# Semantic Analysis

#### The role of semantic analysis is to assign meaning

- "It smells fishy."
- Lexical analysis
  - > Tokenizes "It", "smells", "fishy", "."
  - Determines noun, verb, adjective, punctuation token types
- Syntax analysis
  - > Parses the grammatical structure of the sentence
- Semantic analysis

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  - Determines noun, verb, adjective, punctuation token types
- Syntax analysis
  - > Parses the grammatical structure of the sentence
- Semantic analysis
  - Assigns meaning to the words "It", "smells", "fishy"
  - > Flags error if the sentence does not make sense

#### Semantic Analysis = Binding + Type Inference

- "I don't wanna eat that sushi."
  - "It smells fishy."
    - > "It": the sushi
    - > "smells": feels to my nose
    - "fishy": that the sushi has gone bad
- "The professor says that the exam is going to be easy."
  - "It smells fishy."
    - "It": the situation
    - "smells": feels to my sixth sense
    - "fishy": that it is highly suspicious

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  - "It smells fishy."
    - > "It": the situation
    - "smells": feels to my sixth sense
    - > "fishy": that it is highly suspicious
- Semantic analysis consists of two tasks
  - > Binding: associating a pronoun to an object
  - > Type checking: inferring meaning based on type of object

#### Semantic analysis cannot be done during parsing

- Context Free Grammars (CFGs) cannot recognize bindings
  - Every use of a name needs to be bound to the declaration.
  - Name can refer to a variable, function, class, ...
  - Names are called symbols in semantic analysis
- To do bindings, a CFG must recognize this language:

$$\{\alpha \mathbf{c}\alpha | \alpha \in (\mathbf{a}|\mathbf{b})^*\}$$

The 1st  $\alpha$  represents the declaration, The 2nd  $\alpha$  represents a use.

Above language is a Context Sensitive Language

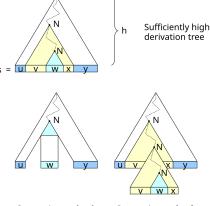
# Why is $\{\alpha c\alpha | \alpha \in (a|b)^*\}$ not a CFG?

- We will base our proof on the **pumping lemma** for CFGs.
- Pumping lemma: a theorem about strings in a grammar
  - > "lemma": a mathematical term for a theorem
  - "pumping": for a sufficiently long enough string, a substring exists within that string that can be "pumped" (repeated 0 or more times and still be in the language).
- For example, for the Regular Language 0(0|1)\*0:
  - ➤ A string longer than 2 will look like 000, 010, 0100, ...
  - ➤ Let's take "010". Here, substring "1" can be pumped.
  - > ("00", "010", "0110", "01110" are all in the language)
- Pumping Lemma applies to CFGs as well.

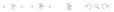
#### Pumping Lemma for CFGs

For a sufficiently long string s derived from a CFG, s can be written as s = uvwxy (u,v,w,x,y are substrings)

Where v and x can be pumped and |vx| > 1.



Generating uv wx y



#### $\{\alpha c\alpha | \alpha \in (a|b)^*\}$ is not a CFG

- Let's say s = uvwxy is a sufficiently long string in language, where v and x can be pumped and  $|vx| \ge 1$ .
  - 1. The substring vwx must bisect s.
    - If vwx is contained in 1st  $\alpha$  (or mostly contained), if we pump v and x 0 times, 1st  $\alpha$  gets shorter than 2nd  $\alpha$ .
    - ightharpoonup string is no longer in  $\{\alpha c\alpha | \alpha \in (a|b)^*\}$ . Contradiction.
    - ightharpoonup The same applies to when vwx is contained in 2nd  $\alpha$ .
- 2. Even when vwx bisects s, pumping fails.
  - Let string s' be the result of pumping v and x 0 times.
  - Let's say s' =  $\alpha_1 c \alpha_2$ , where  $\alpha_1$  and  $\alpha_2$  are shortened versions of the 1st and 2nd  $\alpha$ s.
  - ightharpoonup While  $|\alpha_1|=|\alpha_2|$ , there exist  $\alpha_1$  and  $\alpha_2$  such that  $\alpha_1!=\alpha_2$ .
  - ightharpoonup E.g. s = abcab, and vwx = bca where v = b and x = a. Then,  $\alpha_1$  becomes "a" and  $\alpha_2$  becomes "b". Contradiction.

#### Semantic analysis does binding and type checking

- Semantic analysis performs binding
  - > Since CFGs cannot recognize bindings, as we just proved
  - Done by traversing parse tree produced by syntax analysis
  - Definitions are stored in data structure called symbol table
  - Uses are bound to entries in the symbol table
- Semantic analysis performs type checking
  - ightharpoonup Infer what "a + b" means:
    - If a and b are ints, integer add and return int
    - If a and b are floats, FP add and return float
    - If a and b are strings, concatenate and return string
  - Infer what "a.foo()" means:
    - If object a is an instance of class A, call A.foo()
    - If object a is an instance of class B, call B.foo()
  - $\rightarrow$  Infer what "a[i][j]" means:
    - Offset from a calculated based on type and dimensions

#### Semantic analysis also performs semantic checks

All symbol uses have a corresponding declaration;
All operations are type legal;
Inheritance relationships are correct;
A class is defined only once;
A method in a class is defined only once;

# **Symbol Binding**

#### What is symbol binding?

"Matching symbol declarations with uses"

If there are multiple declarations, which one is matched?

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"Matching symbol declarations with uses"

☐ If there are multiple declarations, which one is matched?

```
void foo()
{
    char x;
    ...
    {
        int x;
        ...
    }
    x = x + 1;
}
```

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}
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```

#### Scope

Ļ	Binding: the association of a use of a symbol to the	е
	declaration of that symbol	

- Which variable (or function) an identifier is referring to
- Scope: section of program where a declaration is valid
  - Uses in the scope of declaration are bound to it
- Some implications of scopes
  - A symbol may have different bindings in different scopes
  - Scopes for the same symbol never overlap
    - there is always exactly one binding per symbol use
- Two types: static scope and dynamic scope

#### Static Scope

Static scope depends on the program text, not run-time behavior (also known as lexical scoping)

```
C/C++, Java, Objective-C
```

Rule: Refer to the closest enclosing declaration

```
void foo()
   char x;
      int x;
      ...
   x = x + 1;
```

#### Static Scope

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- Rule: Refer to the closest enclosing declaration

```
void foo()
{
    char x;
...
    int x;
...
}
x = x + 1;
```

# Dynamic Scope

- Dynamic scoping depends on bindings formed during the execution of the program
  - > LISP, Scheme, Perl
- Rule: Refer to the closest binding in the current execution

```
void foo()
{
    (1) char x;
    (2) if (...) {
     (3) int x;
    (4) ...
     }
    (5) x = x + 1;
}
```

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void foo()
{
    (1) char x;
    (2) if (...) {
     (3) int x;
    (4) ...
    }
    (5) x = x + 1;
}
```

- $\square$  Which x's declaration is the closest?
  - > Execution (a): ...(1)...(2)...(5)
  - > Execution (b): ...(1)...(2)...(3)...(4)...(5)

#### Static vs. Dynamic Scoping

- Most languages that started with dynamic scoping (LISP, Scheme, Perl) added static scoping afterwards
- Why?
  - It is easier for human beings to understand
    - Bindings visible in code without tracing execution
  - It is easier for compilers to understand
    - Compiler can determine bindings at compile time
    - Compiler can translate identifier to a single memory location
    - Results in generation of efficient code
  - With dynamic scoping...
    - There may be multiple possible bindings for a variable
    - Impossible to determine bindings at compile time
    - All bindings have to be done at execution time

# Symbol Table

#### Symbol Table

- Symbol Table: A compiler data structure that tracks information about all identifiers (symbols) in a program
  - Maps symbol uses to declarations given a scope
  - Needs to provide bindings according to the current scope
- Usually discarded after generating the binary code
  - All symbols are mapped to memory locations already
  - > For debugging, symbols may be included in binary
    - To map memory locations back to symbols for debuggers
    - For GCC or Clang, add "-g" flag to include symbol tables

add declaration of x to symbol table

# Maintaining Symbol Table

add symbol(x)

```
    □ Basic idea:
        int x; ... void foo() { int x; ... x=x+1; } ... x=x+1 ...
        ➤ In foo, add x to table, overriding any previous declarations
        ➤ After foo, remove x and restore old declaration if any
    □ Operations
        enter_scope() start a new nested scope
        exit_scope() exit current scope
    find symbol(x) find declaration of x
```

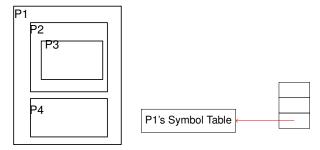
#### Adding Scope Information to the Symbol Table

- To handle multiple scopes in a program,
  - > (Conceptually) need an individual table for each scope
  - Symbols added to the table may not be deleted just because you exited a scope

```
class X { ... void f1() {...} ... } class Y { ... void f2() {...} ... } X v; call v.f1();
```

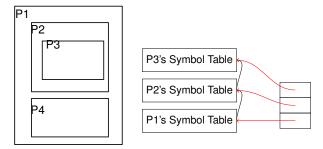
Without deleting symbols, how are scoping rules enforced?
 Keep a list of all scopes in the entire program
 Keep a stack of active scopes at a given point

# Symbol Table with Multiple Scopes



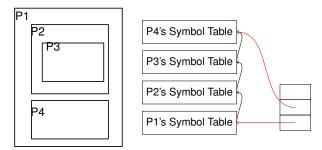
- For nested scopes,
  - Search from top of the active symbol table stack
  - Remove pointer to symbol table when exiting its scope

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#### What Information is Stored in the Symbol Table

Entry in Symbol Table:

string kind attributes

- String the name of identifier
- Kind variable, parameter, function, class, ...
- Attributes vary with the kind of symbol
  - ➤ variable → type, address in memory
  - function → return type, parameter types, address
- Vary with the language
  - Fortran's array → type, dimension, dimension size real A(5) /\* dimension required for static allocation \*/
  - C's array → type, dimension, optional dimension size int A[5]; /\* statically sized array \*/ int A[]; /\* for dynamic allocation of array \*/

# Symbol Table Attribute List

Type information might be arbitrarily complicated

Store all relevant attributes in an attribute list

```
id array 1st dimension upper bound 2nd dimension upper bound id struct total size 1st dimension upper bound field type size field type size
```

# Example application of Type to an operator: Array index operator

# Addressing Array Elements

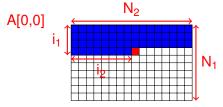
```
int A[0..high];
   A[i] ++;
                                      A[high]
    base=A[0]
                               A[i]
    width — width of element type
    base — address of the first
    high — upper bound of subscript
Addressing an array element:
   address(A[i])
   = base + i * width
```

# Multi-dimensional Arrays

Layout n-dimension items in 1-dimension memory int A[N<sub>1</sub>][N<sub>2</sub>]; /\* int A[0..high<sub>1</sub>][0..high<sub>2</sub>]; \*/  $A[i_1][i_2] ++;$  $N_2$ A[0,0] $N_1$ A[high<sub>1</sub>,high<sub>2</sub>]

# Row Major

Row major — store row by row

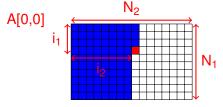


Offset inclues all the "blue" items before A[i1,i2]

address(A[
$$i_1$$
, $i_2$ ])  
= base + ( $i_1 * N_2 + i_2$ ) \* width

# Column Major

Column major — store column by column



 $\Box$  Offset inclues all the "blue" items before A[i<sub>1</sub>,i<sub>2</sub>]

address(A[
$$i_1$$
, $i_2$ ])  
= base + ( $i_2 * N_1 + i_1$ )\*width

# Generalized Row/Column Major

Calculating  $A_k$  = offset of  $A[i_1, i_2, ..., i_k]$  from base address:

```
Row major
```

1-dimension:  $A_1 = i_1^*$  width

2-dimension:  $A_2 = (i_1 * N_2 + i_2) * width = A_1 * N_2 + i_2 * width$ 

3-dimension:  $A_3 = (i_1 * N_2 * N_3 + i_2 * N_3 + i_3) * width = A_2 * N_3 + i_3 * width$ 

..

k-dimension:  $A_k = A_{k-1} * N_k + i_k * width$ 

#### Column major

1-dimension:  $A_1 = i_1*width$ 

2-dimension:  $A_2 = (i_2 * N_1 + i_1) * width = i_2 * N_1 * width + A_1$ 

3-dimension:  $A_3 = ((i_3 * N_2 + i_2) * N_1 + i_1) * width = i_3 * N_2 * N_1 * width + A_2$ 

...

k-dimension:  $A_k = i_k * N_{k-1} * N_{k-2} * ... * N_1 * width + A_{k-1}$ 

# C's implementation

C uses row major
 int fun1(int p[ ][100])
{
 ...
 int a[100][100];
 a[i<sub>1</sub>][i<sub>2</sub>] = p[i<sub>1</sub>][i<sub>2</sub>] + 1;
}

Why is p[][100] allowed?

Why is a[][100] not allowed?

# C's implementation

C uses row major

```
int fun1(int p[][100]) { ... int a[100][100]; a[i_1][i_2] = p[i_1][i_2] + 1; }
```

Why is p[][100] allowed?

- $\rightarrow$  The info is enough to compute p[i<sub>1</sub>][i<sub>2</sub>]'s address
- $\rightarrow$  A<sub>2</sub> = (i<sub>1</sub>\*N<sub>2</sub>+i<sub>2</sub>)\*width (N<sub>1</sub> is not required)

Why is a[][100] not allowed?

> The info is not enough to allocate space for the array

# Type Checking

### What, Why and When

- What is a type?

  Type = a set of values + a set of operations on these values
- What is type checking? Verifying and enforcing type consistency
  - Only legal values are assigned to a type
  - Only legal operations are performed on a type
- Why is compile-time type checking desirable?
  - Type errors easier to debug than malfunctioning programs
  - Dynamic type checking when static checking infeasible
    - E.g. Java null checks and array bounds checks
    - E.g. C++/Java downcasting to a subclass

# Static vs. Dynamic Typing

- - > Types are explicitly declared or can be inferred from code
  - ➤ E.g. int x; /\* type of x is int \*/
  - > Efficient code since runtime type checks are not needed
- Dynamically typed: Python, JavaScript, PHP
  - > Type is a runtime property decided only during execution
  - E.g. var x; /\* type of x is undecided \*/
  - > Type of x changes depending on the type of value it holds
  - More memory since every variable now needs a "type tag"
  - Inefficient code due to runtime checks on type tags

#### Rules of Inference

- What are rules of inference?
  - ➤ Inference rules have the form if Precondition is true, then Conclusion is true
  - Below concise notation used to express above statement

# Precondition Conclusion

- ➤ In the context of type checking: if expressions E1, E2 have certain types (Precondition), expression E3 is legal and has a certain type (Conclusion)
- Type checking via inference
  - Start from variable types and constant types
  - > Repeatedly apply rules until entire program is inferred legal

#### Notation for Inference Rules

By tradition inference rules are written as

# Precondition<sub>1</sub>, ..., Precondition<sub>n</sub> Conclusion

- The precondition/conclusion has the form "e:T"
- Meaning
  - If Precondition₁ and ... and Preconditionn are true, then Conclusion is true.
  - > "e:T" indicates "e is of type T"
  - Example: rule-of-inference for add operation

```
e<sub>1</sub>: int
e<sub>2</sub>: int
e<sub>1</sub>+e<sub>2</sub>:int
```

Rule: If  $e_1$ ,  $e_2$  are ints then  $e_1+e_2$  is legal and is an int

# Two Simple Rules

 $[Add \ operation] \begin{tabular}{ll} i \ is \ an \ integer \\ \hline i: int \\ \hline e_1: int \\ \hline e_2: int \\ \hline e_1+e_2: int \\ \hline \end{tabular}$ 

Example: given "10 is an integer" and "20 is an integer", is the expression "10+20" legal? Then, what is the type?

10 is an integer 20 is an integer 10: int 20: int

10+20:int

This type of reasoning can be applied to the entire program

#### More Rules

```
[New]

new T: T

[Not]

e: Boolean

not e: Boolean

However,

[Var?]

x is an identifier

x: ?
```

- > the expression itself insufficient to determine type
- > solution: provide context for this expression

### Type Environment

- A type environment gives type info for free variables
  - > A variable is *free* if not declared inside the expression
  - ➤ It is a function mapping Symbols to Types
    - Set of declarations active at the current scope
    - Conceptual representation of a symbol table

# Type Environment Notation

Let O be a function from Symbols to Types, the sentence O e:T

is read as "under the assumption of environment O, expression e has type T"

- "if i is an integer, expression i is an int in any environment"
- "if e1 and e2 are ints in O, expression e1+e2 is int in O"
- "if variable x is mapped to int in O, expression x is int in O"

#### **Declaration Rule**

#### [Declaration w/o initialization]

O[
$$T_0/x$$
]  $e_1$ :  $T_1$   
O let x:  $T_0$  in  $e_1$ :  $T_1$ 

 $O[T_0/x]$  means, O is modified to return  $T_0$  on argument x and behaves as O on all other arguments

$$O[T_0/x](x) = T_0$$
  
 $O[T_0/x](y) = O(y)$  when  $x \neq y$ 

Translation: "If expression e<sub>1</sub> is type T<sub>1</sub> when x is mapped to type T<sub>0</sub> in the current environment, expression e<sub>1</sub> is type T<sub>1</sub> when x is declared to be T<sub>0</sub> in the current environment"

#### **Declaration Rule with Initialization**

[Declaration with initialization (initial try)]

The rule is too strict (i.e. correct but not complete)

```
Example class C inherits P ... let x:P ← new C in ...
```

the above rule does not allow this code

### Subtyping

- Subtyping is a relation ≤ on classes
  - > X ≤ X
  - ightharpoonup if X inherits from Y, then X  $\leq$  Y
  - ightharpoonup if  $X \leq Y$  and  $Y \leq Z$ , then  $X \leq Z$
- An improvement of our previous rule

[Declaration with initialization]

$$\label{eq:controller} \begin{array}{c} \textbf{O} \; \textbf{e}_0 \colon \textbf{T} \\ \textbf{T} \leq \textbf{T}_0 \\ \textbf{O}[\textbf{T}_0/\textbf{x}] \; \textbf{e}_1 \colon \textbf{T}_1 \\ \textbf{O} \; \textbf{let} \; \textbf{x} \colon \textbf{T}_0 \leftarrow \textbf{e}_0 \; \textbf{in} \; \textbf{e}_1 \colon \textbf{T}_1 \end{array}$$

- Both versions of declaration rules are correct
- > The improved version checks more programs

### **Assignment**

A correct but too strict rule

```
[Assignment] \\ \textbf{O(id)} = \textbf{T}_0 \\ \textbf{O e_1: T}_1 \\ \textbf{T}_1 \leq \textbf{T}_0 \\ \textbf{O id} \leftarrow \textbf{e_1: T}_0
```

The rule does not allow the below code class C inherits P { only\_in\_C() { ... } } x ← y ← new C x.only in C()

# **Assignment**

An improved rule

```
[Assignment] \\ \textbf{O(id)} = \textbf{T}_0 \\ \textbf{O e_1: T}_1 \\ \textbf{T}_1 \leq \textbf{T}_0 \\ \textbf{O id} \leftarrow \textbf{e_1: T}_1
```

The rule now does allow the below code class C inherits P { only\_in\_C() { ... } }  $x \leftarrow y \leftarrow \text{new C}$  x.only\_in\_C()

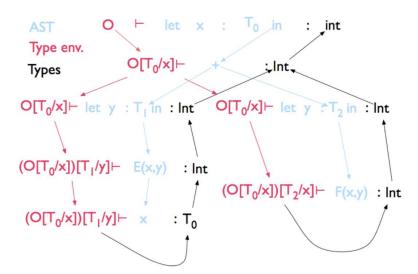
#### If-then-else

- Consider
  - if e<sub>0</sub> then e<sub>1</sub> else e<sub>2</sub>
    - The result can be either e<sub>1</sub> or e<sub>2</sub>
    - The type is either e<sub>1</sub>'s type or e<sub>2</sub>'s type
  - The best that we can do (statically) is the super type larger than e<sub>1</sub>'s type and e<sub>2</sub>'s type
- Least upper bound (LUB)
  - Z = lub(X,Y) Z is defined as the least upper bound of X and Y iff
    - $X \le Z \land Y \le Z$  ; Z is an upper bound
    - $\bullet \ \ X{\le}W \land Y{\le}W \Longrightarrow Z{\le}W \ \ ; Z \ \text{is least among all upper bounds}$

### If-then-else, case

```
[If-then-else]
                             O e<sub>0</sub>: Bool
                              O e<sub>1</sub>: T<sub>1</sub>
                              O e<sub>2</sub>: T<sub>2</sub>
            O if e_0 then e_1 else e_2 fi: lub(T_1,T_2)
☐ The rule allows the below code
        let x:float, y:int, z:float in
        x \leftarrow if (...) then y else z
        /* Assuming lub(int, float) = float */
```

# Implementing Type Checking on AST



### **Error Recovery**

- Just like other errors, we should recover from type errors
  - ➤ Too many errors? let y: int ← x+2 in y+3
    - if x is undefined —- reporting an error "x type undefined"
    - x+2 is undefined reporting an error "x+2 type undefined"
    - ..
- Introducing no-type for ill-typed expressions
  - > It is compatible with all types
  - Report the place where no-type is generated
    - Reduce the number of error messages

# Wrong Declaration Rule (case 1)

Consider a hypothetical let rule

[Wrong Declaration with initialization (case 1)]

```
\begin{array}{c} \textbf{O} \; \textbf{e}_0 \colon \textbf{T} \\ \textbf{T} \leq \textbf{T}_0 \\ \textbf{O} \; \textbf{e}_1 \colon \textbf{T}_1 \\ \textbf{O} \; \textbf{let} \; \textbf{x} \colon \textbf{T}_0 \leftarrow \textbf{e}_0 \; \textbf{in} \; \textbf{e}_1 \colon \textbf{T}_1 \end{array}
```

- > How is it different from the the correct rule?
- The following program does not pass check let x: Int ← 0 in x+1

# Wrong Declaration Rule (case 2)

Consider a hypothetical let rule

[Wrong Declaration with initialization (case 2)]

```
\label{eq:controller} \begin{array}{c} \textbf{O} \; \textbf{e}_0 \colon \textbf{T} \\ \textbf{T}_0 \leq \textbf{T} \\ \textbf{O}[\textbf{T}_0/\textbf{x}] \; \textbf{e}_1 \colon \textbf{T}_1 \\ \textbf{O} \; \overline{\textbf{let} \; \textbf{x} \colon \textbf{T}_0 \leftarrow \textbf{e}_0 \; \text{in} \; \textbf{e}_1 \colon \textbf{T}_1} \end{array}
```

- > How is it different from the the correct rule?
- The following bad program passes the check let  $x: B \leftarrow \text{new A in } x.b()$

# Wrong Declaration Rule (case 3)

Consider a hypothetical let rule

[Wrong Declaration with initialization (case 3)]

```
\label{eq:controller} \begin{array}{c} \textbf{O} \; \textbf{e}_0 \colon \textbf{T} \\ \textbf{T}_0 \leq \textbf{T} \\ \textbf{O}[\textbf{T}/\textbf{x}] \; \textbf{e}_1 \colon \textbf{T}_1 \\ \textbf{O} \; \textbf{let} \; \textbf{x} \colon \textbf{T}_0 \leftarrow \textbf{e}_0 \; \textbf{in} \; \textbf{e}_1 \colon \textbf{T}_1 \end{array}
```

- How is it different from the the correct rule?
- The following bad program passes the check let x: A  $\leftarrow$  new B in {... x  $\leftarrow$  new A; x.a(); }

#### Discussion

- Type rules have to be very carefully constructed
- Virtually any change in a rule either
  - makes the type system unsound (bad programs are accepted as well typed)
  - or, makes the type system less usable (good programs are rejected)
- But some good programs will be rejected anyway
  - > .... what is a "good" program?

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- But some good programs will be rejected anyway
  - > .... what is a "good" program?
  - Good program: A program where all operations performed on all values are type consistent at runtime
  - > Impossible to express all runtime behavior in a type system
    - E.g. Type of if-then-else is LUB of two types, a conservative estimate of runtime behavior

### Designing a Good Type Checking System

- Type system has two conflicting design goals
  - Give flexibility to the programmer (so that she can write a "good" program within the boundaries of the type system)
  - Prevent type-checked programs from "going wrong"
- Should allow maximum flexibility while guaranteeing safety
  - > An example

```
class Count {
  int i = 0;
  Count inc() { i=i+1; return this; }
}
class Stock inherits Count { ... }
class Main {
  Stock a ← (new Stock).inc();
}
```

# What Went Wrong?

- What is (new Stock).inc()'s type?
  - Dynamic type Stock
  - Static type Count
  - > The type checker "looses" the type information
  - > This makes inheriting inc() useless
    - Do we really want to redefine inc() for each subclass returning the correct type?
- SELF\_TYPE to the rescue

### SELF\_TYPE to the Rescue

- What is SELF\_TYPE?
  - inc() returns "self" instead of "Count" type
  - Self could be Count or any subclass of Count, depending on reference type
- SELF\_TYPE is a static type
  - Type violations can still be detected at compile time
  - Expresses runtime behavior accurately w/o undue burden to the programmer
- In practice
  - > C++: made possible by language extension using templates
  - Java: not allowed because there are no templates

### Can Static Type Checking ever be Perfect?

- Many examples where correct programs are disallowed (besides SELF\_TYPE)
  - Why C++ programmers are still forced to downcast (Type of values at runtime are known by programmer but no way to express it in type system)
  - Fundamentally undecidable whether type system is adhered to at runtime
- Solution?
  - Some argue for dynamic type checking instead
    - Philosophy: Maximum expressivity for the programmer
    - Good for scripting languages where expressivity is king
  - Others argue for more expressive static checking rules
    - Philosophy: Too much expressivity is not good for you
    - Good for mission critical code that should never fail
    - Good for performance critical code

### Syntax Directed Translation

### What is Syntax Directed Translation?

To drive semantic analysis tasks based on the language's syntactic structure

- What is meant by semantic analysis tasks?
  - Generate AST (abstract syntax tree)
  - Check type errors
  - Generate intermediate representation (IR)
- What is meant by syntactic structure?
  - Structure of program given by context free grammar (CFG)
  - Structure of the parse tree generated by the parser

#### **How is Syntax Directed Translation Performed?**

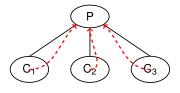
- ☐ How?
  - Attach attributes to grammar symbols/parse tree
  - > Evaluate attribute values using **semantic actions**
- We already did some of this in Project 2:
  - Attached attributes to grammar symbols
    - tptr "tree pointer" of a non-terminal symbol
       By the time program.tptr is evaluated, the parse tree is built
  - > Evaluated attributes using semantic rules (actions)
    - { ... \$\$=makeTree(ProgramOp, leftChild, rightChild); ... }

#### Attributes?

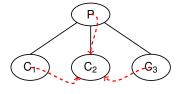
- Attributes can represent anything depending on task
  - A string
  - ➤ A type
  - A number
  - A memory location
  - An attribute grammar is a grammar augmented by associating attributes with each grammar symbol that describes its properties

# Two Types of Attributes

- Synthesized attributes: attributes are computed from attributes of children nodes
  - ightharpoonup P.synthesized\_attr = f(C<sub>1</sub>.attr, C<sub>2</sub>.attr, C<sub>3</sub>.attr)
- Inherited attributes: attributes are computed from attributes of sibling and parent nodes
  - $ightharpoonup C_3.inherited_attr = f(P_1.attr, C_1.attr, C_3.attr)$



Synthesized attribute



Inherited attribute

### Synthesized Attribute Example

#### Example

- Each non-terminal symbol is associated with val attribute
- Each grammar rule is associated with a semantic action

```
\begin{array}{lll} \mathsf{L} \to \mathsf{E} & \{ \; \mathsf{print}(\mathsf{E}.\mathsf{val}) \; \} \\ \mathsf{E} \to \mathsf{E}_1 + \mathsf{T} & \{ \; \mathsf{E}.\mathsf{val} = \mathsf{E}_1.\mathsf{val} + \mathsf{T}.\mathsf{val} \; \} \\ \mathsf{E} \to \mathsf{T} & \{ \; \mathsf{E}.\mathsf{val} = \mathsf{T}.\mathsf{val} \; \} \\ \mathsf{T} \to \mathsf{T}_1 + \mathsf{F} & \{ \; \mathsf{T}.\mathsf{val} = \mathsf{T}_1.\mathsf{val} + \mathsf{F}.\mathsf{val} \; \} \\ \mathsf{T} \to \mathsf{F} & \{ \; \mathsf{T}.\mathsf{val} = \mathsf{F}.\mathsf{val} \; \} \\ \mathsf{F} \to (\; \mathsf{E} \; ) & \{ \; \mathsf{F}.\mathsf{val} = \mathsf{E}.\mathsf{val} \} \\ \mathsf{F} \to \mathsf{digit} & \{ \; \mathsf{F}.\mathsf{val} = \mathsf{digit}.\mathsf{lexval} \} \end{array}
```

### Inherited Attribute Example

### Example:

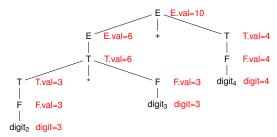
- T synthesized attribute "type"
- L has inherited attribute "in"

```
\begin{array}{lll} D \rightarrow T \; L & \{\; L.in = T.type \;\} \\ T \rightarrow int & \{\; T.type = integer \;\} \\ T \rightarrow real & \{\; T.type = real \;\} \\ L \rightarrow L_1 \; , \; id \; \{\; L_1.in = L.in, \; addtype \; (id.entry, \; L.in) \;\} \\ L \rightarrow id & \{\; addtype \; (id.entry, \; L.in) \;\} \end{array}
```

- > We can use inherited attributes to track type information
- ➤ We can use *inherited attributes* to track whether an identifier appear on the left or right side of an assignment operator ":=" ( e.g. a := a +1 )

### Attribute Parse Tree

- ☐ Parse tree showing values of attributes
  - Parse tree annotated or decorated with attributes
  - Attributes computed at each node
- Properties of attribute parse tree:
  - ➤ Terminal symbols have synthesized attributes only, which are usually provided by the lexical analyzer
  - Start symbol does not have any inherited attributes

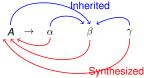


### Two Aspects to Syntax Directed Translation

- Syntax Directed Definitions (SDD)
  - Set of semantic rules attached to each production
  - Semantic rules define values for attributes
  - Specification rather than implementation
- Syntax Directed Ttranslation Scheme (SDTS)
  - Semantic actions are implementations of semantic rules
  - Grammar with semantic actions embedded within RHS of productions (can be anywhere)
  - Semantic actions are fragments of code executed "at that point" in the RHS
    - Top-down: Right after previous symbol has been consumed
    - Bottom-up: Right after previous symbol has been pushed to the stack (when the 'dot' reaches the action)

# Syntax Directed Definition (SDD)

Attribute grammar



SDD has rule of the form for each CFG production  $b = f(c_1, c_2, ..., c_n)$ 

#### either

- If b is a synthesized attributed of A, c₁ (1≤i≤n) are attributes of grammar symbols of its Right Hand Side (RHS); or
- 2. If b is an inherited attribute of one of the symbols of RHS, c<sub>i</sub>'s are attribute of A and/or other symbols on the RHS

- Both inherited and synthesized attributes are used
  - > T synthesized attribute T.val
  - R inherited attribute R.i synthesized attribute R.s
  - ➤ E synthesized attribute E.val

Evaluating attributes using SDTS

```
E \rightarrow T \{R.i=T.val\} R \{E.val=R.s\}
R \rightarrow + T \{R_1.i=R.i+T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow \text{-} \quad T \ \{R_1.i\text{=}R.i\text{-}T.val\} \ R_1 \ \{R.s\text{=}R_1.s\}
R \rightarrow \varepsilon \{R.s=R.i\}
T \rightarrow (E) \{T.val=E.val\}
T \rightarrow num \{T.val=num.val\}
                                              Ε
                                                  → R<sub>1</sub>.i=T.val
                 T.val=num
                                                                                                           R_2
        num
                     num
                                                                                                                         R_3
                                            num
                                                                         num
```

Evaluating attributes using SDTS

```
E \rightarrow T \{R.i=T.val\} R \{E.val=R.s\}
R \rightarrow + T \{R_1.i=R.i+T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow - T \{R_1.i=R.i-T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow \varepsilon \{R.s=R.i\}
T \rightarrow (E) \{T.val=E.val\}
T \rightarrow num \{T.val=num.val\}
                                           Ε
                                                  R<sub>1</sub>.i=T.val R<sub>1</sub>
                                                 T.val = num \longrightarrow R_2.i = R_1.i + T.val
                                                                                                      R_2
        num
                                                                                                                  R_3
                                          num
                                                      num
                                                                     num
```

Evaluating attributes using SDTS

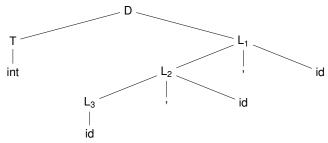
```
E \rightarrow T \{R.i=T.val\} R \{E.val=R.s\}
R \rightarrow + T \{R_1.i=R.i+T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow - T \{R_1.i=R.i-T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow \varepsilon \{R.s=R.i\}
T \rightarrow (E) \{T.val=E.val\}
T \rightarrow num \{T.val=num.val\}
                                                    EE.val=R<sub>1</sub>.s
                                                            R<sub>1</sub>.i=T.val
                                                                                R<sub>1</sub>R<sub>1</sub>.s=R<sub>2</sub>.s
                                                                                      R_2.i=R_1.i+T.val
                                                                                                                         R<sub>2</sub> R<sub>2</sub>.s= R<sub>3</sub>.s
         num
                                                                                     T T.val = num \Rightarrow R<sub>3</sub>.i=R<sub>2</sub>.i+T.valR<sub>3</sub>R<sub>3</sub>.s= R<sub>3</sub>.i
                                                  num
                                                                                  num
                                                                                              num
```

### SDD Implementation using Parse Trees

- Alternative to using Syntax Directed Translation Scheme
  - Goal: create an **annotated parse tree** from the given parse tree
    - Annotated parse tree: tree annotated with attribute values
    - Traverse in a certain order and evaluate semantic rules at each node
    - Traversal order can be arbitrary as long as it adhers to dependency relationships

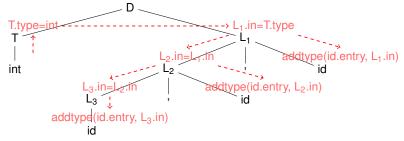
### **Dependency Graph**

- Directed graph where edges are dependency relationships between attributes
  - Needs to be acyclic such that there exists a traversal order for evaluation
    - i.e. all necessary information must be ready when evaluating an attribute at a node



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### SDD Implementation using SDTS

- Tree-based evaluation works for all SDDs unless there is a dependency cycle
  - But involves more work since parse tree must be built initially
  - And the question still remains, how do you build the parse tree itself?
- Is it possible to perform evaluation while parsing?
  - Embed semantic actions in grammar using SDTS
  - What are some potential problems?
    - Parser may not have even "seen" some nodes yet
    - Some dependencies may not exist at time of evaluation
  - Different parsing schemes see nodes in different orders
    - Top-down parsing LL(k) parsing
    - Bottom-up parsing LR(k) parsing
- For certain classes of SDDs, using SDTS is feasible
  - > if dependencies of SDD are amenable to parse order
  - ➤ In other words, an L-Attributed Grammar



### Left-Attributed Grammar

A syntax directed translation is L-attributed if each of its attributes is

#### either

ightharpoonup a synthesized attribute of A in A $\rightarrow$  X<sub>1</sub>... $X_n$ ,

or

- $\rightarrow$  an inherited attribute of  $X_i$  in  $A \rightarrow X_1...X_n$  that
  - depends on attributes of symbols to its left i.e.  $X_1...X_{i-1}$
  - and/or depends on inherited attributes of A

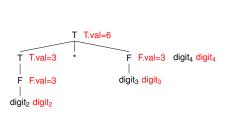
### Left-Attributed Grammar

- An L-Attributed grammar
  - may have synthesized attributes
  - may have inherited attributes but only from left sibling attributes or inherited attributes of the parent
- Evaluation order
  - Left-to-right depth-first traversal of the parse tree
    - Order for both top-down and bottom-up parsers
  - Evaluate inherited attributes while going down the tree
  - > Evaluate synthesized attributes while going up the tree
- Can be evaluated using SDTS w/o parse tree

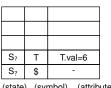
# Syntax Directed Translation Scheme Implementation

When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes

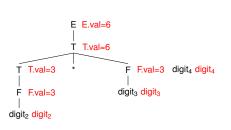


#### parsing stack:

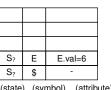


When using LR parsing (bottom-up parsing),

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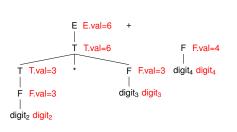


#### parsing stack:



When using LR parsing (bottom-up parsing),

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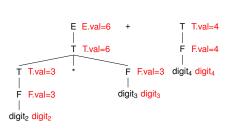


#### parsing stack:

S <sub>?</sub>	F	F.va	al=4	
S <sub>?</sub>	+	-		
S <sub>?</sub>	Е	E.val=6		
S <sub>?</sub>	\$	-		
(state) (symbol) (attribute			iiite	

When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes

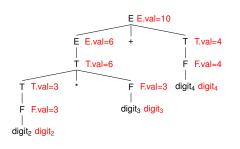


#### parsing stack:

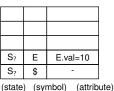
S <sub>?</sub>	Т	T.va	al=4	
S <sub>?</sub>	+	-		
S <sub>?</sub>	Е	E.val=6		
S <sub>?</sub>	\$	-		
(state) (symbol) (attribute				

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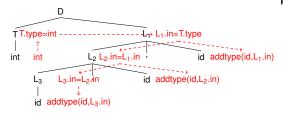


#### parsing stack:

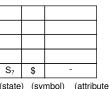


When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes



#### parsing stack:

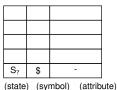


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#### parsing stack:

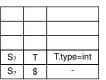


When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes



#### parsing stack:



When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes

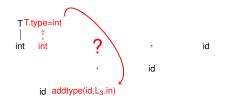


#### parsing stack:

S <sub>?</sub>	id	id.type=L3.in
S <sub>?</sub>	Т	T.type=int
S <sub>?</sub>	\$	-

When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes

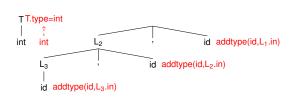


#### parsing stack:

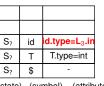
S <sub>?</sub>	id	id.type=L <sub>3</sub> .in
S <sub>?</sub>	Т	T.type=int
S <sub>?</sub>	\$	-

When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes

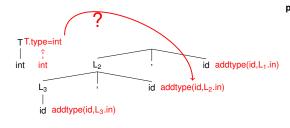


#### parsing stack:



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#### parsing stack:

S <sub>?</sub>	id	id.type=L <sub>3</sub>	.in
S <sub>?</sub>	Т	T T.type=int	
S <sub>?</sub>	\$	-	
(state) (symbol) (attribute			

# **Evaluating Inherited Attributes using LR**

Recall

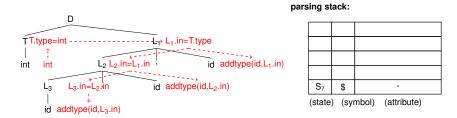
Only applies to L-Attributed grammars

What is L-attributed grammar?

Claim: the information is in the stack, we just do not know the exact location

Solution: let us hack the stack to find the location

```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow int & \{stack[top]=integer\} \\ T \rightarrow real & \{stack[top]=real\} \\ L \rightarrow L & , & id & \{addtype(stack[top],stack[top-3])\} \\ L \rightarrow id & \{addtype(stack[top],stack[top-1])\} \end{array}
```



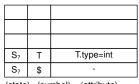
```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow int & \{stack[top]=integer\} \\ T \rightarrow real & \{stack[top]=real\} \\ L \rightarrow L & , & id & \{addtype(stack[top],stack[top-3])\} \\ L \rightarrow id & \{addtype(stack[top],stack[top-1])\} \end{array}
```



(state) (symbol) (attribute)

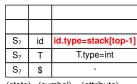
int

```
\begin{split} & D \to T \quad L \\ & T \to int \; \{stack[top] = integer\} \\ & T \to real \; \{stack[top] = real\} \\ & L \to L \; , \quad id \; \{addtype(stack[top], stack[top-3])\} \\ & L \to id \; \{addtype(stack[top], stack[top-1])\} \end{split}
```





```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow int & \{stack[top]=integer\} \\ T \rightarrow real & \{stack[top]=real\} \\ L \rightarrow L & , & id & \{addtype(stack[top],stack[top-3])\} \\ L \rightarrow id & \{addtype(stack[top],stack[top-1])\} \end{array}
```





```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow \text{int } \{stack[top] = \text{integer}\} \\ T \rightarrow \text{real } \{stack[top] = \text{real}\} \\ L \rightarrow L & , & \text{id } \{addtype(stack[top], stack[top-3])\} \\ L \rightarrow \text{id } \{addtype(stack[top], stack[top-1])\} \end{array}
```



	l	
S <sub>?</sub>	id	id.type=stack[top-1]
S <sub>?</sub>	Т	T.type=int
S <sub>?</sub>	\$	-

(state) (symbol) (attribute)

id

TT.type=int

id addtype(id,L3.in)

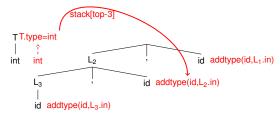
int

```
D \rightarrow T L
T \rightarrow int \{stack[top]=integer\}
T \rightarrow real \{ stack[top] = real \}
L \rightarrow L, id {addtype(stack[top],stack[top-3])}
L \rightarrow id \{addtype(stack[top],stack[top-1])\}
```

#### parsing stack:

	S <sub>?</sub>	id	id.type=stack[top-3]
_	S <sub>?</sub>	,	
id addtype(id,L <sub>1</sub> .in)	S <sub>?</sub>	L <sub>3</sub>	L <sub>3</sub> .in=int
id additype(id,E1.iii)	S <sub>?</sub>	Т	T.type=int
addtype(id,L <sub>2</sub> .in)	S <sub>?</sub>	\$	-
	(state	) (sv	mbol) (attribute)

```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow \text{int } \{stack[top] = \text{integer}\} \\ T \rightarrow \text{real } \{stack[top] = \text{real}\} \\ L \rightarrow L & , & \text{id } \{addtype(stack[top], stack[top-3])\} \\ L \rightarrow \text{id } \{addtype(stack[top], stack[top-1])\} \end{array}
```



S <sub>?</sub>	id	id.type=stack[top-3]
S <sub>?</sub>	,	
S <sub>?</sub>	L <sub>3</sub>	L <sub>3</sub> .in=int
S <sub>?</sub>	Т	T.type=int
S <sub>?</sub>	\$	-

### Marker

 $\Box$  Given the following SDD, where  $|\alpha| != |\beta|$ 

$$A \rightarrow X \alpha Y \mid X \beta Y$$

$$Y \rightarrow \gamma \{ \dots = f(X.s) \}$$

- Problem: cannot generate stack location for X.s since X is at different relative stack locations from Y
- Solution: introduce *markers* M<sub>1</sub> and M<sub>2</sub> that are at the same relative stack locations from Y

$$A \rightarrow X \alpha M_1 Y \mid X \beta M_2 Y$$

$$Y \rightarrow \gamma \{... = f(M_{12}.s)\}$$

$$M_1 \rightarrow \varepsilon \{M_1.s = X.s\}$$

$$M_2 \rightarrow \varepsilon \{M_2.s = X.s\}$$

$$(M_{12} = \text{the stack location of } M_1 \text{ or } M_2, \text{ which are identical})$$

A marker intuitively marks a stack location that is equidistant from the reduced non-terminal

#### Example

How to add the marker?

```
Example 1:
        S \rightarrow a A \{ C.i = A.s \} C
        S \rightarrow b A B \{ C.i = A.s \} C
        C \rightarrow c \{ C.s = f(C.i) \}
Solution:
        S \rightarrow a A \{ C.i = A.s \} C
        S \rightarrow b A B \{ M.i=A.s \} M \{ C.i = M.s \} C
        C \rightarrow c \{ C.s = f(C.i) \}
        M \rightarrow \varepsilon \{ M.s = M.i \}
That is:
        S \rightarrow a A C
        S \rightarrow b A B M C
        C \rightarrow c \{ C.s = f(stack[top-1]) \}
        M \rightarrow \varepsilon \{ M.s = stack[top-2] \}
```

#### How to Add the Marker?

- 1. Identify the stack location(s) to find the desired attribute
- 2. Is there a conflict of location?
  - Yes, add a marker;
  - No, no need to add.
- Add the marker in the place to remove location inconsistency

#### Example:

$$\begin{split} S &\rightarrow a \text{ A B C E D} \\ S &\rightarrow b \text{ A F B C F D} \\ C &\rightarrow c \text{ (/* C.s = f(A.s) */)} \\ D &\rightarrow d \text{ (/* D.s = f(B.s) */)} \end{split}$$

#### Answer

```
S → a A B C E D
S → b A D M B C F D
C → c {/* C.s = f(stack[top-2]) */}
D → d {/* D.s = f(stack[top-3]) */}
M → ε {/* M.s = f(stack[top-2]) */}

Regarding C.s, from stack[top-2], and stack[top-3]
.... add a Marker

Regarding D.s, always from stack[top-2]
... no need to add
```

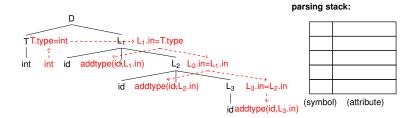
☐ How about Top-Down Parsing?

#### Translation Scheme for Top-Down Parsing

- Predictive Recursive Descent Parsers: Straightforward
  - Synthesized Attribute: Return value of function call for non-terminal is synthesized attribute
    - All function calls for children nodes would have completed by the time this function call returns
    - All dependent values would have been computed
  - Inherited Attribute: Pass as argument to function call for non-terminal inheriting attribute
    - L-Attributed grammar guarantees that dependent attributes come from left sibling attributes or parent inherited attributes
    - Left sibling function calls would have completed and parent inherited attribute would have been passed in as argument
    - All dependent values would have been computed
- Now let's focus on table-driven LL Parsers

When using LL parsing (top-down parsing),

it is natural to evaluate inherited attributes



When using LL parsing (top-down parsing),

it is natural to evaluate inherited attributes

D

#### parsing stack:



When using LL parsing (top-down parsing),

it is natural to evaluate inherited attributes



#### parsing stack:



When using LL parsing (top-down parsing),

it is natural to evaluate inherited attributes



#### parsing stack:



When using LL parsing (top-down parsing),

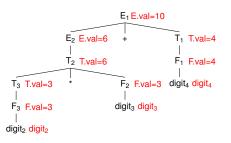
it is natural to evaluate inherited attributes

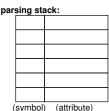


#### parsing stack:



When using LL parsing (top-down parsing),





When using LL parsing (top-down parsing),

it is **not natural** to evaluate synthesized attributes

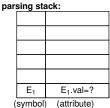
 $E_1$ 

parsing stack:				
	E <sub>1</sub>			
-	evmbol	(attribute)		

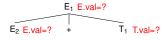
When using LL parsing (top-down parsing),

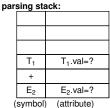
it is **not natural** to evaluate synthesized attributes

E<sub>1</sub> E.val=?

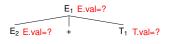


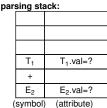
When using LL parsing (top-down parsing),





When using LL parsing (top-down parsing),



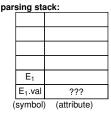


- Solution
  - Always push a 'dummy' stack item below a non-terminal to hold intermediate values for attribute calculation
  - Update dummy item whenever a child node is popped with intermediate value
  - When all children nodes have been popped, compute synthesized attribute from stored values

When using LL parsing (top-down parsing),

it is **not natural** to evaluate synthesized attributes

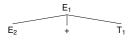
E<sub>1</sub>



#### Solution

- Always push a 'dummy' stack item below a non-terminal to hold intermediate values for attribute calculation
- Update dummy item whenever a child node is popped with intermediate value
- When all children nodes have been popped, compute synthesized attribute from stored values

When using LL parsing (top-down parsing),



parsing stack:				
	T <sub>1</sub>			
	T <sub>1</sub> .val	??		

T <sub>1</sub>		
T <sub>1</sub> .val	???	
+		
E <sub>2</sub>		
E <sub>2</sub> .val	???	
E <sub>1</sub> .val	E2.val + T1.val	
symbol) (attribute)		

- Solution
  - Always push a 'dummy' stack item below a non-terminal to hold intermediate values for attribute calculation
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