Compiler Optimization

Compiler optimizations transform code

- Code optimization transforms code to equivalent code
  - ... but with better performance
- The code transformation can involve either
  - **Replacing** code with more efficient code
  - **Deleting** redundant code
  - **Moving** code to a position where it is more efficient
  - **Inserting** new code to improve performance

The four categories of code transformations

- Replacing code (e.g. **strength reduction**)

  A=2*a; ≡ A=a«1;

- Deleting code (e.g. **dead code elimination**)

  A=2; A=y; ≡ A=y;

- Moving code (e.g. **loop invariant code motion**)

```
for (i = 0; i < 100; i++) { sum += i + x * y; }
≡
t = x * y;
for (i = 0; i < 100; i++) { sum += i + t; }
```

- Inserting code (e.g. **data prefetching**)

```
for (p = head; p != NULL; p = p->next)
{ /* do work on node p */ }
≡
for (p = head; p != NULL; p = p->next)
{ prefetch(p->next); /* do work on node p */ }
```

Compiler optimization categories according to range

- How much code does the compiler view while optimizing?
  - The wider the view, the more powerful the optimization

- Axis 1: optimize across control flow?
  - **Local optimization**: optimizes only within straight line code
  - **Global optimization**: optimizes across control flow (if,for,...)

- Axis 2: optimize across function calls?
  - **Intra-procedural optimization**: only within function
  - **Inter-procedural optimization**: across function calls

- The two axes are orthogonal (any combination is possible)

Local vs. Global Constant Propagation

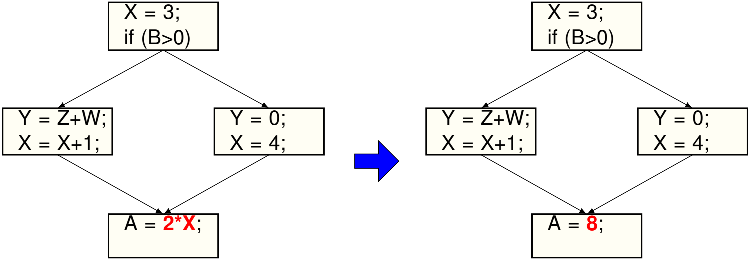
- **Constant propagation**
  - ➢ Optimization: if **x= y op z** and y and z are constants then compute at compile time and replace
- Local Constant Propagation

```
X = 3;
X = X+1;
A = X*2;
```

➡

```
X = 3;
X = X+1;
A = 8;
```

- Global Constant Propagation



Intra- vs. Inter-procedural Constant Propagation

- Intra-procedural Constant Propagation

```
X = 3;
X = X+1;
A = X*2;
```

→

```
X = 3;
X = X+1;
A = 8;
```

- Inter-procedural Constant Propagation

```
X = 3;
foo(X);

void foo(int arg) {
  arg = arg+1;
  A = arg*2;
}
```

→

```
X = 3;
foo(X);

void foo(int arg) {
  arg = arg+1;
  A = 8;
}
```

  - Assuming all other calls to foo always pass in constant 3

Control Flow Analysis

Basic Block

- A function body is composed of one or more **basic blocks.**
- **Basic block**: a maximal sequence of instructions that
  - Has no jumps into the block other than the first instruction
  - Has no jumps out of the block other than the last instruction
- That means:
  - No instruction other than the first is a jump target
  - No instruction other than the last is a jump or branch
- Either all instructions in basic block execute or none
  - Smallest unit of execution in control flow analysis
  - Hence the descriptor "basic" in the name

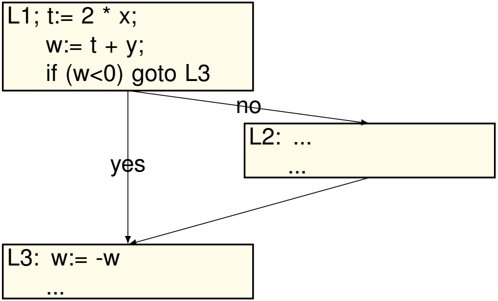
Control Flow Graph

- A **Control Flow Graph (CFG)** is a directed graph in which
  - Nodes are basic blocks
  - Edges represent flows of execution between basic blocks

- CFGs are widely used to represent a program for analysis
- CFGs are especially essential for global optimizations

Control Flow Graph Example

```
L1; t:= 2 * x;
    w:= t + y;
    if (w<0) goto L3
L2:  ...
    ...
L3:  w:= -w
    ...
```

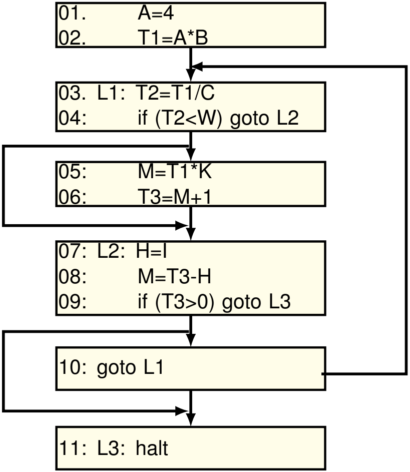


Construction of CFG

- Step 1: partition code into basic blocks
  - Identify **leader** instructions, where a leader is either:
    - the first instruction of a program, or
    - the target of any jump/branch, or
    - an instruction immediately following a jump/branch
  - Create a basic block out of each leader instruction
  - Expand basic block by adding subsequent instructions (Stopping when the next leader instruction is encountered)
- Step 2: add edge between two basic blocks B1 and B2 if
  - there exist a jump/branch from B1 to B2, or
  - B2 follows B1, and B1 does not end with jump/branch

Example

```
01.        A=4
02.        T1=A*B
03. L1: T2=T1/C
04:        if (T2<W) goto L2
05:        M=T1*K
06:        T3=M+1
07: L2: H=I
08:        M=T3-H
09:        if (T3>0) goto L3
10: goto L1
11: L3: halt
```



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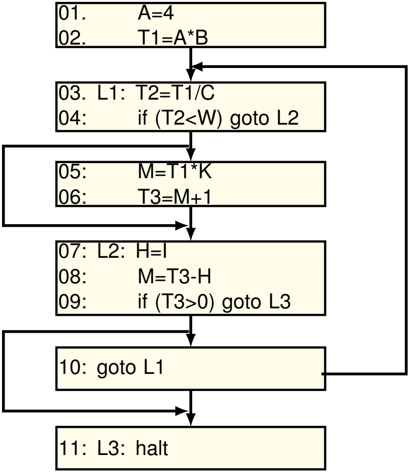
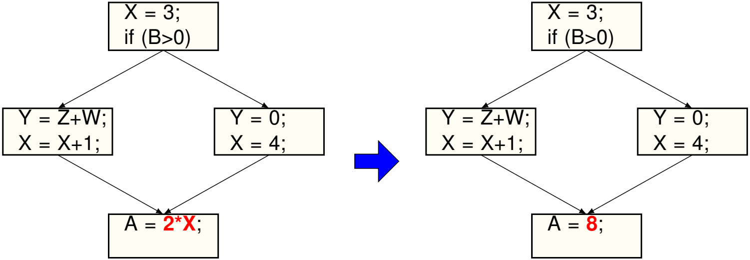


Data Flow Analysis

Global Optimizations

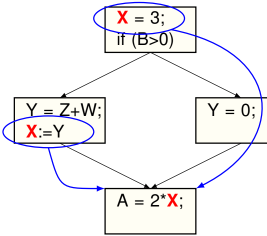
- Extend optimizations across control flows, i.e. CFG
- Like in this example of Global Constant Propagation (GCP):



- How do we know it is OK to globally propagate constants?

Correctness criteria for GCP

- There are situations that prohibit GCP:



- To replace X by a constant C **correctly**, we must know
  - **Along all paths**, the last assignment to X is “X = C”

- Paths may go through loops and/or branches
  - When two paths **meet**, need to make a **conservative** choice

Global Optimizations need to be Conservative

- Many compiler optimizations depend on knowing some property X at a particular point in program execution
  - Need to prove at that point property X holds along all paths

- To ensure correctness, optimization must be **conservative**
  - An optimization is enabled only when X is definitely true
  - If not sure, be conservative and say **don't know**
  - **Don't know** usually disables the optimization

Dataflow Analysis Framework

- **Dataflow analysis**: discovering properties about values
  - ... at certain points in the CFG to enable optimizations
  - E.g. discovering a value is constant at a statement
  - Done by observing the flow of data through the CFG

- **Dataflow analysis framework**:
  - A framework for describing different dataflow analyses
  - Can be defined using the following 4 components:

    $\{$ **D**, **V**, $\wedge$: (V, V) $\rightarrow$ V, **F**: V $\rightarrow$ V $\}$

    - **D**: direction of dataflow (forward or backward)
    - **V**: domain of values denoting property
    - $\wedge$: **meet operator** that merges values when paths meet
    - **F**: **flow propagation function** that propagates values through a basic block

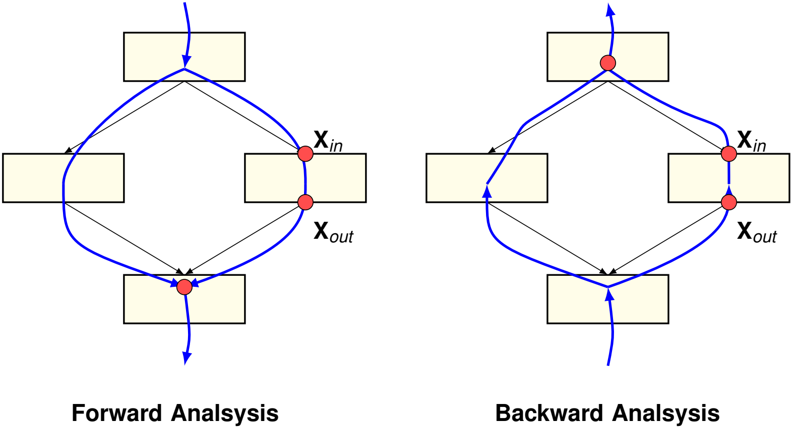
Global Constant Propagation (GCP)

- Let's use **GCP** to study dataflow analysis framework

- We will define each component one by one for GCP
  - **D**: direction of dataflow for constant property
  - **V**: domain of values denoting constant property
  - $\wedge$: **meet operator** that merges values when paths meet
  - **F**: **flow propagation function** for GCP

Direction D for GCP

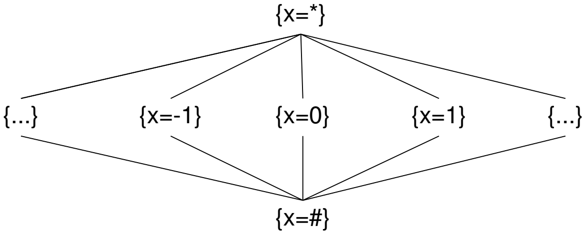
- Is GCP a forward or backward analysis?



- Forward, since "constantness" of a variable flows forward to subsequent instructions starting from assignment

V and meet operator ∧ for GCP

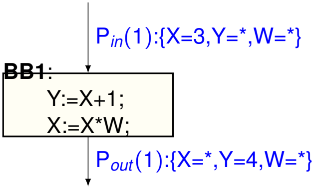
- Given an integer variable x, domain V is the set:
  - ..., {x=-1}, {x=0}, {x=1}, ... /* a constant */
  - {x=*} /* not a constant */
  - {x=#} /* x is not assigned on any path*/
- Meet operator ∧ is given by this **semi-lattice**:
  - a ∧ b = least upper bound in the below graph

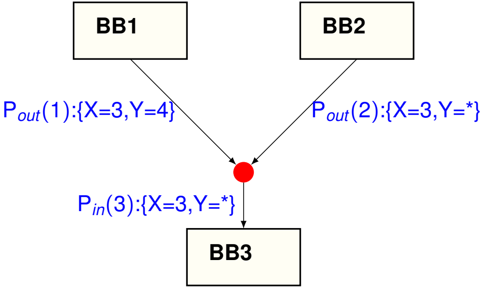


  - {x=0} ∧ {x=1} = {x=*}: Different values on each path
  - {x=#} ∧ {x=1} = {x=1}: Constant definition on one path

Dataflow Equations for GCP

- There are two types of flow functions





- ➢ Flow transfer function F: V $\rightarrow$ V
  - Computes data flow within basic blocks
  - Remove those that become variables, add new constants
- ➢ Meet operator $\wedge$: (V, V) $\rightarrow$ V
  - Computes data flow across basic blocks
  - Merge values from two paths using the previous semi-lattice

Flow Transfer Function F for GCP

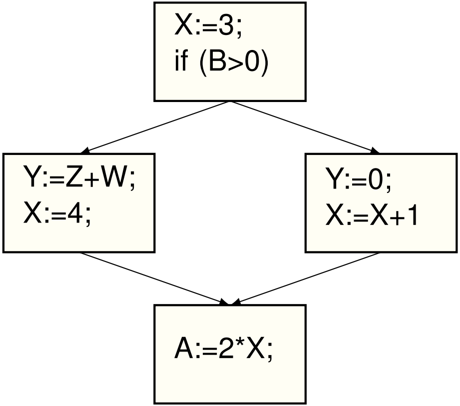
- **X(i)**: dataflow property X of basic block **i**
  - $\mathbf{X_{in}(i)}$: at the entry of basic block **i**
  - $\mathbf{X_{out}(i)}$: at the exit of basic block **i**

- F for Global constant propagation (GCP)

$$\mathbf{GCP_{out}(i) = (\ GCP_{in}(i) - DEF_{v}(i)\ ) \cup DEF_{c}(i)}$$

where DEF_v(i) contains variable definitions in basic block i
DEF_c(i) contains constant definitions in basic block i

GCP Propagation without loops



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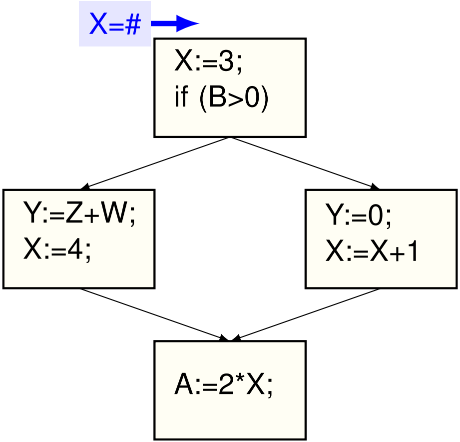

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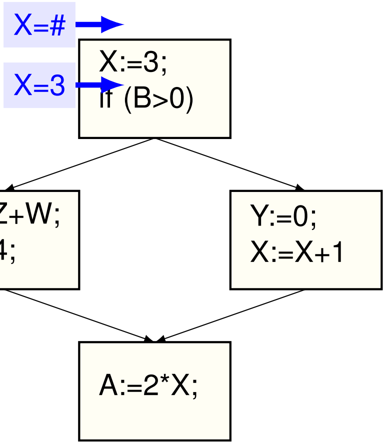

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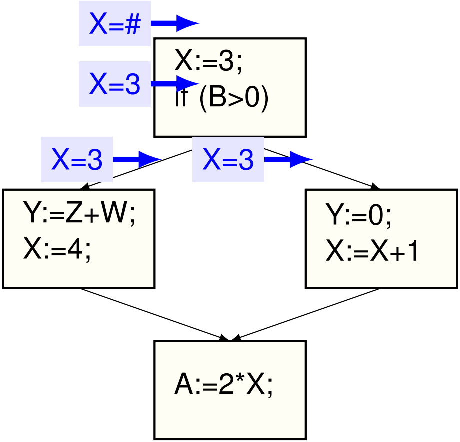

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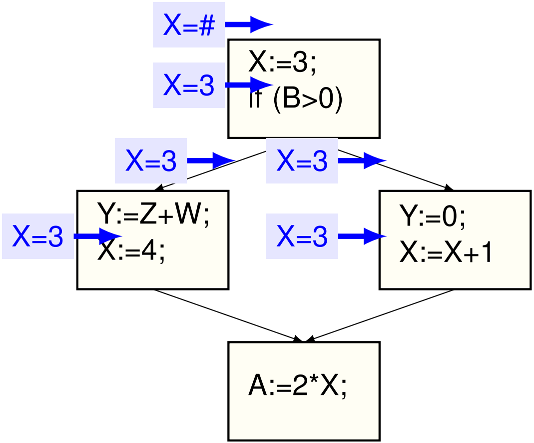

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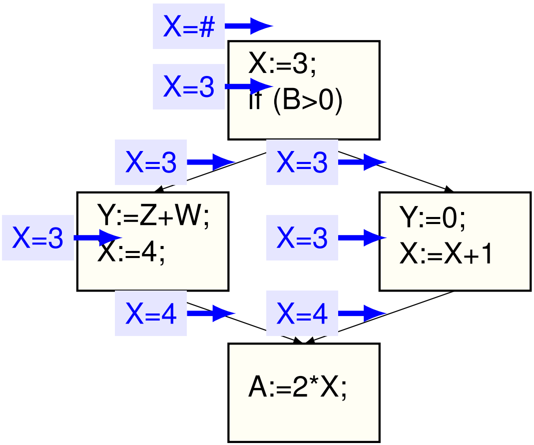

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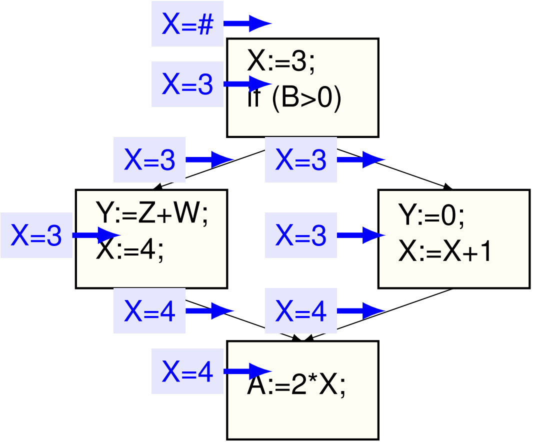

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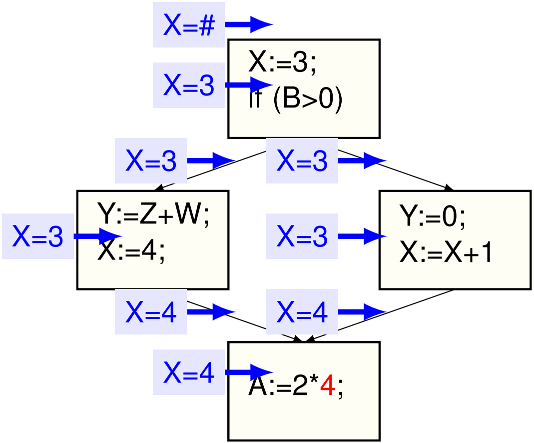
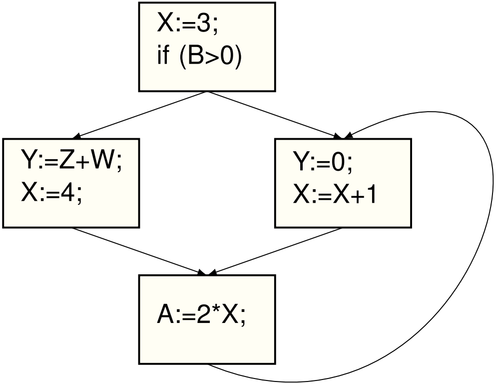


GCP Propagation with loops

- Iterate until there are no changes to values
  - This is called the **maximum fixed point** solution



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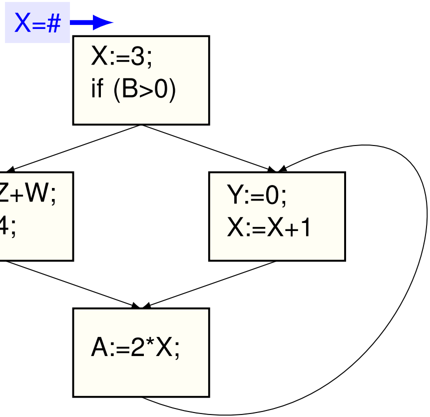

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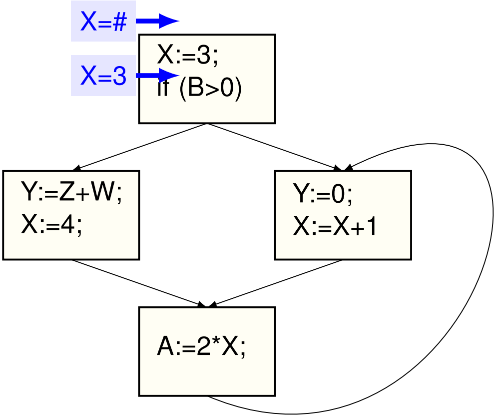

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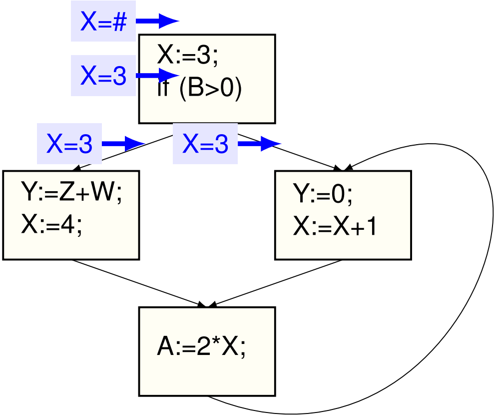

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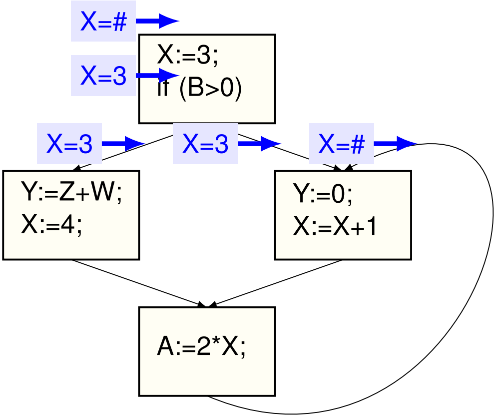

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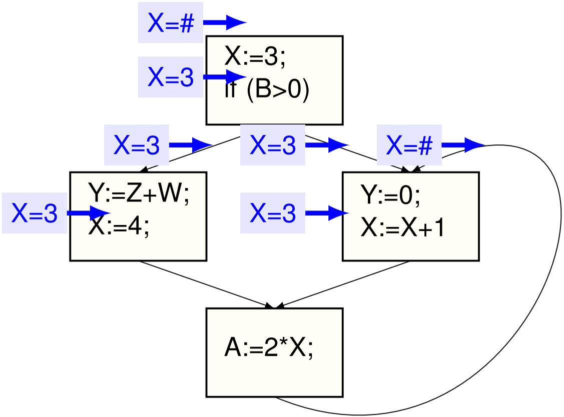

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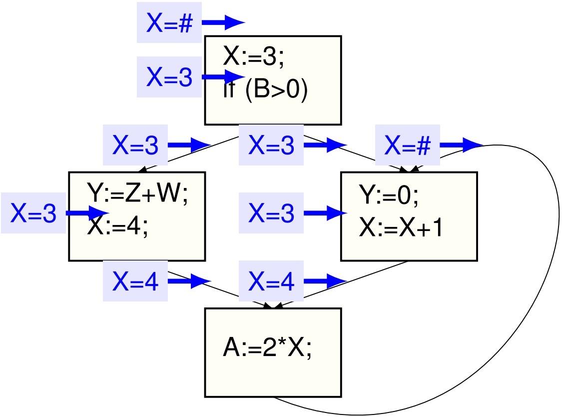

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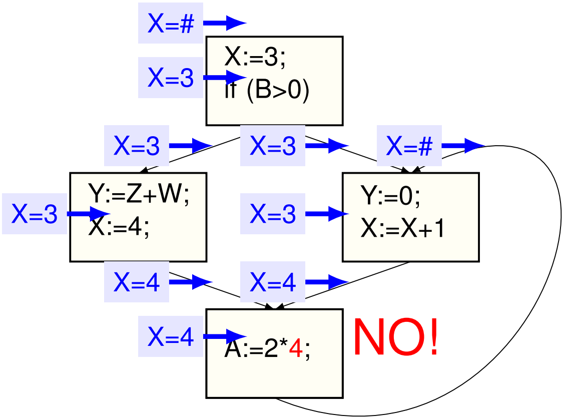

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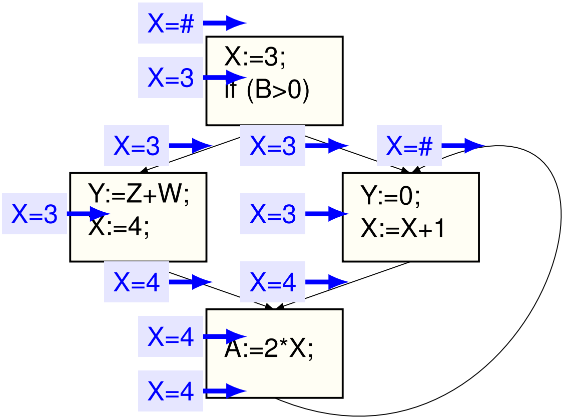

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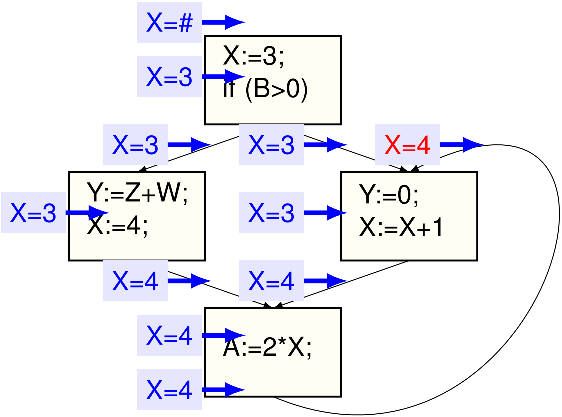

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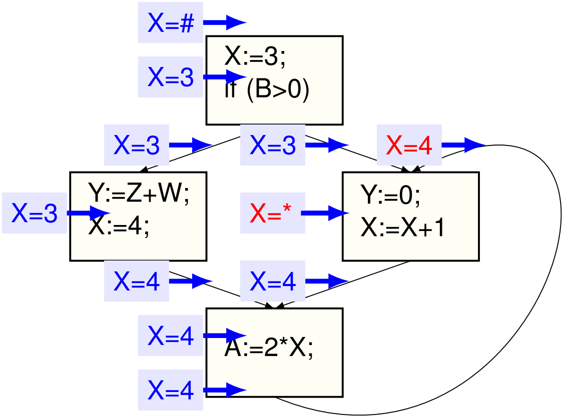

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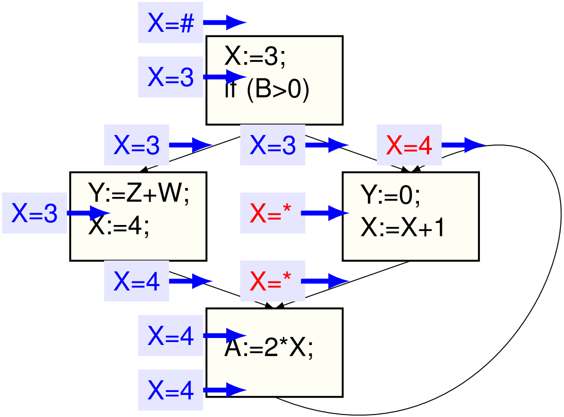

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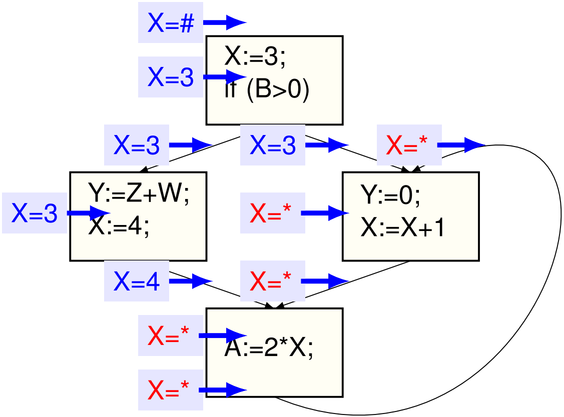


Analysis Algorithm for GCP

- GCP Algorithm
  (1). Set {x=#} at all the points in the procedure
  (2). Propagate the dataflow property along the control flow
  (3). Repeat step (2) until there are no changes

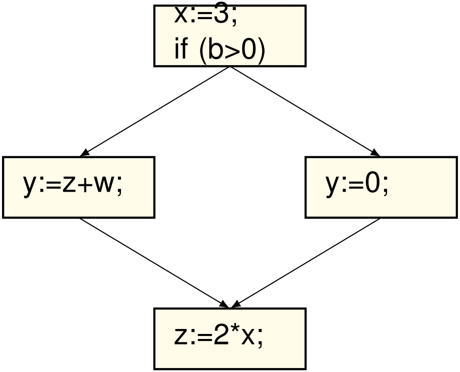
- Will GCP eventually stop?
  - If there are loops, we may propagate the loop many times
  - Is there a possibility to run into an endless loop?

Termination Problem

- **Least upper bound** ensures the termination
  - Values starts from #
  - Values can only increase in the hierarchy
  - Any values can change at most twice in our example
    ... from # to C, and from C to *
- The maximal number of steps is O(program_size)

Another Analysis: Liveness Analysis

- Once constants have been globally propagated, we would like to eliminate the dead code



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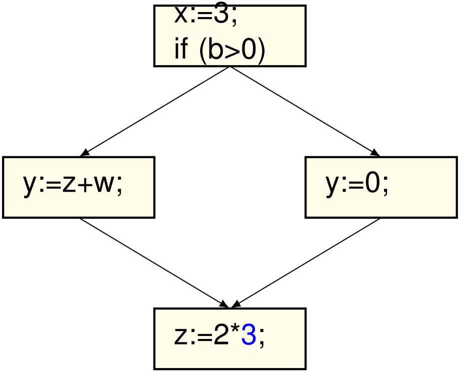

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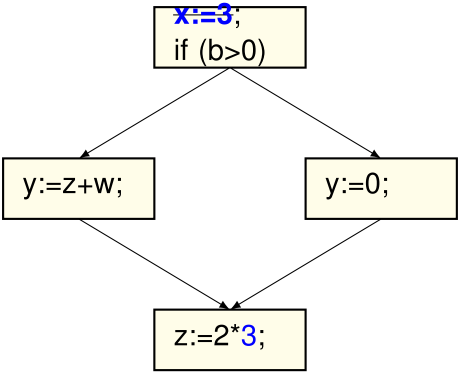
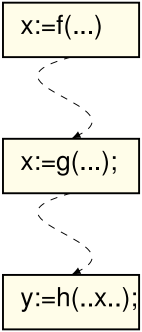


Live/Dead Statment

- A **dead statement** calculates a value that is not used later, or output to file
- Otherwise, it is a **live statement**

In the example,
the 1st statement is dead,
the 2nd statement is live



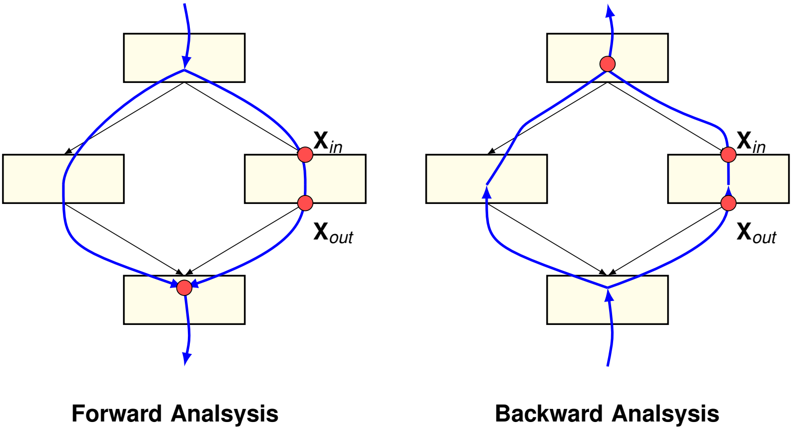
Liveness Analysis

- We can form liveness analysis as a dataflow analysis
  - Propagate the information along control flow
    - “x is dead”, “x is live”, “y is dead”, “y is live”
  - Liveness is simpler than constant propagation
    - It is a boolean property
  - Liveness is different from constant propagation
    - Liveness analysis is a union problem
      ☞ x is alive if x is alive along one path
    - Constant propagation is an intersection problem
      ☞ x is NOT a constant if x is NOT a constant along one path
      ...

Forward and Backward Analysis

- The most significant difference is
  - Liveness analysis wants to know if it is used some time later
    - Use information after this statement to decide
    - **Backward analysis**
  - Constant analysis wants to know if it is constant for all possible executions at this point
    - Use information before this statement to decide
    - **Forward analysis**

Graphic View of Forward and Backward Analysis

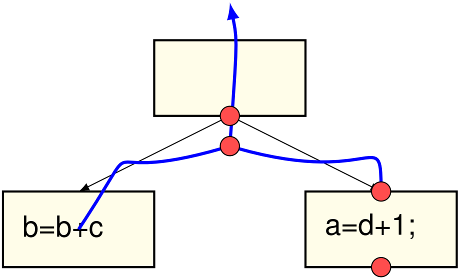


Global Liveness Analysis

- Global liveness analysis
  - A variable x is live at statement s if
    - There exists a statement ss after s that use x
    - There is a path from s to ss
    - That path has no intervening assignment to x

- A backward dataflow analysis
  {**L**, ($\top$, $\bot$), ($\sqcap$, $\sqcup$), **f: L** $\rightarrow$ **L** }
  - Lattice, top and bottowm items
  - Operators
  - Dataflow functions

Liveness Example



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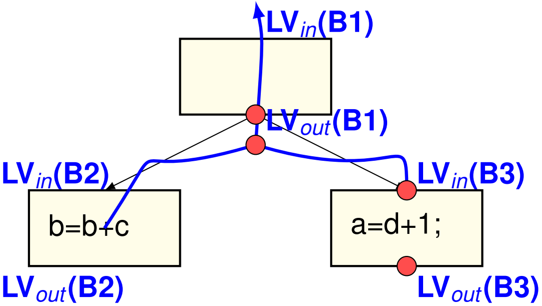

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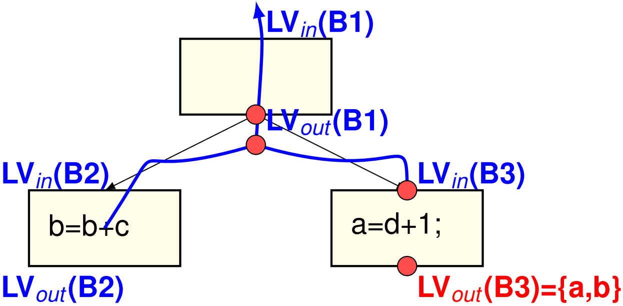

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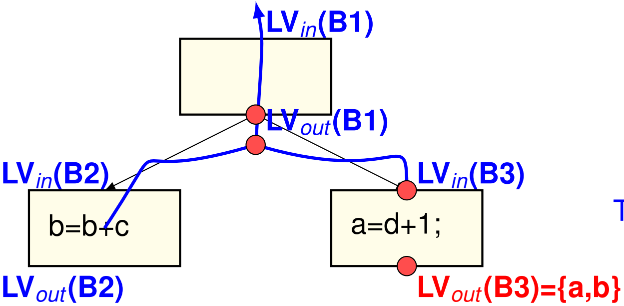
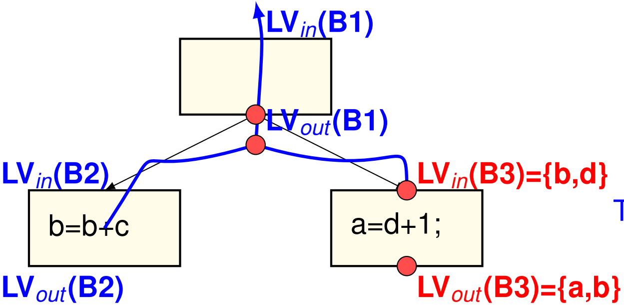


Two sets:
DEF={a}
USE={d}

Liveness Example



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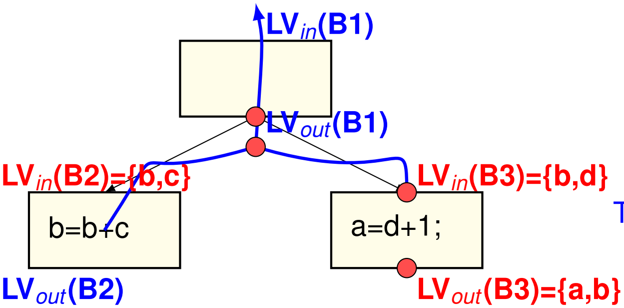
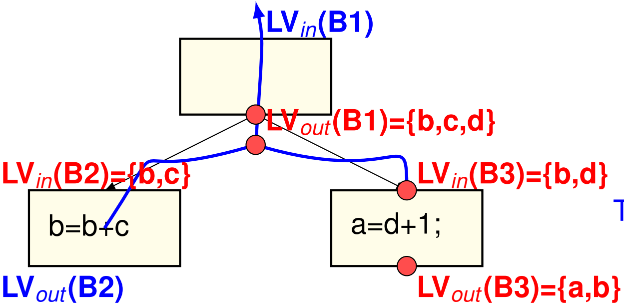
Liveness Example



Two sets:
DEF={a}
USE={d}

Liveness Example



Two sets:
DEF={a}
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Dataflow Equations for Liveness Analysis

- **X(i)** — dataflow property of basic block **i**
  - $\mathbf{X_{in}(i)}$ — at the entry of basic block **i**
  - $\mathbf{X_{out}(i)}$ — at the exit of basic block **i**
- Flow equations
  - flow transfer function
    $\mathbf{LV_{in}(i) = (\ LV_{out}(i) - DEF(i)\ ) \cup USE(i)}$
  - flow propagation function
    $\mathbf{LV_{out}(i) = \cup\ LV_{in}(k)}$ where **k** is successor of **i**

Comparison with Dataflow Equations for GCP

- **X(i)** — dataflow property of basic block **i**
  - $\mathbf{X}_{in}\mathbf{(i)}$ — at the entry of basic block **i**
  - $\mathbf{X}_{out}\mathbf{(i)}$ — at the exit of basic block **i**
- Global constant propagation (GCP)
  - flow transfer function
    $\mathbf{GCP}_{out}\mathbf{(i) = (\ GCP}_{in}\mathbf{(i)\ -\ DEF}_{v}\mathbf{(i)\ )} \cup \mathbf{DEF}_{c}\mathbf{(i)}$

    where DEF_v(i) contains variable definitions in basic block i
    DEF_c(i) contains constant definitions in basic block i
  - flow propagation function
    $\mathbf{GCP}_{in}\mathbf{(i) =} \cap\ \mathbf{GCP}_{out}\mathbf{(k)}$ where **k** is predecessor of **i**

Application of Liveness Analysis

- Global dead code elimination is based on global liveness analysis (GLA)
  - Dead code detection
    - A statement x:= ... is dead code if x is dead after this statement
    - Dead statement can be deleted from the program (Didn't consider side-effect)

- Global register allocation is also based on GLA
  - Live variables should be placed in registers
  - Registers holding dead variables can be reused

Summary of Dataflow Analysis

- There are many other global dataflow analysis
- Classification
  - Forward/Backward analysis
  - Union analysis — some property is true if it is true along at least one path Intersection analysis — some property is true if it is true along all paths
- Very useful in
  - compiler optimization
  - software engineering
  - debugging

Register Allocation

Why Register Allocation?

- Register allocation is a very important optimization
  - Register are important hardware resources
  - RISC computer: all ALU instructions use registers only
  - Fast but a fixed small number of registers
    - from memory: one instruction and 20-100+ cycles
    - from cache: one instruction and 1 cycle
    - from register: same instruction cycle
- Goal of register allocation
  - Improve performance by keeping right values in registers
  - Compute in registers and keep it as long as it will be used

Local Register Allocation

- Allocate registers only to values within basic block
  - Must put values back into memory at the end of each basic block
  - It is helpful to distinguish
    - Short lived temporary variables: only used in basic block
    - Long lived temporary variables: used later in following basic blocks
    - Normal variables
- Backward scan
  - Keep variables in registers if they are used later

Global Register Allocation

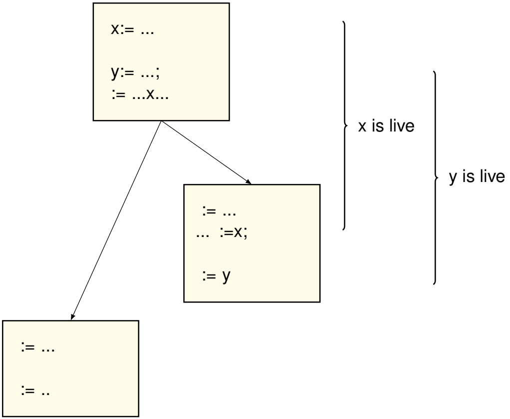
- Allocate registers across basic blocks
- Need liveness information
  - Global liveness analysis
- How to analyze?
  1. Graph coloring
  2. Linear scan
  3. ILP

Allocate Registers using Graph Coloring

- **Algorithm steps**:
  1. Global liveness analysis and identify interference
  2. Build register interference graph
  3. Coalesce nodes of graph
  4. Attempt N-coloring of the graph
     - N is the number of available registers
  5. If none found, modify the program, rebuild graph until N-coloring can be obtained
     - Insert spill code to the program

Global Liveness and Interference Analysis

- A variable is alive till its last usage
- Two variables interfere when their values cannot reside in the same register



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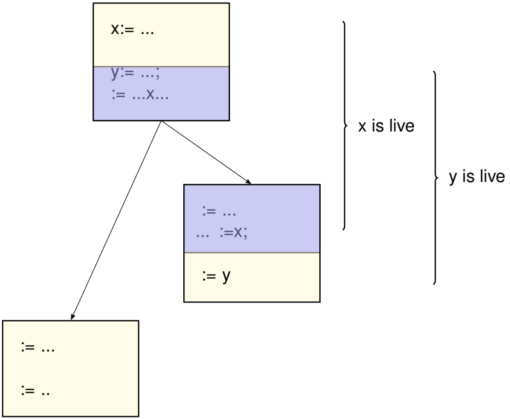

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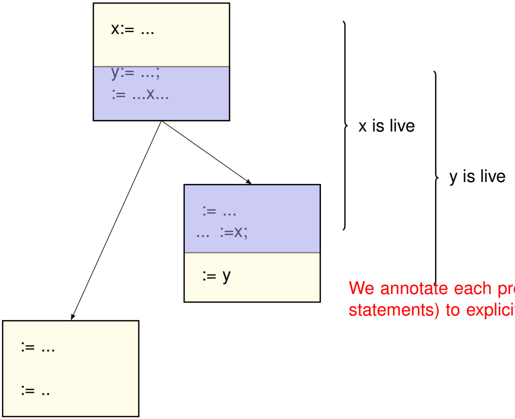
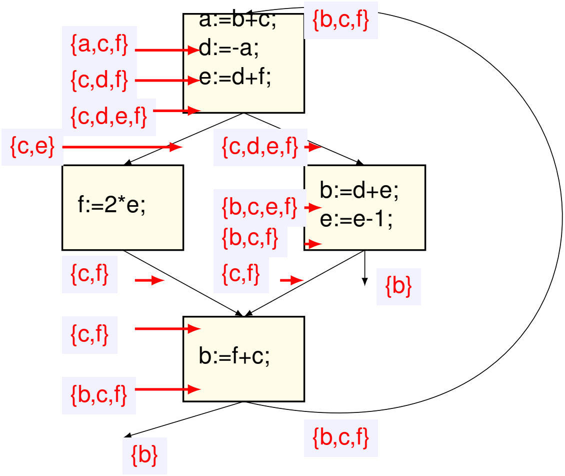


We annotate each program point (between two statements) to explicitly show the interference

Point of Definition Interference

- At each definition point P, compute live variables and determine the interference

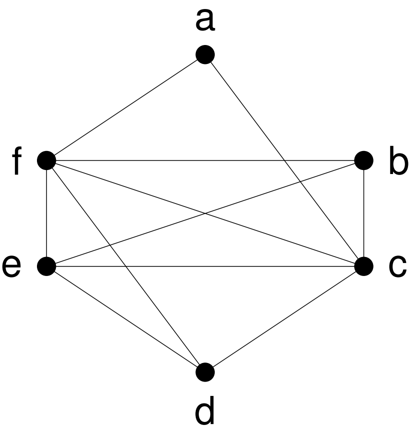


Register Interference Graph

- We can construct **Register Interference Graph (RIG)** such that
  - Each node represents a variable
  - An edge between two nodes V_1 and V_2 if they live simultaneously at some program point
- Based on RIG,
  - Two variables can be allocated in the same register if there is no edge between them
  - Otherwise, they cannot be allocated in the same register

RIG Example

- For our example,
  - b,c cannot be in the same register
  - a,d,d can be in the same register

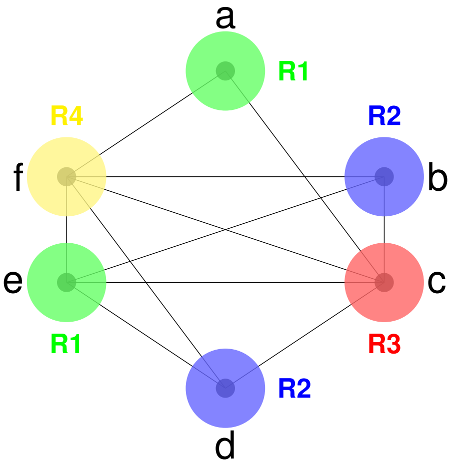


Allocating Registers using Graph Coloring

- Graph coloring is a basic register allocation scheme
  - A coloring of a graph is an assignment of colors to nodes such that nodes connected by an edge have different colors
  - A graph is k-colorable if it has a coloring with k colors
- In the algorithm, colors = registers
  - We need to assign k registers to graph nodes
  - K is the number of available machine registers
  - If the graph is k-colorable, we have a register assignment that uses no more than k registers

Coloring Results

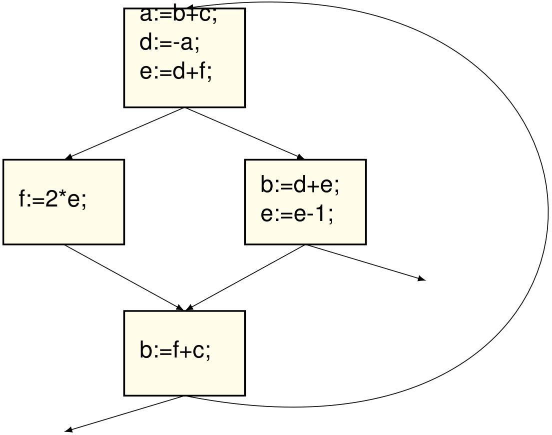
- For our example,
  - There are 4 colors in the coloring result
  - There is no solution with less than 4 colors



After Register Allocation

- Using the coloring result, map it back to the code

a–R1
b–R2
c–R3
d–R2
e–R1
f–R4



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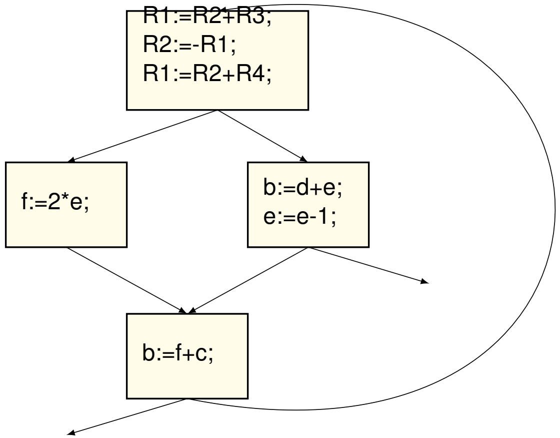

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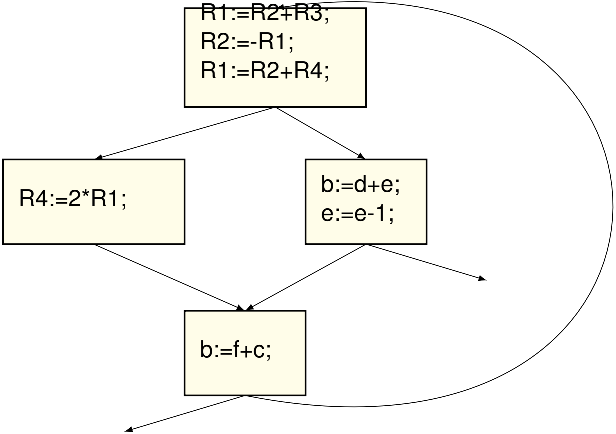

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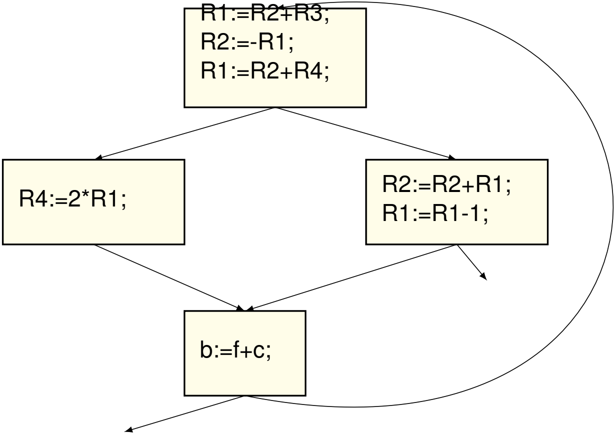

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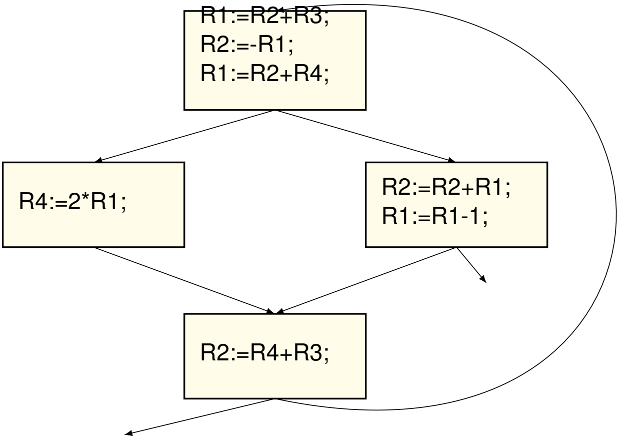
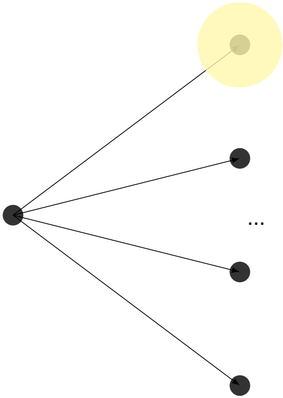


How to Perform Graph Coloring

- Now, the problem is how to compute a coloring from an interference graph
  - For graph G and N>2, determining whether G is N colorable is NP complete
  - A coloring might not exist for a given number or registers
- In practice
  - For the first problem, use heuristics
  - For the second problem, generate spill code

Graph Coloring Heuristics

- **Observation**: given a node with **k-1** neighbors, if there are **k** colors, then no matter how the neighbors were colored, there might be a unused color for this node



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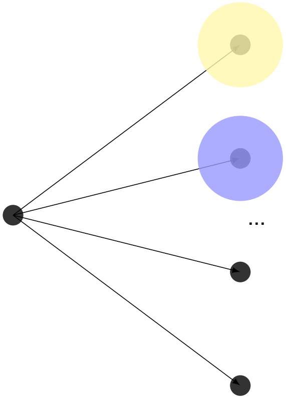

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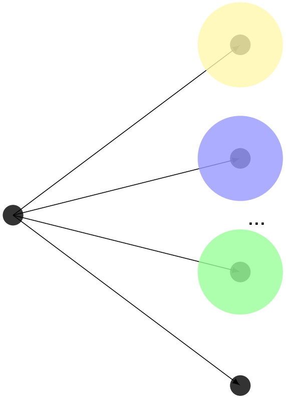

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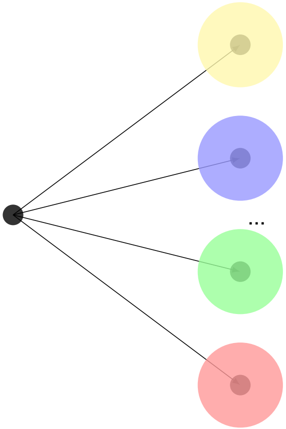

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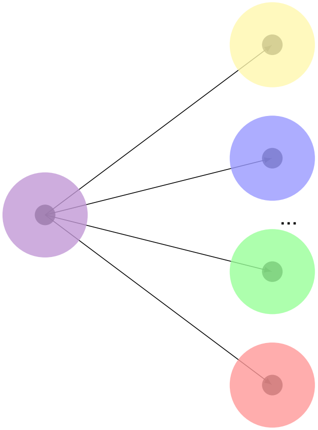


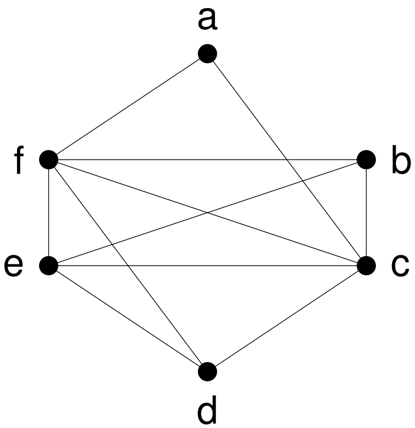
Heuristic Algorithm

To determine if a graph can be colored with **k** colors,

- repeat until there is only one node left
  - Pick a node t with fewer than k neighbors
  - Put t on a stack and remove it and its associated edges from the graph
- Starting assigning colors to nodes on the stack
  - Starting from the last nodes added to the stack
  - Pick a color that is different from its neighbors

Example

- To test if K=4 works in our example



Stack={}

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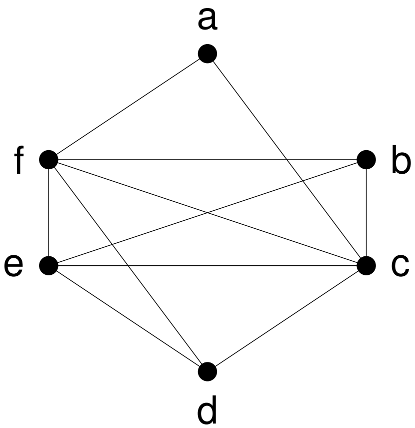
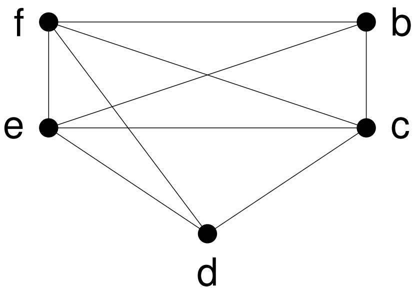


Stack={a}

Example

- To test if K=4 works in our example

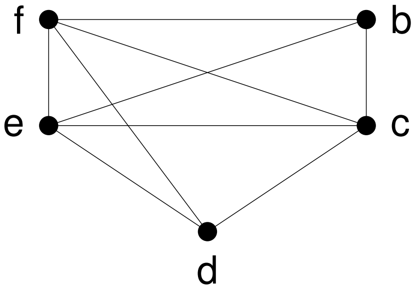
Stack={a}



Example

- To test if K=4 works in our example

Stack={a,d}



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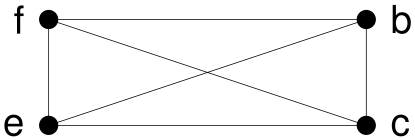
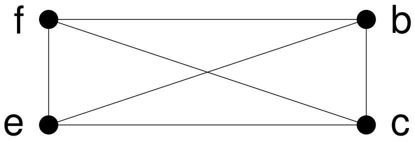


Example

- To test if K=4 works in our example

Stack={a,d,b}



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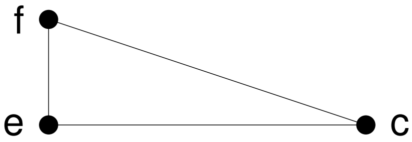

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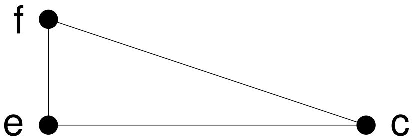

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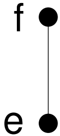
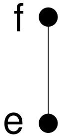


Example

- To test if K=4 works in our example

Stack={a,d,b,c,e}



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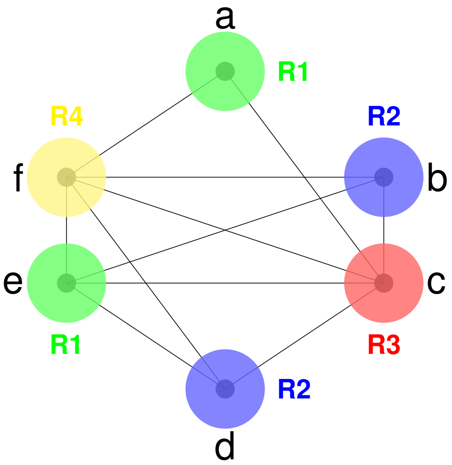
- To test if K=4 works in our example

Stack={a,d,b,c,e}

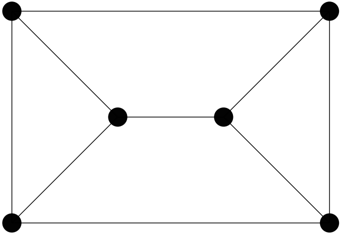
Coloring Results

- Starting assigning colors to **f,e,b,c,d,a**



What if the Heuristic Fails?

- Given the example, is it 3-colorable?
  - Every node has 3 outgoing edges, thus it is not 3-colorable



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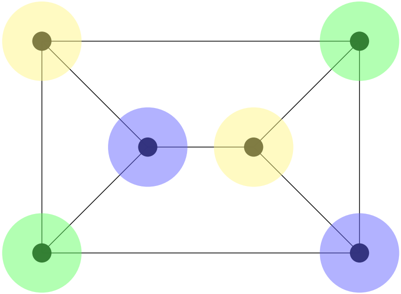

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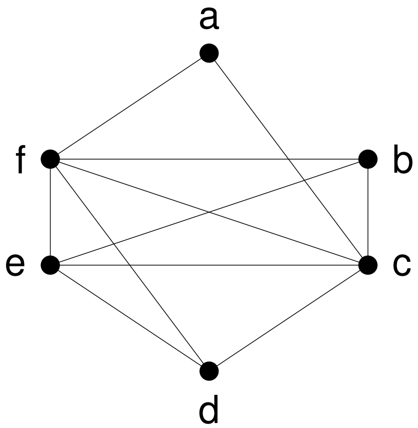
When the Heuristic Fails ...

- Spill the variable to memory
  - a spilled variable temporarily **lives** in memory
  - e.g. to color the previous graph using 3 colors
    - spill "f" into memory



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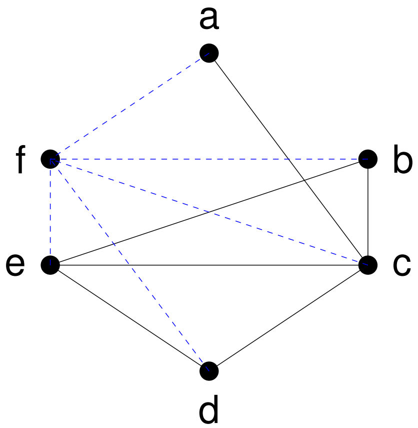

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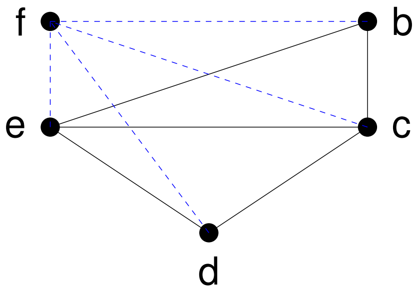

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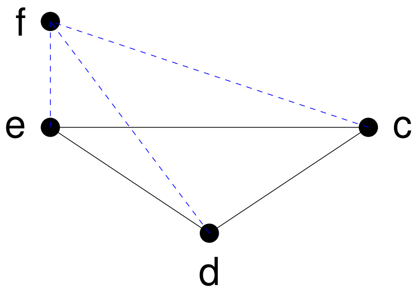

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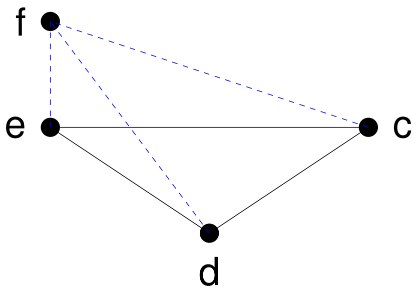
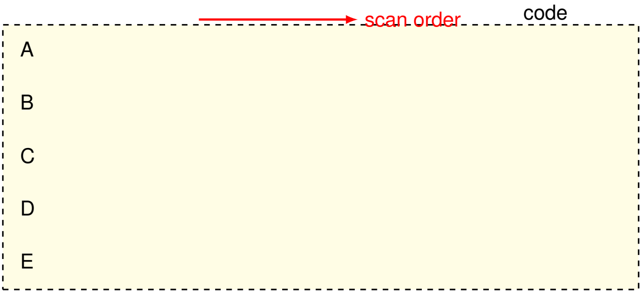


Linear Scan Register Allocation

- On-line compilers need to generate binary code quickly
  - Just-in-time compilation
  - Interactive environments e.g. IDE
  - Dynamic code generation in language extensions
- In these cases, it is beneficial to sacrifice code performance a bit for quicker compilation
  - A faster allocation algorithm
  - Not sacrificing too much

Linear Scan Register Allocation

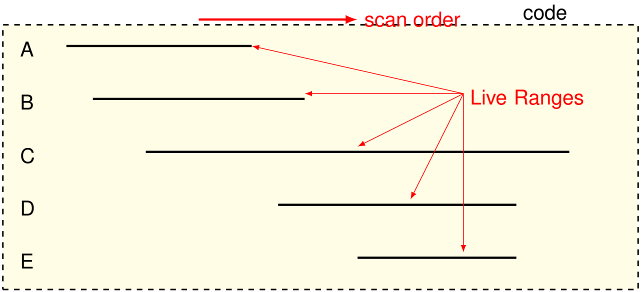
- Live range — the linear code range that a variable is alive



- Allocate at each numbered place
  - **A** and **D** may be allocated to the same register as at location “4”, **A** is dead

Linear Scan Register Allocation

- Live range — the linear code range that a variable is alive



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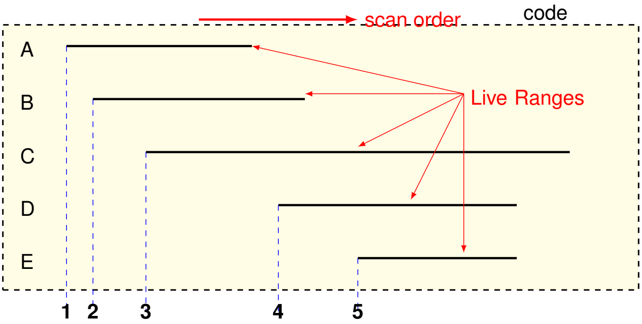

- Allocate at each numbered place
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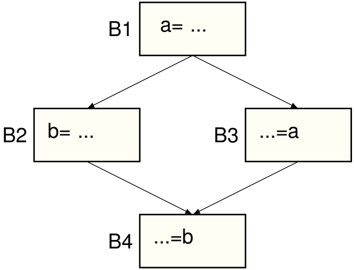
Linear Scan and Live Ranges

- Live range of a = {B1, B3}
- Live range of b = {B2, B4}
  - No interference between a and b, such that only one register is enough
- **However**, if code layout is “B1,B2,B3,B4”, then we need 2 registers



Linear Scan Algorithm

- Linear scan RA consists of four steps

S1. Order the instructions in linear fashion
  - Many have proposed heuristics for finding the best linear order

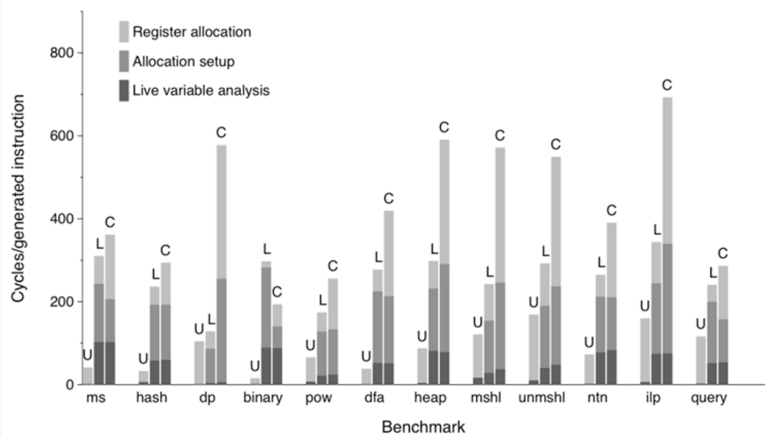
S2. Calculate the set of live intervals
  - Each temporary is given a live interval

S3. Allocate a register to each interval
  - If a register is available then allocation is possible
  - If a register is not available then an already allocated register is chosen (register spill occurs)

S4. Rewrite the code according to the allocation
  - Actual registers replace temporary or virtual registers
  - Spill code is generated

Implementing Type Checking on AST

- **U**sage Counts, **L**inear Scan, and Graph **C**oloring shown
- Linear Scan allocation is always faster than Graph Coloring



ILP-based Register Allocation

- Another types of register allocation that targets at
  - finding “optimal” register allocation result
- Idea and steps:
  1. Convert RA problem to a ILP problem
  2. Solve ILP problem using widely used ILP solvers
  3. Map the ILP solution back to register assignment
- Major problem that restricts its wide adoption
  - ILP problem is NP-hard
  - Solving ILP problem is slow

What is Integer Linear Programming (ILP)?

- Integer Linear Programming (ILP)

  Variables:

  a, b

  Constraints:

  $0 \leq a \leq 10$

  $0 \leq b \leq 29$

  $a + b \leq 36$

  Goal function

  minimize $f(a,b) = 3a + 4b$

- It is trivial if a and b can take real values
- It is NP hard if a and b can only take integer values

How to Convert Register Allocation to ILP?

- An example

  ...
  (10) a = b + ... ;
  ...

  - Want to know to which register b should be allocated i.e.
    load Rx, addr(b)

- Let us form an ILP problem
  - assume there are four free registers R1, R2, R3, R4

S1: Define variables

$X^{Ri}_{var(location)}$ — we allocate **var** at **location** to **Ri**

$X^{R1}_{b(10)}, X^{R2}_{b(10)}, X^{R3}_{b(10)}, X^{R4}_{b(10)}$

Converting Register Allocation to ILP

S2: Constraints: clearly there are constraints for these variables

- $X^{Ri}_{var(location)}$ only takes value 0 or 1
  0 — not allocate to that register at the place
  1 — is allocated to that register at the place
- Any register can hold only one variable at any place
  $X^{R1}_{b(10)} + X^{R1}_{a(10)} \leq 1$
- Any variable just needs to take one register
  $X^{R1}_{b(10)} + X^{R2}_{b(10)} + X^{R3}_{b(10)} + X^{R4}_{b(10)} = 1$
- and many more ...

S3: Define goal function

- to minimize memory operations
  $f_{cost} = (\sum X^{Ri}_{v(mem.p)})$ * $LOAD_{cost}$ + ... (store cost) ...

Conclusion

- Register Allocation is a “must have” optimization in most compilers
  - Because intermediate code uses too many temporaries
  - Because it makes a big difference in performance
- Different algorithms have been developed to satisfy different needs

Instruction Selection

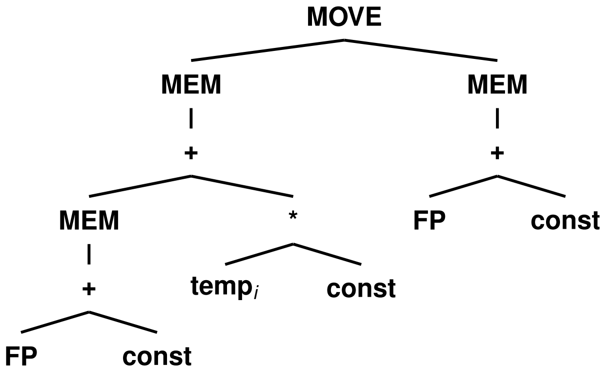
Instruction Selection

- Instruction selection is the task to select appropriate machine instructions to implement the operations in the intermediate representation (IR).
  - ➢ Very important for CISC machines, and machines with special purpose instructions (MMX)
  - ☞ X86, ARM, DSP, ...
- There are many semantically equivalent instruction sequences
  - ➢ How to find the “minimal cost” sequence?

Some Instruction Patterns

Name	Effect	Trees
—	r_i	TEMP
ADD	$d_i \leftarrow d_j + d_k$	d + ; d d
MUL	$d_i \leftarrow d_j \times d_k$	d * ; d d
SUB	$d_i \leftarrow d_j - d_k$	d - ; d d
DIV	$d_i \leftarrow d_j / d_k$	d / ; d d
ADDI	$d_i \leftarrow d_j + c$	d + ; d CONST — d + ; CONST d — d CONST
SUBI	$d_i \leftarrow d_j - c$	d - ; d CONST
MOVEA	$d_j \leftarrow a_i$	d a
MOVED	$a_j \leftarrow d_i$	a d
LOAD	$d_i \leftarrow M[a_j + c]$	d MEM ; + ; a CONST — d MEM ; + ; CONST a — d MEM ; CONST — d MEM ; a
STORE	$M[a_j + c] \leftarrow d_i$	MOVE ; MEM d ; + ; a CONST — MOVE ; MEM d ; + ; CONST a — MOVE ; MEM d ; CONST — MOVE ; MEM d ; a
MOVEM	$M[a_j] \leftarrow M[a_i]$	MOVE ; MEM MEM ; a a

A Parse Tree to be Tiled



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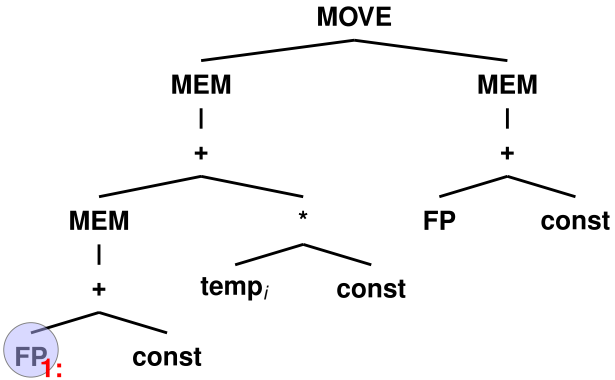

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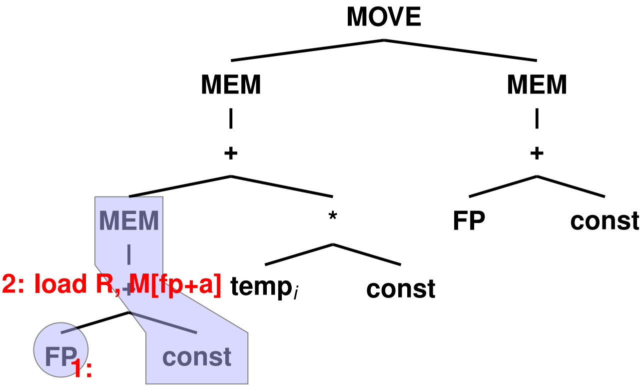

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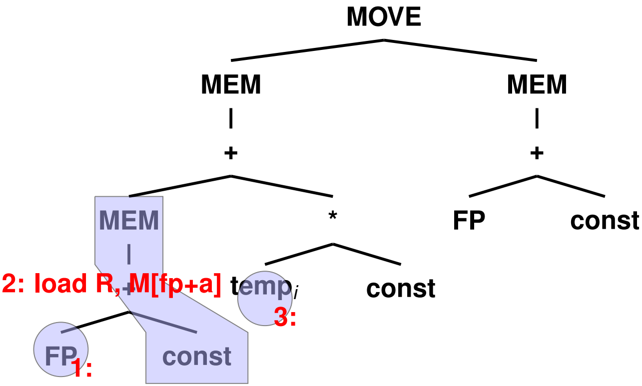

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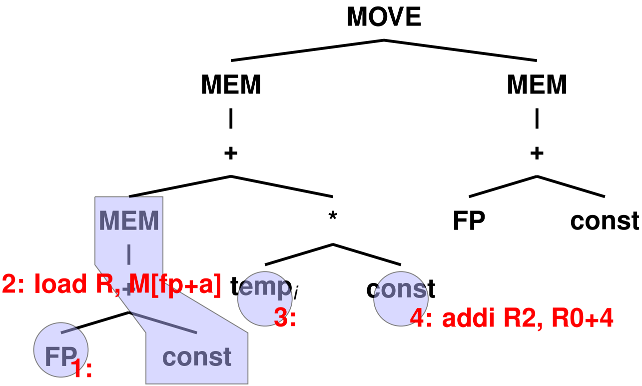

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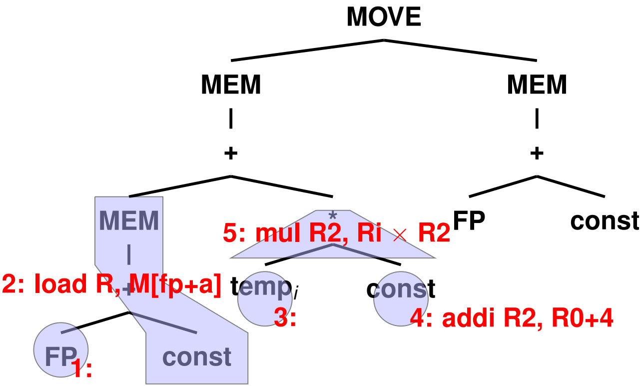

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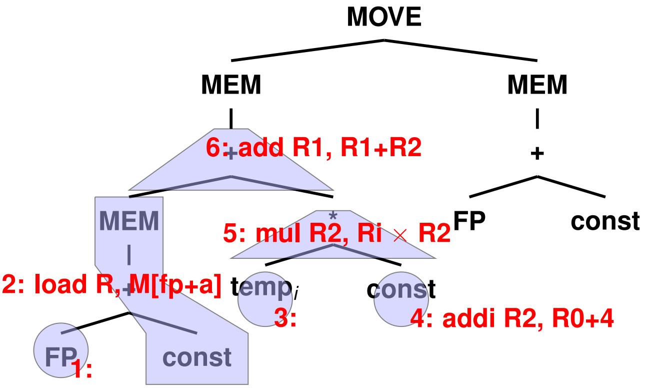

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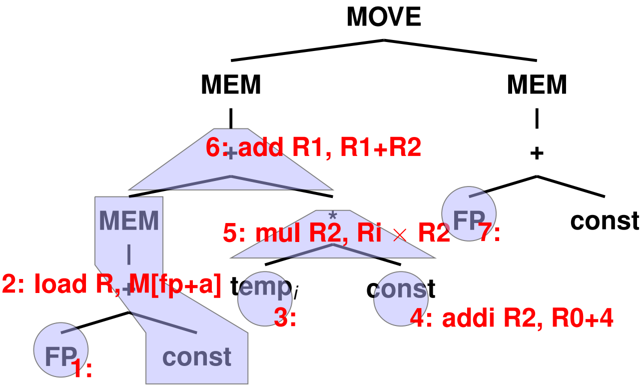

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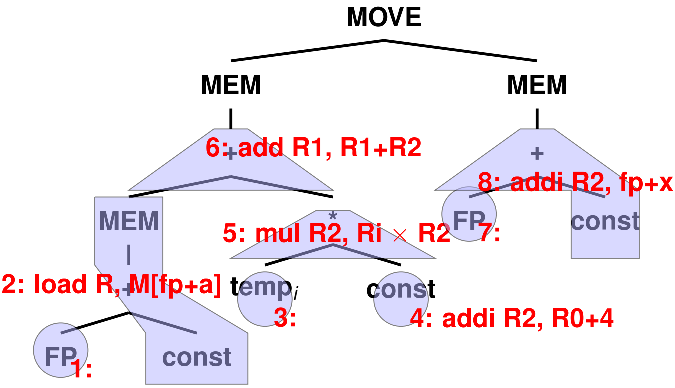

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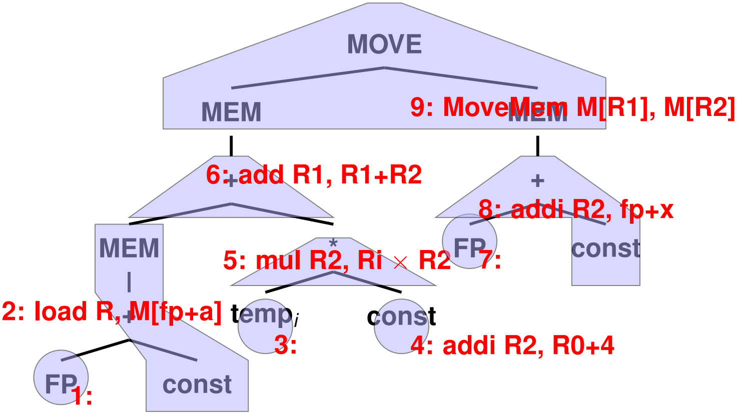

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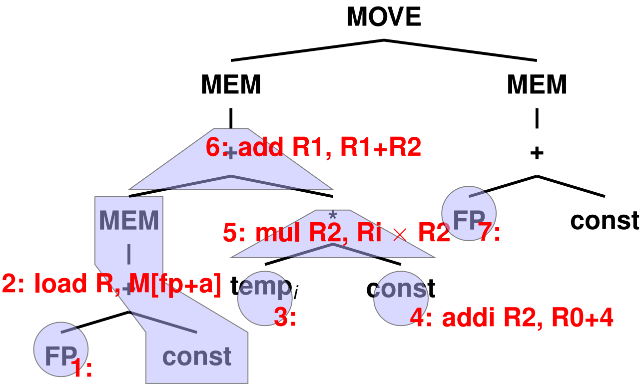

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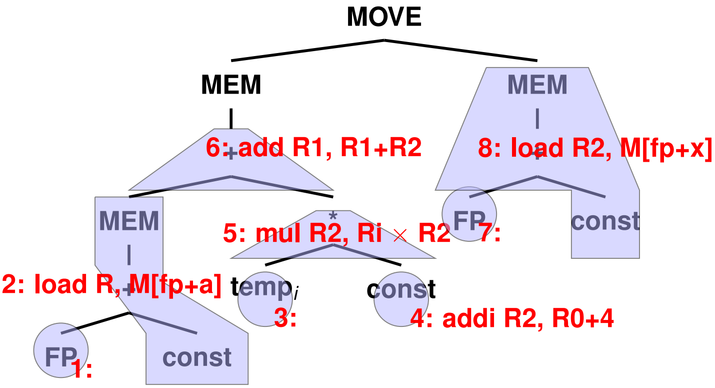

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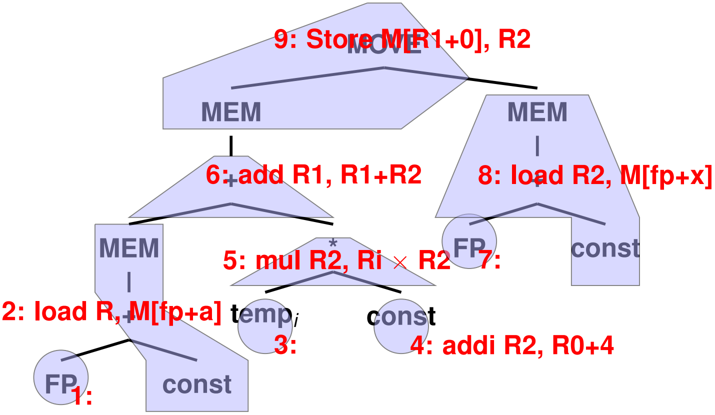

The END !