

Bottom Up Parsing

PITT CS 1622

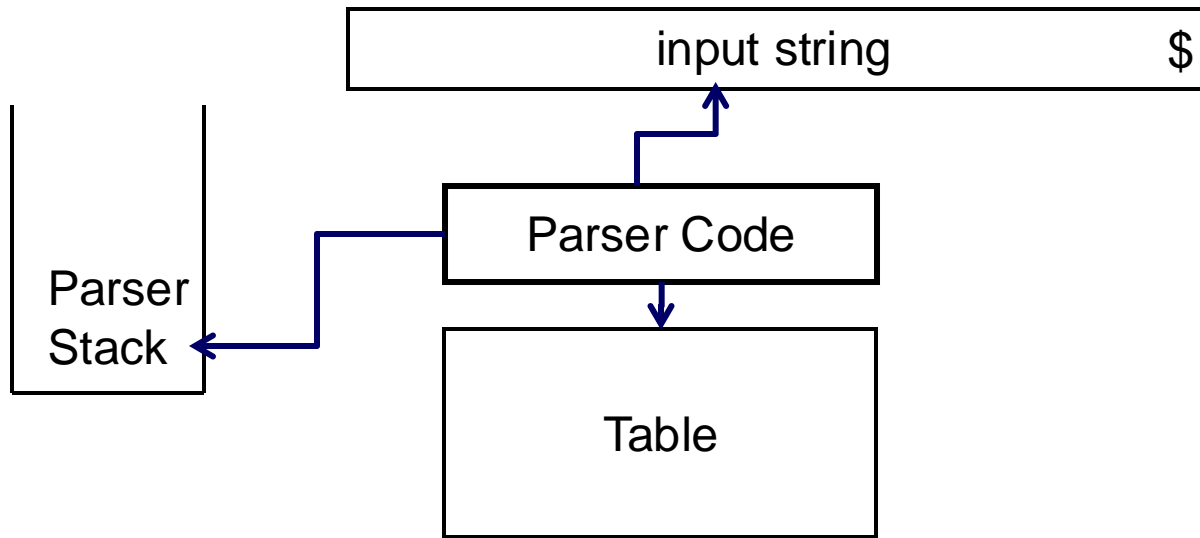
Bottom Up Parsing

- ❑ More powerful than top down
 - Don't need left factored grammars
 - Can handle left recursion
 - Can parse a larger set of grammars (and languages)

- ❑ Begins at leaves and works to the top
 - In reverse order of rightmost derivation
(In effect, builds tree from left to right)

- ❑ Also known as Shift-Reduce parsing
 - Involves two types of operations: shift and reduce

Parser Implementation

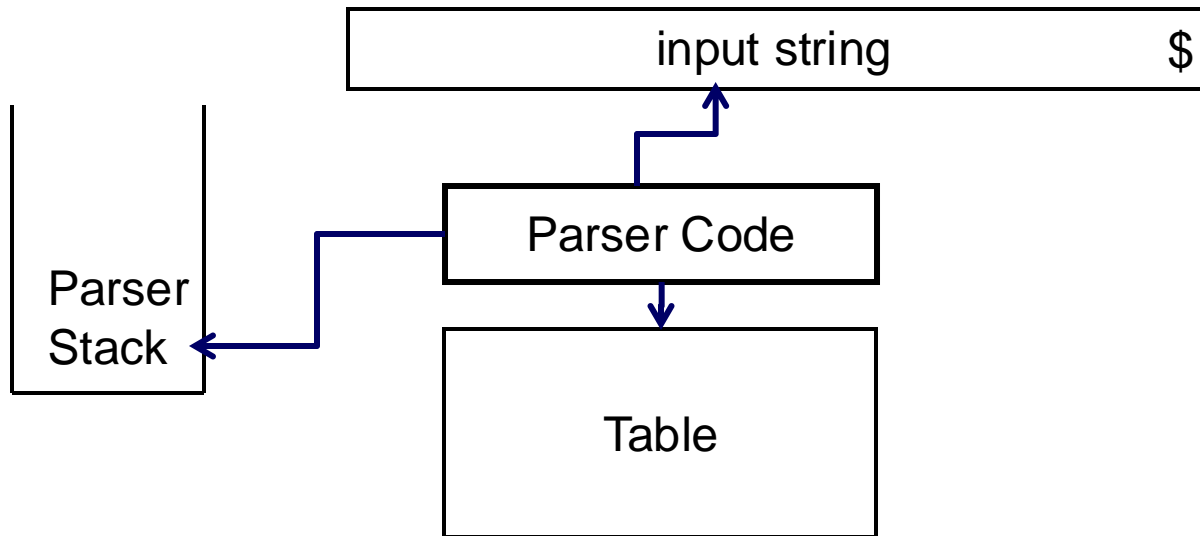


Parser Stack – holds consumed portion of derivation string

Table – “actions” performed based on rules of grammar, and current state of stack and input string

Parser Code – next action based on (**current token, stack top**)

Parser Implementation



Actions

1. **Shift** – consume input symbol and push symbol onto the stack
2. **Reduce** – pop RHS at stack top and push LHS of a production rule, reducing stack contents
3. **Accept** – success (when reduced to start symbol and input at \$)
4. **Error**

Bottom-Up compared to Top-Down

❑ Conceptual difference in how the stack works:

	Stack Content At Input Start	Stack Content At Input End	Stack Represents	Key Operations
Top-Down	Start Symbol	Nothing	Unconsumed input	Match / Expand
Bottom-up	Nothing	Start Symbol	Consumed input	Shift / Reduce

❑ But both use a stack to parse languages with nested structures

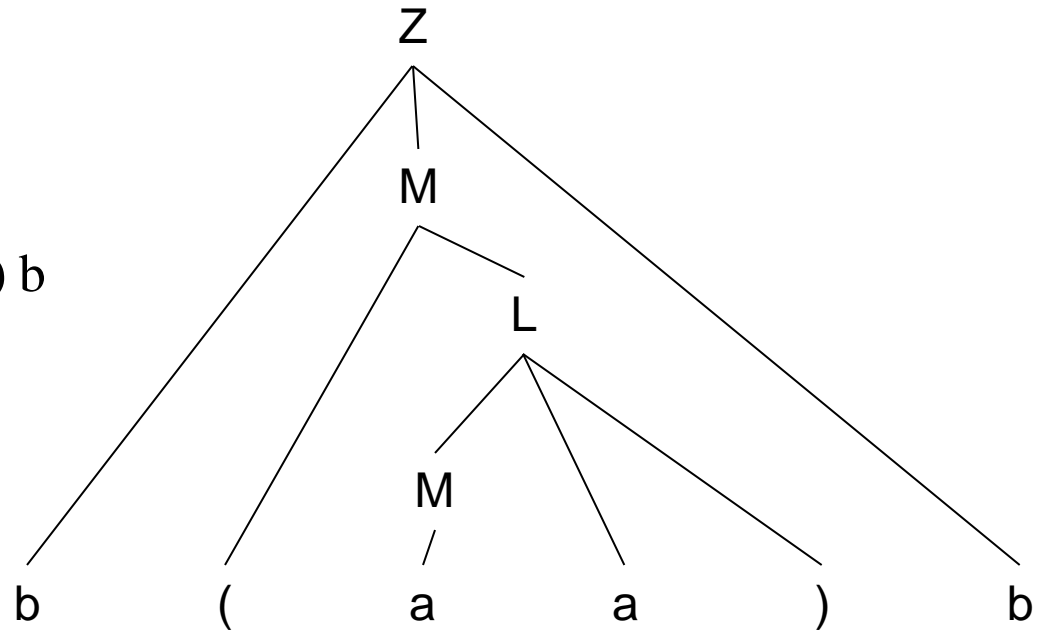
➤ Not surprising since CFGs are parsed using Pushdown Automata!

$Z \rightarrow b M b$

$M \rightarrow (L \mid a$

$L \rightarrow M a \mid)$

Considering string: $w = b (a a) b$



The rightmost derivation of this parse tree:

$Z \Rightarrow b M b \Rightarrow b (L b \Rightarrow b (M a) b \Rightarrow b (a a) b$

Bottom up parsing involves finding “handles” (RHSs) to reduce

$b (a a) b \Rightarrow b (M a) b \Rightarrow b (L b \Rightarrow b M b \Rightarrow Z$

$Z \rightarrow b M b$
 $M \rightarrow (L \mid a$
 $L \rightarrow M a) \mid)$

String
 $b (a a) \$$

Stack	Input	Action
\$	b (a a) b \$	shift
\$ b	(a a) b \$	shift
\$ b (a a) b \$	shift
\$ b (a	a) b \$	reduce
\$ b (M	a) b \$	shift
\$ b (M a) b \$	shift
\$ b (M a)	b \$	reduce
\$ b (L	b \$	reduce
\$ b M	b \$	shift
\$ b M b	\$	reduce
\$ Z	\$	accept

Sentential Form and Handle

- ❑ **Sentential form:** Any string derivable from the start symbol
- ❑ **Handle:** RHS of a production rule that, when reduced to LHS in a sentential form, will lead to another sentential form

- ❑ **Definition:**
 - Let $\alpha\beta w$ be a sentential form where
 - α, β is a string of terminals and non-terminals
 - w is a string of terminals
 - $X \rightarrow \beta$ is a production rule
 - Then β is a handle of $\alpha\beta w$ if
 - $S \Rightarrow^* \alpha X w \Rightarrow \alpha\beta w$ by a rightmost derivation
 - Handles formalize the intuition “ β should be reduced to X for a successful parse”, but does not really say how to find them

Single Pass Left-to-Right Scan

- Note in the formulation of a handle $S \Rightarrow^* \alpha X w \Rightarrow \alpha \beta w$
 - α is a string of terminals and non-terminals
 - w is a string of only terminals

- Why is this so?
 - Let's assume w contained a non-terminal Y
 - $S \Rightarrow^* \alpha X w_1 Y w_2 \Rightarrow \alpha \beta w_1 Y w_2$, where $w = w_1 Y w_2$
 - Above is not a sentential form in a rightmost derivation.
 - Contradiction!

- What are the implications?
 - $\alpha \beta$ is consumed input and w is unconsumed input
 - Reduction only happens at the frontier of consumed input
 - Amenable to single pass left-to-right scan

Handle Always Occurs at Top of Stack

□ Grammar

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow (E)$

$E \rightarrow id$

Sentential form	Handle	Production
$id_1 + id_2 * id_3$	id_1	$E \rightarrow id$
$E + id_2 * id_3$	id_2	$E \rightarrow id$
$E + E * id_3$	id_3	$E \rightarrow id$
$E + E * E$	$E * E$	$E \rightarrow E * E$
$E + E$	$E + E$	$E \rightarrow E + E$
E		

□ # indicates top of stack (at the frontier of reduction where the handle is)

Left of # : stack contents, Right of # : unconsumed input string

$$\begin{aligned}
 id_1 \# + id_2 * id_3 &\Rightarrow E \# + id_2 * id_3 \Rightarrow E + \# id_2 * id_3 \Rightarrow E + id_2 \# * id_3 \\
 &\Rightarrow E + E \# * id_3 \Rightarrow E + E * id_3 \# \Rightarrow E + E * E \# \Rightarrow E + E \# \Rightarrow E
 \end{aligned}$$

□ Stack works because reducing $X \rightarrow \beta$ involves popping recently pushed symbols

Handle Always Occurs at Top of Stack

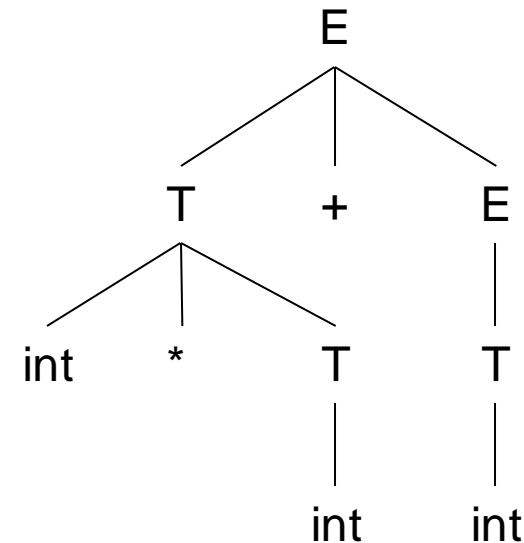
- Consider our usual grammar

$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

Consider the string: $\text{int} * \text{int} + \text{int}$

<i>sentential form</i>	<i>production</i>
$\text{int} * \text{int} \# + \text{int}$	$T \rightarrow \text{int}$
$\text{int} * T \# + \text{int}$	$T \rightarrow \text{int} * T$
$T + \text{int} \#$	$T \rightarrow \text{int}$
$T + T \#$	$E \rightarrow T$
$T + E \#$	$E \rightarrow T + E$
$E \#$	



- Reduction of a handle always happens at the top of the stack
- Makes life easier for parser (no need to access middle of stack)

Ambiguous Grammars

❑ Conflicts arise with ambiguous grammars

- Just like LL parsing, bottom up parsing tries to predict the correct action, but if there are multiple correct actions, conflicts arise

❑ Example:

- Consider the ambiguous grammar

$$E \rightarrow E * E \mid E + E \mid (E) \mid \text{int}$$

Sentential form	Actions	Sentential form	Actions
int * int + int	shift	int * int + int	shift
...
E * E # + int	reduce $E \rightarrow E * E$	E * E # + int	shift
E # + int	shift	E * E + # int	shift
E + # int	shift	E * E + int #	reduce $E \rightarrow \text{int}$
E + int #	reduce $E \rightarrow \text{int}$	E * E + E #	reduce $E \rightarrow E + E$
E + E #	reduce $E \rightarrow E + E$	E * E #	reduce $E \rightarrow E * E$
E #		E #	

Ambiguity

- ❑ Previous shift-reduce conflict occurred because of ambiguity
 - Due to lack of precedence between + and * in the grammar
 - Ambiguity shows up as “conflicts” in the parsing table
(More than one action in parse table, just like for LL parsers)
- ❑ Shift-reduce conflict also occurs with input “int + int + int”
 - Due to ambiguous associativity of * and +
- ❑ Can always rewrite to encode precedence and associativity
 - But can sometimes result in convoluted grammars
 - Tools have other means to encode precedence and association
 - %left ‘+’ ‘-’
 - %left ‘*’ ‘/’

Properties of Bottom Up Parsing

- ❑ Handles always appear at the top of the stack
 - Never in middle of stack
 - Justifies use of stack in shift – reduce parsing
- ❑ Results in an easily generalized shift – reduce strategy
 - If there is no handle at the top of the stack, shift
 - If there is a handle, reduce to the non-terminal
 - Easy to automate the synthesis of the parser using a table
- ❑ Can have conflicts
 - If it is legal to either shift or reduce then there is a shift-reduce conflict.
 - If there are two legal reductions, then there is a reduce-reduce conflict.
 - Most often occur because of ambiguous grammars
 - In rare cases, because of non-ambiguous grammars not amenable to parser

Types of Bottom Up Parsers

- ❑ Types of bottom up parsers
 - Simple precedence parsers
 - Operator precedence parsers
 - LR family parsers
 - ...

- ❑ In this course, we will only discuss LR family parsers
 - Most automated tools generate either LL or LR parsers
 - Precedence parsers are weaker siblings of LR parsers

LR Parsers are more powerful than LL

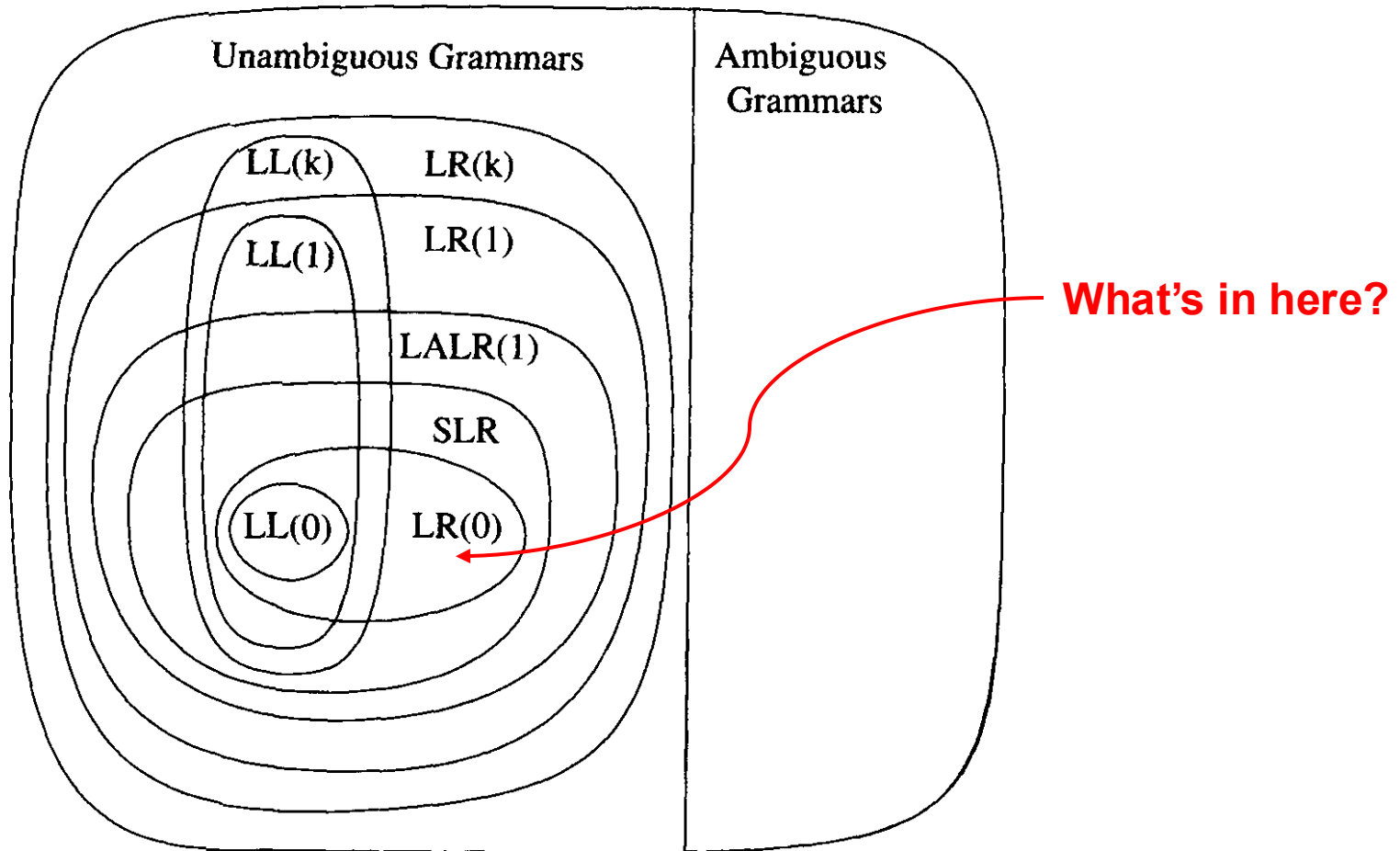
❑ LR family of parsers

- LR(k) L – left to right scan
 R – rightmost derivation in reverse
 k elements of look ahead

❑ Pros in comparison to LL(k)

1. More powerful than LL(k)
 - Handles more grammars: no left recursion removal, no left factoring needed
 - Handles more languages: $LL(k) \subset LR(k)$
2. As efficient as LL(k)
 - Linear in time and space to length of input (same as LL(k))
3. As convenient as LL(k)
 - Can generate automatically from grammar – YACC, Bison

A Hierarchy of Grammar Classes



LR Parsers are harder to deal with

❑ Cons in comparison to LL(k)

1. More complex in structure compared to LL(k)
 - Structure of parser looks nothing like grammar
 - Parse conflicts are hard to understand and debug
2. Harder to emit informative error messages and recover from errors
 - LR is a bottom-up while LL is a top-down parser
 - When parse error occurs,
LR: Knows only of currently reduced non-terminal
LL: Knows how upper levels of tree look like and context of error
 - LL can emit smart error messages referring to context of error
 - LL can perform better error recovery according to context

Implementation

-- LR Parsing

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Viable Prefix

□ Definition: α is a viable prefix if

- There is a w where αw is a rightmost sentential form, where w is the unconsumed input string
- In other words, if there is a w where $\alpha \# w$ is a configuration of a shift-reduce parser

$b (a \# a) b \Rightarrow b (M \# a) b \Rightarrow b (L \# b \Rightarrow b M \# b \Rightarrow Z \#$

□ If contents of parse stack is a viable prefix, that means the parser is on the right track (at least for the consumed input)

□ Shift-reduce parsing is the process of massaging the contents of the parse stack from viable prefix to viable prefix

- Error if neither shifting or reducing results in a viable prefix

Massaging into a Viable Prefix

□ How do you know what results in a viable prefix?

➤ Example grammar

$S \rightarrow a B S \mid b$

$B \rightarrow b$

➤ Example shift and reduce on: $a \# b b$

Shift: $a \# b b \Rightarrow a b \# b$ How do you know shifting is the answer?

Reduce: $a b \# b \Rightarrow a B \# b$ Should I apply $B \rightarrow b$ (and not $S \rightarrow b$)?

□ You need to keep track of where you are on the RHS of rules

➤ In example

Shift: $a \# b b \Rightarrow a b \# b$

$S \rightarrow a \# B S, B \rightarrow \# b \Rightarrow S \rightarrow a \# B S, B \rightarrow b \#$

Reduce: $a b \# b \Rightarrow a B \# b$

$S \rightarrow a \# B S, B \rightarrow b \# \Rightarrow S \rightarrow a B \# S, S \rightarrow \# a B S, S \rightarrow \# b$

LR(0) Item Notation

□ LR(0) Item: a production + a dot on the RHS

- Dot indicates extent of production already seen
- In example grammar

Items for production $S \rightarrow a B S$

$S \rightarrow \cdot a B S$

$S \rightarrow a \cdot B S$

$S \rightarrow a B \cdot S$

$S \rightarrow a B S \cdot$

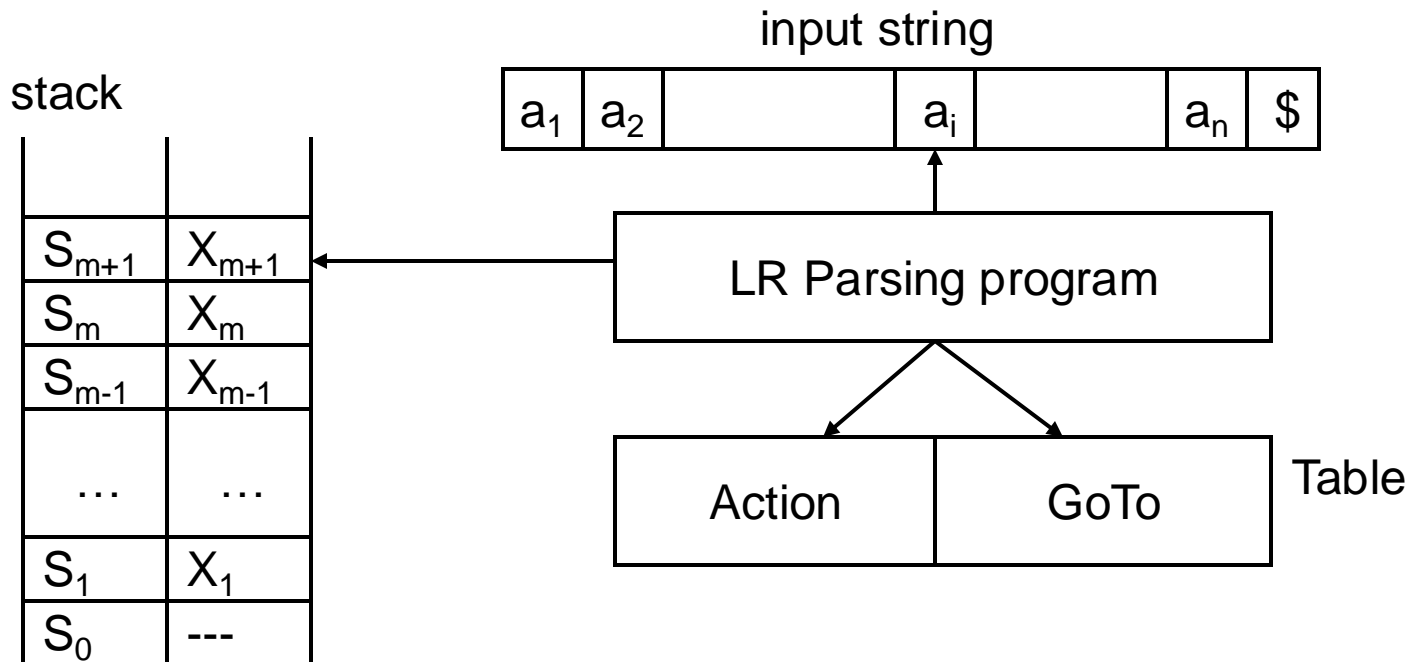
□ Items denote the idea of the viable prefix. E.g.

- $S \rightarrow \cdot a B S$: to be a viable prefix, terminal 'a' needs to be shifted
- $S \rightarrow a \cdot B S$: to be a viable prefix, a set of terminals need to be shifted and reduced to non-terminal 'B'

States in the LR Parser

- ❑ State of LL parser is simply tuple (stack top symbol, input)
- ❑ State of LR parser is more complex
 - Must keep track of where we are in the RHS production rules
 - State is denoted by a set of LR(0) items
- ❑ Why a *set* of LR(0) items?
 - There may be multiple candidate RHSs for the prefix. E.g.
For grammar $S \rightarrow a b \mid a c$,
 $S \rightarrow a . b$ and $S \rightarrow a . c$ would be items in the same set
 - Even for one RHS, you may need multiple items expressing the same position, if the following symbol is a non-terminal. E.g.
For grammar $S \rightarrow a B, B \rightarrow b$
 $S \rightarrow a . B$ and $B \rightarrow . b$ would be items in the same set
- ❑ LR parsers keep track of states alongside symbols in stack

Parser Implementation in More Detail



- Each grammar symbol X_i is associated with a state S_i
- Contents of stack ($X_1X_2\dots X_m$) is a viable prefix
- Contents of stack + input ($X_1X_2\dots X_ma_1\dots a_n$) is a right sentential form
 - If the input string is a member of the language
- Uses **state** at the top of stack and current input to index into parsing table to determine whether to shift or reduce

Parser Actions

S_m	X_m
S_{m-1}	X_{m-1}
...	...
S_1	X_1
S_0	---

...	X_{m+1}	...	\$
-----	-----------	-----	----

$S_m : A \rightarrow X_m \cdot X_{m+1}$
 $S_{m-1} : A \rightarrow \cdot X_m X_{m+1}$
 $B \rightarrow X_{m-1} \cdot A$

Shift

S_{m+1}	X_{m+1}
S_m	X_m
S_{m-1}	X_{m-1}
...	...
S_1	X_1
S_0	---

...	X_{m+1}	...	\$
-----	-----------	-----	----

$S_{m+1} : A \rightarrow X_m X_{m+1} \cdot$
 $S_m : A \rightarrow X_m \cdot X_{m+1}$
 $S_{m-1} : A \rightarrow \cdot X_m X_{m+1}$
 $B \rightarrow X_{m-1} \cdot A$

Reduce(1)

$S_{m+1} : A \rightarrow X_m X_{m+1} \cdot$
 $S_m : A \rightarrow X_m \cdot X_{m+1}$
 $S_{m-1} : A \rightarrow \cdot X_m X_{m+1}$
 $B \rightarrow X_{m-1} \cdot A$

...	X_{m+2}	...	\$
-----	-----------	-----	----

S_{m+1}	X_{m+1}
S_m	X_m
S_{m-1}	X_{m-1}
...	...
S_1	X_1
S_0	---

(2)

	A
S_{m-1}	X_{m-1}
...	...
S_1	X_1
S_0	---

GOTO

$S_A : B \rightarrow X_{m-1} A \cdot$
 $S_{m-1} : A \rightarrow \cdot X_m X_{m+1}$
 $B \rightarrow X_{m-1} \cdot A$

S_A	A
S_{m-1}	X_{m-1}
...	...
S_1	X_1
S_0	---

Parser Actions

□ Assume configuration = $S_0X_1S_1X_2S_2\ldots X_mS_m\#a_ia_{i+1}\ldots a_n\$$

□ Actions can be one of:

1. Shift input a_i and push new state **S**
 - New configuration = $S_0X_1S_1X_2S_2\ldots X_mS_m a_i S \# a_{i+1}\ldots \$$
 - Where **Action** $[S_m, a_i] = s[S]$
2. Reduce using Rule **R** ($A \rightarrow \beta$) and push new state **S**
 - Let $k = |\beta|$, pop $2*k$ symbols and push A
 - New configuration = $S_0X_1S_1\ldots X_{m-k}S_{m-k}A S \# a_i a_{i+1}\ldots \$$
 - Where **Action** $[S_m, a_i] = r[R]$ and **GoTo** $[S_{m-k}, A] = [S]$
3. Accept – parsing is complete (**Action** $[S_m, a_i] = \text{accept}$)
4. Error – report and stop (**Action** $[S_m, a_i] = \text{error}$)

Parse Table: Action and Goto

□ Action $[S_m, a_i]$ can be one of:

- **s[S]: shift** input symbol a_i and push **state S**
(One item in S_m must be of the form $A \rightarrow \alpha \cdot a_i \beta$)
- **r[R]: reduce** using **rule R** on seeing input symbol a_i
(One item in S_m must be $R: A \rightarrow \alpha \cdot$, where $a_i \in \text{Follow}(A)$)
 - Use $\text{GoTo}[S_{m-|\alpha|}, A]$ to figure out state to push with A
- **Accept** (One item in S_m must be $S' \rightarrow S \cdot$ where S is the original start symbol, and a_i must be $\$$)
- **Error** (Cannot shift, reduce, accept on symbol a_i in state S_m)

□ $\text{GoTo}[S_m, X_i]$ is **[S]**:

- Next **state** to push when pushing nonterminal X_i from a reduction
(At least one item in S_m must be of the form $A \rightarrow \alpha \cdot X_i \beta$)
- Similar to shifting input except now we are “shifting” a nonterminal

□ Grammar

1. $S \rightarrow E$
2. $E \rightarrow E + T$
3. $E \rightarrow T$
4. $T \rightarrow \text{id}$
5. $T \rightarrow (E)$

Non-terminal	Follow
S	\$
E	+) \$
T	+) \$

ACTION

	+	id	()	\$
S0		s3	s4		
S1	s7				accept
S2	r3			r3	r3
S3	r4			r4	r4
S4		s3	s4		
S5	s7			s6	
S6	r5			r5	r5
S7		s3	s4		
S8	r2			r2	r2

GOTO

	E	T	S
S0	1	2	
S1			
S2			
S3			
S4	5	2	
S5			
S6			
S7		8	
S8			

□ Grammar

1. $S \rightarrow E$
2. $E \rightarrow E + T$
3. $E \rightarrow T$
4. $T \rightarrow id$
5. $T \rightarrow (E)$

Non-terminal	Follow
S	\$
E	+) \$
T	+) \$

ACTION

	+	id	()	\$
S0		s3	s4		
S1	s7				accept
S2	r3			r3	r3
S3	r4			r4	r4
S4		s3	s4		
S5	s7			s6	
S6	r5			r5	r5
S7		s3	s4		
S8	r2			r2	r2

GOTO

	E	T	S
S0	1	2	
S1			
S2			
S3			
S4	5	2	
S5			
S6			
S7		8	
S8			

Parse Table in Action

□ Example input string

id + id + id

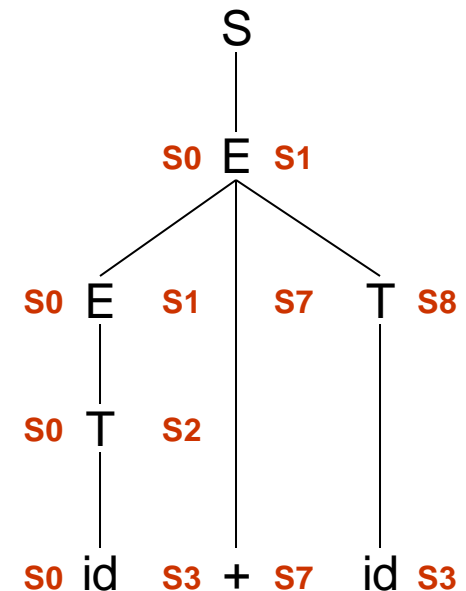
□ Parser actions

Stack	Input	Actions
S0	id + id + id \$	Action[S0, id] (s3): Shift “id”, Push S3
S0 id S3	+ id + id \$	Action[S3, +] (r4): Reduce rule 4 (T→id) GoTo[S0, T] (2): Push S2
S0 T S2	+ id + id \$	Action[S2, +] (r3): Reduce rule 3 (E→T) GoTo[S0, E] (1): Push S1
S0 E S1	+ id + id \$	Action[S1, +] (s7): Shift “+”, Push S7
S0 E S1 + S7	id + id \$	Action[S7, id] (s7): Shift “id”, Push S3
...
S0 E S1 + S7 T S8	+ id \$	Action[S8, +] (r2): Reduce rule 2 (E→E+T) GoTo[S0, E] (1): Push S1
...

Power Added to DFA by Stack

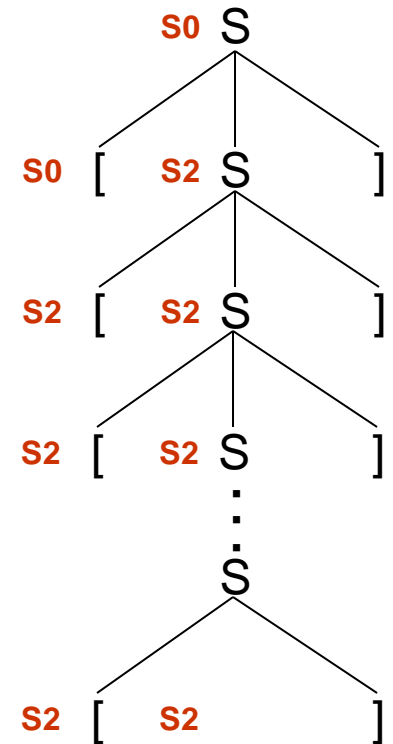
- ❑ LR parser is basically DFA+Stack (Pushdown Automaton)
- ❑ DFA: can only remember one state (“dot” in current rule)
- ❑ DFA + Stack: remembers current state and all past states (“dots” in rules higher up in the tree waiting for next symbol)

Stack	Input	Action
S0	id + id \$	s3
S0 id S3	+ id \$	r4, goto[S0, T]
S0 T S2	+ id \$	r3, goto[S0, E]
S0 E S1	+ id \$	s7
S0 E S1 + S7	id \$	s3
S0 E S1 + S7 id S3	\$	r4, goto[S7, T]
S0 E S1 + S7 T S8	\$	r2, goto[S0, E]
S0 E S1	\$	Accept



Power Added to DFA by Stack

- Remember the following CFG for the language $\{ [^i]^i \mid i \geq 1 \}$?
 $S \rightarrow [S] \mid []$
- Regular grammars (or DFAs) could not recognize language because the state machine had to “count”
- LR parser stack counts number of **[symbols**
- Q: Is this language LL(1)?
 - Yes. After left-factoring.
 $S \rightarrow [S' , S' \rightarrow S] \mid []$
 - LL parser stack counts number of **] symbols**
 - Same pushdown automaton but different usage



LR Parse Table Construction

- ❑ Must be able to decide on action from:
 - State at the top of stack
 - Next k input symbols (In practice, $k = 1$ is often sufficient)
- ❑ To construct LR parse table from grammar
 - Two phases
 - Build deterministic finite state automaton to go from state to state
 - Express DFA using Action and GoTo tables
- ❑ State: Where we are currently in the structure of the grammar
 - Expressed as a set of LR(0) items
 - Each item expresses position in terms of the RHS of a rule

Construction of LR States

1. Create augmented grammar G' for G
 - Given $G: S \rightarrow \alpha \mid \beta$, create $G': S' \rightarrow S \mid S \rightarrow \alpha \mid \beta$
 - Creates a single rule $S' \rightarrow S$ that when reduced, signals acceptance
2. Create first state by performing a *closure* on initial item $S' \rightarrow \cdot S$
 - **Closure(I)**: computes set of items expressing the same position as I
 - $\text{Closure}(\{S' \rightarrow \cdot S\}) = \{S' \rightarrow \cdot S, S \rightarrow \cdot \alpha, S \rightarrow \cdot \beta\}$
3. Create additional states by performing a *goto* on each symbol
 - **Goto(I, X)**: creates state that can be reached by advancing X
 - If α was single symbol, the following new state would be created:
 $\text{Goto}(\{S' \rightarrow \cdot S, S \rightarrow \cdot \alpha, S \rightarrow \cdot \beta\}, \alpha) =$
 $\text{Closure}(\{S \rightarrow \alpha \cdot\}) = \{S \rightarrow \alpha \cdot\}$
4. Repeatedly perform gotos until there are no more states to add

Closure Function

□ Closure(I) where I is a set of items

- Returns the state (set of items) that express the same position as I
- Items in I are called **kernel items**
- Rest of items in closure(I) are called **non-kernel items**

□ Let N be a non-terminal

- If dot is in front of N, then add each production for that N and put dot at the beginning of the RHS
 - $A \rightarrow \alpha . B \beta$ is in I ; we expect to see a string derived from B
 - $B \rightarrow . \gamma$ is added to the closure, where $B \rightarrow \gamma$ is a production
 - Apply rule until nothing is added

➤ Given

$$\begin{aligned} S &\rightarrow E \\ E &\rightarrow E + T \\ E &\rightarrow T \\ T &\rightarrow \text{id} \mid (E) \end{aligned}$$

$\text{Closure}(\{ S \rightarrow . E \}) = \{ S \rightarrow . E, E \rightarrow . E + T, E \rightarrow . T, T \rightarrow . \text{id}, T \rightarrow . (E) \}$

Kernel and Non-kernel Items

□ Two kinds of items

➤ Kernel items

- Items that act as “seed” items when creating a state
- What items act as seed items when states are created?
 - Initial state: $S' \rightarrow \cdot S$
 - Additional state: from $\text{goto}(I, X)$ so has X at left of dot
- Besides $S' \rightarrow \cdot S$, all kernel items have dot in middle of RHS

➤ Non-kernel items

- Items added during the closure of kernel items
- All non-kernel items have dot at the beginning of RHS
- Trivially derivable from kernel items so often not explicitly expressed as part of state to save memory footprint

Goto Function

- ❑ Goto (I, X) where I is a set of items and X is a symbol
 - Returns state (set of items) that can be reached by advancing X
 - For each $A \rightarrow \alpha . X \beta$ in I,
Closure($A \rightarrow \alpha X . \beta$) is added to goto(I, X)
 - X can be a terminal or non-terminal
 - Terminal if obtained from input string by shifting
 - Non-terminal if obtained from reduction
 - Example
 - $\text{Goto}(\{T \rightarrow . (E)\}, () = \text{closure}(\{T \rightarrow (. E)\})$
- ❑ Ensures every symbol consumption results in a viable prefix
 - If parser configuration is $\alpha\#\beta$, then $\text{goto}(I, X)$ results in $\alpha X\#\beta'$
 - If α was a viable prefix, αX is trivially a viable prefix

Construction of DFA

- ❑ Algorithm to compute set C (set of all states in DFA)

void items (G') {

$C = \{\text{closure}(\{S \rightarrow \cdot S\})\}$ // Add initial state to C

 repeat

 for (each state I in C)

 for (each grammar symbol X)

 if (goto(I, X) is not empty and not in C)

 add goto(I, X) to C

 until no new states are added to C

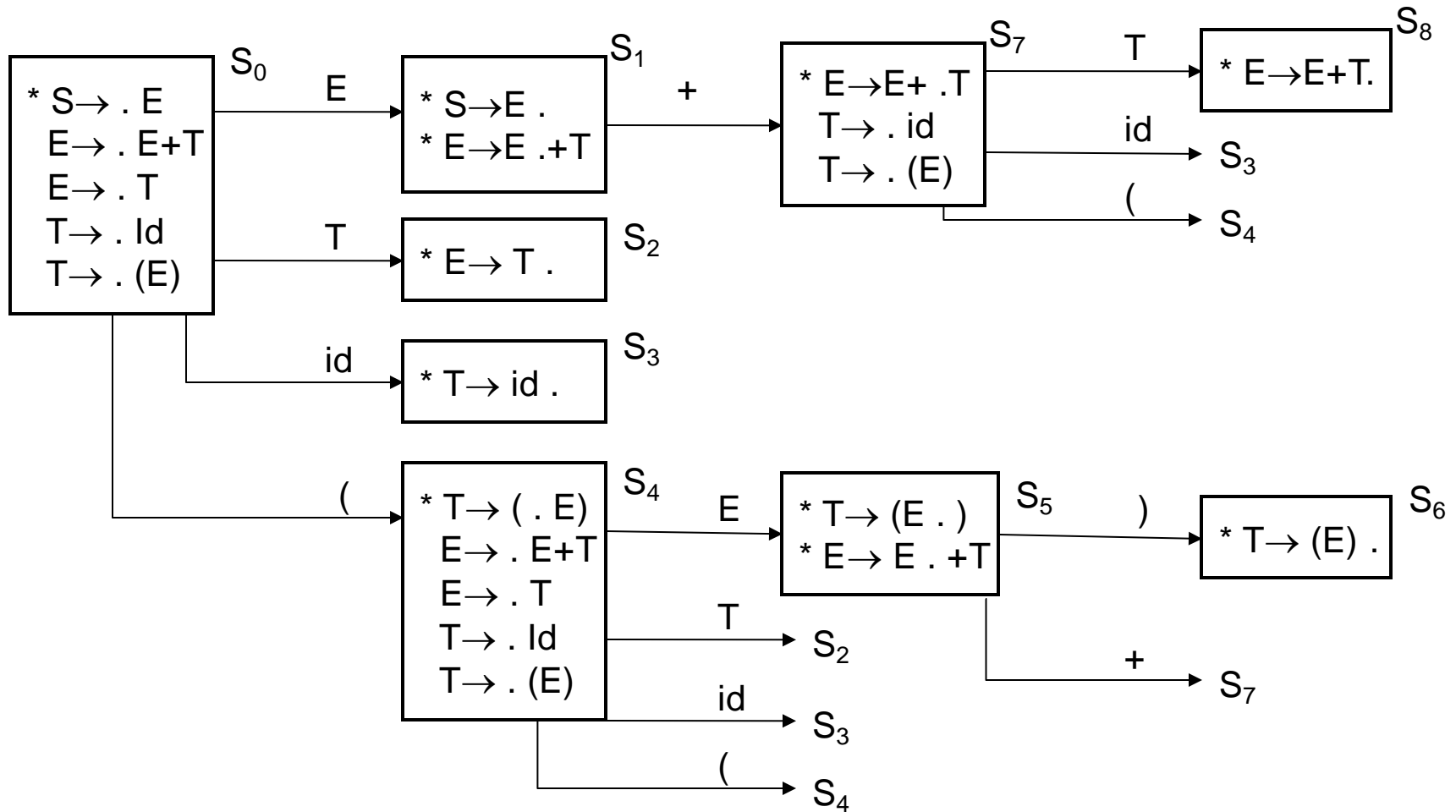
- ❑ All new states are added through goto(I, X)

➤ States transitions are done on symbol X

□ Example: $S \rightarrow E$
 $E \rightarrow E + T \mid T$
 $T \rightarrow \text{id} \mid (E)$

- $S_0 = \text{closure}(\{S \rightarrow . E\})$
 $= \{S \rightarrow . E, E \rightarrow . E + T, E \rightarrow . T, T \rightarrow . \text{id}, T \rightarrow . (E)\}$
- $\text{goto}(S_0, E) = \text{closure}(\{S \rightarrow E ., S \rightarrow E . + T\})$
 $S_1 = \{S \rightarrow E ., S \rightarrow E . + T\}$
- $\text{goto}(S_0, T) = \text{closure}(\{E \rightarrow T .\})$
 $S_2 = \{E \rightarrow T .\}$
- $\text{goto}(S_0, \text{id}) = \text{closure}(\{T \rightarrow \text{id} .\})$
 $S_3 = \{T \rightarrow \text{id} .\}$
-
- $S_8 = \dots$

□ DFA for the previous grammar
 (* are closures applied to kernel items)



Building Parse Table from DFA

- ACTION [state, input symbol/terminal symbol]
- GOTO [state, non-terminal symbol]

□ ACTION:

1. If $[A \rightarrow \alpha \bullet a \beta]$ is in S_i and $\text{goto}(S_i, a) = S_j$, where “a” is a terminal
then $\text{ACTION}[S_i, a] = \text{shift } j$
 2. If $[A \rightarrow \alpha \bullet]$ is in S_i
then $\text{ACTION}[S_i, a] = \text{reduce } A \rightarrow \alpha$ for all $a \in \text{Follow}(A)$
 3. If $[S' \rightarrow S_0 \bullet]$ is in S_i
then $\text{ACTION}[S_i, \$] = \text{accept}$
- If no conflicts among 1 and 2
then it is said that this parser is able to parse the given grammar

□ GOTO

1. if $\text{goto}(S_i, A) = S_j$ then $\text{GOTO}[S_i, A] = S_j$

□ All entries not filled are errors

□ Grammar

1. $S \rightarrow E$
2. $E \rightarrow E + T$
3. $E \rightarrow T$
4. $T \rightarrow \text{id}$
5. $T \rightarrow (E)$

Non-terminal	Follow
S	\$
E	+) \$
T	+) \$

ACTION

	+	id	()	\$
S0		s3	s4		
S1	s7				accept
S2	r3			r3	r3
S3	r4			r4	r4
S4		s3	s4		
S5	s7			s6	
S6	r5			r5	r5
S7		s3	s4		
S8	r2			r2	r2

GOTO

	E	T	S
S0	1	2	
S1			
S2			
S3			
S4	5	2	
S5			
S6			
S7		8	
S8			

Types of LR Parsers

- ❑ SLR – simple LR (what we saw so far was SLR(1))
 - Easiest to implement
 - Not as powerful
- ❑ Canonical LR
 - Most powerful
 - Expensive to implement
- ❑ LALR
 - Look ahead LR
 - In between the 2 previous ones in power and overhead

Overall parsing algorithm is the same – table is different

Conflict

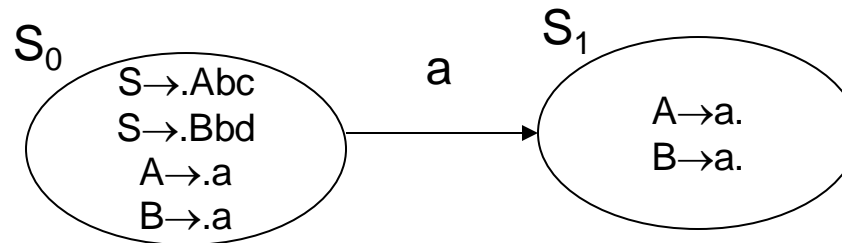
- Consider the grammar G

$S \rightarrow A b c \mid B b d$

$A \rightarrow a$

$b \in \text{Follow}(A)$ and also $b \in \text{Follow}(B)$

$B \rightarrow a$



- What is reduced when “a b” is seen? reduce to A or B?

- Reduce-reduce conflict

- G is not SLR(1) but SLR(2)

- We need 2 symbols of look ahead to look past b:

- b c – reduce to A

- b d – reduce to B

- Possible to extend SLR(1) to k symbols of look ahead – allows larger class of CFGs to be parsed

SLR(k)

□ Extend SLR(1) definition to SLR(k) as follows

let $\alpha, \beta \in V^*$

- $\text{First}_k(\alpha) = \{ x \in V_T^* \mid (\alpha \rightarrow^* x\beta \text{ and } |x| \leq k) \}$ gives
 - all terminal strings of size $\leq k$ derivable from α
 - all k -symbol terminal prefixes of strings derivable from α
- $\text{Follow}_k(B) = \{ w \in V_T^* \mid S \rightarrow^* \alpha B \gamma \text{ and } w \in \text{First}_k(\gamma) \}$ gives
 - all k symbol terminal strings that can follow B in some derivation
 - all shorter terminal strings that can follow B in a sentential form

Parse Table

Let S be a state and lookahead $b \in V_T^*$ such that $|b| \leq k$

1. If $A \rightarrow \alpha. \in S$ and $b \in \text{Follow}_k(A)$ then
 - $\text{Action}(S, b)$ – reduce using production $A \rightarrow \alpha$,
2. If $D \rightarrow \alpha.a \gamma \in S$ and $a \in V_T$ and $b \in \text{First}_k(a \gamma \text{Follow}_k(D))$
 - $\text{Action}(S, b) = \text{shift “a” and push state goto}(S, a)$

For $k = 1$, this definition reduces to SLR(1)

Reduce: Trivially true

Shift: $\text{First}_1(a \gamma \text{Follow}_1(D)) = \{a\}$

SLR(k)

□ Consider

$$S \rightarrow A b^{k-1} c \mid B b^{k-1} d$$

$$A \rightarrow a$$

$$B \rightarrow a$$

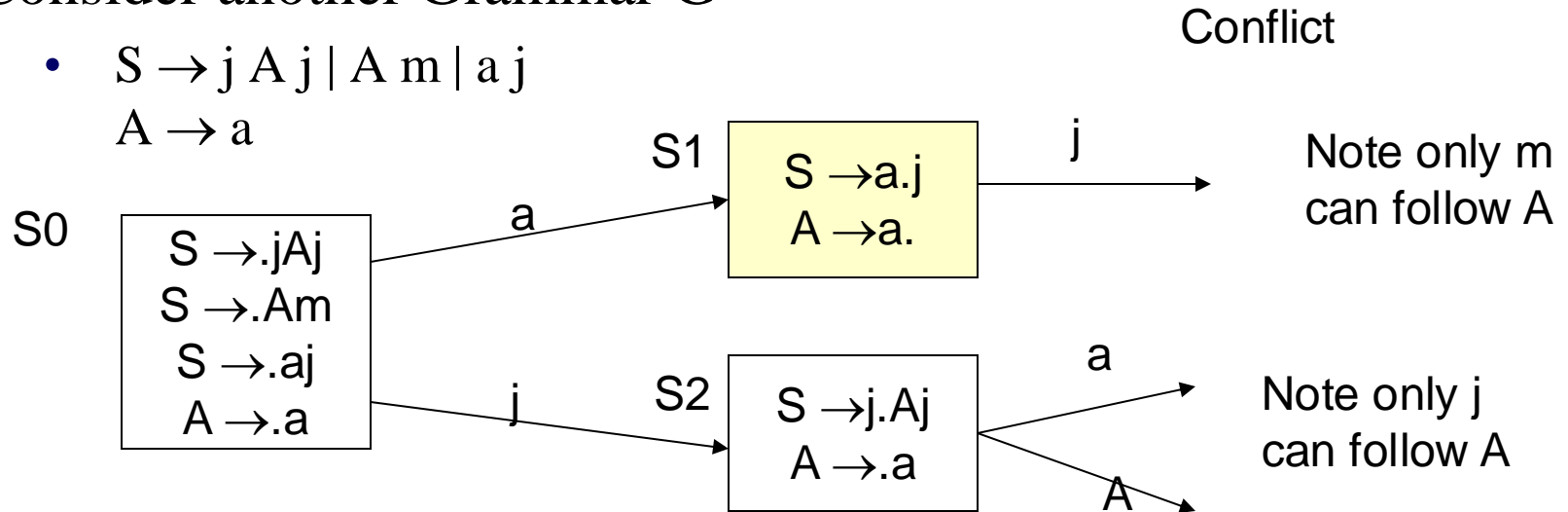
SLR(k) not SLR(k-1)

- cannot decide what to reduce,
- reduce a to A or B depends the next k symbols
 $b^{k-1} c$ or $b^{k-1} d$

Non SLR(k)

□ Consider another Grammar G

- $S \rightarrow j A j \mid A m \mid a j$
 $A \rightarrow a$



$\text{Follow}(A) = \{j, m\}$

State S1: $[A \rightarrow a.]$ – reduce using this production (on j or m)

$[S \rightarrow a.j]$ – shift j ➔ shift-reduce conflict ➔ not SLR(1)

? SLR(k)?

For reducing $A \rightarrow a.$: $\text{Follow}_k(A) = \text{First}_k(j) + \text{First}_k(m) = \{j, m\}$,

For shifting $S \rightarrow a.j$: $\text{First}_k(j\text{Follow}_k(S)) = \{j\}$ so not SLR(k) for any k !!!

Why?

❑ Look ahead is too crude

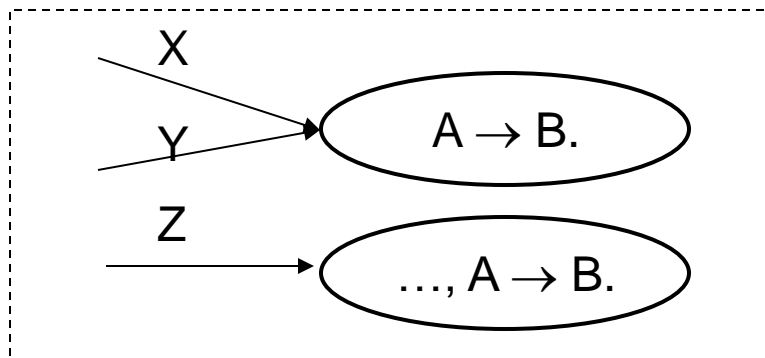
- In $S1$, if $A \rightarrow a$ is reduced then $\{m\}$ is the only possible symbol that can be seen – the only valid look ahead
- Fact that $\{j\}$ can follow A in another context is irrelevant

❑ Want to compare look ahead in a state to those symbols that might actually occur in the context represented by the state.

❑ Done in Canonical LR !!!

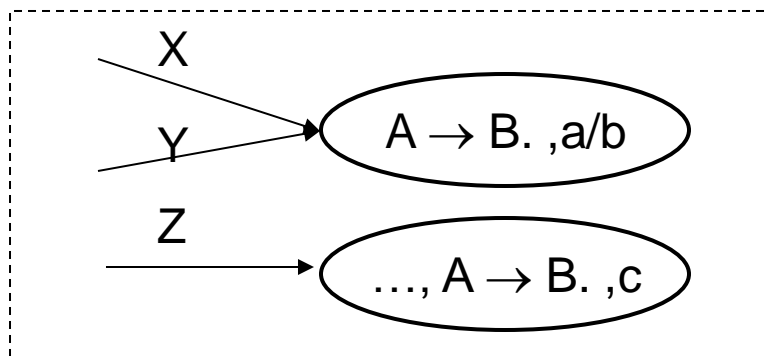
- Determine look ahead appropriate to the context of the state
- States contains a look ahead – will be used only for reductions

SLR(1)



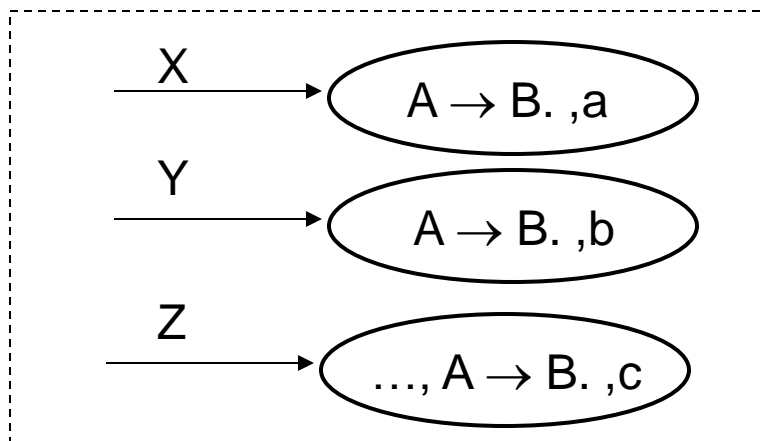
$\text{Follow}(A) = \{a, b, c\}$

LALR(1)



state merging

LR(1)



state splitting

Constructing Canonical LR

- ❑ Problem: Follow set in SLR is not precise enough
 - Follow set ignores context where reduction for item occurs
- ❑ Solution: Define a more precise follow set
 - For each item, encode a more precise follow set according to context
 - Use more precise follow set when deciding whether to reduce item
- ❑ LR(1) item: LR item with one lookahead
 - $[A \rightarrow \alpha.\beta, a]$ where $A \rightarrow \alpha\beta$ is a production and a is a terminal or $\$$
 - Meaning: Only terminal a can follow A in this context
 - Interpretation: After β is shifted and eventually we reach $[A \rightarrow \alpha\beta., a]$, only reduce A if lookahead matches terminal a
 - Second lookahead component will always be a subset of $\text{Follow}(A)$

Constructing Canonical LR

- Essentially the same as LR(0) items only adding lookahead

- Modify closure and goto function

- Changes for **closure**

- $[A \rightarrow \alpha.B\beta, a]$ and $B \rightarrow \delta$ then

- $[B \rightarrow \cdot \delta, c]$ where $c \in \text{First}(\beta a)$

- Changes for **goto** function

- Carry over look ahead

- $[A \rightarrow \alpha.X\beta, a] \in I$ then $\text{goto}(I, X) = [A \rightarrow \alpha X \cdot \beta, a]$

Example

□ Grammar

$$S' \rightarrow S$$
$$S \rightarrow CC$$
$$C \rightarrow eC \mid d$$

□ S0: closure($S' \rightarrow .S$, \$)

$$[S' \rightarrow .S, \$]$$
$$[S \rightarrow .CC, \$]$$
$$\text{first}(\epsilon\$) = \{ \$ \}$$
$$[C \rightarrow .eC, e/d]$$
$$\text{first}(C\$) = \{ e, d \}$$
$$[C \rightarrow .d, e/d]$$
$$\text{first}(C\$) = \{ e, d \}$$

□ S1: goto($S0, S$) = closure($S' \rightarrow S.$, \$)

$$[S' \rightarrow S., \$]$$

□ S2: goto($S0, C$) = closure($S \rightarrow C.C$, \$)

$$[S \rightarrow C.C, \$]$$
$$[C \rightarrow .eC, \$]$$
$$\text{first}(\epsilon\$) = \{ \$ \}$$
$$[C \rightarrow .d, \$]$$
$$\text{first}(\epsilon\$) = \{ \$ \}$$

- S3: $\text{goto}(S0, e) = \text{closure}(C \rightarrow e.C, e/d)$

$[C \rightarrow e.C, e/d]$

$[C \rightarrow .eC, e/d] \qquad \text{first}(\epsilon e/d) = \{e, d\}$

$[C \rightarrow .d, e/d] \qquad \text{first}(\epsilon e/d) = \{e, d\}$
- S4: $\text{goto}(S0, d) = \text{closure}(C \rightarrow d., e/d)$

$[C \rightarrow d., e/d]$
- S5: $\text{goto}(S2, C) = \text{closure}(S \rightarrow CC., \$)$

$[S \rightarrow CC., \$]$
- S6: $\text{goto}(S2, e) = \text{closure}(C \rightarrow e.C, \$)$

$[C \rightarrow e.C, \$]$

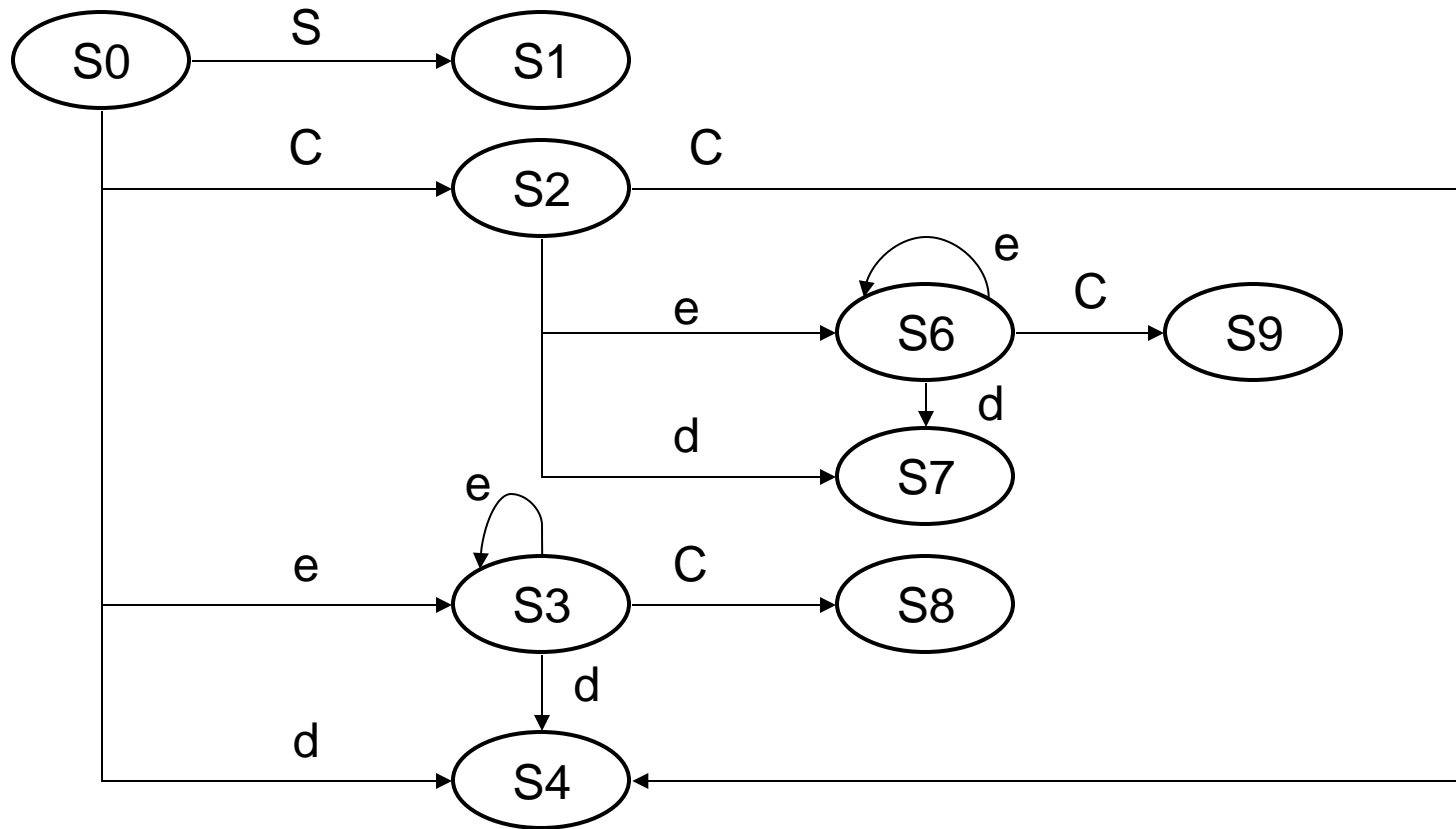
$[C \rightarrow .eC, \$] \qquad \text{first}(\epsilon \$) = \{\$ \}$

$[C \rightarrow .d, \$] \qquad \text{first}(\epsilon \$) = \{\$ \}$
- S7: $\text{goto}(S2, d) = \text{closure}(C \rightarrow d., \$)$

$[C \rightarrow d., \$]$
- S8: $\text{goto}(S3, C) = \text{closure}(C \rightarrow eC., e/d)$

$[C \rightarrow eC., e/d]$
- S9: $\text{goto}(S6, C) = \text{closure}(C \rightarrow eC., \$)$

$[C \rightarrow eC., \$]$



Note S3, S6 are same except for lookahead (also true for S4, S7 and S8, S9)
In SLR(1) – one state represents both

Constructing Canonical LR Parse Table

- ❑ Shifting: same as before

- ❑ Reducing:

 - Don't use follow set (too coarse grain)

 - Reduce only if input matches lookahead for item

- ❑ Action and GOTO

1. if $[A \rightarrow \alpha \bullet a \beta, b] \in S_i$ and $\text{goto}(S_i, a) = S_j$,

 Action[I,a] = s[Sj] – shift and goto state j if input matches a

Note: same as SLR

1. if $[A \rightarrow \alpha \bullet, a] \in S_i$

 Action[I,a] = r[R] – reduce R: $A \rightarrow \alpha$ if input matches a

Note: for SLR, reduced if input matches Follow(A)

2. if $[S' \rightarrow S., \$] \in S_i$,

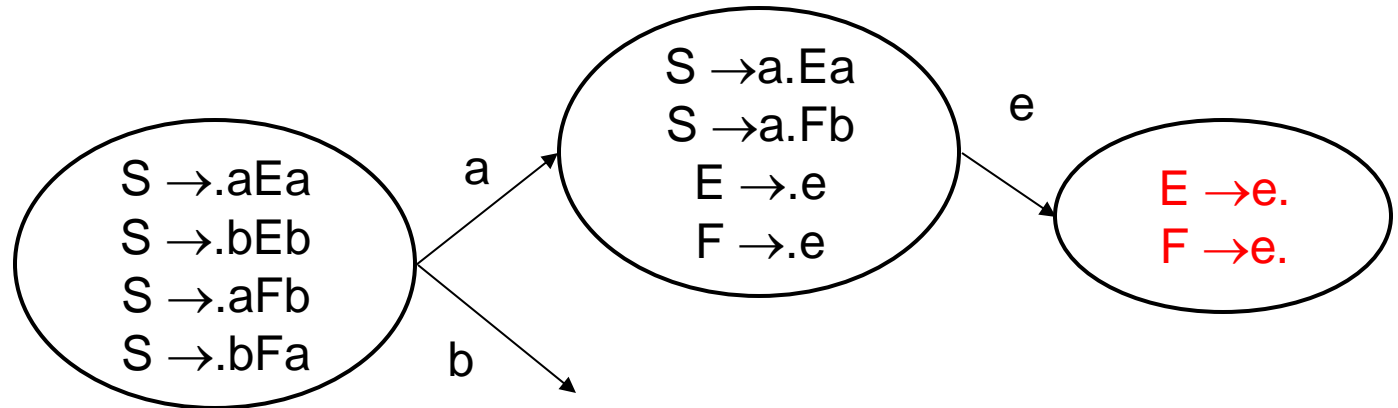
 Action[i,\$] = accept

□ Revisit SLR and LR

➤ $S \rightarrow aEa \mid bEb \mid aFb \mid bFa$

$E \rightarrow e$

$F \rightarrow e$



□ SLR: reduce/reduce conflict $\text{Follow}(E, F) = \{a, b\}$

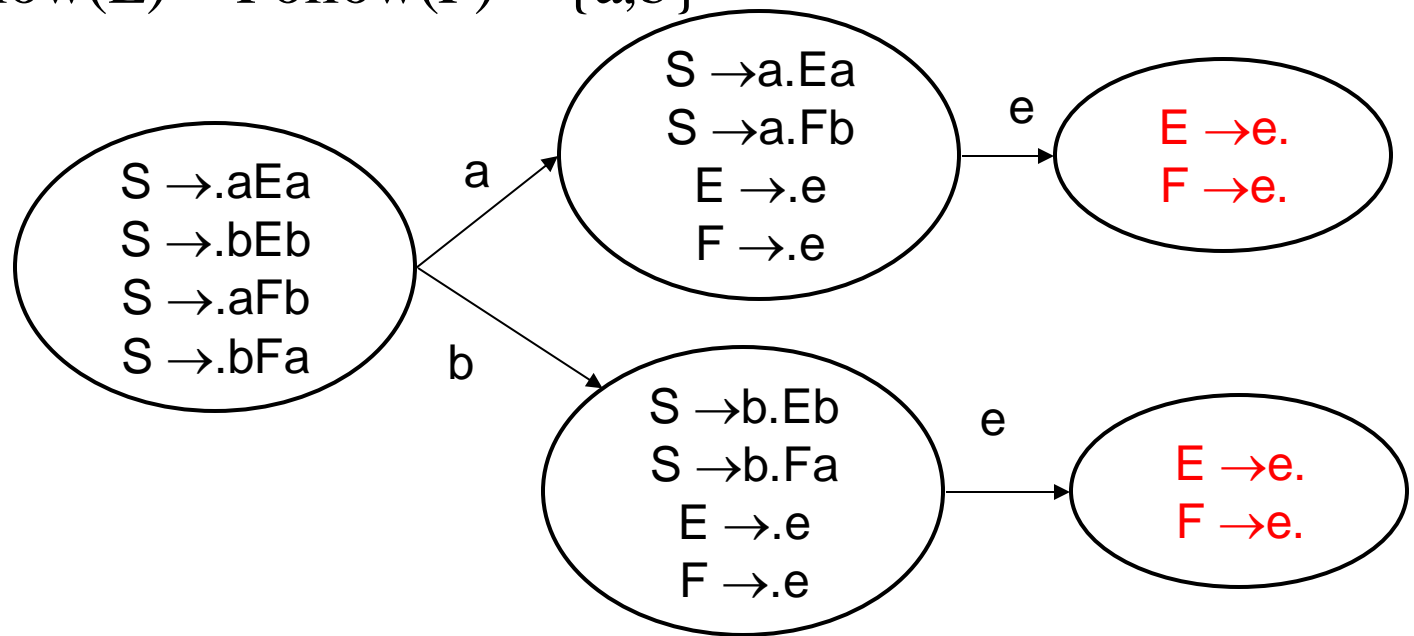
$aea \Rightarrow E \rightarrow e$ and $bea \Rightarrow F \rightarrow e$

□ Is this LR(1)? Will not have a conflict because states will be split to take into account this context

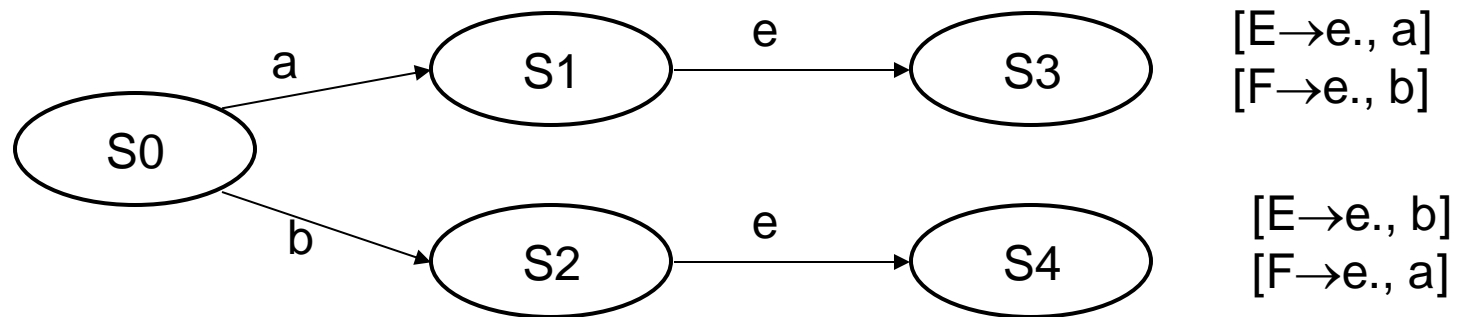
E if followed by a/b preceded by a/b, respectively

F if followed by a/b preceded by b/a, respectively

❑ SLR: $\text{Follow}(E) = \text{Follow}(F) = \{a, b\}$



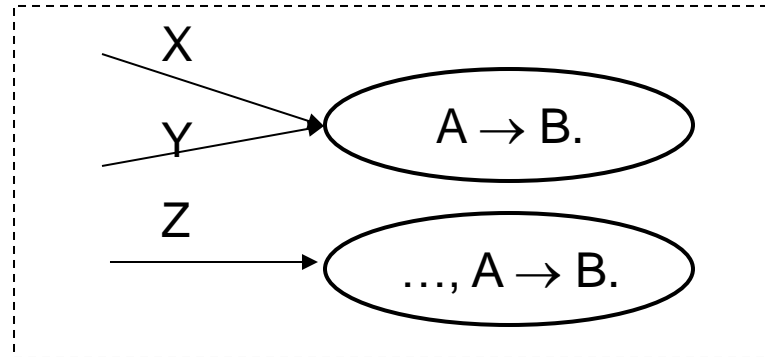
❑ LR: Follow sets more precise



SLR(1) and LR(1)

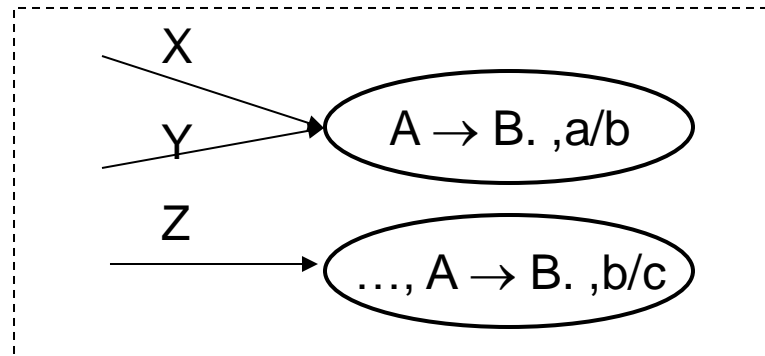
- ❑ LR(1) more powerful than SLR(1) – can parse more grammars
- ❑ But LR(1) may end up with many more states than SLR(1)
 - One LR(0) item may split up to many LR(1) items
(As many as all combinations of lookahead possible – potentially powerset of entire alphabet)
- ❑ LALR(1) – compromise between LR(1) and SLR(1)
 - Constructed by merging LR(1) states with the same core
 - Ends up with same number of states as SLR(1)
 - But items still retain some lookahead info – still better than SLR(1)
 - Used in practice because most programming language syntactic structures can be represented by LALR (not true for SLR)

SLR(1)



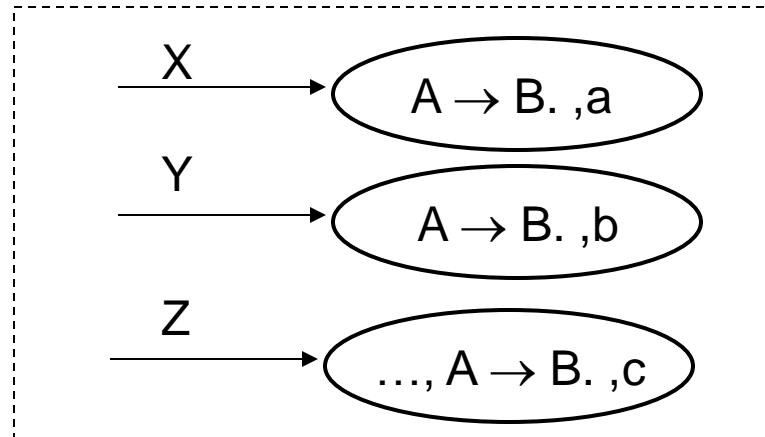
$\text{Follow}(A) = \{a, b, c\}$

LALR(1)



state merging

LR(1)



state splitting

Example

□ Grammar

$$S' \rightarrow S$$
$$S \rightarrow CC$$
$$C \rightarrow eC \mid d$$

S3: goto(S0,e)=closure($C \rightarrow e.C$, e/d)

[$C \rightarrow e.C$, e/d]

[$C \rightarrow .eC$, e/d]

[$C \rightarrow .d$, e/d]

S4: goto(S0, d)=closure($C \rightarrow d.$, e/d)

[$C \rightarrow d.$, e/d]

S8: goto(S3, C)=closure($C \rightarrow eC.$, e/d)

[$C \rightarrow eC.$, e/d]

S6: goto(S2,e)=closure($C \rightarrow e.C$, \$)

[$C \rightarrow e.C$, \$]

[$C \rightarrow .eC$, \$]

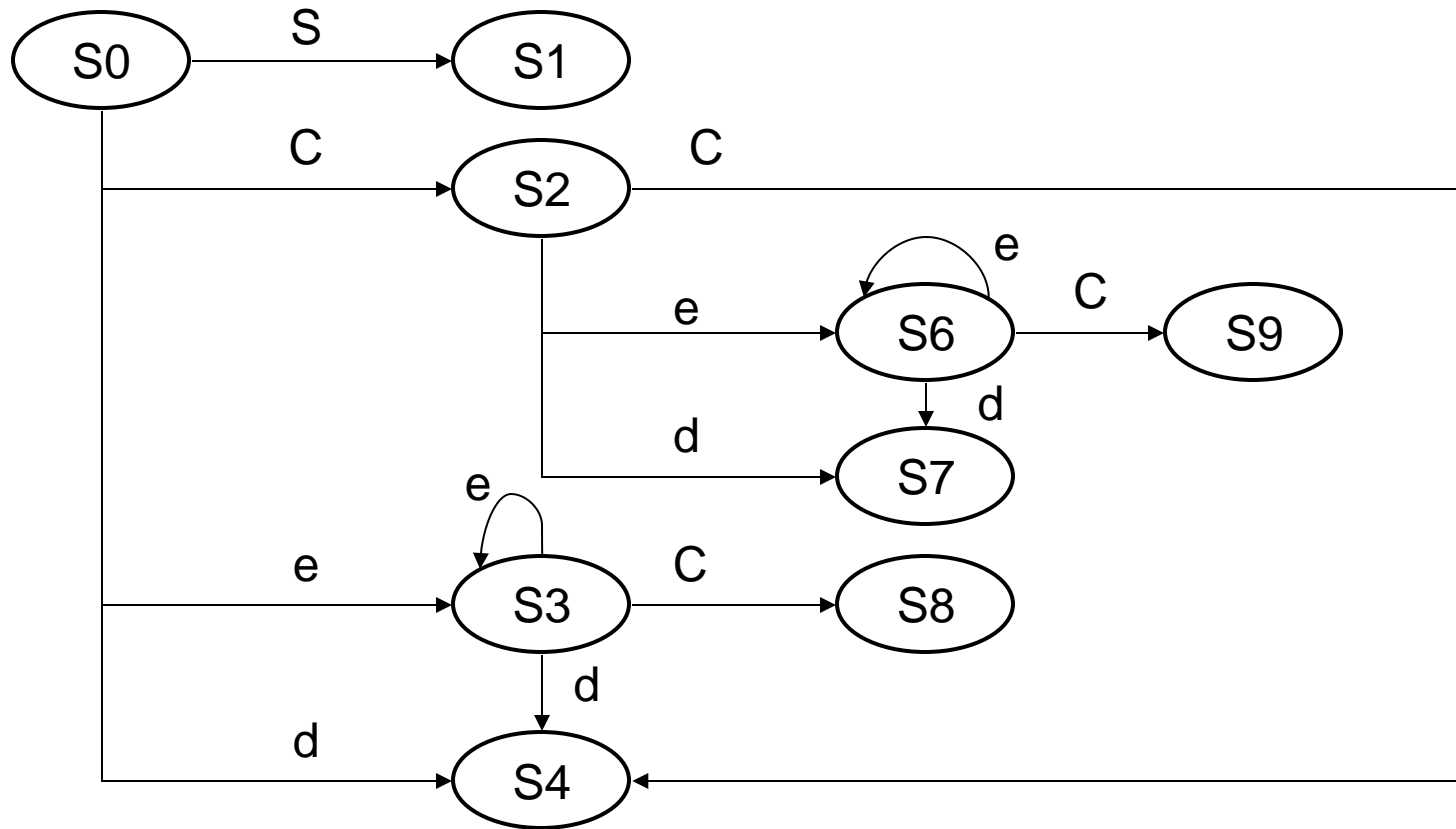
[$C \rightarrow .d$, \$]

S7: goto(S2, d)=closure($C \rightarrow d.$, \$)

[$C \rightarrow d.$, \$]

S9: goto(S6,C)=closure($C \rightarrow eC.$, \$)

[$C \rightarrow eC.$, \$]



Note S3 and S6 are the same except for look ahead (same for S4 and S7)
In SLR(1) – one state represents both

Merging states

❑ Can merge S3 and S6

S3: goto(S0,e)=closure($C \rightarrow e.C$, e/d)

$[C \rightarrow e.C, e/d]$

$[C \rightarrow .eC, e/d]$

$[C \rightarrow .d, e/d]$

S6: goto(S2,e)=closure($C \rightarrow e.C$, \$)

$[C \rightarrow e.C, \$]$

$[C \rightarrow .eC, \$]$

$[C \rightarrow .d, \$]$

S36: $[C \rightarrow e.C, e/d/\$]$

$[C \rightarrow .eC, e/d/\$]$

$[C \rightarrow .d, e/d/\$]$

❑ Similarly

- S47: $[C \rightarrow d., e/d/\$]$
- S89: $[C \rightarrow eC., e/d/\$]$

Effects of Merging

1. Detection of errors may be delayed

- On error, LALR parsers will not perform shifts beyond an LR parser but may perform more reductions before finding error

- Example: $S' \rightarrow S$ $S \rightarrow CC$

$C \rightarrow eC \mid d$

and input string eed\$

- Canonical LR: Parse Stack S0 e S3 e S3 d S4

State S4 on \$ input = error S4: {C \rightarrow d., e/d}

- LALR:

stack: S0 e S36 e S36 d S47 \rightarrow state S47 input \$, reduce C \rightarrow d

stack: S0e S36 e S36 C S89 \rightarrow reduce C \rightarrow eC

stack: S0 e S36 C S89 \rightarrow reduce C \rightarrow eC

stack: S0 C S2 \rightarrow state S2 on input \$, error

Effects of Merging

2. Merging of states can introduce conflicts

- cannot introduce shift-reduce conflicts
- can introduce reduce-reduce conflicts

□ Shift-reduce conflicts

Suppose S_{ij} : $[A \rightarrow \alpha., a]$ reduce on input a
 $[B \rightarrow \beta.a\delta, b]$ shift on input a

formed by merging S_i and S_j

Cores are the same for S_i , S_j and one of them must contain

$[A \rightarrow \alpha., a]$ and $[B \rightarrow \beta.a\delta, b]$

➔ shift-reduce conflicts were already present in either S_i and S_j (or both) and not newly introduced by merging

Reduce-reduce Conflicts

$S \rightarrow aEa \mid bEb \mid aFb \mid bFa$

$E \rightarrow e$

$F \rightarrow e$

S3: $[E \rightarrow e., a]$ viable prefix ae

$[F \rightarrow e., b]$

S4: $[E \rightarrow e., b]$ viable prefix be

$[F \rightarrow e., a]$

Merging S34: $[E \rightarrow e., a/b]$

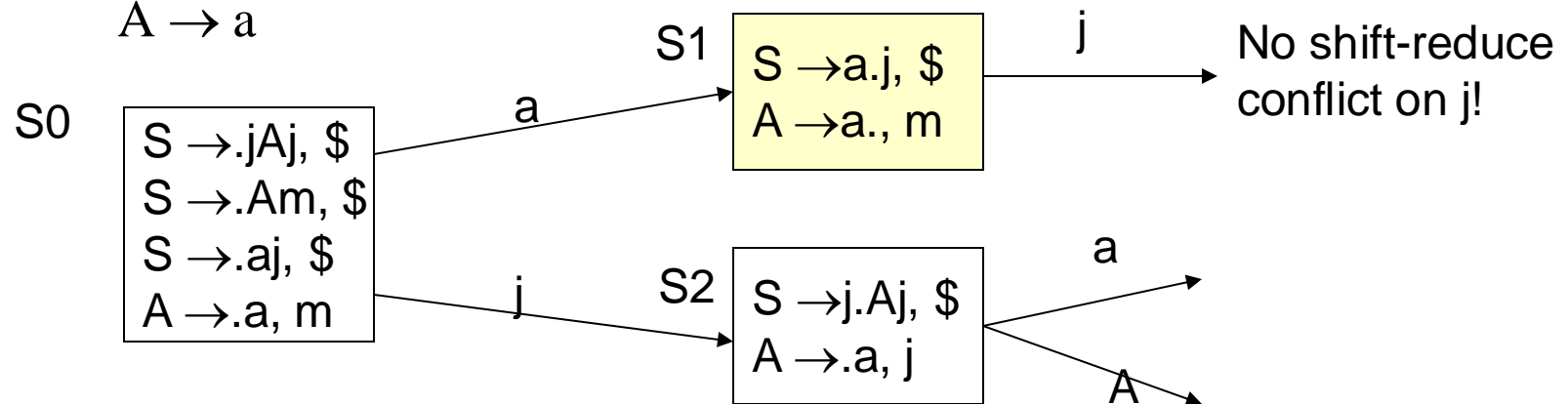
$[F \rightarrow e., a/b]$

- both reductions are called on inputs a and b ,
i.e. reduce-reduce conflict

Non SLR(k) but LALR(1)

□ Let's consider the Non SLR(k) Grammar G again

- $S \rightarrow j A j \mid A m \mid a j$
 $A \rightarrow a$



$\text{Follow}(A) = \{j, m\}$

State S_1 : $[A \rightarrow a \cdot]$ – reduce using this production (on j or m)

$[S \rightarrow a \cdot j]$ – shift j \rightarrow not SLR(1)

But LALR(1)

For reducing $A \rightarrow a \cdot$: lookahead must be $\{m\}$

For shifting $S \rightarrow a \cdot j$: $\text{First}_k(j\text{Follow}_k(S)) = \{j\} \rightarrow$ so LALR(1)

Construction on LALR Parser

❑ One solution:

- Construct LR(1) states
- Merge states with same core
- if no conflicts, you have a LALR parser

❑ Inefficient because of building LR(1) items are expensive in time and space (then what is the point of using LALR?)

❑ Efficient construction of LALR parsers

- Avoids initial construction of LR(1) states
- Merges states on-the-fly (step-by-step merging)
 - States are created as in LR(1)
 - On state creation, immediately merge if there is an opportunity

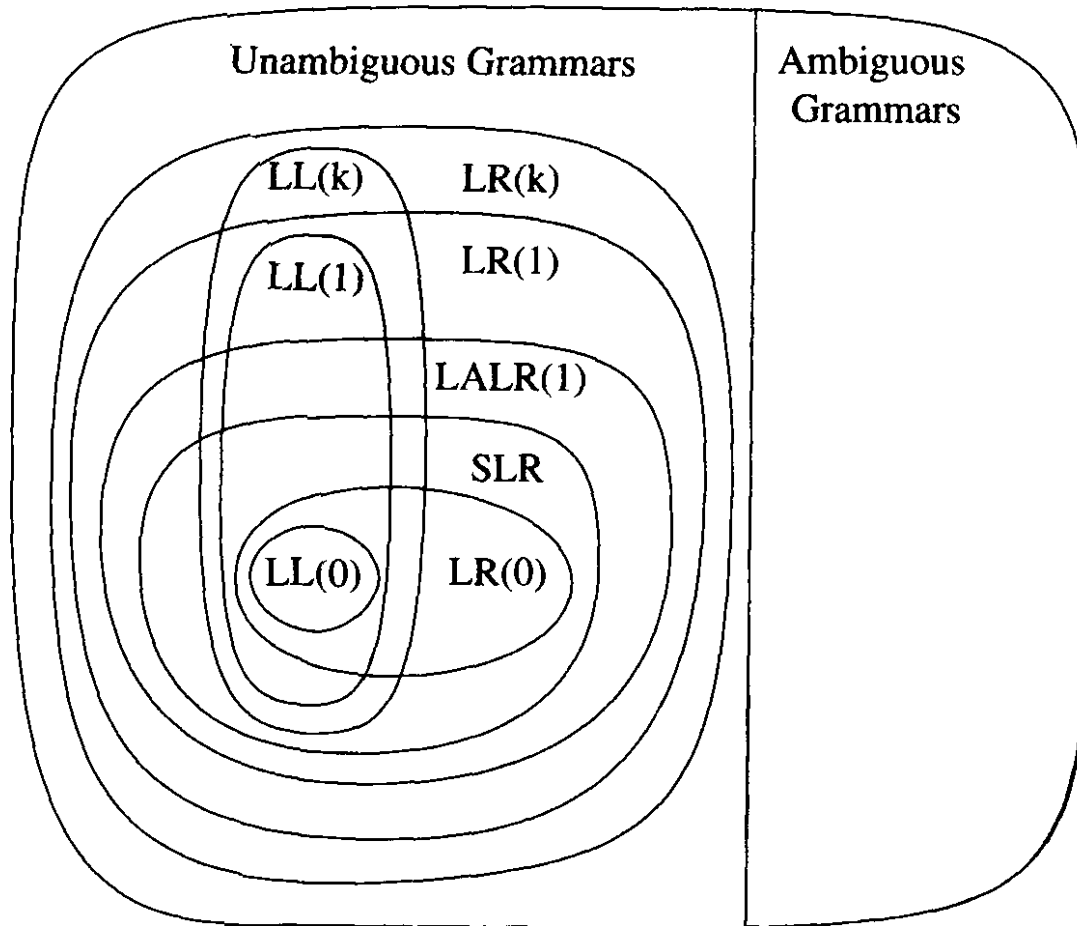
Compaction of LALR Parse Table

- ❑ A typical language grammar with 50-100 terminals and 100 productions may have an LALR parser with several hundred states and thousands of action entries
 - Often multiple rows of table are identical so share the rows
 - Make states point to an array of unique rows
 - Often most entries in a row are empty
 - Instead of an actual row array, use row lists of (input, action) pairs
 - Slows access to the table but can reduce memory footprint

Error Recovery

- ❑ Error recovery: How does the parser uncover multiple errors?
- ❑ Error detected when parser consults parsing table and hits an empty entry
 - Compared to LR, SLR and LALR parser may go through several reductions before detecting an error
 - Due to more coarse-grained use of lookahead
 - But never shifts beyond an erroneous symbol
- ❑ Simple error recovery (by discarding offending code sequence)
 1. Decide on non-terminal A: candidate for discarding
 - Typically an expression, statement, or block
 2. Continue to scan down the stack until a state S with a goto on a particular non-terminal A is found
 3. Discard input symbols until a symbol 'a' is found that can follow A
 - E.g. if A is a statement then 'a' would be ';'
 4. Push state Goto[a,A] on stack and continue parsing

A Hierarchy of Grammar Classes



LALR vs. LR Parsing (LALR < LR)

- ❑ LR(k) is strictly more powerful compared to LALR(k)
 - LALR merges and reduces number of states.
- ❑ Slightly unintuitive since unlike LL, LR since no formal definition exists on what is LALR
 - Definition by construction: if LALR parser has no conflicts
 - If there is a reduce-reduce conflict, is it because of merging?
- ❑ However, LALR(1) has become a standard for programming languages and for parser generators
 - YACC, Bison, etc.
 - Most PLs have an LALR(1) grammar
 - Reduce-reduce conflicts due to merging are rare (mostly due to ambiguity)

LL vs. LR Parsing ($LL < LR$)

- ❑ LL(k) parser, each expansion $A \rightarrow \alpha$ is decided on the basis of
 - Current non-terminal at the top of the stack
 - Which LHS to produce
 - k terminals of lookahead at *beginning* of RHS
 - Must guess which RHS by peeking at first few terminals of RHS
- ❑ LR(k) parser, each reduction $A \rightarrow \alpha \bullet$ is decided on the basis of
 - RHS at the top of the stack
 - Can postpone choice of RHS until entire RHS is seen
 - Common left factor is okay – waits until entire RHS is seen anyway
 - Left recursion is okay – does not impede forming RHS for reduction
 - k terminals of lookahead *beyond* RHS
 - Can decide on RHS after looking at entire RHS plus lookahead

LL vs. SLR Parsing (LL \neq SLR)

- ❑ Neither is strictly more powerful than the other
- ❑ Advantage of SLR: can delay decision until entire RHS seen
 - LL must decide RHS with a few symbols of lookahead
- ❑ Disadvantage of SLR: lookahead applied out of context
 - Consider grammar: $S \rightarrow Bb \mid Cc \mid aBc$, $B \rightarrow \varepsilon$, $C \rightarrow \varepsilon$
 - Initial state $S_0 = \{S \rightarrow . Bb \mid . Cc \mid . aBc, B \rightarrow ., C \rightarrow .\}$
 - For SLR(1), reduce-reduce conflict on $B \rightarrow .$ and $C \rightarrow .$
 - $\text{Follow}(B) = \{b, \mathbf{c}\}$ and $\text{Follow}(C) = \{\mathbf{c}\}$
 - For LL(1), no conflict
 - $\text{First}(Bb) = \{b\}$, $\text{First}(Cc) = \{c\}$, $\text{First}(aBC) = \{a\}$
- ❑ For the same reason, LL \neq LALR

$LR(0) == LALR(0) == SLR(0) > LL(0)$

□ $LR(0) == LALR(0) == SLR(0)$

- No lookahead for reducing
 - Lookahead components are meaningless (hence $LR == LALR == SLR$)
 - Must reduce regardless of Follow sets
 - If a state contains a reduce item, there can be no other reduce items or shift items for that state, or there will be a conflict
- Makes grammars very restrictive. Not used very much.

□ $LL(0) < LR(0)$

- $LL(0)$ can only have one RHS per non-terminal to avoid conflict
- $LR(0)$ can still have multiple RHSs per non-terminal
- E.g. $S \rightarrow a \mid b$ is not $LL(0)$ but is $LR(0)$

$L(\text{Rec. Descent}) == L(\text{GLR}) == L(\text{CFG})$

□ $L(\text{Recursive Descent}) == L(\text{CFG})$

- Can parse all CFGs by trial-and-error until input string match
- Including ambiguous CFGs (accepts first encountered parse tree)
- A general top-down parser for all CFGs can be constructed by using LL(k) parsing table, and falling back on recursive descent

□ Does that make top-down parsers superior to bottom-up?

- No. Same trial-and-error strategy can be employed for bottom-up
- GLR (Generalized LR) parser: a parser for all CFGs that relies on an LR parsing table, but falls back on trial-and-error on conflict
- $L(\text{GLR}(k)) == L(\text{CFG})$
- Any LR table (e.g. SLR, LALR, Canonical LR) can be used
- GLR implementations: GNU Bison etc. (but not Yacc)

Using Automatic Tools

-- YACC

Pitt, CS 1622

Using a Parser Generator

- ❑ YACC is an LALR(1) parser generator
 - YACC: Yet Another Compiler-Compiler
- ❑ YACC constructs an LALR(1) table and reports an error when a table entry is multiply defined
 - A shift and a reduce – reports shift/reduce conflict
 - Multiple reduces – reports reduce/reduce conflict
 - Most conflicts are due to ambiguous grammars
 - Must resolve conflicts
 - By specifying associativity or precedence rules
 - By modifying the grammar
 - YACC outputs detail about where the conflict occurred (by default, in the file “y.output”)

Shift/Reduce Conflicts

- Typically due to ambiguities in the grammar
- Classic example: the dangling else

$S \rightarrow \text{if } E \text{ then } S \mid \text{if } E \text{ then } S \text{ else } S \mid \text{OTHER}$

will have DFA state containing

$[S \rightarrow \text{if } E \text{ then } S. , \text{else}]$

$[S \rightarrow \text{if } E \text{ then } S. \text{ else } S , \text{else}]$

so on ‘else’ we can shift or reduce

- Default (YACC, bison, etc.) behavior is to shift
 - Default behavior is the correct one in this case
 - Better not to rely on this and remove ambiguity

More Shift/Reduce Conflicts

□ Consider the ambiguous grammar

$$E \rightarrow E + E \mid E * E \mid \text{int}$$

we will have the states containing

$$\begin{array}{ll} [E \rightarrow E * . E, +/*] & [E \rightarrow E * E . , +/*] \\ [E \rightarrow . E + E, +/*] & \xRightarrow{E} [E \rightarrow E . + E, +/*] \end{array}$$

...

...

Again we have a shift/reduce conflict on input +

- In this case, we need to reduce (* is higher than +)
- Easy (better) solution: declare precedence rules for * and +
- Hard solution: rewrite grammar to be unambiguous

More Shift/Reduce Conflicts

□ Declaring precedence and associativity in YACC

`%left '+' '-'`

`%left '*' '/'`

➤ Interpretation:

- `+`, `-`, `*`, `/` are left associative
- `+`, `-` have lower precedence compared to `*`, `/`
(associativity declarations are in the order of increasing precedence)
- Precedence of a candidate rule for reduction is the precedence of the last terminal in that rule (e.g. For `'E → E+E.'`, level is same as `'+'`)

➤ Resolve shift/reduce conflict with a shift if:

- No precedence declared for either rule or terminal
- Input terminal has higher precedence than the rule
- The precedence levels are the same and right associative

Use Precedence to Solve S/R Conflict

$$\begin{array}{ccc} [E \rightarrow E^* \cdot E, +/*] & & [E \rightarrow E^* E \cdot, +/*] \\ [E \rightarrow \cdot E + E, +/*] & \xRightarrow{E} & [E \rightarrow E \cdot + E, +/*] \\ \dots & & \dots \end{array}$$

- we will choose reduce because precedence of rule $E \rightarrow E^* E$ is higher than that of terminal $+$

$$\begin{array}{ccc} [E \rightarrow E + \cdot E, +/*] & & [E \rightarrow E + E \cdot, +/*] \\ [E \rightarrow \cdot E + E, +/*] & \xRightarrow{E} & [E \rightarrow E \cdot + E, +/*] \\ \dots & & \dots \end{array}$$

- we will choose reduce because $E \rightarrow E + E$ and $+$ have the same precedence and $+$ is left-associative

❑ Back to our dangling else example

$[S \rightarrow \text{if } E \text{ then } S. , \text{else}]$

$[S \rightarrow \text{if } E \text{ then } S. \text{ else } S, \text{else}]$

- Can eliminate conflict by declaring ‘else’ with higher precedence than ‘then’
- But this looks much less intuitive compared to arithmetic operator precedence
- Best to avoid overuse of precedence declarations that do not enhance the readability of your code

Reduce/Reduce Conflicts

- Usually due to ambiguity in the grammar

- Example: a sequence of identifiers

$$S \rightarrow \varepsilon \mid \text{id} \mid \text{id } S$$

There are two parse trees for the string 'id'

$$S \rightarrow \text{id}$$

$$S \rightarrow \text{id } S \rightarrow \text{id}$$

How does this confuse the parser?

Reduce/Reduce Conflicts

□ Consider the states

$[S' \rightarrow .S, \$]$

$[S \rightarrow ., \$]$

$[S \rightarrow .id, \$]$

$[S \rightarrow .id S, \$]$

\xRightarrow{id}

$[S \rightarrow id. , \$]$

$[S \rightarrow id.S, \$]$

$[S \rightarrow ., \$]$

$[S \rightarrow .id, \$]$

$[S \rightarrow .id S, \$]$

Reduce/reduce conflict on input “id\$”

$S' \rightarrow S \rightarrow id$

$S' \rightarrow S \rightarrow id S \rightarrow id$

Better rewrite the grammar: $S \rightarrow \varepsilon \mid id S$

Semantic Actions

- ❑ Semantic actions are implemented for LR parsing
 - keep attributes on the semantic stack – parallel to the parse stack
 - on shift a , push attribute for a on semantic stack
 - on reduce $X \rightarrow \alpha$
 - pop attributes for α
 - compute attribute for X
 - push it on the semantic stack
- ❑ Creating an AST
 - Bottom up
 - Create leaf node from attribute values of token(s) in RHS
 - Create internal node from subtree(s) passed on from RHS

Performing Semantic Actions

□ Compute the value

$E \rightarrow T + E1$	$\{E.val = T.val + E1.val\}$
$\quad \quad T$	$\{E.val = T.val\}$
$T \rightarrow int * T1$	$\{T.val = int.val * T1.val\}$
$\quad \quad int$	$\{T.val = int.val\}$

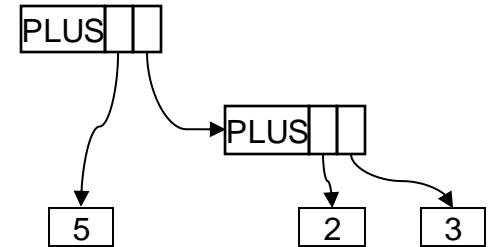
consider the parsing of the string $3 * 5 + 8$

□ Recall: creating the AST

$E \rightarrow int$	$E.ast = mkleaf(int.lexval)$
$\quad \quad E1 + E2$	$E.ast = mktree(plus, E1.ast, E2.ast)$
$\quad \quad (E1)$	$E.ast = E1.ast$

➤ a bottom-up evaluation of the ast attribute:

$E.ast = mktree(plus, mkleaf(5),$
 $\quad mktree(plus, mkleaf(2), mkleaf(3)))$



Notes on Parsing

□ Parsing

- A solid foundation: CFG
- A simple parser: LL(1)
- A more powerful parser: LR(1)
- An efficient compromise: LALR(1)
- LALR(1) parser generators