

Syntax Analysis

Syntax Analysis is the second phase of compilation

❑ Comparison with lexical analysis:

Phase	Input	Output
Lexer	string of characters	string of tokens
Parser	string of tokens	Parse tree/AST

❑ Syntax analysis is also called **parsing**

- Because it produces a parse tree.
- AST (Abstract Syntax Tree) is a simplified parse tree.

What is a Parse Tree?

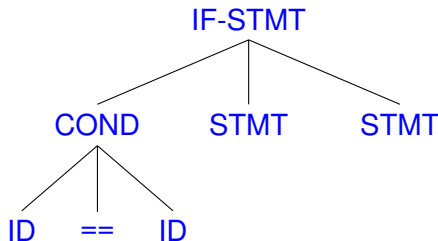
- ❑ **Parse tree:** a tree that represents grammatical structure
- ❑ Language constructs often have recursive structures

If-stmt \equiv **if** (EXPR) **then Stmt else Stmt fi**

Stmt \equiv **If-stmt** | **While-stmt** | ...

A Parse Tree Example

- ❏ Code to be compiled:
... if x==y then else ... fi
- ❏ Lexer:
- ❏ Parser:
 - Input: sequence of tokens
... IF ID==ID THEN ... ELSE ... FI
 - Desired output:



REs cannot express recursive program constructs

□ Example of a recursive construct is matching parenthesis:

of "(" must equal # of ")"

✓ $(x+y)^*z$

✓ $((x+y)+y)^*z$

...

✓ $(\dots(((x+y)+y)+y)\dots)$

✗ $((x+y)+y)+y)^*z$

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✓ $(...(((x+y)+y)+y)...)z$

✗ $((x+y)+y)+y)^*z$

□ Can regular expressions express this construct?

- Recall $RL \equiv L(\text{Regular Expression}) \equiv L(\text{Finite Automata})$
- Boils down to whether an FA can accept this construct

RE/FA is Not Powerful Enough

Describe strings with pattern $[i]^i$ ($i \geq 1$)

REs are not suitable for Syntax Analysis

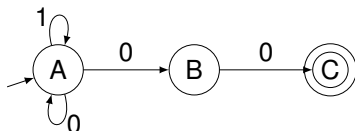
- ❑ REs cannot express recursive language constructs
- ❑ Programming languages belong to a category called CFLs
 - CFL is short for Context Free Language
 - CFLs are a strictly larger set than RLs
- ❑ To express CFLs, we need a new formalism: Grammars
- ❑ Grammars are general enough to express most languages
 - Regular Languages
 - Context Free Languages
 - Context Sensitive Languages
 - Recursively Enumerable Languages

A Grammar defines a Language

- ❑ A grammar, along with tokens, defines a language
 - Like how English grammar defines the English language
- ❑ Grammars are defined using rigorous math just like for REs
- ❑ Recall the following definitions
 - Language: A set of strings over alphabet
 - Alphabet: A finite set of symbols
 - Empty string: ϵ
- ❑ We will also start calling strings in the language **sentences**

An Example Grammar

- Language $L = \{ \text{any string with "00" at the end} \}$



- Grammar $G = \{ T, N, s, \delta \}$

where $T = \{ 0, 1 \}$, $N = \{ A, B \}$, $s = A$, and
production rules $\delta = \{ A \rightarrow 0A \mid 1A \mid 0B, B \rightarrow 0 \}$

- Derivation:** from grammar to language

- $A \Rightarrow 0A \Rightarrow 00B \Rightarrow 000$
- $A \Rightarrow 1A \Rightarrow 10B \Rightarrow 100$
- $A \Rightarrow 0A \Rightarrow 00A \Rightarrow 000B \Rightarrow 0000$
- $A \Rightarrow 0A \Rightarrow 01A \Rightarrow \dots$

Grammar, formally defined

- ❏ A **grammar** consists of 4 components (**T**, **N**, **S**, δ)
- **T** — set of **terminal** symbols
 - Leaves in the parse tree — essentially tokens
 - **N** — set of **non-terminal** symbols
 - Internal nodes in the parse tree that expands into tokens
 - Language construct composed of one or more tokens like: statements, loops, functions, classes, ...
 - **S** — A special non-terminal **start symbol**
 - Every string in language is derived from it
 - δ — a set of **production** rules
 - “LHS \rightarrow RHS”: left-hand-side *produces* right-hand-side

Production Rule and Derivation

□ “LHS \rightarrow RHS”

- Production rule to replace LHS with RHS
- Applied repeatedly to derive sentence from **S**

□ $\beta \Rightarrow \alpha$: string β derives α

- $\beta \Rightarrow \alpha$ — 1 step
- $\beta \Rightarrow^* \alpha$ — 0 or more steps
- $\beta \Rightarrow^+ \alpha$ — 0 or more steps

➤ example:

$A \Rightarrow 0A \Rightarrow 00B \Rightarrow 000$

$A \Rightarrow^* 000$

$A \Rightarrow^+ 000$

Noam Chomsky Grammars

- Chomsky classified grammars into 4 types:

Type 0: recursive grammar

Type 1: context sensitive grammar

Type 2: context free grammar

Type 3: regular grammar

(Classification done based on form of production rules)

- The grammars produce the corresponding languages:

$L(\text{recursive grammar}) \equiv \text{recursively enumerable language}$

$L(\text{context sensitive grammar}) \equiv \text{context sensitive language}$

$L(\text{context free grammar}) \equiv \text{context free language}$

$L(\text{regular grammar}) \equiv \text{regular language}$

Type 0: Unrestricted/Recursive Grammar

□ Type 0 grammar — unrestricted or recursive grammar

➤ Form of rules

$$\alpha \rightarrow \beta$$

where $\alpha \in (N \cup T)^+$, $\beta \in (N \cup T)^*$

➤ No restrictions on form of grammar rules

➤ Example:

$$aAB \rightarrow aCD$$

$$aAB \rightarrow aB$$

$$A \rightarrow \varepsilon$$

; erase rule is allowed

Type 1: Context Sensitive Grammar

□ Type 1 grammar — context sensitive grammar

➤ Form of rules

$$\alpha A \beta \rightarrow \alpha \gamma \beta$$

where $A \in N$, $\alpha, \beta \in (N \cup T)^*$, $\gamma \in (N \cup T)^+$

➤ Replace A by γ only if found in the context of α and β

➤ No erase rule

➤ Example:

$$aAB \rightarrow aCB$$

Type 2: Context Free Grammar

□ Type 2 grammar — context free grammar

➤ Form of rules

$$A \rightarrow \gamma$$

where $A \in N$, $\gamma \in (N \cup T)^+$

➤ Can replace A by γ at any time — cannot specify context

Type 2: Context Free Grammar

□ Type 2 grammar — context free grammar

- Form of rules

$$A \rightarrow \gamma$$

where $A \in N$, $\gamma \in (N \cup T)^+$

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□ Are programming languages (PLs) context free ?

- Some PL constructs are context free: If-stmt, declaration
- Many are not: **def-before-use**, **matching formal/actual parameters**, etc.

Type 3: Regular Grammar

□ Type 3 grammar — regular grammar

➤ Form of rules

$$A \rightarrow \alpha, \text{ or } A \rightarrow \alpha B$$

where $A, B \in N, \alpha \in T$

➤ Regular grammar defines RE

➤ Can be used to define tokens for lexical analysis

➤ Example:

$$A \rightarrow 1A \mid 0$$

Differentiate Type 2 and 3 Grammars

Language $L1 = \{ [^i j^j \mid i, j \geq 1] \}$

➤ Regular grammar

$$\begin{aligned} S &\rightarrow [S \mid [T \\ T &\rightarrow] T \mid] \end{aligned}$$

Language $L2 = \{ [^i j^i \mid i \geq 1] \}$

➤ Context free grammar

$$S \rightarrow [S] \mid []$$

Differentiate Type 1 and 2 Grammars

□ Type 2 grammar (context free)

$$S \rightarrow D U$$
$$D \rightarrow \text{int } x; \mid \text{int } y;$$
$$U \rightarrow x=1; \mid y=1;$$

□ Type 1 grammar (context sensitive)

$$S \rightarrow D U$$
$$D \rightarrow \text{int } x; \mid \text{int } y;$$
$$\text{int } x; U \rightarrow \text{int } x; x=1;$$
$$\text{int } y; U \rightarrow \text{int } y; y=1;$$

Are Programming Languages Really Context Free?

Language from type 2 grammar

- $S \Rightarrow DU \Rightarrow \text{int } x; U \Rightarrow \text{int } x; x=1;$
- $S \Rightarrow DU \Rightarrow \text{int } x; U \Rightarrow \text{int } x; y=1;$
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Language from type 1 grammar

- $S \Rightarrow DU \Rightarrow \text{int } x; U \Rightarrow \text{int } x; x=1;$
- $S \Rightarrow DU \Rightarrow \text{int } y; U \Rightarrow \text{int } y; y=1;$

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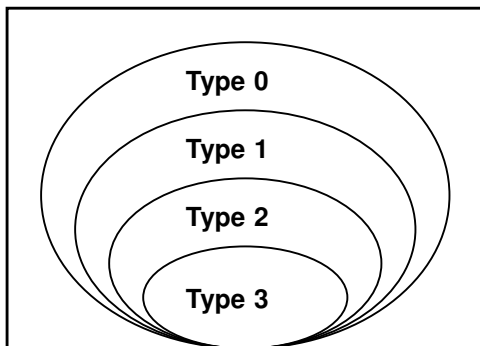
Language from type 1 grammar

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PLs are context sensitive, why use CFG in parsing?

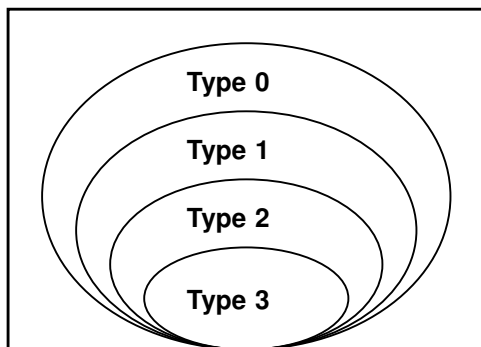
The Chomsky Hierarchy of Grammars

■ $RL \subset CFL \subset CSL \subset L(\text{Recursive Grammar})$



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■ However, $L_y \subset L_x$ where $L_x:[^i]^k$ —RG, $L_y:[^i]^i$ —CFG

➤ Is it a problem?

Context Free Grammars

Syntax Analysis is a process of derivation

- ❑ Grammar is used to derive string or construct parser
- ❑ A **derivation** is a sequence of applications of rules
 - Starting from the **start symbol**
 - $S \Rightarrow \dots \Rightarrow \dots \Rightarrow \dots \Rightarrow (\text{sentence})$
- ❑ **Leftmost** and **Rightmost** derivations
 - At each derivation step, **leftmost** derivation always replaces the leftmost non-terminal symbol
 - **Rightmost** derivation always replaces the rightmost one

Examples

$$E \rightarrow E * E \mid E + E \mid (E) \mid \text{id}$$

➤ leftmost derivation

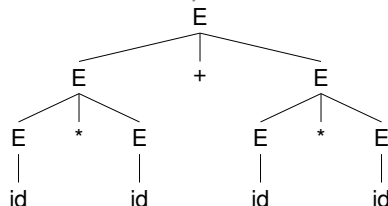
$$\begin{aligned} E &\Rightarrow E + E \Rightarrow E * E + E \Rightarrow \text{id} * E + E \Rightarrow \text{id} * \text{id} + E \Rightarrow \dots \\ &\Rightarrow \text{id} * \text{id} + \text{id} * \text{id} \end{aligned}$$

➤ rightmost derivation

$$\begin{aligned} E &\Rightarrow E + E \Rightarrow E + E * E \Rightarrow E + E * \text{id} \Rightarrow E + \text{id} * \text{id} \Rightarrow \dots \\ &\Rightarrow \text{id} * \text{id} + \text{id} * \text{id} \end{aligned}$$

A Parse Tree represent the Derivation

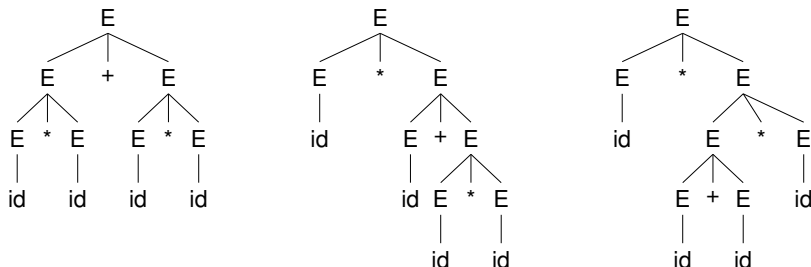
- This is the parse tree that represents both derivations:



- A parse tree
 - describes program structure (defined by the rules applied)
 - is agnostic of leftmost or rightmost derivation (as long as the same rules are applied in both)
- There are two types of nodes in a parse tree:
 - Leaves: terminals that form the sentence
 - Non-leaves: intermediate non-terminals in the derivation

Different Rules result in different Parse Trees

Application of different rules result in different parse trees:



Note: each parse tree has a unique leftmost derivation

- First: $E \Rightarrow E + E \Rightarrow E * E + E \Rightarrow id * E + E \Rightarrow id * id + E \Rightarrow \dots$
- Second: $E \Rightarrow E * E \Rightarrow id * E \Rightarrow id * E + E \Rightarrow id * id + E \Rightarrow \dots$
- Third: $E \Rightarrow E * E \Rightarrow id * E \Rightarrow id * E * E \Rightarrow id * E + E * E \Rightarrow \dots$

Same goes for rightmost derivations

Ambiguity

- ❑ A grammar G is **ambiguous** if
 - there exist a string $str \in L(G)$ such that
 - more than one parse tree derives str
 - \equiv there is more than leftmost derivation for str
 - \equiv there is more than rightmost derivation for str

- ❑ Grammars that produce multiple parse trees is a problem
 - Each parse tree is a different interpretation of program

- ❑ Likely, there is an unambiguous version of the grammar
 - That accepts the same programming language
 - Programming languages are rarely inherently ambiguous

Grammar can be rewritten to remove ambiguity

Method I: to specify **precedence**

- build precedence into grammar, have different non-terminal for each precedence level
 - Lower precedence — relatively higher in tree (close to root)
 - Higher precedence — relatively lower in tree (far from root)
 - Same precedence — depends on associativity

$E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid E \wedge E \mid (E) \mid id$

rewrite it to

$$E \rightarrow E + T \mid E - T \mid T$$
$$T \rightarrow T * F \mid T / F \mid F$$
$$F \rightarrow P \wedge F \mid P$$
$$P \rightarrow id \mid (E)$$

How to Remove Ambiguity?

❏ Method II: to specify **associativity**

- Allow recursion only on either left or right non-terminal
 - Left associative — recursion on left non-terminal
 - Right associative — recursion on right non-terminal

❏ For the previous example,

$E \rightarrow E + E \dots$; allows both left/right associativity

rewrite it to

$E \rightarrow E + T \dots$; only left associativity

$F \rightarrow P \wedge F \dots$; only right associativity

Ambiguity is undecidable for CFGs

- ❑ Decidable: computable using a Turing Machine
- ❑ It is **decidable** if a string is in a context free language
 - Implementing a parser is feasible for every CFL
- ❑ It is **undecidable** if a CFG is ambiguous
 - Checking ambiguity at compile time is impossible
 - Can only be checked reliably at runtime for a given string
 - In practice, tools like Yacc check for a more restricted grammar (e.g. LALR(1)) instead
 - LALR(1) is a subset of unambiguous grammars
 - Can be done easily at compile time

The Two Outcomes of Parsing

- ❑ Outcome 1: Parser is able to derive input from grammar
 - Parser builds parse tree that represents the derivation

- ❑ Outcome 2: Parser is unable to derive input from grammar
 - Parser emits a syntax error with source code location

The Two Outcomes of Parsing

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- ❑ How would you write a parser that does both well?

Types of Parsers

❏ Universal parser

- Can parse any CFG e.g. Early's algorithm
- Powerful but extremely inefficient ($O(N^3)$ where N is length of string)

❏ Top-down parser

- Tries to *expand* start symbol to input string
- Finds leftmost derivation
- Only works for a certain class of grammars
- Starts from root and expands into leaves
- Parser structure closely mimics grammar
 - Amenable to implementation by hand

Types of Parsers (cont.)



Bottom-up parser

- Tries to *reduce* the input string to the start symbol
- Finds reverse order of the rightmost derivation
- Works for wider class of grammars
- Starts at leaves and build tree in bottom-up fashion
- More amenable to generation by an automated tool

What Output do We Want?

- ❑ The output of parsing is
 - parse tree, or
 - abstract syntax tree
- ❑ An **abstract syntax tree** is
 - similar to a parse tree but ignores some details
 - internal nodes may contain terminal symbols

An Example

- Consider the grammar

$$E \rightarrow \text{int} \mid (E) \mid E + E$$

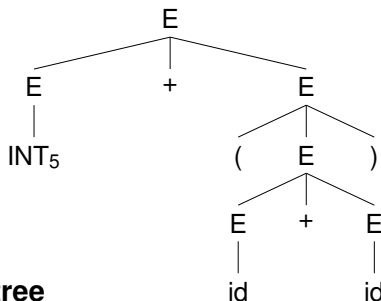
and an input

$$5 + (2 + 3)$$

- After lexical analysis, we have a sequence of tokens

$$\text{INT}_5 \text{ ' + ' } (\text{INT}_2 \text{ ' + ' INT}_3 \text{ ') '}$$

Parse Tree of the Input



A parse tree

- Traces the operation of the parser
- Does capture the nested structure

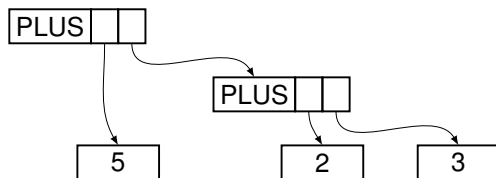


but contains too much information

- parentheses
- single-successor nodes

Abstract Syntax Tree

■ An **Abstract Syntax Tree (AST)** for the input



- **AST** also captures the nested structure
- **AST** abstracts from parse tree (a.k.a. concrete syntax tree)
- **AST** is more compact and contains only relevant info
- **ASTs** are used in most compilers rather than parse trees

How are ASTs Constructed?

- ❑ Through implementation of **semantic actions**
- ❑ We already used them in project 1 to return token tuples
- ❑ To construct AST, we attach an **attribute** to each symbol X
 - **$X.ast$** — the constructed AST for symbol X
- ❑ Extend each production rule with semantic actions, i.e.

$$X \rightarrow Y_1 Y_2 \dots Y_n \quad \{ \text{actions} \}$$

actions may define or use $X.ast$, $Y_i.ast$ ($1 \leq i \leq n$)

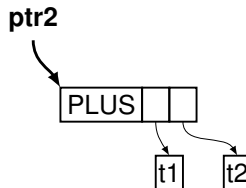
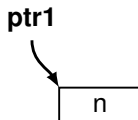
Example

For the previous example, we have

$E \rightarrow$	<code>int</code>	<code>{ E.ast = mkleaf(int.lval) }</code>
	<code> E1 + E2</code>	<code>{ E.ast = mkplus(E1.ast, E2.ast) }</code>
	<code> (E1)</code>	<code>{ E.ast = E1.ast }</code>

Here, we use two pre-defined functions

- `ptr1=mkleaf(n)` — create a leaf node and assign value “n”
- `ptr2=mkplus(t1, t2)` — create a tree node and assign the root value “PLUS”, and two subtrees as t1 and t2



AST Construction Steps

For input $\text{INT}_5 \text{ ' + ' (' INT}_2 \text{ ' + ' INT}_3 \text{ ') '}$

Construction order given is for a top-down LL(1) parser
(Order can change depending on parser implementation)

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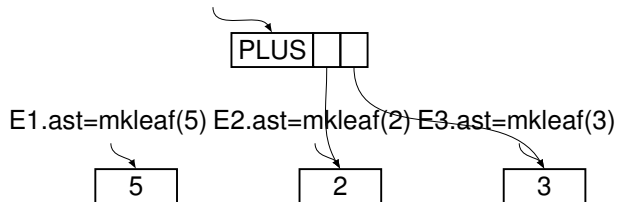


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$E4.\text{ast} = \text{mkplus}(E2.\text{ast}, E3.\text{ast})$



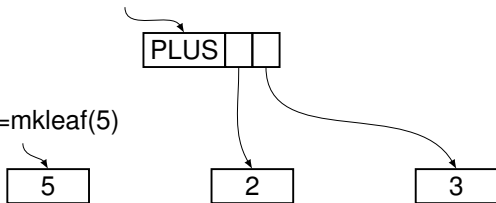
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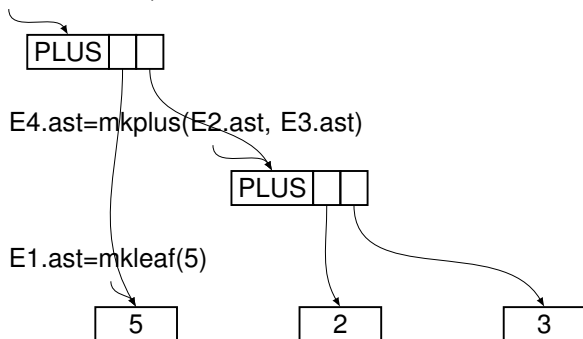


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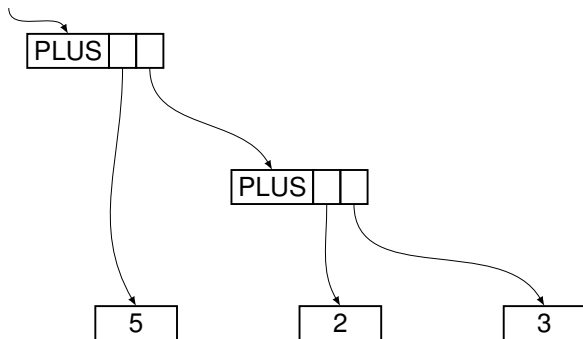


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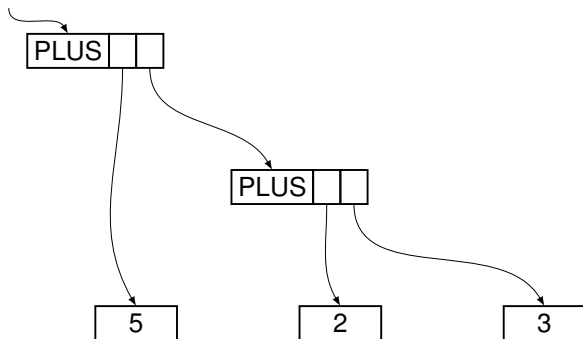


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Summary

- ❏ Compilers specify program structure using CFG
 - Most programming languages are not context free
 - Context sensitive analysis can easily separate out to semantic analysis phase

- ❏ A parser uses CFG to
 - ... answer if an input $str \in L(G)$
 - ... and build a parse tree
 - ... or build an AST instead
 - ... and pass it to the rest of compiler

Parsing

Parsing

- ❑ We will study two approaches
- ❑ Top-down
 - Easier to understand and implement manually
- ❑ Bottom-up
 - More powerful, can be implemented automatically

Example

Consider a CFG grammar G

$$S \rightarrow A B \quad A \rightarrow a C \quad B \rightarrow b D$$
$$D \rightarrow d \quad C \rightarrow c$$

Actually, this language has only one sentence, i.e.

$$L(G) = \{ acbd \}$$

Leftmost Derivation:

$S \Rightarrow AB$ (1)
 $\Rightarrow aCB$ (2)
 $\Rightarrow acB$ (3)
 $\Rightarrow acbD$ (4)
 $\Rightarrow acbd$ (5)

S

Rightmost Derivation:

$S \Rightarrow AB$ (5)
 $\Rightarrow AbD$ (4)
 $\Rightarrow Abd$ (3)
 $\Rightarrow aCbD$ (2)
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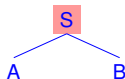
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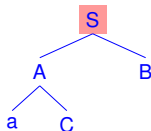
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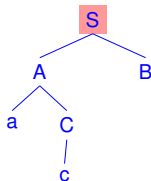
$$S \rightarrow AB \quad A \rightarrow aC \quad B \rightarrow bD$$
$$D \rightarrow d \quad C \rightarrow c$$

Actually, this language has only one sentence, i.e.

$$L(G) = \{acbd\}$$

Leftmost Derivation:

$S \Rightarrow AB$ (1)
 $\Rightarrow aCB$ (2)
 $\Rightarrow acB$ (3)
 $\Rightarrow acbD$ (4)
 $\Rightarrow acbd$ (5)



Rightmost Derivation:

$S \Rightarrow AB$ (5)
 $\Rightarrow AbD$ (4)
 $\Rightarrow Abd$ (3)
 $\Rightarrow aCbD$ (2)
 $\Rightarrow acbd$ (1)

Example

Consider a CFG grammar G

$$S \rightarrow AB \quad A \rightarrow aC \quad B \rightarrow bD$$
$$D \rightarrow d \quad C \rightarrow c$$

Actually, this language has only one sentence, i.e.

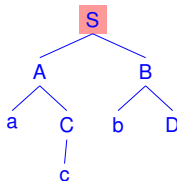
$$L(G) = \{acbd\}$$

Leftmost Derivation:

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Rightmost Derivation:

$S \Rightarrow AB$ (5)
 $\Rightarrow AbD$ (4)
 $\Rightarrow Abd$ (3)
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 $\Rightarrow acbd$ (1)



Example

Consider a CFG grammar G

$$S \rightarrow AB \quad A \rightarrow aC \quad B \rightarrow bD$$
$$D \rightarrow d \quad C \rightarrow c$$

Actually, this language has only one sentence, i.e.

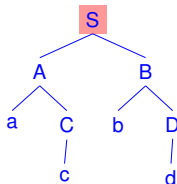
$$L(G) = \{acbd\}$$

Leftmost Derivation:

$S \Rightarrow AB$ (1)
 $\Rightarrow aCB$ (2)
 $\Rightarrow acB$ (3)
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Rightmost Derivation:

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Example

Consider a CFG grammar G

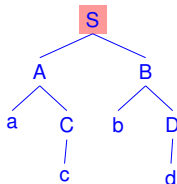
$$\begin{array}{lll} S \rightarrow AB & A \rightarrow aC & B \rightarrow bD \\ D \rightarrow d & C \rightarrow c & \end{array}$$

Actually, this language has only one sentence, i.e.

$$L(G) = \{ acbd \}$$

Leftmost Derivation:

$S \Rightarrow AB$ (1)
 $\Rightarrow aCB$ (2)
 $\Rightarrow acB$ (3)
 $\Rightarrow acbD$ (4)
 $\Rightarrow acbd$ (5)



Rightmost Derivation:

$S \Rightarrow AB$ (5)
 $\Rightarrow AbD$ (4)
 $\Rightarrow Abd$ (3)
 $\Rightarrow aCbd$ (2)
 $\Rightarrow acbd$ (1)

a c b d

Example

Consider a CFG grammar G

$$S \rightarrow AB \quad A \rightarrow aC \quad B \rightarrow bD$$

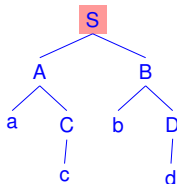
$$D \rightarrow d \quad C \rightarrow c$$

Actually, this language has only one sentence, i.e.

$$L(G) = \{acbd\}$$

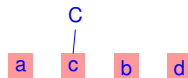
Leftmost Derivation:

$S \Rightarrow AB$ (1)
 $\Rightarrow aCB$ (2)
 $\Rightarrow acB$ (3)
 $\Rightarrow acbD$ (4)
 $\Rightarrow acbd$ (5)



Rightmost Derivation:

$S \Rightarrow AB$ (5)
 $\Rightarrow AbD$ (4)
 $\Rightarrow Abd$ (3)
 $\Rightarrow aCbD$ (2)
 $\Rightarrow acbd$ (1)



Example

Consider a CFG grammar G

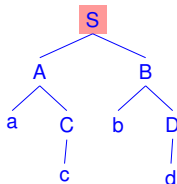
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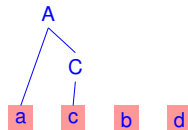
Leftmost Derivation:

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Rightmost Derivation:

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Example

Consider a CFG grammar G

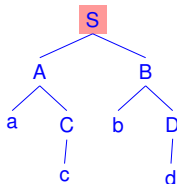
$S \rightarrow AB$ $A \rightarrow aC$ $B \rightarrow bD$
 $D \rightarrow d$ $C \rightarrow c$

Actually, this language has only one sentence, i.e.

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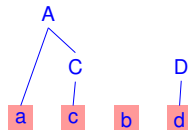
Leftmost Derivation:

$S \Rightarrow AB$ (1)
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Example

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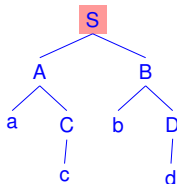
$S \rightarrow AB$ $A \rightarrow aC$ $B \rightarrow bD$
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Actually, this language has only one sentence, i.e.

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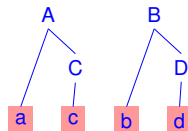
Leftmost Derivation:

$S \Rightarrow AB$ (1)
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Example

Consider a CFG grammar G

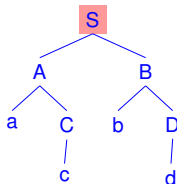
$S \rightarrow AB$ $A \rightarrow aC$ $B \rightarrow bD$
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Actually, this language has only one sentence, i.e.

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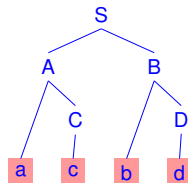
Leftmost Derivation:

$S \Rightarrow AB$ (1)
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Rightmost Derivation:

$S \Rightarrow AB$ (5)
 $\Rightarrow AbD$ (4)
 $\Rightarrow Abd$ (3)
 $\Rightarrow aCbd$ (2)
 $\Rightarrow acbd$ (1)



Top Down Parsers

Backtracking or Predictive?

- How does a parser choose between production rules?
 - Given $A \rightarrow \alpha|\beta$, expand A to α or β ?

- Backtracking parser: exhaustively tries all rules
 - When input mismatch, backtrack to alternative rule
 - Con Non-linear time due to exhaustive search
 - Con Complex to roll back semantic actions on backtrack
 - Pro Can parse most CFGs (except left-recursion)

- Predictive parser: predict correct rule using lookahead
 - Looks ahead k input symbols to make prediction
 - Con Can parse only a subset of CFGs (dependent on k)
 - Pro Linear time as only correct derivations are done
 - Pro Simple structure as there is no need to backtrack

- Parsers can be backtracking or predictive (or both).

Recursive Descent or Table Driven?

- ❑ How is the parser implementation done?
 - Hand-coded parsers are typically recursive descent
 - Auto-generated parsers are table driven

- ❑ Recursive descent parser: each non-terminal is a function
 - Function is in charge of expanding non-terminal
 - Descends parse tree via recursive calls to non-terminals
 - Hand-written but easier to customize and control
 - Typically uses backtracking rather than prediction

- ❑ Table driven parser: uses a table of predictions
 - Similar to lexer, uses a table to decide on next production
 - Table indexed by non-terminal and k lookahead symbols
 - Similar to lexer, table can be generated from grammar
 - Always predictive but can use backtracking if needed

Backtracking Example

$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

input string: `int * int`

start symbol: `E`

initial parse tree is `E`

Backtracking Example

$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

input string: `int * int`

start symbol: `E`

initial parse tree is `E`

 Assume: when there are alternative rules, try right rule first

Parsing Sequence using Backtracking

E

Parsing Sequence using Backtracking

$E \Rightarrow T$

– pick right most rule $E \rightarrow T$

Parsing Sequence using Backtracking

$$E \Rightarrow T \Rightarrow (E)$$

- pick right most rule $E \rightarrow T$
- pick right most rule $T \rightarrow (E)$

Parsing Sequence using Backtracking

$E \Rightarrow T \Rightarrow (E)$

- pick right most rule $E \rightarrow T$
- pick right most rule $T \rightarrow (E)$
- “(” does not match “int”

Parsing Sequence using Backtracking

$E \Rightarrow T \Rightarrow (E)$

- pick right most rule $E \rightarrow T$
- pick right most rule $T \rightarrow (E)$
- “(” does not match “int”
- failure, backtrack one level

Parsing Sequence using Backtracking

$E \Rightarrow T \Rightarrow \cancel{(E)}$

- pick right most rule $E \rightarrow T$
- pick right most rule $T \rightarrow (E)$
- “(” does not match “int”
- failure, backtrack one level

Parsing Sequence using Backtracking

$E \Rightarrow T \Rightarrow \cancel{(E)}$

$\Rightarrow \text{int}$

- pick right most rule $E \rightarrow T$
- pick right most rule $T \rightarrow (E)$
- “(” does not match “int”
- **failure, backtrack one level**
- pick $T \rightarrow \text{int}$
- “int” matches input “int”

Parsing Sequence using Backtracking

$E \Rightarrow T \Rightarrow (E)$

$\Rightarrow \text{int}$

- pick right most rule $E \rightarrow T$
- pick right most rule $T \rightarrow (E)$
- “(” does not match “int”
- failure, backtrack one level
- pick $T \rightarrow \text{int}$
- “int” matches input “int”
- however, we expect more tokens
- failure, backtrack one level

Parsing Sequence using Backtracking

$E \Rightarrow T \Rightarrow (E)$

$\Rightarrow \text{int}$

- pick right most rule $E \rightarrow T$
- pick right most rule $T \rightarrow (E)$
- “(” does not match “int”
- failure, backtrack one level
- pick $T \rightarrow \text{int}$
- “int” matches input “int”
- however, we expect more tokens
- failure, backtrack one level

Parsing Sequence using Backtracking

$E \Rightarrow T \Rightarrow (E)$

$\Rightarrow \text{int}$

$\Rightarrow \text{int} * T$

- pick right most rule $E \rightarrow T$
- pick right most rule $T \rightarrow (E)$
- “(” does not match “int”
- **failure, backtrack one level**
- pick $T \rightarrow \text{int}$
- “int” matches input “int”
- however, we expect more tokens
- **failure, backtrack one level**
- pick $T \rightarrow \text{int} * T$

Parsing Sequence using Backtracking

$$E \Rightarrow T \Rightarrow \cancel{(E)}$$

$$\Rightarrow \cancel{\text{int}}$$

$$\Rightarrow \text{int} * T \Rightarrow \text{int} * (E)$$

- pick right most rule $E \rightarrow T$
- pick right most rule $T \rightarrow (E)$
- “(” does not match “int”
- **failure, backtrack one level**
- pick $T \rightarrow \text{int}$
- “int” matches input “int”
- however, we expect more tokens
- **failure, backtrack one level**
- pick $T \rightarrow \text{int} * T$
- pick $T \rightarrow \text{int} * (E)$

Parsing Sequence using Backtracking

$$E \Rightarrow T \Rightarrow (E)$$

$$\Rightarrow \text{int}$$

$$\Rightarrow \text{int} * T \Rightarrow \text{int} * (E)$$

- pick right most rule $E \rightarrow T$
- pick right most rule $T \rightarrow (E)$
- “(” does not match “int”
- **failure, backtrack one level**
- pick $T \rightarrow \text{int}$
- “int” matches input “int”
- however, we expect more tokens
- **failure, backtrack one level**
- pick $T \rightarrow \text{int} * T$
- pick $T \rightarrow \text{int} * (E)$
- “(” does not match input “int”
- **failure, backtrack one level**

Parsing Sequence using Backtracking

$$E \Rightarrow T \Rightarrow (E)$$

$$\Rightarrow \text{int}$$

$$\Rightarrow \text{int} * T \Rightarrow \text{int} * (E)$$

- pick right most rule $E \rightarrow T$
- pick right most rule $T \rightarrow (E)$
- “(” does not match “int”
- **failure, backtrack one level**
- pick $T \rightarrow \text{int}$
- “int” matches input “int”
- however, we expect more tokens
- **failure, backtrack one level**
- pick $T \rightarrow \text{int} * T$
- pick $T \rightarrow \text{int} * (E)$
- “(” does not match input “int”
- **failure, backtrack one level**

Parsing Sequence using Backtracking

$$E \Rightarrow T \Rightarrow (E)$$

$$\Rightarrow \text{int}$$

$$\Rightarrow \text{int} * T \Rightarrow \text{int} * (E)$$

$$\Rightarrow \text{int} * \text{int}$$

- pick right most rule $E \rightarrow T$
- pick right most rule $T \rightarrow (E)$
- “(” does not match “int”
- **failure, backtrack one level**
- pick $T \rightarrow \text{int}$
- “int” matches input “int”
- however, we expect more tokens
- **failure, backtrack one level**
- pick $T \rightarrow \text{int} * T$
- pick $T \rightarrow \text{int} * (E)$
- “(” does not match input “int”
- **failure, backtrack one level**
- pick $T \rightarrow \text{int}$

Parsing Sequence using Backtracking

$$E \Rightarrow T \Rightarrow \cancel{(E)}$$

$$\Rightarrow \cancel{\text{int}}$$

$$\Rightarrow \text{int} * T \Rightarrow \cancel{\text{int} * (E)}$$

$$\Rightarrow \text{int} * \text{int}$$

- pick right most rule $E \rightarrow T$
- pick right most rule $T \rightarrow (E)$
- “(” does not match “int”
- **failure, backtrack one level**
- pick $T \rightarrow \text{int}$
- “int” matches input “int”
- however, we expect more tokens
- **failure, backtrack one level**
- pick $T \rightarrow \text{int} * T$
- pick $T \rightarrow \text{int} * (E)$
- “(” does not match input “int”
- **failure, backtrack one level**
- pick $T \rightarrow \text{int}$
- **match, accept**

Recursive Descent Parser with Backtracking

Recursive Descent Parsing Implementation

- ❑ When expanding a non-terminal, try all productions until
 - A production is found that generates a portion of the input, or
 - No production is found that generates a portion of the input, in which case backtrack to previous non-terminal

- ❑ Create a function for each non-terminal
 1. For RHS of each production rule,
 - a. For a terminal, match with input symbol and consume
 - b. For a non-terminal, call function for that non-terminal
 - c. If match succeeds for entire RHS, return success
 - d. If match fails, regurgitate input and try next RHS
 2. If match succeeds for any rule, apply that rule to LHS

- ❑ If entire input string matched with start symbol, success!

A Hand-coded Recursive Descent Parser

Sample implementation of parser for previous grammar:

$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

```
char fetchNext() {
    // Fetch one character
}
void regurgitate(int n) {
    // Unfetch n characters
}
bool expr() {
rule1:
    if(!term()) goto rule2;
    if (fetchNext()!=AddNum) {
        regurgitate(1);
        goto rule2;
    }
    if(!expr()) {
        regurgitate(1);
        goto rule2;
    }
    return true;
rule2:
    if (!term()) return false;
    return true;
}
```

```
bool term() {
rule1:
    if (fetchNext()!=IntNum) {
        regurgitate(1);
        goto rule2;
    }
    if (fetchNext()!=StarNum) {
        regurgitate(2);
        goto rule2;
    }
    if(!term()) {
        regurgitate(2);
        goto rule2;
    }
    return true;
rule2:
    ...
    return true;
rule3:
    ...
    return true;
}
```

Recursive Descent has a Left Recursion Problem

❑ Recursive descent doesn't work if grammar is left recursive

❑ Why is left recursion a problem?

- For left recursive grammar

$$A \rightarrow A b \mid c$$

- We may repeatedly choose to apply $A b$

$$A \Rightarrow A b \Rightarrow A b b \dots$$

- Sentence can grow indefinitely w/o consuming input
- How do you know when to stop recursion and choose c ?

Recursive Descent has a Left Recursion Problem

- ❑ Recursive descent doesn't work if grammar is left recursive
- ❑ Why is left recursion a problem?
 - For left recursive grammar
$$A \rightarrow A b \mid c$$
 - We may repeatedly choose to apply $A b$
$$A \Rightarrow A b \Rightarrow A b b \dots$$
 - Sentence can grow indefinitely w/o consuming input
 - How do you know when to stop recursion and choose c ?
- ❑ Rewrite the grammar so that it is right recursive
 - Which expresses the same language

Removing Left Recursion

- All immediate left recursion can be eliminated this way:

$$A \rightarrow A x \mid y$$

change to

$$A \rightarrow y A'$$

$$A' \rightarrow x A' \mid \epsilon$$

- Not all left recursion is immediate
(Recursion may involve multiple non-terminals)

$$A \rightarrow BC \mid D$$

$$B \rightarrow AE \mid F$$

... see Section 4.3 for *elimination of general left recursion*

... (not required for this course)

Table Driven Parser using Predictions

Predictive Parsers can avoid Backtracking

- ❑ Predict correct production rule based on k lookahead
 - Backtracking can be avoided if grammar limited to LL(k)

- ❑ LL(k) Parser
 - L — left to right scan
 - L — leftmost derivation
 - k — k symbols of lookahead
 - A predictive parser that uses k lookahead tokens

- ❑ LL(k) Grammar
 - A grammar parse-able by LL(k) parser with no backtracking

- ❑ LL(k) Language
 - A language that can be expressed as a LL(k) grammar
 - LL(k) languages are a restricted subset of CFLs
 - But many languages are LL(k). In fact, many are LL(1)!

Left factoring can make grammars LL(1)

□ An LL(1) grammar

- First terminal of every alternative production is unique

$$\begin{aligned} A &\rightarrow a B D \mid b B B \\ B &\rightarrow c \mid b c e \\ D &\rightarrow d \end{aligned}$$

□ What if no LL(1)? Left factor to make it LL(1)!

- What if production rules for A was changed to below?

$$A \rightarrow a B D \mid a B B$$

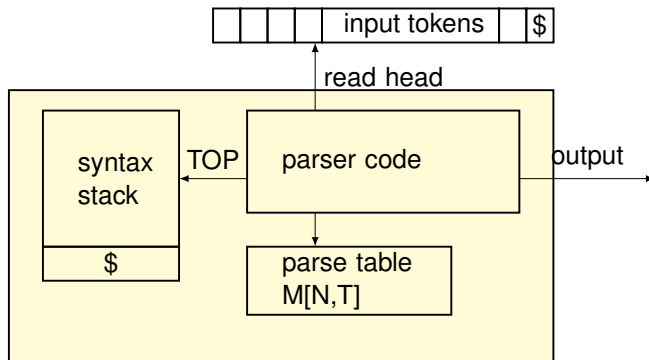
- Left factor $a B$ to enable prediction:

$$\begin{aligned} A &\rightarrow a B A' \mid a B A' \\ A' &\rightarrow D \mid B \end{aligned}$$

□ In general, if you see $A \rightarrow \alpha\beta \mid \alpha\gamma$, change to:

$$\begin{aligned} A &\rightarrow \alpha A' \\ A' &\rightarrow \beta \mid \gamma \end{aligned}$$

A Table Driven Pushdown Automaton



Syntax stack — hold right hand side (RHS) of grammar rules

Parse table $M[A,b]$ — an entry containing rule “ $A \rightarrow \dots$ ” or error

Parser code — next action based on **(current token, stack top)**

Table can be automatically generated from grammar (just like lexers)

A Sample Parse Table

	int	*	+	()	\$
E	$E \rightarrow TX$			$E \rightarrow TX$		
X			$X \rightarrow +E$		$X \rightarrow \epsilon$	$X \rightarrow \epsilon$
T	$T \rightarrow \text{int } Y$			$T \rightarrow (E)$		
Y		$Y \rightarrow *T$	$Y \rightarrow \epsilon$		$Y \rightarrow \epsilon$	$Y \rightarrow \epsilon$

☐ Predicts rule based on (current non-terminal, lookahead)

- **First column** lists all non-terminals
- **First row** lists all possible terminals and \$
- A table entry contains one production (one prediction)

☐ What if an entry has more than one production?

- Means that this grammar is not LL(1)
- A parser can handle this situation by either:
 - Throwing an error to grammar writer to fix the problem
 - Resorting to backtracking to try out both productions

Pseudocode for Table-Driven Parser

X — symbol at the top of the syntax stack

a — current input symbol

Parsing based on **(X,a)**

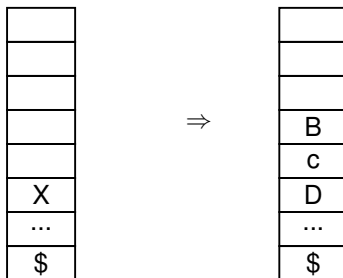
- If $X == a == \$$, then
 - parser halts with “success”
- If $X == a \neq \$$, then
 - pop X from stack **and** advance input head
- If $X \neq a$, then
 - Case (a): if $X \in T$, then
 - parser halts with “failed”, input rejected
 - Case (b): if $X \in N$, $M[X,a] = “X \rightarrow RHS”$
 - pop X **and** push RHS to stack in reverse order


Push RHS in Reverse Order

X — symbol at the top of the syntax stack

a — current input symbol

if $M[X,a] = "X \rightarrow B \ c \ D"$



 Why? Because that is the order of leftmost derivation.

Applying LL(1) Parsing to a Grammar

❏ Given our old grammar

$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

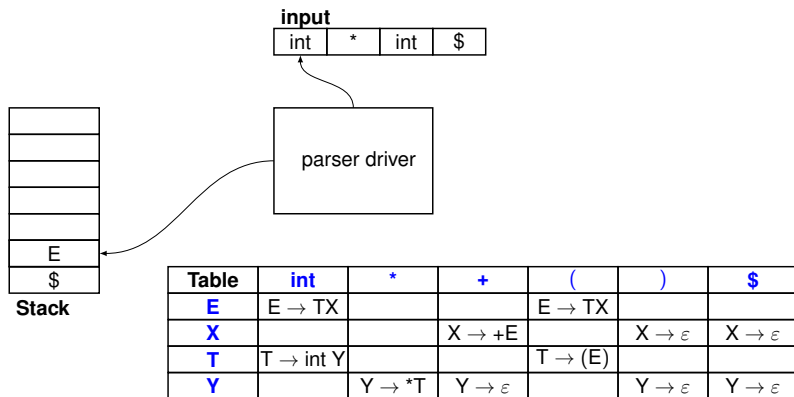
➤ Requires left factoring of T and int

❏ After rewriting grammar, we have

$$E \rightarrow T X$$
$$X \rightarrow + E \mid \epsilon$$
$$T \rightarrow \text{int} Y \mid (E)$$
$$Y \rightarrow * T \mid \epsilon$$

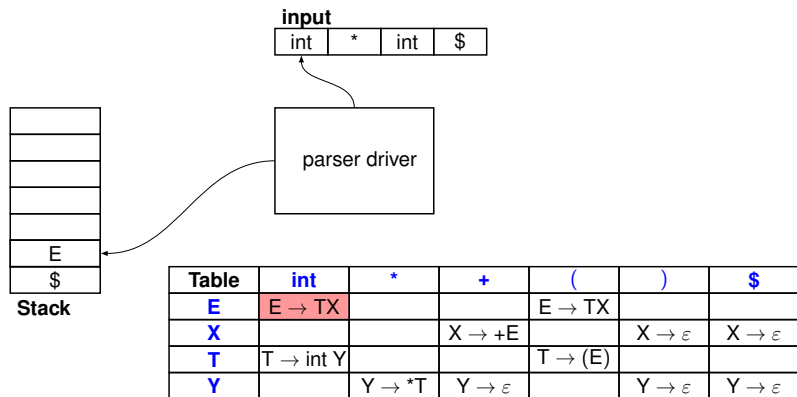
Using the Parse Table

□ To recognize “int * int”



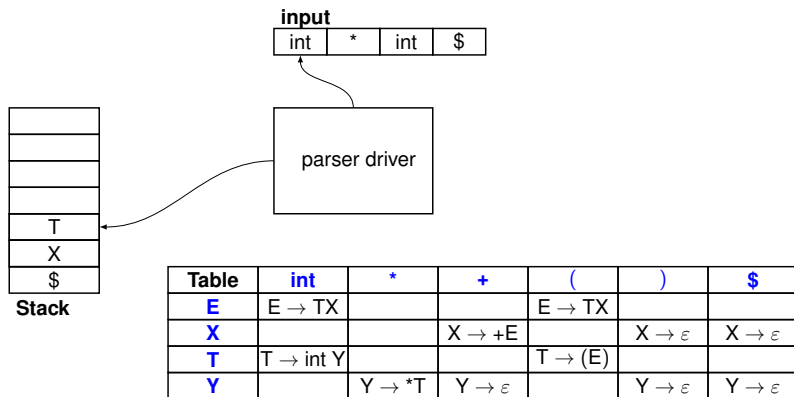
Using the Parse Table

■ To recognize “int * int”



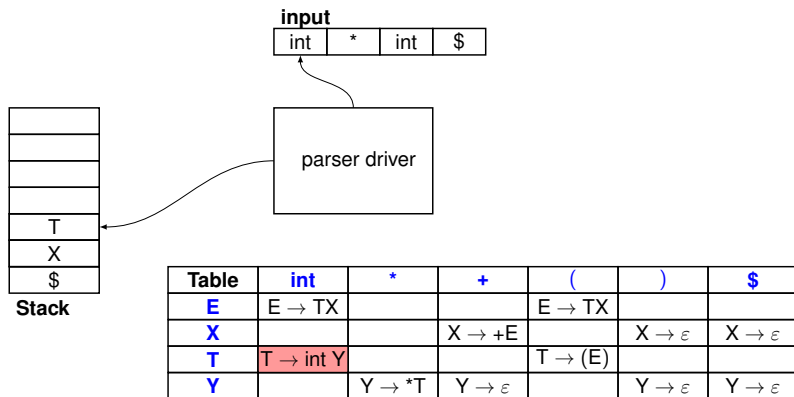
Using the Parse Table

■ To recognize “int * int”



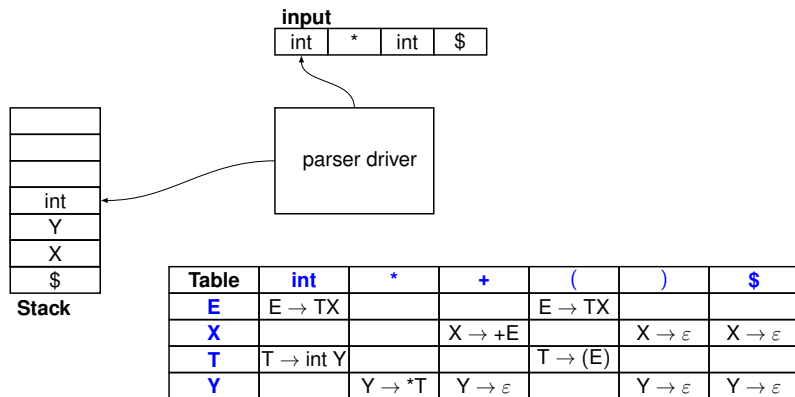
Using the Parse Table

■ To recognize “int * int”



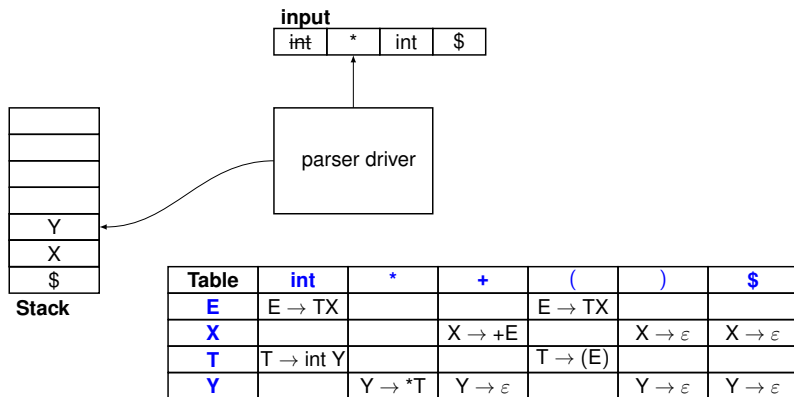
Using the Parse Table

■ To recognize “int * int”



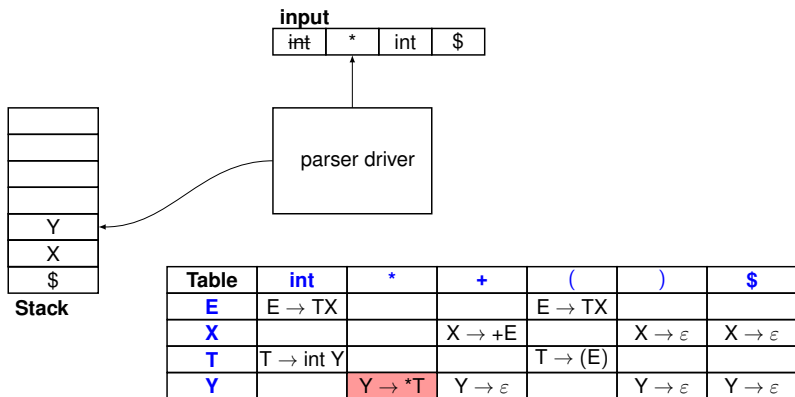
Using the Parse Table

■ To recognize “int * int”



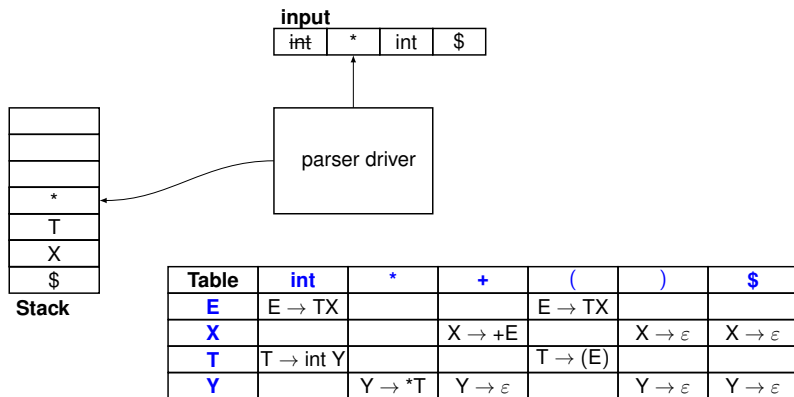
Using the Parse Table

- To recognize “int * int”



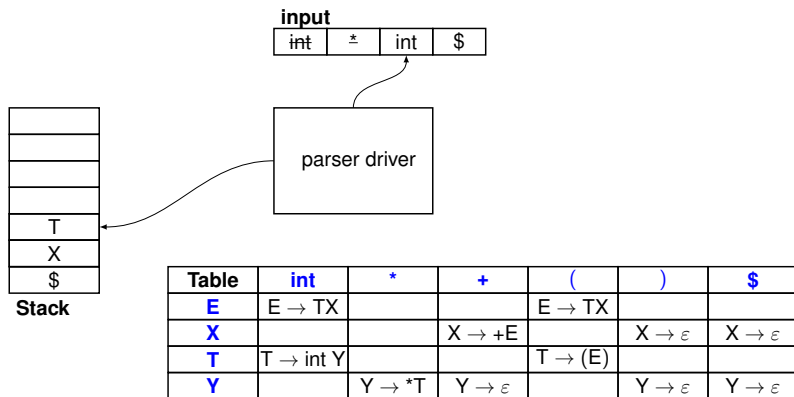
Using the Parse Table

□ To recognize “int * int”



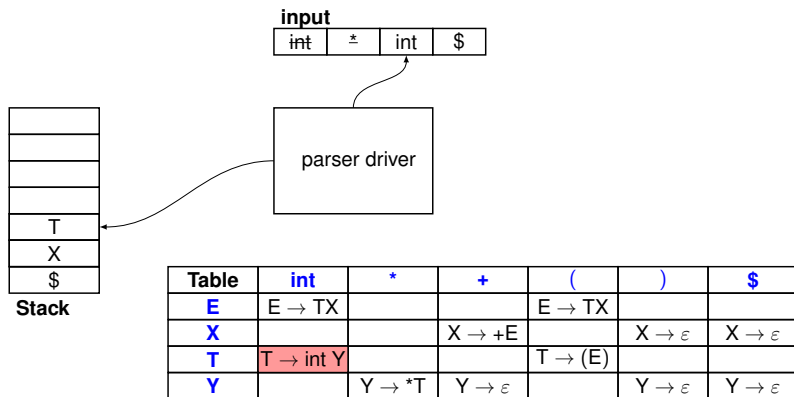
Using the Parse Table

■ To recognize “int * int”



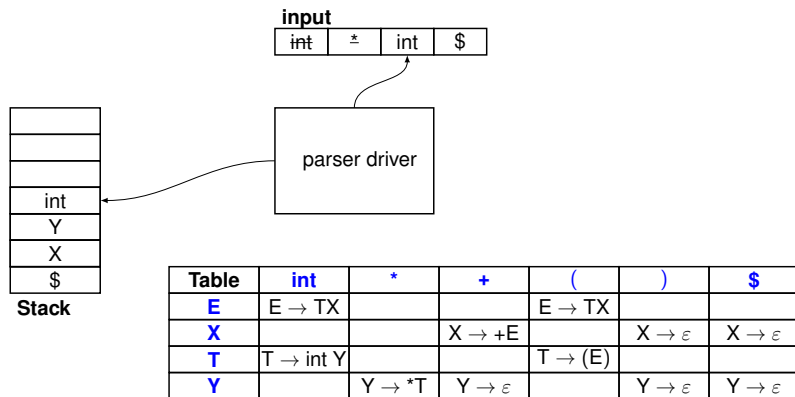
Using the Parse Table

■ To recognize “int * int”



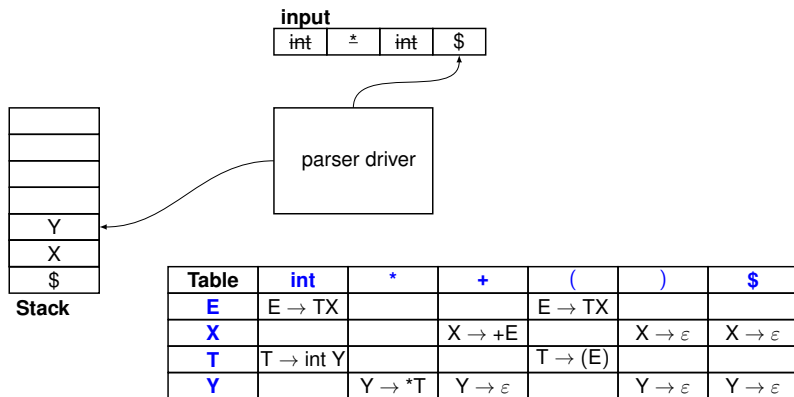
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□ To recognize “int * int”



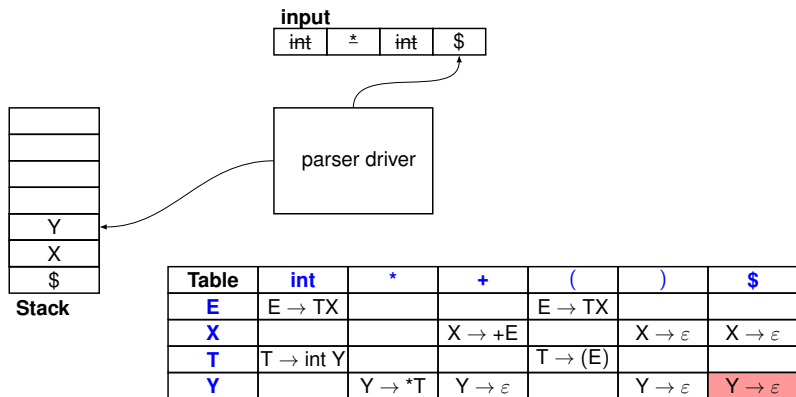
Using the Parse Table

■ To recognize “int * int”



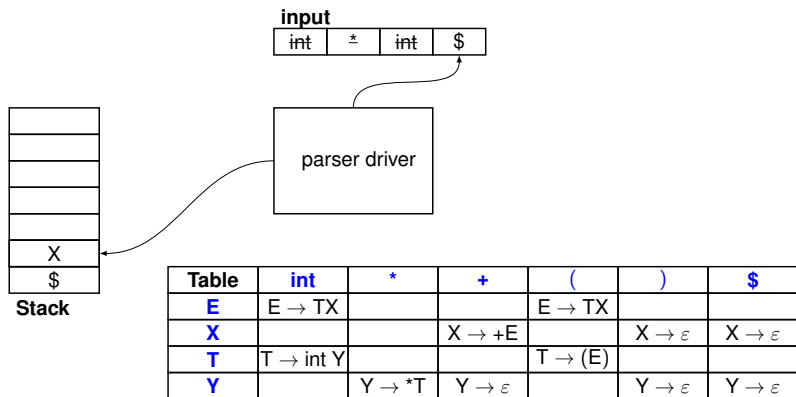
Using the Parse Table

■ To recognize “int * int”



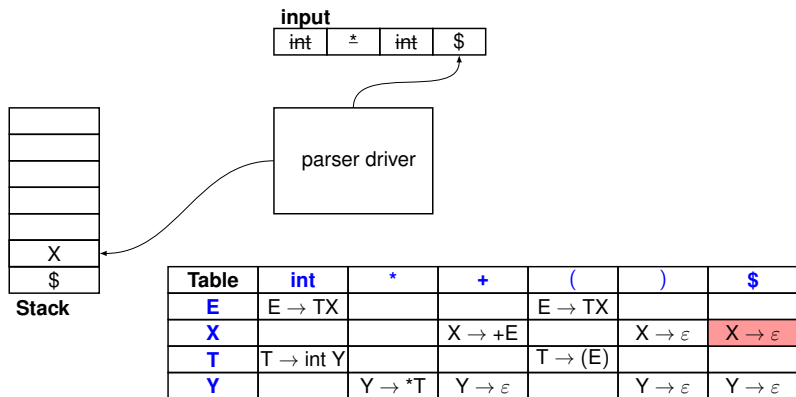
Using the Parse Table

□ To recognize “int * int”



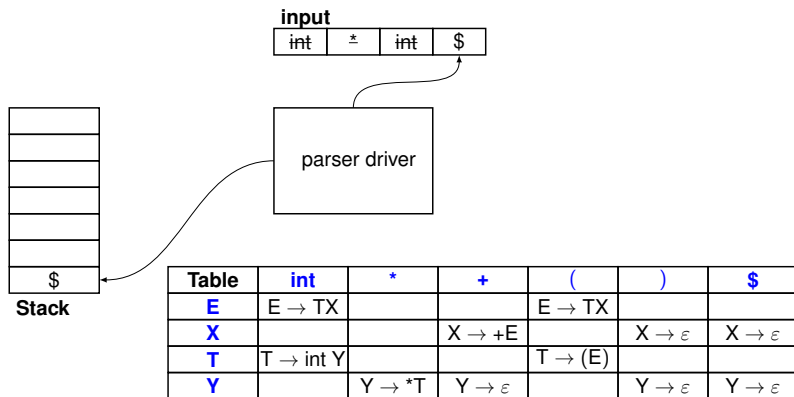
Using the Parse Table

□ To recognize “int * int”



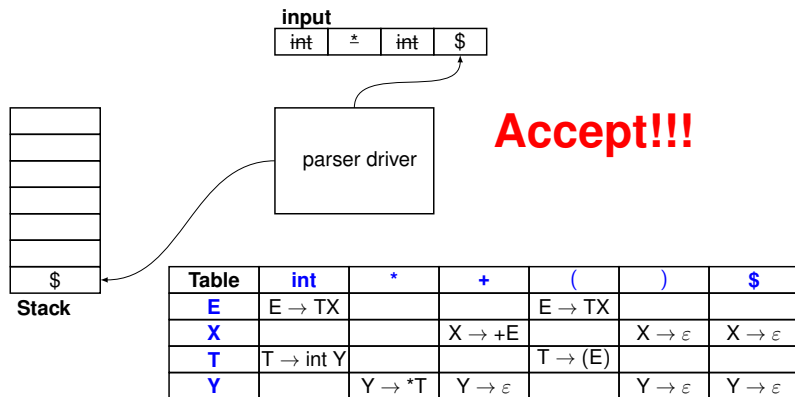
Using the Parse Table

■ To recognize “int * int”



Using the Parse Table

□ To recognize “int * int”



Recognition Sequence

It is possible to write in a action list

Stack	Input	Action
E \$	int * int \$	$E \rightarrow TX$
T X \$	int * int \$	$T \rightarrow \text{int } Y$
int Y X \$	int * int \$	terminal
Y X \$	* int \$	$Y \rightarrow * T$
* T X \$	* int \$	terminal
T X \$	int \$	$T \rightarrow \text{int } Y$
int Y X \$	int \$	terminal
Y X \$	\$	$Y \rightarrow \epsilon$
X \$	\$	$X \rightarrow \epsilon$
\$	\$	halt and accept

First step in building Parse Table: First and Follow Sets

□ $\text{First}(\alpha) = \{t \mid \alpha \Rightarrow *t\beta\}$

➤ Set of terminals that can start a string derived from α .

□ $\text{Follow}(\alpha) = \{t \mid S \Rightarrow *\alpha t\beta\}$

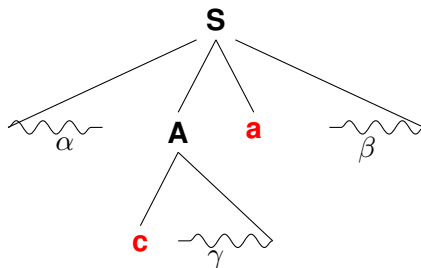
➤ Set of terminals that can follow α in some derivation.

□ Given rule $A \rightarrow \alpha$,

➤ Choose $A \rightarrow \alpha$ for all terminals in $\text{First}(\alpha)$

➤ Choose $A \rightarrow \alpha$ for all terminals in $\text{Follow}(A)$,
if and only if $\alpha \Rightarrow *\epsilon$

Intuitive Meaning of **First** and **Follow**



$c \in \text{First}(A)$

$a \in \text{Follow}(A)$

 Why is the Follow Set important?

Calculating $\text{First}(\alpha)$

- Given $A \rightarrow \alpha$, let's calculate $\text{First}(\alpha)$.
 - α is string $Y_1 Y_2 Y_3 \dots Y_m$ of terminals and non-terminals.
 - 1) For all Y_i , if Y_i is a terminal t , then $\text{First}(Y_i) = t$
 - 2) For all non-terminal Y_i , recursively calculate $\text{First}(Y_i)$
 - 3) Calculate $\text{First}(\alpha)$ based on $\text{First}(Y_i)$ where $i = 1 \dots m$

- Apply following rules until no terminal or ε can be added
 - 1) Add $(\text{First}(Y_1) - \varepsilon)$ to $\text{First}(\alpha)$.
 - 2) If $\text{First}(Y_1), \dots, \text{First}(Y_{k-1})$ all contain ε , then add $(\sum_{1 \leq i \leq k} \text{First}(Y_i) - \varepsilon)$ to $\text{First}(\alpha)$.
 - 3) If $\text{First}(Y_1), \dots, \text{First}(Y_m)$ all contain ε , then add ε to $\text{First}(\alpha)$.

Calculating Follow(A)

$$\text{Follow}(\alpha) = \{t \mid S \Rightarrow * \alpha t \beta\}$$

Intuition: if $X \rightarrow A B$, then $\text{First}(B) \subseteq \text{Follow}(A)$

little trickier because B may be ε i.e. $B \Rightarrow * \varepsilon$

Apply following rules until no terminal or ε can be added

- 1). $\$ \in \text{Follow}(S)$, where S is the start symbol.
e.g. $\text{Follow}(E) = \{\$ \dots\}$.
- 2). Look at the occurrence of a non-terminal on the right hand side of a production which is followed by something
If $\dots \rightarrow \alpha A \beta$, then $\text{First}(\beta) - \{\varepsilon\} \subseteq \text{Follow}(A)$
- 3). Look at N on the RHS that is not followed by anything,
if $(X \rightarrow \alpha A)$ or $(X \rightarrow \alpha A \beta \text{ and } \varepsilon \in \text{First}(\beta))$,
then $\text{Follow}(X) \subseteq \text{Follow}(A)$

Calculating First and Follow Sets for the example

$$\begin{array}{lcl} E & \rightarrow & T X \\ X & \rightarrow & + E \mid \varepsilon \\ T & \rightarrow & \text{int } Y \mid (E) \\ Y & \rightarrow & * T \mid \varepsilon \end{array}$$

□ Start by calculating the First Sets for all RHSs

- First($T X$)
- First($+ E$), First(ε)
- First($\text{int } Y$), First((E))
- First($* T$), First(ε)

□ If any of the above First Sets contains ε ,
calculate the Follow Set for corresponding non-terminal

Calculating First and Follow Sets for the example

$$\begin{aligned}
 E &\rightarrow TX \\
 X &\rightarrow +E \mid \varepsilon \\
 T &\rightarrow \text{int } Y \mid (E) \\
 Y &\rightarrow *T \mid \varepsilon
 \end{aligned}$$

Symbol	First
((
))
+	+
*	*
int	int
E	(, int
X	+, ε
T	(, int
Y	*, ε

RHS	First
TX	(, int
$+E$	+
ε	ε
$\text{int } Y$	int
(E)	(
$*T$	*
ε	ε

Calculating First and Follow Sets for the example

$$\begin{aligned}
 E &\rightarrow TX \\
 X &\rightarrow +E \mid \epsilon \\
 T &\rightarrow \text{int } Y \mid (E) \\
 Y &\rightarrow *T \mid \epsilon
 \end{aligned}$$

Symbol	First
((
))
+	+
*	*
int	int
E	(, int
X	+, ϵ
T	(, int
Y	*, ϵ

RHS	First
TX	(, int
$+E$	+
ϵ	ϵ
$\text{int } Y$	int
(E)	(
$*T$	*
ϵ	ϵ

Non-terminal	Follow
X	\$,)
Y	\$,),+
E	\$,)
T	\$,),+

Construction of LL(1) Parse Table

- To construct the parse table, we check each $A \rightarrow \alpha$
- For each terminal $a \in \text{First}(\alpha)$, then add $A \rightarrow \alpha$ to $M[A, a]$.
 - If $\varepsilon \in \text{First}(\alpha)$, then
for each terminal $b \in \text{Follow}(A)$, add $A \rightarrow \alpha$ to $M[A, b]$.
 - If $\varepsilon \in \text{First}(\alpha)$ and $\$ \in \text{Follow}(A)$, then add $A \rightarrow \alpha$ to $M[A, \$]$.

Example

$E \rightarrow TX$
 $X \rightarrow +E \mid \epsilon$
 $T \rightarrow \text{int } Y \mid (E)$
 $Y \rightarrow *T \mid \epsilon$

RHS	First
TX	(,int
$+E$	+
ϵ	ϵ
$\text{int } Y$	int
(E)	(
$*T$	*
ϵ	ϵ

Non-terminal	Follow
X	\$,)
Y	\$,),+

Table	int	*	+	()	\$
E	$E \rightarrow TX$			$E \rightarrow TX$		
X			$X \rightarrow +E$		$X \rightarrow \epsilon$	$X \rightarrow \epsilon$
T	$T \rightarrow \text{int } Y$			$T \rightarrow (E)$		
Y		$Y \rightarrow *T$	$Y \rightarrow \epsilon$		$Y \rightarrow \epsilon$	$Y \rightarrow \epsilon$

Determine if Grammar G is LL(1)

Observation

If a grammar is LL(1), then each of its LL(1) table entry contains at most one rule. Otherwise, it is not LL(1).

Two methods to determine if a grammar is LL(1) or not

(1) Construct LL(1) table, and check if there is a multi-rule entry
or

(2) Checking each rule as if the table is getting constructed.

G is LL1(1) **iff** for a rule $A \rightarrow \alpha | \beta$

➤ $\text{First}(\alpha) \cap \text{First}(\beta) = \phi$

➤ at most one of α and β can derive ε

➤ If β derives ε , then $\text{First}(\alpha) \cap \text{Follow}(A) = \phi$

Left-recursion disqualifies grammar for LL(1)

- ❑ Recall recursive descent had trouble with left-recursion.
- ❑ Table-driven parsers have a similar problem.
- ❑ Left-recursion is of the form $A \rightarrow Ab|a$ or $A \rightarrow Ab|\varepsilon$
 - For $A \rightarrow Ab|a$, $\text{First}(Ab) \cap \text{First}(a) = \{a\}$
 - For $A \rightarrow Ab|\varepsilon$, $\text{First}(Ab) \cap \text{Follow}(A) = \{b\}$
 - Either way, an ambiguity in prediction
- ❑ Even if prediction can be made with more lookahead,
 - Sentence can grow indefinitely w/o consuming input
 - We may repeatedly choose to apply $A \rightarrow Ab$:
 $A \Rightarrow A b \Rightarrow A b b \dots$
 - Same stack explosion problem as with recursive descent

Dealing with Non-LL(1) Grammars

(1) Likely still an LL(1) language. Massage to LL(1) grammar:

- Apply left-factoring
- Apply left-recursion removal

(2) If (1) fails, the possibilities are...

- Grammar just needs a little more lookahead
(May need LL(k) parser where $k > 1$ or backtracking)
- Grammar is ambiguous (multiple parse trees)

□ How do we deal with ambiguous grammars then?

- Note: left-factoring and left-recursion removal don't help
- Expressing precedence and associativity in grammar helps

Ambiguous not just non-LL(1)

- Some grammars are not LL(1) even after left-factoring and left-recursion removal

$S \rightarrow \text{if } C \text{ then } S \mid \text{if } C \text{ then } S \text{ else } S \mid a \text{ (other statements)}$

$C \rightarrow b$

change to

$S \rightarrow \text{if } C \text{ then } S X \mid a$

$X \rightarrow \text{else } S \mid \epsilon$

$C \rightarrow b$

problem sentence: “if b then if b then a else a”

$\text{First}(X) = \{\text{else}, \epsilon\}$

From $S \rightarrow \text{if } C \text{ then } S X$, $\text{Follow}(S) \subseteq \text{Follow}(X)$

$\text{Follow}(X) = \{\text{else}, \$\}$

For $X \rightarrow \text{else } S \mid \epsilon$, $\text{First}(\text{else } S) \cap \text{Follow}(X) = \{\text{else}\}$

- Such grammars are potentially ambiguous

Removing Ambiguity

- ❑ We want to express precedence of if-then-else over if-then.
- ❑ How would you rewrite grammar to express precedence?

$S \rightarrow \text{if } C \text{ then } S \mid S_2$

$S_2 \rightarrow \text{if } C \text{ then } S_2 \text{ else } S \mid a$

$C \rightarrow b$

- ❑ Now grammar is unambiguous but it is not LL(k) for any k
 - Intuitively, must lookahead until 'else' to choose rule for 'S'
 - That lookahead may be an arbitrary number of tokens

- ❑ Changing the grammar to be perfectly unambiguous
 - Can be very taxing for programmers to specify correctly
 - May still result in grammar not suitable for LL(1) parsing

- ❑ More practical to encode precedence rules into parser
 - E.g. Always choose $X \rightarrow \text{else } S$ over $X \rightarrow \epsilon$ on 'else' token

LL(1) Time and Space Complexity

- ❑ LL(1) parsers operate in linear time and space relative to the length of input.
- ❑ Time: each token is processed constant number of times
 - Why?
- ❑ Space: stack space required is at max the length of input
 - If $X \rightarrow \varepsilon$ rules removed (easily done by substitution)
 - Why?
- ❑ How about LL(k)?
 - Same time complexity as the same argument applies
 - Space complexity is $O(T^k)$, where T is number of terminals (if constructing the parse table naively)

ANTLR: A modern LL(k) parser

- ❑ A free open source top-down LL(k) parser (antlr.org)
 - Used in Apache Groovy, Jython, MySQL Workbench, ...

- ❑ Reduces table space by expressing lookahead as DFA
 - A DFA decides on which rule for each non-terminal
 - DFA can express arbitrarily long lookahead compactly

- ❑ If you are interested, refer to this paper:
LL(*): The Foundation of the ANTLR Parser Generator
<https://www.antlr.org/papers/LL-star-PLDI11.pdf>