

Lexical Analysis

What is Lexical Analysis

□ What do we want to do?

Example:

```
if (i==j)
    z = 0;
else
    z = 1;
```

□ The input is just a string of characters

"if(i = j)\n\t\tz = 0; \telse\n\t\tz = 1; \n"

□ Goal: partition input string into substrings

➤ where these substrings are **tokens**

What is a Token ?

- ❑ A syntactic category of entities over alphabet
 - In English:
noun, verb, adjective, ...
 - In a programming language:
identifier, integer, keyword, whitespace, ...

- ❑ Tokens correspond to sets of strings
 - Identifier: strings of letters and digits, starting with a letter
 - Integer: a non-empty string of digits
 - Keyword: “else”, “if”, “while”, ...
 - Whitespace: a non-empty sequence of blanks, newlines, and tabs

What are Tokens For ?

- ❑ Classify program substrings according to role
- ❑ Output of lexical analysis is a stream of tokens
- ❑ Tokens are the input to the Parser
 - Parser relies on token distinctions
a keyword is treated differently than an identifier

Lexical Analyzer in FORTRAN

- ❑ FORTRAN compilation rule: **whitespace is insignificant**
 - rule was motivated from the inaccuracy of card punching by operators
- ❑ Consider
 - DO 5I=1,25
 - DO 5I=1.25
- ❑ We have
 - The first: a loop iterates from 1 to 25 with step 5
 - The second: an assignment
- ❑ Reading left-to-right, cannot tell if DO5I is a variable or DO statement; Have to continue until “,” or “.” is reached.

Lesson Learned

Two important observations:

- The goal is to partition the string. This is implemented by reading left-to-right, recognizing one token at a time.
- “lookahead” may be required to decide where one token ends and the next one begins.

Lexical Analysis in C++

- Unfortunately, the problems continue today

- C++ template syntax

`FOO<Bar>`

- C++ stream syntax

`cin> >var`

- Now, the problem

`FOO<Bar<Bazz> >`

Regular Languages

- ❑ Unfortunately, the problems continue today
- ❑ To describe tokens, we adopt **Regular Languages** formalism
 - Simple and useful theory
 - Easy to understand
 - Efficient implementations

Languages

Definition

Let Σ be a set of characters, a **language** over Σ is a set of strings of the characters drawn from Σ

Examples of Languages



Alphabet = English characters
Language = English sentences



Alphabet = ASCII
Language = C programs

- Not every string on English characters is an English sentence
- Note all ASCII strings are valid C programs

Notation

- ❑ Languages are **sets of strings**
- ❑ Need some notation for specifying which set we want to designate a language
 - Regular languages are those with some special properties
 - The standard notation for regular language is **regular expression**

Atomic Regular Expressions

- Single character denotes a set of one string
 $'c' = \{ "c" \}$
- Epsilon* or ϵ character denotes a zero length string $\epsilon = \{ "" \}$
- Empty set is $\{ \} = \phi$, not the same as ϵ
 $\text{size}(\phi) = 0$
 $\text{size}(\epsilon) = 1$
 $\text{length}(\epsilon) = 0$

Compound Regular Expressions

- Union: if A and B are REs, then

$$A + B = \{ s \mid s \in A \text{ or } s \in B \}$$

- Concatenation of sets/strings

$$AB = \{ ab \mid a \in A \text{ and } b \in B \}$$

- Iteration (Kleene closure)

$$A^* = \bigcup_{i \geq 0} A^i \quad \text{where } A^i = A \dots A \text{ (} i \text{ times)}$$

in particular

$$A^* = \{ \epsilon \} + A + AA + AAA + \dots$$

$$A^+ = A + AA + AAA + \dots = A A^*$$

Regular Expressions

Definition

The **regular expressions (REs)** over Σ are the total set of expressions that can be constructed using the following components:


- ϵ
- 'c' where $c \in \Sigma$
- $A + B$ where A, B are **RE** over Σ
- AB where A, B are **RE** over Σ
- A^* where A is a **RE** over Σ

Notation

 This notation means

- $L(\varepsilon) = \{ \text{""} \}$
- $L('c') = \{ \text{"c"} \}$
- $L(A+B) = L(A) \cup L(B)$
- $L(AB) = \{ ab \mid a \in L(A) \text{ and } b \in L(B) \}$
- $L(A^*) = \bigcup_{i \geq 0} L(A^i)$

Examples

 Keywords: “else” or “if” or “while” or ...

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➤ ‘else’ + ‘if’ + ‘while’ + ...

➤ ‘else’ abbreviates

‘e’ (concatenate) ‘l’ (concatenate) ‘s’ (concatenate) ‘e’

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Integer

- digit = ‘0’ + ‘1’ + ‘2’ + ‘3’ + ‘4’ + ‘5’ + ‘6’ + ‘7’ + ‘8’ + ‘9’
- integer = digit digit*

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- integer = digit digit*
 - **Q:** is ‘000’ an integer?
 - **Q:** how to define another integer RE that excludes sequences with leading 0s?

More Examples

❏ Identifier: strings of letters or digits, starting with a letter

- letter = 'A' + ... + 'Z' + 'a' + ... + 'z'
- Identifier = letter (letter + digit)*

❏ Whitespace: a non-empty sequence of blanks, newlines and tabs

- whitespace = (' ' + '\n' + '\t') +

More Examples

❑ Identifier: strings of letters or digits, starting with a letter

- letter = 'A' + ... + 'Z' + 'a' + ... + 'z'
- Identifier = letter (letter + digit)*
 - **Q:** is (letter* + digit*) the same?

❑ Whitespace: a non-empty sequence of blanks, newlines and tabs

- whitespace = (' ' + '\n' + '\t') +

More Examples

☐ Phones number: consider (412) 624-0000

- $\Sigma = \text{digit} \cup \{ -, (,) \}$
- $\text{area} = \text{digit}^3$
- $\text{exchange} = \text{digit}^3$
- $\text{phone} = \text{digit}^4$
- $\text{phoneNumber} = '(' \text{ area } ')' \text{ exchange } '-' \text{ phone}$

☐ Email address: student @ pitt.edu

- $\Sigma = \text{letter} \cup \{ ., @ \}$
- $\text{name} = \text{letter}^+$
- $\text{emailAddress} = \text{name } '@' \text{ name } '.' \text{ name}$

More Examples in Practice

RE used in languages

- By itself, it is a string, but semantically gets interpreted as a RE

- RE in PERL,

`if ($str =~ /\d+/) ...`

here,

- `$str` denotes a variable
- `=~` denotes RE matching
- `(\d+)` defines a RE pattern

- RE in C#,

`Match m = Regex.Match("abrabceaab", "(a|b|r)+");`

Some Common REs in Programming Languages

	Meaning		Meaning		Meaning
<code>\d</code>	Digits	<code>\w</code>	Any word char	<code>\s</code>	Space char
<code>\D</code>	Non-digits	<code>\W</code>	Non-word char	<code>\S</code>	Non-space char
<code>[a-f]</code>	Char range	<code>[^a-f]</code>	Exclude range	<code>^</code>	Matching string start
<code>?</code>	Optional	<code>{n,m}</code>	Appear n-m times	<code>\$</code>	Matching string end
<code>.</code>	Any char	<code>(...)</code>	Capture matches	<code>\(,\{</code>	Matching (, { ...
<code>\.</code>	Matching “.”	<code>+</code>	Appear ≥ 1 times	<code>*</code>	Appear 0 or many times

Implementation of Lexical Analysis

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- We have learnt the formalism for lexical analysis
 - Regular expression (RE)

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- How to actually get the lexical analyzer?

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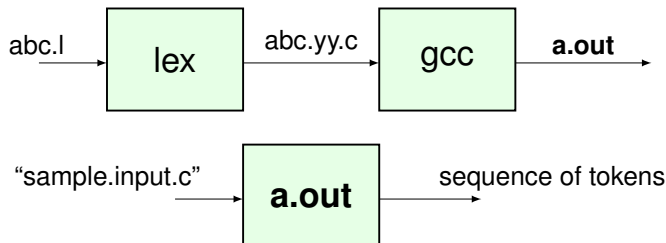
- ❑ How to actually get the lexical analyzer?
 - **Solution 1:** to implement using a tool — Lex (for C), Flex (for C++), Jlex (for java)
 - Programmer specifies the interesting tokens using REs
 - The tool generates the source code from the given REs

Implementation of Lexical Analysis

- ❑ We have learnt the formalism for lexical analysis
 - Regular expression (RE)

- ❑ How to actually get the lexical analyzer?
 - **Solution 1:** to implement using a tool — Lex (for C), Flex (for C++), Jlex (for java)
 - Programmer specifies the interesting tokens using REs
 - The tool generates the source code from the given REs
 - **Solution 2:** to write the code starting from scratch
 - This is also the code that the tool generates
 - The code is table-driven and is based on finite state automaton

Lex: a Tool for Lexical Analysis



- ❑ Big difference from your previous coding experience
 - Writing REs instead of the code itself
 - Writing actions associated with each RE
- ❑ A specification file has well-defined structures
- ❑ The detailed implementation will be discussed later

Lex Specifications

```
%{ /* include, extern, etc. */  
extern int yytext, yylino;  
#include "token.h"  
%}  
/* declarations : declare variables, constants & regular definitions, */  
digit      [0-9]  
number     [0-9]+  
%%  
/* transition rules: regular expressions and actions */  
/* R1 action where actions are program fragments written in C. */  
number     { printf("Token: int const %s and %d", yytext, yylino); }  
%%  
/* auxiliary procedures */  
myTableInsert()  
{ ... }
```

Implementation Notes

- ❑ Write regular expressions for all tokens in language
- ❑ Comments: keep track of nesting level if nesting allowed
- ❑ String table maintained by you to detect duplicate identifiers / strings
- ❑ *yyline*, *yycolumn* maintained by you
- ❑ *yytext*, *yytext* maintained by lex library
- ❑ Special characters
 - `'\n'` — newline
 - `'\t'` — tab
 - `'\''` — single quote
 - `'\\'` — backslash

Discussion of RE and Lexical Analysis

- ❑ We use RE to assist lexical analysis
- ❑ Regular Expressions describe many useful languages
- ❑ Regular Expressions is a language specification
 - An implementation is still needed
- ❑ The problem we face is

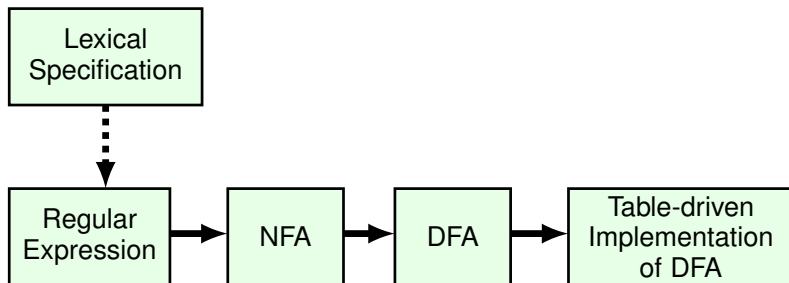
Given a string **s** and a regular expression **RE**, is

$$\mathbf{s} \in \mathbf{L(RE)} ?$$

Implementing Lexical Analysis with Finite Automata

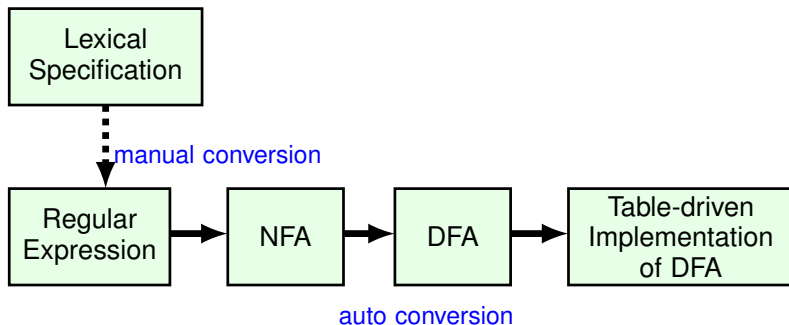
An Overview of RE to FA

Our implementation sketch



An Overview of RE to FA

Our implementation sketch



Implementation Outline

- ❏ RE \Rightarrow NFA \Rightarrow DFA \Rightarrow Table-driven Implementation
 - Specifying lexical structure using regular expressions
 - Finite automata
 - Deterministic Finite Automata (DFAs)
 - Non-deterministic Finite Automata (NFAs)
 - Table implementations

Notations

- In the following discussion, we use some alternative notations

Union: $A \mid B \equiv A + B$

Option: $A \varepsilon \equiv A$

Range: $'a' + 'b' + \dots + 'z' \equiv [a-z]$

Excluded range:
complement of $[a-z] \equiv [^a-z]$

Finite Automata

□ A finite automata consists of 5 components
 $(\Sigma, S, n, F, \delta)$

- (1). An input alphabet Σ
- (2). A set of states S
- (3). A start state $n \in S$
- (4). A set of accepting states $F \subseteq S$
- (5). A set of transitions $\delta: S_a \xrightarrow{\text{input}} S_b$

□ For lexical analysis

- Specification — Regular expression
- Implementation — Finite automata

More About Transition

Transition $\delta: S_a \xrightarrow{\text{input}} S_b$

read as

in state S1 on input “a” go to state S2

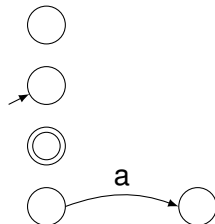
At the end of input (or no transition possible), if current state X

- $X \in$ accepting set F , then \Rightarrow **accept**
- otherwise, \Rightarrow **reject**

State Graph

- Sometimes we use **state graph** to represent a FA
- A **state graph** includes

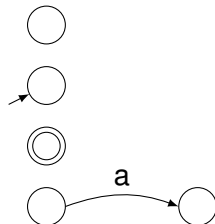
- A set of states
- A start state
- A set of accepting states
- A set of transitions



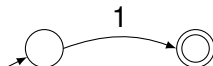
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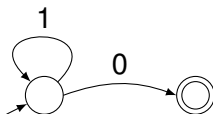


- Example: a finite state automata that accepts only "1"



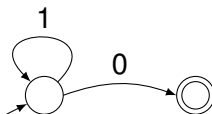
More Examples

- A finite automata accepting any number of **1**s followed by a single **0**. Here we have Alphabet = $\{0,1\}$



More Examples

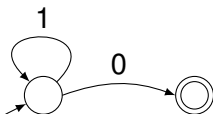
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- Example: What language does the following state graph recognize? Here we have Alphabet = $\{0,1\}$

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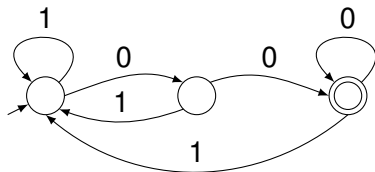


Table Implementation of a DFA

Given the state graph of a DFA,

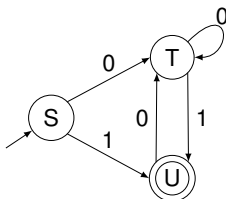
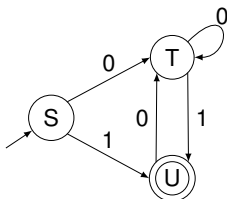


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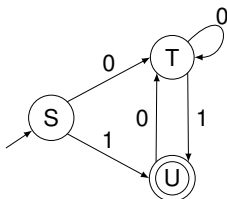
→ input characters

state ↓

	0	1
S		
T		
U		

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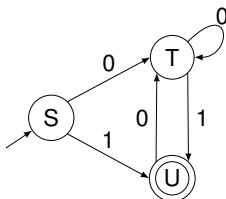
→ input characters

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	0	1
S	T	U
T	T	U
U	T	x

Table Implementation of a DFA

Given the state graph of a DFA,



→ input characters

state ↓

	0	1
S	T	U
T	T	U
U	T	x

Table-driven Code:

```

DFA() {
    state = "S";
    while (!done) {
        ch = fetch_input();
        state = Table[state][ch];
        if (state == "x")
            perror("error");
    }
    if (state ∈ F)
        printf("accept");
    else
        printf("reject");
}
  
```

Discussion

- ❑ Each RE has a different DFA / state graph
- ❑ For different REs,
 - their tables are different
 - their DFA recognition code is the same

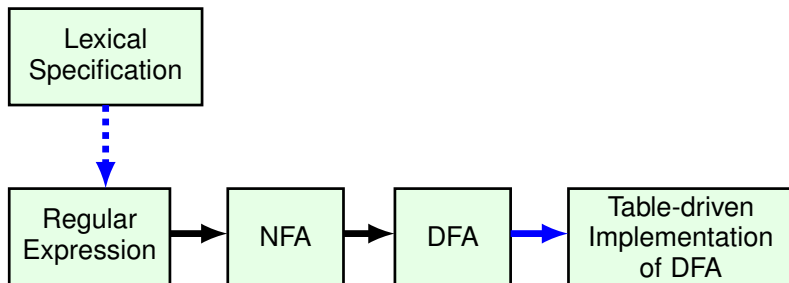
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- ❑ Revisit our implementation outline
 - RE \Rightarrow NFA \Rightarrow **DFA** \Rightarrow **Table-driven Implementation**

From RE to FA

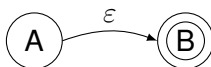
Our implementation sketch



Epsilon Moves

□ Another kind of transition: ε -moves

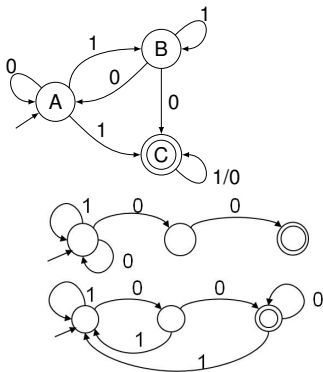
- Machine can move from state A to state B without reading any input



Deterministic and Nondeterministic Automata

- ❑ Deterministic Finite Automata (DFA)
 - One transition per input per state
 - No ϵ -moves
- ❑ Non-deterministic Finite Automata (NFA)
 - Can have multiple transitions for one input in a given state
 - Can have ϵ -moves
- ❑ Finite automata have finite memory
 - Need only to encode the current state

Examples

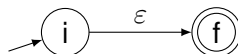


Converting RE to NFA

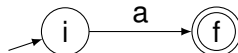
McNaughton-Yamada-Thompson Algorithm

Step 1: processing atomic REs

➤ ϵ expression



➤ single character RE a



Converting RE to NFA (cont.)

Step 2: processing compound REs

➤ $r = s \mid t$

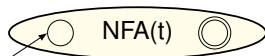
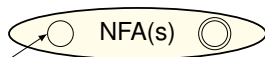
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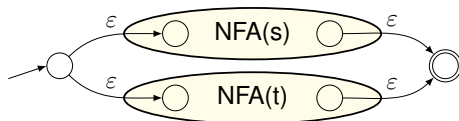
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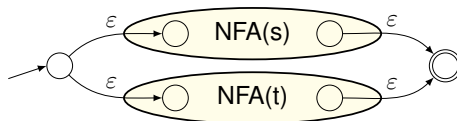
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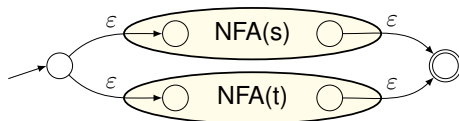


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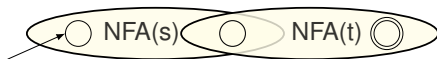
Converting RE to NFA (cont.)

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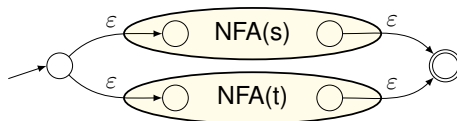


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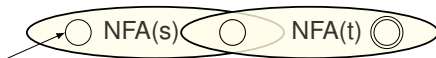
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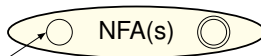
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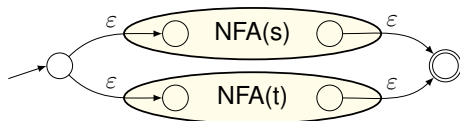
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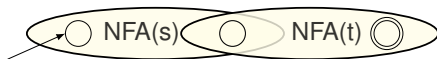
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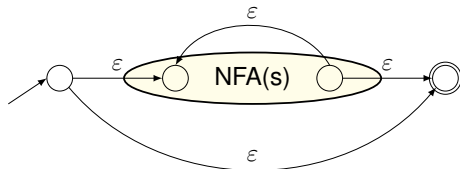
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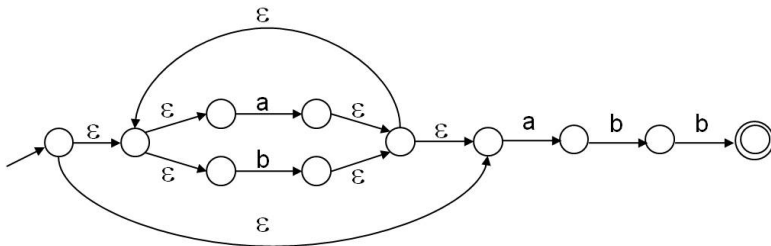


In-class Practice

 Convert “**(a|b)* a b b**” to NFA

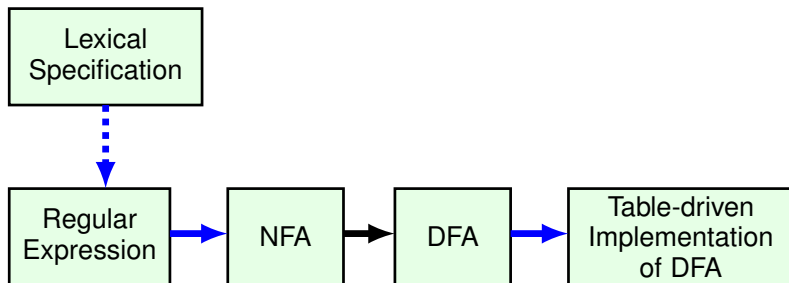
In-class Practice

Convert “**(a|b)* a b b**” to NFA



From RE to FA

Our implementation sketch



Execution of Finite Automata

- ❑ A DFA can take only one path through the state graph
 - Completely determined by input
- ❑ A NFA can take
 - Whether to make ϵ -moves
 - Which of multiple transitions for a single input to take
 - Acceptance of NFAs
 - An NFA can get into multiple states
 - **Rule:** the NFA accepts it if can get in a final state

Execution of Finite Automata

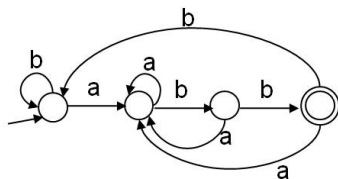
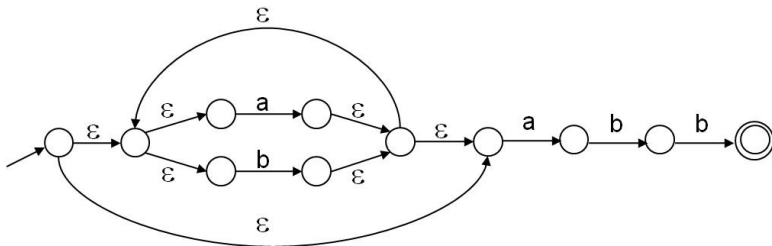
- ❑ A DFA can take only one path through the state graph
 - Completely determined by input
- ❑ A NFA can take
 - Whether to make ϵ -moves
 - Which of multiple transitions for a single input to take
 - Acceptance of NFAs
 - An NFA can get into multiple states
 - **Rule:** the NFA accepts it if can get in a final state
- ❑ **Question:** which one is more powerful?

Comparing NFA and DFA

- ❑ **Theorem:** NFAs and DFAs recognize the same set of languages
- ❑ Both recognize regular languages
- ❑ DFAs are faster to execute
 - There are no choices to consider
- ❑ For a given language, NFA can be simpler than DFA
- ❑ DFA can be exponentially larger than NFA
 - Example: DFA and NFA that accept **$(a|b)^* a b b$**

NFA and DFA

Both accept “**(a|b)* a b b**”



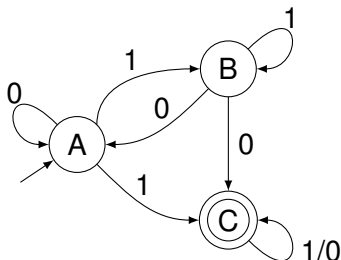
How to Convert NFA to DFA

- ❑ Basic idea: Given a NFA, simulate its execution using a DFA
 - At step n , the NFA may be in any of multiple possible states
- ❑ The new DFA is constructed as follows,
 - A state of DFA \equiv a non-empty subset of states of the NFA
 - Start state \equiv the set of NFA states reachable through ε -moves from NFA start state
 - A transition $S_a \xrightarrow{c} S_b$ is added **iff**

 S_b is the set of NFA states reachable from any state in S_a after seeing the input c , considering ε -moves as well

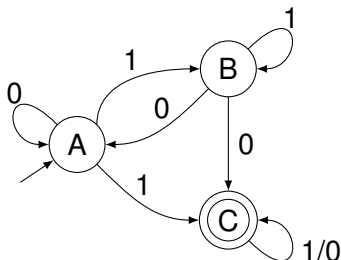
Example NFA to DFA

What is the Equivalent DFA ?



Example NFA to DFA

What is the Equivalent DFA ?

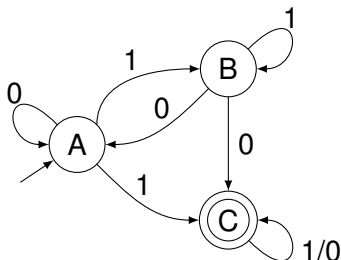


state ↓ → input characters

	0	1
A		
B		
C		

Example NFA to DFA

What is the Equivalent DFA ?

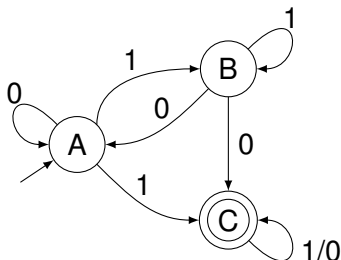


state ↓ → input characters

	0	1
A	A	BC
B	AC	B
C	C	C

Example NFA to DFA

What is the Equivalent DFA ?

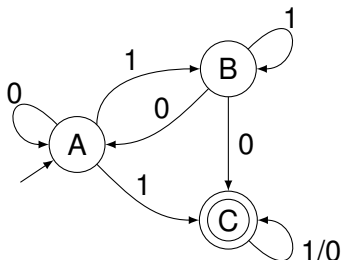


state ↓ → input characters

	0	1
A	A	BC
B	AC	B
C	C	C

Example NFA to DFA

What is the Equivalent DFA ?

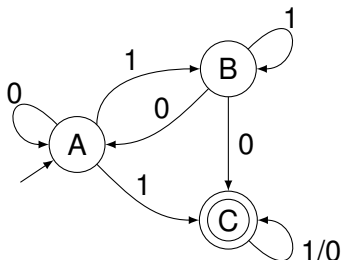


state ↓ → input characters

	0	1
A	A	BC
B	AC	B
C	C	C
AC		
BC		

Example NFA to DFA

What is the Equivalent DFA ?

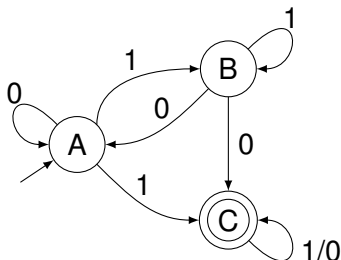


state ↓ → input characters

	0	1
A	A	BC
B	AC	B
C	C	C
AC	AC	BC
BC	AC	BC

Example NFA to DFA

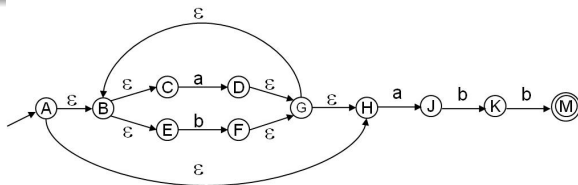
What is the Equivalent DFA ?



state ↓ → input characters

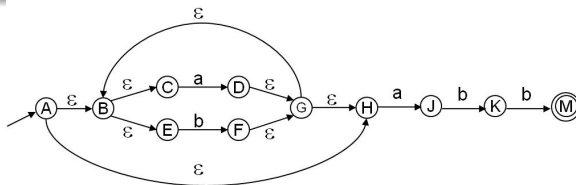
	0	1
A	A	BC
B	AC	B
C	C	C
AC	AC	BC
BC	AC	BC
AB	x	x
ABC	x	x

Algorithm Illustrated: Converting NFA to DFA



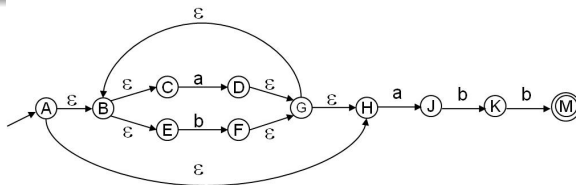
	ϵ	a	b
A			
B			
C			
D			
E			
F			
G			
H			
J			
K			
M			

Step 1: Construct the Table



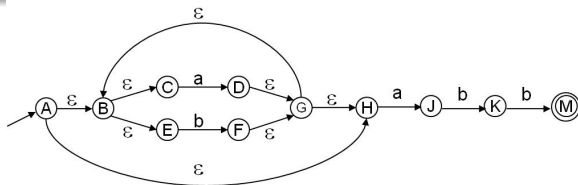
	ϵ	a	b
A	BH		
B	CE		
C		D	
D	G		
E			F
F	G		
G	BH		
H		J	
J			K
K			M
M			

Step 2: Construct ϵ -closure



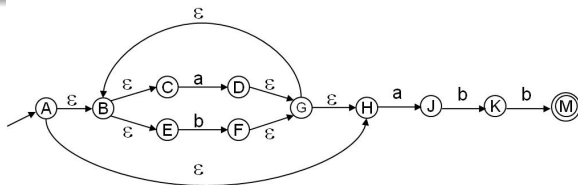
	ϵ	a	b
A	BHCE		
B	CE		
C		D	
D	GBHCE		
E			F
F	GBHCE		
G	BHCE		
H		J	
J			K
K			M
M			

Step 3: Update Other Columns



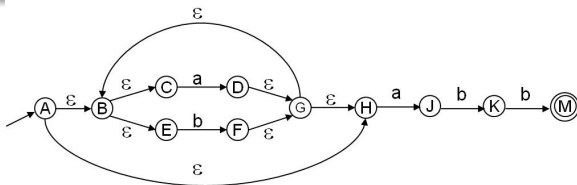
	ϵ	a	b
A	BHCE	DJ	F
B	CE	D	F
C		D	
D	GBHCE	DJ	F
E			F
F	GBHCE	DJ	F
G	BHCE	DJ	F
H		J	
J			K
K			M
M			

Step 4: Construct a New Table



	ϵ	a	b
A	BHCE	DJ	F
B	CE	D	F
C		D	
D	GBHCE	DJ	F
E			F
F	GBHCE	DJ	F
G	BHCE	DJ	F
H		J	
J			K
K			M
M			

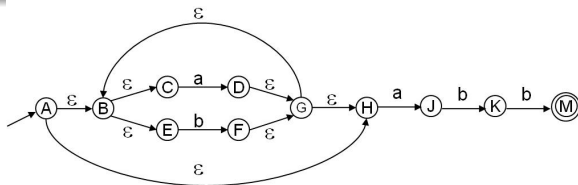
Step 4: Construct a New Table



	ϵ	a	b
A	BHCE	DJ	F
B	CE	D	F
C		D	
D	GBHCE	DJ	F
E			F
F	GBHCE	DJ	F
G	BHCE	DJ	F
H		J	
J			K
K			M
M			

	a	b
ABHCE	DJ	F

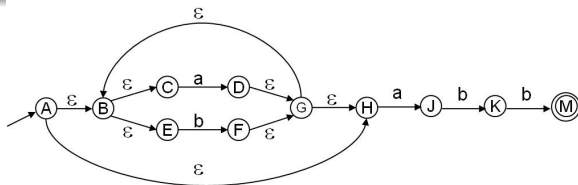
Step 4: Construct a New Table



	ϵ	a	b
A	BHCE	DJ	F
B	CE	D	F
C		D	
D	GBHCE	DJ	F
E			F
F	GBHCE	DJ	F
G	BHCE	DJ	F
H		J	
J			K
K			M
M			

	a	b
ABHCE	DJ	F
DJ	DJ	FK
F	DJ	F

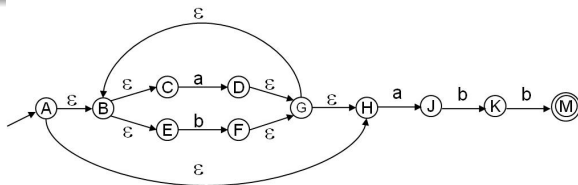
Step 4: Construct a New Table



	ϵ	a	b
A	BHCE	DJ	F
B	CE	D	F
C		D	
D	GBHCE	DJ	F
E			F
F	GBHCE	DJ	F
G	BHCE	DJ	F
H		J	
J			K
K			M
M			

	a	b
ABHCE	DJ	F
DJ	DJ	FK
F	DJ	F
FK	DJ	FM

Step 4: Construct a New Table



	ϵ	a	b
A	BHCE	DJ	F
B	CE	D	F
C		D	
D	GBHCE	DJ	F
E			F
F	GBHCE	DJ	F
G	BHCE	DJ	F
H		J	
J			K
K			M
M			


	a	b
ABHCE	DJ	F
DJ	DJ	FK
F	DJ	F
FK	DJ	FM
FM	DJ	F

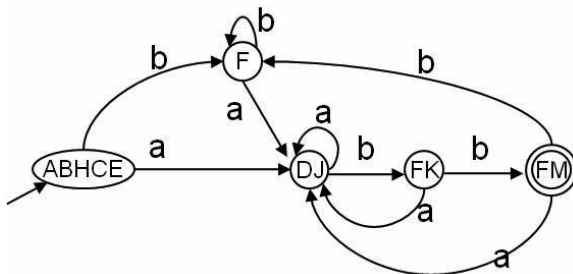
Step 5: Generate the DFA

	a	b
ABHCE	DJ	F
DJ	DJ	FK
F	DJ	F
FK	DJ	FM
FM	DJ	F

Step 5: Generate the DFA

	a	b
ABHCE	DJ	F
DJ	DJ	FK
F	DJ	F
FK	DJ	FM
FM	DJ	F

 Note: the number of states is not minimized



NFA to DFA. Space Complexity

- ❑ An NFA may be in many states at any time
- ❑ How many different possible states?
 - If there are N states, the NFA must be in some subset of those N states
 - How many non-empty subsets are there ?
 - $2^N - 1$ many states
- ❑ The resulting DFA has $O(2^N)$ space complexity where N is the number of original states


NFA to DFA Time Complexity

- ❑ A DFA can be implemented by a 2D table T
 - One dimension is “states”, the other dimension is “input characters”
 - For $S_a \xrightarrow{c} S_b$, we have $T[S_a, c] = S_b$
- ❑ DFA execution
 - If the current state is S_a and input is c , then read $T[S_a, c]$
 - Update the current state to S_b , assuming $S_b = T[S_a, c]$
 - Requires $O(|X|)$ steps, where $|X|$ is the length of input
- ❑ NFA execution
 - At a given step, there is a set of possible states, up to N
 - On input c , must access table for each possible state to get set of next possible states
 - Requires $O(|X| * N)$ steps

Implementation in Practice

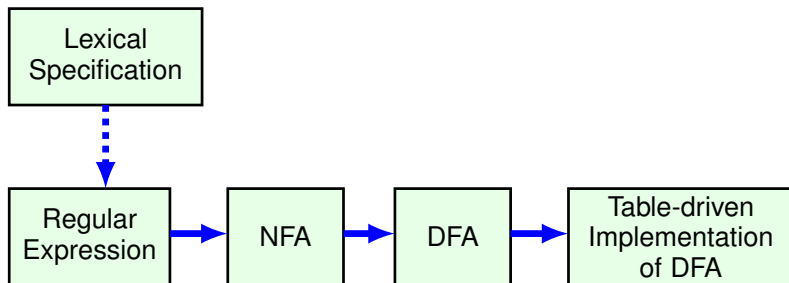
GNU **lex**

- Convert regular expression to NFA
- Convert NFA to DFA
- Perform DFA state minimization to reduce space
- Generate transition table from DFA
- Perform table compression to further reduce space

 Most other automated lexers also trade off space for speed by choosing DFA over NFA

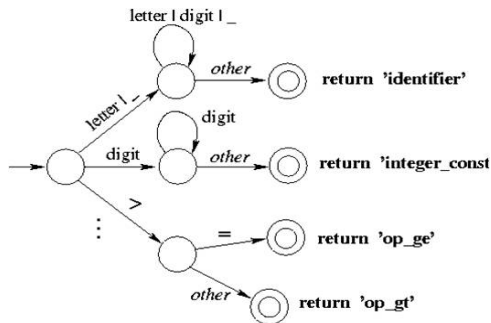
From RE to FA

Our implementation sketch



Structure of a Scanner Automaton

■ A scanner recognize multiple REs



How much should we match?

- In general, find the longest match possible
- If same length, rule appearing first in lex file takes precedence

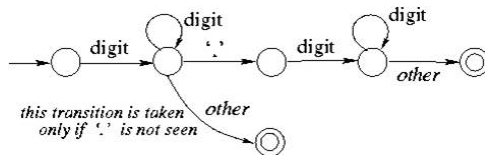
Example:

on input **123.45**, we match it as

(numConst, 123.45)

rather than

(numConst, 123), (dot, "."), (numConst, 45)



How to Match Keywords?

- ❑ Approach 1: Hardcode the keywords
- ❑ Approach 2: When the token is identified, check a special table

Example: to recognize the following tokens

Identifiers: `letter(letter|digit)*`

Keywords: `if, then, else`

Beyond Regular Languages

- ❑ Regular languages are expressive enough to describe tokens during lexical analysis
- ❑ Regular languages can express identifiers, strings, comments, etc.
- ❑ However, it is the weakest (least expressive) formal language
 - Many languages are not regular
 - C programming language is not
 - “(((...)))” is also not
 - Finite automata cannot remember # of times
- ❑ We need a more powerful language for parsing
 - In the next lecture, we will discuss **context-free languages**