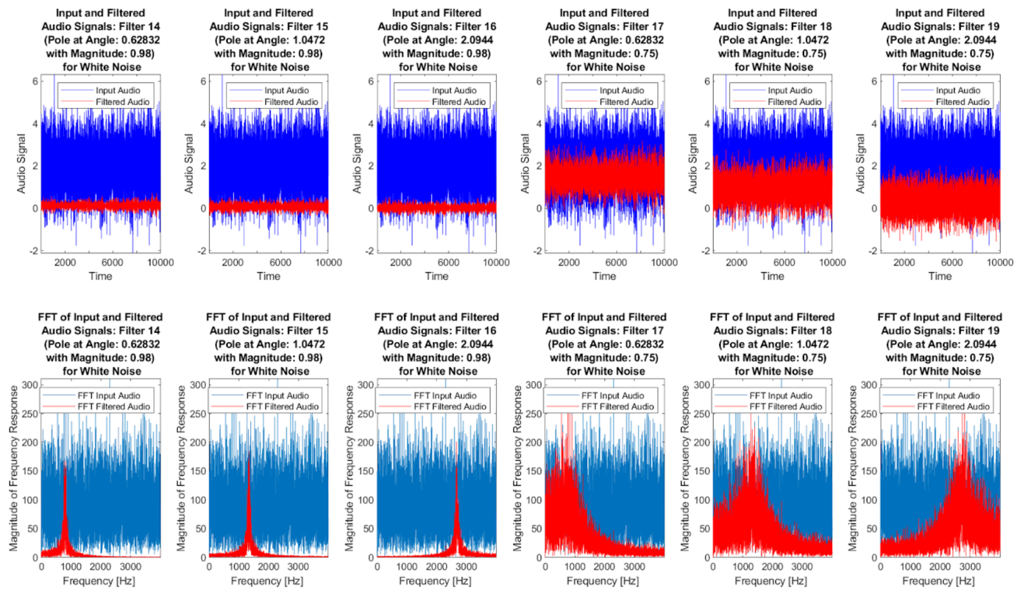


Figure 15

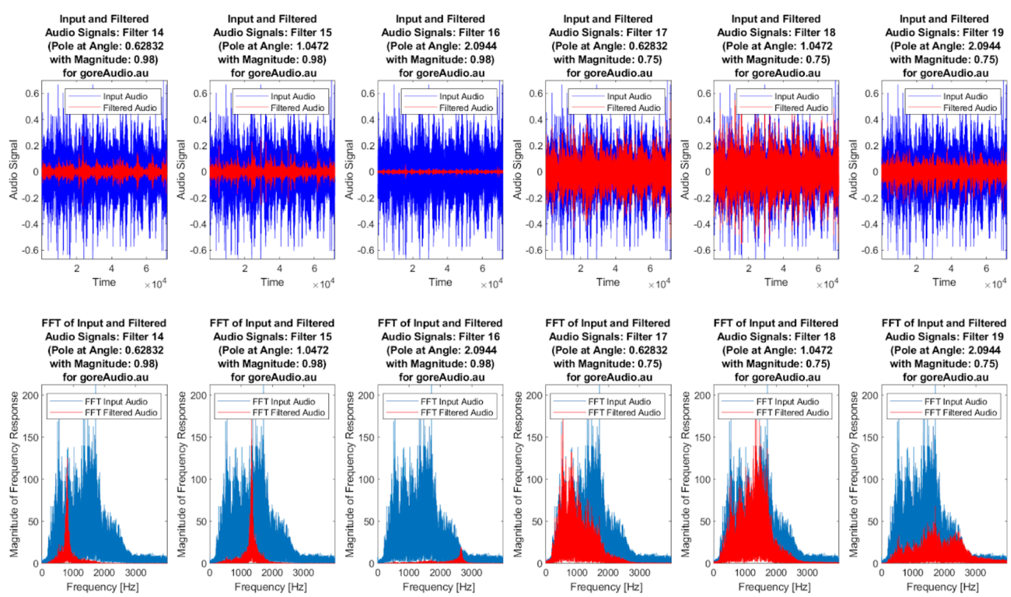


Figure 16

These filters with singular zeros provide even less overall attenuation than the Lab 8 & Lab 9. The filtered signals in the time domain have larger amplitudes. In the FFT of the filtered white noise signals, the decays from passband to stopband on both sides of the zero are more gradual (Figure 13).

The moving zero filters demonstrate the effects of altering angle position of a singular zero. From the 20Hzto20kHz.mp3 audio, we could see that as the zero angle goes from 0 to π, the amplitude of the low frequencies increases and the amplitude of the high frequencies decreases. The position of the zero determines which frequencies are attenuated. The zero causes the filter to change from a highpass to bandstop to lowpass as it moves counter clockwise along the unit circle. This type of filter also provides the most preserved high frequency content from all the previous filters, when lowpass.

The magnitude response of the moving poles show the effects of angle and magnitude for poles. As the angle of the pole increases, the peak position of the magnitude response moves to the right (increased frequency). As the magnitude of the pole decreases, the width of the peak increases (Figure 15). This is because the frequency response only considers values of z on the unit circle. As the magnitude of the pole decreases, it moves away from the unit circle and its usual amplifying effect on the magnitude response decreases.

This adjustable width behavior of the poles could be useful when trying to find a balance between frequency preservation and frequency accentuation. As seen in Figure 16, larger pole magnitudes preserve a thin frequency range while a smaller pole magnitude merely accentuates a certain frequency range while more or less preserving spectrum shape.