Lab 8 - FM Modulation and Demodulation Edward Enriquez
ECE 4705-L

Introduction

Background Theory

In frequency modulation (FM), the message signal is encoded in the carrier signal by changing the carrier's instantaneous frequency according to the message signal's magnitude. The equation for the modulated signal's frequency follows the form $f_i(t) = f_c + k_f m(t)$. Where f_c is the carrier of the signal, $m(t)$ is the modulating/message signal, and k_f is a constant determining how much the frequency changes per magnitude of the message signal. A frequency modulated signal only contains instantaneous frequencies within a certain range of the original carrier frequency, giving it higher frequency content, making it viable for transmission.

An important property to know about frequency modulation is the max frequency deviation. The max frequency deviation is defined as $k_f \cdot [max(|m(t)|)]$ and is denoted as Δf. The modulation index β is also another important parameter which relates the maximum frequency deviation of a modulated signal and the frequency of the modulating signal. The modulation index β, is defined as $\frac{\Delta f}{f_m}$, where f_m is the original modulating frequency.

Using the modulation index, the following time domain equation of the FM signal can be constructed, $FM(t) = A_c \sum_n J_n(\beta) \cos[(\omega_c + n \cdot \omega_m)t]$. The $J_n(\beta)$ function is the Bessel function of 1st kind, order n, argument β. Since the outputs of this function are constants, this equation gives us a way to express the frequency modulated signal as the sum of cosines. This will make it possible to find the spectrum from the time domain equation.

To demodulate the message, a bandpass filter will be applied to the FM signal. The bandpass filter will have its lower cutoff frequency somewhere near the carrier signal's frequency so that the lower frequency content can be attenuated while the higher frequency content gets passed. The filter should be set up in such a way that frequency content of the FM signal linearly increases in magnitude. When done correctly, this should cause the original signal to be embedded in the envelope of the filtered signal. After this, the signal could pass through an envelope detector to retrieve the original signal.

Prelab

In the prelab, the frequency spectrum of FM modulated waves were plotted for different modulation indices β. Values of $\beta = 2, 5,$ and 10 where plotted. As seen in the plots, as the modulation index increases, so does the apparent bandwidth of the modulated signal. Although the Bessel function is never exactly 0, it has the property of dying down quickly after a certain value of n. Theoretically, there is still frequency content at all frequency, but in practice, equations like Carson's rule ($B_T = 2\Delta f(1 + \frac{1}{\beta})$) is used to find practical bandwidth measurements.

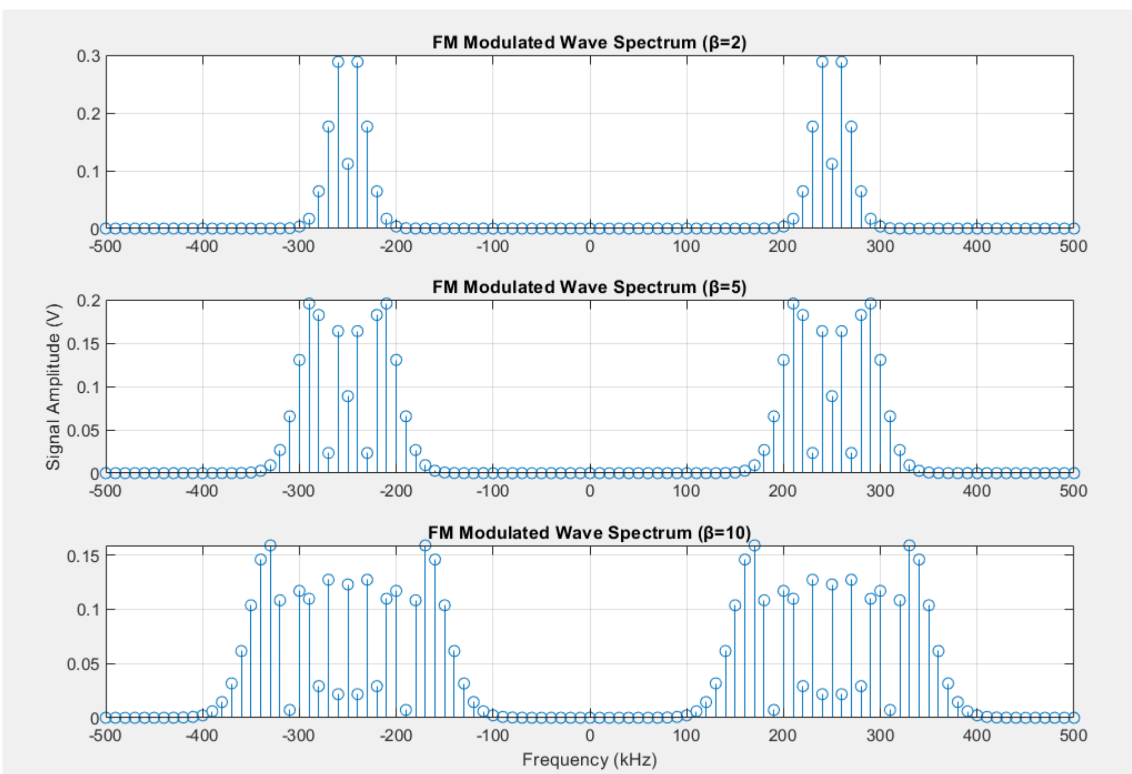

Figure 1 - Prelab FM Spectrum Plots

Lab

In the lab an HP3312A Function Generator will be set to output a carrier cosine signal of 250kHz. Using the voltage controlled oscillator (VOC) of the HP3312A, a message cosine signal of 10kHz will control the frequency shift of the carrier signal producing a frequency modulated signal.

To do proper measurements and analysis, before hooking up the message signal to the VOC, a DC voltage supply was hooked up to the VOC to find the relationship between input voltage and output frequency. Plotting multiple points, the value of k_f for the HP3312A's VOC could be found. This value will then be used to solve for the certain message signal amplitudes which correspond to modulation indices β of 2,5, and 10.

Once the amplitudes are found, the 10kHz message signals with their amplitudes corresponding to certain β will be input to the VOC and the spectrums of the FM signals will be sent to the spectrum analyzer and compared to prelab results.

One of the FM signals will then be sent through a bandpass filter to demonstrate the start of FM demodulation. The output of the bandpass filter will be an FM signal which has as its envelope the original 10kHz message signal as discussed in the background theory.

Voltage (V)	Frequency (kHz)	Frequency (Hz)
0.111	264.5	264500
0.384	301.2	301200
0.484	314.5	314500
0.94	375.9	375900
1.98	520.8	520800
3.01	657.9	657900

β	Am	2*Am
2	0.146845034	0.293690069
5	0.367112586	0.734225172
10	0.734225172	1.468450344

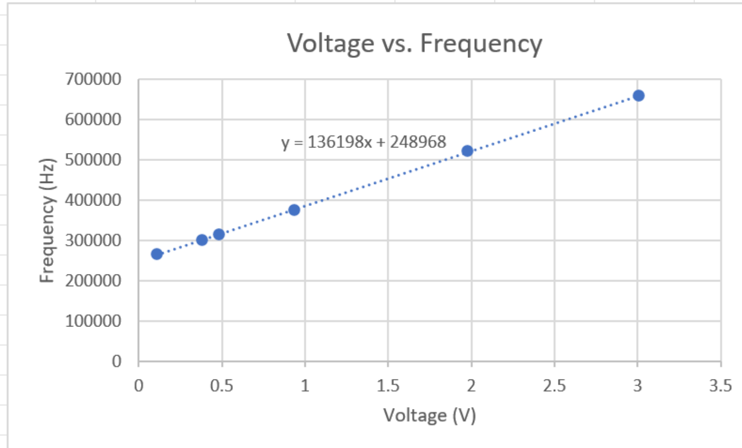

Figure 2 - Voltage and Frequency (VOC) Measurements and Plots / β and A_m calculations

Figure 3 - Spectrum of FM Modulated Signal ($\beta = 2$)

Figure 4 - Spectrum of FM Modulated Signal ($\beta = 5$)

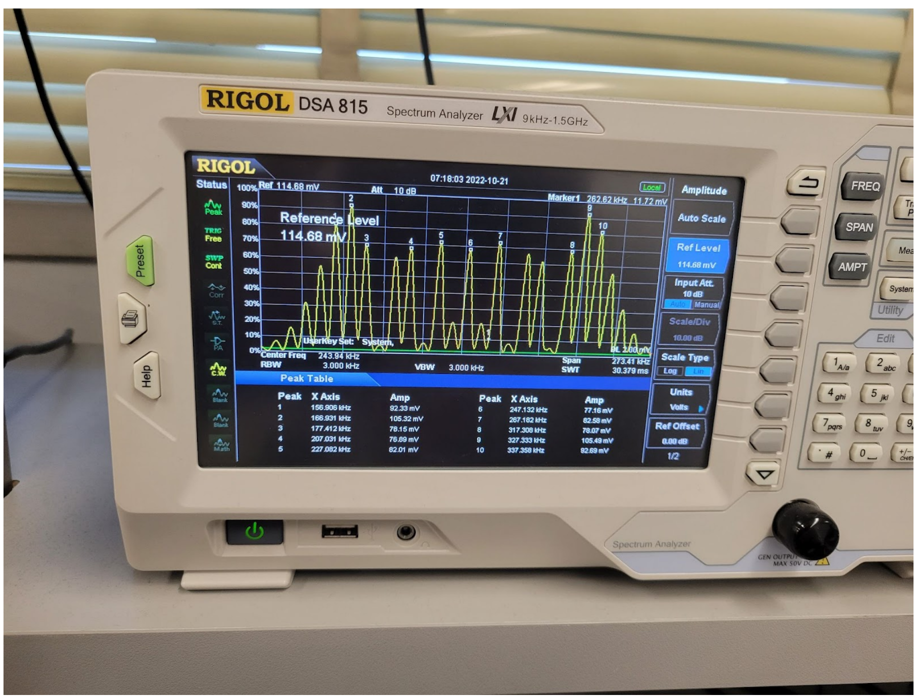

Figure 5 - Spectrum of FM Modulated Signal ($\beta = 10$)

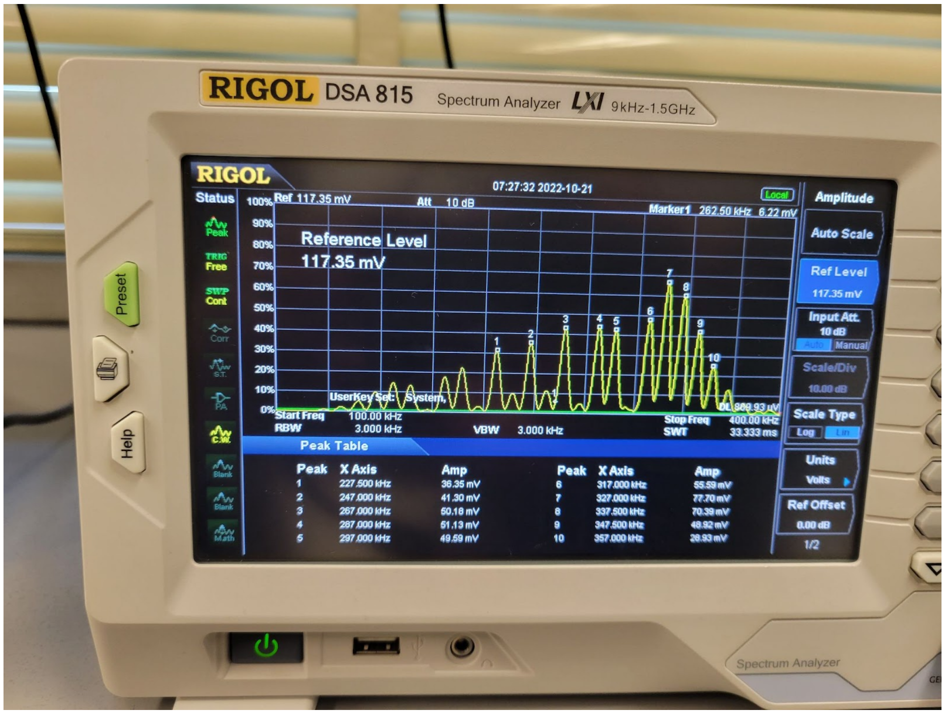

Figure 6 - Spectrum of Bandpass Filtered FM Modulated Signal ($\beta = 10$)

Figure 7 - Bandpass Filtered FM Modulated Signal ($\beta = 10$) in Time Domain

Conclusion/Analysis

As seen in Figure 2, the VOC had a linear relationship between input voltage and frequency of the output signal. The slope of the line formed by multiple measurement points was equal to around 136198 Hz/V. This is our k_f. Rearranging the equation for the modulation index $\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$, we can solve for the message amplitude to achieve a specific β. Solving for A_m, the equation is $A_m = \frac{\beta f_m}{k_f}$. Solving this equation with a message frequency of 10kHz and for modulation indices of 2,5, and 10, the message signal amplitudes, in Figure 2 were obtained.

The HP3312A Function Generator was set to output a cosine carrier signal of 250kHz. When message signals were applied to the VOC at 10kHz and the amplitudes corresponding to a specific β, the spectrums shown in Figures 3, 4, and 5 were shown. These spectrums match ,in shape, the theoretical spectrums that were derived in the prelab. They both have peaks which are centered about the carrier frequency and distanced by the message frequency of 10kHz. This is what is dictated by $cos[(\omega_c + n\cdot\ \omega_m)t]$ portion of

$FM(t) = A_c \sum_n J_n(\beta)\, cos[(\omega_c + n\cdot\ \omega_m)t]$. The rest of the equation just defines the magnitudes.

In the lab, the magnitude of the peaks are different from the theoretical peaks even though the relationship between peaks is the same. This is because the oscilloscope measures the root mean square or RMS magnitude of the signal which is the theoretical magnitude divided by $\sqrt{2}$. If the magnitude of the peaks on the oscilloscope are multiplied by $\sqrt{2}$, then they will match up with the theoretical plots from the prelab.

Figure 6 shows the spectrum of $\beta = 10$ FM modulation after a bandpass filter was applied to it. The lower cutoff frequency was fine tuned so that the peak amplitudes of the spectrum would now increase linearly as frequency increased. The resulting filtered waveform is shown in Figure 7, demonstrating the 10kHz message cosine embedded into the envelope of the frequency modulated signal. Because the content of all the frequencies are still present, just attenuated, the filtered signal still has the instantaneous frequencies of the unfiltered signal and the FM characteristics could still be seen. What changes is the amplitude of the signals as its instantaneous frequency changes. Where the signal had higher frequency content, the bandpass filter does not attenuate as much, so that part of the signal has larger amplitude. Where the signal had lower frequency content, the bandpass filter attenuated more and so this part of the signal has smaller amplitude. Altering the amplitude of the signal this way causes the message signal to be placed as the envelope of the FM signal. With an envelope detector the original signal will be able to be reconstructed.