Semantic Analysis

The role of semantic analysis is to assign meaning

- "It smells fishy."
- Lexical analysis
 - > Tokenizes "It", "smells", "fishy", "."
 - Determines noun, verb, adjective, punctuation token types
- Syntax analysis
 - > Parses the grammatical structure of the sentence
- Semantic analysis

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 - > Tokenizes "It", "smells", "fishy", "."
 - Determines noun, verb, adjective, punctuation token types
- Syntax analysis
 - > Parses the grammatical structure of the sentence
- Semantic analysis
 - Assigns meaning to the words "It", "smells", "fishy"
 - > Flags error if the sentence does not make sense

Semantic Analysis = Binding + Type Inference

- "I don't wanna eat that sushi."
 - "It smells fishy."
 - > "It": the sushi
 - > "smells": feels to my nose
 - "fishy": that the sushi has gone bad
- "The professor says that the exam is going to be easy."
 - "It smells fishy."
 - "It": the situation
 - "smells": feels to my sixth sense
 - "fishy": that it is highly suspicious

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 - > "It": the situation
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 - > "fishy": that it is highly suspicious
- Semantic analysis consists of two tasks
 - > Binding: associating a pronoun to an object
 - > Type checking: inferring meaning based on type of object

Semantic analysis cannot be done during parsing

- Context Free Grammars (CFGs) cannot recognize bindings
 - Every use of a name needs to be bound to the declaration.
 - Name can refer to a variable, function, class, ...
 - Names are called symbols in semantic analysis
- To do bindings, a CFG must recognize this language:

$$\{\alpha \mathbf{c}\alpha | \alpha \in (\mathbf{a}|\mathbf{b})^*\}$$

The 1st α represents the declaration, The 2nd α represents a use.

Above language is a Context Sensitive Language

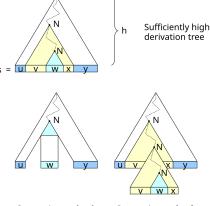
Why is $\{\alpha c\alpha | \alpha \in (a|b)^*\}$ not a CFG?

- We will base our proof on the **pumping lemma** for CFGs.
- Pumping lemma: a theorem about strings in a grammar
 - > "lemma": a mathematical term for a theorem
 - "pumping": for a sufficiently long enough string, a substring exists within that string that can be "pumped" (repeated 0 or more times and still be in the language).
- For example, for the Regular Language 0(0|1)*0:
 - ➤ A string longer than 2 will look like 000, 010, 0100, ...
 - ➤ Let's take "010". Here, substring "1" can be pumped.
 - > ("00", "010", "0110", "01110" are all in the language)
- Pumping Lemma applies to CFGs as well.

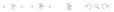
Pumping Lemma for CFGs

For a sufficiently long string s derived from a CFG, s can be written as s = uvwxy (u,v,w,x,y are substrings)

Where v and x can be pumped and |vx| > 1.



Generating uv wx y



$\{\alpha \boldsymbol{c}\alpha | \alpha \in (\boldsymbol{a}|\boldsymbol{b})^*\}$ is not a CFG

- Let's say s = uvwxy is a sufficiently long string in language $\{\alpha c\alpha | \alpha \in (a|b)^*\},$
 - where v, x can be pumped and $|vx| \ge 1$.
 - 1. The substring vwx must bisect $\alpha c\alpha$.
 - If vwx is contained in 1st α (or mostly contained), if we pump v and x 0 times, 1st α gets shorter than 2nd α .
 - ightharpoonup string is no longer in $\{\alpha c\alpha | \alpha \in (a|b)^*\}$. Contradiction.
 - ightharpoonup The same applies to when vwx is contained in 2nd α .
- 2. Even when vwx bisects $\alpha c\alpha$, pumping fails.
 - Let string s' be the result of pumping v and x 0 times.
 - Let's say s' = $\alpha_1 c \alpha_2$, where α_1 and α_2 are shortened versions of the 1st and 2nd α s.
 - ightharpoonup While $|\alpha_1|=|\alpha_2|$, there exist α_1 and α_2 such that $\alpha_1!=\alpha_2$.
 - ightharpoonup E.g. s = abcab, and vwx = bca where v = b and x = a. Then, α_1 becomes "a" and α_2 becomes "b". Contradiction.

Semantic analysis does binding and type checking

- Semantic analysis performs binding
 - > Since CFGs cannot recognize bindings, as we just proved
 - Done by traversing parse tree produced by syntax analysis
 - Definitions are stored in data structure called symbol table
 - Uses are bound to entries in the symbol table
- Semantic analysis performs type checking
 - ightharpoonup Infer what "a + b" means:
 - If a and b are ints, integer add and return int
 - If a and b are floats, FP add and return float
 - If a and b are strings, concatenate and return string
 - Infer what "a.foo()" means:
 - If object a is an instance of class A, call A.foo()
 - If object a is an instance of class B, call B.foo()
 - \rightarrow Infer what "a[i][j]" means:
 - Offset from a calculated based on type and dimensions

Semantic analysis also performs semantic checks

All symbol uses have a corresponding declaration;
All operations are type legal;
Inheritance relationships are correct;
A class is defined only once;
A method in a class is defined only once;

Symbol Binding

What is symbol binding?

"Matching symbol declarations with uses"

If there are multiple declarations, which one is matched?

What is symbol binding?

"Matching symbol declarations with uses"

☐ If there are multiple declarations, which one is matched?

```
void foo()
{
    char x;
    ...
    {
        int x;
        ...
    }
    x = x + 1;
}
```

What is symbol binding?

"Matching symbol declarations with uses"

☐ If there are multiple declarations, which one is matched?

```
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    ...
    {
        int x;
}
    x = x + 1;
}
```

Scope

| Ļ | Binding: the association of a use of a symbol to the | е |
|---|--|---|
| | declaration of that symbol | |

- Which variable (or function) an identifier is referring to
- Scope: section of program where a declaration is valid
 - Uses in the scope of declaration are bound to it
- Some implications of scopes
 - A symbol may have different bindings in different scopes
 - Scopes for the same symbol never overlap
 - there is always exactly one binding per symbol use
- Two types: static scope and dynamic scope

Static Scope

Static scope depends on the program text, not run-time behavior (also known as lexical scoping)

```
C/C++, Java, Objective-C
```

Rule: Refer to the closest enclosing declaration

```
void foo()
   char x;
      int x;
      ...
   x = x + 1;
```

Static Scope

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 - ➤ C/C++, Java, Objective-C
- Rule: Refer to the closest enclosing declaration

```
void foo()
{
    char x;
...
    int x;
...
}
x = x + 1;
```

Dynamic Scope

- Dynamic scoping depends on bindings formed during the execution of the program
 - > LISP, Scheme, Perl
- Rule: Refer to the closest binding in the current execution

```
void foo()
{
    (1) char x;
    (2) if (...) {
     (3) int x;
    (4) ...
     }
    (5) x = x + 1;
}
```

Dynamic Scope

- Dynamic scoping depends on bindings formed during the execution of the program
 - LISP, Scheme, Perl
- Rule: Refer to the closest binding in the current execution

```
void foo()
{
    (1) char x;
    (2) if (...) {
     (3) int x;
    (4) ...
    }
    (5) x = x + 1;
}
```

- \square Which x's declaration is the closest?
 - > Execution (a): ...(1)...(2)...(5)
 - > Execution (b): ...(1)...(2)...(3)...(4)...(5)

Static vs. Dynamic Scoping

- Most languages that started with dynamic scoping (LISP, Scheme, Perl) added static scoping afterwards
- **₩** Why?
 - It is easier for human beings to understand
 - Bindings are visible in code without tracing execution
 - It is easier for compilers to understand
 - Compiler can determine bindings at compile time
 - Compiler can translate identifier to a single memory location
 - Results in generation of efficient code
 - With dynamic scoping...
 - There may be multiple possible bindings for a variable
 - Impossible to determine bindings at compile time
 - All bindings have to be done at execution time (Typically with the help of a hash table)

Symbol Table

Symbol Table

- Symbol Table: A compiler data structure that tracks information about all identifiers (symbols) in a program
 - Maps symbol uses to declarations given a scope
 - Needs to provide bindings according to the current scope
- Usually discarded after generating the binary code
 - All symbols are mapped to memory locations already
 - > For debugging, symbols may be included in binary
 - To map memory locations back to symbols for debuggers
 - For GCC or Clang, add "-g" flag to include symbol tables

Maintaining Symbol Table

find symbol(x)

add symbol(x)

```
    ■ Basic idea:
        int x; ... void foo() { int x; ... x=x+1; } ... x=x+1 ...
        In foo, add x to table, overriding any previous declarations
        After foo, remove x and restore old declaration if any

    ■ Operations
        enter_scope() start a new nested scope
        exit_scope() exit current scope
```

find declaration of x

add declaration of x to symbol table

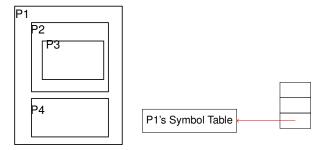
Adding Scope Information to the Symbol Table

- To handle multiple scopes in a program,
 - > (Conceptually) need an individual table for each scope
 - Symbols added to the table may not be deleted just because you exited a scope

```
class X { ... void f1() {...} ... } class Y { ... void f2() {...} ... } X v; call v.f1();
```

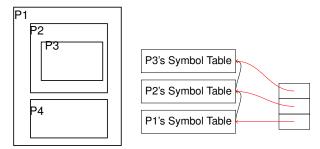
Without deleting symbols, how are scoping rules enforced?
 Keep a list of all scopes in the entire program
 Keep a stack of active scopes at a given point

Symbol Table with Multiple Scopes



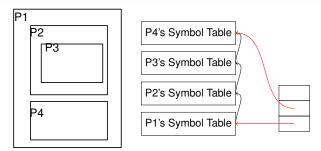
- For nested scopes,
 - Search from top of the active symbol table stack
 - Remove pointer to symbol table when exiting its scope

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Symbol Table with Multiple Scopes



- For nested scopes,
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 - Remove pointer to symbol table when exiting its scope

What Information is Stored in the Symbol Table

Entry in Symbol Table:

string kind attributes

- String the name of identifier
- Kind variable, parameter, function, class, ...
- Attributes vary with the kind of symbol
 - ➤ variable → type, address in memory
- Vary with the language
 - Fortran's array → type, dimension, dimension size real A(5) /* dimension required for static allocation */
 - C's array → type, dimension, optional dimension size char A[5]; /* statically sized array */ char A[]="hello"; /* dynamically sized to fit content */

Symbol Table Attribute List

```
Type information might be arbitrarily complicated
     ➤ In C:
                  struct {
                         int a[10];
                         char b;
                         float c;
    Store all relevant attributes in an attribute list
                      1st upper bound
                                                  2nd upper bound
      array
 id
                      field₁
                            type
                                                  field<sub>2</sub> | type
 id
                                  size
                                                              size
      struct
```

Example application of Type to an operator: Array index operator

A[high]

A[i]

Addressing Array Elements

base=A[0]

```
int A[0..high];
A[i] ++;
```

- > width width of element type
- base address of the first
- > high upper bound of subscript
- Addressing an array element:

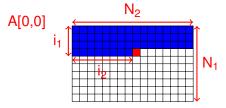
```
address(A[i]) = base + i * width
offset(A[i]) = i * width
```

Multi-dimensional Arrays

Layout n-dimension items in 1-dimension memory int A[N₁][N₂]; /* int A[0..high₁][0..high₂]; */ $A[i_1][i_2] ++;$ N_2 A[0,0] N_1 A[high₁,high₂]

Row Major

Row major — store row by row

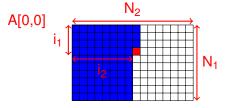


 \Box Offset inclues all the "blue" items before A[i₁,i₂]

$$\begin{array}{l} \text{offset}(A[i_1,i_2]) = (i_1 \ ^*\ N_2 + i_2\)\ ^*\ \text{width} \\ = i_1 \ ^*\ N_2 \ ^*\ \text{width} \ + i_2 \ ^*\ \text{width} \\ = \text{offset}(A[i_1])\ ^*\ N_2 \ + i_2 \ ^*\ \text{width} \\ \end{array}$$

Column Major

Column major — store column by column



 \Box Offset inclues all the "blue" items before A[i₁,i₂]

offset(A[i₁,i₂]) =
$$(i_2 * N_1 + i_1)*$$
width
= $i_2 * N_1 *$ width + $i_1 *$ width
= $i_2 * N_1 *$ width + offset(A[i₁])

Generalized Row/Column Major

```
Let A_k = \text{offset}(A[i_1, i_2, ..., i_k]). Then,
```

Row major

1-dimension: $A_1 = i_1*width$

2-dimension: $A_2 = (i_1 * N_2 + i_2) * width = A_1 * N_2 + i_2 * width$

3-dimension: $A_3 = (i_1 * N_2 * N_3 + i_2 * N_3 + i_3) * width = A_2 * N_3 + i_3 * width$

k-dimension: $A_k = A_{k-1} N_k + i_k \text{width}$

Type needs to provide $N_2...N_k$ and width for offset

Column major

1-dimension: $A_1 = i_1*width$

2-dimension: $A_2 = (i_2 * N_1 + i_1) * width = i_2 * N_1 * width + A_1$

3-dimension: $A_3 = ((i_3*N_2+i_2)*N_1+i_1)*width = i_3*N_2*N_1*width + A_2$

k-dimension: $A_k = i_k * N_{k-1} * N_{k-2} * ... * N_1 * width + A_{k-1}$

Type needs to provide $N_1...N_{k-1}$ and width for offset

C's implementation

C uses row major
 int fun1(int p[][100])
{
 ...
 int a[100][100];
 a[i₁][i₂] = p[i₁][i₂] + 1;
}

Why is p[][100] allowed?

Why is a[][100] not allowed?

C's implementation

C uses row major

```
int fun1(int p[][100]) { ... int a[100][100]; a[i_1][i_2] = p[i_1][i_2] + 1; }
```

Why is p[][100] allowed?

- \rightarrow The info is enough to compute p[i₁][i₂]'s address
- \rightarrow A₂ = (i₁*N₂+i₂)*width (N₁ is not required)

Why is a[][100] not allowed?

> The info is not enough to allocate space for the array

Type Checking

What, Why and When

- What is a type?

 Type = a set of values + a set of operations on these values
- What is type checking? Verifying and enforcing type consistency
 - Only legal values are assigned to a type
 - Only legal operations are performed on a type
- Why is compile-time type checking desirable?
 - Type errors easier to debug than malfunctioning programs
 - Dynamic type checking when static checking infeasible
 - E.g. Java null checks and array bounds checks
 - E.g. C++/Java downcasting to a subclass

Static vs. Dynamic Typing

- - > Types are explicitly declared or can be inferred from code
 - ➤ E.g. int x; /* type of x is int */
 - > Efficient code since runtime type checks are not needed
- Dynamically typed: Python, JavaScript, PHP
 - > Type is a runtime property decided only during execution
 - E.g. var x; /* type of x is undecided */
 - Type of x changes depending on the type of value it holds
 - More memory since every variable now needs a "type tag"
 - Inefficient code due to runtime checks on type tags

Rules of Inference

- What are rules of inference?
 - ➤ Inference rules have the form if Precondition is true, then Conclusion is true
 - Below concise notation used to express above statement

Precondition Conclusion

- ➤ In the context of type checking: if expressions E1, E2 have certain types (Precondition), expression E3 is legal and has a certain type (Conclusion)
- Type checking via inference
 - Start from variable types and constant types
 - > Repeatedly apply rules until entire program is inferred legal

Notation for Inference Rules

By tradition inference rules are written as

Precondition₁, ..., Precondition_n Conclusion

- The precondition/conclusion has the form "e:T"
- Meaning
 - If Precondition₁ and ... and Preconditionn are true, then Conclusion is true.
 - > "e:T" indicates "e is of type T"
 - Example: rule-of-inference for add operation

```
e<sub>1</sub>: int
e<sub>2</sub>: int
e<sub>1</sub>+e<sub>2</sub>:int
```

Rule: If e_1 , e_2 are ints then e_1+e_2 is legal and is an int

Two Simple Rules

 $[Add \ operation] \begin{tabular}{ll} i \ is \ an \ integer \\ \hline i: int \\ \hline e_1: int \\ \hline e_2: int \\ \hline e_1+e_2: int \\ \hline \end{tabular}$

Example: given "10 is an integer" and "20 is an integer", is the expression "10+20" legal? Then, what is the type?

10 is an integer 20 is an integer 10: int 20: int

10+20:int

This type of reasoning can be applied to the entire program

More Rules

```
[New]

new T: T

[Not]

e: Boolean

not e: Boolean

However,

[Var?]

x is an identifier

x: ?
```

- > the expression itself insufficient to determine type
- > solution: provide context for this expression

Type Environment

- A type environment gives type info for free variables
 - > A variable is *free* if not declared inside the expression
 - ➤ It is a function mapping Symbols to Types
 - Set of declarations active at the current scope
 - Conceptual representation of a symbol table

Type Environment Notation

Let O be a function from Symbols to Types, the sentence O e:T

is read as "under the assumption of environment O, expression e has type T"

- "if i is an integer, expression i is an int in any environment"
- "if e1 and e2 are ints in O, expression e1+e2 is int in O"
- "if variable x is mapped to int in O, expression x is int in O"

Declaration Rule

[Declaration w/o initialization]

O[
$$T_0/x$$
] e_1 : T_1
O let x: T_0 in e_1 : T_1

 $O[T_0/x]$ means, O is modified to return T_0 on argument x and behaves as O on all other arguments

$$O[T_0/x](x) = T_0$$

 $O[T_0/x](y) = O(y)$ when $x \neq y$

Translation: "If expression e₁ is type T₁ when x is mapped to type T₀ in the current environment, expression e₁ is type T₁ when x is declared to be T₀ in the current environment"

Declaration Rule with Initialization

[Declaration with initialization (initial try)]

```
\begin{array}{c} \textbf{O} \ \textbf{e}_0 \colon \textbf{T}_0 \\ \textbf{O}[\textbf{T}_0/\textbf{x}] \ \textbf{e}_1 \colon \textbf{T}_1 \\ \textbf{O} \ \textbf{let} \ \textbf{x} \colon \textbf{T}_0 \leftarrow \textbf{e}_0 \ \textbf{in} \ \textbf{e}_1 \colon \textbf{T}_1 \end{array}
```

The rule is too strict (i.e. correct but not complete)

```
Example
class C inherits P ...
let x:P ← new C in ...
```

the above rule does not allow this code

Subtyping

- Subtyping is a relation ≤ on classes
 - > X ≤ X
 - ightharpoonup if X inherits from Y, then X \leq Y
 - ightharpoonup if $X \leq Y$ and $Y \leq Z$, then $X \leq Z$
- An improvement of our previous rule

[Declaration with initialization]

$$\label{eq:controller} \begin{array}{c} \textbf{O} \; \textbf{e}_0 \colon \textbf{T} \\ \textbf{T} \leq \textbf{T}_0 \\ \textbf{O}[\textbf{T}_0/\textbf{x}] \; \textbf{e}_1 \colon \textbf{T}_1 \\ \textbf{O} \; \textbf{let} \; \textbf{x} \colon \textbf{T}_0 \leftarrow \textbf{e}_0 \; \textbf{in} \; \textbf{e}_1 \colon \textbf{T}_1 \end{array}$$

- Both versions of declaration rules are correct
- > The improved version checks more programs

Assignment

A correct but too strict rule

```
[Assignment] \\ \textbf{O(id)} = \textbf{T}_0 \\ \textbf{O e_1: T}_1 \\ \textbf{T}_1 \leq \textbf{T}_0 \\ \textbf{O id} \leftarrow \textbf{e_1: T}_0
```

The rule does not allow the below code class C inherits P { only_in_C() { ... } } x ← y ← new C x.only in C()

Assignment

An improved rule

```
[Assignment] \\ \textbf{O(id)} = \textbf{T}_0 \\ \textbf{O e_1: T}_1 \\ \textbf{T}_1 \leq \textbf{T}_0 \\ \textbf{O id} \leftarrow \textbf{e_1: T}_1
```

The rule now does allow the below code class C inherits P { only_in_C() { ... } } $x \leftarrow y \leftarrow \text{new C}$ x.only_in_C()

If-then-else

- Consider
 - if e₀ then e₁ else e₂
 - The result can be either e₁ or e₂
 - The type is either e₁'s type or e₂'s type
 - The best that we can do (statically) is the super type larger than e₁'s type and e₂'s type
- Least upper bound (LUB)
 - Z = lub(X,Y) Z is defined as the least upper bound of X and Y iff
 - $X \le Z \land Y \le Z$; Z is an upper bound
 - $\bullet \ \ X{\le}W \land Y{\le}W \Longrightarrow Z{\le}W \ \ ; Z \ \text{is least among all upper bounds}$

If-then-else, case

```
[If-then-else]
                             O e<sub>0</sub>: Bool
                              O e<sub>1</sub>: T<sub>1</sub>
                              O e<sub>2</sub>: T<sub>2</sub>
            O if e_0 then e_1 else e_2 fi: lub(T_1,T_2)
The rule allows the below code
        let x:float, y:int, z:float in
        x \leftarrow if (...) then y else z
        /* Assuming lub(int, float) = float */
```

Error Recovery

- Just like other errors, we should recover from type errors
 - ➤ Too many errors? let y: int ← x+2 in y+3
 - if x is undefined —- reporting an error "x type undefined"
 - x+2 is undefined reporting an error "x+2 type undefined"
 - ...
- Introduce no-type for ill-typed expressions
 - > It is compatible with all types
 - > Report the place where no-type is generated
 - Reduce the number of error messages

Wrong Declaration Rule (case 1)

Consider a hypothetical let rule

[Wrong Declaration with initialization (case 1)]

```
\label{eq:controller} \begin{array}{c} \textbf{O} \; \textbf{e}_0 \colon \textbf{T} \\ \textbf{T} \leq \textbf{T}_0 \\ \textbf{O} \; \textbf{e}_1 \colon \textbf{T}_1 \\ \textbf{O} \; \textbf{let} \; \textbf{x} \colon \textbf{T}_0 \leftarrow \textbf{e}_0 \; \textbf{in} \; \textbf{e}_1 \colon \textbf{T}_1 \end{array}
```

- > How is it different from the the correct rule?
- The following program does not pass check let x: int ← 0 in x+1

Wrong Declaration Rule (case 2)

Consider a hypothetical let rule

[Wrong Declaration with initialization (case 2)]

```
\label{eq:controller} \begin{array}{c} \textbf{O} \; \textbf{e}_0 \colon \textbf{T} \\ \textbf{T}_0 \leq \textbf{T} \\ \textbf{O}[\textbf{T}_0/\textbf{x}] \; \textbf{e}_1 \colon \textbf{T}_1 \\ \textbf{O} \; \overline{\textbf{let} \; \textbf{x} \colon \textbf{T}_0 \leftarrow \textbf{e}_0 \; \text{in} \; \textbf{e}_1 \colon \textbf{T}_1} \end{array}
```

- How is it different from the the correct rule?
- The following bad program passes the check class B inherits A { only_in_B() { ... } } let x: B ← new A in x.only_in_B()

Wrong Declaration Rule (case 3)

Consider a hypothetical let rule

[Wrong Declaration with initialization (case 3)]

```
\label{eq:controller} \begin{array}{c} \textbf{O} \; \textbf{e}_0 \colon \textbf{T} \\ \textbf{T} \leq \textbf{T}_0 \\ \textbf{O}[\textbf{T}/\textbf{x}] \; \textbf{e}_1 \colon \textbf{T}_1 \\ \textbf{O} \; \textbf{let} \; \textbf{x} \colon \textbf{T}_0 \leftarrow \textbf{e}_0 \; \textbf{in} \; \textbf{e}_1 \colon \textbf{T}_1 \end{array}
```

- How is it different from the the correct rule?
- The following bad program passes the check class B inherits A { only_in_B() { ... } } let x: A ← new B in x.only_in_B()

Discussion

- Type rules have to be very carefully constructed
- Virtually any change in a rule either
 - makes the type system unsound (bad programs are accepted as well typed)
 - or, makes the type system less usable (good programs are rejected)
- But some good programs will be rejected anyway
 - > what is a "good" program?

Discussion

- Type rules have to be very carefully constructed
- Virtually any change in a rule either
 - makes the type system unsound (bad programs are accepted as well typed)
 - or, makes the type system less usable (good programs are rejected)
- But some good programs will be rejected anyway
 - > what is a "good" program?
 - Good program: A program where all operations performed on all values are type consistent at runtime

Discussion

- Type rules have to be very carefully constructed
- Virtually any change in a rule either
 - makes the type system unsound (bad programs are accepted as well typed)
 - or, makes the type system less usable (good programs are rejected)
- But some good programs will be rejected anyway
 - > what is a "good" program?
 - Good program: A program where all operations performed on all values are type consistent at runtime
 - Impossible to express all runtime behavior in a type system
 - E.g. Type of if-then-else is LUB of two types, a conservative estimate of runtime behavior

Designing a Good Type Checking System

- Type system has two conflicting design goals
 - Give flexibility to the programmer (so that she can write a "good" program within the boundaries of the type system)
 - Prevent type-checked programs from "going wrong"
- Should allow maximum flexibility while guaranteeing safety
 - > An example

```
class Count { int i = 0; Count inc() { i=i+1; return this; } } class Stock inherits Count { ... } class Main { Stock a \leftarrow (new Stock).inc(); }
```

What Went Wrong?

- What is (new Stock).inc()'s type?
 - Dynamic type Stock
 - Static type Count
 - > The type checker "looses" the type information
 - > This makes inheriting inc() useless
 - Do we really want to redefine inc() for each subclass returning the correct type?
- SELF_TYPE to the rescue

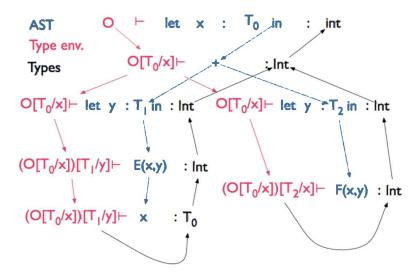
SELF_TYPE to the Rescue

- What is SELF_TYPE?
 - inc() returns "self" instead of "Count" type
 - Self could be Count or any subclass of Count, depending on reference type
- SELF_TYPE is a static type
 - Type violations can still be detected at compile time
 - Expresses runtime behavior accurately w/o undue burden to the programmer
- In practice
 - > C++: made possible by language extension using templates
 - Java: not allowed because there are no templates

Can Static Type Checking ever be Perfect?

- Many examples where correct programs are disallowed (besides SELF_TYPE)
 - Why C++ programmers are still forced to downcast (Type of values at runtime are known by programmer but no way to express it in type system)
 - Fundamentally undecidable whether type system is adhered to at runtime
- Solution?
 - Some argue for dynamic type checking instead
 - Philosophy: Maximum expressivity for the programmer
 - Good for scripting languages where expressivity is king
 - Others argue for more expressive static checking rules
 - Philosophy: Too much expressivity is not good for you
 - Good for mission critical code that should never fail
 - Good for performance critical code

Implementing Type Checking on AST



Syntax Directed Translation

What is Syntax Directed Translation?

To drive semantic analysis tasks based on the language's syntactic structure

- What is meant by semantic analysis tasks?
 - Generate AST (abstract syntax tree)
 - Check type errors
 - Generate intermediate representation (IR)
- What is meant by syntactic structure?
 - Structure of program given by context free grammar (CFG)
 - Structure of the parse tree generated by the parser

How is Syntax Directed Translation Performed?

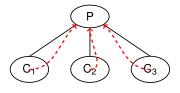
- ☐ How?
 - Attach attributes to grammar symbols/parse tree
 - > Evaluate attribute values using **semantic actions**
- We already did some of this in Project 2:
 - Attached attributes to grammar symbols
 - tptr "tree pointer" of a non-terminal symbol
 By the time program.tptr is evaluated, the parse tree is built
 - > Evaluated attributes using semantic rules (actions)
 - { ... \$\$=makeTree(ProgramOp, leftChild, rightChild); ... }

Attributes?

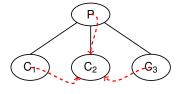
- Attributes can represent anything depending on task
 - A string
 - ➤ A type
 - A number
 - A memory location
 - An attribute grammar is a grammar augmented by associating attributes with each grammar symbol that describes its properties

Two Types of Attributes

- Synthesized attributes: attributes are computed from attributes of children nodes
 - ightharpoonup P.synthesized_attr = f(C₁.attr, C₂.attr, C₃.attr)
- Inherited attributes: attributes are computed from attributes of sibling and parent nodes
 - $ightharpoonup C_3.inherited_attr = f(P_1.attr, C_1.attr, C_3.attr)$



Synthesized attribute



Inherited attribute

Synthesized Attribute Example

Example

- Each non-terminal symbol is associated with val attribute
- Each grammar rule is associated with a semantic action

```
\begin{array}{lll} \mathsf{L} \to \mathsf{E} & \{ \; \mathsf{print}(\mathsf{E}.\mathsf{val}) \; \} \\ \mathsf{E} \to \mathsf{E}_1 + \mathsf{T} & \{ \; \mathsf{E}.\mathsf{val} = \mathsf{E}_1.\mathsf{val} + \mathsf{T}.\mathsf{val} \; \} \\ \mathsf{E} \to \mathsf{T} & \{ \; \mathsf{E}.\mathsf{val} = \mathsf{T}.\mathsf{val} \; \} \\ \mathsf{T} \to \mathsf{T}_1 + \mathsf{F} & \{ \; \mathsf{T}.\mathsf{val} = \mathsf{T}_1.\mathsf{val} + \mathsf{F}.\mathsf{val} \; \} \\ \mathsf{T} \to \mathsf{F} & \{ \; \mathsf{T}.\mathsf{val} = \mathsf{F}.\mathsf{val} \; \} \\ \mathsf{F} \to (\; \mathsf{E} \; ) & \{ \; \mathsf{F}.\mathsf{val} = \mathsf{E}.\mathsf{val} \} \\ \mathsf{F} \to \mathsf{digit} & \{ \; \mathsf{F}.\mathsf{val} = \mathsf{digit}.\mathsf{lexval} \} \end{array}
```

Inherited Attribute Example

Example:

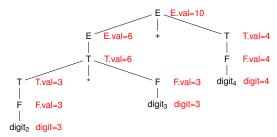
- T synthesized attribute "type"
- L has inherited attribute "in"

```
\begin{array}{lll} D \rightarrow T \; L & \{\; L.in = T.type \;\} \\ T \rightarrow int & \{\; T.type = integer \;\} \\ T \rightarrow real & \{\; T.type = real \;\} \\ L \rightarrow L_1 \; , \; id \; \{\; L_1.in = L.in, \; addtype \; (id.entry, \; L.in) \;\} \\ L \rightarrow id & \{\; addtype \; (id.entry, \; L.in) \;\} \end{array}
```

- > We can use inherited attributes to track type information
- ➤ We can use *inherited attributes* to track whether an identifier appear on the left or right side of an assignment operator ":=" (e.g. a := a +1)

Attribute Parse Tree

- ☐ Parse tree showing values of attributes
 - Parse tree annotated or decorated with attributes
 - Attributes computed at each node
- Properties of attribute parse tree:
 - ➤ Terminal symbols have synthesized attributes only, which are usually provided by the lexical analyzer
 - Start symbol does not have any inherited attributes

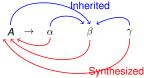


Two Aspects to Syntax Directed Translation

- Syntax Directed Definitions (SDD)
 - Set of semantic rules attached to each production
 - Semantic rules define values for attributes
 - Specification rather than implementation
- Syntax Directed Ttranslation Scheme (SDTS)
 - Semantic actions are implementations of semantic rules
 - Grammar with semantic actions embedded within RHS of productions (can be anywhere)
 - Semantic actions are fragments of code executed "at that point" in the RHS
 - Top-down: Right after previous symbol has been consumed
 - Bottom-up: Right after previous symbol has been pushed to the stack (when the 'dot' reaches the action)

Syntax Directed Definition (SDD)

Attribute grammar



SDD has rule of the form for each CFG production $b = f(c_1, c_2, ..., c_n)$

either

- If b is a synthesized attributed of A, c₁ (1≤i≤n) are attributes of grammar symbols of its Right Hand Side (RHS); or
- 2. If b is an inherited attribute of one of the symbols of RHS, c_i's are attribute of A and/or other symbols on the RHS

- Both inherited and synthesized attributes are used
 - > T synthesized attribute T.val
 - R inherited attribute R.i synthesized attribute R.s
 - ➤ E synthesized attribute E.val

Evaluating attributes using SDTS

```
E \rightarrow T \{R.i=T.val\} R \{E.val=R.s\}
R \rightarrow + T \{R_1.i=R.i+T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow \text{-} \quad T \ \{R_1.i\text{=}R.i\text{-}T.val\} \ R_1 \ \{R.s\text{=}R_1.s\}
R \rightarrow \varepsilon \{R.s=R.i\}
T \rightarrow (E) \{T.val=E.val\}
T \rightarrow num \{T.val=num.val\}
                                              Ε
                                                  → R<sub>1</sub>.i=T.val
                 T.val=num
                                                                                                           R_2
        num
                     num
                                                                                                                         R_3
                                            num
                                                                         num
```

Evaluating attributes using SDTS

```
E \rightarrow T \{R.i=T.val\} R \{E.val=R.s\}
R \rightarrow + T \{R_1.i=R.i+T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow - T \{R_1.i=R.i-T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow \varepsilon \{R.s=R.i\}
T \rightarrow (E) \{T.val=E.val\}
T \rightarrow num \{T.val=num.val\}
                                           Ε
                                                  R<sub>1</sub>.i=T.val R<sub>1</sub>
                                                 T.val = num \longrightarrow R_2.i = R_1.i + T.val
                                                                                                      R_2
        num
                                                                                                                  R_3
                                          num
                                                      num
                                                                     num
```

Evaluating attributes using SDTS

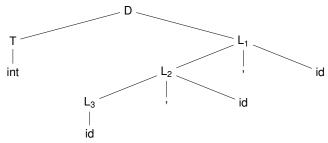
```
E \rightarrow T \{R.i=T.val\} R \{E.val=R.s\}
R \rightarrow + T \{R_1.i=R.i+T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow - T \{R_1.i=R.i-T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow \varepsilon \{R.s=R.i\}
T \rightarrow (E) \{T.val=E.val\}
T \rightarrow num \{T.val=num.val\}
                                                    EE.val=R<sub>1</sub>.s
                                                            R<sub>1</sub>.i=T.val
                                                                                R<sub>1</sub>R<sub>1</sub>.s=R<sub>2</sub>.s
                                                                                      R_2.i=R_1.i+T.val
                                                                                                                         R<sub>2</sub> R<sub>2</sub>.s= R<sub>3</sub>.s
         num
                                                                                     T T.val = num \Rightarrow R<sub>3</sub>.i=R<sub>2</sub>.i+T.valR<sub>3</sub>R<sub>3</sub>.s= R<sub>3</sub>.i
                                                  num
                                                                                  num
                                                                                              num
```

SDD Implementation using Parse Trees

- Alternative to using Syntax Directed Translation Scheme
 - Goal: create an **annotated parse tree** from the given parse tree
 - Annotated parse tree: tree annotated with attribute values
 - Traverse in a certain order and evaluate semantic rules at each node
 - Traversal order can be arbitrary as long as it adhers to dependency relationships

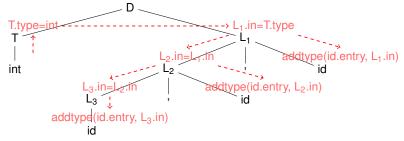
Dependency Graph

- Directed graph where edges are dependency relationships between attributes
 - Needs to be acyclic such that there exists a traversal order for evaluation
 - i.e. all necessary information must be ready when evaluating an attribute at a node



Dependency Graph

- Directed graph where edges are dependency relationships between attributes
 - Needs to be acyclic such that there exists a traversal order for evaluation
 - i.e. all necessary information must be ready when evaluating an attribute at a node



SDD Implementation using SDTS

- Tree-based evaluation works for all SDDs unless there is a dependency cycle
 - But involves more work since parse tree must be built initially
 - And the question still remains, how do you build the parse tree itself?
- Is it possible to perform evaluation while parsing?
 - Embed semantic actions in grammar using SDTS
 - What are some potential problems?
 - Parser may not have even "seen" some nodes yet
 - Some dependencies may not exist at time of evaluation
 - Different parsing schemes see nodes in different orders
 - Top-down parsing LL(k) parsing
 - Bottom-up parsing LR(k) parsing
- For certain classes of SDDs, using SDTS is feasible
 - > if dependencies of SDD are amenable to parse order
 - ➤ In other words, an L-Attributed Grammar



Left-Attributed Grammar

A syntax directed translation is L-attributed if each of its attributes is

either

ightharpoonup a synthesized attribute of A in A \rightarrow X₁... X_n ,

or

- \rightarrow an inherited attribute of X_i in $A \rightarrow X_1...X_n$ that
 - depends on attributes of symbols to its left i.e. $X_1...X_{i-1}$
 - and/or depends on inherited attributes of A

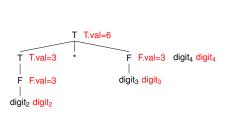
Left-Attributed Grammar

- An L-Attributed grammar
 - may have synthesized attributes
 - may have inherited attributes but only from left sibling attributes or inherited attributes of the parent
- Evaluation order
 - Left-to-right depth-first traversal of the parse tree
 - Order for both top-down and bottom-up parsers
 - Evaluate inherited attributes while going down the tree
 - > Evaluate synthesized attributes while going up the tree
- Can be evaluated using SDTS w/o parse tree

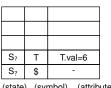
Syntax Directed Translation Scheme Implementation

When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes

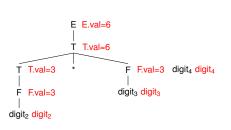


parsing stack:

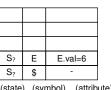


When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes

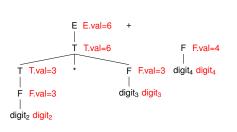


parsing stack:



When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes

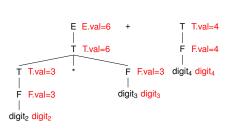


parsing stack:

| S _? | F | F.va | al=4 | |
|-----------------------------|----|---------|-------|--|
| S _? | + | - | | |
| S _? | Е | E.val=6 | | |
| S _? | \$ | - | | |
| (state) (symbol) (attribute | | | iiite | |

When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes

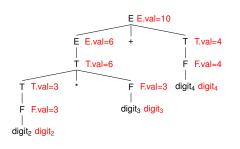


parsing stack:

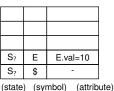
| S _? | Т | T.va | al=4 | |
|-----------------------------|----|---------|------|--|
| S _? | + | - | | |
| S _? | Е | E.val=6 | | |
| S _? | \$ | - | | |
| (state) (symbol) (attribute | | | | |

When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes

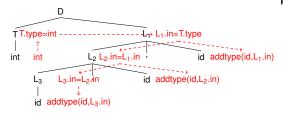


parsing stack:

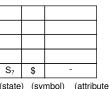


When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes



parsing stack:

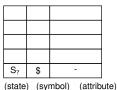


When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes



parsing stack:

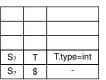


When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes



parsing stack:



When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes

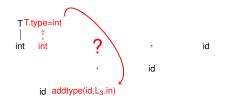


parsing stack:

| S _? | id | id.type=L3.in |
|----------------|----|---------------|
| S _? | Т | T.type=int |
| S _? | \$ | - |

When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes

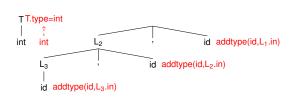


parsing stack:

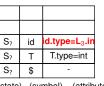
| S _? | id | id.type=L ₃ .in |
|----------------|----|----------------------------|
| S _? | Т | T.type=int |
| S _? | \$ | - |

When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes

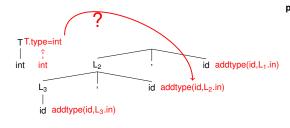


parsing stack:



When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes



parsing stack:

| S _? | id | id.type=L ₃ | .in |
|-----------------------------|----|------------------------|-----|
| S _? | Т | T T.type=int | |
| S _? | \$ | - | |
| (state) (symbol) (attribute | | | |

Evaluating Inherited Attributes using LR

Recall

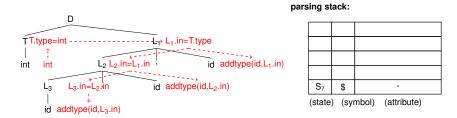
Only applies to L-Attributed grammars

What is L-attributed grammar?

Claim: the information is in the stack, we just do not know the exact location

Solution: let us hack the stack to find the location

```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow int & \{stack[top]=integer\} \\ T \rightarrow real & \{stack[top]=real\} \\ L \rightarrow L & , & id & \{addtype(stack[top],stack[top-3])\} \\ L \rightarrow id & \{addtype(stack[top],stack[top-1])\} \end{array}
```



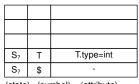
```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow int & \{stack[top]=integer\} \\ T \rightarrow real & \{stack[top]=real\} \\ L \rightarrow L & , & id & \{addtype(stack[top],stack[top-3])\} \\ L \rightarrow id & \{addtype(stack[top],stack[top-1])\} \end{array}
```



(state) (symbol) (attribute)

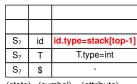
int

```
\begin{split} & D \to T \quad L \\ & T \to int \; \{stack[top] = integer\} \\ & T \to real \; \{stack[top] = real\} \\ & L \to L \; , \quad id \; \{addtype(stack[top], stack[top-3])\} \\ & L \to id \; \{addtype(stack[top], stack[top-1])\} \end{split}
```





```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow int & \{stack[top]=integer\} \\ T \rightarrow real & \{stack[top]=real\} \\ L \rightarrow L & , & id & \{addtype(stack[top],stack[top-3])\} \\ L \rightarrow id & \{addtype(stack[top],stack[top-1])\} \end{array}
```





```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow \text{int } \{stack[top] = \text{integer}\} \\ T \rightarrow \text{real } \{stack[top] = \text{real}\} \\ L \rightarrow L & , & \text{id } \{addtype(stack[top], stack[top-3])\} \\ L \rightarrow \text{id } \{addtype(stack[top], stack[top-1])\} \end{array}
```



| | l | |
|----------------|----|----------------------|
| | | |
| | | |
| S _? | id | id.type=stack[top-1] |
| S _? | Т | T.type=int |
| S _? | \$ | - |
| | | |

(state) (symbol) (attribute)

id

TT.type=int

id addtype(id,L3.in)

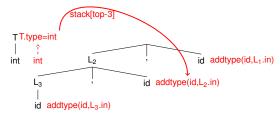
int

```
D \rightarrow T L
T \rightarrow int \{stack[top]=integer\}
T \rightarrow real \{ stack[top] = real \}
L \rightarrow L, id {addtype(stack[top],stack[top-3])}
L \rightarrow id \{addtype(stack[top],stack[top-1])\}
```

parsing stack:

| | S _? | id | id.type=stack[top-3] |
|-----------------------------------|----------------|----------------|------------------------|
| _ | S _? | , | |
| id addtype(id,L ₁ .in) | S _? | L ₃ | L ₃ .in=int |
| id additype(id,E1.iii) | S _? | Т | T.type=int |
| addtype(id,L ₂ .in) | S _? | \$ | - |
| | (state |) (sv | mbol) (attribute) |

```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow \text{int } \{stack[top] = \text{integer}\} \\ T \rightarrow \text{real } \{stack[top] = \text{real}\} \\ L \rightarrow L & , & \text{id } \{addtype(stack[top], stack[top-3])\} \\ L \rightarrow \text{id } \{addtype(stack[top], stack[top-1])\} \end{array}
```



| S _? | id | id.type=stack[top-3] |
|----------------|----------------|------------------------|
| S _? | , | |
| S _? | L ₃ | L ₃ .in=int |
| S _? | Т | T.type=int |
| S _? | \$ | - |
| | | |

Marker

 \Box Given the following SDD, where $|\alpha| != |\beta|$

$$A \rightarrow X \alpha Y \mid X \beta Y$$

$$Y \rightarrow \gamma \{ \dots = f(X.s) \}$$

- Problem: cannot generate stack location for X.s since X is at different relative stack locations from Y
- Solution: introduce *markers* M₁ and M₂ that are at the same relative stack locations from Y

$$A \rightarrow X \alpha M_1 Y \mid X \beta M_2 Y$$

$$Y \rightarrow \gamma \{... = f(M_{12}.s)\}$$

$$M_1 \rightarrow \varepsilon \{M_1.s = X.s\}$$

$$M_2 \rightarrow \varepsilon \{M_2.s = X.s\}$$

$$(M_{12} = \text{the stack location of } M_1 \text{ or } M_2, \text{ which are identical})$$

A marker intuitively marks a stack location that is equidistant from the reduced non-terminal

Example

How to add the marker?

```
Example 1:
        S \rightarrow a A \{ C.i = A.s \} C
        S \rightarrow b A B \{ C.i = A.s \} C
        C \rightarrow c \{ C.s = f(C.i) \}
Solution:
        S \rightarrow a A \{ C.i = A.s \} C
        S \rightarrow b A B \{ M.i=A.s \} M \{ C.i = M.s \} C
        C \rightarrow c \{ C.s = f(C.i) \}
        M \rightarrow \varepsilon \{ M.s = M.i \}
That is:
        S \rightarrow a A C
        S \rightarrow b A B M C
        C \rightarrow c \{ C.s = f(stack[top-1]) \}
        M \rightarrow \varepsilon \{ M.s = stack[top-2] \}
```

How to Add the Marker?

- 1. Identify the stack location(s) to find the desired attribute
- 2. Is there a conflict of location?
 - Yes, add a marker;
 - No, no need to add.
- Add the marker in the place to remove location inconsistency

Example:

$$\begin{split} S &\rightarrow a \text{ A B C E D} \\ S &\rightarrow b \text{ A F B C F D} \\ C &\rightarrow c \text{ (/* C.s = f(A.s) */)} \\ D &\rightarrow d \text{ (/* D.s = f(B.s) */)} \end{split}$$

Answer

```
S → a A B C E D
S → b A D M B C F D
C → c {/* C.s = f(stack[top-2]) */}
D → d {/* D.s = f(stack[top-3]) */}
M → ε {/* M.s = f(stack[top-2]) */}

Regarding C.s, from stack[top-2], and stack[top-3]
.... add a Marker

Regarding D.s, always from stack[top-2]
... no need to add
```

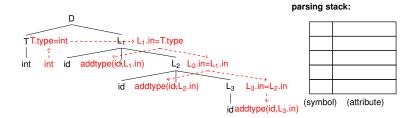
☐ How about Top-Down Parsing?

Translation Scheme for Top-Down Parsing

- Predictive Recursive Descent Parsers: Straightforward
 - Synthesized Attribute: Return value of function call for non-terminal is synthesized attribute
 - All function calls for children nodes would have completed by the time this function call returns
 - All dependent values would have been computed
 - Inherited Attribute: Pass as argument to function call for non-terminal inheriting attribute
 - L-Attributed grammar guarantees that dependent attributes come from left sibling attributes or parent inherited attributes
 - Left sibling function calls would have completed and parent inherited attribute would have been passed in as argument
 - All dependent values would have been computed
- Now let's focus on table-driven LL Parsers

When using LL parsing (top-down parsing),

it is natural to evaluate inherited attributes



When using LL parsing (top-down parsing),

it is natural to evaluate inherited attributes

D

parsing stack:



When using LL parsing (top-down parsing),

it is natural to evaluate inherited attributes



parsing stack:



When using LL parsing (top-down parsing),

it is natural to evaluate inherited attributes



parsing stack:



When using LL parsing (top-down parsing),

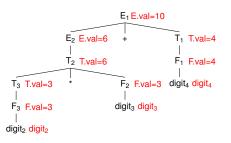
it is natural to evaluate inherited attributes

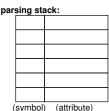


parsing stack:



When using LL parsing (top-down parsing),





When using LL parsing (top-down parsing),

it is **not natural** to evaluate synthesized attributes

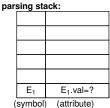
 E_1

| parsing stack: | | | | |
|----------------|----------------|-------------|--|--|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | E ₁ | | | |
| - | evmbol | (attribute) | | |

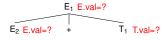
When using LL parsing (top-down parsing),

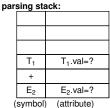
it is **not natural** to evaluate synthesized attributes

E₁ E.val=?

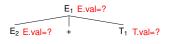


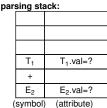
When using LL parsing (top-down parsing),





When using LL parsing (top-down parsing),



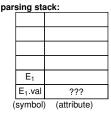


- Solution
 - Always push a 'dummy' stack item below a non-terminal to hold intermediate values for attribute calculation
 - Update dummy item whenever a child node is popped with intermediate value
 - When all children nodes have been popped, compute synthesized attribute from stored values

When using LL parsing (top-down parsing),

it is **not natural** to evaluate synthesized attributes

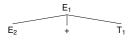
E₁



Solution

- Always push a 'dummy' stack item below a non-terminal to hold intermediate values for attribute calculation
- Update dummy item whenever a child node is popped with intermediate value
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When using LL parsing (top-down parsing),



| parsing stack: | | | | |
|----------------|---------------------|----|--|--|
| | T ₁ | | | |
| | T ₁ .val | ?? | | |

| T ₁ | | |
|---------------------|-----------------|--|
| T ₁ .val | ??? | |
| + | | |
| E ₂ | | |
| E ₂ .val | ??? | |
| E ₁ .val | E2.val + T1.val | |
| symbol) (attribute) | | |

- Solution
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