

Semantic Analysis

The role of semantic analysis is to assign meaning

❏ "It smells fishy."

❏ Lexical analysis

- Tokenizes "It", "smells", "fishy", "."
- Determines noun, verb, adjective, punctuation token types

❏ Syntax analysis

- Parses the grammatical structure of the sentence

❏ Semantic analysis

The role of semantic analysis is to assign meaning

❑ "It smells fishy."

❑ Lexical analysis

- Tokenizes "It", "smells", "fishy", "."
- Determines noun, verb, adjective, punctuation token types

❑ Syntax analysis

- Parses the grammatical structure of the sentence

❑ Semantic analysis

- Assigns meaning to the words "It", "smells", "fishy"
- Flags error if the sentence does not make sense

Semantic Analysis = Binding + Type Checking

- ❑ "I don't wanna eat that sushi."
 - "It smells fishy."
 - "It": the sushi
 - "smells": feels to my nose
 - "fishy": that the sushi has gone bad
- ❑ "The professor says that the exam is going to be easy."
 - "It smells fishy."
 - "It": the situation
 - "smells": feels to my sixth sense
 - "fishy": that it is highly suspicious

Semantic Analysis = Binding + Type Checking

- ❑ "I don't wanna eat that sushi."
"It smells fishy."
 - "It": the sushi
 - "smells": feels to my nose
 - "fishy": that the sushi has gone bad
- ❑ "The professor says that the exam is going to be easy."
"It smells fishy."
 - "It": the situation
 - "smells": feels to my sixth sense
 - "fishy": that it is highly suspicious
- ❑ Semantic analysis consists of two tasks
 - **Binding**: associating a pronoun to an object
 - **Type checking**: inferring meaning based on type of object

Semantic Analysis = Binding + Type Checking



Semantic analysis performs binding

- Done by traversing parse tree produced by syntax analysis
- Declarations are stored in data structure called **symbol table**
- Uses are bound to entries in the symbol table



Semantic analysis performs type checking

- Infers what " $a + b$ " means:
 - If a and b are ints, integer add and return int
 - If a and b are floats, FP add and return float
 - If a and b are strings, concatenate and return string
- Infers what " $x.foo()$ " means:
 - If object x is a reference of class A , call to $foo()$ in A
 - If object x is a reference of class B , call to $foo()$ in B
- Infers what " $a[i][j]$ " means:
 - Offset from a based on array type and dimensions

Semantic analysis also performs semantic checks

- ❑ All symbol uses have corresponding declarations
- ❑ All symbols defined only once
 - Where symbols can be variables, methods, classes
 - Declaration: provides type information for a symbol
 - Definition: allocates a symbol in program memory
- ❑ All statements do not violate type rules
 - Operators (+, -, *, /, =, >, <, ==, ...) have legal parameters
 - Method calls have correct numbers of legal parameters
 - Private methods are not called by external classes
 - ...

Symbol Binding

What is symbol binding?

“Matching symbol **declarations** with **uses**”

 If there are multiple declarations, which one is matched?

What is symbol binding?

“Matching symbol **declarations** with **uses**”

❑ If there are multiple declarations, which one is matched?

```
void foo()
{
    char x;
    ...
    {
        int x;
        ...
    }
    x = x + 1;
}
```

What is symbol binding?

“Matching symbol **declarations** with **uses**”

❑ If there are multiple declarations, which one is matched?

```
void foo()
{
    char x;
    ...
    {
        int x; ?
    }
    x = x + 1;
}
```

Scope

- ❑ **Binding**: associating a symbol use to its declaration
 - Which variable (or function) an identifier is referring to

- ❑ **Scope**: section of program where a declaration is valid
 - Uses in the scope of declaration are bound to it

- ❑ Some implications of scopes
 - A symbol may have different bindings in different scopes
 - Scopes for the same symbol never overlap
 - there is always exactly one binding per symbol use

- ❑ Two types: static scope and dynamic scope

Static Scope

❏ **Static Scope**: scope expressed in program text

- Also called **Lexical Scope**
- C/C++, Java, JavaScript, Python

❏ Rule: bind to the closest enclosing declaration

```
void foo()  
{  
    char x;  
  
    ...  
    {  
        int x;  
  
        ...  
    }  
    x = x + 1;  
}
```

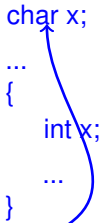
Static Scope

❏ **Static Scope**: scope expressed in program text

- Also called **Lexical Scope**
- C/C++, Java, JavaScript, Python

❏ Rule: bind to the closest enclosing declaration

```
void foo()  
{  
    char x;  
    ...  
    {  
        int x;  
        ...  
    }  
    x = x + 1;  
}
```



Dynamic Scope

❏ **Dynamic Scope:** bindings formed during code execution

➤ LISP, Scheme, Perl

❏ **Rule:** bind to the most recent declaration during execution

```
void foo()
{
  (1) char x;
  (2) if (...) {
  (3)   int x;
  (4)   ...
      }
  (5) x = x + 1;
}
```

Dynamic Scope

❑ **Dynamic Scope:** bindings formed during code execution

➤ LISP, Scheme, Perl

❑ **Rule:** bind to the most recent declaration during execution

```
void foo()  
{  
  (1) char x;  
  (2) if (...) {  
    (3)  int x;  
    (4)  ...  
  }  
  (5) x = x + 1;  
}
```

❑ Which *x*'s declaration is the closest?

➤ Execution (a): ...**(1)**...(2)...(5)

➤ Execution (b): ...(1)...(2)...**(3)**...(4)...(5)

Static vs. Dynamic Scoping

- ❑ Most languages that started with dynamic scoping (LISP, Scheme, Perl) added static scoping afterwards
- ❑ Why?
 - It is easier for human beings to understand
 - Bindings are visible in code without tracing execution
 - It is easier for compilers to understand
 - Compiler can determine bindings at compile time
 - Compiler can translate identifier to a single memory location
 - Results in generation of efficient code
 - With dynamic scoping...
 - There may be multiple possible bindings for a variable
 - Impossible to determine bindings at compile time
 - All bindings have to be done at execution time (Typically with the help of a hash table)

Symbol Table

Symbol Table

- ❑ **Symbol Table:** A compiler data structure that tracks information about all identifiers (symbols) in a program
 - Maps symbol uses to declarations given a scope
 - Needs to provide bindings according to the current scope

- ❑ Usually discarded after generating the binary code
 - All symbols are mapped to memory locations already
 - For debugging, symbols may be included in binary
 - To map memory locations back to symbols for debuggers
 - For GCC or Clang, add “-g” flag to include symbol tables

Maintaining Symbol Table

Basic idea:

```
int x; ... void foo() { int x; ... x=x+1; } ... x=x+1 ...
```

- In *foo*, add *x* to table, overriding any previous declarations
- After *foo*, remove *x* and restore old declaration if any

Operations

`enter_scope()` start a new nested scope

`exit_scope()` exit current scope

`find_symbol(x)` find declaration of *x*

`add_symbol(x)` add declaration of *x* to symbol table

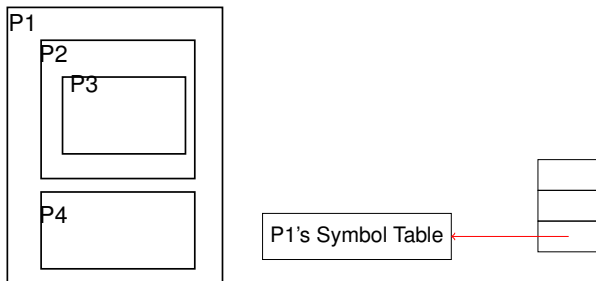
Adding Scope Information to the Symbol Table

- ❏ To handle multiple scopes in a program,
 - (Conceptually) need an individual table for each scope
 - Symbols added to the table may not be deleted just because you exited a scope

```
class X { ... void f1() {...} ... }  
class Y { ... void f2() {...} ... }  
class Z { ... void f3() {  
    X v;  
    v.f1();  
} ... }
```

- Without deleting symbols, how are scoping rules enforced?
 - ☞ Keep a list of all scopes in the entire program
 - ☞ Keep a stack of active scopes at a given point

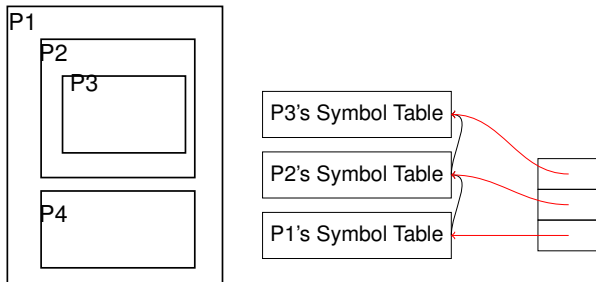
Symbol Table with Multiple Scopes



For nested scopes,

- Search from top of the active symbol table stack
- Remove pointer to symbol table when exiting its scope

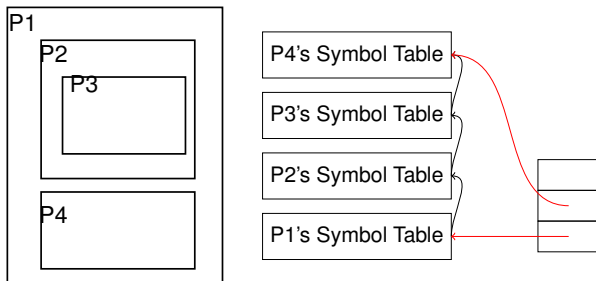
Symbol Table with Multiple Scopes



For nested scopes,

- Search from top of the active symbol table stack
- Remove pointer to symbol table when exiting its scope

Symbol Table with Multiple Scopes



For nested scopes,

- Search from top of the active symbol table stack
- Remove pointer to symbol table when exiting its scope

What Information is Stored in the Symbol Table

Entry in Symbol Table:

string	kind	attributes
--------	------	------------

- String — the name of identifier
- Kind — variable, parameter, function, class, ...

Attributes vary with the kind of symbol

- variable → type, address in memory
- function → return type, parameter types, address

Vary with the language

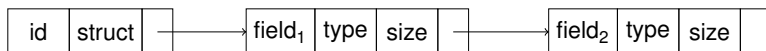
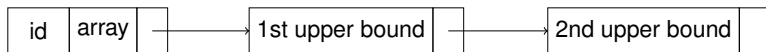
- Fortran's array → type, dimension, dimension size
`real A(5) /* dimension required for static allocation */`
- C's array → type, dimension, optional dimension size
`char A[5]; /* statically sized array */`
`char A[]="hello"; /* dynamically sized to fit content */`

Symbol Table Attribute List

❑ Type information might be arbitrarily complicated

➤ In C: struct {
 int a[10];
 char b;
 float c;
 }

❑ Store all relevant attributes in an attribute list

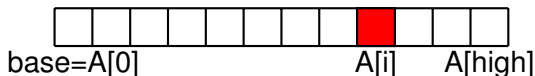


Example application of Type to an operator: Array index operator


Addressing Array Elements

```
int A[0..high];
```

```
A[i] ++;
```



- width — width of element type
- base — address of the first
- high — upper bound of subscript

 Addressing an array element:

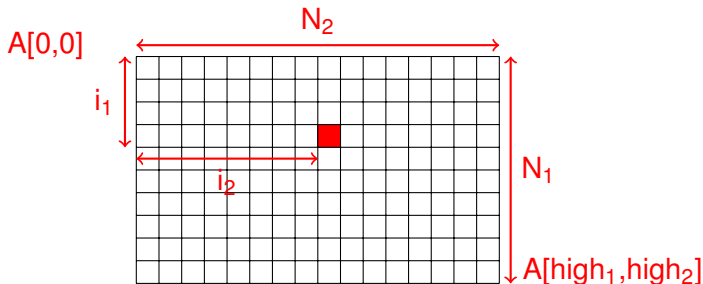
$$\text{address}(A[i]) = \text{base} + i * \text{width}$$
$$\text{offset}(A[i]) = i * \text{width}$$

Multi-dimensional Arrays

- Layout n-dimension items in 1-dimension memory

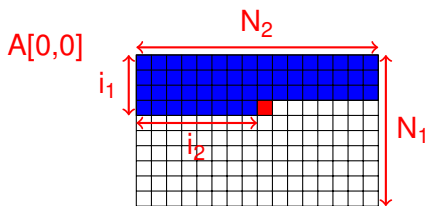
```
int A[N1][N2]; /* int A[0..high1][0..high2]; */
```

```
A[i1][i2] ++;
```



Row Major

Row major — store row by row

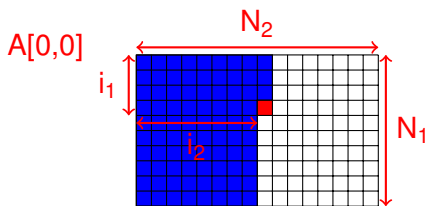


□ Offset includes all the “blue” items before $A[i_1, i_2]$

$$\begin{aligned}
 \text{offset}(A[i_1, i_2]) &= (i_1 * N_2 + i_2) * \text{width} \\
 &= i_1 * N_2 * \text{width} + i_2 * \text{width} \\
 &= \text{offset}(A[i_1]) * N_2 + i_2 * \text{width}
 \end{aligned}$$

Column Major

Column major — store column by column



□ Offset includes all the “blue” items before $A[i_1, i_2]$

$$\begin{aligned} \text{offset}(A[i_1, i_2]) &= (i_2 * N_1 + i_1) * \text{width} \\ &= i_2 * N_1 * \text{width} + i_1 * \text{width} \\ &= i_2 * N_1 * \text{width} + \text{offset}(A[i_1]) \end{aligned}$$

Generalized Row/Column Major

Let $A_k = \text{offset}(A[i_1, i_2, \dots, i_k])$. Then,

Row major (C/C++, C#, Objective-C)

1-dimension: $A_1 = i_1 * \text{width}$

2-dimension: $A_2 = (i_1 * N_2 + i_2) * \text{width} = A_1 * N_2 + i_2 * \text{width}$

3-dimension: $A_3 = (i_1 * N_2 * N_3 + i_2 * N_3 + i_3) * \text{width} = A_2 * N_3 + i_3 * \text{width}$

k-dimension: $A_k = A_{k-1} * N_k + i_k * \text{width}$

Type needs to provide $N_2 \dots N_k$ and width for offset

Column major (Fortran, Matlab, R)

1-dimension: $A_1 = i_1 * \text{width}$

2-dimension: $A_2 = (i_2 * N_1 + i_1) * \text{width} = i_2 * N_1 * \text{width} + A_1$

3-dimension: $A_3 = ((i_3 * N_2 + i_2) * N_1 + i_1) * \text{width} = i_3 * N_2 * N_1 * \text{width} + A_2$

k-dimension: $A_k = i_k * N_{k-1} * N_{k-2} * \dots * N_1 * \text{width} + A_{k-1}$

Type needs to provide $N_1 \dots N_{k-1}$ and width for offset

C's implementation

❏ C uses row major

```
int fun1(int p[ ][100])  
{  
  ...  
  int a[100][100];  
  a[i1][i2] = p[i1][i2] + 1;  
}
```

Why is p[][100] allowed?

Why is a[][100] not allowed?

C's implementation

❏ C uses row major

```
int fun1(int p[][100])  
{  
...  
    int a[100][100];  
    a[i1][i2] = p[i1][i2] + 1;  
}
```

Why is `p[][100]` allowed?

- The info is enough to compute `p[i1][i2]`'s address
- $A_2 = (i_1 * N_2 + i_2) * \text{width}$ (N_1 is not required)

Why is `a[][100]` not allowed?

- The info is not enough to allocate space for the array

Type Checking

What, When, and Why

What?

- **Type**: a set of values + a set of operations on values
- **Type Checking**: Verifying and enforcing type consistency
 - Only legal values are assigned to a type
 - Only legal operations are performed on a type

When?

- **Static Type Checking**: Type checking at compile-time
- **Dynamic Type Checking**: Type checking at execution time

Static type checking is more desirable. Why?

- Better to fail at compile time than during deployment
- More memory since every variable now needs a "type tag"
- Longer runtime since type tag needs checking at runtime
- Check dynamically only when static checking is infeasible
 - E.g. Java array bounds checks
 - E.g. Type checks to verify C++/Java downcasting

Static vs. Dynamic Typing

❑ Statically typed: C/C++, Java

👉 Our discussion

- Types are explicitly declared or can be inferred from code
`int x; /* type of x is int */`
- Better compiler error detection due to static type checks
- Efficient code since dynamic type checks are not needed

❑ Dynamically typed: Python, JavaScript, PHP

- Type is a runtime property decided only during execution
`var x; /* type of x is undecided */`
`x = 42; /* type of x is int */`
`x = "forty two"; /* type of x is now string */`
`/* Type of x changes depending on the value it holds */`
- Static type checking and error reporting is impossible
- Inefficient code due to dynamic checks on type tags

Rules of Inference

□ What are *rules of inference*?

- Inference rules have the form
if **Precondition** is true, then **Conclusion** is true
- Below concise notation used to express above statement

Precondition
Conclusion

- In the context of type checking:
if expressions E1, E2 have certain types (Precondition),
expression E3 is legal and has a certain type (Conclusion)

□ Type checking via inference

- Start from variable types and constant types
- Repeatedly apply rules until entire program is inferred legal

Notation for Inference Rules

- By tradition inference rules are written as

$$\frac{\text{Precondition}_1, \dots, \text{Precondition}_n}{\text{Conclusion}}$$

- The precondition/conclusion has the form “**e:T**”

- Meaning

- If **Precondition**₁ and ... and **Precondition**_n are true, then **Conclusion** is true.
- “**e:T**” indicates “**e is of type T**”
- Example: rule-of-inference for add operation

$$\frac{e_1: \text{int} \quad e_2: \text{int}}{e_1 + e_2: \text{int}}$$

Rule: If e_1, e_2 are ints then $e_1 + e_2$ is legal and is an int

Two Simple Rules

[Constant]

$$\frac{\text{**i is an integer**}}{\text{**i: int**}}$$

[Add operation]

$$\frac{\begin{array}{l} \text{**e}_1\text{: int} \\ \text{**e}_2\text{: int} \end{array}}{\text{**e}_1\text{+e}_2\text{:int}}}******$$

□ Example: given “10 is an integer” and “20 is an integer”, is the expression “10+20” legal? Then, what is the type?

$$\frac{\frac{\text{**10 is an integer**}}{\text{**10: int**}} \quad \frac{\text{**20 is an integer**}}{\text{**20: int**}}}{\text{**10+20:int**}}$$

□ This type of reasoning can be applied to the entire program

More Rules

[New]

$$\frac{}{\text{new T: T}}$$

[Not]

$$\frac{e: \text{Boolean}}{\text{not } e: \text{Boolean}}$$

 However,

[Var?]

$$\frac{x \text{ is an identifier}}{x: ?}$$

- the expression itself insufficient to determine type
- **solution:** provide context for this expression

Type Environment

- ❏ A *type environment* gives type info for free variables
 - A variable is *free* if not declared inside the expression
 - It is a function mapping **Symbols** to **Types**
 - Set of declarations active at the current scope
 - Conceptual representation of a symbol table

Type Environment Notation

Let \mathcal{O} be a function from **Symbols** to **Types**,
the sentence $\mathcal{O} \ e:T$

is read as “under the assumption of environment \mathcal{O} ,
expression e has type T ”

$$\frac{i \text{ is an intger}}{\mathcal{O} \ i: \text{int}}$$

$$\frac{\begin{array}{l} \mathcal{O} \ e1: \text{int} \\ \mathcal{O} \ e2: \text{int} \end{array}}{\mathcal{O} \ e1+e2: \text{int}}$$

$$\frac{\mathcal{O}(x) == T}{\mathcal{O} \ x: T}$$

- “if i is an integer, expression i is an int in any environment”
- “if $e1$ and $e2$ are ints in \mathcal{O} , expression $e1+e2$ is int in \mathcal{O} ”
- “if variable x is mapped to int in \mathcal{O} , expression x is int in \mathcal{O} ”

Declaration Rule

[Declaration w/o initialization]

$$\frac{O[T_0/x] \ e_1 : T_1}{O \text{ let } x : T_0 \text{ in } e_1 : T_1}$$

$O[T_0/x]$: environment O modified so that it return T_0 on argument x and behaves as O on all other arguments:

$$O[T_0/x](x) = T_0$$

$$O[T_0/x](y) = O(y) \text{ when } x \neq y$$

- Translation: "If expression e_1 is type T_1 when x is mapped to type T_0 in the current environment, expression e_1 is type T_1 when x is declared to be T_0 in the current environment"

Declaration Rule with Initialization

[Declaration with initialization (initial try)]

$$\frac{\begin{array}{c} \mathbf{O} \ e_0 : T_0 \\ \mathbf{O}[T_0/x] \ e_1 : T_1 \end{array}}{\mathbf{O} \ \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

❏ The rule is too strict (i.e. correct but not complete)

Example

class C inherits P ...

let x:P \leftarrow new C in ...

👉 the above rule does not allow this code

Subtype

□ A subtype is a relation \leq on classes

- $X \leq X$
- if X inherits from Y , then $X \leq Y$
- if $X \leq Y$ and $Y \leq Z$, then $X \leq Z$

□ An improvement of our previous rule

[Declaration with initialization]

$$\frac{\begin{array}{c} O\ e_0: T \\ T \leq T_0 \\ O[T_0/x]\ e_1: T_1 \end{array}}{O\ \text{let } x: T_0 \leftarrow e_0\ \text{in } e_1: T_1}$$

- Both versions of declaration rules are correct
- The improved version checks more programs

Wrong Declaration Rule (case 1)

❏ Consider a hypothetical let rule

[Wrong Declaration with initialization (case 1)]

$$\frac{\begin{array}{c} \text{O } e_0: T \\ T \leq T_0 \\ \text{O } e_1: T_1 \end{array}}{\text{O let } x: T_0 \leftarrow e_0 \text{ in } e_1: T_1}$$

- How is it different from the the correct rule?
- The following good program does not pass check
let x: int \leftarrow 0 in x+1

Wrong Declaration Rule (case 2)

❑ Consider a hypothetical let rule

[Wrong Declaration with initialization (case 2)]

$$\frac{\begin{array}{c} \mathbf{O} \ e_0: \mathbf{T} \\ \mathbf{T}_0 \leq \mathbf{T} \\ \mathbf{O}[T_0/x] \ e_1: \mathbf{T}_1 \end{array}}{\mathbf{O} \ \text{let } x: \mathbf{T}_0 \leftarrow e_0 \ \text{in } e_1: \mathbf{T}_1}$$

- How is it different from the the correct rule?
- The following bad program passes the check


```
class B inherits A { only_in_B() { ... } }
let x: B ← new A in x.only_in_B()
```


Assignment

❏ A correct but too strict rule

[Assignment]

$O(id) = T_0$

$O e_1 : T_1$

$T_1 \leq T_0$

$O id \leftarrow e_1 : T_0$

- The rule does not allow the below code
- ```
class C inherits P { only_in_C() { ... } }
let x:C in
let y:P in
x ← y ← new C
x.only_in_C()
```

# Assignment

□ An improved rule

[Assignment]

$O(id) = T_0$

$O\ e_1: T_1$

$T_1 \leq T_0$

---

$O\ id \leftarrow e_1: T_1$

- The rule now does allow the below code
- ```
class C inherits P { only_in_C() { ... } }
let x:C in
let y:P in
x ← y ← new C
x.only_in_C()
```

If-then-else

- Let's say semantics of "if e_0 then e_1 else e_2 " is:
 - Returns the value of either e_1 or e_2 , depending on e_0 .

- What is the type of the above expression?
 - The type is either e_1 's type or e_2 's type.
 - Best compiler can do is to assign a super type of e_1 and e_2 .

- Least upper bound (LUB): the super type of two types
 - $Z = \text{lub}(X, Y)$ — Z is the least upper bound of X and Y iff
 - $X \leq Z \wedge Y \leq Z$; Z is an upper bound
 - $X \leq W \wedge Y \leq W \implies Z \leq W$; Z is least among all upper bounds

If-then-else

[If-then-else]

$$\frac{\begin{array}{l} \text{O } e_0: \text{Bool} \\ \text{O } e_1: T_1 \\ \text{O } e_2: T_2 \end{array}}{\text{O if } e_0 \text{ then } e_1 \text{ else } e_2 \text{ fi: lub}(T_1, T_2)}$$

□ The rule allows the below code

let x:float, y:int, z:float in

x \leftarrow if (...) then y else z

/* Assuming lub(int, float) = float */

Discussion

- ❑ Type rules have to be carefully constructed, or
 - The type system becomes unsound
(ill-behaved programs are accepted as well typed)
 - The type system becomes unusable
(well-behaved programs are rejected as badly typed)

Discussion

- ❑ Type rules have to be carefully constructed, or
 - The type system becomes unsound
(ill-behaved programs are accepted as well typed)
 - The type system becomes unusable
(well-behaved programs are rejected as badly typed)
- ❑ What is a “well-behaved” program anyway?
 - Program that performs no forbidden operations **at runtime**

Discussion

- ❑ Type rules have to be carefully constructed, or
 - The type system becomes unsound
(ill-behaved programs are accepted as well typed)
 - The type system becomes unusable
(well-behaved programs are rejected as badly typed)
- ❑ What is a “well-behaved” program anyway?
 - Program that performs no forbidden operations **at runtime**
- ❑ Static type system cannot accurately capture behavior
 - Here is a well-behaved program rejected by the type system
`obj ← if (x > y) then new Child else new Parent`
`if (x > y) then obj.only_in_Child()`
 - LUB type makes a choice of soundness over usability

Designing a Good Type Checking System

- ❑ A good type system achieves two opposing goals:
 - Prevents **false negative** type errors, that is, runtime errors that are missed by type checking
 - Minimizes **false positive** type errors, that is, type errors that do not cause runtime errors
- ❑ A good type system should allow the following code:

```
class Parent {  
    Parent clone() { return new this.getClass(); }  
}  
class Child inherits Parent { ... }  
void main() {  
    // Error! Assignment of parent to child reference.  
    Child c ← (new Child).clone();  
}  
}
```


What Went Wrong?

- ❑ What is `(new Child).clone()`'s type?
 - Dynamic type — Child
 - Static type — Parent
 - Type system is not able to express runtime types precisely
 - This makes inheriting `clone()` not very useful
 - `clone()` needs redefinition to return correct type anyway
- ❑ A "SELF_TYPE" would be useful in these situations.

SELF_TYPE expresses runtime types precisely

❏ What is SELF_TYPE?

- `clone()` returns “self” instead of “Parent” type
- Self can be Parent or any subclass of Parent

❏ SELF_TYPE is a static type

- Type reflects precise runtime behavior for each class
- Type violations can still be detected at compile time

❏ In practice

- Python, Rust, Scala: language support for self types
- C++: can emulate using C++ templates
- Java: can emulate to a lesser degree using Java generics

Can Static Type Checking ever be Perfect?

❑ Many cases where well-behaved programs are rejected

- Reason for elaborate type systems like generics
- Why programmers must sometimes typecast anyway

❑ Solution? Can't have your cake and eat it too.

➤ Dynamic type checking

- + Allows all runtime behaviors that are type consistent
- Type errors occur at runtime rather than compile time

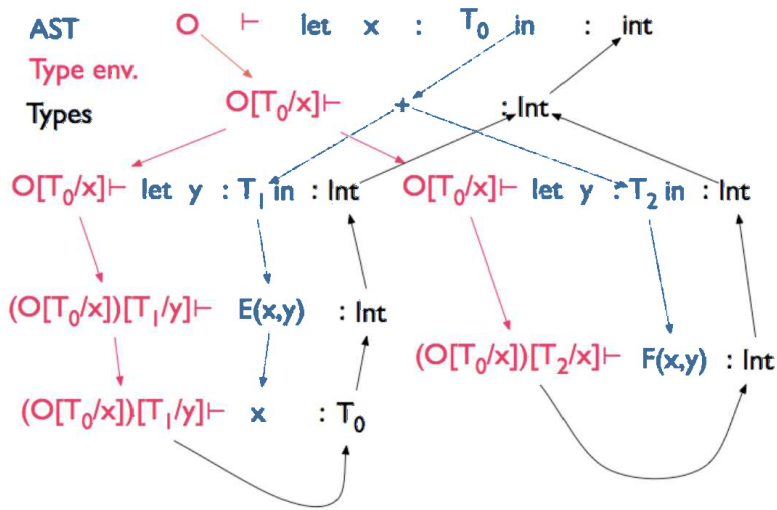
👉 Best used for fast prototyping (scripting languages)

➤ Static type checking

- + Type errors can be caught at compile time
- Effort to express well-behaved programs using type system

👉 Best used when reliability is important

Implementing Type Checking on AST



Error Recovery

❑ Compiler must recover from type errors like syntax errors

➤ Or else, below code results in multiple cascading errors

let y: int \leftarrow x+2 in y+3

- Reports error “x is undefined”
- Reports error “Type of x+2 is undefined”
- Reports error “Type of let y: int \leftarrow x+2 in y+3 is undefined”
- ...

❑ Solution: introduce **no-type** for ill-typed expressions

- It is compatible with all types \rightarrow no cascading errors
- Report only the place where **no-type** is generated

Syntax Directed Definitions (SDDs)

SDD: Definitions of attributes and rules

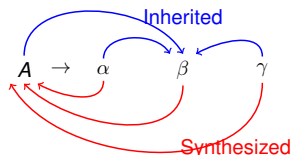
- ❑ Syntax Directed Definitions (SDD):
 1. Set of **attributes** attached to each grammar symbol
 2. Set of **semantic rules** attached to each production
 - Semantic rules define values of attributes

- ❑ Attribute Grammar:
 - An SDD where rules depend only on other attributes (i.e. An SDD that does not rely on any side-effects)
 - Think of it as a "grammar" for semantic analysis

- ❑ Example: let's say we want to define type checking
 - SDD can have semantic rules to access a symbol table
 - Attribute grammar must transmit type info through attributes

Synthesized vs. Inherited Attributes

 Semantic rule:



SDD has rule of the form for each CFG production

$$b = f(c_1, c_2, \dots, c_n)$$

either

1. If b is a synthesized attribute of A ,
 c_i ($1 \leq i \leq n$) are attributes of grammar symbols of its Right Hand Side (RHS); or
2. If b is an inherited attribute of one of the symbols of RHS,
 c_i 's are attribute of A and/or other symbols on the RHS

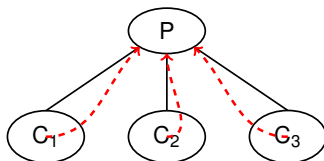
Synthesized vs. Inherited Attributes

❑ **Synthesized attributes:** computed from children nodes

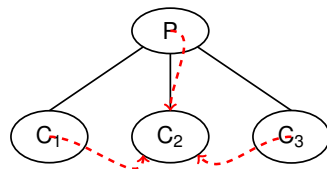
➤ $P.\text{synthesized_attr} = f(C_1.\text{attr}, C_2.\text{attr}, C_3.\text{attr})$

❑ **Inherited attributes:** computed from sibling/parent nodes

➤ $C_3.\text{inherited_attr} = f(P_1.\text{attr}, C_1.\text{attr}, C_3.\text{attr})$



Synthesized attribute



Inherited attribute

Synthesized Attribute Example

Example

- Each non-terminal symbol is associated with **val** attribute
- The **val** attribute is computed solely from children attributes

[Grammar Rules]

$L \rightarrow E$

$E \rightarrow E_1 + T$

$E \rightarrow T$

$T \rightarrow T_1 * F$

$T \rightarrow F$

$F \rightarrow (E)$

$F \rightarrow \text{digit}$

[Semantic Rules]

$\text{print}(E.\text{val})$

$E.\text{val} = E_1.\text{val} + T.\text{val}$

$E.\text{val} = T.\text{val}$

$T.\text{val} = T_1.\text{val} * F.\text{val}$

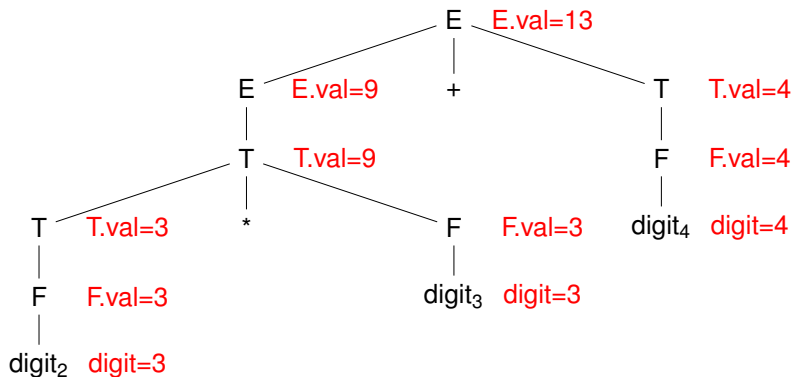
$T.\text{val} = F.\text{val}$

$F.\text{val} = E.\text{val}$

$F.\text{val} = \text{digit}.\text{lexval}$

Synthesized Attribute Example: Attribute Parse Tree

 **Attribute parse tree:** Parse tree decorated with attributes



Inherited Attribute Example

Example:

- T.type: synthesized attribute
- L.in: inherited attribute
- id.type: inherited attribute

[Grammar Rules]

$D \rightarrow T L$

$T \rightarrow \text{int}$

$T \rightarrow \text{real}$

$L \rightarrow L_1, \text{id}$

$L \rightarrow \text{id}$

[Semantic Rules]

$L.in = T.type$

$T.type = \text{integer}$

$T.type = \text{real}$

$L_1.in = L.in, \text{id.type} = L.in$

$\text{id.type} = L.in$

Why is L.in an inherited attribute?

- L.in is computed from a sibling T.type
- $L_1.in$ is computed from a parent L.in

SDD Implementation

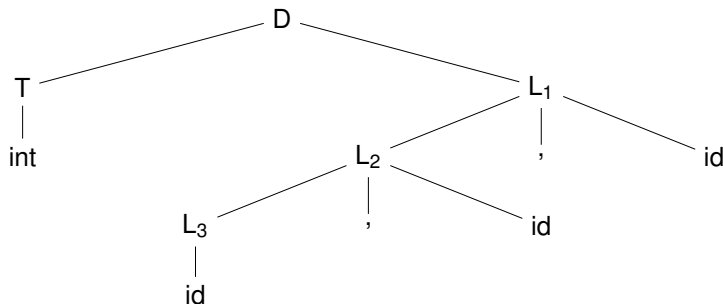
SDD Implementation using Parse Trees

- ❑ Assumes a previous parse stage
 - Input: a parse tree with no attribute annotations
 - Output: an attribute parse tree

- ❑ Goal: compute attribute values from leaf token values
 - Traverse in some order, apply semantic rules at each node
 - Traversal order must consider attribute dependencies

Dependency Graph

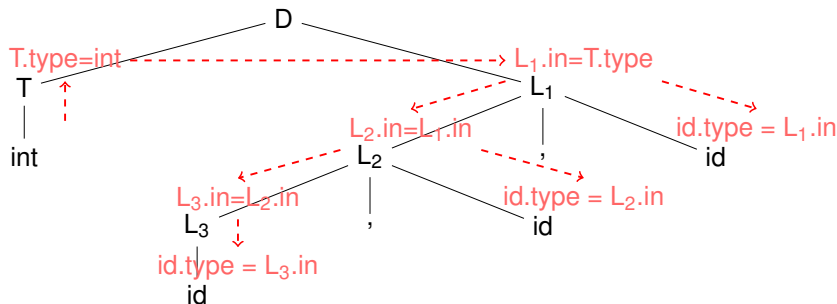
- Directed graph where edges are attribute dependencies
 - "To" attribute is computed base on "from" attribute
 - Must be **acyclic** such that there exists "a" traversal order



Dependency Graph

□ Directed graph where edges are attribute dependencies

- "To" attribute is computed base on "from" attribute
- Must be **acyclic** such that there exists "a" traversal order



SDD Implementation using SDT

- ❑ Syntax Directed Translation (SDT)
 - Applying semantic rules as part of syntax analysis (parsing)
 - Does NOT assume a pre-existing parse tree

- ❑ Syntax Directed Translation Scheme (SDTS)
 - A "scheme" or plan to perform SDT
 - A grammar specification embedded with **semantic actions**
 - Specific to choice of parser (top-down or bottom-up)

An SDTS is specific to choice of parser

□ Semantic action:

- Code between curly braces embedded into RHS
- Executed “at that point” in the RHS
 - Top-down: After previous symbol has been fully matched
 - Bottom-up: After previous symbol has been pushed to stack (when the 'dot' reaches the semantic action)

□ Example: Type declaration

- Given the following SDD:
 $L \rightarrow L_1, id \quad L_1.in = L.in, id.type = L.in$
- SDTS for top-down parser:
 $L \rightarrow \{L_1.in=L.in\} L_1, \{id.type=L.in\} id$
 - Doing $\{L_1.in=L.in\}$ before L_1 is expanded allows type attribute to flow down L_1 tree, when it is eventually expanded
- Using above SDTS for a bottom-up parser is not feasible
 - Symbol L is not on the stack when semantic actions are run
 - Don't know whether RHS is the handle until 'dot' reaches end (Hence cannot perform semantic actions in middle of RHS)

What are the dependencies allowed in SDTS?

- ❑ Parse trees: dependencies only required to be acyclic
- ❑ What is required of dependencies for SDTS?
 - Different parsing schemes see nodes in different orders
 - Top-down parsing — LL(k) parsing
 - Bottom-up parsing — LR(k) parsing
 - What if dependent node has not been seen yet?
- ❑ **L-Attributed Grammars:**
 - Short for Left-Attributed Grammar
 - Class of SDDs where LL(k) and LR(k) SDTS is feasible

Left-Attributed Grammar

■ An SDD is L-attributed if each of its attributes is either:

- a synthesized attribute of A in $A \rightarrow X_1 \dots X_n$,

or

- an inherited attribute of X_j in $A \rightarrow X_1 \dots X_n$ that
 - depends on attributes of siblings to its left i.e. $X_1 \dots X_{j-1}$
 - and/or depends on parent A

■ Evaluation order amenable to LL(k) and LR(k) parsing

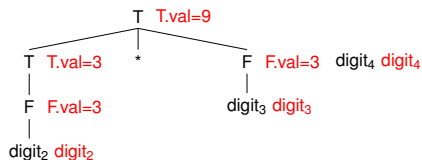
- All attribute values originate from token values
- L-Attributed Grammar dependencies flow from left to right
 - No attributes depend on (unscanned) tokens to the right
- There's a way to compute an attribute from scanned tokens

Syntax Directed Translation Scheme (SDTS)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is natural and easy to evaluate synthesized attributes



parsing stack:

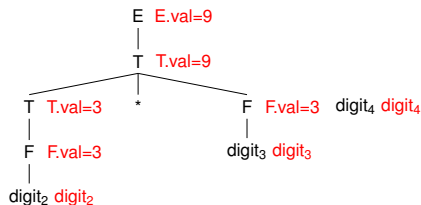
S ₇	T	T.val=9
S ₇	\$	-

(state) (symbol) (attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

□ it is natural and easy to evaluate synthesized attributes



parsing stack:

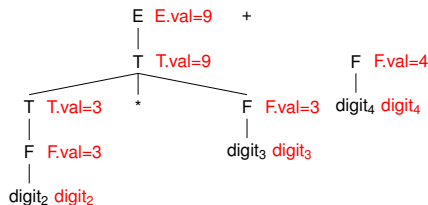
S ₇	E	E.val=9
S ₇	\$	-

(state) (symbol) (attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

□ it is natural and easy to evaluate synthesized attributes



parsing stack:

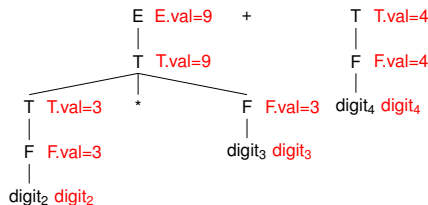
S?	F	F.val=4
S?	+	-
S?	E	E.val=9
S?	\$	-

(state) (symbol) (attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes



parsing stack:

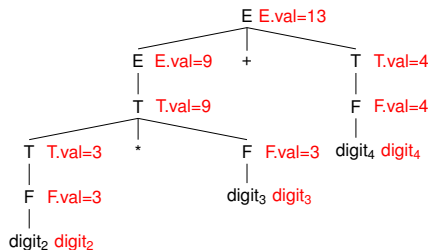
S?	T	T.val=4
S?	+	-
S?	E	E.val=9
S?	\$	-

(state) (symbol) (attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes



parsing stack:

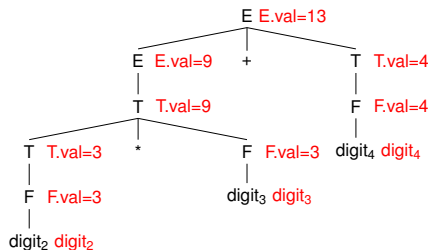
S ₇	E	E.val=13
S ₇	\$	-

(state) (symbol) (attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is natural and easy to evaluate synthesized attributes



parsing stack:

S ₇	E	E.val=13
S ₇	\$	-

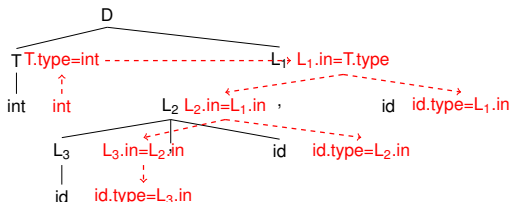
(state) (symbol) (attribute)

- Grammars with only synthesized attributes are called **S-Attributed Grammars**

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes



parsing stack:

$S_?$	$\$$	-

(state) (symbol) (attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

□ it is **not natural** to evaluate inherited attributes

```

int          ,          id
          ,          id
id
  
```

parsing stack:

S?	\$	-

(state) (symbol) (attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

 it is **not natural** to evaluate inherited attributes

T T.type=int

|

int

↑

int

,

id

,

id

id

parsing stack:

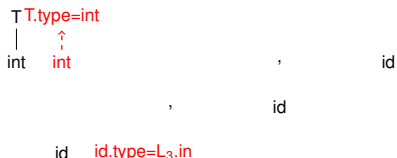
S?	T	T.type=int
S?	\$	-

(state) (symbol) (attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes



parsing stack:

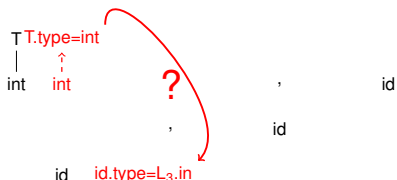
S?	id	id.type=L ₃ .in
S?	T	T.type=int
S?	\$	-

(state) (symbol) (attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

□ it is **not natural** to evaluate inherited attributes



parsing stack:

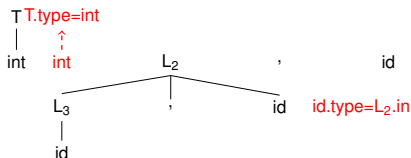
S _?	id	id.type=L ₃ .in
S _?	T	T.type=int
S _?	\$	-

(state) (symbol) (attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

it is **not natural** to evaluate inherited attributes



parsing stack:

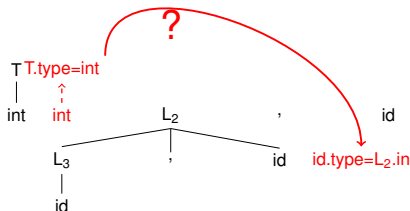
$S_?$	id	$id.type=L_2.in$
$S_?$,	
$S_?$	L_3	$L_3.in=L_2.in$
$S_?$	T	$T.type=int$
$S_?$	$\$$	-

(state) (symbol) (attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

□ it is **not natural** to evaluate inherited attributes




parsing stack:

S _?	id	id.type=L ₂ .in
S _?	,	
S _?	L ₃	L ₃ .in=L ₂ .in
S _?	T	T.type=int
S _?	\$	-


(state) (symbol) (attribute)

Evaluating Inherited Attributes using LR

 **Claim:** Given an L-Attributed grammar, inherited attributes needed for the computation are already on the stack

 Recall: What is an L-Attributed grammar?

- May have synthesized attributes
- May have inherited attributes but only from:
 - **Left** sibling attributes
 - Parent attribute

 Finding inherited attributes on the stack

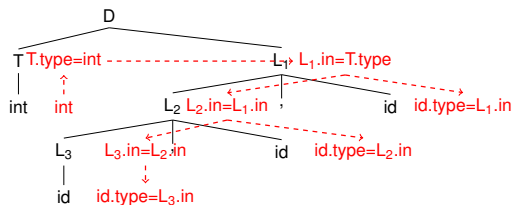
- Left siblings: previously reduced, so already on the stack
- Parent: not yet reduced, but left siblings of the parent used to compute the parent attribute are on the stack

$$D \rightarrow T \quad L$$

$$T \rightarrow \text{int} \quad \{T.\text{type}=\text{int}\}$$

$$T \rightarrow \text{real} \quad \{T.\text{type}=\text{real}\}$$

$$L \rightarrow L \ , \ \text{id} \quad \{\text{id}.\text{type}=\text{stack}[\text{top}-3].\text{type}\}$$

$$L \rightarrow \text{id} \quad \{\text{id}.\text{type}=\text{stack}[\text{top}-1].\text{type}\}$$


parsing stack:

S_7	\$	-

(state) (symbol) (attribute)

$$D \rightarrow T \mid L$$

$$T \rightarrow \text{int} \quad \{\text{T.type}=\text{int}\}$$

$$T \rightarrow \text{real} \quad \{\text{T.type}=\text{real}\}$$

$$L \rightarrow L, \text{id} \quad \{\text{id.type}=\text{stack}[\text{top}-3].\text{type}\}$$

$$L \rightarrow \text{id} \quad \{\text{id.type}=\text{stack}[\text{top}-1].\text{type}\}$$

int

,

id

,

id

id

parsing stack:

S ₇	\$	-

(state) (symbol) (attribute)

$$L \rightarrow \text{id} \quad \{\text{id.type} = \text{stack}[\text{top}-1].\text{type}\}$$

id

S _?	T	T.type=int
S _?	\$	-

A set of small navigation icons typically found in Beamer presentations, including symbols for back, forward, search, and other slide controls.

$$L \rightarrow \text{id} \quad \{\text{id.type} = \text{stack}[\text{top}-1].\text{type}\}$$

```
T.T.type=int
|      ^
int    int
```

S ₇	id	id.type=stack[top-1]
S ₇	T	T.type=int
S ₇	\$	-

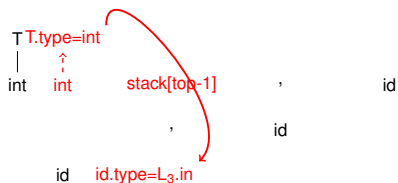
A set of small navigation icons typically found in Beamer presentations, including symbols for back, forward, search, and other slide controls.

$$D \rightarrow T \ L$$

$$T \rightarrow \text{int} \ \{T.\text{type}=\text{int}\}$$

$$T \rightarrow \text{real} \ \{T.\text{type}=\text{real}\}$$

$$L \rightarrow L \ , \ \text{id} \ \{\text{id}.\text{type}=\text{stack}[\text{top}-3].\text{type}\}$$

$$L \rightarrow \text{id} \ \{\text{id}.\text{type}=\text{stack}[\text{top}-1].\text{type}\}$$


parsing stack:

$S_?$	id	$\text{id}.\text{type}=\text{stack}[\text{top}-1]$
$S_?$	T	$T.\text{type}=\text{int}$
$S_?$	\$	-

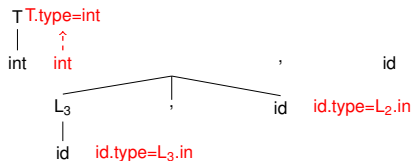
(state) (symbol) (attribute)

$$D \rightarrow T \mid L$$

$$T \rightarrow \text{int} \quad \{\text{T.type}=\text{int}\}$$

$$T \rightarrow \text{real} \quad \{\text{T.type}=\text{real}\}$$

$$L \rightarrow L \mid \text{id} \quad \{\text{id.type}=\text{stack}[\text{top}-3].\text{type}\}$$

$$L \rightarrow \text{id} \quad \{\text{id.type}=\text{stack}[\text{top}-1].\text{type}\}$$


parsing stack:

S _?	id	id.type=stack[top-3]
S _?	,	
S _?	L ₃	L ₃ .in=int
S _?	T	T.type=int
S _?	\$	-

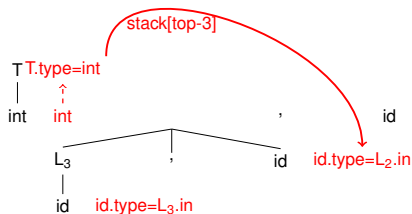
(state) (symbol) (attribute)

$$D \rightarrow T \ L$$

$$T \rightarrow \text{int} \ \{T.\text{type}=\text{int}\}$$

$$T \rightarrow \text{real} \ \{T.\text{type}=\text{real}\}$$

$$L \rightarrow L \ , \ \text{id} \ \{\text{id.type}=\text{stack}[\text{top}-3].\text{type}\}$$

$$L \rightarrow \text{id} \ \{\text{id.type}=\text{stack}[\text{top}-1].\text{type}\}$$


parsing stack:

S _?	id	id.type=stack[top-3]
S _?	,	
S _?	L ₃	L ₃ .in=int
S _?	T	T.type=int
S _?	\$	-

(state) (symbol) (attribute)

Marker

- Given the following SDD, where $|\alpha| \neq |\beta|$

$$A \rightarrow X \alpha Y \mid X \beta Y$$

$$Y \rightarrow \gamma \{ \dots = f(X.s) \}$$
- Problem: cannot generate stack location for $X.s$ since X is at different relative stack locations from Y
- Solution: introduce *markers* M_1 and M_2 that are at the same relative stack locations from Y

$$A \rightarrow X \alpha M_1 Y \mid X \beta M_2 Y$$

$$Y \rightarrow \gamma \{ \dots = f(M_{12}.s) \}$$

$$M_1 \rightarrow \varepsilon \{ M_1.s = X.s \}$$

$$M_2 \rightarrow \varepsilon \{ M_2.s = X.s \}$$

(M_{12} = the stack location of M_1 or M_2 , which are identical)
- A marker intuitively marks a stack location that is equidistant from the reduced non-terminal

Example

□ When is a marker necessary and how is it added?

Example 1:

$$\begin{aligned} S &\rightarrow a A \{ C.i = A.s \} C \\ S &\rightarrow b A B \{ C.i = A.s \} C \\ C &\rightarrow c \{ C.s = f(C.i) \} \end{aligned}$$

Solution:

$$\begin{aligned} S &\rightarrow a A \{ C.i = A.s \} C \\ S &\rightarrow b A B \{ M.i=A.s \} M \{ C.i = M.s \} C \\ C &\rightarrow c \{ C.s = f(C.i) \} \\ M &\rightarrow \varepsilon \{ M.s = M.i \} \end{aligned}$$

That is:

$$\begin{aligned} S &\rightarrow a A C \\ S &\rightarrow b A B M C \\ C &\rightarrow c \{ C.s = f(\text{stack}[\text{top}-1]) \} \\ M &\rightarrow \varepsilon \{ M.s = \text{stack}[\text{top}-2] \} \end{aligned}$$

When and how to add a marker

1. Identify the stack offset(s) to find the desired attribute
2. If stack offsets are different, add a marker
3. Add marker where it would result in uniform stack offsets

Example:

$S \rightarrow a \ A \ B \ C \ E \ D$

$S \rightarrow b \ A \ F \ B \ C \ F \ D$

$C \rightarrow c \ \{ /* \ C.s = f(A.s) \ */ \}$

$D \rightarrow d \ \{ /* \ D.s = f(B.s) \ */ \}$

Answer


$S \rightarrow a A B C E D$


$S \rightarrow b A D M B C F D$

$C \rightarrow c \text{ /* C.s = f(stack[top-2]) */}$

$D \rightarrow d \text{ /* D.s = f(stack[top-3]) */}$

$M \rightarrow \varepsilon \text{ /* M.s = f(stack[top-2]) */}$

 Regarding C.s, from stack[top-2], and stack[top-3]
.... add a Marker

 Regarding D.s, always from stack[top-2]
... no need to add

 How about Top-Down Parsing?

Translation Scheme for Top-Down Parsing

❑ Recursive Descent Parsers: Straightforward

➤ Synthesized Attribute

- Say function for non-terminal returns synthesized attribute
- Compute attribute from children function call return values

➤ Inherited Attribute

- Pass as argument to function call for inheriting non-terminal
- Left sibling attributes: left sibling calls already complete
- Parent attributes: passed in as arguments to parent function

❑ How about table-driven LL parsers?

Translation Scheme for LL Parsing

□ it is natural to evaluate inherited attributes

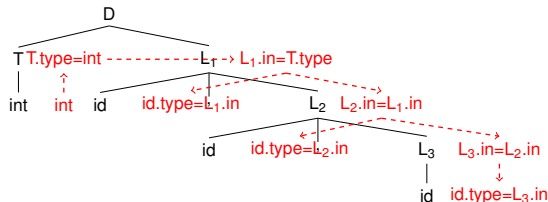
$D \rightarrow T \{L.in = T.type\} L$

$T \rightarrow \text{int} \{T.type = \text{int}\}$

$T \rightarrow \text{real} \{T.type = \text{real}\}$

$L \rightarrow \{id.type = L.in\} id, \{L_1.in = L.in\} L_1$

$L \rightarrow \{id.type = L.in\} id$



parsing stack:

(symbol) (attribute)

Translation Scheme for LL Parsing

□ it is natural to evaluate inherited attributes

$D \rightarrow T \{L.in = T.type\} L$

$T \rightarrow \text{int} \{T.type = \text{int}\}$

$T \rightarrow \text{real} \{T.type = \text{real}\}$

$L \rightarrow \{id.type = L.in\} id, \{L_1.in = L.in\} L_1$

$L \rightarrow \{id.type = L.in\} id$

D

parsing stack:

D	

(symbol) (attribute)

Translation Scheme for LL Parsing

□ it is natural to evaluate inherited attributes

$D \rightarrow T \{L.in=T.type\} L$

$T \rightarrow \text{int} \{T.type=\text{int}\}$

$T \rightarrow \text{real} \{T.type=\text{real}\}$

$L \rightarrow \{id.type=L.in\} id, \{L_1.in=L.in\} L_1$

$L \rightarrow \{id.type=L.in\} id$



parsing stack:

T	T.type=int
	{L ₁ .in=T.type}
L	L ₁ .in=???

(symbol) (attribute)

Translation Scheme for LL Parsing

□ it is natural to evaluate inherited attributes

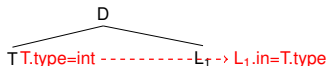
$D \rightarrow T \{L.in = T.type\} L$

$T \rightarrow \text{int} \{T.type = \text{int}\}$

$T \rightarrow \text{real} \{T.type = \text{real}\}$

$L \rightarrow \{id.type = L.in\} id, \{L_1.in = L.in\} L_1$

$L \rightarrow \{id.type = L.in\} id$



parsing stack:

	$\{L_1.in = \text{int}\}$
L	$L_1.in = ???$

(symbol) (attribute)

Translation Scheme for LL Parsing

□ it is natural to evaluate inherited attributes

$D \rightarrow T \{L.in = T.type\} L$

$T \rightarrow \text{int} \{T.type = \text{int}\}$

$T \rightarrow \text{real} \{T.type = \text{real}\}$

$L \rightarrow \{id.type = L.in\} id, \{L_1.in = L.in\} L_1$

$L \rightarrow \{id.type = L.in\} id$



parsing stack:

	$L_1.in = \text{int}$

(symbol) (attribute)

Translation Scheme for LL Parsing

□ it is natural to evaluate inherited attributes

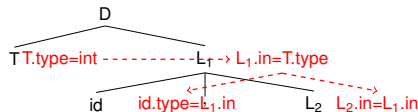
$D \rightarrow T \{L.in = T.type\} L$

$T \rightarrow \text{int} \{T.type = \text{int}\}$

$T \rightarrow \text{real} \{T.type = \text{real}\}$

$L \rightarrow \{id.type = L.in\} id, \{L_1.in = L.in\} L_1$

$L \rightarrow \{id.type = L.in\} id$



parsing stack:

	{id.type=L ₁ .in}
id	id.type=???
,	
	{L ₂ .in=L ₁ .in}
L ₂	L ₂ .in=???

(symbol) (attribute)

Translation Scheme for LL Parsing

□ it is natural to evaluate inherited attributes

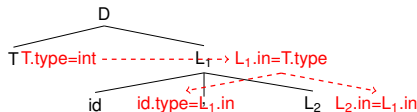
$D \rightarrow T \{L.in = T.type\} L$

$T \rightarrow \text{int} \{T.type = \text{int}\}$

$T \rightarrow \text{real} \{T.type = \text{real}\}$

$L \rightarrow \{id.type = L.in\} id, \{L_1.in = L.in\} L_1$

$L \rightarrow \{id.type = L.in\} id$



parsing stack:

	{id.type=int}
id	id.type=???
,	
	{L ₂ .in=int}
L ₂	L ₂ .in=???

(symbol) (attribute)

Translation Scheme for LL Parsing

□ it is natural to evaluate inherited attributes

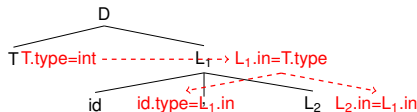
$D \rightarrow T \{L.in = T.type\} L$

$T \rightarrow \text{int} \{T.type = \text{int}\}$

$T \rightarrow \text{real} \{T.type = \text{real}\}$

$L \rightarrow \{id.type = L.in\} id, \{L_1.in = L.in\} L_1$

$L \rightarrow \{id.type = L.in\} id$



parsing stack:

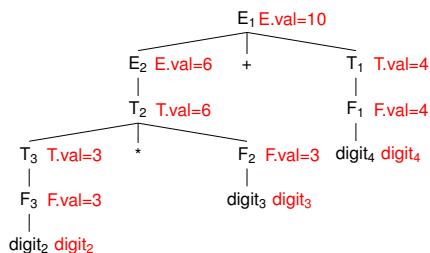
	{id.type=int}
id	id.type=???
,	
	{L2.in=int}
L2	L2.in=???

(symbol) (attribute)

□ Semantic actions on the stack are called **action-records**.

Translation Scheme for LL Parsing

it is **not natural** to evaluate synthesized attributes



parsing stack:

(symbol)	(attribute)

Translation Scheme for LL Parsing

□ it is **not natural** to evaluate synthesized attributes

E_1

parsing stack:

E_1	

(symbol) (attribute)

Translation Scheme for LL Parsing

□ it is **not natural** to evaluate synthesized attributes

E_1 $E.val=?$

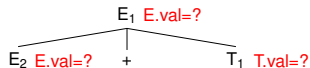
parsing stack:

E_1	$E_1.val=?$

(symbol) (attribute)

Translation Scheme for LL Parsing

it is **not natural** to evaluate synthesized attributes



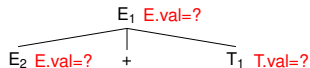
parsing stack:

E ₂	E ₂ .val=?
+	
T ₁	T ₁ .val=?

(symbol) (attribute)

Translation Scheme for LL Parsing

it is **not natural** to evaluate synthesized attributes



parsing stack:

E ₂	E ₂ .val=?
+	
T ₁	T ₁ .val=?

(symbol) (attribute)

Translation Scheme for LL Parsing

□ it is **not natural** to evaluate synthesized attributes

E_1

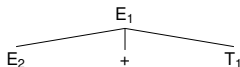
parsing stack:

E_1	
$E_1.val$???

(symbol) (attribute)

Translation Scheme for LL Parsing

it is **not natural** to evaluate synthesized attributes



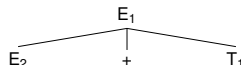
parsing stack:

E ₂	
E ₂ .val	???
+	
T ₁	
T ₁ .val	???
E ₁ .val	E ₂ .val + T ₁ .val

(symbol) (attribute)

Translation Scheme for LL Parsing

- it is **not natural** to evaluate synthesized attributes



parsing stack:

E_2	
$E_2.val$???
$+$	
T_1	
$T_1.val$???
$E_1.val$	$E_2.val + T_1.val$

(symbol) (attribute)

- Synthesized attributes on the stack: **synthesize-records**.
(Inserted below non-terminal with synthesized attribute)
- In synthesize-record $E_1.val = E_2.val + T_1.val$,
 $E_2.val$ and $T_1.val$ are place holders for pending values.
(Updated when records $E_2.val$ and $T_1.val$ are popped.)