

Compiler Optimization

Compiler optimizations transform code

- ❑ Code optimization transforms code to equivalent code
 - ... but with better performance

- ❑ The code transformation can involve either
 - **Replacing** code with more efficient code
 - **Deleting** redundant code
 - **Moving** code to a position where it is more efficient
 - **Inserting** new code to improve performance

The four categories of code transformations

- Replacing code (e.g. **strength reduction**)

$A = 2 * a;$ \equiv $A = a \ll 1;$

- Deleting code (e.g. **dead code elimination**)

$A = 2; A = y;$ \equiv $A = y;$

- Moving code (e.g. **loop invariant code motion**)

for (i = 0; i < 100; i++) { sum += i + $x * y$; }

\equiv

$t = x * y;$

for (i = 0; i < 100; i++) { sum += i + t ; }

- Inserting code (e.g. **data prefetching**)

for (p = head; p != NULL; p = p->next)
{ /* do work on node p */ }

\equiv

for (p = head; p != NULL; p = p->next)
{ $\text{prefetch}(p \rightarrow \text{next})$; /* do work on node p */ }

Compiler optimization categories according to range

- ❑ How much code does the compiler view while optimizing?
 - The wider the view, the more powerful the optimization

- ❑ Axis 1: optimize across control flow?
 - **Local optimization**: optimizes only within straight line code
 - **Global optimization**: optimizes across control flow (if,for,...)

- ❑ Axis 2: optimize across function calls?
 - **Intra-procedural optimization**: only within function
 - **Inter-procedural optimization**: across function calls

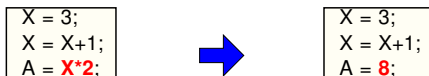
- ❑ The two axes are orthogonal (any combination is possible)

Local vs. Global Constant Propagation

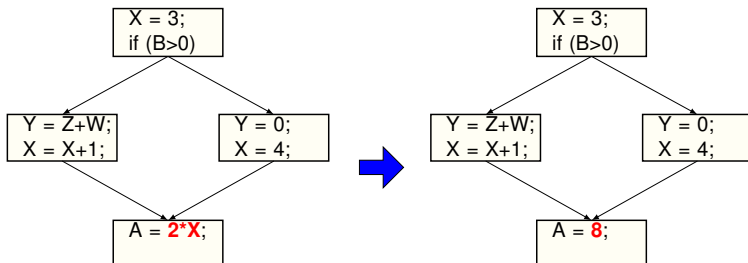
Constant propagation

- Optimization: if $x = y \text{ op } z$ and y and z are constants then compute at compile time and replace

Local Constant Propagation

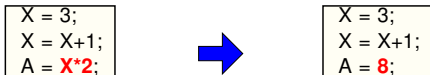


Global Constant Propagation

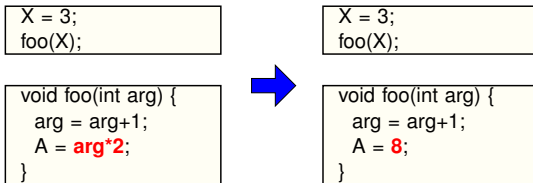


Intra- vs. Inter-procedural Constant Propagation

□ Intra-procedural Constant Propagation



□ Inter-procedural Constant Propagation



➤ Assuming all other calls to foo always pass in constant 3

Control Flow Analysis

Basic Block

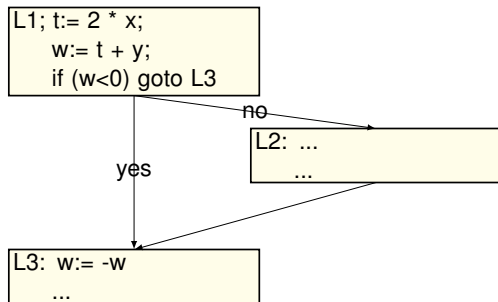
- ❑ A function body is composed of one or more **basic blocks**.
- ❑ **Basic block**: a maximal sequence of instructions that
 - Has no jumps into the block other than the first instruction
 - Has no jumps out of the block other than the last instruction
- ❑ That means:
 - No instruction other than the first is a jump target
 - No instruction other than the last is a jump or branch
- ❑ Either all instructions in basic block execute or none
 - Smallest unit of execution in control flow analysis
 - Hence the descriptor "basic" in the name

Control Flow Graph

- ❑ A **Control Flow Graph (CFG)** is a directed graph in which
 - Nodes are basic blocks
 - Edges represent flows of execution between basic blocks
- ❑ CFGs are widely used to represent a program for analysis
- ❑ CFGs are especially essential for global optimizations

Control Flow Graph Example

```
L1; t:= 2 * x;  
    w:= t + y;  
    if (w<0) goto L3  
L2: ...  
    ...  
L3: w:= -w  
    ...
```



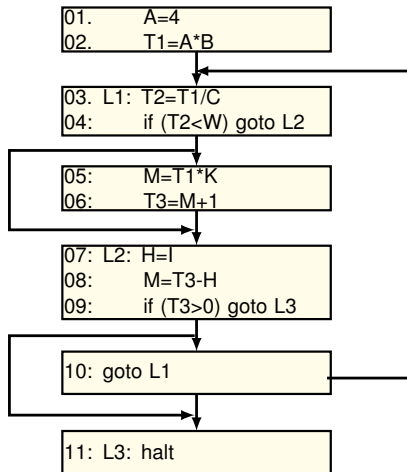
Construction of CFG

- ❑ Step 1: partition code into basic blocks
 - Identify **leader** instructions, where a leader is either:
 - the first instruction of a program, or
 - the target of any jump/branch, or
 - an instruction immediately following a jump/branch
 - Create a basic block out of each leader instruction
 - Expand basic block by adding subsequent instructions (Stopping when the next leader instruction is encountered)

- ❑ Step 2: add edge between two basic blocks B1 and B2 if
 - there exist a jump/branch from B1 to B2, or
 - B2 follows B1, and B1 does not end with jump/branch

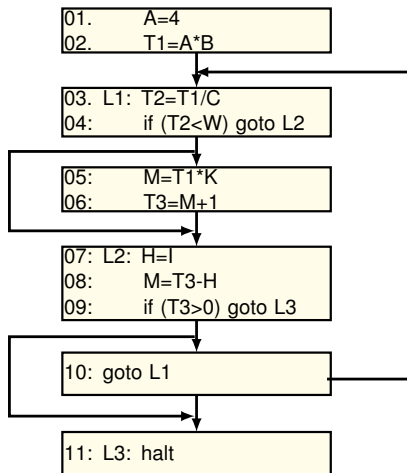
Example

```
01.    A=4
02.    T1=A*B
03. L1: T2=T1/C
04.    if (T2<W) goto L2
05.    M=T1*K
06.    T3=M+1
07. L2: H=I
08.    M=T3-H
09.    if (T3>0) goto L3
10.    goto L1
11. L3: halt
```



Example

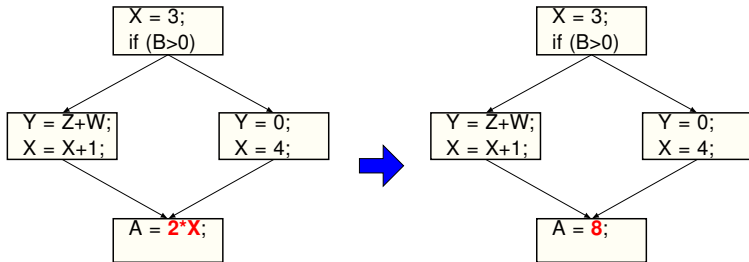
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Data Flow Analysis

Global Optimizations

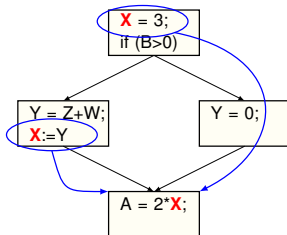
- Extend optimizations across control flows, i.e. CFG
- Like in this example of Global Constant Propagation (GCP):



- How do we know it is OK to globally propagate constants?

Correctness criteria for GCP

- There are situations that prohibit GCP:



- To replace `X` by a constant `C` **correctly**, we must know
 - **Along all paths**, the last assignment to `X` is "`X = C`"
- Paths may go through loops and/or branches
 - When two paths **meet**, need to make a **conservative** choice

Global Optimizations need to be Conservative

- ❑ Many compiler optimizations depend on knowing some property X at a particular point in program execution
 - Need to prove at that point property X holds along all paths

- ❑ To ensure correctness, optimization must be **conservative**
 - An optimization is enabled only when X is definitely true
 - If not sure, be conservative and say **don't know**
 - **Don't know** usually disables the optimization

Dataflow Analysis Framework

❏ **Dataflow analysis:** discovering properties about values

- ... at certain points in the CFG to enable optimizations
- E.g. discovering a value is constant at a statement
- Done by observing the flow of data through the CFG

❏ **Dataflow analysis framework:**

- A framework for describing different dataflow analyses
- Can be defined using the following 4 components:

$$\{ \mathbf{D}, \mathbf{V}, \wedge: (\mathbf{V}, \mathbf{V}) \rightarrow \mathbf{V}, \mathbf{F}: \mathbf{V} \rightarrow \mathbf{V} \}$$

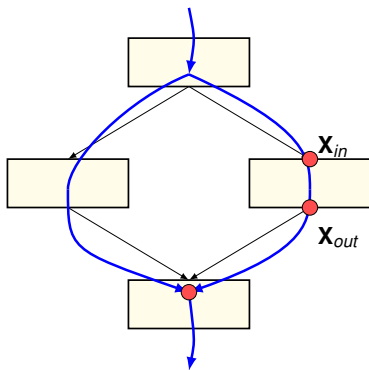
- **D**: direction of dataflow (forward or backward)
- **V**: domain of values denoting property
- \wedge : **meet operator** that merges values when paths meet
- **F**: **flow propagation function** that propagates values through a basic block

Global Constant Propagation (GCP)

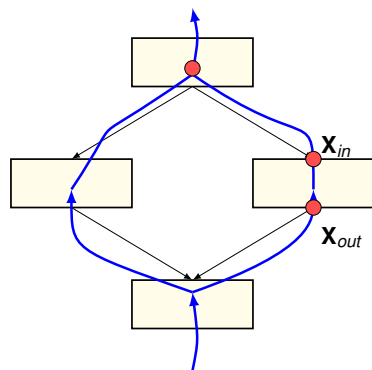
- Let's use **GCP** to study dataflow analysis framework
- We will define each component one by one for GCP
 - **D**: direction of dataflow for constant property
 - **V**: domain of values denoting constant property
 - \wedge : **meet operator** that merges values when paths meet
 - **F**: **flow propagation function** for GCP

Direction D for GCP

Is GCP a forward or backward analysis?



Forward Analysis



Backward Analysis

Forward, since "constantness" of a variable flows forward to subsequent instructions starting from assignment

V and meet operator \wedge for GCP

- Given an integer variable x , domain V is the set:

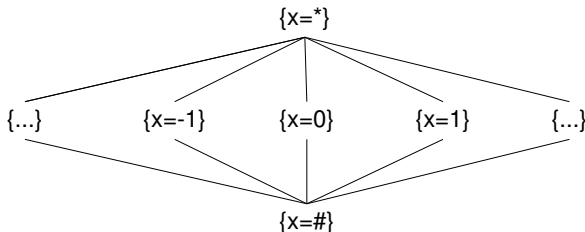
..., $\{x=-1\}$, $\{x=0\}$, $\{x=1\}$, ... /* a constant */

$\{x=*\}$ /* not a constant */

$\{x=\#\}$ /* x is not assigned on any path */

- Meet operator \wedge is given by this **semi-lattice**:

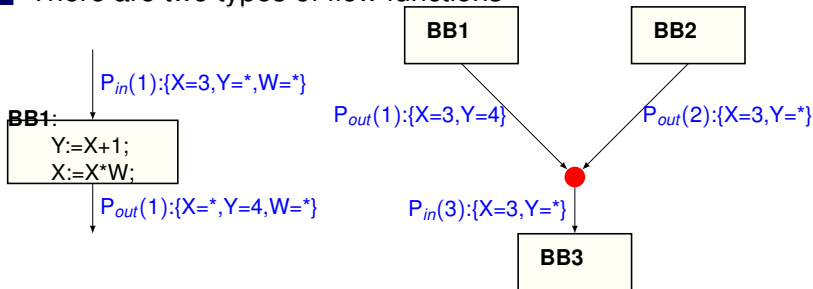
➤ $a \wedge b$ = least upper bound in the below graph



- $\{x=0\} \wedge \{x=1\} = \{x=*\}$: Different values on each path
- $\{x=\#\} \wedge \{x=1\} = \{x=1\}$: Constant definition on one path

Dataflow Equations for GCP

There are two types of flow functions



- Flow transfer function $F: V \rightarrow V$
 - Computes data flow within basic blocks
 - Remove those that become variables, add new constants
- Meet operator $\wedge: (V, V) \rightarrow V$
 - Computes data flow across basic blocks
 - Merge values from two paths using the previous semi-lattice

Flow Transfer Function F for GCP

□ **X(i)**: dataflow property X of basic block i

➤ **X_{in}(i)**: at the entry of basic block i

➤ **X_{out}(i)**: at the exit of basic block i

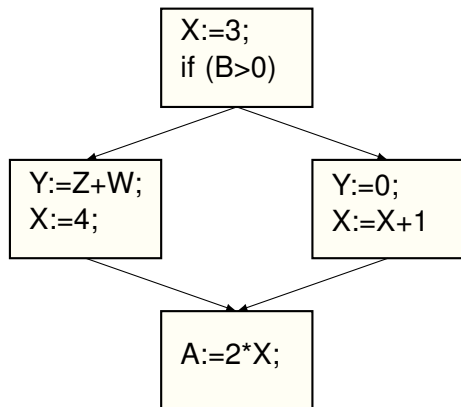
□ F for Global constant propagation (GCP)

$$\mathbf{GCP}_{out}(i) = (\mathbf{GCP}_{in}(i) - \mathbf{DEF}_v(i)) \cup \mathbf{DEF}_c(i)$$

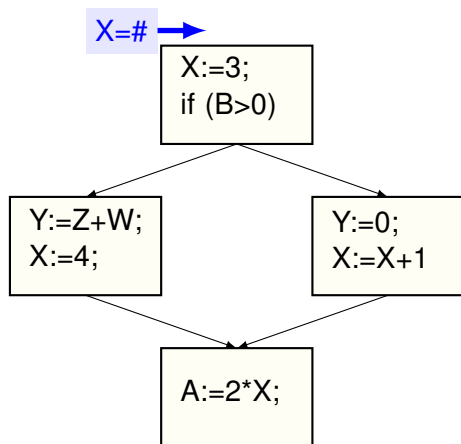
where $\mathbf{DEF}_v(i)$ contains variable definitions in basic block i

$\mathbf{DEF}_c(i)$ contains constant definitions in basic block i

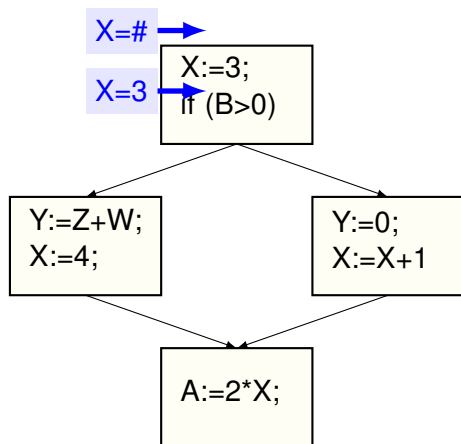
GCP Propagation without loops



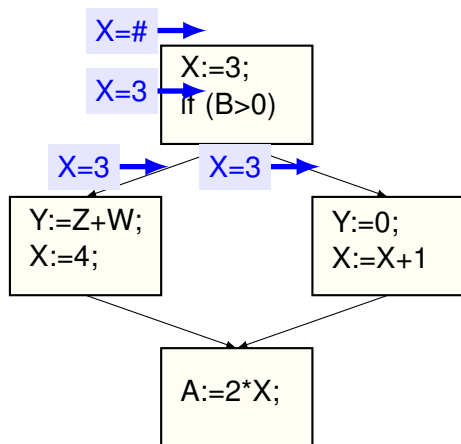
GCP Propagation without loops



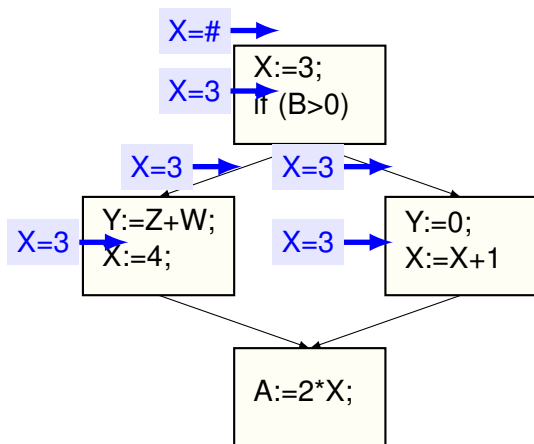
GCP Propagation without loops



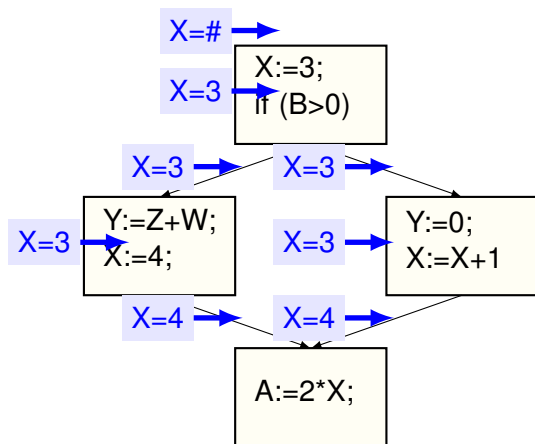
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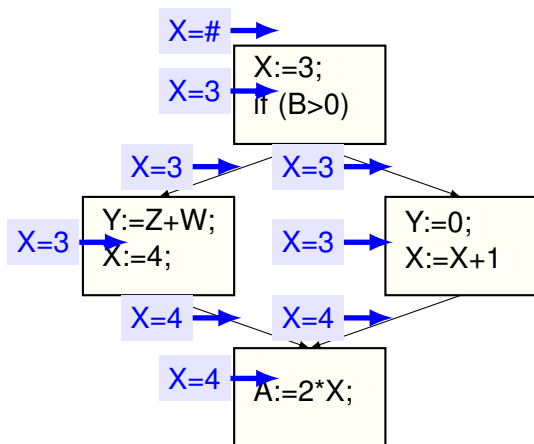
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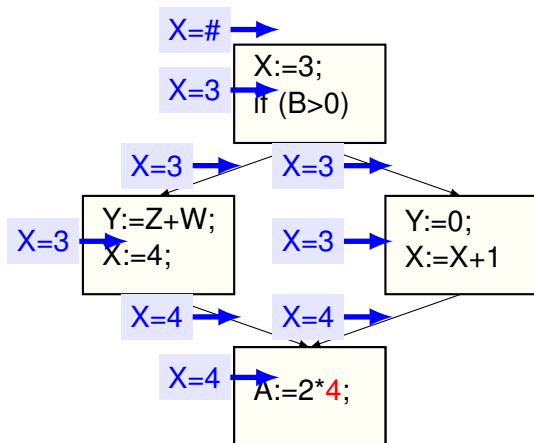
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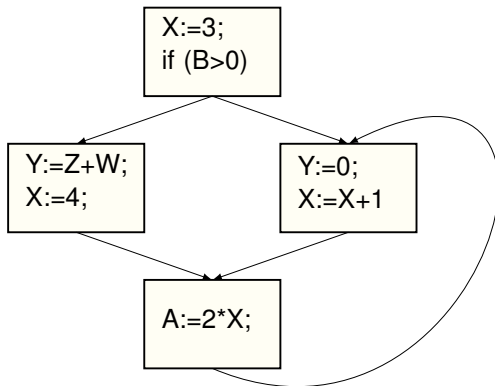


GCP Propagation without loops



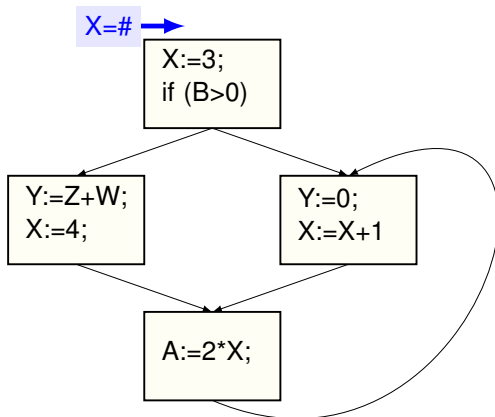
GCP Propagation with loops

- Iterate until there are no changes to values
 - This is called the **maximum fixed point** solution



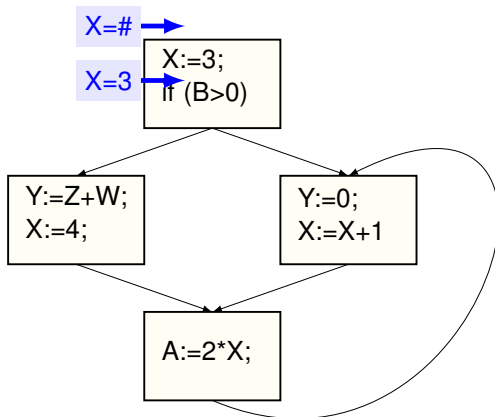
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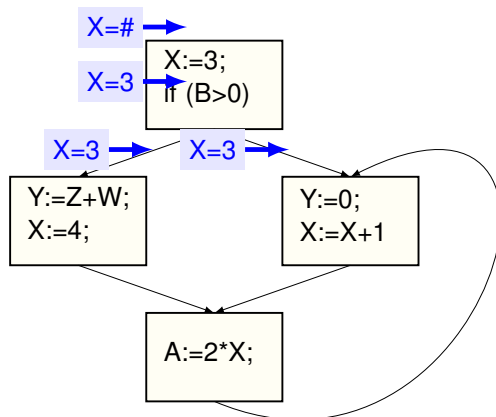
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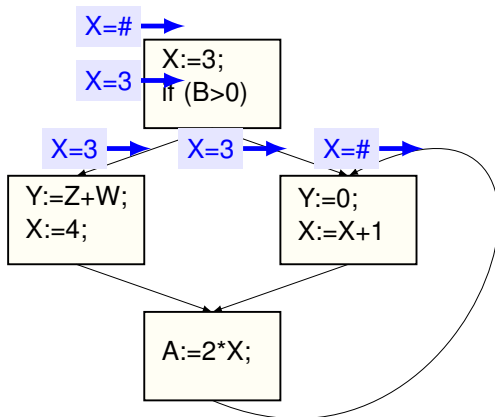
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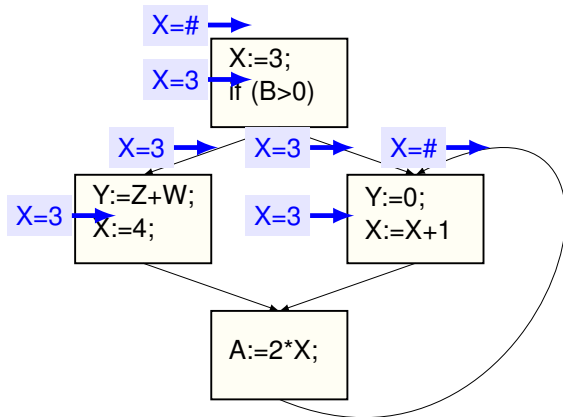
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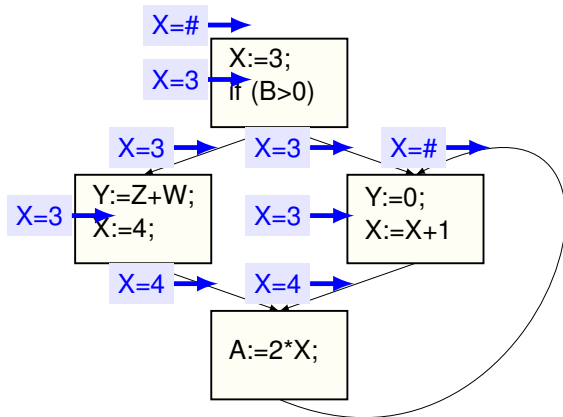
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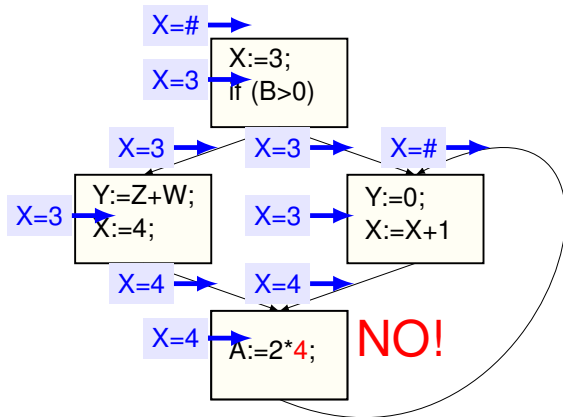
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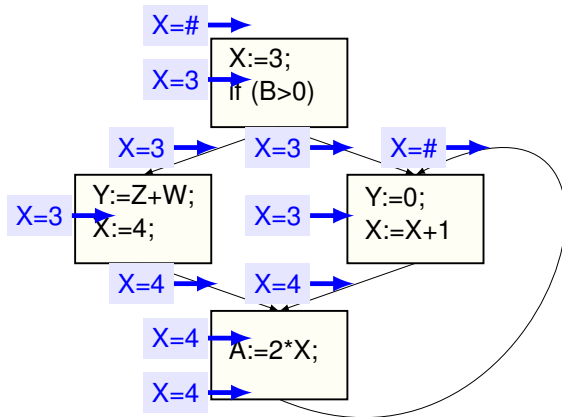
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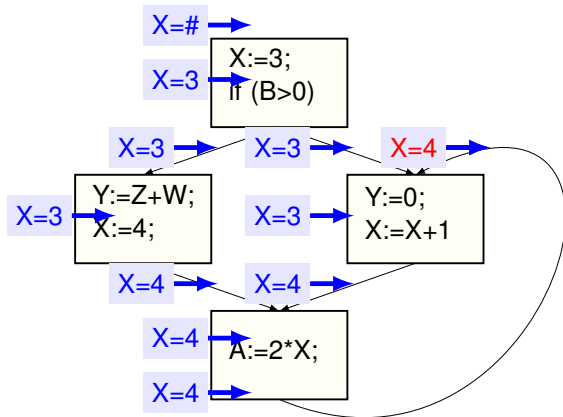
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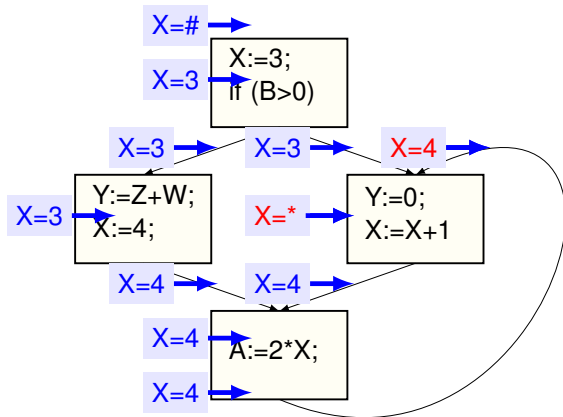
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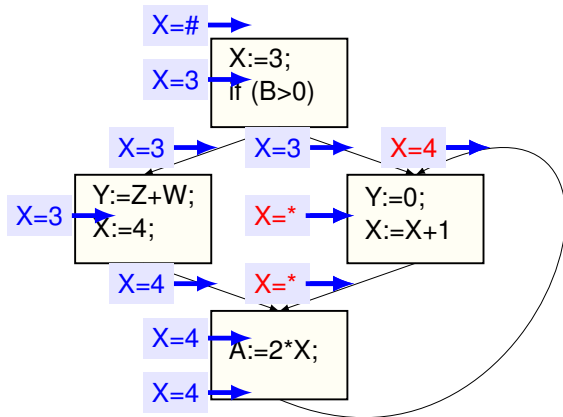
GCP Propagation with loops

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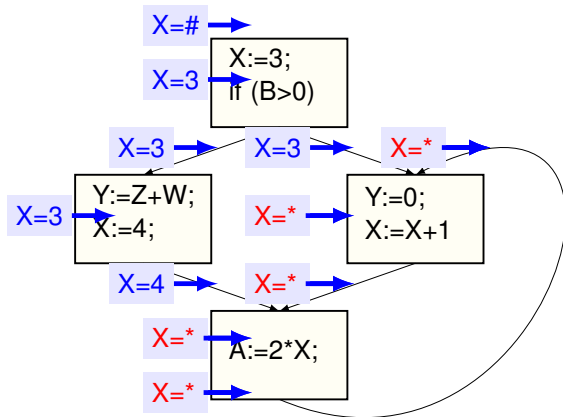
GCP Propagation with loops

- Iterate until there are no changes to values
 - This is called the **maximum fixed point** solution



GCP Propagation with loops

- Iterate until there are no changes to values
 - This is called the **maximum fixed point** solution



Analysis Algorithm for GCP

□ GCP Algorithm

- (1). Set $\{x=\#\}$ at all the points in the procedure
- (2). Propagate the dataflow property along the control flow
- (3). Repeat step (2) until there are no changes

□ Will GCP eventually stop?

- If there are loops, we may propagate the loop many times
- Is there a possibility to run into an endless loop?

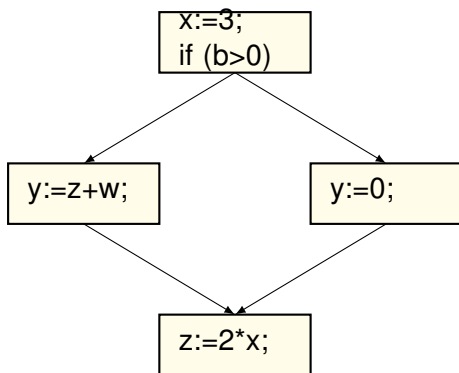
Termination Problem

- ❑ **Least upper bound** ensures the termination
 - Values starts from #
 - Values can only increase in the hierarchy
 - Any values can change at most twice in our example
... from # to C, and from C to *

- ❑ The maximal number of steps is $O(\text{program_size})$

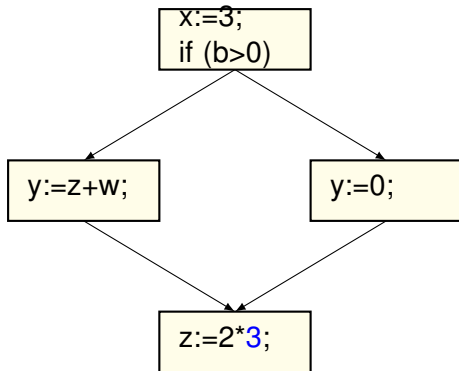
Another Analysis: Liveness Analysis

- Once constants have been globally propagated, we would like to eliminate the dead code



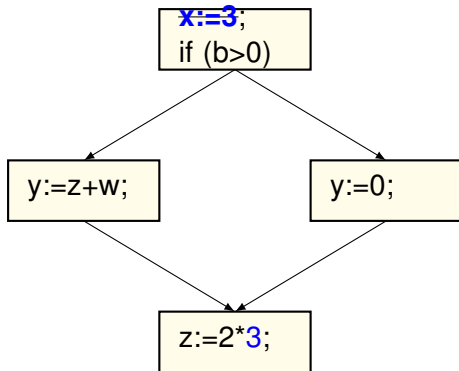
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Another Analysis: Liveness Analysis

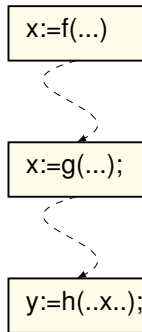
- Once constants have been globally propagated, we would like to eliminate the dead code



Live/Dead Statment

- ❑ A **dead statement** calculates a value that is not used later, or output to file
- ❑ Otherwise, it is a **live statement**

In the example,
the 1st statement is dead,
the 2nd statement is live



Liveness Analysis

- ❏ We can form liveness analysis as a dataflow analysis
 - Propagate the information along control flow
 - “x is dead”, “x is live”, “y is dead”, “y is live”
 - Liveness is simpler than constant propagation
 - It is a boolean property
 - Liveness is different from constant propagation
 - Liveness analysis is a union problem
 - ☞ x is alive if x is alive along one path
 - Constant propagation is an intersection problem
 - ☞ x is NOT a constant if x is NOT a constant along one path
 - ...

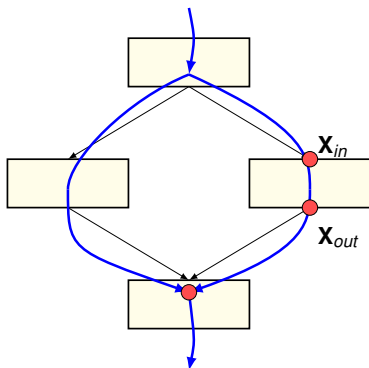
Forward and Backward Analysis



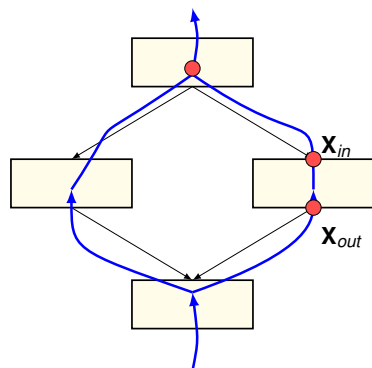
The most significant difference is

- Liveness analysis wants to know if it is used some time later
 - Use information after this statement to decide
 - **Backward analysis**
- Constant analysis wants to know if it is constant for all possible executions at this point
 - Use information before this statement to decide
 - **Forward analysis**

Graphic View of Forward and Backward Analysis



Forward Analysis



Backward Analysis

Global Liveness Analysis

□ Global liveness analysis

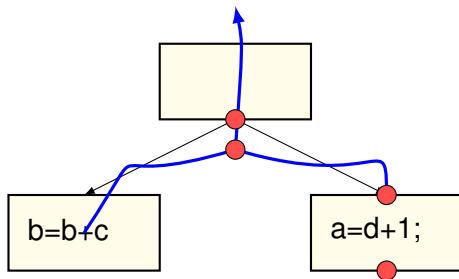
- A variable x is live at statement s if
 - There exists a statement ss after s that use x
 - There is a path from s to ss
 - That path has no intervening assignment to x

□ A backward dataflow analysis

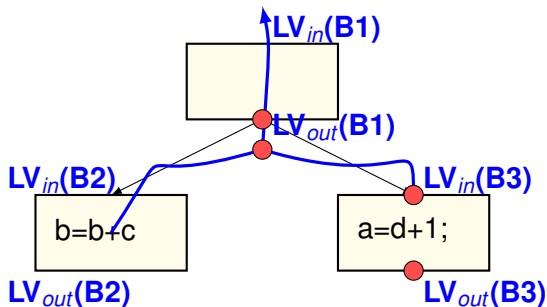
$\{\mathbf{L}, (\top, \perp), (\sqcap, \sqcup), \mathbf{f}: \mathbf{L} \rightarrow \mathbf{L}\}$

- Lattice, top and bottom items
- Operators
- Dataflow functions

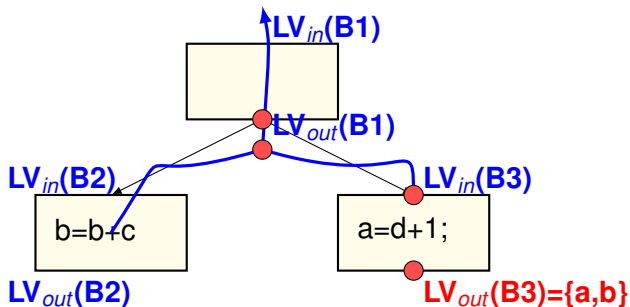
Liveness Example



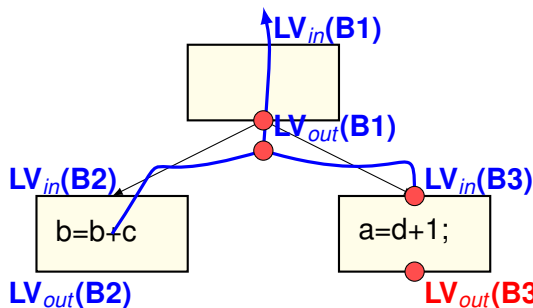
Liveness Example



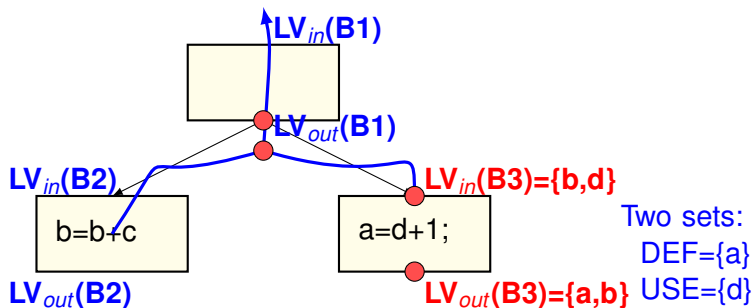
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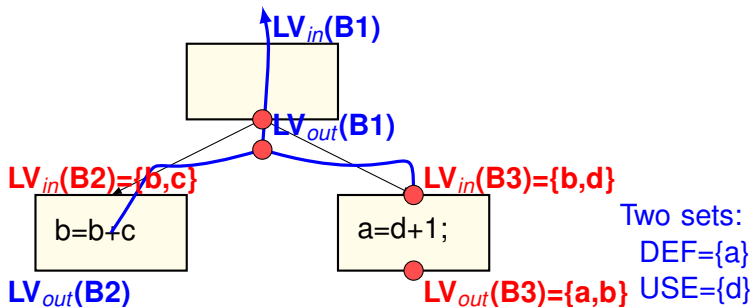
Liveness Example



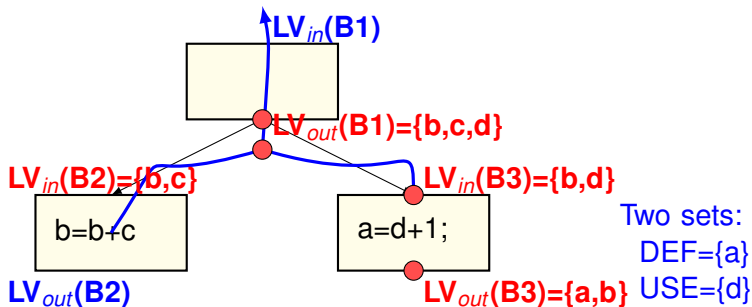
Liveness Example



Liveness Example



Liveness Example



Dataflow Equations for Liveness Analysis

❏ **X(i)** — dataflow property of basic block **i**

➤ **X_{in}(i)** — at the entry of basic block **i**

➤ **X_{out}(i)** — at the exit of basic block **i**

❏ Flow equations

➤ flow transfer function

$$\mathbf{LV}_{in}(\mathbf{i}) = (\mathbf{LV}_{out}(\mathbf{i}) - \mathbf{DEF}(\mathbf{i})) \cup \mathbf{USE}(\mathbf{i})$$

➤ flow propagation function

$$\mathbf{LV}_{out}(\mathbf{i}) = \cup \mathbf{LV}_{in}(\mathbf{k}) \quad \text{where } \mathbf{k} \text{ is successor of } \mathbf{i}$$

Comparison with Dataflow Equations for GCP

□ **X(i)** — dataflow property of basic block **i**

➤ **X_{in}(i)** — at the entry of basic block **i**

➤ **X_{out}(i)** — at the exit of basic block **i**

□ Global constant propagation (GCP)

➤ flow transfer function

$$\mathbf{GCP}_{out}(i) = (\mathbf{GCP}_{in}(i) - \mathbf{DEF}_v(i)) \cup \mathbf{DEF}_c(i)$$

where $\mathbf{DEF}_v(i)$ contains variable definitions in basic block **i**

$\mathbf{DEF}_c(i)$ contains constant definitions in basic block **i**

➤ flow propagation function

$$\mathbf{GCP}_{in}(i) = \cap \mathbf{GCP}_{out}(k) \quad \text{where } k \text{ is predecessor of } i$$

Application of Liveness Analysis

❑ Global dead code elimination is based on global liveness analysis (GLA)

➤ Dead code detection

- A statement $x := \dots$ is dead code if x is dead after this statement
- Dead statement can be deleted from the program (Didn't consider side-effect)

❑ Global register allocation is also based on GLA

- Live variables should be placed in registers
- Registers holding dead variables can be reused

Summary of Dataflow Analysis

- ❑ There are many other global dataflow analysis
- ❑ Classification
 - Forward/Backward analysis
 - Union analysis — some property is true if it is true along at least one path
 - Intersection analysis — some property is true if it is true along all paths
- ❑ Very useful in
 - compiler optimization
 - software engineering
 - debugging

The END !