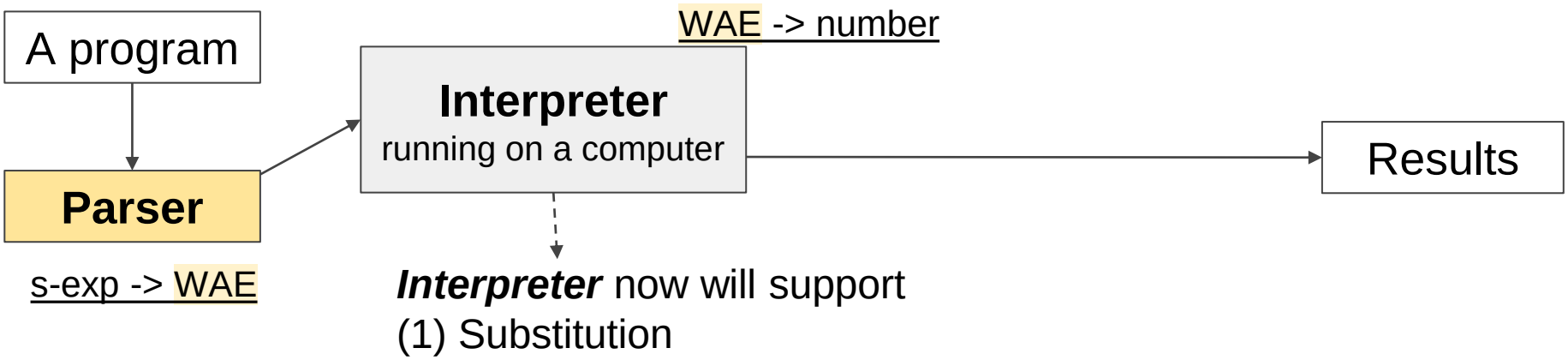
ITP20005 L6/L7
Substitution

Lecture06
JC

Q&A

- What are we doing now? Are we learning Racket of PL?
- A language called AE
  - arithmetic (addition and subtraction) 3 + (5 - 2)
  - concrete syntax {+ 3 {- 5 2}}
  - abstract syntax AE
    (add (num 3) (sub (num 5) (num 2)))
  - parser parse: sexp -> AE
    (test (parse '{+ 3 {- 5 2}})
    (add (num 3) (sub (num 5) (num 2))))
  - interpreter interp: AE -> number
    (test (interp (parse '{+ 3 {- 5 2}})) 6)

Big Picture (modeling languages: substitution)



* **WAE**: AE that support identifiers

Agenda

- Identifiers
- Substitution
- Binding
- Scope

Motivating Example

Repeated expressions

{+ {+ 5 5} {+ 5 5}}

Repeated expressions

{+ {+ {+ 5 5} {+ 5 5}} {+ {+ 5 5} {+ 5 5}}}

Repeated expressions

{+ {+ {+ 5 5} {+ 5 5}} {+ {+ 5 5} {+ 5 5}}}

When we have any repeated expressions, we might make a mistake and evaluating them wastes computational cycles.

See some example in your favorite language

```
// sum from 1 to 10 and repeat it three times to get total sum
int totalSum = (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10) + (1 + 2 + 3
+ 4 + 5 + 6 + 7 + 8 + 9 + 10) + (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 +
10);
```

See some example in your favorite language

```
// sum from 1 to 10 and repeat it three times to get total sum
int partialSum = 1 + 2 + 3+ 4 + 5 + 6 + 7 + 8 + 9 + 10;
int totalSum = partialSum + partialSum + partialSum;
```

See some example in your favorite language

```
// sum from 1 to 10 and repeat it three times to get total sum
int partialSum = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10;
int totalSum = partialSum + partialSum + partialSum;
```

1. **Computational benefit**: *partialSum* is calculated once.
2. We call *partialSum* an **identifier**.
3. **Substitution**: To get the result of totalSum, *partialSum* needs to be replaced with 55 while computing the total sum.

Identifier

to avoid this redundancy

Identifiers

- Name/identify the value of an expression
- Reuse its name in place of the larger computation
- Similar to variables
  - But variables imply the value of the identifier can change.
- Note that in ***our current language,*** we do not initially offer any way of changing the associated value with the identifier.

  ⇒ Identifier (works like a **constant** in our current language)

Topics we cover and schedule (tentative)

- Racket tutorials (L2,3)
- Modeling languages (L4)
- Interpreting arithmetic (L5)
- Language principles
  - **Substitution** (L6,7)
  - Function (L8)
  - Deferring Substitution (L9)
  - First-class Functions (L10)
  - Laziness (L11,12)
  - Recursion (L13,14)
  - Representation choices (L15)
  - Mutable data structures (L16)
  - Variables (L17)
  - Continuations (L18,19,20,21)
  - Garbage collection (L22)
  - Semantics (L23,24)
  - Type (L25,26,27)
- Guest Video Lecture (L28)

No Class: September 25 (Tue, Chuseok) and October 9 (Tue, Hangul day)

Sample program that uses identifiers

{+ {+ 5 5} {+ 5 5}}

How do you want to deal with this expression with an identifier *<u>in our new language</u>*??

* Remember that we use an uniform syntax: parenthesized {} prefix in this class.

Sample program that uses identifiers

{+ {+ 5 5} {+ 5 5}}

;; use an identifier for the repeated expressions, {+ 5 5}
;; We would like to use '**with**' keyword to define **an identifier** for **an arithmetic expression** and use the identifier **for another arithmetic expression**.
{with {x {+ 5 5}} {+ x x}}

A more elaborate example

{+ {- {+ 5 5} 3} {- {+ 5 5} 3}}

⇒ Write this expression with 'with'.

* descend: move to the inner expression to continue calculating.

A more elaborate example

{+ {- {+ 5 5} 3} {- {+ 5 5} 3}}

```
{with {x {+ 5 5 }}
        {with {y {- x 3}}
                {+ y y}}}
= {with {x 10} {with {y {- x 3}} {+ y y}}}
= {with {x 10} {with {y {- 10 3}} {+ y y}}}
= {with {y {- 10 3}} {+ y y}}}
= {with {y 7} {+ y y}}
= {with {y 7} {+ 7 7}}
={+ 7 7}
= 14
```

[+ operator]
[substitution]
[descend*]
[- operator]
[substitution]
[descend]
[+ operator]

* descend: move to the inner expression to continue calculating.

Then, improve AE to support identifiers!

- 'with' with arithmetic expressions ⇒ WAE in BNF

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | ...
        | ...
```

Then, improve AE to support identifiers!

- 'with' with arithmetic expressions ⇒ WAE in BNF

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

* We have now two additional rules to AE in BNF:
  (1) for associating values with an identifier
  (2) for using the identifier

Then, improve AE to support identifiers!

- Identifiers

  <id> ::= x, y, plus, factorial, swap, interp,...

Then, improve AE to support identifiers!

- Identifiers

  <id> ::= x, y, plus, factorial, swap, interp,...

  or (in scheme: a bit different from Racket)

  <id> ::= <initial> <subsequent>* | + | - |

  ...

  <initial> ::= <letter> | ! | $ | % | & | * | : | < | = | > | ? |

  ~ | _ | ^

  <subsequent> ::= <initial> | <digit> | . | + | -

  <letter> ::= a | b | … | z

  <digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

* zero or more occurrences

Then, improve AE to support identifiers!

- 'with' with arithmetic expressions

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

```
{with {x {+ 1 2}}
      {+ x x}}
```

Then, improve AE to support identifiers!

- 'with' with arithmetic expressions

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

Binding instance (identifier)

```
{with {x {+ 1 2}}
      {+ x x}}
```

$\Rightarrow$ 6

Bound instance (identifier)

Then, improve AE to support identifiers!

- 'with' with arithmetic expressions

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

```
{with {x {+ 1 2}}
      {+ x y}}
```

⇒ error: free

identifier

Free identifier (instance)

Then, improve AE to support identifiers!

- 'with' with arithmetic expressions

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

x ⇒ error: free identifier

Then, improve AE to support identifiers!

- 'with' with arithmetic expressions

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

```
{+ {with {x {+ 1 2}}
         {+ x x}}
   {with {x {- 4 3}}
         {+ x x}}}
```

Then, improve AE to support identifiers!

- 'with' with arithmetic expressions

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

```
{+ {with {x {+ 1 2}}
         {+ x x}}
   {with {x {- 4 3}}
         {+ x x}}}
```

$\Rightarrow$ 8

Then, improve AE to support identifiers!

- 'with' with arithmetic expressions

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

```
{with {x {+ 1 2}}
        {with {x {- 4 3}}
                {+ x x}}}
```

Then, improve AE to support identifiers!

- 'with' with arithmetic expressions

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

```
{with {x {+ 1 2}}
      {with {x {- 4 3}}
            {+ x x}}}
```

⇒ 2

Then, improve AE to support identifiers!

- 'with' with arithmetic expressions

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

```
{with {x {+ 1 2}}
      {with {y {- 4 3}}
            {+ x x}}}
```

Then, improve AE to support identifiers!

- 'with' with arithmetic expressions

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

```
{with {x {+ 1 2}}
      {with {y {- 4 3}}
            {+ x x}}}
```

⇒ 6

1. Define type WAE!

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

```
(define-type WAE
    [num (n number?)]
    [add (lhs WAE?) (rhs WAE?)]
    [sub (lhs WAE?) (rhs WAE?)]
    [with (name symbol?) (named-expr WAE?) (body WAE?)]
    [id (name symbol?)])
```

2. Implement a parser for WAE!

```
;; [contract] parse: sexp -> WAE
;; [purpose] to convert s-expression into WAE
(define (parse sexp)
  (cond
        [(number? sexp) (num sexp)]
        [(and (= 3 (length sexp)) (eq? (first sexp) '+))
                                                    (add (parse (second sexp))
(parse (third sexp)))]
        [(and (= 3 (length sexp)) (eq? (first sexp) '-))
                                                    (sub (parse (second sexp))
(parse (third sexp)))]
        [... (with ...)]
        [... (id ...)]
        [else (error 'parse "bad syntax:~a" sexp)]))
```

2. Implement a parser for WAE! (by using match)

```
;; [contract] parse: sexp -> WAE
;; [purpose] to convert s-expression into WAE
(define (parse sexp)
  (match sexp
        [(? number?)                    (num sexp)]
        [(list '+ l r)                  (add (parse l) (parse r))]
        [(list '- l r)                  (sub (parse l) (parse r))]
        [... (with ...)]
        [... (id ...)]
        [else (error 'parse "bad syntax:~a" sexp)]))
```

https://docs.racket-lang.org/reference/match.html

2. Implement a parser for WAE! (by using match)

```
;; [contract] parse: sexp -> WAE
;; [purpose] to convert s-expression into WAE
(define (parse sexp)
  (match sexp
        [(? number?)                    (num sexp)]
        [(list '+ l r)                  (add (parse l) (parse r))]
        [(list '- l r)                  (sub (parse l) (parse r))]
        [... (with ...)]
        [... (id ...)]
        [else (error 'parse "bad syntax:~a" sexp)]))

(test (parse '{+ {- 3 4} 7}) (add (sub (num 3) (num 4)) (num 7)))
(test (parse '{with {x 5} {+ 8 2}}) (with 'x (num 5) (add (num 8) (num 2))))
(test (parse '{with {x 5} {+ x x}}) (with 'x (num 5) (add (id 'x) (id 'x))))
```

https://docs.racket-lang.org/reference/match.html

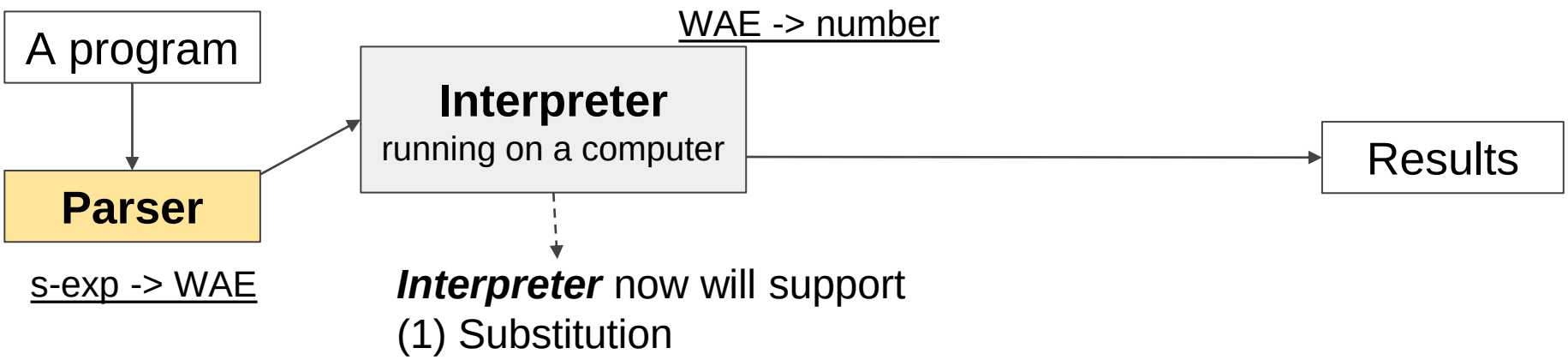
2. Implement a parser for WAE! (by using match)

```
;; [contract] parse: sexp -> WAE
;; [purpose] to convert s-expression into WAE
(define (parse sexp)
  (match sexp
        [(? number?)                          (num sexp)]
        [(list '+ l r)                        (add (parse l) (parse r))]
        [(list '- l r)                        (sub (parse l) (parse r))]
        [(list 'with (list i v) e)      (with i (parse v) (parse e))]
        [(? symbol?)                          (id sexp)]
        [else (error 'parse "bad syntax:~a" sexp)]))

(test (parse '{+ {- 3 4} 7}) (add (sub (num 3) (num 4)) (num 7)))
(test (parse '{with {x 5} {+ 8 2}}) (with 'x (num 5) (add (num 8) (num 2))))
(test (parse '{with {x 5} {+ x x}}) (with 'x (num 5) (add (id 'x) (id 'x))))
```

https://docs.racket-lang.org/reference/match.html

Big Picture (modeling languages: substitution)



2. Implement a parser for WAE! (by using match)

```
;; [contract] parse: sexp -> WAE
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(define (parse sexp)
  (match sexp
        [(? number?)                          (num sexp)]
        [(list '+ l r)                        (add (parse l) (parse r))]
        [(list '- l r)                        (sub (parse l) (parse r))]
        [(list 'with (list i v) e)      (with i (parse v) (parse e))]
        [(? symbol?)                          (id sexp)]
        [else (error 'parse "bad syntax:~a" sexp)]))

(test (parse '{+ {- 3 4} 7}) (add (sub (num 3) (num 4)) (num 7)))
(test (parse '{with {x 5} {+ 8 2}}) (with 'x (num 5) (add (num 8) (num 2))))
(test (parse '{with {x 5} {+ x x}}) (with 'x (num 5) (add (id 'x) (id 'x))))
```

https://docs.racket-lang.org/reference/match.html

Recall this example

{+ {- {+ 5 5} 3} {- {+ 5 5} 3}}

```
{with {x {+ 5 5 }}
        {with {y {- x 3}}
                {+ y y}}}                                  [+ operator]
= {with {x 10} {with {y {- x 3}} {+ y y}}}                 [substitution]
= {with {x 10} {with {y {- 10 3}} {+ y y}}}                [descend*]
= {with {y {- 10 3}} {+ y y}}}                             [- operator]
= {with {y 7} {+ y y}}                                     [substitution]
= {with {y 7} {+ 7 7}}                                     [descend]
={+ 7 7}                                                   [+ operator]
= 14
```

* descend: move to the inner expression to continue calculating.

Before implementing an WAE interpreter, we need to think how to deal with identifiers in the interpreter.

$\Rightarrow$ Substitution

Defining Substitution

- Definition 1 (Substitution)
  - To substitute identifier ***i*** in ***e*** with the expression ***v***, replace all identifiers in e that have the name ***i***, with the expression ***v***.

    ; Our code
    ; ***i***: x
    ; ***v***: 5
    ; ***e***: {+ x x}
    {with {x 5} {+ x x}}

  - Substitution based on Definition 1
    ?

Defining Substitution

- Definition 1 (Substitution)
  - To substitute identifier ***i*** in ***e*** with the expression ***v***, replace all identifiers in e that have the name ***i***, with the expression ***v***.

    ; Our code
    ; ***i***: x
    ; ***v***: 5
    ; ***e***: {+ x x}
    {with {x 5} {+ x x}}

  - Substitution based on Definition 1
    {with {x 5} {+ 5 5}}

Defining Substitution

- How about this code?

  {with {x 5} {+ 10 y}} ;; [substitution]

  to

  {with {x 5} {+ 10 y}}

  * No substitutions occur since there is no instances of x in the expression.

Defining Substitution

- How about this based on Definition 1?

  {with {x 5} {+ x {with {x 3} 10}}} ;; [substitution]

  to

  {with {x 5} {+ 5 {with {5 3} 10}}}

Defining Substitution

- How about this based on Definition 1?
  {with {x 5} {+ x {with {x 3} 10}}} ;; [substitution]

  to

  {with {x 5} {+ 5 {with {5 3} 10}}}

  ???? Syntactically illegal! Our parser will reject this expression!
  Recall WAE in BNF: | {with {<id> <WAE>} <WAE>

Defining Substitution

- So, we need other detailed definitions to make our algorithm for Substitution ***precisely***.
  - Definition 2 (Binding Instance)

    A binding instance of an identifier is the instance of the identifier that gives it its value. In WAE, ***the <id> position of a 'with' is the only binding instance***.

    {with {x 5} {+ x 5}}

Defining Substitution

- So, we need other detailed definitions to make our algorithm for Substitution ***precisely***.
  - Definition 2 (Binding Instance)

    A binding instance of an identifier is the instance of the identifier that gives it its value. In WAE, ***the <id> position of a 'with' is the only binding instance***.
  - Definition 3 (Scope)

    The scope of a binding instance is the region of program text in which instances of the identifier refer to the value bound by the binding instance.

{with {x 5} {+ x x}}

{with {x 5} {+ x {with {x 3} x}}}

Scope of outer x Scope of inner x

Defining Substitution

- So, we need other detailed definitions to make our algorithm for Substitution ***precisely***.
  - Definition 3 (Scope)

    The scope of a binding instance is the region of program text in which instances of the identifier refer to the value bound by the binding instance.
  - Definition 4 (Bound Instance)

    An identifier is bound if it is contained within the scope of a binding instance of its name.

{with {x 5} {+ x x}}

Defining Substitution

- So, we need other detailed definitions to make our algorithm for Substitution ***precisely***.
  - Definition 5 (Free identifier/Instance)

    An identifier not contained in the scope of any binding instance of its name is said to be free.

{with {x 5} {+ x {with {**y** 3} x}}}

free instance as there is no binding instance of it within its scope.

Defining Substitution

- Definition 6 (Substitution, take 2)
  - To substitute identifier ***i*** in ***e*** with the expression ***v***, replace all identifiers in ***e*** which are not binding instances that have the name i with expression ***v***.

    ; Our code
    ; ***i***: x
    ; ***v***: 5
    ; ***e***: {+ x {with {x 3} 10}}
    {with {x 5} {+ x {with {x 3} 10}}}

  - Based on Definition 6

    ?

Defining Substitution

- Definition 6 (Substitution, take 2)
  - To substitute identifier ***i*** in ***e*** with the expression ***v***, replace all identifiers in ***e*** which are not binding instances that have the name i with expression ***v***.

    ; Our code
    ; ***i***: x
    ; ***v***: 5
    ; ***e***: {+ x {with {x 3} 10}}
    {with {x 5} {+ x {with {x 3} 10}}}

  - Based on Definition 6
    {with {x 5} {+ 5 {with {x 3} 10}}}

Defining Substitution

- Definition 6 (Substitution, take 2)
  - To substitute identifier ***i*** in ***e*** with the expression ***v***, replace all identifiers in ***e*** which are not binding instances that have the name ***i*** with expression ***v***.

    How about the following code?

    ; Our code
    ; ***i***: x
    ; ***v***: 5
    ; ***e***: {+ x {with {x 3} x}}
    {with {x 5} {+ x {with {x 3} x}}} ;; the value of the program is 8

  - But….based on Definition 6, the substitution is?

Defining Substitution

- Definition 6 (Substitution, take 2)
  - To substitute identifier ***i*** in ***e*** with the expression ***v***, replace all identifiers in ***e*** which are not binding instances that have the name ***i*** with expression ***v***.

    How about the following code?

    ; Our code
    ; ***i***: x
    ; ***v***: 5
    ; ***e***: {+ x {with {x 3} x}}
    {with {x 5} {+ x {with {x 3} x}}} ;; the value of the program is 8

  - But….based on Definition 6, the substitution is

    {with {x 5} {+ 5 {with {x 3} 5}}} ;; the value is 10

    ????What's wrong with here????

Defining Substitution

- Definition 7 (Substitution, take 3)
  - To substitute identifier ***i*** in ***e*** with the expression ***v***, replace all non-binding identifiers in ***e*** having the name ***i*** with the expression ***v***, unless the identifier is in a scope different from that introduced by ***i***.

```
; Our code
; i: x
; v: 5
; e: {+ x {with {x 3} x}}
{with {x 5} {+ x {with {x 3} x}}}  ;; the value of the program is 8
```

  - But….based on Definition 6, the substitution is

```
{with {x 5} {+ 5 {with {x 3} 3}}} ;; the value is 8
```

Defining Substitution

- Definition 7 (Substitution, take 3)
  - To substitute identifier ***i*** in ***e*** with the expression ***v***, replace all non-binding identifiers in ***e*** having the name ***i*** with the expression ***v***, unless the identifier is in a scope different from that introduced by ***i***.

    How about the following code?

    ; Our code
    ; ***i***: x
    ; ***v***: 5
    ; ***e***: {+ x {with {y 3} x}}
    {with {x 5} {+ x {with {y 3} x}}} ;; the value of the program is 10

  - But….based on Definition 7, the substitution cannot be done (error) because of x has no value.

    {with {x 5} {+ 5 {with {y 3} x}}} ;; error

Defining Substitution (final)

- Definition 8 (Substitution, take 4)
  - To substitute identifier ***i*** in ***e*** with the expression ***v***, <u>replace all bound instances</u> and <u>replace all non-binding identifiers in ***e*** having the name ***i***</u> with the expression ***v***, except within nested scopes of ***i***.

    ; Our code
    ; ***i***: x
    ; ***v***: 5
    ; ***e***: {+ x {with {y 3} x}}
    {with {x 5} {+ x {with {y 3} x}}} ;; the value of the program is 10

  - Based on Definition 8...
    {with {x 5} {+ 5 {with {y 3} 5}}} ;; the value is 10.

Defining Substitution (final)

- Definition 9 (Substitution, take 5)
  - To substitute identifier ***i*** in ***e*** with the expression ***v***, <u>replace all bound instances of ***i***</u> and **replace all free instances of *i* in *e* with *v***.

    ; Our code
    ; ***i***: x
    ; ***v***: 5
    ; ***e***: {+ x {with {y 3} x}}
    {with {x 5} {+ x {with {y 3} x}}} ;; the value of the program is 10

  - Based on Definition 9...
    {with {x 5} {+ 5 {with {y 3} 5}}} ;; the value is 10.

TODO
Read Chapter 3. Substitution

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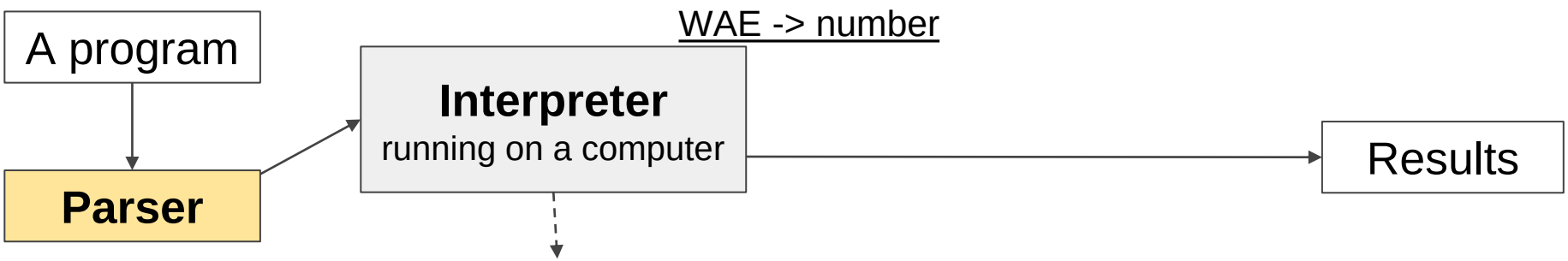
https://lifove.github.io

* Slides are from Prof. Sukyoung Ryu's PL class in 2018 Spring or created by JC based on the main text book.

ITP20005 L7
Substitution (2)

Lecture07
JC

Big Picture (modeling languages: substitution)



Interpreter now will support
(1) **Substitution**

See this example...

- Find Bound/Binding/Free instances and scopes of binding identifiers

{with {x 5} {+ x {with {y x} x}}}

{with {x 5} {+ x {with {x {+ x 1}} x}}}

Defining Substitution (final)

- Definition 9 (Substitution, take 5)
  - To substitute identifier ***i*** in ***e*** with the expression ***v***, <u>replace all bound instances of ***i***</u> and **replace all free instances of *i* in *e* with *v***.

    ; Our code
    ; ***i***: x
    ; ***v***: 5
    ; ***e***: {+ x {with {y x} x}}
    {with {x 5} {+ x {with {y x} x}}} ;; the value of the program is 10

  - Based on Definition 9...
    {with {x 5} {+ 5 {with {y 5} 5}}} ;; the value is 10.

1. Define type WAE!

```
<WAE> ::= <num>
        | {+ <WAE> <WAE>}
        | {- <WAE> <WAE>}
        | {with {<id> <WAE>} <WAE>}
        | <id>
```

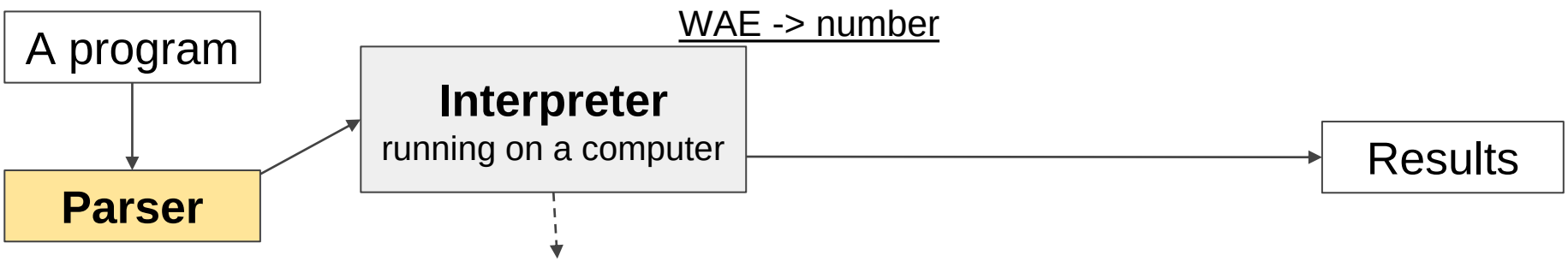
```
(define-type WAE
    [num (n number?)]
    [add (lhs WAE?) (rhs WAE?)]
    [sub (lhs WAE?) (rhs WAE?)]
    [with (name symbol?) (named-expr WAE?) (body WAE?)]
    [id (name symbol?)])
```

2. Implement a parser for WAE! (by using match)

```
;; [contract] parse: sexp -> WAE
;; [purpose] to convert s-expression into WAE
(define (parse sexp)
  (match sexp
        [(? number?)                          (num sexp)]
        [(list '+ l r)                        (add (parse l) (parse r))]
        [(list '- l r)                        (sub (parse l) (parse r))]
        [(list 'with (list i v) e)      (with i (parse v) (parse e))]
        [(? symbol?)                          (id sexp)]
        [else (error 'parse "bad syntax:~a" sexp)]))

(test (parse '{+ {- 3 4} 7}) (add (sub (num 3) (num 4)) (num 7)))
(test (parse '{with {x 5} {+ 8 2}}) (with 'x (num 5) (add (num 8) (num 2))))
(test (parse '{with {x 5} {+ x x}}) (with 'x (num 5) (add (id 'x) (id 'x))))
```

https://docs.racket-lang.org/reference/match.html

Big Picture (modeling languages: substitution)



Interpreter now will support
(1) **Substitution**

WAE and its interpretation

(with 'x (num 5) (add (id 'x) (id 'x)) [Substitution]
(with 'x (num 5) (add (num 5) (num 5)) [Descend]
(add (num 5) (num 5))
[add operator]
10 [Result]

⇒ We need an interpreter function as well as a substitution function.

* descend: move to the inner expression to continue calculating.

3. Implement substitution for WAE Interpreter

```
                          e        i        v
                          ↓        ↓        ↓
; [contract] subst: WAE symbol number -> WAE
; (here, symbol is an identifier and number is the value for the identifier)
; [purpose] to substitute second argument with third argument in first argument,
; as per the rules of substitution; the resulting expression contains
; no free instances of the second argument
(define (subst wae i val)
...)

; [tests]
; {with {x 10} 5}                ⇒    10 for x in 5          ⇒ 5
(test (subst (num 5) 'x 10)   (num 5))
; {with {x 10} {+ 1 x}}      ⇒    10 for x in {+ 1 x}      ⇒ {+ 1 10}
(test (subst (add (num 1) (id 'x)) 'x 10)   (add (num 1) (num 10)))
; {with {x 10} x}            ⇒    10 for x in x                  ⇒ 10
(test (subst (id 'x) 'x 10)   (num 10))
; {with {x 10} y}            ⇒                 10 for x in y                ⇒   y (no
substitution)
(test (subst (id 'y) 'x 10)   (id 'y))
```

3. Implement substitution for WAE Interpreter

; [contract] subst: WAE symbol number -> WAE
; ...

; [tests]
; {with {x 10} 5} ⇒ 10 for x in 5 ⇒ 5
(test (subst (num 5) 'x 10) (num 5))
; {with {x 10} {+ 1 x}} ⇒ 10 for x in {+ 1 x} ⇒ {+ 1 10}
(test (subst (add (num 1) (id 'x)) 'x 10) (add (num 1) (num 10)))
; {with {x 10} x} ⇒ 10 for x in x ⇒ 10
(test (subst (id 'x) 'x 10) (num 10))
; {with {x 10} y} ⇒ 10 for x in y ⇒ y (no substitution)
(test (subst (id 'y) 'x 10) (id 'y))
; {with {y 10} {- x 1}} ⇒ 10 for y in {- x 1} => {- x 1} (no substitution)
(test (subst (sub (id 'x) (num 1)) 'y 10) (sub (id 'x) (num 1)))

3. Implement substitution for WAE Interpreter

```
; [contract] subst: WAE symbol number -> WAE
; [...]
(define (subst wae id val)
        (type-case WAE wae
                [num    (n)             wae]
                [add    (l r)           ...]
                [sub            (l r)           ...]
                [with   (i v e)         ...]
                [id             (s)             ...]))

; [tests]
; {with {x 10} 5}                 ⇒   10 for x in 5       ⇒ 5
(test (subst (num 5) 'x 10)  (num 5))
; {with {x 10} {+ 1 x}}     ⇒   10 for x in {+ 1 x}  ⇒  {+ 1 10}
(test (subst (add (num 1) (id 'x)) 'x 10)  (add (num 1) (num 10)))
; {with {x 10} x}           ⇒   10 for x in x                ⇒ 10
(test (subst (id 'x) 'x 10)  (num 10))
; {with {x 10} y}           ⇒           10 for x in y                    ⇒  y (no substitution)
(test (subst (id 'y) 'x 10)  (id 'y))
; {with {y 10} {- x 1}}                ⇒           10 for y in {- x 1}      => {- x 1} (no substitution)
(test (subst (sub (id 'x) (num 1)) 'y 10)   (sub (id 'x) (num 1)))
```

3. Implement substitution for WAE Interpreter

```
; [contract] subst: WAE symbol number -> WAE
(define (subst wae idtf val)
        (type-case WAE wae
                [num    (n)             wae]
                [add        (l r)       (add (subst l idtf val) (subst r idtf val))]
                [sub            (l r)           (sub (subst l idtf val) (subst r
idtf val))]
                [with   (i v e)     ...]
                [id             (s)             ...]))
```

```
; [tests]
; {with {x 10} {+ 1 x}}     ⇒     10 for x in {+ 1 x}  ⇒  {+ 1 10}
(test (subst (add (num 1) (id 'x)) 'x 10)  (add (num 1) (num 10)))
; {with {x 10} x}           ⇒     10 for x in x                ⇒ 10
(test (subst (id 'x) 'x 10)  (num 10))
; {with {x 10} y}           ⇒         10 for x in y                    ⇒  y (no substitution)
(test (subst (id 'y) 'x 10)  (id 'y))
; {with {y 10} {- x 1}}            ⇒            10 for y in {- x 1}      => {- x 1} (no substitution)
(test (subst (sub (id 'x) (num 1)) 'y 10)   (sub (id 'x) (num 1)))
```

3. Implement substitution for WAE Interpreter

```
; [contract] subst: WAE symbol number -> WAE
(define (subst wae idtf val)
        (type-case WAE wae
                [num    (n)             wae]
                [add    (l r)           (add (subst l idtf val) (subst r idtf
val))]
                [sub            (l r)           (sub (subst l idtf val) (subst r
idtf val))]
                [with   (i v e)  ...]
                [id             (s)             (if (symbol=? s idtf) (num val)
wae)]))

; [tests]
; {with {x 10} {+ 1 x}}      ⇒     10 for x in {+ 1 x}  ⇒  {+ 1 10}
(test (subst (add (num 1) (id 'x))  'x 10)   (add (num 1) (num 10)))
; {with {x 10} x}            ⇒     10 for x in x                ⇒ 10
(test (subst (id 'x)  'x 10)   (num 10))
; {with {x 10} y}            ⇒             10 for x in y                ⇒   y (no substitution)
(test (subst (id 'y)  'x 10)   (id 'y))
```

3. Implement substitution for WAE Interpreter

```
; [contract] subst: WAE symbol number -> WAE
(define (subst wae idtf val)
        (type-case WAE wae
                [num    (n)             wae]
                [add    (l r)           (add (subst l idtf val) (subst r idtf
val))]
                [sub            (l r)           (sub (subst l idtf val) (subst r
idtf val))]
                [with   (i v e)  ...]
                [id             (s)             (if (symbol=? s idtf) (num val)
wae)]))

; {with {x 10} {...{with {y 17} x}} ⇒ 10 for x in {with {y 17} x}    ⇒  {with {y 17} 10}
(test (subst (with 'y (num 17) (id 'x)) 'x 10) (with 'y (num 17) (num 10)))
; {with {x 10} {...{with {y x} y}}}  ⇒ 10 for x in {with {y x} y}     ⇒  {with {y 10} y}
(test (subst (with 'y (id 'x) (id 'y)) 'x 10) (with 'y (num 10) (id 'y)))
; {with {x 10} {...{with {x y} x}}}  ⇒ 10 for x in {with {x y} x}     ⇒ {with {x y} x}
(test (subst (with 'x (id 'y) (id 'x)) 'x 10) (with 'x (id 'y) (id 'x)))
```

3. Implement substitution for WAE Interpreter

```
; [contract] subst: WAE symbol number -> WAE
(define (subst wae idtf val)
          (type
                                                                                st r idtf
val))]
                                                                                val) (subst r
idtf val))]
                    [with      (r v e)      ...]
                    [id                     (s)                  (if (symbol=? s idtf) (num val)
wae)]))

; {with {x 10} {...{with {y 17} x}} ⇒ 10 for x in {with {y 17} x}      ⇒  {with {y 17} 10}
(test (subst (with 'y (num 17) (id 'x)) 'x 10) (with 'y (num 17) (num 10)))
; {with {x 10} {...{with {y x} y}}}  ⇒ 10 for x in {with {y x} y}       ⇒  {with {y 10} y}
(test (subst (with 'y (id 'x) (id 'y)) 'x 10) (with 'y (num 10) (id 'y)))
; {with {x 10} {...{with {x y} x}}}  ⇒ 10 for x in {with {x y} x}       ⇒ {with {x y} x}
(test (subst (with 'x (id 'y) (id 'x)) 'x 10) (with 'x (id 'y) (id 'x)))
```

One subst function call conducts substitutions for one binding identifier!

3. Implement substitution for WAE Interpreter

```
; [contract] subst: WAE symbol number -> WAE
(define (subst wae idtf val)
        (type-case WAE wae
                [num    (n)             wae]
                [add    (l r)           (add (subst l idtf val) (subst r idtf
val))]
                [sub            (l r)           (sub (subst l idtf val) (subst r
idtf val))]
                [with   (i v e)  (with i (subst v idtf val) (if (symbol=? i idtf) e

        (subst e idtf val)))]
                [id             (s)             (if (symbol=? s idtf) (num val)
wae)]))

; {with {x 10} {...{with {y 17} x}} ⇒ 10 for x in {with {y 17} x}    ⇒  {with {y 17} 10}
(test (subst (with 'y (num 17) (id 'x)) 'x 10) (with 'y (num 17) (num 10)))
; {with {x 10} {...{with {y x} y}}}  ⇒ 10 for x in {with {y x} y}      ⇒  {with {y 10} y}
(test (subst (with 'y (id 'x) (id 'y)) 'x 10) (with 'y (num 10) (id 'y)))
```

3. Implement substitution for WAE Interpreter

```
; [contract] subst: WAE symbol number -> WAE
(define (subst wae idtf val)
        (type-case WAE wae
                [num    (n)             wae]
                [add    (l r)           (add (subst l idtf val) (subst r idtf
val))]
                [sub            (l r)           (sub (subst l idtf val) (subst r
idtf val))]
                [with   (i v e)  (with i (subst v idtf val) (if (symbol=? i idtf) e

        (subst e idtf val)))]
                [id             (s)             (if (symbol=? s idtf) (num val)
wae)]))
```

4. Implement WAE interpreter

```
; interp: WAE -> number
(define (interp wae)
        (type-case WAE wae
                [num (n) n]
                [add (l r) (+ (interp l) (interp r))]
                [sub (l r) (- (interp l) (interp r))]))
                ...
                ... ))
```

4. Implement WAE interpreter

```
; interp: WAE -> number
(define (interp wae)
        (type-case WAE wae
                [num (n) n]
                [add (l r) (+ (interp l) (interp r))]
                [sub (l r) (- (interp l) (interp r))]))
                [with (i v e) …]
                [id (s) …]))
```

4. Implement WAE interpreter

```
; interp: WAE -> number
(define (interp wae)
        (type-case WAE wae
                [num (n) n]
                [add (l r) (+ (interp l) (interp r))]
                [sub (l r) (- (interp l) (interp r))]))
                [with (i v e) … ]
                [id (s)          (error 'interp "free identifier")]))
```

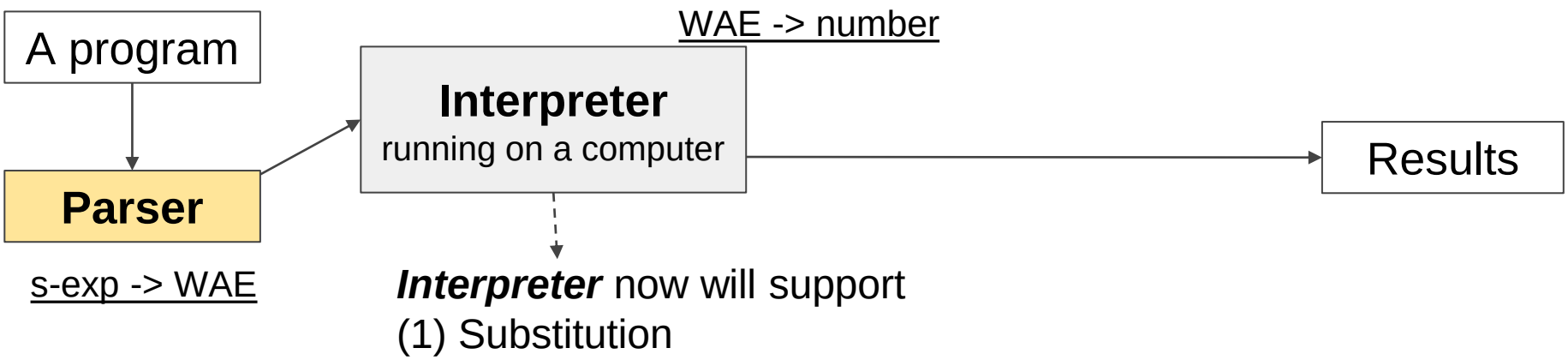
4. Implement WAE interpreter

```
; interp: WAE -> number
(define (interp wae)
        (type-case WAE wae
                [num (n) n]
                [add (l r) (+ (interp l) (interp r))]
                [sub (l r) (- (interp l) (interp r))]))
                [with (i v e) … (interp v) … (interp e) ...]
                [id (s)          (error 'interp "free identifier")]))


(test (interp (with 'x (num 5) (add (id 'x) (id 'x)))) 10)
```

4. Implement WAE interpreter

```
; interp: WAE -> number
(define (interp wae)
        (type-case WAE wae
                [num (n) n]
                [add (l r) (+ (interp l) (interp r))]
                [sub (l r) (- (interp l) (interp r))]
                [with (i v e) (interp (subst e i (interp v)))]
                [id (s)          (error 'interp "free identifier")]))


(test (interp (with 'x (num 5) (add (id 'x) (id 'x)))) 10)
```

Big Picture (modeling languages: substitution)



Topics we cover and schedule (tentative)

- Racket tutorials (L2,3)
- Modeling languages (L4)
- Interpreting arithmetic (L5)
- Language principles
  - **Substitution** (L6,7)
  - Function (L8,9)
  - Deferring Substitution (L9,10)
  - First-class Functions (L10,11)
  - Laziness (L11,12)
  - Recursion (L13,14)
  - Representation choices (L15)
  - Mutable data structures (L16)
  - Variables (L17)
  - Continuations (L18,19,20,21)
  - Garbage collection (L22)
  - Semantics (L23,24)
  - Type (L25,26,27)
- Guest Video Lecture (L28)

No class: October 2 (Fri, Chuseok), October 9 (Fri, Hangul day)
Online only class can be provided.

TODO

Read Chapter 4. An Introduction to Functions

JC

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* Slides are from Prof. Sukyoung Ryu's PL class in 2018 Spring or created by JC based on the main text book.