Semantic Analysis

The role of semantic analysis is to assign meaning

- "It smells fishy."
- Lexical analysis
  - Tokenizes "It", "smells", "fishy", "."
  - Determines noun, verb, adjective, punctuation token types
- Syntax analysis
  - Parses the grammatical structure of the sentence
- Semantic analysis

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  - Determines noun, verb, adjective, punctuation token types
- Syntax analysis
  - Parses the grammatical structure of the sentence
- Semantic analysis
  - Assigns meaning to the words "It", "smells", "fishy"
  - Flags error if the sentence does not make sense

Semantic Analysis = Binding + Type Checking

- "I don't wanna eat that sushi."
  "It smells fishy."
  - "It": the sushi
  - "smells": feels to my nose
  - "fishy": that the sushi has gone bad
- "The professor says that the exam is going to be easy."
  "It smells fishy."
  - "It": the situation
  - "smells": feels to my sixth sense
  - "fishy": that it is highly suspicious

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  "It smells fishy."
  - "It": the situation
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- Semantic analysis consists of two tasks
  - **Binding**: associating a pronoun to an object
  - **Type checking**: inferring meaning based on type of object

Semantic analysis cannot be done during parsing

- Context Free Grammars (CFGs) cannot recognize bindings
  - Every use of a name needs to be bound to the declaration.
  - Name can refer to a variable, function, class, ...
  - Names are called **symbols** in semantic analysis
- To do bindings, a CFG must recognize this language:

$$\{\alpha c \alpha | \alpha \in (a|b)^*\}$$

The 1st α represents the declaration,
The 2nd α represents a use.

- Above language is a Context Sensitive Language

Why is $\{\alpha c \alpha | \alpha \in (a|b)^*\}$ not a CFG?

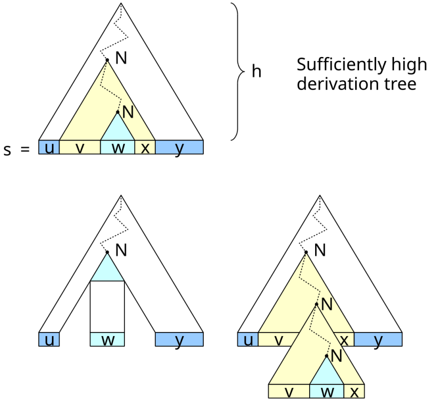
- We will base our proof on the **pumping lemma** for CFGs.
- **Pumping lemma**: a theorem about strings in a grammar
  - "lemma": a mathematical term for a theorem
  - "pumping": for a sufficiently long enough string,
    a substring exists within that string that can be "pumped"
    (repeated 0 or more times and still be in the language).

- For example, for the Regular Language 0(0|1)*0:
  - A string longer than 2 will look like 000, 010, 0100, ...
  - Let's take "010". Here, substring "1" can be pumped.
  - ("00", "010", "0110", "01110" are all in the language)

- Pumping Lemma applies to CFGs as well.

Pumping Lemma for CFGs

- For a sufficiently long string s derived from a CFG, s can be written as s = uvwxy (u,v,w,x,y are substrings)
  - Where v and x can be pumped and $|vx| \geq 1$.



Generating uv^0wx^0y Generating uv^2wx^2y

$\{\alpha c\alpha | \alpha \in (a|b)^*\}$ is not a CFG

- Let's say s = uvwxy is a sufficiently long string in language $\{\alpha c\alpha | \alpha \in (a|b)^*\}$,
  where v, x can be pumped and $|vx| \geq 1$.

1. The substring vwx must bisect $\alpha c\alpha$.
   - If vwx is contained in 1st α (or mostly contained), if we pump v and x 0 times, 1st α gets shorter than 2nd α.
   - string is no longer in $\{\alpha c\alpha | \alpha \in (a|b)^*\}$. Contradiction.
   - The same applies to when vwx is contained in 2nd α.
2. Even when vwx bisects $\alpha c\alpha$, pumping fails.
   - Let string s' be the result of pumping v and x 0 times.
   - Let's say s' = $\alpha_1 c\alpha_2$, where α_1 and α_2 are shortened versions of the 1st and 2nd αs.
   - While $|\alpha_1| = |\alpha_2|$, there exist α_1 and α_2 such that $\alpha_1 != \alpha_2$.
   - E.g. s = abcab, and vwx = bca where v = b and x = a. Then, α_1 becomes "a" and α_2 becomes "b". Contradiction.

Semantic analysis does binding and type checking

- Semantic analysis performs binding
  - Since CFGs cannot recognize bindings, as we just proved
  - Done by traversing parse tree produced by syntax analysis
  - Definitions are stored in data structure called **symbol table**
  - Uses are bound to entries in the symbol table
- Semantic analysis performs type checking
  - Infer what "$a + b$" means:
    - If a and b are ints, integer add and return int
    - If a and b are floats, FP add and return float
    - If a and b are strings, concatenate and return string
  - Infer what "$a.foo()$" means:
    - If object a is an instance of class A, call A.foo()
    - If object a is an instance of class B, call B.foo()
  - Infer what "$a[i][j]$" means:
    - Offset from a calculated based on type and dimensions

Semantic analysis also performs semantic checks

- All symbol uses have a corresponding declaration;
- All operations are type legal;
- Inheritance relationships are correct;
- A class is defined only once;
- A method in a class is defined only once;
- ...

Symbol Binding

What is symbol binding?

"Matching symbol **declarations** with **uses**"

- If there are multiple declarations, which one is matched?

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"Matching symbol **declarations** with **uses**"

- If there are multiple declarations, which one is matched?

```
void foo()
{
  char x;
  ...
  {
     int x;
     ...
  }
  x = x + 1;
}
```

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```

?

?

Scope

- Binding: the association of a use of a symbol to the declaration of that symbol
  - Which variable (or function) an identifier is referring to
- Scope: section of program where a declaration is valid
  - Uses in the scope of declaration are bound to it
- Some implications of scopes
  - A symbol may have different bindings in different scopes
  - Scopes for the same symbol never overlap
    - there is always exactly one binding per symbol use
- Two types: static scope and dynamic scope

Static Scope

- Static scope depends on the program text, not run-time behavior (also known as lexical scoping)
  - C/C++, Java, Objective-C
- Rule: Refer to the closest enclosing declaration

```
void foo()
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    char x;
  ...
  {
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void foo()
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  ...
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  }
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```

Dynamic Scope

- Dynamic scoping depends on bindings formed during the execution of the program
  - LISP, Scheme, Perl
- Rule: Refer to the closest binding in the current execution

```
void foo()
{
   (1) char x;
   (2) if (...) {
   (3)     int x;
   (4)      ...
        }
   (5) x = x + 1;
}
```

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{
   (1) char x;
   (2) if (...) {
   (3)     int x;
   (4)      ...
         }
   (5) x = x + 1;
}
```

- Which *x*'s declaration is the closest?
  - Execution (a): ...(1)...(2)...(5)
  - Execution (b): ...(1)...(2)...(3)...(4)...(5)

Static vs. Dynamic Scoping

- Most languages that started with dynamic scoping (LISP, Scheme, Perl) added static scoping afterwards
- Why?
  - It is easier for human beings to understand
    - Bindings are visible in code without tracing execution
  - It is easier for compilers to understand
    - Compiler can determine bindings at compile time
    - Compiler can translate identifier to a single memory location
    - Results in generation of efficient code
  - With dynamic scoping...
    - There may be multiple possible bindings for a variable
    - Impossible to determine bindings at compile time
    - All bindings have to be done at execution time (Typically with the help of a hash table)

Symbol Table

Symbol Table

- **Symbol Table**: A compiler data structure that tracks information about all identifiers (symbols) in a program
  - Maps symbol uses to declarations given a scope
  - Needs to provide bindings according to the current scope

- Usually discarded after generating the binary code
  - All symbols are mapped to memory locations already
  - For debugging, symbols may be included in binary
    - To map memory locations back to symbols for debuggers
    - For GCC or Clang, add “-g” flag to include symbol tables

Maintaining Symbol Table

- Basic idea:

  int x; ... void foo() { int x; ... x=x+1; } ... x=x+1 ...

  - In *foo*, add *x* to table, overriding any previous declarations
  - After *foo*, remove *x* and restore old declaration if any

- Operations

enter_scope()	start a new nested scope
exit_scope()	exit current scope
find_symbol(x)	find declaration of *x*
add_symbol(x)	add declaration of *x* to symbol table

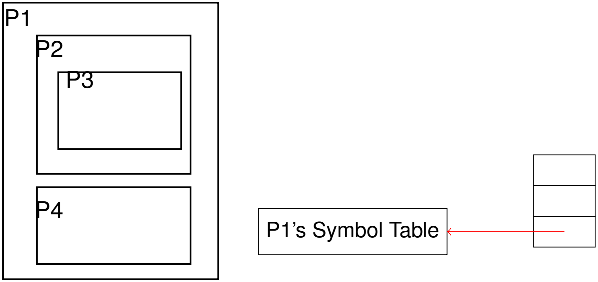
Adding Scope Information to the Symbol Table

- To handle multiple scopes in a program,
  - (Conceptually) need an individual table for each scope
  - Symbols added to the table may not be deleted just because you exited a scope

```
class X { ... void f1() {...} ... }
class Y { ... void f2() {...} ... }
X v;
call v.f1();
```

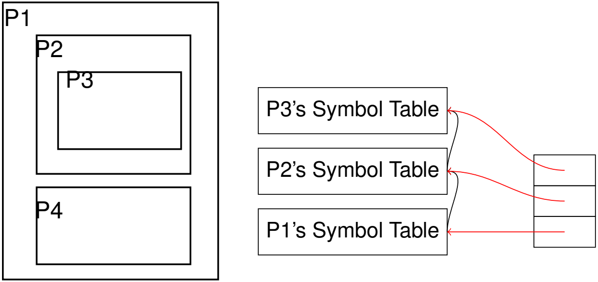
  - Without deleting symbols, how are scoping rules enforced?
    - Keep a list of all scopes in the entire program
    - Keep a stack of active scopes at a given point

Symbol Table with Multiple Scopes



- For nested scopes,
  - Search from top of the active symbol table stack
  - Remove pointer to symbol table when exiting its scope

Symbol Table with Multiple Scopes



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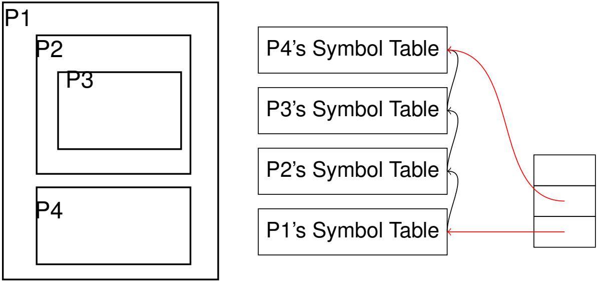

- For nested scopes,
  - Search from top of the active symbol table stack
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What Information is Stored in the Symbol Table

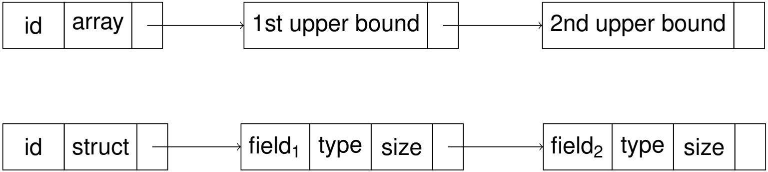
- Entry in Symbol Table:

string	kind	attributes

  - String — the name of identifier
  - Kind — variable, parameter, function, class, ...
- Attributes vary with the kind of symbol
  - variable $\rightarrow$ type, address in memory
  - function $\rightarrow$ return type, parameter types, address
- Vary with the language
  - Fortran's array $\rightarrow$ type, dimension, dimension size
    real A(5) /* dimension required for static allocation */
  - C's array $\rightarrow$ type, dimension, optional dimension size
    char A[5]; /* statically sized array */
    char A[]="hello"; /* dynamically sized to fit content */

Symbol Table Attribute List

- Type information might be arbitrarily complicated
  - In C:
```
struct {
    int a[10];
    char b;
    float c;
}
```
- Store all relevant attributes in an attribute list

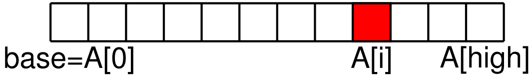


Example application of Type to an operator: Array index operator

Addressing Array Elements

int A[0..high];

A[i] ++;



- ➢ width — width of element type
- ➢ base — address of the first
- ➢ high — upper bound of subscript

- ❑ Addressing an array element:

address(A[i]) = base + i * width

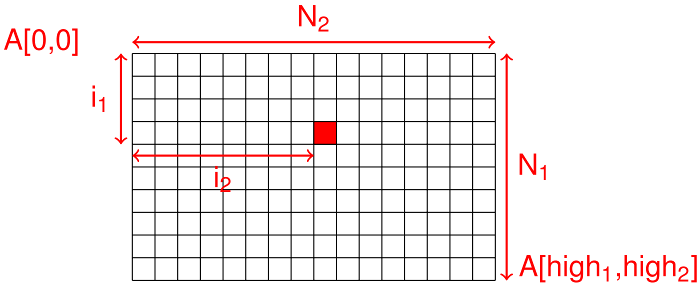
offset(A[i]) = i * width

Multi-dimensional Arrays

- Layout n-dimension items in 1-dimension memory

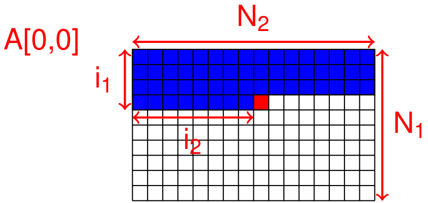
  int $A[N_1][N_2]$; /* int $A[0..high_1][0..high_2]$; */

  $A[i_1][i_2]$ ++;



Row Major

Row major — store row by row

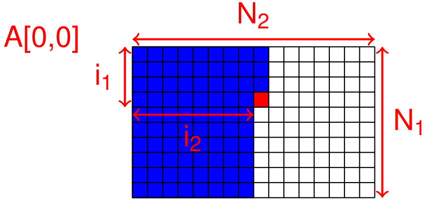


- Offset inclues all the “blue” items before $A[i_1,i_2]$

$$\text{offset}(A[i_1,i_2]) = (i_1 * N_2 + i_2) * \text{width}$$
$$= i_1 * N_2 * \text{width} + i_2 * \text{width}$$
$$= \text{offset}(A[i_1]) * N_2 + i_2 * \text{width}$$

Column Major

Column major — store column by column



- Offset inclues all the “blue” items before $A[i_1,i_2]$

$\text{offset}(A[i_1,i_2]) = (i_2 * N_1 + i_1)*\text{width}$
$= i_2 * N_1 * \text{width} + i_1 * \text{width}$
$= i_2*N_1*\text{width} + \text{offset}(A[i_1])$

Generalized Row/Column Major

- Let A_k = offset(A[i_1, i_2, ..., i_k]). Then,

- Row major
  1-dimension: $A_1 = i_1$*width
  2-dimension: $A_2 = (i_1*N_2+i_2)$*width = $A_1*N_2+i_2$*width
  3-dimension: $A_3 = (i_1*N_2*N_3+i_2*N_3+i_3)$*width = $A_2*N_3+i_3$*width
  k-dimension: $A_k = A_{k-1}*N_k + i_k$*width
  ☞**Type** needs to provide $N_2...N_k$ and width for offset

- Column major
  1-dimension: $A_1 = i_1$*width
  2-dimension: $A_2 = (i_2*N_1 + i_1)$*width = i_2*N_1*width + A_1
  3-dimension: $A_3 = ((i_3*N_2+i_2)*N_1+i_1)$*width = $i_3*N_2*N_1$*width + A_2
  k-dimension: $A_k = i_k*N_{k-1}*N_{k-2}*...*N_1$*width+$A_{k-1}$
  ☞**Type** needs to provide $N_1...N_{k-1}$ and width for offset

C's implementation

- C uses row major

```
int fun1(int p[ ][100])
{
...
   int a[100][100];
   a[i1][i2] = p[i1][i2] + 1;
}
```

Why is p[][100] allowed?

Why is a[][100] not allowed?

C's implementation

- C uses row major

```
int fun1(int p[ ][100])
{
...
   int a[100][100];
   a[i1][i2] = p[i1][i2] + 1;
}
```

Why is p[][100] allowed?

- The info is enough to compute $p[i_1][i_2]$'s address
- $A_2 = (i_1 * N_2 + i_2) * \text{width}$ (N_1 is not required)

Why is a[][100] not allowed?

- The info is not enough to allocate space for the array

Type Checking

What, Why and When

- What is a type?
  Type = a set of values + a set of operations on these values
- What is type checking?
  Verifying and enforcing type consistency
  - Only legal values are assigned to a type
  - Only legal operations are performed on a type
- Why is compile-time type checking desirable?
  - Runtime errors may go unnoticed while testing
  - Dynamic type checking when static checking infeasible
    - E.g. Java array bounds checks
    - E.g. Type checks to verify C++/Java downcasting

Static vs. Dynamic Typing

- Statically typed: C/C++, Java ☜Our discussion
  - Types are explicitly declared or can be inferred from code
  - E.g. int x; /* type of x is int */
  - Efficient code since runtime type checks are not needed
- Dynamically typed: Python, JavaScript, PHP
  - Type is a runtime property decided only during execution
  - E.g. var x; /* type of x is undecided */
  - Type of x changes depending on the type of value it holds
  - More memory since every variable now needs a "type tag"
  - Inefficient code due to runtime checks on type tags

Rules of Inference

- What are *rules of inference*?
  - Inference rules have the form
    if **Precondition** is true, then **Conclusion** is true
  - Below concise notation used to express above statement

$$\frac{\textbf{Precondition}}{\textbf{Conclusion}}$$

  - In the context of type checking:
    if expressions E1, E2 have certain types (Precondition),
    expression E3 is legal and has a certain type (Conclusion)
- Type checking via inference
  - Start from variable types and constant types
  - Repeatedly apply rules until entire program is inferred legal

Notation for Inference Rules

- By tradition inference rules are written as

$$\frac{\textbf{Precondition}_1\textbf{, ..., Precondition}_n}{\textbf{Conclusion}}$$

  - The precondition/conclusion has the form **"e:T"**
- Meaning
  - If **Precondition**$_1$ and ... and **Precondition**$_n$ are true, then **Conclusion** is true.
  - **"e:T" indicates "e is of type T"**
  - Example: rule-of-inference for add operation

$$\frac{\begin{array}{c}\mathbf{e_1 : int} \\ \mathbf{e_2 : int}\end{array}}{\mathbf{e_1 + e_2 : int}}$$

Rule: If e_1, e_2 are ints then e_1+e_2 is legal and is an int

Two Simple Rules

[Constant]

$$\frac{\textbf{i is an integer}}{\textbf{i: int}}$$

[Add operation]

$$\frac{e_1: \text{int} \quad e_2: \text{int}}{e_1+e_2: \text{int}}$$

- Example: given “10 is an integer” and “20 is an integer”, is the expression “10+20” legal? Then, what is the type?

$$\frac{\dfrac{\text{10 is an integer}}{\text{10: int}} \qquad \dfrac{\text{20 is an integer}}{\text{20: int}}}{\text{10+20:int}}$$

- This type of reasoning can be applied to the entire program

More Rules

[New]

$$\frac{}{\textbf{new T: T}}$$

[Not]

$$\frac{\textbf{e: Boolean}}{\textbf{not e: Boolean}}$$

- However,

  [Var?]

  $$\frac{\textbf{x is an identifier}}{\textbf{x: ?}}$$

  - the expression itself insufficient to determine type
  - **solution:** provide context for this expression

Type Environment

- A *type environment* gives type info for free variables
  - A variable is *free* if not declared inside the expression
  - It is a function mapping Symbols to Types
    - Set of declarations active at the current scope
    - Conceptual representation of a symbol table

Type Environment Notation

Let O be a function from Symbols to Types,

the sentence **O e:T**

is read as "under the assumption of environment O, expression e has type T"

$$\frac{\textbf{i is an intger}}{\textbf{O i: int}} \qquad \frac{\textbf{O e1: int} \quad \textbf{O e2: int}}{\textbf{O e1+e2: int}} \qquad \frac{\textbf{O(x) == T}}{\textbf{O x: T}}$$

– "if i is an integer, expression i is an int in any environment"

– "if e1 and e2 are ints in O, expression e1+e2 is int in O"

– "if variable x is mapped to int in O, expression x is int in O"

Declaration Rule

[Declaration w/o initialization]

$$\frac{\mathbf{O[T_0/x]\ e_1 : T_1}}{\mathbf{O\ let\ x{:}\ T_0\ in\ e_1 : T_1}}$$

$O[T_0/x]$: environment O modified so that it return T_0 on argument x and behaves as O on all other arguments:

$O[T_0/x](x) = T_0$

$O[T_0/x](y) = O(y)$ when $x \neq y$

- Translation: "If expression e_1 is type T_1 when x is mapped to type T_0 in the current environment, expression e_1 is type T_1 when x is declared to be T_0 in the current environment"

Declaration Rule with Initialization

[Declaration with initialization (initial try)]

$$\frac{O \vdash e_0 : T_0 \qquad O[T_0/x] \vdash e_1 : T_1}{O \vdash \textbf{let } x : T_0 \leftarrow e_0 \textbf{ in } e_1 : T_1}$$

- The rule is too strict (i.e. correct but not complete)

  Example

  class C inherits P ...

  let x:P ← new C in ...

  ☞ the above rule does not allow this code

Subtyping

- Subtyping is a relation $\leq$ on classes
  - $X \leq X$
  - if X inherits from Y, then $X \leq Y$
  - if $X \leq Y$ and $Y \leq Z$, then $X \leq Z$

- An improvement of our previous rule

  [Declaration with initialization]

$$\frac{\begin{array}{c}\mathbf{O}\ \mathbf{e}_0 : \mathbf{T} \\ \mathbf{T} \leq \mathbf{T}_0 \\ \mathbf{O}[\mathbf{T}_0/\mathbf{x}]\ \mathbf{e}_1 : \mathbf{T}_1\end{array}}{\mathbf{O}\ \mathbf{let}\ \mathbf{x} : \mathbf{T}_0 \leftarrow \mathbf{e}_0\ \mathbf{in}\ \mathbf{e}_1 : \mathbf{T}_1}$$

  - Both versions of declaration rules are correct
  - The improved version checks more programs

Assignment

- A correct but too strict rule

[Assignment]

$$\frac{\begin{array}{c}\mathbf{O(id) = T_0} \\ \mathbf{O\ e_1 : T_1} \\ \mathbf{T_1 \leq T_0}\end{array}}{\mathbf{O\ id \leftarrow e_1 : T_0}}$$

- The rule does not allow the below code

```
class C inherits P { only_in_C() { ... } }
x ← y ← new C
x.only_in_C()
```

Assignment

- An improved rule

[Assignment]

$$\frac{\begin{array}{c} O(id) = T_0 \\ O \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O \vdash id \leftarrow e_1 : T_1}$$

- The rule now does allow the below code

  class C inherits P { only_in_C() { ... } }
  x ← y ← new C
  x.only_in_C()

If-then-else

- Consider

  if e_0 then e_1 else e_2

  - The result can be either e_1 or e_2
  - The type is either e_1's type or e_2's type

  ➢ The best that we can do (statically) is the super type larger than e_1's type and e_2's type

- Least upper bound (LUB)

  ➢ Z = lub(X,Y) — Z is defined as the least upper bound of X and Y *iff*

  - $X \leq Z \wedge Y \leq Z$; Z is an upper bound
  - $X \leq W \wedge Y \leq W \Longrightarrow Z \leq W$; Z is least among all upper bounds

If-then-else, case

[If-then-else]

$$\frac{\begin{array}{c} O\ e_0 : \textbf{Bool} \\ O\ e_1 : T_1 \\ O\ e_2 : T_2 \end{array}}{O\ \textbf{if}\ e_0\ \textbf{then}\ e_1\ \textbf{else}\ e_2\ \textbf{fi} : \text{lub}(T_1, T_2)}$$

- The rule allows the below code

  let x:float, y:int, z:float in

  x ← if (...) then y else z

  /* Assuming lub(int, float) = float */

Error Recovery

- Just like other errors, we should recover from type errors
  - Too many errors?
    let y: int $\leftarrow$ x+2 in y+3
    - if x is undefined —- reporting an error "x type undefined"
    - x+2 is undefined —- reporting an error "x+2 type undefined"
    - ...
- Introduce no-type for ill-typed expressions
  - It is compatible with all types
  - Report the place where no-type is generated
    - Reduce the number of error messages

Wrong Declaration Rule (case 1)

- Consider a hypothetical let rule

[Wrong Declaration with initialization (case 1)]

$$\frac{O\ e_0 : T \quad T \le T_0 \quad O\ e_1 : T_1}{O\ \text{let}\ x : T_0 \leftarrow e_0\ \text{in}\ e_1 : T_1}$$

- ➢ How is it different from the the correct rule?
- ➢ The following good program does not pass check

  let x: int ← 0 in x+1

Wrong Declaration Rule (case 2)

- Consider a hypothetical let rule

[Wrong Declaration with initialization (case 2)]

$$\frac{\mathbf{O} \vdash \mathbf{e_0 : T} \quad \mathbf{T_0 \leq T} \quad \mathbf{O[T_0/x]} \vdash \mathbf{e_1 : T_1}}{\mathbf{O} \vdash \mathbf{let\ x : T_0 \leftarrow e_0\ in\ e_1 : T_1}}$$

- ➢ How is it different from the the correct rule?
- ➢ The following bad program passes the check

  class B inherits A { only_in_B() { ... } }

  let x: B $\leftarrow$ new A in x.only_in_B()

Discussion

- Type rules have to be carefully constructed, or
  - The type system becomes unsound
    (bad programs are accepted as well typed)
  - The type system becomes unusable
    (good programs are rejected as badly typed)

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Discussion

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  - The type system becomes unusable
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- What is a “good” program anyway?
  - Good program: a program where all operations on all values are type consistent **at runtime**
- All runtime behavior not expressed in a static type system
  - At below is a good program rejected by the type system
    obj ← if (x > y) then new Child else new Parent
    if (x > y) then obj.only_in_Child()
  - LUB type makes a choice of soundness over usability

Designing a Good Type Checking System

- A good type system achieves two opposing goals:
  - Prevents **false negative** type errors, that is, runtime errors that are missed by type checking
  - Minimizes **false positive** type errors, that is, type errors that do not cause runtime errors
- A good type system should allow the following code:

```
class Parent {
  Parent clone() { return new this.getClass(); }
}
class Child inherits Parent { ... }
  void main() {
    // Error! Assignment of parent to child reference.
    Child c ← (new Child).clone();
  }
}
```

What Went Wrong?

- What is (new Child).clone()'s type?
  - Dynamic type — Child
  - Static type — Parent

  - Type system is not able to express runtime types precisely
  - This makes inheriting clone() not very useful
    - clone() needs redefinition to return correct type anyway

- A "SELF_TYPE" would be useful in these situations.

SELF_TYPE expresses runtime types precisely

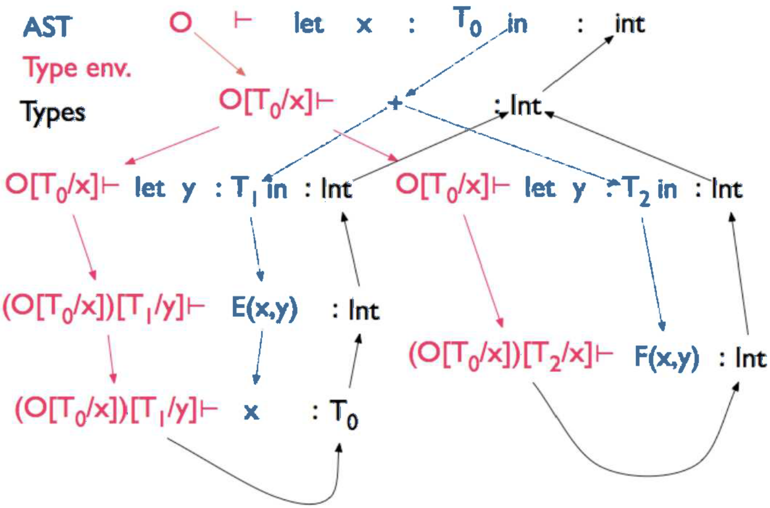
- What is SELF_TYPE?
  - clone() returns “self” instead of “Parent” type
  - Self can be Parent or any subclass of Parent
- SELF_TYPE is a static type
  - Type reflects precise runtime behavior for each class
  - Type violations can still be detected at compile time
- In practice
  - Python, Rust, Scala: language support for self types
  - C++: can emulate using C++ templates
  - Java: can emulate to a lesser degree using Java generics

Can Static Type Checking ever be Perfect?

- Many examples where "good" programs are disallowed
  - ➢ Reason for elaborate type systems like templates and generics
  - ➢ Why programmers are sometimes forced to typecast anyway

- Solution? Can't have your cake and eat it too.
  - ➢ Dynamic typing: values have types, variables do not
    - \+ Allows all runtime behaviors that are type consistent
    - \- Type errors occur at runtime rather than compile time
    - ☞ Best used for fast prototyping (scripting languages)
  - ➢ Static typing: variables have declared (or inferred) types
    - \+ Type errors can be caught at compile time
    - \- Effort needed to express "good" programs using type system
    - ☞ Best used when reliability is important

Implementing Type Checking on AST



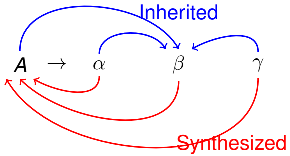
Syntax Directed Definitions (SDDs)

SDD: Definitions of attributes and rules

- Syntax Directed Definitions (SDD):
  1. Set of **attributes** attached to each grammar symbol
  2. Set of **semantic rules** attached to each production
  - Semantic rules define values of attributes
- Attribute Grammar:
  - An SDD where rules depend only on other attributes (i.e. An SDD that does not rely on any side-effects)
  - Think of it as a "grammar" for semantic analysis
- Example: let's say we want to define type checking
  - SDD can have semantic rules to access a symbol table
  - Attribute grammar must transmit type info through attributes

Syntax Directed Definition (SDD)

- Semantic rule:



SDD has rule of the form for each CFG production

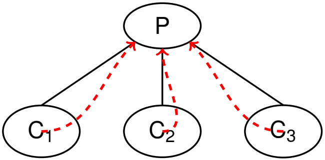
$$b = f(c_1, c_2, ..., c_n)$$

either

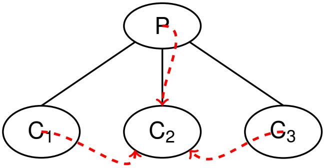
1. If b is a synthesized attribute of A,
   c_1 ($1 \leq i \leq n$) are attributes of grammar symbols of its Right Hand Side (RHS); or
2. If b is an inherited attribute of one of the symbols of RHS,
   c_i's are attribute of A and/or other symbols on the RHS

Two Types of Attributes

- **Synthesized attributes:** attributes are computed from attributes of children nodes
  - P.synthesized_attr = f(C_1.attr, C_2.attr, C_3.attr)
- **Inherited attributes:** attributes are computed from attributes of sibling and parent nodes
  - C_3.inherited_attr = f(P_1.attr, C_1.attr, C_3.attr)



Synthesized attribute



Inherited attribute

Synthesized Attribute Example

- Example
  - Each non-terminal symbol is associated with **val** attribute
  - The **val** attribute is computed soley from children attributes

[Grammar Rules]	[Semantic Rules]
$L \rightarrow E$	print(E.val)
$E \rightarrow E_1 + T$	$E.val = E_1.val + T.val$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T_1 * F$	$T.val = T_1.val * F.val$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow digit$	$F.val = digit.lexval$

- Is this an SDD or an attribute grammar?

Inherited Attribute Example

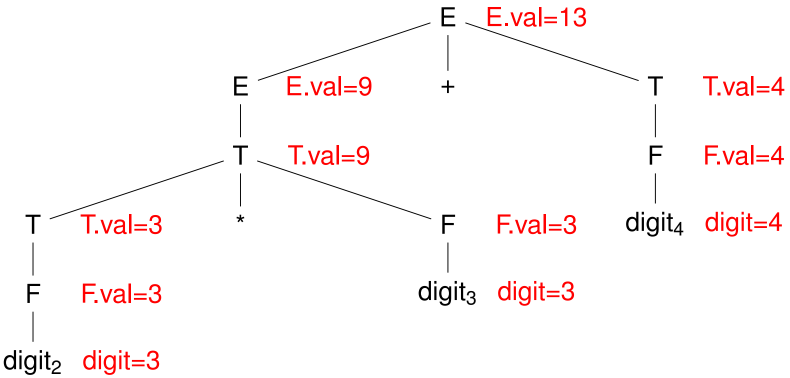
- Example:
  - T.type: synthesized attribute
  - L.in: inherited attribute

[Grammar Rules]	[Semantic Rules]
$D \rightarrow T\ L$	L.in = T.type
$T \rightarrow$ int	T.type = integer
$T \rightarrow$ real	T.type = real
$L \rightarrow L_1$, id	L_1.in = L.in, addtype (id.entry, L.in)
$L \rightarrow$ id	addtype(id.entry, L.in)

- Why is L.in an inherited attribute?
  - L.in is computed from a sibling T.type
  - L_1.in is computed from a parent L.in
- Is this an SDD or an attribute grammar?

Attribute Parse Tree

- SDDs produce an attribute parse tree
  - **Attribute parse tree**: Parse tree annotated with attributes



SDD Implementation

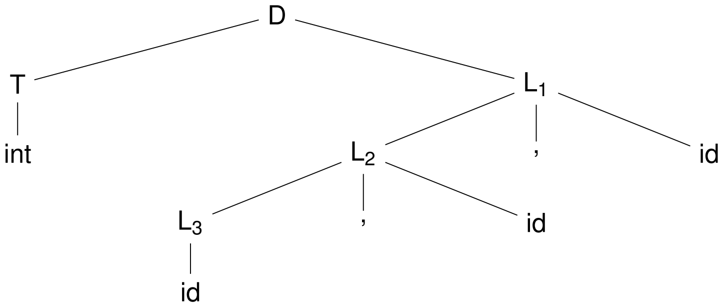
SDD Implementation using Parse Trees

- Assumes a previous parse stage
  - Input: a parse tree with no attribute annotations
  - Output: an attribute parse tree

- Goal: compute attribute values from leaf token values
  - Traverse in some order, apply semantic rules at each node
  - Traversal order must consider attribute dependencies

Dependency Graph

- Directed graph where edges are attribute dependencies
  - "To" attribute is computed base on "from" attribute
  - Must be **acyclic** such that there exists "a" traversal order



Dependency Graph

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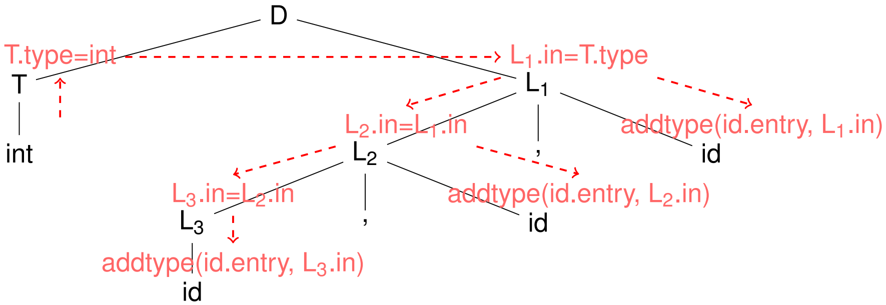


SDD Implementation using SDT

- Syntax Directed Translation (SDT)
  - Applying semantic rules as part of syntax analysis (parsing)
  - Does NOT assume a pre-existing parse tree
  - Done through **semantic actions** embedded in grammar
- Semantic action:
  - Code between curly braces embedded into RHS
  - Executed “at that point” in the RHS
    - Top-down: Right after previous symbol has been consumed
    - Bottom-up: After previous symbol has been pushed to stack (when the ’dot’ reaches the semantic action)
  - Example of building a parse tree:
    - Program : Program IDNum ; Classes { $$=makeTree(ProgramOp, $2, $4); }
  - $2 and $4 are indices into the parse stack
    - RHS is currently at top of stack waiting to be reduced
    - $2 is attribute value for IDNum and $4 is Classes

Syntax Directed Translation Scheme (SDTS)

- Syntax Directed Translation Scheme (SDTS)
  - A "scheme" or plan to perform SDT
  - A grammar specification embedded with semantic actions
  - Depends heavily on choice of top-down or bottom-up parser
- Example:

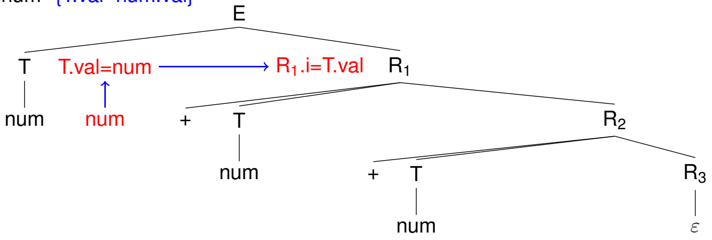
$$\begin{aligned}
&E \rightarrow T \quad R \\
&R \rightarrow + \quad T \quad R \\
&R \rightarrow - \quad T \quad R \\
&R \rightarrow \varepsilon \\
&T \rightarrow (\; E \;) \\
&T \rightarrow \text{num}
\end{aligned}$$

- Both inherited and synthesized attributes are used
  - T — synthesized attribute T.val
  - R — inherited attribute R.i
    synthesized attribute R.s
  - E — synthesized attribute E.val

Syntax Directed Translation Scheme (SDTS)

- Evaluating attributes using SDTS

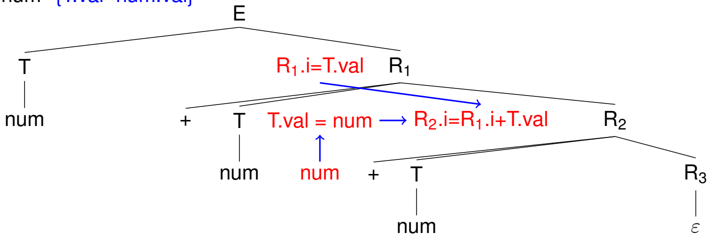
$E \rightarrow T$ {R.i=T.val} R {E.val=R.s}
$R \rightarrow + \; T$ {R_1.i=R.i+T.val} R_1 {R.s=R_1.s}
$R \rightarrow - \; T$ {R_1.i=R.i-T.val} R_1 {R.s=R_1.s}
$R \rightarrow \varepsilon$ {R.s=R.i}
$T \rightarrow (\; E \;)$ {T.val=E.val}
$T \rightarrow$ num {T.val=num.val}



Syntax Directed Translation Scheme (SDTS)

- Evaluating attributes using SDTS

$E \rightarrow T$ {R.i=T.val} R {E.val=R.s}
$R \rightarrow +\ T$ {R_1.i=R.i+T.val} R_1 {R.s=R_1.s}
$R \rightarrow -\ T$ {R_1.i=R.i-T.val} R_1 {R.s=R_1.s}
$R \rightarrow \varepsilon$ {R.s=R.i}
$T \rightarrow (\ E\ )$ {T.val=E.val}
$T \rightarrow$ num {T.val=num.val}



Syntax Directed Translation Scheme (SDTS)

- Evaluating attributes using SDTS

$E \rightarrow T$ {R.i=T.val} R {E.val=R.s}
$R \rightarrow +\ T$ {R_1.i=R.i+T.val} R_1 {R.s=R_1.s}
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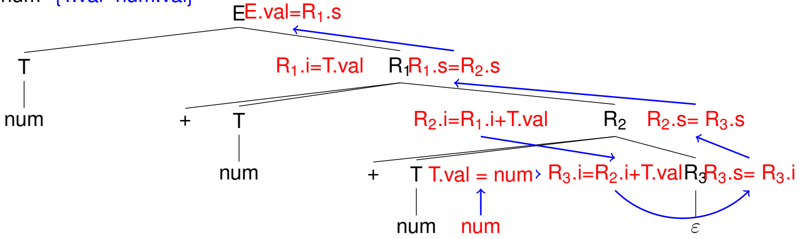


What are the dependencies allowed in SDTS?

- Parse trees: dependencies only required to be acyclic

- What is required of dependencies for SDTS?
  - Different parsing schemes see nodes in different orders
    - Top-down parsing — LL(k) parsing
    - Bottom-up parsing — LR(k) parsing
  - What if dependency node has not been seen yet?

- For certain classes of SDDs, using SDTS is feasible
  - If dependencies of SDD are amenable to parse order
  - This class of SDDs are called L-Attributed Grammars

Left-Attributed Grammar

- An SDD is L-attributed if each of its attributes is either:
  - a synthesized attribute of A in A$\rightarrow$ $X_1...X_n$,

    or

  - an inherited attribute of X_j in A$\rightarrow$ $X_1...X_n$ that
    - depends on attributes of siblings to its left i.e. $X_1...X_{j-1}$
    - and/or depends on parent A

Left-Attributed Grammar

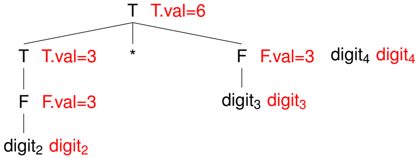
- An L-Attributed grammar
  - may have synthesized attributes
  - may have inherited attributes but only from left sibling attributes or inherited attributes of the parent
- Evaluation order
  - Left-to-right depth-first traversal of the parse tree
    - Order for both top-down and bottom-up parsers
  - Evaluate inherited attributes while going down the tree
  - Evaluate synthesized attributes while going up the tree
- Can be evaluated using SDTS w/o parse tree

Syntax Directed Translation Scheme (SDTS)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is natural and easy to evaluate synthesized attributes



parsing stack:

$S_?$	T	T.val=6
$S_?$	$	-
(state)	(symbol)	(attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

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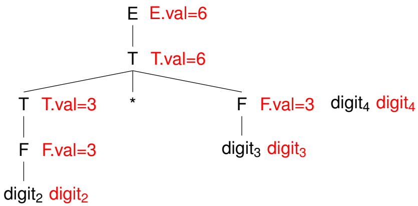

parsing stack:

(state)	(symbol)	(attribute)
$S_?$	E	E.val=6
$S_?$	$	-

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

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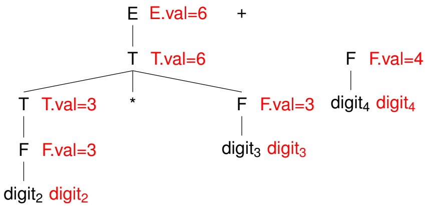

parsing stack:

(state)	(symbol)	(attribute)
S?	F	F.val=4
S?	+	-
S?	E	E.val=6
S?	$	-

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

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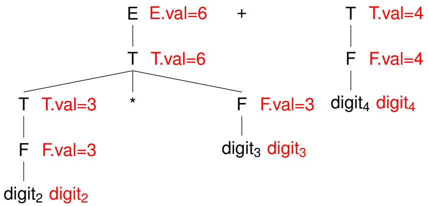

parsing stack:

S?	T	T.val=4
S?	+	-
S?	E	E.val=6
S?	$	-
(state)	(symbol)	(attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

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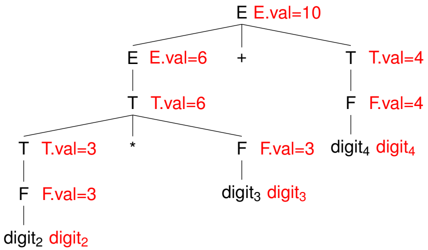


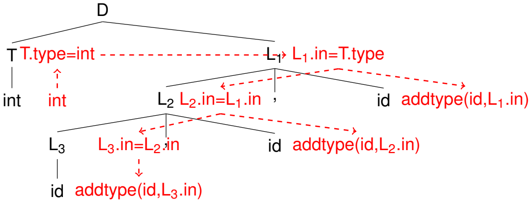
parsing stack:

$S_?$	E	E.val=10
$S_?$	$	-
(state)	(symbol)	(attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is **not natural** to evaluate inherited attributes



parsing stack:

$S_?$	\$	-

(state) (symbol) (attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is **not natural** to evaluate inherited attributes

int , id

, id

id

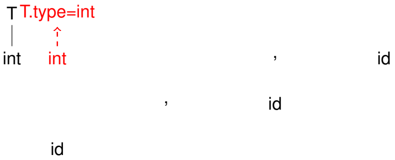
parsing stack:

(state)	(symbol)	(attribute)
$S_?$	$	-

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is **not natural** to evaluate inherited attributes



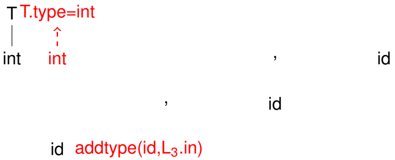
parsing stack:

$S_?$	T	T.type=int
$S_?$	$	-
(state)	(symbol)	(attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is **not natural** to evaluate inherited attributes



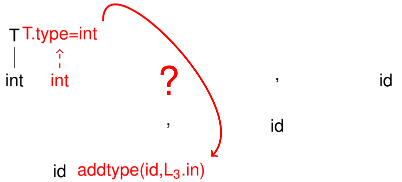
parsing stack:

(state)	(symbol)	(attribute)
$S_?$	id	**id.type=L_3.in**
$S_?$	T	T.type=int
$S_?$	\$	-

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is **not natural** to evaluate inherited attributes



parsing stack:

(state)	(symbol)	(attribute)
$S_?$	id	**id.type=L_3.in**
$S_?$	T	T.type=int
$S_?$	\$	-

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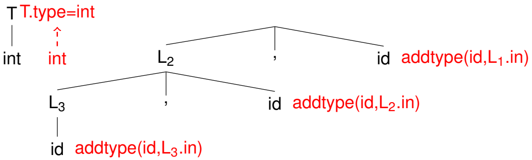

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(state)	(symbol)	(attribute)
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$S_?$	T	T.type=int
$S_?$	$	-

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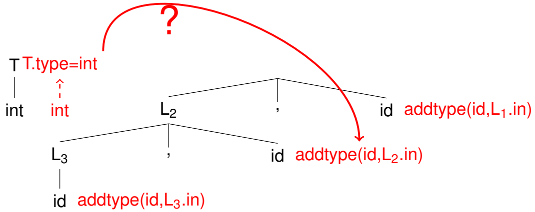


parsing stack:

(state)	(symbol)	(attribute)
$S_?$	id	**id.type=L_3.in**
$S_?$	T	T.type=int
$S_?$	\$	-

Evaluating Inherited Attributes using LR

- Recall
- Only applies to L-Attributed grammars
  - What is L-attributed grammar?

- **Claim:** the information is in the stack, we just do not know the exact location
- **Solution:** let us hack the stack to find the location

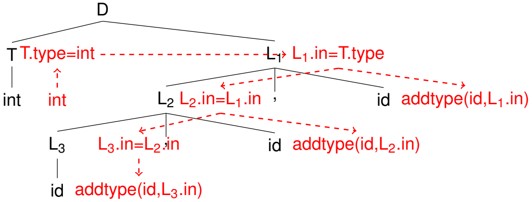
$D \rightarrow T \quad L$

$T \rightarrow$ int {stack[top]=integer}

$T \rightarrow$ real {stack[top]=real}

$L \rightarrow L$, id {addtype(stack[top],stack[top-3])}

$L \rightarrow$ id {addtype(stack[top],stack[top-1])}



parsing stack:

$S_?$	$	-
(state)	(symbol)	(attribute)

D → T L
T → int {stack[top]=integer}
T → real {stack[top]=real}
L → L , id {addtype(stack[top],stack[top-3])}
L → id {addtype(stack[top],stack[top-1])}

int , id

, id

id

parsing stack:

$S_?$	$	-
(state)	(symbol)	(attribute)

D $\rightarrow$ T L
T $\rightarrow$ int {stack[top]=integer}
T $\rightarrow$ real {stack[top]=real}
L $\rightarrow$ L , id {addtype(stack[top],stack[top-3])}
L $\rightarrow$ id {addtype(stack[top],stack[top-1])}

T T.type=int
int int , id
, id
id

parsing stack:

$S_?$	T	T.type=int
$S_?$	$	-
(state)	(symbol)	(attribute)

D → T L
T → int {stack[top]=integer}
T → real {stack[top]=real}
L → L , id {addtype(stack[top],stack[top-3])}
L → id {addtype(stack[top],stack[top-1])}

T T.type=int
int int
, id
, id
id addtype(id,L_3.in)

parsing stack:

$S_?$	id	**id.type=stack[top-1]**
$S_?$	T	T.type=int
$S_?$	$	-
(state)	(symbol)	(attribute)

D → T L
T → int {stack[top]=integer}
T → real {stack[top]=real}
L → L , id {addtype(stack[top],stack[top-3])}
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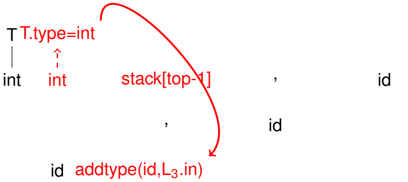
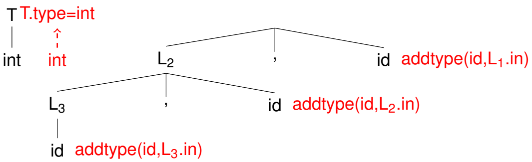


parsing stack:

$S_?$	id	**id.type=stack[top-1]**
$S_?$	T	T.type=int
$S_?$	$	-
(state)	(symbol)	(attribute)

D $\rightarrow$ T L
T $\rightarrow$ int {stack[top]=integer}
T $\rightarrow$ real {stack[top]=real}
L $\rightarrow$ L , id {addtype(stack[top],stack[top-3])}
L $\rightarrow$ id {addtype(stack[top],stack[top-1])}



parsing stack:

$S_?$	id	**id.type=stack[top-3]**
$S_?$	,	
$S_?$	L_3	L_3.in=int
$S_?$	T	T.type=int
$S_?$	$	-
(state)	(symbol)	(attribute)

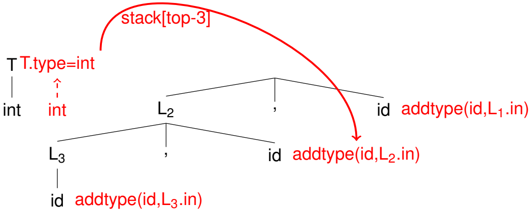
D $\rightarrow$ T L

T $\rightarrow$ int {stack[top]=integer}

T $\rightarrow$ real {stack[top]=real}

L $\rightarrow$ L , id {addtype(stack[top],stack[top-3])}

L $\rightarrow$ id {addtype(stack[top],stack[top-1])}



parsing stack:

$S_?$	id	**id.type=stack[top-3]**
$S_?$	,	
$S_?$	L_3	L_3.in=int
$S_?$	T	T.type=int
$S_?$	\$	-
(state)	(symbol)	(attribute)

Marker

- Given the following SDD, where $|\alpha|$!= $|\beta|$

  A$\rightarrow$ X α Y | X β Y
  Y$\rightarrow$ γ {... = f(X.s)}

- Problem: cannot generate stack location for X.s since X is at different relative stack locations from Y
- Solution: introduce *markers* M_1 and M_2 that are at the same relative stack locations from Y

  A$\rightarrow$ X α M_1 Y | X β M_2 Y
  Y$\rightarrow$ γ {... = f(M_{12}.s)}
  $M_1\rightarrow$ ε {M_1.s = X.s}
  $M_2\rightarrow$ ε {M_2.s = X.s}
  (M_{12} = the stack location of M_1 or M_2, which are identical)

- A marker intuitively marks a stack location that is equidistant from the reduced non-terminal

Example

- How to add the marker ?

Example 1:

$S \rightarrow$ a A { C.i = A.s } C
$S \rightarrow$ b A B { C.i = A.s } C
$C \rightarrow$ c { C.s = f(C.i) }

Solution:

$S \rightarrow$ a A { C.i = A.s } C
$S \rightarrow$ b A B { M.i=A.s } M { C.i = M.s } C
$C \rightarrow$ c { C.s = f(C.i) }
$M \rightarrow \varepsilon$ { M.s = M.i }

That is:

$S \rightarrow$ a A C
$S \rightarrow$ b A B M C
$C \rightarrow$ c { C.s = f(stack[top-1]) }
$M \rightarrow \varepsilon$ { M.s = stack[top-2] }

How to Add the Marker?

1. Identify the stack location(s) to find the desired attribute
2. Is there a conflict of location?
   - Yes, add a marker;
   - No, no need to add.
3. Add the marker in the place to remove location inconsistency

Example:

S $\rightarrow$ a A B C E D
S $\rightarrow$ b A F B C F D
C $\rightarrow$ c {/* C.s = f(A.s) */}
D $\rightarrow$ d {/* D.s = f(B.s) */}

Answer

S $\rightarrow$ a A B C E D
S $\rightarrow$ b A D M B C F D
C $\rightarrow$ c {/* C.s = f(stack[top-2]) */}
D $\rightarrow$ d {/* D.s = f(stack[top-3]) */}
M $\rightarrow$ ε {/* M.s = f(stack[top-2]) */}

- Regarding C.s, from stack[top-2], and stack[top-3]
  .... add a Marker
- Regarding D.s, always from stack[top-2]
  ... no need to add

- How about Top-Down Parsing?

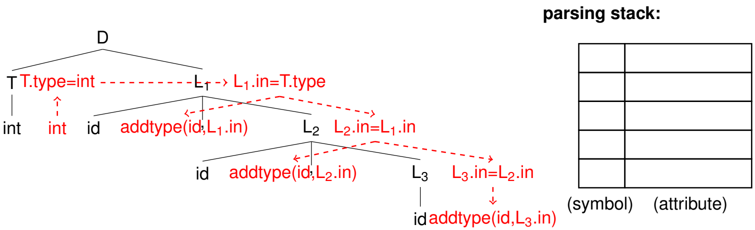
Translation Scheme for Top-Down Parsing

- Predictive Recursive Descent Parsers: Straightforward
  - Synthesized Attribute: Return value of function call for non-terminal is synthesized attribute
    - All function calls for children nodes would have completed by the time this function call returns
    - All dependent values would have been computed
  - Inherited Attribute: Pass as argument to function call for non-terminal inheriting attribute
    - L-Attributed grammar guarantees that dependent attributes come from left sibling attributes or parent inherited attributes
    - Left sibling function calls would have completed and parent inherited attribute would have been passed in as argument
    - All dependent values would have been computed
- Now let's focus on table-driven LL Parsers

Translation Scheme for LL Parsing

When using LL parsing (top-down parsing),

- it is natural to evaluate inherited attributes



Translation Scheme for LL Parsing

When using LL parsing (top-down parsing),

- it is natural to evaluate inherited attributes

D

parsing stack:

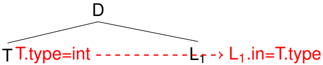
D	

(symbol) (attribute)

Translation Scheme for LL Parsing

When using LL parsing (top-down parsing),

- it is natural to evaluate inherited attributes



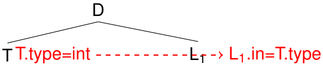
parsing stack:

T	T.type=int
L_1	L_1.in=()
(symbol)	(attribute)

Translation Scheme for LL Parsing

When using LL parsing (top-down parsing),

- it is natural to evaluate inherited attributes



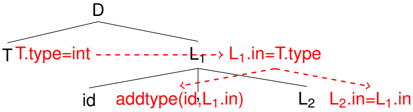
parsing stack:

L_1	L_1.in=int
(symbol)	(attribute)

Translation Scheme for LL Parsing

When using LL parsing (top-down parsing),

- it is natural to evaluate inherited attributes



parsing stack:

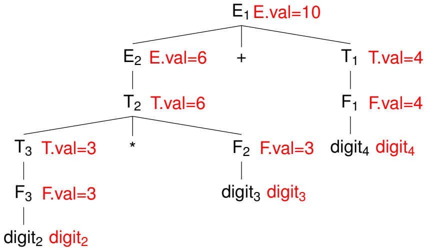
id	id.type=L_1.in
,	
L_2	L_2.in=intL_1.in

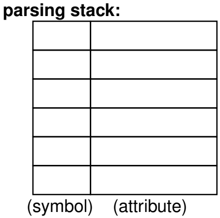
(symbol) (attribute)

Translation Scheme for LL Parsing

When using LL parsing (top-down parsing),

- it is **not natural** to evaluate synthesized attributes





Translation Scheme for LL Parsing

When using LL parsing (top-down parsing),

- it is **not natural** to evaluate synthesized attributes

E_1

parsing stack:

(symbol)	(attribute)
E_1	

Translation Scheme for LL Parsing

When using LL parsing (top-down parsing),

- it is **not natural** to evaluate synthesized attributes

E_1 E.val=?

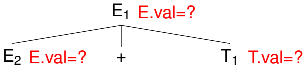
parsing stack:

(symbol)	(attribute)
E_1	E_1.val=?

Translation Scheme for LL Parsing

When using LL parsing (top-down parsing),

- it is **not natural** to evaluate synthesized attributes



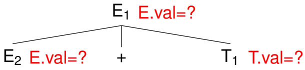
parsing stack:

(symbol)	(attribute)
T_1	T_1.val=?
+	
E_2	E_2.val=?

Translation Scheme for LL Parsing

When using LL parsing (top-down parsing),

- it is **not natural** to evaluate synthesized attributes



parsing stack:

(symbol)	(attribute)
T_1	T_1.val=?
+	
E_2	E_2.val=?

- Solution
  - Always push a 'dummy' stack item below a non-terminal to hold intermediate values for attribute calculation
  - Update dummy item whenever a child node is popped with intermediate value
  - When all children nodes have been popped, compute synthesized attribute from stored values

Translation Scheme for LL Parsing

When using LL parsing (top-down parsing),

- it is **not natural** to evaluate synthesized attributes

E_1

parsing stack:

E_1	
E_1.val	???

(symbol) (attribute)

- Solution
  - ➢ Always push a 'dummy' stack item below a non-terminal to hold intermediate values for attribute calculation
  - ➢ Update dummy item whenever a child node is popped with intermediate value
  - ➢ When all children nodes have been popped, compute synthesized attribute from stored values

Translation Scheme for LL Parsing

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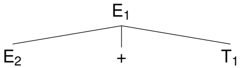

parsing stack:

T_1	
T_1.val	???
+	
E_2	
E_2.val	???
E_1.val	E_2.val + T_1.val
(symbol)	(attribute)

- Solution
  - Always push a 'dummy' stack item below a non-terminal to hold intermediate values for attribute calculation
  - Update dummy item whenever a child node is popped with intermediate value
  - When all children nodes have been popped, compute synthesized attribute from stored values