# Semantic Analysis

#### The role of semantic analysis is to assign meaning

- "It smells fishy."
- Lexical analysis
  - > Tokenizes "It", "smells", "fishy", "."
  - Determines noun, verb, adjective, punctuation token types
- Syntax analysis
  - > Parses the grammatical structure of the sentence
- Semantic analysis

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  - Determines noun, verb, adjective, punctuation token types
- Syntax analysis
  - > Parses the grammatical structure of the sentence
- Semantic analysis
  - > Assigns meaning to the words "It", "smells", "fishy"
  - > Flags error if the sentence does not make sense

#### Semantic Analysis = Binding + Type Checking

- "I don't wanna eat that sushi."
  - "It smells fishy."
    - > "It": the sushi
    - > "smells": feels to my nose
    - "fishy": that the sushi has gone bad
- "The professor says that the exam is going to be easy."
  - "It smells fishy."
    - "It": the situation
    - "smells": feels to my sixth sense
    - > "fishy": that it is highly suspicious

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- "The professor says that the exam is going to be easy."
  - "It smells fishy."
    - "It": the situation
    - "smells": feels to my sixth sense
    - > "fishy": that it is highly suspicious
- Semantic analysis consists of two tasks
  - > Binding: associating a pronoun to an object
  - > Type checking: inferring meaning based on type of object

#### Semantic Analysis = Binding + Type Checking

- Semantic analysis performs binding
  - Done by traversing parse tree produced by syntax analysis
  - Declarations are stored in data structure called symbol table
  - Uses are bound to entries in the symbol table
- Semantic analysis performs type checking
  - $\rightarrow$  Infers what "a + b" means:
    - If a and b are ints, integer add and return int
    - If a and b are floats, FP add and return float
    - If a and b are strings, concatenate and return string
  - Infers what "x.foo()" means:
    - If object x is a reference of class A, call to foo() in A
    - If object x is a reference of class B, call to foo() in B
  - Infers what "a[i][j]" means:
    - Offset from a based on array type and dimensions

#### Semantic analysis also performs semantic checks

- All symbol uses have corresponding declarations
- All symbols defined only once
  - Where symbols can be variables, methods, classes
  - Declaration: provides type information for a symbol
  - Definition: allocates a symbol in program memory
- All statements do not violate type rules
  - $\rightarrow$  Operators (+, -, \*, /, =, >, <, ==, ...) have legal parameters
  - Method calls have correct numbers of legal parameters
  - Private methods are not called by external classes
  - **>** ..

#### **Symbol Binding**

#### What is symbol binding?

"Matching symbol declarations with uses"

If there are multiple declarations, which one is matched?

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If there are multiple declarations, which one is matched?

```
void foo()
{
    char x;
    ...
    {
        int x;
        ...
    }
    x = x + 1;
}
```

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```

#### Scope

- Binding: associating a symbol use to its declaration
  - Which variable (or function) an identifier is referring to
- Scope: section of program where a declaration is valid
  - Uses in the scope of declaration are bound to it
- Some implications of scopes
  - A symbol may have different bindings in different scopes
  - Scopes for the same symbol never overlap
    - there is always exactly one binding per symbol use
- Two types: static scope and dynamic scope

#### Static Scope

- Static Scope: scope expressed in program text
  - Also called Lexical Scope
  - > C/C++, Java, JavaScript, Python
- Rule: bind to the closest enclosing declaration

```
void foo()
{
    char x;
    ...
    {
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    }
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#### Dynamic Scope

- Dynamic Scope: bindings formed during code execution
  - > LISP, Scheme, Perl
- Rule: bind to the most recent declaration during execution

```
void foo()
{
    (1) char x;
    (2) if (...) {
     (3) int x;
    (4) ...
    }
    (5) x = x + 1;
```

# Dynamic Scope

- Dynamic Scope: bindings formed during code execution
  - > LISP, Scheme, Perl
- Rule: bind to the most recent declaration during execution

```
void foo()
{
    (1) char x;
    (2) if (...) {
     (3) int x;
    (4) ...
    }
    (5) x = x + 1;
}
```

- $\square$  Which x's declaration is the closest?
  - > Execution (a): ...(1)...(2)...(5)
  - > Execution (b): ...(1)...(2)...(3)...(4)...(5)

#### Static vs. Dynamic Scoping

- Most languages that started with dynamic scoping (LISP, Scheme, Perl) added static scoping afterwards
- Why?
  - > It is easier for human beings to understand
    - Bindings are visible in code without tracing execution
  - It is easier for compilers to understand
    - Compiler can determine bindings at compile time
    - Compiler can translate identifier to a single memory location
    - Results in generation of efficient code
  - > With dynamic scoping...
    - There may be multiple possible bindings for a variable
    - Impossible to determine bindings at compile time
    - All bindings have to be done at execution time (Typically with the help of a hash table)

# Symbol Table

#### Symbol Table

- Symbol Table: A compiler data structure that tracks information about all identifiers (symbols) in a program
  - Maps symbol uses to declarations given a scope
  - Needs to provide bindings according to the current scope
- Usually discarded after generating the binary code
  - All symbols are mapped to memory locations already
  - > For debugging, symbols may be included in binary
    - To map memory locations back to symbols for debuggers
    - For GCC or Clang, add "-g" flag to include symbol tables

# Maintaining Symbol Table

```
Basic idea:
```

```
int x; ... void foo() { int x; ... x=x+1; } ... x=x+1 ...
```

- ➤ In *foo*, add *x* to table, overriding any previous declarations
- After foo, remove x and restore old declaration if any

```
Operations
```

```
enter_scope() start a new nested scope
```

exit\_scope() exit current scope

```
find_symbol(x) find declaration of x
```

add\_symbol(x) add declaration of x to symbol table

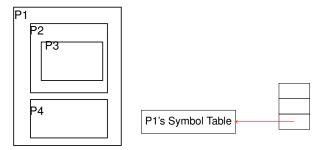
#### Adding Scope Information to the Symbol Table

- To handle multiple scopes in a program,
  - > (Conceptually) need an individual table for each scope
  - Symbols added to the table may not be deleted just because you exited a scope

```
class X { ... void f1() {...} ... } class Y { ... void f2() {...} ... } class Z { ... void f3() { ... void f3() { ... void f3() { ... void f3(); } ... }
```

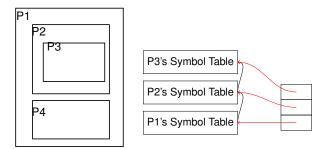
Without deleting symbols, how are scoping rules enforced?
 Keep a list of all scopes in the entire program
 Keep a stack of active scopes at a given point

#### Symbol Table with Multiple Scopes



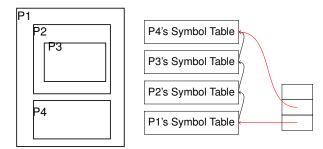
- For nested scopes,
  - Search from top of the active symbol table stack
  - Remove pointer to symbol table when exiting its scope

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#### What Information is Stored in the Symbol Table

Entry in Symbol Table:

string kind attributes

- String the name of identifier
- ➤ Kind variable, parameter, function, class, ...
- Attributes vary with the kind of symbol
  - ➤ variable → type, address in memory
  - → function → return type, parameter types, address
- Vary with the language
  - ➤ Fortran's array → type, dimension, dimension size real A(5) /\* dimension required for static allocation \*/
  - C's array → type, dimension, optional dimension size char A[5]; /\* statically sized array \*/ char A[]="hello"; /\* dynamically sized to fit content \*/

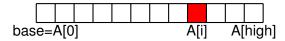
#### Symbol Table Attribute List

```
Type information might be arbitrarily complicated
     ➤ In C:
                  struct {
                         int a[10];
                         char b;
                         float c;
    Store all relevant attributes in an attribute list
                      1st upper bound
                                                  2nd upper bound
      array
 id
                      field₁
                            type
                                                  field<sub>2</sub> | type
 id
                                  size
                                                              size
      struct
```

# Example application of Type to an operator: Array index operator

#### Addressing Array Elements

```
int A[0..high];
A[i] ++;
```



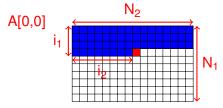
- > width width of element type
- > base address of the first
- > high upper bound of subscript
- Addressing an array element:

#### Multi-dimensional Arrays

Layout n-dimension items in 1-dimension memory int A[N<sub>1</sub>][N<sub>2</sub>]; /\* int A[0..high<sub>1</sub>][0..high<sub>2</sub>]; \*/  $A[i_1][i_2] ++;$  $N_2$ A[0,0] $N_1$ A[high<sub>1</sub>,high<sub>2</sub>]

# Row Major

Row major — store row by row

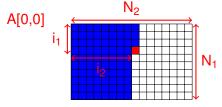


Offset inclues all the "blue" items before A[i1,i2]

$$\begin{split} & \text{offset}(A[i_1,i_2]) = (i_1 \ ^*\ N_2 + i_2\ )\ ^*\ width \\ & = i_1 \ ^*\ N_2\ ^*\ width + i_2\ ^*\ width \\ & = \text{offset}(A[i_1])\ ^*\ N_2 + i_2\ ^*\ width \end{split}$$

# Column Major

Column major — store column by column



 $\Box$  Offset inclues all the "blue" items before A[i<sub>1</sub>,i<sub>2</sub>]

offset(A[i<sub>1</sub>,i<sub>2</sub>]) = 
$$(i_2 * N_1 + i_1)*$$
width  
=  $i_2 * N_1 *$  width +  $i_1 *$  width  
=  $i_2 * N_1 *$ width + offset(A[i<sub>1</sub>])

# Generalized Row/Column Major

```
Let A_k = \text{offset}(A[i_1, i_2, ..., i_k]). Then,
```

Row major (C/C++, C#, Objective-C)

1-dimension:  $A_1 = i_1^*$ width

2-dimension:  $A_2 = (i_1 * N_2 + i_2) * width = A_1 * N_2 + i_2 * width$ 

3-dimension:  $A_3 = (i_1 * N_2 * N_3 + i_2 * N_3 + i_3) * width = A_2 * N_3 + i_3 * width$ 

k-dimension:  $A_k = A_{k-1} N_k + i_k \text{width}$ 

**Type** needs to provide  $N_2...N_k$  and width for offset

Column major (Fortran, Matlab, R)

1-dimension:  $A_1 = i_1^*$ width

2-dimension:  $A_2 = (i_2 * N_1 + i_1) * width = i_2 * N_1 * width + A_1$ 

3-dimension:  $A_3 = ((i_3*N_2+i_2)*N_1+i_1)*width = i_3*N_2*N_1*width + A_2$ 

k-dimension:  $A_k = i_k * N_{k-1} * N_{k-2} * ... * N_1 * width + A_{k-1}$ 

**Type** needs to provide  $N_1...N_{k-1}$  and width for offset

# C's implementation

```
C uses row major
   int fun1(int p[ ][100])
   {
      ...
      int a[100][100];
      a[i<sub>1</sub>][i<sub>2</sub>] = p[i<sub>1</sub>][i<sub>2</sub>] + 1;
}
```

Why is p[][100] allowed?

Why is a[][100] not allowed?

#### C's implementation

C uses row major
 int fun1(int p[ ][100])
{
 ...
 int a[100][100];
 a[i<sub>1</sub>][i<sub>2</sub>] = p[i<sub>1</sub>][i<sub>2</sub>] + 1;
}

Why is p[][100] allowed?

- ➤ The info is enough to compute p[i₁][i₂]'s address
- $\rightarrow$  A<sub>2</sub> = (i<sub>1</sub>\*N<sub>2</sub>+i<sub>2</sub>)\*width (N<sub>1</sub> is not required)

Why is a[][100] not allowed?

The info is not enough to allocate space for the array

# Type Checking

#### What, When, and Why

- What?
  - > Type: a set of values + a set of operations on values
  - Type Checking: Verifying and enforcing type consistency
    - Only legal values are assigned to a type
    - Only legal operations are performed on a type
- When?
  - Static Type Checking: Type checking at compile-time
  - Dynamic Type Checking: Type checking at execution time
- Static type checking is more desirable. Why?
  - > Better to fail at compile time than during deployment
  - > Dynamic type checking can impact runtime performance
  - Check dynamically only when static checking is infeasible
    - E.g. Java array bounds checks
    - E.g. Type checks to verify C++/Java downcasting

# Static vs. Dynamic Typing

- - Our discussion
  - Types are explicitly declared or can be inferred from code int x; /\* type of x is int \*/
  - Better compiler error detection due to static type checks
  - Efficient code since dynamic type checks are not needed
- Dynamically typed: Python, JavaScript, PHP
  - Type is a runtime property decided only during execution var x; /\* type of x is undecided \*/
    - x = 42; /\* type of x is int \*/
    - x = "forty two"; /\* type of x is now string \*/
    - /\* Type of x changes depending on the value it holds \*/
  - > Static type checking and error reporting is impossible
  - > More memory since every variable now needs a "type tag"
  - Inefficient code due to dynamic checks on type tags

#### Rules of Inference

- What are *rules of inference*?
  - ➤ Inference rules have the form if Precondition is true, then Conclusion is true
  - Below concise notation used to express above statement

# Precondition Conclusion

- ➤ In the context of type checking: if expressions E1, E2 have certain types (Precondition), expression E3 is legal and has a certain type (Conclusion)
- Type checking via inference
  - Start from variable types and constant types
  - > Repeatedly apply rules until entire program is inferred legal

### Notation for Inference Rules

By tradition inference rules are written as

# Precondition<sub>1</sub>, ..., Precondition<sub>n</sub> Conclusion

- The precondition/conclusion has the form "e:T"
- Meaning
  - If Precondition₁ and ... and Preconditionn are true, then Conclusion is true.
  - > "e:T" indicates "e is of type T"
  - > Example: rule-of-inference for add operation

```
e<sub>1</sub>: int
e<sub>2</sub>: int
e<sub>1</sub>+e<sub>2</sub>:int
```

Rule: If  $e_1$ ,  $e_2$  are ints then  $e_1+e_2$  is legal and is an int

# Two Simple Rules

 $[Constant] \begin{tabular}{ll} i is an integer \\ \hline i: int \\ \hline [Add operation] \begin{tabular}{ll} e_1: int \\ \hline e_2: int \\ \hline e_1+e_2: int \\ \hline \end{tabular}$ 

Example: given "10 is an integer" and "20 is an integer", is the expression "10+20" legal? Then, what is the type?

10 is an integer 20 is an integer 20: int 20: int

10+20:int

This type of reasoning can be applied to the entire program

#### More Rules

[New]

new T: T

[Not]

e: Boolean

not e: Boolean

However,

[Var?]

x is an identifier

x: ?

- > the expression itself insufficient to determine type
- > solution: provide context for this expression

# Type Environment

- □ A type environment gives type info for free variables
  - > A variable is *free* if not declared inside the expression
  - ➤ It is a function mapping Symbols to Types
    - Set of declarations active at the current scope
    - Conceptual representation of a symbol table

# Type Environment Notation

Let O be a function from Symbols to Types, the sentence O e:T

is read as "under the assumption of environment O, expression e has type T"

- "if i is an integer, expression i is an int in any environment"
- "if e1 and e2 are ints in O, expression e1+e2 is int in O"
- "if variable x is mapped to int in O, expression x is int in O"

### **Declaration Rule**

#### [Declaration w/o initialization]

O[T<sub>0</sub>/x] 
$$e_1$$
: T<sub>1</sub>  
O let x: T<sub>0</sub> in  $e_1$ : T<sub>1</sub>

 $O[T_0/x]$ : environment O modified so that it return  $T_0$  on argument x and behaves as O on all other arguments:

$$O[T_0/x](x) = T_0$$
  
 $O[T_0/x](y) = O(y)$  when  $x \neq y$ 

Translation: "If expression  $e_1$  is type  $T_1$  when x is mapped to type  $T_0$  in the current environment, expression  $e_1$  is type  $T_1$  when x is declared to be  $T_0$  in the current environment"

### **Declaration Rule with Initialization**

[Declaration with initialization (initial try)]

```
\begin{array}{c} \textbf{O} \ \textbf{e}_0 \colon \textbf{T}_0 \\ \hline \textbf{O}[\textbf{T}_0/\textbf{x}] \ \textbf{e}_1 \colon \textbf{T}_1 \\ \textbf{O} \ \textbf{let} \ \textbf{x} \colon \textbf{T}_0 \leftarrow \textbf{e}_0 \ \textbf{in} \ \textbf{e}_1 \colon \textbf{T}_1 \end{array}
```

The rule is too strict (i.e. correct but not complete)

```
Example class C inherits P ... let x:P ← new C in ...
```

the above rule does not allow this code

### Subtype

- lacktriangle A subtype is a relation  $\leq$  on classes
  - $\rightarrow$  X  $\leq$  X
  - ightharpoonup if X inherits from Y, then  $X \leq Y$
  - ightharpoonup if  $X \leq Y$  and  $Y \leq Z$ , then  $X \leq Z$
- An improvement of our previous rule

[Declaration with initialization]

- Both versions of declaration rules are correct
- > The improved version checks more programs

# Wrong Declaration Rule (case 1)

Consider a hypothetical let rule

[Wrong Declaration with initialization (case 1)]

```
\label{eq:controller} \begin{array}{c} \textbf{O} \; \textbf{e}_0 \colon \textbf{T} \\ \textbf{T} \leq \textbf{T}_0 \\ \textbf{O} \; \textbf{e}_1 \colon \textbf{T}_1 \\ \\ \textbf{O} \; \textbf{let} \; \textbf{x} \colon \textbf{T}_0 \leftarrow \textbf{e}_0 \; \textbf{in} \; \textbf{e}_1 \colon \textbf{T}_1 \end{array}
```

- > How is it different from the the correct rule?
- ➤ The following good program does not pass check let x: int ← 0 in x+1

# Wrong Declaration Rule (case 2)

Consider a hypothetical let rule

[Wrong Declaration with initialization (case 2)]

```
\label{eq:controller} \begin{array}{c} \textbf{O} \; \textbf{e}_0 \colon \textbf{T} \\ \textbf{T}_0 \leq \textbf{T} \\ \\ \textbf{O}[\textbf{T}_0/\textbf{x}] \; \textbf{e}_1 \colon \textbf{T}_1 \\ \\ \textbf{O} \; \textbf{let} \; \textbf{x} \colon \textbf{T}_0 \leftarrow \textbf{e}_0 \; \textbf{in} \; \textbf{e}_1 \colon \textbf{T}_1 \end{array}
```

- > How is it different from the the correct rule?
- The following bad program passes the check class B inherits A { only\_in\_B() { ... } } let x: B ← new A in x.only\_in\_B()

# **Assignment**

A correct but too strict rule

```
[Assignment] \\ \textbf{O(id)} = \textbf{T}_0 \\ \textbf{O e_1: T}_1 \\ \textbf{T}_1 \leq \textbf{T}_0 \\ \textbf{O id} \leftarrow \textbf{e_1: T}_0
```

The rule does not allow the below code class C inherits P { only\_in\_C() { ... } } let x:C in let y:P in x ← y ← new C x.only in C()

# **Assignment**

An improved rule

```
[Assignment] \\ \textbf{O(id)} = \textbf{T}_0 \\ \textbf{O e_1: T}_1 \\ \textbf{T}_1 \leq \textbf{T}_0 \\ \textbf{O id} \leftarrow \textbf{e_1: T}_1
```

The rule now does allow the below code class C inherits P { only\_in\_C() { ... } } let x:C in let y:P in x ← y ← new C x.only in C()

#### If-then-else

- $\square$  Let's say semantics of "if  $e_0$  then  $e_1$  else  $e_2$ " is:
  - > Returns the value of either e<sub>1</sub> or e<sub>2</sub>, depending on e<sub>0</sub>.
- What is the type of the above expression?
  - ➤ The type is either e<sub>1</sub>'s type or e<sub>2</sub>'s type.
  - Best compiler can do is to assign a super type of e<sub>1</sub> and e<sub>2</sub>.
- Least upper bound (LUB): the super type of two types
  - $\geq$  Z = lub(X,Y) Z is the least upper bound of X and Y iff

    - $X < Z \land Y < Z$  ; Z is an upper bound
    - $X < W \land Y < W \Longrightarrow Z < W$ ; Z is least among all upper bounds

#### If-then-else

```
[If-then-else]
```

```
\begin{array}{c} \text{O } \textbf{e}_0 \text{: Bool} \\ \text{O } \textbf{e}_1 \text{: } \textbf{T}_1 \\ \text{O } \textbf{e}_2 \text{: } \textbf{T}_2 \\ \hline \text{O } \text{if } \textbf{e}_0 \text{ then } \textbf{e}_1 \text{ else } \textbf{e}_2 \text{ fi: lub}(\textbf{T}_1, \textbf{T}_2) \end{array}
```

The rule allows the below code let x:float, y:int, z:float in x ← if (...) then y else z /\* Assuming lub(int, float) = float \*/

#### Discussion

- Type rules have to be carefully constructed, or
  - The type system becomes unsound (ill-behaved programs are accepted as well typed)
  - The type system becomes unusable (well-behaved programs are rejected as badly typed)

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  - Program that performs no forbidden operations at runtime

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- Type rules have to be carefully constructed, or
  - The type system becomes unsound (ill-behaved programs are accepted as well typed)
  - The type system becomes unusable (well-behaved programs are rejected as badly typed)
- What is a "well-behaved" program anyway?
  - Program that performs no forbidden operations at runtime
- Static type system cannot accurately capture behavior
  - → Here is a well-behaved program rejected by the type system obj ← if (x > y) then new Child else new Parent if (x > y) then obj.only\_in\_Child()
  - LUB type makes a choice of soundness over usability

# Designing a Good Type Checking System

- A good type system achieves two opposing goals:
  - Prevents false negative type errors, that is, runtime errors that are missed by type checking
  - Minimizes false positive type errors, that is, type errors that do not cause runtime errors
- A good type system should allow the following code:

```
class Parent {
    Parent clone() { return new this.getClass(); }
}
class Child inherits Parent { ... }
    void main() {
        // Error! Assignment of parent to child reference.
        Child c ← (new Child).clone();
    }
```

# What Went Wrong?

- What is (new Child).clone()'s type?
  - Dynamic type Child
  - Static type Parent
  - > Type system is not able to express runtime types precisely
  - > This makes inheriting clone() not very useful
    - clone() needs redefinition to return correct type anyway
- A "SELF\_TYPE" would be useful in these situations.

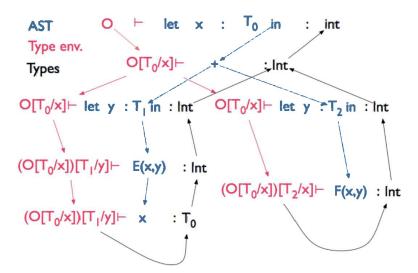
# SELF\_TYPE expresses runtime types precisely

- What is SELF\_TYPE?
  - clone() returns "self" instead of "Parent" type
  - > Self can be Parent or any subclass of Parent
- SELF\_TYPE is a static type
  - Type reflects precise runtime behavior for each class
  - Type violations can still be detected at compile time
- In practice
  - > Python, Rust, Scala: language support for self types
  - C++: can emulate using C++ templates
  - Java: can emulate to a lesser degree using Java generics

### Can Static Type Checking ever be Perfect?

- Many cases where well-behaved programs are rejected
  - > Reason for elaborate type systems like generics
  - Why programmers must sometimes typecast anyway
- Solution? Can't have your cake and eat it too.
  - Dynamic typing: values have types, variables do not
    - + Allows all runtime behaviors that are type consistent
    - Type errors occur at runtime rather than compile time
    - Best used for fast prototyping (scripting languages)
  - Static typing: variables have declared (or inferred) types
    - + Type errors can be caught at compile time
    - Effort needed to express "good" programs using type system
    - Best used when reliability is important

# Implementing Type Checking on AST



### **Error Recovery**

- ☐ Compiler must recover from type errors like syntax errors
  - ➤ Or else, below code results in multiple cascading errors let y: int ← x+2 in y+3
    - Reports error "x is undefined"
    - Reports error "Type of x+2 is undefined"
    - Reports error "Type of let y: int ← x+2 in y+3 is undefined"
    - ..
- Solution: introduce no-type for ill-typed expressions
  - ightharpoonup It is compatible with all types ightharpoonup no cascading errors
  - Report only the place where no-type is generated

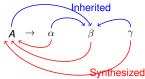
# Syntax Directed Definitions (SDDs)

#### SDD: Definitions of attributes and rules

- Syntax Directed Definitions (SDD):
  - Set of attributes attached to each grammar symbol
  - 2. Set of **semantic rules** attached to each production
  - Semantic rules define values of attributes
- Attribute Grammar:
  - An SDD where rules depend only on other attributes (i.e. An SDD that does not rely on any side-effects)
  - > Think of it as a "grammar" for semantic analysis
- Example: let's say we want to define type checking
  - > SDD can have semantic rules to access a symbol table
  - > Attribute grammar must transmit type info through attributes

### Synthesized vs. Inherited Attributes

Semantic rule:



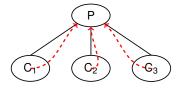
SDD has rule of the form for each CFG production  $b = f(c_1, c_2, ..., c_n)$ 

#### either

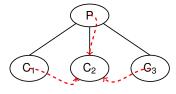
- If b is a synthesized attribute of A, c₁ (1≤i≤n) are attributes of grammar symbols of its Right Hand Side (RHS); or
- 2. If b is an inherited attribute of one of the symbols of RHS, c<sub>i</sub>'s are attribute of A and/or other symbols on the RHS

# Synthesized vs. Inherited Attributes

- Synthesized attributes: computed from children nodes
  - P.synthesized\_attr = f(C<sub>1</sub>.attr, C<sub>2</sub>.attr, C<sub>3</sub>.attr)
- ☐ Inherited attributes: computed from sibling/parent nodes
  - $ightharpoonup C_3.inherited_attr = f(P_1.attr, C_1.attr, C_3.attr)$



Synthesized attribute



Inherited attribute

# Synthesized Attribute Example

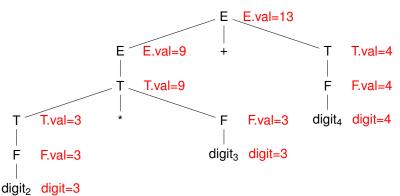
### Example

- > Each non-terminal symbol is associated with val attribute
- > The val attribute is computed soley from children attributes

```
[Grammar Rules]
                                     [Semantic Rules]
\mathsf{I} \to \mathsf{F}
                                     print(E.val)
\mathsf{E} \to \mathsf{E_1} + \mathsf{T}
                                     E.val = E_1.val + T.val
\mathsf{F} \to \mathsf{T}
                                      E.val = T.val
T \rightarrow T_1 * F
                                     T.val = T_1.val * F.val
\mathsf{T} \to \mathsf{F}
                                     T.val = F.val
\mathsf{F} \to (\mathsf{E})
                                     F.val = F.val
F → digit
                                      F.val = digit.lexval
```

### Synthesized Attribute Example: Attribute Parse Tree

Attribute parse tree: Parse tree decorated with attributes



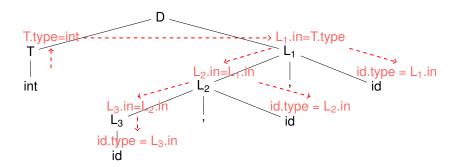
### Inherited Attribute Example

- Example:
  - T.type: synthesized attribute
  - L.in: inherited attribute
  - id.type: inherited attribute

- Why is L.in an inherited attribute?
  - > L.in is computed from a sibling T.type
  - ➤ L<sub>1</sub>.in is computed from a parent L.in

### Inherited Attribute Example: Attribute Parse Tree

- Red arrows denote dependencies between attributes
- Arrows for inherited attributes go sideways or downwards
- Arrows for synthesized attributes go upwards



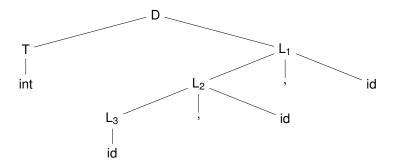
# **SDD** Implementation

# SDD Implementation using Parse Trees

- Assumes a previous parse stage
  - Input: a parse tree with no attribute annotations
  - Output: an attribute parse tree
- Goal: compute attribute values from leaf token values
  - > Traverse in some order, apply semantic rules at each node
  - > Traversal order must consider attribute dependencies

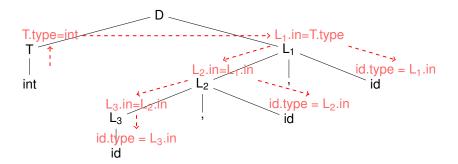
### Dependency Graph

- ☐ Directed graph where edges are attribute dependencies
  - "To" attribute is computed base on "from" attribute
  - > Must be **acyclic** such that there exists "a" traversal order



### **Dependency Graph**

- ☐ Directed graph where edges are attribute dependencies
  - "To" attribute is computed base on "from" attribute
  - ➤ Must be **acyclic** such that there exists "a" traversal order



# SDD Implementation using SDT

- Syntax Directed Translation (SDT)
  - Applying semantic rules as part of syntax analysis (parsing)
  - Does NOT assume a pre-existing parse tree
  - Done through semantic actions embedded in grammar
- Semantic action:
  - Code between curly braces embedded into RHS
  - Executed "at that point" in the RHS
    - Top-down: Right after previous symbol has been consumed
    - Bottom-up: After previous symbol has been pushed to stack (when the 'dot' reaches the semantic action)
  - Example of building a parse tree:
    - Program : Program IDNum ; Classes { \$\$=makeTree(ProgramOp, \$2, \$4); }
  - > \$2 and \$4 are indices into the parse stack
    - RHS is currently at top of stack waiting to be reduced
    - \$2 is attribute value for IDNum and \$4 is Classes

- Syntax Directed Translation Scheme (SDTS)
  - A "scheme" or plan to perform SDT
  - > A grammar specification embedded with semantic actions
  - Depends on choice of top-down or bottom-up parser
- **Example:**

- Both inherited and synthesized attributes are used
  - T synthesized attribute T.val
  - R inherited attribute R.i synthesized attribute R.s
  - ➤ E synthesized attribute E.val

Evaluating attributes using SDTS

```
E \rightarrow T \{R.i=T.val\} R \{E.val=R.s\}
R \rightarrow + T \{R_1.i=R.i+T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow \text{-} \quad T \ \{R_1.i\text{=}R.i\text{-}T.val\} \ R_1 \ \{R.s\text{=}R_1.s\}
R \rightarrow \varepsilon \{R.s=R.i\}
T \rightarrow (E) \{T.val=E.val\}
T \rightarrow num \{T.val=num.val\}
                                              Ε
                 T.val=num
                                                  \rightarrow R_1.i=T.val
                                                                                                            R_2
        num
                     num
                                                                                                                         R_3
                                            num
                                                                         num
```

Evaluating attributes using SDTS

```
E \rightarrow T \{R.i=T.val\} R \{E.val=R.s\}
R \rightarrow + T \{R_1.i=R.i+T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow - T \{R_1.i=R.i-T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow \varepsilon \{R.s=R.i\}
T \rightarrow (E) \{T.val=E.val\}
T \rightarrow num \{T.val=num.val\}
                                           Ε
                                                 R<sub>1</sub>.i=T.val R<sub>1</sub>
                                                T.val = num \rightarrow R_2.i=R_1.i+T.val
                                                                                                    R_2
       num
                                                                                                                R_3
                                         num
                                                     num
                                                                    num
```

Evaluating attributes using SDTS

```
E \rightarrow T \{R.i=T.val\} R \{E.val=R.s\}
R \rightarrow + T \{R_1.i=R.i+T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow - T \{R_1.i=R.i-T.val\} R_1 \{R.s=R_1.s\}
R \rightarrow \varepsilon \{R.s=R.i\}
T \rightarrow (E) \{T.val=E.val\}
T \rightarrow num \{T.val=num.val\}
                                                    EE.val=R<sub>1</sub>.s
                                                            R<sub>1</sub>.i=T.val
                                                                                R<sub>1</sub>R<sub>1</sub>.s=R<sub>2</sub>.s
                                                                                      R_2.i=R_1.i+T.val
                                                                                                                         R<sub>2</sub> R<sub>2</sub>.s= R<sub>3</sub>.s
         num
                                                                                     T T.val = num \Rightarrow R<sub>3</sub>.i=R<sub>2</sub>.i+T.valR<sub>3</sub>R<sub>3</sub>.s= R<sub>3</sub>.i
                                                  num
                                                                                  num
                                                                                              num
```

### What are the dependencies allowed in SDTS?

- Parse trees: dependencies only required to be acyclic
- What is required of dependencies for SDTS?
  - Different parsing schemes see nodes in different orders
    - Top-down parsing LL(k) parsing
    - Bottom-up parsing LR(k) parsing
  - What if dependency node has not been seen yet?
- For certain classes of SDDs, using SDTS is feasible
  - > If dependencies of SDD are amenable to parse order
  - > This class of SDDs are called L-Attributed Grammars

### Left-Attributed Grammar

- An SDD is L-attributed if each of its attributes is either:
  - ightharpoonup a synthesized attribute of A in A $\rightarrow$  X<sub>1</sub>... $X_n$ ,

or

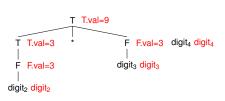
- $\rightarrow$  an inherited attribute of  $X_j$  in  $A \rightarrow X_1...X_n$  that
  - depends on attributes of siblings to its left i.e.  $X_1...X_{j-1}$
  - and/or depends on parent A

### Left-Attributed Grammar

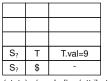
- An L-Attributed grammar
  - may have synthesized attributes
  - may have inherited attributes but only from left sibling attributes or inherited attributes of the parent
- Evaluation order
  - Left-to-right depth-first traversal of the parse tree
    - Order for both top-down and bottom-up parsers
  - Evaluate inherited attributes while going down the tree
  - > Evaluate synthesized attributes while going up the tree
- Can be evaluated using SDTS w/o parse tree

When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes

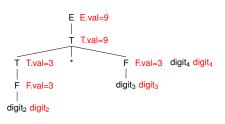


#### parsing stack:

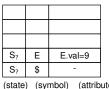


When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes



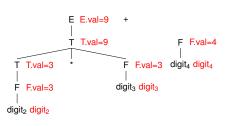
#### parsing stack:



(attribute)

When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes

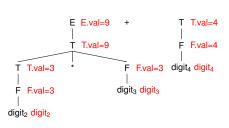


#### parsing stack:

S <sub>?</sub>	F	F.val=4	
S <sub>?</sub>	+	-	
S <sub>?</sub>	Е	E.val=9	
S <sub>?</sub>	\$	-	
(state) (symbol) (attribute			

When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes

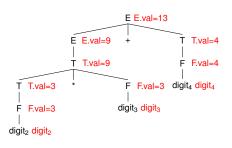


#### parsing stack:

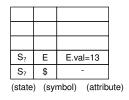
S <sub>?</sub>	Т	T.val=4	
S <sub>?</sub>	+	-	
S <sub>?</sub>	Е	E.val=9	
S <sub>?</sub>	\$	-	
(state) (symbol) (attribute			

When using LR parsing (bottom-up parsing),

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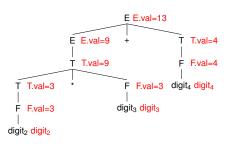


#### parsing stack:

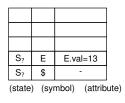


When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes



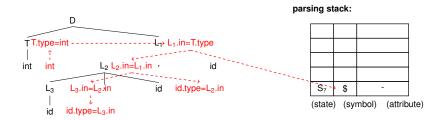
#### parsing stack:



Grammars with only synthesized attributes are called S-Attributed Grammars

When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes

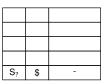


When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes



#### parsing stack:

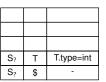


When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes

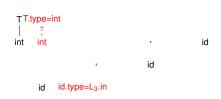


#### parsing stack:

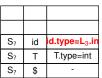


When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes

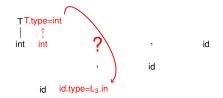


#### parsing stack:

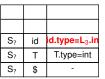


When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes

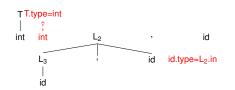


#### parsing stack:



When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes

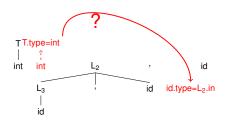


#### parsing stack:

S <sub>?</sub>	id	id.type=L2.in
S <sub>?</sub>	,	
S <sub>?</sub>	L <sub>3</sub>	L <sub>3</sub> .in=L <sub>2</sub> .in
S <sub>?</sub>	Т	T.type=int
S <sub>?</sub>	\$	-

When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes



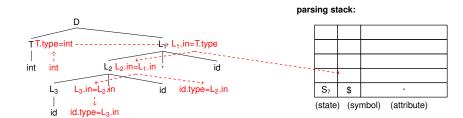
#### parsing stack:

S <sub>?</sub>	id	id.type=L2.in
S <sub>?</sub>	,	
S <sub>?</sub>	L <sub>3</sub>	L <sub>3</sub> .in=L <sub>2</sub> .in
S <sub>?</sub>	Т	T.type=int
S <sub>?</sub>	\$	-

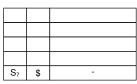
# **Evaluating Inherited Attributes using LR**

- Claim: Given an L-Attributed grammar, inherited attributes needed for the computation are already on the stack
- Recall: What is an L-Attributed grammar?
  - May have synthesized attributes
  - > May have inherited attributes but only from:
    - Left sibling attributes
    - Parent attribute
- Finding inherited attributes on the stack
  - Left siblings: previously reduced, so already on the stack
  - > Parent: not yet reduced, but left siblings of the parent used to compute the parent attribute are on the stack

```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow int & \{T.type=int\} \\ T \rightarrow real & \{T.type=real\} \\ L \rightarrow L & , & id & \{id.type=stack[top-3].type\} \\ L \rightarrow id & \{id.type=stack[top-1].type\} \end{array}
```



```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow int & \{T.type=int\} \\ T \rightarrow real & \{T.type=real\} \\ L \rightarrow L & , & id & \{id.type=stack[top-3].type\} \\ L \rightarrow id & \{id.type=stack[top-1].type\} \end{array}
```

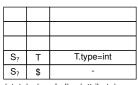


(state) (symbol) (attribute)

, id id id

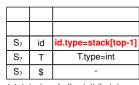
int

```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow int & \{T.type=int\} \\ T \rightarrow real & \{T.type=real\} \\ L \rightarrow L & , & id & \{id.type=stack[top-3].type\} \\ L \rightarrow id & \{id.type=stack[top-1].type\} \end{array}
```



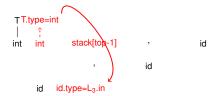


```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow \text{int} & \{\text{T.type=int}\} \\ T \rightarrow \text{real} & \{\text{T.type=real}\} \\ L \rightarrow L & , & \text{id} & \{\text{id.type=stack[top-3].type}\} \\ L \rightarrow \text{id} & \{\text{id.type=stack[top-1].type}\} \end{array}
```





```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow int & \{T.type=int\} \\ T \rightarrow real & \{T.type=real\} \\ L \rightarrow L & , & id & \{id.type=stack[top-3].type\} \\ L \rightarrow id & \{id.type=stack[top-1].type\} \end{array}
```



S <sub>?</sub>	id	id.type=stack[top-1]
S <sub>?</sub>	Т	T.type=int
S <sub>?</sub>	\$	-

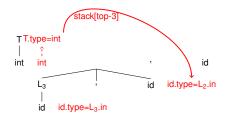
```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow int & \{T.type=int\} \\ T \rightarrow real & \{T.type=real\} \\ L \rightarrow L & , & id & \{id.type=stack[top-3].type\} \\ L \rightarrow id & \{id.type=stack[top-1].type\} \end{array}
```

#### 

#### parsing stack:

S <sub>?</sub>	id	id.type=stack[top-3]
S <sub>?</sub>	,	
S <sub>?</sub>	L <sub>3</sub>	L <sub>3</sub> .in=int
S <sub>?</sub>	Т	T.type=int
S <sub>?</sub>	\$	-

```
\begin{array}{lll} D \rightarrow T & L \\ T \rightarrow int & \{T.type=int\} \\ T \rightarrow real & \{T.type=real\} \\ L \rightarrow L & , & id & \{id.type=stack[top-3].type\} \\ L \rightarrow id & \{id.type=stack[top-1].type\} \end{array}
```



S <sub>?</sub>	id	id.type=stack[top-3]
S <sub>?</sub>	,	
S <sub>?</sub>	L <sub>3</sub>	L <sub>3</sub> .in=int
S <sub>?</sub>	Т	T.type=int
S <sub>?</sub>	\$	-

### Marker

 $\Box$  Given the following SDD, where  $|\alpha| != |\beta|$ 

 $A \rightarrow X \alpha Y \mid X \beta Y$ 

 $Y \rightarrow \gamma \{... = f(X.s)\}$ 

- Problem: cannot generate stack location for X.s since X is at different relative stack locations from Y
- Solution: introduce *markers* M<sub>1</sub> and M<sub>2</sub> that are at the same relative stack locations from Y

 $A \rightarrow X \alpha M_1 Y \mid X \beta M_2 Y$ 

 $Y \rightarrow \gamma \{ \dots = f(M_{12}.s) \}$ 

 $M_1 \rightarrow \varepsilon \{M_1.s = X.s\}$ 

 $M_2 \rightarrow \varepsilon \{M_2.s = X.s\}$ 

 $(M_{12} = \text{the stack location of } M_1 \text{ or } M_2, \text{ which are identical})$ 

A marker intuitively marks a stack location that is equidistant from the reduced non-terminal

### Example

■ When is a marker necessary and how is it added?

```
Example 1:
        S \rightarrow a A \{ C.i = A.s \} C
        S \rightarrow b A B \{ C.i = A.s \} C
        C \rightarrow c \{ C.s = f(C.i) \}
Solution:
        S \rightarrow a A \{ C.i = A.s \} C
        S \rightarrow b A B \{ M.i=A.s \} M \{ C.i = M.s \} C
        C \rightarrow c \{ C.s = f(C.i) \}
        M \rightarrow \varepsilon \{ M.s = M.i \}
That is:
        S \rightarrow a A C
        S \rightarrow b A B M C
        C \rightarrow c \{ C.s = f(stack[top-1]) \}
        M \rightarrow \varepsilon \{ M.s = stack[top-2] \}
```

### When and how to add a marker

- 1. Identify the stack offset(s) to find the desired attribute
- 2. If stack offsets are different, add a marker
- 3. Add marker where it would result in uniform stack offsets

### Example:

```
\begin{split} S &\rightarrow a \ A \ B \ C \ E \ D \\ S &\rightarrow b \ A \ F \ B \ C \ F \ D \\ C &\rightarrow c \ \{/^* \ C.s = f(A.s) \ ^*/\} \\ D &\rightarrow d \ \{/^* \ D.s = f(B.s) \ ^*/\} \end{split}
```

### Answer

```
S → a A B C E D
S → b A D M B C F D
C → c {/* C.s = f(stack[top-2]) */}
D → d {/* D.s = f(stack[top-3]) */}
M → ε {/* M.s = f(stack[top-2]) */}

Regarding C.s, from stack[top-2], and stack[top-3]
.... add a Marker

Regarding D.s, always from stack[top-2]
... no need to add
```

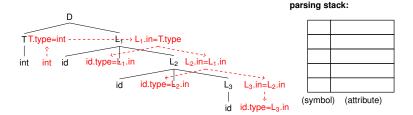
How about Top-Down Parsing?

### Translation Scheme for Top-Down Parsing

- ☐ Recursive Descent Parsers: Straightforward
  - Synthesized Attribute
    - Say function for non-terminal returns synthesized attribute
    - Compute attribute from children function call return values
  - Inherited Attribute
    - Pass as argument to function call for inheriting non-terminal
    - Left sibling attributes: left sibling calls already complete
    - Parent attributes: passed in as arguments to parent function
- How about table-driven LL parsers?

it is natural to evaluate inherited attributes

```
\begin{array}{lll} D \rightarrow T & \{L.in=T.type\} \ L \\ T \rightarrow int & \{T.type=int\} \\ T \rightarrow real & \{T.type=real\} \\ L \rightarrow \{id.type=L.in\} \ id \ , & \{L_1.in=L.in\} \ L_1 \\ L \rightarrow \{id.type=L.in\} \ id \end{array}
```



it is natural to evaluate inherited attributes

```
\begin{array}{lll} D \rightarrow T & \{L.in=T.type\} \ L \\ T \rightarrow int & \{T.type=int\} \\ T \rightarrow real & \{T.type=real\} \\ L \rightarrow \{id.type=L.in\} \ id \\ L \rightarrow \{id.type=L.in\} \ id \\ \end{array}
```

D

### parsing stack:



it is natural to evaluate inherited attributes

```
\begin{array}{lll} D \rightarrow T & \{L.in=T.type\} \ L \\ T \rightarrow int & \{T.type=int\} \\ T \rightarrow real & \{T.type=real\} \\ L \rightarrow \{id.type=L.in\} \ id \ , & \{L_1.in=L.in\} \ L_1 \\ L \rightarrow \{id.type=L.in\} \ id \end{array}
```



### parsing stack:



it is natural to evaluate inherited attributes

```
\begin{array}{l} D \rightarrow T \ \{L.in=T.type\} \ L \\ T \rightarrow int \ \{T.type=int\} \\ T \rightarrow real \ \{T.type=real\} \\ L \rightarrow \{id.type=L.in\} \ id \ , \ \{L_1.in=L.in\} \ L_1 \\ L \rightarrow \{id.type=L.in\} \ id \end{array}
```



### parsing stack:



it is natural to evaluate inherited attributes

```
\begin{array}{l} D \rightarrow T \ \{L.in=T.type\} \ L \\ T \rightarrow int \ \{T.type=int\} \\ T \rightarrow real \ \{T.type=real\} \\ L \rightarrow \{id.type=L.in\} \ id \ , \ \{L_1.in=L.in\} \ L_1 \\ L \rightarrow \{id.type=L.in\} \ id \end{array}
```

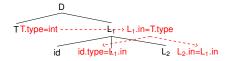


### parsing stack:



it is natural to evaluate inherited attributes

```
\begin{array}{lll} D \rightarrow T & \{L.in=T.type\} \ L \\ T \rightarrow int & \{T.type=int\} \\ T \rightarrow real & \{T.type=real\} \\ L \rightarrow \{id.type=L.in\} \ id \ , & \{L_1.in=L.in\} \ L_1 \\ L \rightarrow \{id.type=L.in\} \ id \end{array}
```

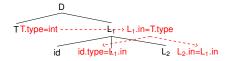


#### parsing stack:

	{id.type=L <sub>1</sub> .in}
id	id.type=???
,	
	$\{L_2.in=L_1.in\}$
L <sub>2</sub>	L <sub>2</sub> .in=???

it is natural to evaluate inherited attributes

```
\begin{array}{l} D \rightarrow T \ \{L.in=T.type\} \ L \\ T \rightarrow int \ \{T.type=int\} \\ T \rightarrow real \ \{T.type=real\} \\ L \rightarrow \{id.type=L.in\} \ id \ , \ \{L_1.in=L.in\} \ L_1 \\ L \rightarrow \{id.type=L.in\} \ id \end{array}
```



#### parsing stack:

	{id.type=int}
id	id.type=???
,	
	{L <sub>2</sub> .in=int}
L <sub>2</sub>	L <sub>2</sub> .in=???

it is natural to evaluate inherited attributes

```
\begin{array}{l} D \rightarrow T \ \{L.in=T.type\} \ L \\ T \rightarrow int \ \{T.type=int\} \\ T \rightarrow real \ \{T.type=real\} \\ L \rightarrow \{id.type=L.in\} \ id \ , \ \{L_1.in=L.in\} \ L_1 \\ L \rightarrow \{id.type=L.in\} \ id \end{array}
```

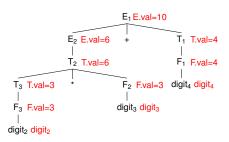


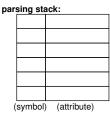
#### parsing stack:

		{id.type=int}
	id	id.type=???
	,	
		{L <sub>2</sub> .in=int}
	L <sub>2</sub>	L <sub>2</sub> .in=???
(5	vmbo	l) (attribute)

Semantic actions on the stack are called action-records.

it is **not natural** to evaluate synthesized attributes





it is **not natural** to evaluate synthesized attributes

 $E_1$ 

parsing stack:			
	E <sub>1</sub>		
(	symbol)	(attribute)	

pai

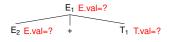
### Translation Scheme for LL Parsing

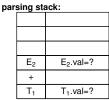
it is **not natural** to evaluate synthesized attributes

E<sub>1</sub> E.val=?

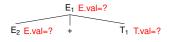
r	rsing stack:		
	E <sub>1</sub>	E <sub>1</sub> .val=?	

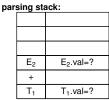
it is **not natural** to evaluate synthesized attributes





it is **not natural** to evaluate synthesized attributes



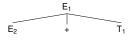


it is **not natural** to evaluate synthesized attributes

 $E_1$ 

parsing stack:			
	E <sub>1</sub>		
	E <sub>1</sub> .val	???	
(	symbol	(attribute)	

it is **not natural** to evaluate synthesized attributes



### pars

sing stack:		
E <sub>2</sub>		
E <sub>2</sub> .val	???	
+		
T <sub>1</sub>		
T <sub>1</sub> .val	???	
E <sub>1</sub> .val	E2.val + T1.val	
symbol	(attribute)	

it is **not natural** to evaluate synthesized attributes



parsing stack

sing stack:		
E <sub>2</sub>		
E <sub>2</sub> .val	???	
+		
T <sub>1</sub>		
T <sub>1</sub> .val	???	
E <sub>1</sub> .val	E2.val + T1.val	
symbol	(attribute)	

- Synthesized attributes on the stack: **synthesize-records**. (Inserted below non-terminal with synthesized attribute)
- In synthesize-record E<sub>1</sub>.val = E<sub>2</sub>.val + T<sub>1</sub>.val, E<sub>2</sub>.val and T<sub>1</sub>.val are place holders for pending values. (Updated when records E<sub>2</sub>.val and T<sub>1</sub>.val are popped.)