ITP20005 L8
Introduction to Functions

Lecture08
JC

Topics we cover and schedule (tentative)

- Racket tutorials (L2,3)
- Modeling languages (L4)
- Interpreting arithmetic (L5)
- Language principles
  - **Substitution** (L6,7)
  - **Function** (L8)
  - Deferring Substitution (L9)
  - First-class Functions (L10)
  - Laziness (L11,12)
  - Recursion (L13,14)
  - Representation choices (L15)
  - Mutable data structures (L16)
  - Variables (L17)
  - Continuations (L18,19,20,21)
  - Garbage collection (L22)
  - Semantics (L23,24)
  - Type (L25,26,27)
- Guest Video Lecture (L28)

No class: October 2 (Fri, Chuseok), October 9 (Fri, Hangul day)
Online only class can be provided.

Q&A

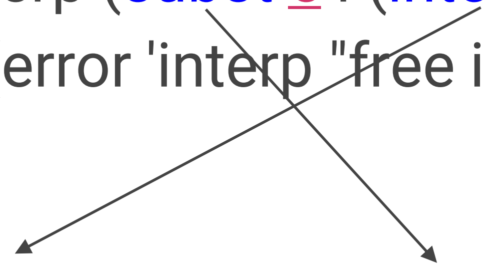
- The 'with' part of the function, subst.

```
; [contract] subst: WAE symbol number -> WAE
(define (subst wae idtf val)
        (type-case WAE wae
                [num    (n)      wae]
                [add    (l r)     (add (subst l idtf val) (subst r idtf val))]
                [sub    (l r)     (sub (subst l idtf val) (subst r idtf val))]
                [with   (i v e)   (with i (subst v idtf val) (if (symbol=? i idtf) e
                                  (subst e idtf val)))]
                [id      (s)       (if (symbol=? s idtf) (num val) wae)]))
```

Q&A

- Calling subst in the interpreter.

```
; interp: WAE -> number
(define (interp wae)
     (type-case WAE wae
               [num (n) n]
               [add (l r) (+ (interp l) (interp r))]
               [sub (l r) (- (interp l) (interp r))]
               [with (i v e) (interp (subst e i (interp v)))]
               [id (s)            (error 'interp "free identifier")]))
```



```
(test (interp (with 'x (sub (num 7) (num 2))) (add (id 'x) (id 'x)))) 10)
```

Q&A

Target expression for substitution

- The 'with' part of the function, subst.

```
; [contract] subst: WAE symbol number -> WAE
(define (subst wae idtf val)
          (type-case WAE wae
                    [num      (n)         wae]
                    [add      (l r)       (add (subst l idtf val) (subst r idtf val))]
                    [sub      (l r)       (sub (subst l idtf val) (subst r idtf val))]
                    [with      (i v e)     (with i (subst v idtf val)
                                                   (if (symbol=? i idtf) e
                                                              (subst e idtf val)))]
                    [id        (s)        (if (symbol=? s idtf) (num val) wae)]))
```

```
; {with {x 10} {...{with {y 17} x}}              ⇒ 10 for x in {with {y 17} x}        ⇒  {with {y 17} 10}
(test (subst (with 'y (num 17) (id 'x)) 'x 10) (with 'y (num 17) (num 10)))
; {with {x 10} {...{with {y x} y}}}              ⇒ 10 for x in {with {y x} y}         ⇒  {with {y 10} y}
(test (subst (with 'y (id 'x) (id 'y)) 'x 10) (with 'y (num 10) (id 'y)))
; {with {x 10} {...{with {y x} x}}}              ⇒ 10 for x in {with {y x} x}         ⇒  {with {y 10} 10}
(test (subst (with 'y (id 'x) (id 'x)) 'x 10) (with 'y (num 10) (num  10)))
```

Q&A

- Calling subst in the interpreter.

```
; interp: WAE -> number
(define (interp wae)
     (type-case WAE wae
               [num (n) n]
               [add (l r) (+ (interp l) (interp r))]
               [sub (l r) (- (interp l) (interp r))]
               [with (i v e) (interp (subst e i (interp v)))]
               [id (s)          (error 'interp "free identifier")]))
```

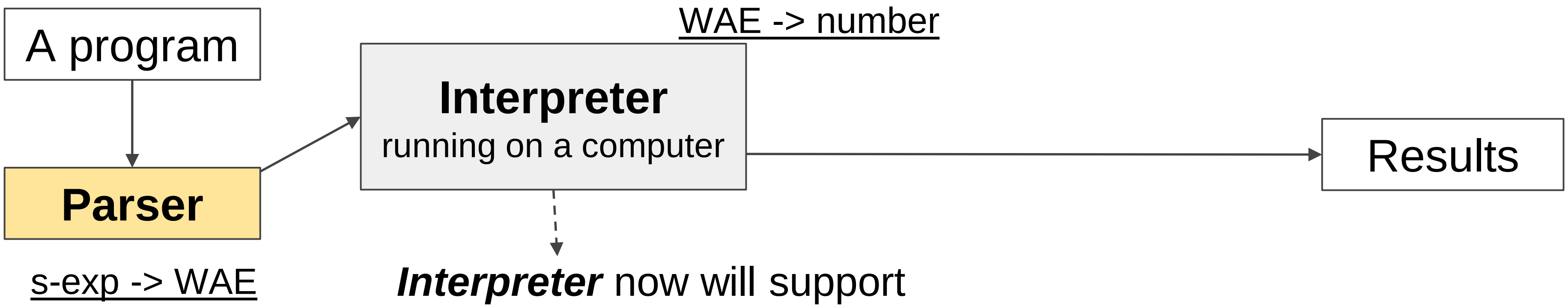
Target expression for substitution

WAE in a value expression

Identifiers (i) in e will be substituted with an actual value from (interp v).

```
(test (interp (with 'x (sub (num 7) (num 2))) (add (id 'x) (id 'x)))) 10)
```

Big Picture (modeling languages: substitution)



A program

Parser

s-exp -> WAE

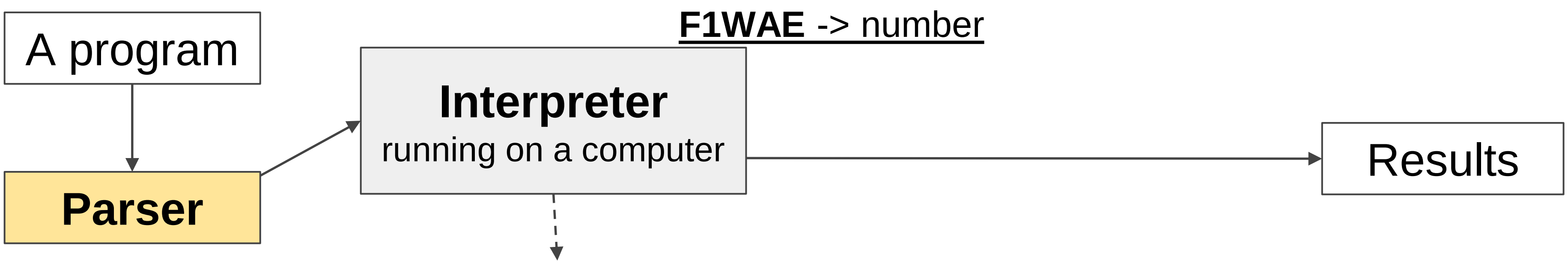
WAE -> number

Interpreter
running on a computer

Results

Interpreter now will support
(1) Substitution

Big Picture (modeling languages: substitution)



F1WAE -> number

A program

Parser

Interpreter

running on a computer

Results

s-exp -> F1WAE

Interpreter now will support

(1) Substitution

(2) **Function**

Expression with an identifier

```
{with {x 5} {+ x 5}}
```

How about?

$f(x) = x + 5$

$1 + 5$	; $f(1)$
$2 + 5$	; $f(2)$
$3 + 5$	; $f(3)$
....	

Parameterized Expression

{+ x 5}

```
1 + 5          ; f(1)
2 + 5          ; f(2)
3 + 5          ; f(3)
....
```

Functions are useful in PL?!

Let's add functions to WAE

We need to define its concrete and abstract syntax.

$\Rightarrow$ Our new language, AE → WAE → F1WAE

Think about simple functions

- $identity(x) = x$
- $twice(x) = x + x$

Think about simple functions

- *identity(x) = x*
- ***twice(x) = x + x***

- AE

  ```
  {- 20 {+ 10 10}
  {- 20 {+ 17 17}}
  {- 20 {+ 3 3}}
  ```

Think about simple functions

- *identity(x) = x*
- *twice(x) = x + x*

- AE
  ```
  {- 20 {+ 10 10}
  {- 20 {+ 17 17}}
  {- 20 {+ 3 3}}
  ```
- WAE
  ```
  {with {x 10} {- 20 {+ x x}}}
  {with {x 17} {- 20 {+ x x}}}
  {with {x 3} {- 20 {+ x x}}}
  ```

Think about simple functions

- *identity(x) = x*
- *twice(x) = x + x*

- AE

```
{- 20 {+ 10 10}
{- 20 {+ 17 17}}
{- 20 {+ 3 3}}
```

- WAE

```
{with {x 10} {- 20 {+ x x}}}
{with {x 17} {- 20 {+ x x}}}
{with {x 3} {- 20 {+ x x}}}
```

- F1WAE

```
{deffun {identity x}
    x}
```

```
{deffun {twice x}
    {+ x x}}
```

Think about simple functions

- *identity(x) = x*
- *twice(x) = x + x*

- AE
  {- 20 {+ 10 10}
  {- 20 {+ 17 17}}
  {- 20 {+ 3 3}}
- WAE
  {with {x 10} {- 20 {+ x x}}}
  {with {x 17} {- 20 {+ x x}}}
  {with {x 3} {- 20 {+ x x}}}
- F1WAE

```
{deffun {identity x}
    x}
{identity 8}
```

```
{deffun {twice x}
             {+ x x}}
               {twice 10}
                     {twice
17}
                     {twice
```

WAE: Concrete Syntax

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

F1WAE: Concrete Syntax

```
<FunDef> ::= {deffun {<id> <id>} <F1WAE>}
<F1WAE> ::= <num>
          | {+ <F1WAE> <F1WAE>}
          | {- <F1WAE> <F1WAE>}
          | {with {<id> <F1WAE>} <F1WAE>}
          | <id>
          | {<id> <F1WAE>}
```

F1WAE: Concrete Syntax in BNF

```
<FunDef> ::= {deffun {<id> <id>} <F1WAE>}          ← for function definition
<F1WAE> ::= <num>
            | {+ <F1WAE> <F1WAE>}
            | {- <F1WAE> <F1WAE>}
            | {with {<id> <F1WAE>} <F1WAE>}
            | <id>
            | {<id> <F1WAE>}                       ← for function call
```

F1WAE: Concrete Syntax

```
<FunDef> ::= {deffun {<id> <id>} <F1WAE>}        ← for function definition
<F1WAE> ::= <num>
          | {+ <F1WAE> <F1WAE>}
          | {- <F1WAE> <F1WAE>}
          | {with {<id> <F1WAE>} <F1WAE>}
          | <id>
          | {<id> <F1WAE>}                       ← for function call
```

```
{deffun {identity x} x}
{identity 8}
```

```
{deffun {twice x} {+ x x}}
{- 20 {twice 10}}
{- 20 {twice 17}}
{- 20 {twice 3}}
```

How about this??? F1WAE: Concrete Syntax

```
<F1WAE> ::= <num>
          | {+ <F1WAE> <F1WAE>}
          | {- <F1WAE> <F1WAE>}
          | {with {<id> <F1WAE>} <F1WAE>}
          | <id>
          | {<F1WAE> <F1WAE>}              ← function call??
          | {deffun {<id>} <F1WAE>}        ← for function definition
```

F1WAE: Concrete Syntax

```
<FunDef> ::= {deffun {<id> <id>} <F1WAE>}          ← for function definition
<F1WAE> ::= <num>
          | {+ <F1WAE> <F1WAE>}
          | {- <F1WAE> <F1WAE>}
          | {with {<id> <F1WAE>} <F1WAE>}
          | <id>
          | {<id> <F1WAE>}                          ← for function call
```

```
{deffun {identity x} x}
{identity 8}

{deffun {twice x} {+ x x}}
{- 20 {twice 10}}
{- 20 {twice 17}}
{- 20 {twice 3}}
```

F1WAE: Abstract Syntax

```
(define-type FunDef
        [fundef  (fun-name symbol?)
                 (arg-name symbol?)
                 (body F1WAE?)])

(define-type F1WAE
        [num     (n number?)]
        [add     (lhs F1WAE?) (rhs F1WAE?)]
        [sub     (lhs F1WAE?) (rhs F1WAE?)]
        [with    (name symbol?) (named-expr F1WAE?) (body F1WAE?)]
        [id            (name symbol?)]
        [app     (ftn symbol?)     (arg F1WAE?)])
```

F1WAE: Abstract Syntax

```
(define-type FunDef
            [fundef   (fun-name symbol?) (arg-name symbol?) (body F1WAE?)])
(define-type F1WAE
            [num      (n number?)]
            [add      (lhs F1WAE?) (rhs F1WAE?)]
            [sub      (lhs F1WAE?) (rhs F1WAE?)]
            [with     (name symbol?) (named-expr F1WAE?) (body F1WAE?)]
            [id                   (name symbol?)]
            [app      (ftn symbol?)       (arg F1WAE?)])
```

```
(fundef   'identify 'x (id 'x))
(app 'identity (num 8))

(fundef 'twice 'x (add (id 'x) (id 'x)))
(app 'twice (num 10))
(app 'twice (num 17))
(app 'twice (num 3))
```

← Abstract syntax representation of the example code written in our new language.

F1WAE Parser

F1WAE: Concrete Syntax in BNF

```
<FunDef> ::= {deffun {<id> <id>} <F1WAE>}          ← for function definition
<F1WAE> ::= <num>
                     | {+ <F1WAE> <F1WAE>}
                     | {- <F1WAE> <F1WAE>}
                     | {with {<id> <F1WAE>} <F1WAE>}
                     | <id>
                     | {<id> <F1WAE>}              ← for function call
```

F1WAE: Abstract Syntax

```
(define-type FunDef
        [fundef   (fun-name symbol?) (arg-name symbol?) (body F1WAE?)])
(define-type F1WAE
        [num      (n number?)]
        [add      (lhs F1WAE?) (rhs F1WAE?)]
        [sub      (lhs F1WAE?) (rhs F1WAE?)]
        [with     (name symbol?) (named-expr F1WAE?) (body F1WAE?)]
        [id                 (name symbol?)]
        [app      (ftn symbol?)      (arg F1WAE?)])
```

```
(fundef   'identify 'x (id 'x))
(app 'identity (num 8))

(fundef 'twice 'x (add (id 'x) (id 'x)))
(app 'twice (num 10))
(app 'twice (num 17))
(app 'twice (num 3))
```

← Abstract syntax representation of the example code written in our new language.

F1WAE Parser

```
; parse-fd: sexp -> FunDef
…

; parse : sexp -> F1WAE
```

F1WAE: Parser

```
; parse-fd: sexp -> FunDef
```

```
; parse : sexp -> F1WAE
(define (parse sexp)
        (match sexp
                [(? number?)                            (num sexp)]
                [(list '+ l r)                          (add (parse l) (parse r))]
                [(list '- l r)                                    (sub (parse l) (parse
r))]
                [(list 'with (list i v) e)        (with i (parse v) (parse e))]
                [(? symbol?)                            (id sexp)]
                [(list f a)                             (app f (parse a))]
                [else                                   (error 'parse "bad syntax: ~a"
sexp)]))
```

F1WAE: Parser

```
; parse-fd: sexp -> FunDef
(define (parse-fd sexp)
          (match sexp
                    [(list 'deffun (list f x) b)    (fundef f x (parse b))]))

; parse : sexp -> F1WAE
(define (parse sexp)
          (match sexp
                    [(? number?)                            (num sexp)]
                    [(list '+ l r)                          (add (parse l) (parse r))]
                    [(list '- l r)                                    (sub (parse l) (parse r))]
                    [(list 'with (list i v) e)    (with i (parse v) (parse e))]
                    [(? symbol?)                            (id sexp)]
                    [(list f a)                             (app f (parse a))]
                    [else                                   (error 'parse "bad syntax: ~a" sexp)]))
```

F1WAE: Parser

Function body ← F1WAE

```
; parse-fd: sexp -> FunDef
(define (parse-fd sexp)
          (match sexp
                    [(list 'deffun (list f x) b)    (fundef f x (parse b))]))

; parse : sexp -> F1WAE
(define (parse sexp)
          (match sexp
                    [(? number?)                              (num sexp)]
                    [(list '+ l r)                            (add (parse l) (parse r))]
                    [(list '- l r)                                      (sub (parse l) (parse
r))]
                    [(list 'with (list i v) e)     (with i (parse v) (parse e))]
                    [(? symbol?)                              (id sexp)]
                    [(list f a)                               (app f (parse a))]
                    [else                                     (error 'parse "bad syntax: ~a"
sexp)]))
```

F1WAE Interpreter

```
; interp: F1WAE ?????? -> number
```

F1WAE Interpreter

; interp: F1WAE ?????? -> number

```
(fundef   'identify 'x (id 'x))
(fundef 'twice 'x (add (id 'x) (id 'x)))

(app 'identity (num 8))
(app 'twice (num 10))
```

F1WAE Interpreter

; interp: F1WAE list-of-FuncDef -> number

```
(fundef   'identify 'x (id 'x))
(fundef 'twice 'x (add (id 'x) (id 'x)))

(app 'identity (num 8))
(app 'twice (num 10))
```

F1WAE: Interpreter

```
; interp: F1WAE list-of-FuncDef -> number
(define (interp f1wae fundefs)
        (type-case F1WAE f1wae
                [num    (n)                 n]
                [add    (l r)               (+ (interp l fundefs) (interp r
fundefs))]
                [sub    (l r)               (- (interp l fundefs) (interp r
fundefs))]
                [with   (x i b)   (interp (subst b x (interp i fundefs))
fundefs)]
                [id               (s)                 (error 'interp "free
identifier")]
                [app    (f a)     …]))
```

Let's make examples/tests first

F1WAE: Interpreter

```
(test (interp (add (num 1) (num 1))
                                                            empty)
          ?)

; interp: F1WAE list-of-FuncDef -> number
(define (interp f1wae fundefs)
          (type-case F1WAE f1wae
                    [num    (n)                 n]
                    [add    (l r)               (+ (interp l fundefs) (interp r
fundefs))]
                    [sub    (l r)               (- (interp l fundefs) (interp r
fundefs))]
                    [with   (x i b)   (interp (subst b x (interp i fundefs))
fundefs)]
                    [id               (s)                 (error 'interp "free
identifier")]
```

F1WAE: Interpreter

```
(test (interp (add (num 1) (num 1))
                                                  empty)
          2)

; interp: F1WAE list-of-FuncDef -> number
(define (interp f1wae fundefs)
          (type-case F1WAE f1wae
                    [num     (n)                    n]
                    [add     (l r)                  (+ (interp l fundefs) (interp r
fundefs))]
                    [sub     (l r)                  (- (interp l fundefs) (interp r
fundefs))]
                    [with    (x i b)    (interp (subst b x (interp i fundefs))
fundefs)]
                    [id                 (s)                  (error 'interp "free
```

F1WAE: Interpreter

```
(test (interp (add (num 1) (num 1))
                                        (list (fundef 'f 'x (add (id 'x) (num 3)))))
          ?)

; interp: F1WAE list-of-FuncDef -> number
(define (interp f1wae fundefs)
          (type-case F1WAE f1wae
                    [num     (n)                n]
                    [add     (l r)              (+ (interp l fundefs) (interp r
fundefs))]
                    [sub     (l r)              (- (interp l fundefs) (interp r
fundefs))]
                    [with    (x i b)   (interp (subst b x (interp i fundefs))
fundefs)]
                    [id                (s)                (error 'interp "free
identifier")]
```

F1WAE: Interpreter

```
(test (interp (add (num 1) (num 1))
                                        (list (fundef 'f 'x (add (id 'x) (num 3)))))
          2)

; interp: F1WAE list-of-FuncDef -> number
(define (interp f1wae fundefs)
          (type-case F1WAE f1wae
                    [num    (n)                 n]
                    [add    (l r)               (+ (interp l fundefs) (interp r
fundefs))]
                    [sub    (l r)               (- (interp l fundefs) (interp r
fundefs))]
                    [with   (x i b)   (interp (subst b x (interp i fundefs))
fundefs)]
                    [id               (s)                 (error 'interp "free
identifier")]
```

F1WAE: Interpreter

```
(test (interp (app 'f (num 1))
                                        (list (fundef 'f 'x (add (id 'x) (num 3)))))
          ?)

; interp: F1WAE list-of-FuncDef -> number
(define (interp f1wae fundefs)
          (type-case F1WAE f1wae
                    [num     (n)                n]
                    [add     (l r)              (+ (interp l fundefs) (interp r
fundefs))]
                    [sub     (l r)              (- (interp l fundefs) (interp r
fundefs))]
                    [with    (x i b)    (interp (subst b x (interp i fundefs))
fundefs)]
                    [id                 (s)                 (error 'interp "free
identifier")]
```

F1WAE: Interpreter

```
(test (interp (app 'f (num 1))
                                        (list (fundef 'f 'x (add (id 'x) (num 3)))))
          4)

; interp: F1WAE list-of-FuncDef -> number
(define (interp f1wae fundefs)
          (type-case F1WAE f1wae
                    [num    (n)                  n]
                    [add    (l r)                (+ (interp l fundefs) (interp r
fundefs))]
                    [sub    (l r)                (- (interp l fundefs) (interp r
fundefs))]
                    [with   (x i b)    (interp (subst b x (interp i fundefs))
fundefs)]
                    [id                (s)                 (error 'interp "free
identifier")]
```

F1WAE: Interpreter

```
(test (interp (app 'f (num 10)
                                        (list (fundef 'f 'x (sub (num 20)

          (app 'twice (id 'x))))
                                                                          (fundef 'twice 'y
(add (id 'y) (id 'y))))))
          ?)

; interp: F1WAE list-of-FuncDef -> number
(define (interp f1wae fundefs)
          (type-case F1WAE f1wae
                    [num      (n)                 n]
                    [add      (l r)               (+ (interp l fundefs) (interp r
fundefs))]
                    [sub      (l r)               (- (interp l fundefs) (interp r fundefs))]
                    [with     (x i b)     (interp (subst b x (interp i fundefs)) fundefs)]
```

F1WAE: Interpreter

```
(test (interp (app 'f (num 10))
                                        (list (fundef 'f 'x (sub (num 20)

          (app 'twice (id 'x))))
                                                                          (fundef 'twice 'y
(add (id 'y) (id 'y)))))
          0)
```

```
; interp: F1WAE list-of-FuncDef -> number
(define (interp f1wae fundefs)
          (type-case F1WAE f1wae
                    [num     (n)                  n]
                    [add     (l r)                (+ (interp l fundefs) (interp r
fundefs))]
                    [sub     (l r)                (- (interp l fundefs) (interp r fundefs))]
                    [with    (x i b)    (interp (subst b x (interp i fundefs)) fundefs)]
```

F1WAE: Interpreter

```
; interp: F1WAE list-of-FuncDef -> number
(define (interp f1wae fundefs)
        (type-case F1WAE f1wae
                [num    (n)              n]
                [add    (l r)            (+ (interp l fundefs) (interp r
fundefs))]
                [sub    (l r)            (- (interp l fundefs) (interp r
fundefs))]
                [with   (x i b)   (interp (subst b x (interp i fundefs))
fundefs)]
                [id           (s)              (error 'interp "free
identifier")]
                [app    (f a)     ... (interp a fundefs) ...]))
```

F1WAE: Interpreter

```
; interp: F1WAE list-of-FuncDef -> number
(define (interp f1wae fundefs)
          (type-case F1WAE f1wae
                    [num    (n)                  n]
                    [add    (l r)                (+ (interp l fundefs) (interp r
fundefs))]
                    [sub    (l r)                (- (interp l fundefs) (interp r
fundefs))]
                    [with   (x i b)   (interp (subst b x (interp i fundefs))
fundefs)]
                    [id               (s)                  (error 'interp "free
identifier")]
                    [app    (f a)     … (interp a fundefs) ...]))

; lookup-fundef: symbol list-of-FunDef -> FunDef
```

F1WAE: Interpreter

```
; interp: F1WAE list-of-FuncDef -> number
(define (interp f1wae fundefs)
        (type-case F1WAE f1wae
                [num    (n)     n]
                [add    (l r)        (+ (interp l fundefs) (interp r fundefs))]
                ...
                [app    (f a)
                   (local
                              [(define a_fundef (lookup-fundef f
fundefs))]
                              (interp (subst (fundef-body
a_fundef)
                                                                  (interp a
fundefs))
```

Get the function body from the look-up function

(fundef 'f 'x (add (id 'x) (num 3)))

* local: to implement a local logic. https://docs.racket-lang.org/reference/local.html
In our case, we need a local logic for the result expression in a branch of type-case

Lookup

```
; lookup-fundef: symbol list-of-FunDef -> FunDef
(define (lookup-fundef name fundefs)
          ...)
```

Lookup

```
; lookup-fundef: symbol list-of-FunDef -> FunDef
(define (lookup-fundef name fundefs)
        (cond
                [(empty? fundefs)
                        …]
                [else
                        ... (first fundefs)
                        ... (lookup-fundef name (rest fundefs)) …]))
```

Lookup

```
; lookup-fundef: symbol list-of-FunDef -> FunDef
(define (lookup-fundef name fundefs)
    (cond
        [(empty? fundefs)
            (error 'lookup-fundef "unknown function")]
        [else
            (if (symbol=? name (fundef-fun-name (first fundefs)))
                  (first fundefs)
                  (lookup-fundef name (rest fundefs)))]))
```

Substitution for F1WAE Interpreter

```
; [contract] subst: F1WAE symbol number -> F1WAE
(define (subst f1wae idtf val)
          (type-case F1WAE f1wae
                    [num     (n)                 f1wae]
                    [add     (l r)               (add (subst l idtf val) (subst r idtf
val))]
                    [sub               (l r)               (sub (subst l idtf val) (subst r
idtf val))]
                    [with    (i v e)   (with i (subst v idtf val) (if (symbol=? i idtf) e

          (subst e idtf val)))]
                    [id                (s)                 (if (symbol=? s idtf) (num val)
f1wae)]
                    [app     (f a)               (app f    (subst a idtf val))]))

; {with {x 1} {fn x}}    <- function call in the body of 'with'.
(subst (app 'fn (id 'x)) 'x 1) ; (app 'fn (num 1))
```

Topics we cover and schedule (tentative)

- Racket tutorials (L2,3)
- Modeling languages (L4)
- Interpreting arithmetic (L5)
- Language principles
  - **Substitution** (L6,7)
  - **Function** (L8)
  - Deferring Substitution (L9)
  - First-class Functions (L10)
  - Laziness (L11,12)
  - Recursion (L13,14)
  - Representation choices (L15)
  - Mutable data structures (L16)
  - Variables (L17)
  - Continuations (L18,19,20,21)
  - Garbage collection (L22)
  - Semantics (L23,24)
  - Type (L25,26,27)
- Guest Video Lecture (L28)

No class: October 2 (Fri, Chuseok), October 9 (Fri, Hangul day)
Online only class can be provided.

TODO
Read PLAI (first edition) Chapter 5. Deferring Substitution

Second edition Ch 6. From Substitution to Environments
http://cs.brown.edu/courses/cs173/2012/book/From_Substitution_to_Environments.html

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* Slides are from Prof. Sukyoung Ryu's PL class in 2018 Spring
or created by JC based on the main text book.