Lab 3 - Sampling

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ECE 4705-L

Introduction

Background Theory

Sampling is the process of measuring select values of a continuous signal. In signal processing, this is necessary to represent a continuous valued signal with finite values. The finite values could be easily stored and manipulated in the digital realm. Ideally, sampling a signal will retrieve individual points of the signal. This would require an impulse train which can not be realized in the lab. In practice, a rectangular pulse train is used to retrieve individual slices of the signal. These sampled slices could be analyzed using analog techniques or methods could be used to obtain a singular value for digital analysis. The original signal could be completely recovered from the sampled signal as long as the sampling rate used was twice the frequency of the highest frequency in the signal. This doubled frequency is known as the Nyquist sampling rate.

When the signal is sampled, the resulting frequency spectrum is the convolution of the original signal's spectrum with the spectrum of the sampling pulse train. A rectangular pulse train's spectrum has value at every integer multiple of the fundamental frequency. This means that a copy of each signal spectrum will be placed at each integer multiple of the fundamental, scaled by some sinc based factor. Using a low pass filter, one copy of the spectrum could be recovered giving the original signal.

Prelab:

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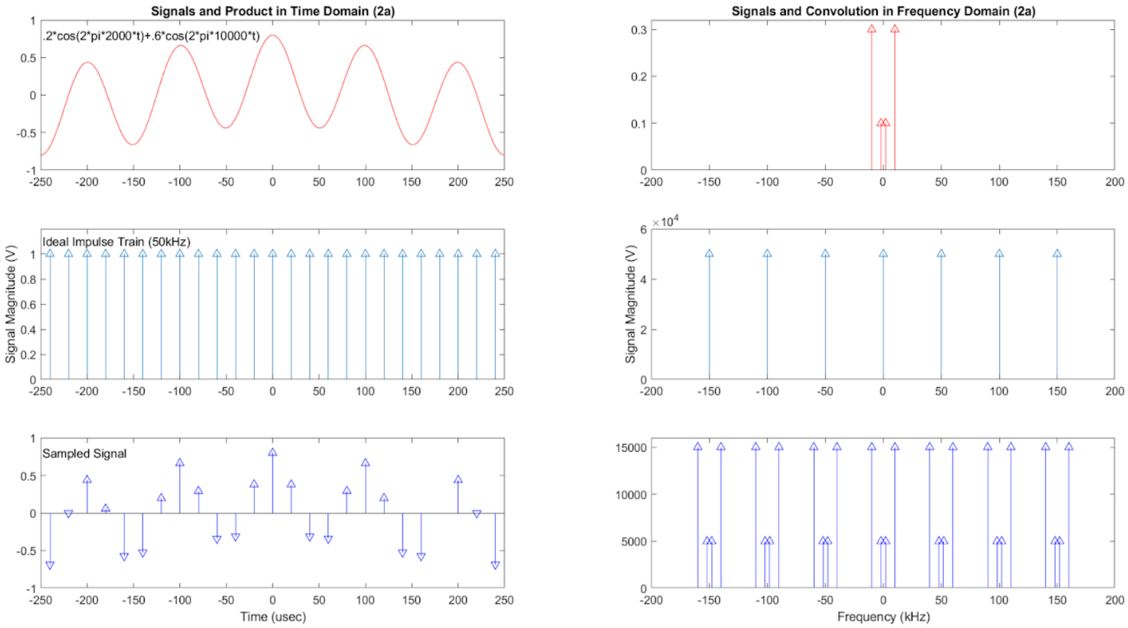

Prelab Plots (Ideal Sampling)

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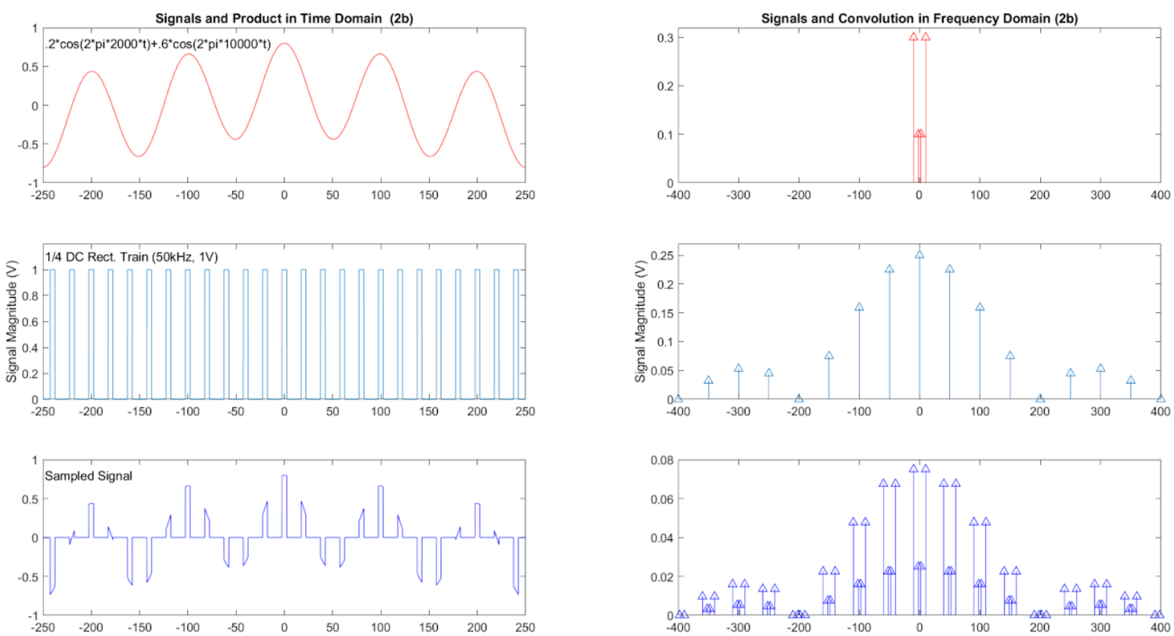

Prelab Plots (Practical Sampling)

The prelab demonstrated the convolution properties of sampling. As seen in the figures below, spectral copies of the original signal are made at every interval of the sampling frequency. Since the double sided spectrum is copied, each copy needs a bandwidth which is double that of its highest frequency. This is the basic theory behind the Nyquist sampling frequency. Using a low pass filter on the sampled signal, the middle copy of the spectrum could be obtained which is enough to recover the original signal. Depending on the sampling frequency, the range where the cutoff frequency needs to fall in can change , but in general the cutoff frequency should be near half the sampling frequency.

Lab and Results:

In the lab, a 10,000 Hz cosine wave and a 10,000 Hz 50% duty cycle triangle wave were sampled using a 50,000 Hz 75% duty cycle rectangular pulse train. The circuit below was used to obtain the sampled signals. Due to the design of the circuit, the output will be a signal which contains 25% of the original. This means the signal is considered to be multiplied by a 25% duty cycle rectangular pulse train. After sampling, the signals were then filtered to obtain a single copy of its original frequency spectrum as discussed in the prelab section.

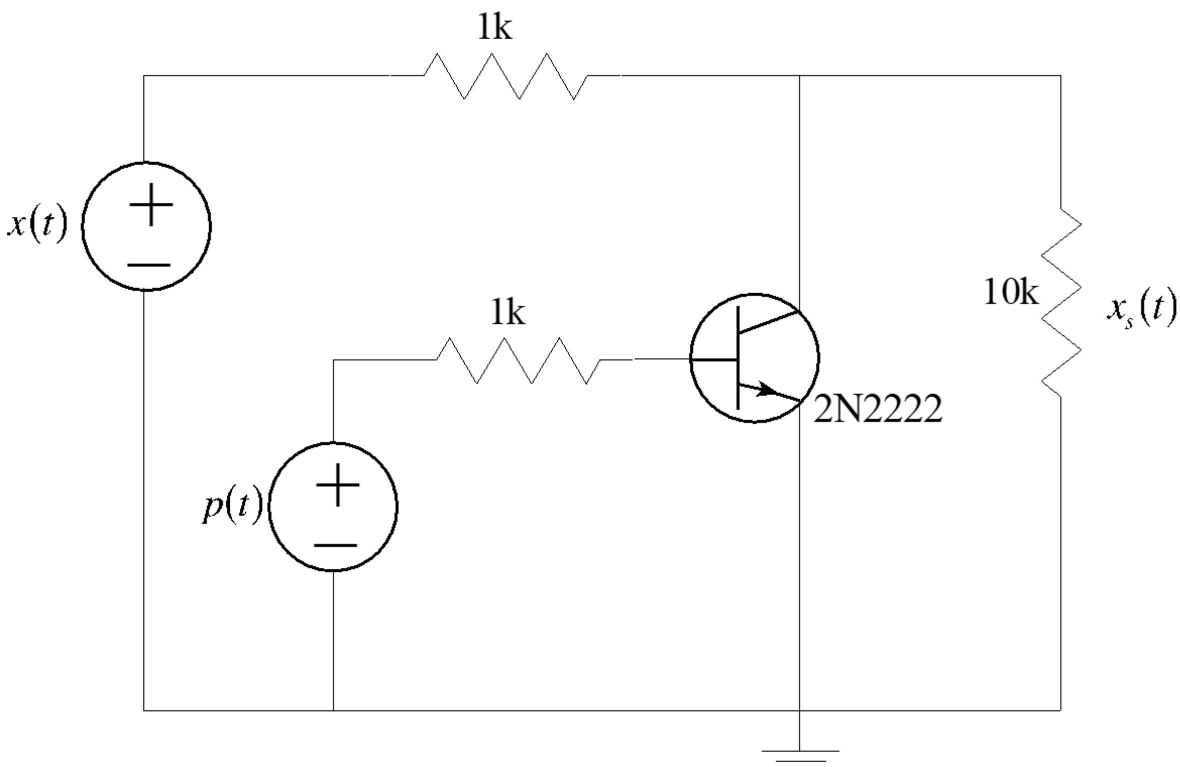

Figure 1 - Sampling Circuit

Figure 2 - Sampled Cosine Wave

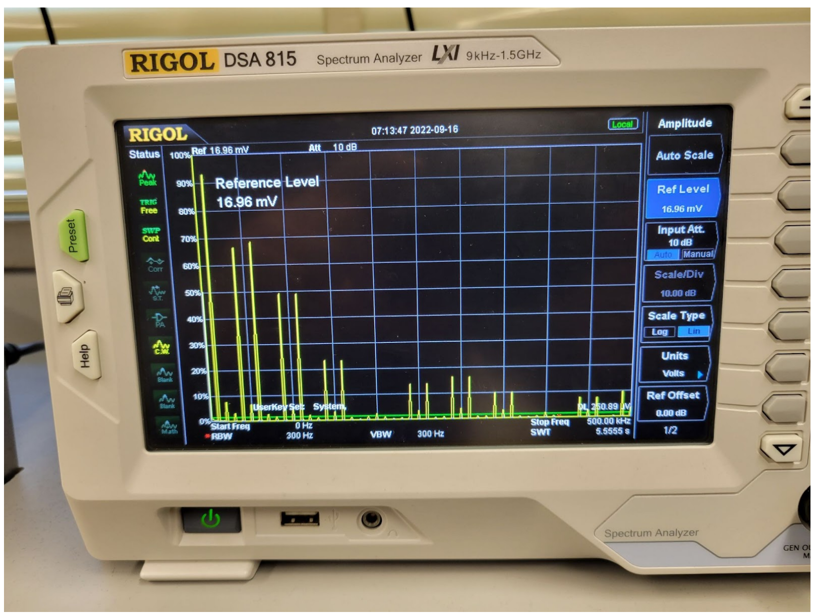

Figure 3 - Frequency Spectrum of Sampled Cosine Wave

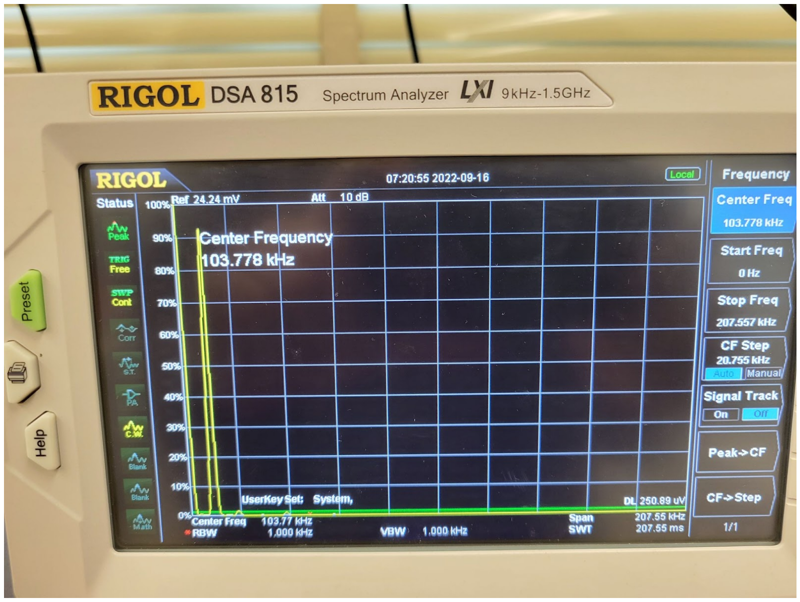

Figure 4 - Frequency Spectrum of Low Pass Filtered Sampled Cosine Wave (Recovered Signal)

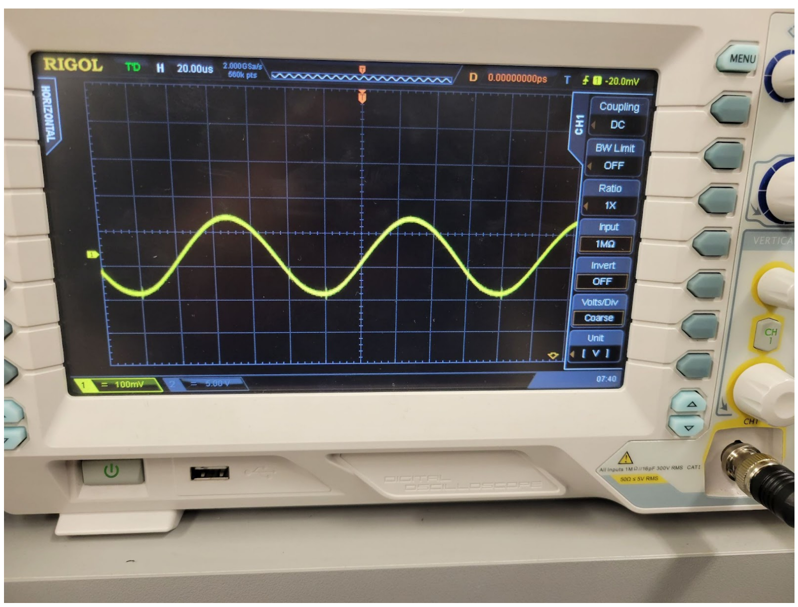

Figure 5 - Low Pass Filtered Sampled Cosine Wave (Recovered Signal)

Figure 6 - Sampled Triangle Wave

Figure 7 - Frequency Spectrum of Sampled Triangle Wave

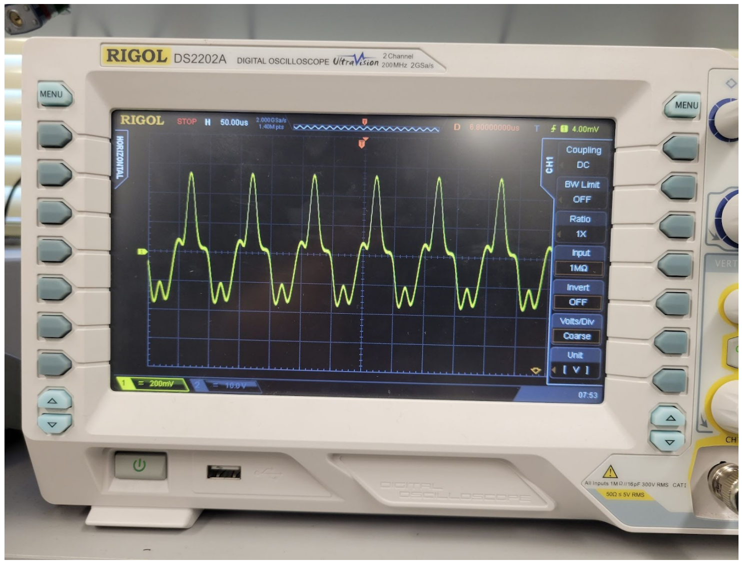

Figure 8 - Low Pass Filtered Sampled Triangle Wave (Larger Cutoff Frequency)

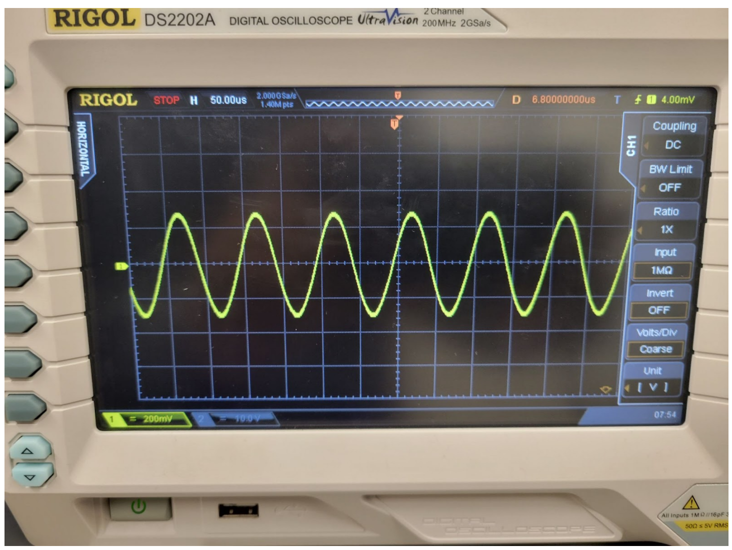

Figure 9 - Low Pass Filtered Sampled Triangle Wave (Smaller Cutoff Frequency)

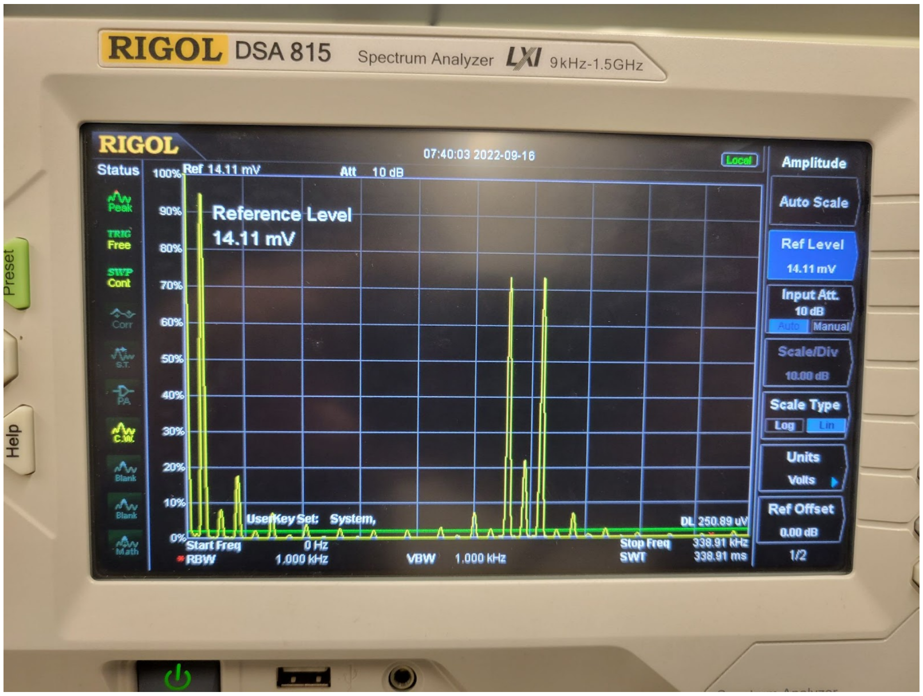

Figure 10 - Frequency Spectrum of Sampled Triangle Wave (Increased Sampling Frequency)

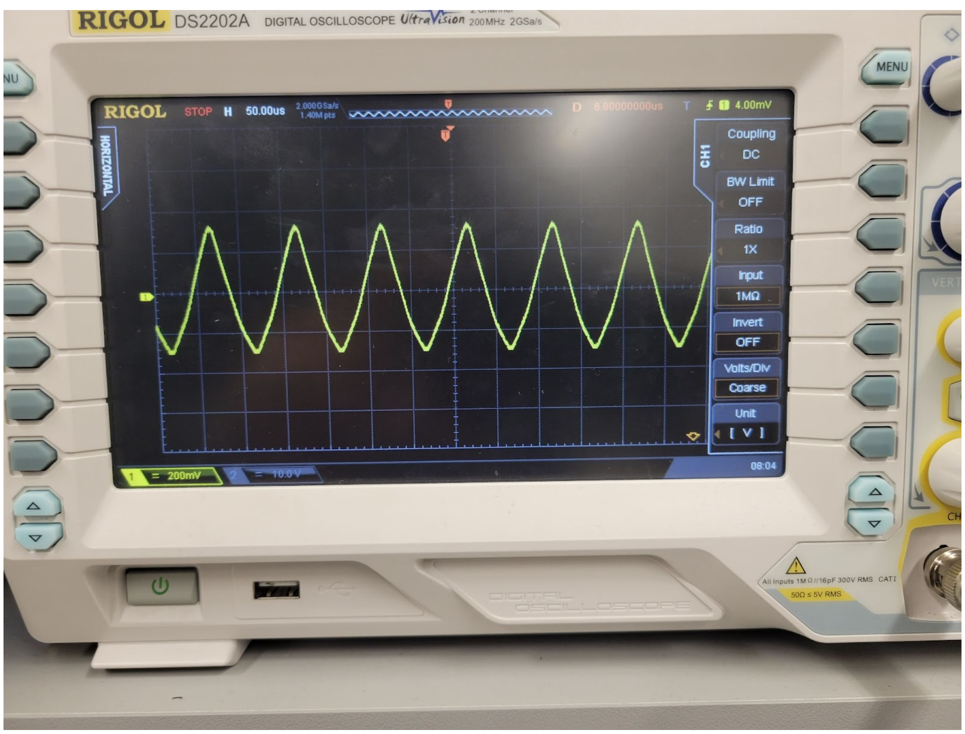

Figure 11 - Low Pass Filtered Sampled Triangle Wave (Increased Sampling Frequency)

Results / Analysis:

The sampled cosine spectrum accurately showed the convolution of the cosine wave with the rectangular pulse train spectrum. At each integer multiple of the sampling frequency, there was a scaled copy of the double impulses representative of a cosine wave (Figure 3). The scale factor for each integer multiple is the A*d*sinc(n*d) fourier expression of the pulse train, causing every 4th copy to be scaled by 0. When the low pass filter was applied, only a single peak was kept, recreating the single sided spectrum for a cosine wave (Figure 4). Showing the filtered signal on the oscilloscope demonstrated that the cosine wave was successfully reconstructed (Figure 5). If the sampling frequency was not at least twice the rate of the cosine frequency, then the copies of each cosine spectrum will overlap with each other. Overlapping spectrums are ambiguous and their original signal can not be retrieved. This is why the Nyquist sampling frequency is twice that of the highest frequency component.

For the sampled triangle wave, the expected spectrum was obtained. The sinc squared valued peaks were present at the corresponding multiples. When a low pass filter was applied to obtain a single copy of the original spectrum, the resulting waveform was not a triangle wave. The signal was distorted and had extra curves which did not seem to be correlated to the original triangle wave's behavior (Figure 8). Lowering the cutoff frequency of the filter converted the recovered signal into what seemed to be a cosine wave (Figure 9). This was due to aliasing of the sampled signal. Unlike the cosine wave, the triangle wave has a theoretically infinite bandwidth. There is no 'highest' frequency in the signal. This means that no matter the sampling rate, a triangle wave could not be perfectly reconstructed using low pass filtering. Each copy of the triangle wave spectrum (sinc squared peaks) will interact with one another and a single copy could not be isolated. The solution to this is to increase the frequency of sampling. This would increase the distance between copies as they are spread apart by the sampling frequency. Since the spectrum of the triangle wave goes to near zero values quickly, the higher sampling rate allows for spectral copies of the triangle wave to overlap only within these high frequency ranges. The sampling rate in the lab was increased to 80,000 Hz. The spectral copies were now more spaced out (Figure 10) and so more content was left unaltered (altered by near zero values). Now when the lowpass filter was applied, there was enough unaltered frequency content for a single copy to allow for an accurate reconstruction of the signal (Figure 11).