

Bottom Up Parsing

PITT CS 1622

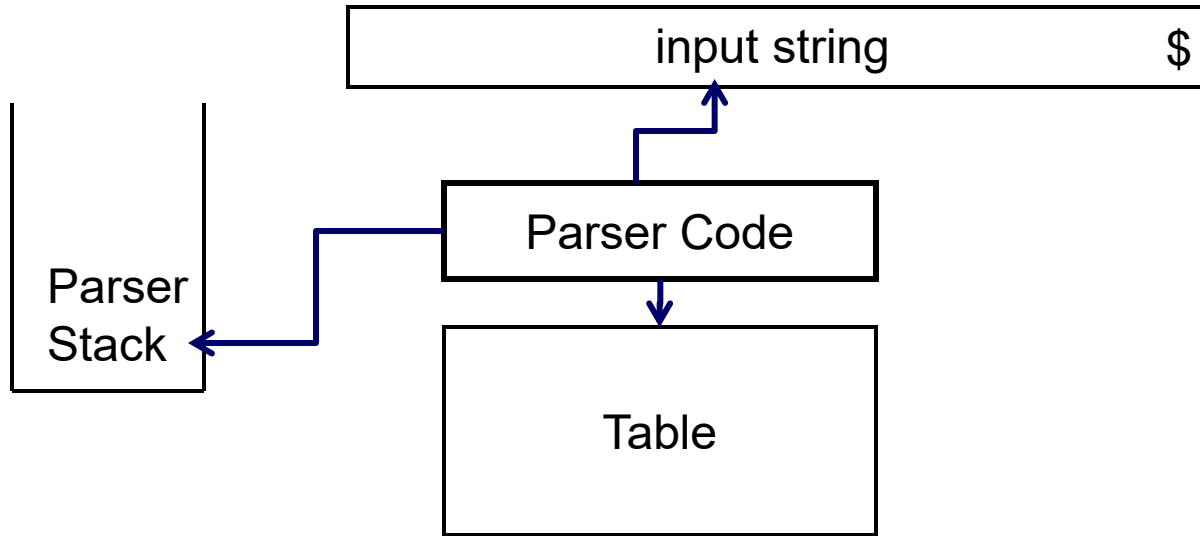
Bottom Up Parsing

- ❑ More powerful than top down
 - Don't need left factored grammars
 - Can handle left recursion
 - Can parse a larger set of grammars (and languages)

- ❑ Begins at leaves and works to the top
 - In reverse order of rightmost derivation
(In effect, builds tree from left to right)

- ❑ Also known as Shift-Reduce parsing
 - Involves two types of operations: shift and reduce

Parser Implementation

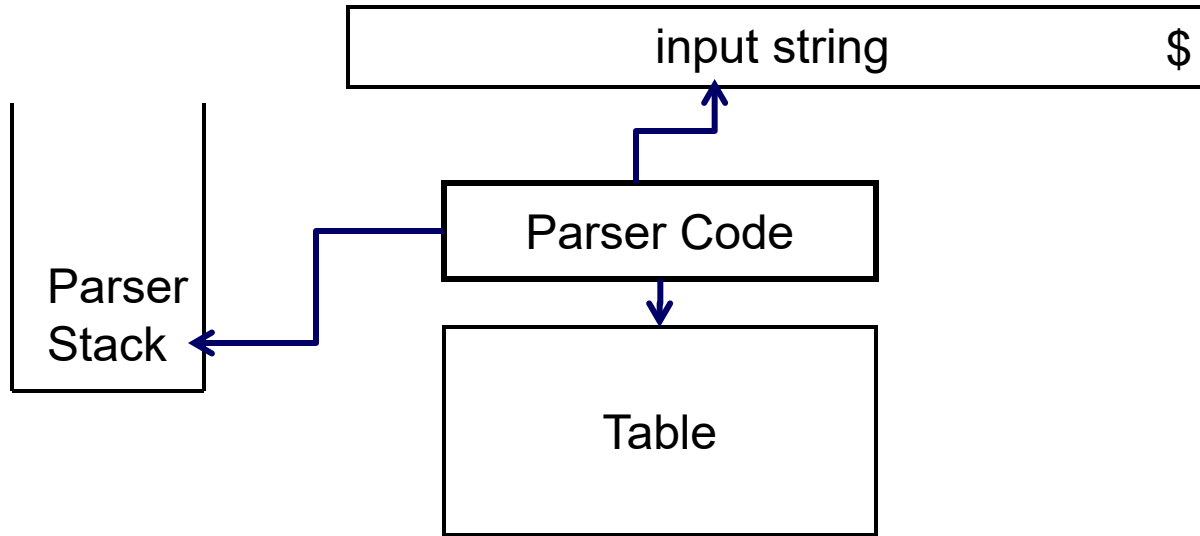


Parser Stack – holds consumed portion of derivation string

Table – “actions” performed based on rules of grammar, and current state of stack and input string

Parser Code – next action based on [**stack top, lookahead token(s)**]

Parser Implementation



Actions

1. **Shift** – consume input symbol and push symbol onto the stack
2. **Reduce** – pop RHS at stack top and push LHS of a production rule, reducing stack contents
3. **Accept** – success (when reduced to start symbol and input at \$)
4. **Error**

Bottom-Up compared to Top-Down

❑ Conceptual difference in how the stack works:

	Stack Content At Input Start	Stack Content At Input End	Stack Represents	Key Operations
Top-Down	Start Symbol	Nothing	Unconsumed input	Match / Expand
Bottom-up	Nothing	Start Symbol	Consumed input	Shift / Reduce

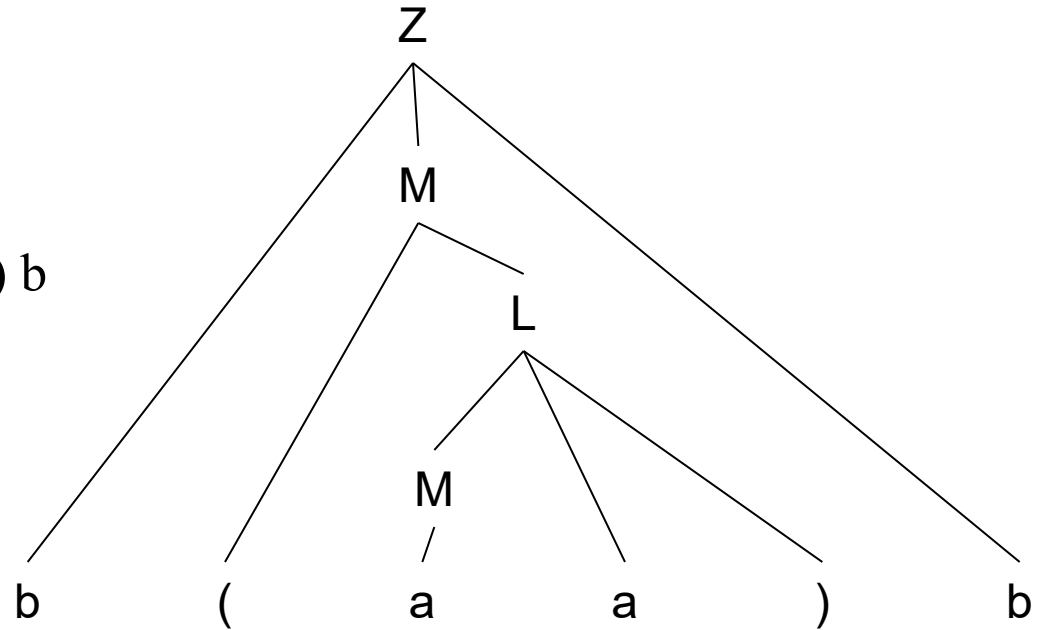
- ❑ But both use a stack to parse languages with nested structures
- Not surprising since CFGs are parsed using Pushdown Automata!

$Z \rightarrow b M b$

$M \rightarrow (L \mid a$

$L \rightarrow M a \mid)$

Considering string: $w = b (a a) b$



The rightmost derivation of this parse tree:

$Z \Rightarrow b M b \Rightarrow b (L b \Rightarrow b (M a) b \Rightarrow b (a a) b$

Bottom up parsing involves finding “handles” (RHSs) to reduce

$b (a a) b \Rightarrow b (M a) b \Rightarrow b (L b \Rightarrow b M b \Rightarrow Z$

$$Z \rightarrow b M b$$

$$M \rightarrow (L \mid a$$

$$L \rightarrow M a) \mid)$$

String
 $b (a a) \$$

Stack	Input	Action
\$	b (a a) b \$	shift
\$ b	(a a) b \$	shift
\$ b (a a) b \$	shift
\$ b (a	a) b \$	reduce
\$ b (M	a) b \$	shift
\$ b (M a) b \$	shift
\$ b (M a)	b \$	reduce
\$ b (L	b \$	reduce
\$ b M	b \$	shift
\$ b M b	\$	reduce
\$ Z	\$	accept

Sentential Form and Handle

- ❑ **Sentential form:** Any string derivable from the start symbol
- ❑ **Handle:** RHS of a production rule that, when reduced to LHS in a sentential form, will lead to another sentential form

- ❑ **Definition:**
 - Let $\alpha\beta w$ be a sentential form where
 - α, β is a string of terminals and non-terminals
 - w is a string of terminals
 - $X \rightarrow \beta$ is a production ruleThen β is a handle of $\alpha\beta w$ if
 - $S \Rightarrow^* \alpha X w \Rightarrow \alpha\beta w$ by a rightmost derivation
 - Handles formalize the intuition “ β should be reduced to X for a successful parse”, but does not really say how to find them

Single Pass Left-to-Right Scan

□ Note in the formulation of a handle $S \Rightarrow^* \alpha X w \Rightarrow \alpha \beta w$

- α is a string of terminals and non-terminals
- w is a string of only terminals
- Why is this so?

□ Proof by example

- Given $S \rightarrow ABCD$, $A \rightarrow a$, $B \rightarrow b$, $C \rightarrow c$, $D \rightarrow d$
- $S \Rightarrow ABCD \Rightarrow ABCd \Rightarrow ABcd \Rightarrow Abcd \Rightarrow abcd$

□ Mathematical proof

- Let's assume w contained a non-terminal Y ($w = w_1 Y w_2$)
- Then, $S \Rightarrow^* \alpha X w_1 Y w_2 \Rightarrow \alpha \beta w_1 Y w_2$
- Above is not a rightmost derivation (you derived X before Y)
- Contradiction!

Single Pass Left-to-Right Scan

□ Note in the formulation of a handle $S \Rightarrow^* \alpha X w \Rightarrow \alpha \beta w$

- α is a string of terminals and non-terminals
- w is a string of only terminals

□ Why is this important?

- $\alpha \beta$ is consumed input in the stack and w is unconsumed input
- The reduced handle β is always at the top of the stack
- No need to view middle of the stack to reduce!
- No need to view unconsumed input to reduce!
- Amenable to single pass left-to-right scan using a stack

Handle Always Occurs at Top of Stack

□ Grammar

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow (E)$

$E \rightarrow id$

Sentential form	Handle	Production
$id_1 + id_2 * id_3$	id_1	$E \rightarrow id$
$E + id_2 * id_3$	id_2	$E \rightarrow id$
$E + E * id_3$	id_3	$E \rightarrow id$
$E + E * E$	$E * E$	$E \rightarrow E * E$
$E + E$	$E + E$	$E \rightarrow E + E$
E		

□ # indicates top of stack (at the frontier of reduction where the handle is)

Left of # : stack contents, Right of # : unconsumed input string

$$\begin{aligned}
 id_1 \# + id_2 * id_3 &\Rightarrow E \# + id_2 * id_3 \Rightarrow E + \# id_2 * id_3 \Rightarrow E + id_2 \# * id_3 \\
 &\Rightarrow E + E \# * id_3 \Rightarrow E + E * id_3 \# \Rightarrow E + E * E \# \Rightarrow E + E \# \Rightarrow E
 \end{aligned}$$

□ Stack works because the reduction $X \rightarrow \beta$ always happens at the top of the stack

Handle Always Occurs at Top of Stack

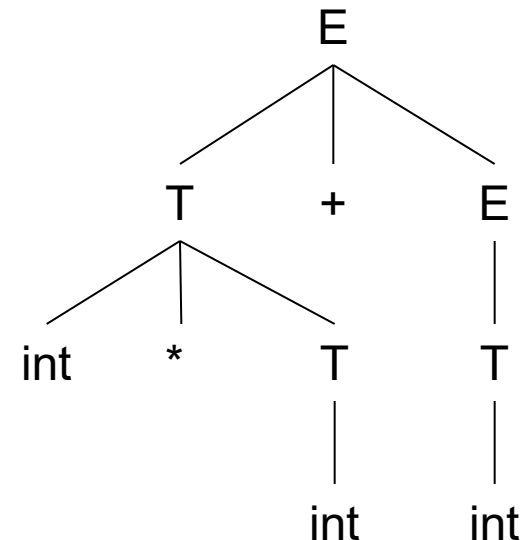
□ Consider our usual grammar

$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

Consider the string: $\text{int} * \text{int} + \text{int}$

<i>sentential form</i>	<i>production</i>
$\text{int} * \text{int} \# + \text{int}$	$T \rightarrow \text{int}$
$\text{int} * T \# + \text{int}$	$T \rightarrow \text{int} * T$
$T + \text{int} \#$	$T \rightarrow \text{int}$
$T + T \#$	$E \rightarrow T$
$T + E \#$	$E \rightarrow T + E$
$E \#$	



□ Reduction of a handle always happens at the top of the stack

Ambiguous Grammars

❑ Conflicts arise with ambiguous grammars

- Just like LL parsing, bottom-up parsing tries to predict the correct action
- But if there are multiple correct actions, conflicts arise

❑ Example:

- Consider the ambiguous grammar

$$E \rightarrow E * E \mid E + E \mid (E) \mid \text{int}$$

Sentential form	Actions	Sentential form	Actions
int * int + int	shift	int * int + int	shift
...
E * E # + int	reduce $E \rightarrow E * E$	E * E # + int	shift
E # + int	shift	E * E + # int	shift
E + # int	shift	E * E + int #	reduce $E \rightarrow \text{int}$
E + int #	reduce $E \rightarrow \text{int}$	E * E + E #	reduce $E \rightarrow E + E$
E + E #	reduce $E \rightarrow E + E$	E * E #	reduce $E \rightarrow E * E$
E #		E #	

Ambiguity

- ❑ Previous shift-reduce conflict occurred because of ambiguity
 - Due to lack of precedence between $+$ and $*$ in the grammar
 - Ambiguity shows up as “conflicts” in the parsing table
(More than one action in parse table, just like for LL parsers)
- ❑ Shift-reduce conflict also occurs with input “int + int + int”
 - Due to ambiguous associativity of $*$ and $+$
- ❑ Can always rewrite to encode precedence and associativity
 - But can sometimes result in convoluted grammars
 - Tools have other means to encode precedence and associativity
 - %left ‘+’ ‘-’
 - %left ‘*’ ‘/’

Properties of Bottom Up Parsing

- ❑ Handles always appear at the top of the stack
 - Never in middle of stack
- ❑ Easily generalized shift – reduce strategy
 - If there is a handle at top of stack, reduce to LHS non-terminal
 - If there is no handle at the top of the stack, shift input token
 - Easy to automate parser using a table [stack top, lookahead token(s)]
- ❑ Can have conflicts
 - If it is legal to either shift or reduce then there is a **shift-reduce** conflict.
 - If there are two legal reductions, then there is a **reduce-reduce** conflict.
 - Most often occur because of ambiguous grammars
 - In rare cases, because of non-ambiguous grammars not amenable to parser

Types of Bottom Up Parsers

- ❑ Types of bottom up parsers
 - Simple precedence parsers
 - Operator precedence parsers
 - LR family parsers
 - ...

- ❑ In this course, we will only discuss LR family parsers
 - Most automated tools generate either LL or LR parsers
 - Precedence parsers are weaker siblings of LR parsers

LR Parsers are more powerful than LL

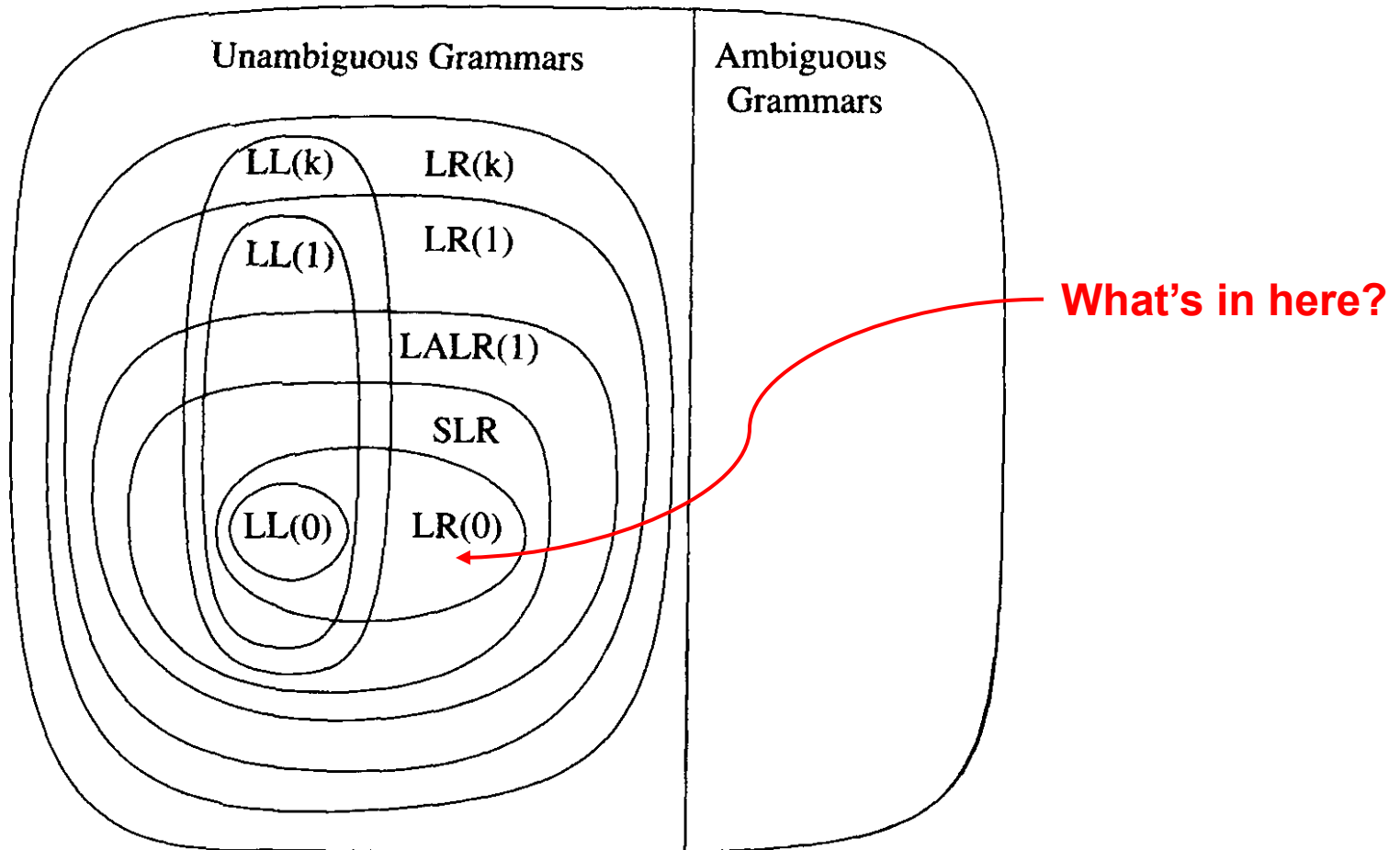
❑ LR family of parsers

- LR(k) L – left to right scan
 R – rightmost derivation in reverse
 k elements of look ahead

❑ Pros in comparison to LL(k)

1. More powerful than LL(k)
 - Handles more grammars: no left recursion removal, no left factoring needed
 - Handles more languages: $LL(k) \subset LR(k)$
2. As efficient as LL(k)
 - Linear in time and space to length of input (same as LL(k))
3. As convenient as LL(k)
 - Can generate automatically from grammar – YACC, Bison

A Hierarchy of Grammar Classes



LR Parsers are harder to deal with

❑ Cons in comparison to LL(k)

1. More complex in structure compared to LL(k)
 - Structure of parser looks nothing like grammar
 - Parse conflicts are hard to understand and debug
2. Harder to emit informative error messages and recover from errors
 - LR is a bottom-up while LL is a top-down parser
 - When parse error occurs,
LR: Knows only of currently reduced non-terminal
LL: Knows how upper levels of tree look like and context of error
 - LL can emit smart error messages referring to context of error
 - LL can perform better error recovery according to context

Implementation

-- LR Parsing

Pitt, CS 1622

Viable Prefix

□ Definition: α is a viable prefix if

- There is a w where αw is a rightmost sentential form, where w is the unconsumed input string
- In other words, if there is a w where $\alpha \# w$ is a configuration of a shift-reduce parser

$b (a \# a) b \Rightarrow b (M \# a) b \Rightarrow b (L \# b \Rightarrow b M \# b \Rightarrow Z \#$

□ If contents of parse stack is a viable prefix, that means the parser is on the right track (at least for the consumed input)

□ Shift-reduce parsing is the process of massaging the contents of the parse stack from viable prefix to viable prefix

- Error if neither shifting or reducing results in a viable prefix

Massaging into a Viable Prefix

□ How do you know what results in a viable prefix?

➤ Example grammar

$S \rightarrow a B S \mid b$

$B \rightarrow b$

➤ Example shift and reduce on: $a \# b b$

Shift: $a \# b b \Rightarrow a b \# b$ How do you know shifting is the answer?

Reduce: $a b \# b \Rightarrow a B \# b$ Should I apply $B \rightarrow b$ (and not $S \rightarrow b$)?

□ You need to keep track of where you are on the RHS of rules

➤ In example

Shift: $a \# b b \Rightarrow a b \# b$

$S \rightarrow a \# B S, B \rightarrow \# b \Rightarrow S \rightarrow a \# B S, B \rightarrow b \#$

Reduce: $a b \# b \Rightarrow a B \# b$

$S \rightarrow a \# B S, B \rightarrow b \# \Rightarrow S \rightarrow a B \# S, S \rightarrow \# a B S, S \rightarrow \# b$

LR(0) Item Notation

□ LR(0) Item: a production + a dot on the RHS

- Dot indicates extent of production already seen
- In example grammar

Items for production $S \rightarrow a B S$

$S \rightarrow \cdot a B S$

$S \rightarrow a \cdot B S$

$S \rightarrow a B \cdot S$

$S \rightarrow a B S \cdot$

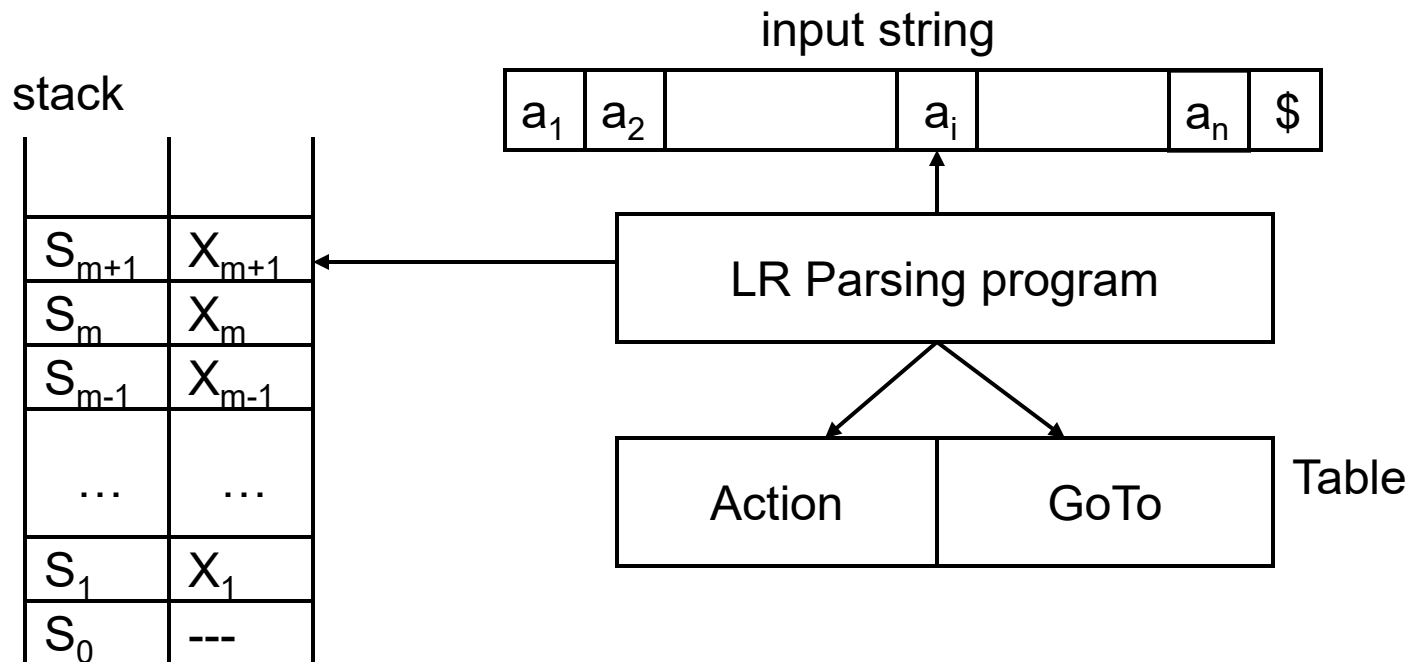
□ Items denote the idea of the viable prefix. E.g.

- $S \rightarrow \cdot a B S$: to be a viable prefix, terminal 'a' needs to be shifted
- $S \rightarrow a \cdot B S$: to be a viable prefix, a set of terminals need to be shifted and reduced to non-terminal 'B'

States in the LR Parser

- ❑ State of LL parser is simply tuple (stack top symbol, input)
- ❑ State of LR parser is more complex
 - Must keep track of where we are in the RHS production rules
 - State is denoted by a set of LR(0) items
- ❑ Why a *set* of LR(0) items?
 - There may be multiple candidate RHSs for the prefix. E.g.
For grammar $S \rightarrow a b \mid a c$,
 $S \rightarrow a . b$ and $S \rightarrow a . c$ would be items in the same set
 - Even for one RHS, you may need multiple items expressing the same position, if the following symbol is a non-terminal. E.g.
For grammar $S \rightarrow a B, B \rightarrow b$
 $S \rightarrow a . B$ and $B \rightarrow . b$ would be items in the same set
- ❑ LR parsers keep track of states alongside symbols in stack

Parser Implementation in More Detail



- Each grammar symbol X_i is associated with a state S_i
- Contents of stack ($X_1X_2\dots X_m$) is a viable prefix
- Contents of stack + input ($X_1X_2\dots X_ma_1\dots a_n$) is a right sentential form
 - If the input string is a member of the language
- Uses **state** at the top of stack and current input to index into parsing table to determine whether to shift or reduce

Parser Actions

S_m	X_m
S_{m-1}	X_{m-1}
...	...
S_1	X_1
S_0	---

...	X_{m+1}	...	\$
-----	-----------	-----	----

$S_m : A \rightarrow X_m \cdot X_{m+1}$
 $S_{m-1} : A \rightarrow \cdot X_m X_{m+1}$
 $B \rightarrow X_{m-1} \cdot A$

Shift

S_{m+1}	X_{m+1}
S_m	X_m
S_{m-1}	X_{m-1}
...	...
S_1	X_1
S_0	---

...	X_{m+1}	...	\$
-----	-----------	-----	----

$S_{m+1} : A \rightarrow X_m X_{m+1} \cdot$
 $S_m : A \rightarrow X_m \cdot X_{m+1}$
 $S_{m-1} : A \rightarrow \cdot X_m X_{m+1}$
 $B \rightarrow X_{m-1} \cdot A$

Reduce(1)

$S_{m+1} : A \rightarrow X_m X_{m+1} \cdot$
 $S_m : A \rightarrow X_m \cdot X_{m+1}$
 $S_{m-1} : A \rightarrow \cdot X_m X_{m+1}$
 $B \rightarrow X_{m-1} \cdot A$

...	X_{m+2}	...	\$
-----	-----------	-----	----

S_{m+1}	X_{m+1}
S_m	X_m
S_{m-1}	X_{m-1}
...	...
S_1	X_1
S_0	---

(2)

	A
S_{m-1}	X_{m-1}
...	...
S_1	X_1
S_0	---

GOTO

$S_A : B \rightarrow X_{m-1} A \cdot$
 $S_{m-1} : A \rightarrow \cdot X_m X_{m+1}$
 $B \rightarrow X_{m-1} \cdot A$

S_A	A
S_{m-1}	X_{m-1}
...	...
S_1	X_1
S_0	---

Parser Actions

□ Assume configuration = $S_0X_1S_1X_2S_2\dots X_mS_m\#a_ia_{i+1}\dots a_n\$$

□ Actions can be one of:

1. Shift input a_i and push new state **S**
 - New configuration = $S_0X_1S_1X_2S_2\dots X_mS_m a_i S \# a_{i+1}\dots\$$
 - Where **Action** $[S_m, a_i] = s[S]$
2. Reduce using Rule **R** ($A \rightarrow \beta$) and push new state **S**
 - Let $k = |\beta|$, pop $2*k$ symbols and push A
 - New configuration = $S_0X_1S_1\dots X_{m-k}S_{m-k}A S \# a_i a_{i+1}\dots\$$
 - Where **Action** $[S_m, a_i] = r[R]$ and **GoTo** $[S_{m-k}, A] = [S]$
3. Accept – parsing is complete (**Action** $[S_m, a_i] = \text{accept}$)
4. Error – report and stop (**Action** $[S_m, a_i] = \text{error}$)

Parse Table: Action and Goto

□ Action $[S_m, a_i]$ can be one of:

- **s[S]: shift** input symbol a_i and push **state S**
(One item in S_m must be of the form $A \rightarrow \alpha \cdot a_i \beta$)
- **r[R]: reduce** using **rule R** on seeing input symbol a_i
(One item in S_m must be $R: A \rightarrow \alpha \cdot$, where $a_i \in \text{Follow}(A)$)
 - Use $\text{GoTo}[S_{m-|\alpha|}, A]$ to figure out state to push with A
- **Accept** (One item in S_m must be $S' \rightarrow S \cdot$ where S is the original start symbol, and a_i must be $\$$)
- **Error** (Cannot shift, reduce, accept on symbol a_i in state S_m)

□ $\text{GoTo}[S_m, X_i]$ is **[S]**:

- Next **state** to push when pushing nonterminal X_i from a reduction
(At least one item in S_m must be of the form $A \rightarrow \alpha \cdot X_i \beta$)
- Similar to shifting input except now we are “shifting” a nonterminal

□ Grammar

1. $S \rightarrow E$
2. $E \rightarrow E + T$
3. $E \rightarrow T$
4. $T \rightarrow \text{id}$
5. $T \rightarrow (E)$

Non-terminal	Follow
S	\$
E	+) \$
T	+) \$

ACTION

	+	id	()	\$
S0		s3	s4		
S1	s7				accept
S2	r3			r3	r3
S3	r4			r4	r4
S4		s3	s4		
S5	s7			s6	
S6	r5			r5	r5
S7		s3	s4		
S8	r2			r2	r2

GOTO

	E	T	S
S0	1	2	
S1			
S2			
S3			
S4	5	2	
S5			
S6			
S7		8	
S8			

□ Grammar

1. $S \rightarrow E$
2. $E \rightarrow E + T$
3. $E \rightarrow T$
4. $T \rightarrow \text{id}$
5. $T \rightarrow (E)$

Non-terminal	Follow
S	\$
E	+) \$
T	+) \$

ACTION

	+	id	()	\$
S0		s3	s4		
S1	s7				accept
S2	r3			r3	r3
S3	r4			r4	r4
S4		s3	s4		
S5	s7			s6	
S6	r5			r5	r5
S7		s3	s4		
S8	r2			r2	r2

GOTO

	E	T	S
S0	1	2	
S1			
S2			
S3			
S4	5	2	
S5			
S6			
S7		8	
S8			

Parse Table in Action

□ Example input string

id + id + id

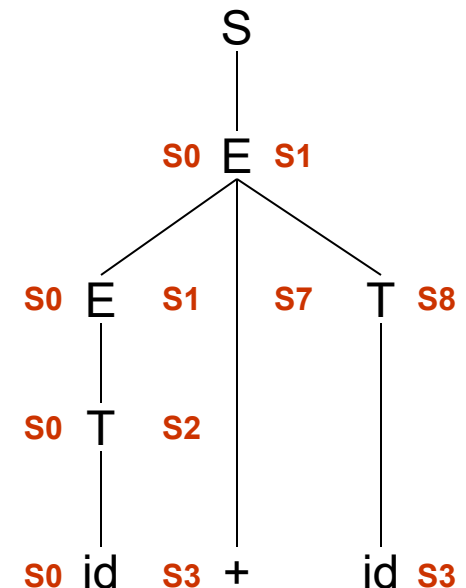
□ Parser actions

Stack	Input	Actions
S0	id + id + id \$	Action[S0, id] (s3): Shift “id”, Push S3
S0 id S3	+ id + id \$	Action[S3, +] (r4): Reduce rule 4 (T→id) GoTo[S0, T] (2): Push S2
S0 T S2	+ id + id \$	Action[S2, +] (r3): Reduce rule 3 (E→T) GoTo[S0, E] (1): Push S1
S0 E S1	+ id + id \$	Action[S1, +] (s6): Shift “+”, Push S7
S0 E S2 + S7	id + id \$	Action[S7, id] (s3): Shift “id”, Push S3
...
S0 E S2 + S7 T S8	+ id \$	Action[S8, +] (r1): Reduce rule 2 (E→E+T) GoTo[S0, E] (1): Push S1
...

Power Added to DFA by Stack

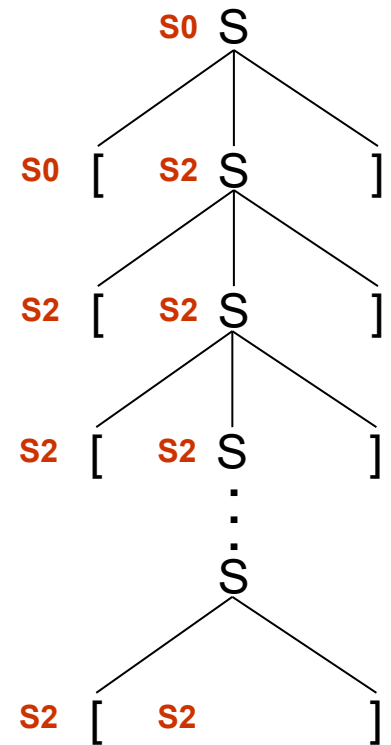
- ❑ LR parser is basically DFA+Stack (Pushdown Automaton)
- ❑ DFA: can only remember one state (“dot” in current rule)
- ❑ DFA + Stack: remembers current state and all past states (“dots” in rules higher up in the tree waiting for next symbol)

Stack	Input	Action
S0	id + id \$	s3
S0 id S3	+ id \$	r4, goto[S0, T]
S0 T S2	+ id \$	r3, goto[S0, E]
S0 E S1	+ id \$	s7
S0 E S2 + S7	id \$	s3
S0 E S2 + S7 id S3	\$	r4, goto[S7, T]
S0 E S2 + S7 T S8	\$	r2, goto[S0, E]
S0 E S1	\$	Accept



Power Added to DFA by Stack

- ❑ Remember the following CFG for the language $\{ [^i]^i \mid i \geq 1 \}$?
 $S \rightarrow [S] \mid []$
- ❑ Regular grammars (or DFAs) could not recognize language because the state machine had to “count”
- ❑ LR parser stack counts number of **[symbols**
- ❑ Q: Is this language LL(1)?
 - Yes. After left-factoring.
 $S \rightarrow [S', S' \rightarrow S] \mid []$
 - LL parser stack counts number of **] symbols**
 - Same pushdown automaton but different usage



LR Parse Table Construction

- ❑ Must be able to decide on action from:
 - State at the top of stack
 - Next k input symbols (In practice, $k = 1$ is often sufficient)

- ❑ To construct LR parse table from grammar
 1. Build deterministic finite automaton (DFA) using LR(0) items
 2. Express DFA using Action and GoTo tables

- ❑ State: Where we are currently in the structure of the grammar
 - Expressed as a set of LR(0) items
 - Each item expresses position in the RHS of a rule using a dot

Construction of LR States

1. Create augmented grammar G' for G
 - Given $G: S \rightarrow \alpha \mid \beta$, create $G': S' \rightarrow S \mid S \rightarrow \alpha \mid \beta$
 - Creates a single rule $S' \rightarrow S$ that when reduced, signals acceptance
2. Create first state by performing a *closure* on initial item $S' \rightarrow \cdot S$
 - **Closure(I)**: computes set of items expressing the same position as I
 - $\text{Closure}(\{S' \rightarrow \cdot S\}) = \{S' \rightarrow \cdot S, S \rightarrow \cdot \alpha, S \rightarrow \cdot \beta\}$
3. Create additional states by performing a *goto* on each symbol
 - **Goto(I, X)**: creates state that can be reached by advancing X
 - If α was single symbol, the following new state would be created:
 $\text{Goto}(\{S' \rightarrow \cdot S, S \rightarrow \cdot \alpha, S \rightarrow \cdot \beta\}, \alpha) =$
 $\text{Closure}(\{S \rightarrow \alpha \cdot\}) = \{S \rightarrow \alpha \cdot\}$
4. Repeatedly perform gotos until there are no more states to add

Closure Function

□ Closure(I) where I is a set of items

- Returns the state (set of items) that express the same position as I
- Items in I are called **kernel items**
- Rest of items in closure(I) are called **non-kernel items**

□ Let N be a non-terminal

- If dot is in front of N, then add each production for that N and put dot at the beginning of the RHS
 - $A \rightarrow \alpha . B \beta$ is in I ; we expect to see a string derived from B
 - $B \rightarrow . \gamma$ is added to the closure, where $B \rightarrow \gamma$ is a production
 - Apply rule until nothing is added

➤ Given

$$\begin{aligned} S &\rightarrow E \\ E &\rightarrow E + T \\ E &\rightarrow T \\ T &\rightarrow \text{id} \mid (E) \end{aligned}$$

$\text{Closure}(\{ S \rightarrow . E \}) = \{ S \rightarrow . E, E \rightarrow . E + T, E \rightarrow . T, T \rightarrow . \text{id}, T \rightarrow . (E) \}$

Kernel and Non-kernel Items

□ Two kinds of items

➤ Kernel items

- Items that act as “**seed**” items when creating a state
- What items act as seed items when states are created?
 - Initial state: $S' \rightarrow \cdot S$
 - Additional state: from $\text{goto}(I, X)$ so has X at left of dot
- Besides $S' \rightarrow \cdot S$, all kernel items have **dot in the middle of RHS**

➤ Non-kernel items

- Items added during the **closure** of kernel items
- All non-kernel items have **dot at the beginning of RHS**

Goto Function

- Goto (I, X) where I is a set of items and X is a symbol
 - Returns state (set of items) that can be reached by advancing X
 - For each $A \rightarrow \alpha . X \beta$ in I,
Closure($A \rightarrow \alpha X . \beta$) is added to goto(I, X)
 - X can be a terminal or non-terminal
 - Terminal if obtained from input string by shifting
 - Non-terminal if obtained from reduction
 - Example
 - $\text{Goto}(\{T \rightarrow . (E)\}, () = \text{closure}(\{T \rightarrow (. E)\})$
- Ensures every symbol consumption results in a viable prefix

Construction of DFA

- ❑ Algorithm to compute set C (set of all states in DFA)

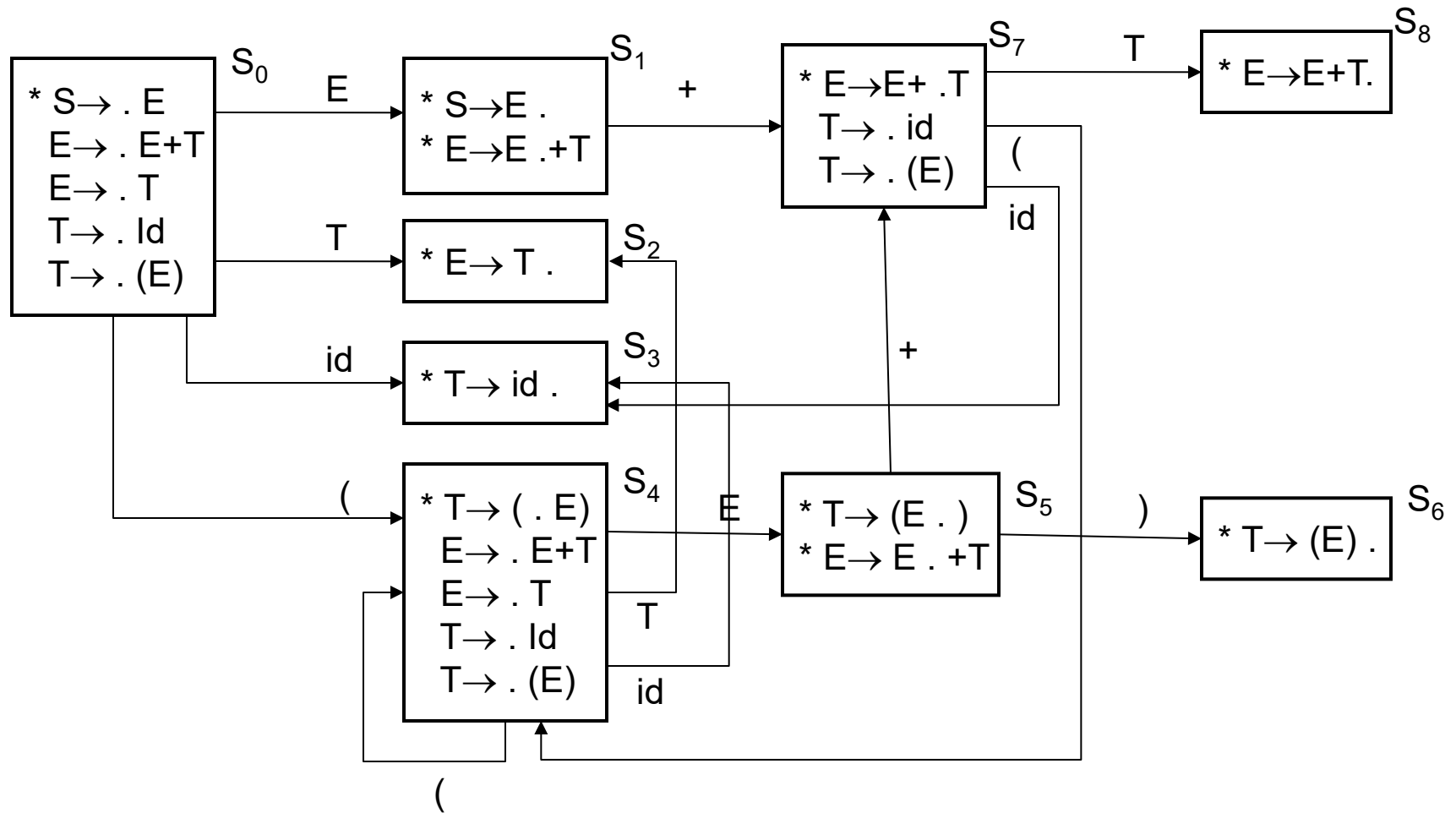
```
void constructDFA (G') {  
    C = {closure({S' → . S})} // Add initial state to C  
    repeat  
        for (each state I in C)  
            for (each grammar symbol X)  
                if (goto(I, X) is not empty and not in C)  
                    add goto(I, X) to C  
    until no new states are added to C  
}
```

- ❑ Add transitions from I to goto(I, X) on symbol X

□ Example: $S \rightarrow E$
 $E \rightarrow E + T \mid T$
 $T \rightarrow \text{id} \mid (E)$

- $S_0 = \text{closure}(\{S \rightarrow . E\})$
 $= \{S \rightarrow . E, E \rightarrow . E + T, E \rightarrow . T, T \rightarrow . \text{id}, T \rightarrow . (E)\}$
- $\text{goto}(S_0, E) = \text{closure}(\{S \rightarrow E ., E \rightarrow E . + T\})$
 $S_1 = \{S \rightarrow E ., E \rightarrow E . + T\}$
- $\text{goto}(S_0, T) = \text{closure}(\{E \rightarrow T .\})$
 $S_2 = \{E \rightarrow T .\}$
- $\text{goto}(S_0, \text{id}) = \text{closure}(\{T \rightarrow \text{id} .\})$
 $S_3 = \{T \rightarrow \text{id} .\}$
-
- $S_8 = \dots$

□ DFA for the previous grammar
 (* are closures applied to kernel items)



Building Parse Table from DFA

- ACTION [state, terminal symbol]
- GOTO [state, non-terminal symbol]
- Filling in the ACTION and GOTO cells
 1. If $[A \rightarrow \alpha \bullet a \beta]$ is in S_i and $\text{goto}(S_i, a) = S_j$, where “a” is a terminal
then $\text{ACTION}[S_i, a] = \text{shift } j$
 2. If $[A \rightarrow \alpha \bullet A \beta]$ is in S_i and $\text{goto}(S_i, A) = S_j$, where “A” is a non-terminal
then $\text{GOTO}[S_i, A] = S_j$
 3. If $[A \rightarrow \alpha \bullet]$ is in S_i
then $\text{ACTION}[S_i, a] = \text{reduce } A \rightarrow \alpha$ for all $a \in \text{Follow}(A)$
 4. If $[S' \rightarrow S_0 \bullet]$ is in S_i
then $\text{ACTION}[S_i, \$] = \text{accept}$
- Two potential prediction conflicts
 - Reduce-reduce conflict: when an ACTION cell has two 3s
 - Shift-reduce conflict: when an ACTION cell has both 1 and 3
 - ☞ More lookahead in $\text{Follow}(A)$ may improve prediction accuracy

□ Grammar

1. $S \rightarrow E$
2. $E \rightarrow E + T$
3. $E \rightarrow T$
4. $T \rightarrow \text{id}$
5. $T \rightarrow (E)$

Non-terminal	Follow
S	\$
E	+) \$
T	+) \$

ACTION

	+	id	()	\$
S0		s3	s4		
S1	s7				accept
S2	r3			r3	r3
S3	r4			r4	r4
S4		s3	s4		
S5	s7			s6	
S6	r5			r5	r5
S7		s3	s4		
S8	r2			r2	r2

GOTO

	E	T	S
S0	1	2	
S1			
S2			
S3			
S4	5	2	
S5			
S6			
S7		8	
S8			

Types of LR Parsers

- ❑ SLR – simple LR (what we saw so far was SLR(1))
 - Small parse table
 - Not as powerful
- ❑ Canonical LR
 - Much larger parse table
 - More powerful (can parse more grammars)
- ❑ LALR
 - Look ahead LR
 - In between the 2 previous ones in power and overhead

Overall parsing algorithm is the same – table is different

Conflict due to not enough lookahead

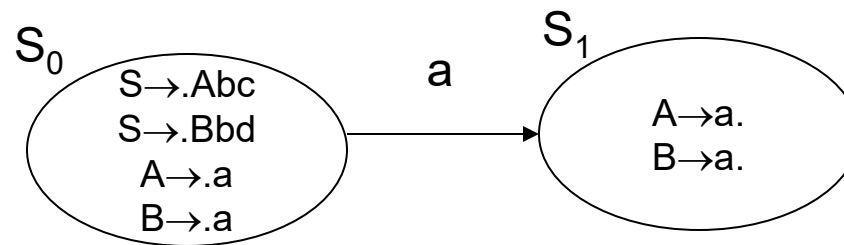
- Consider the grammar G

$S \rightarrow A b c \mid B b d$

$A \rightarrow a$

$B \rightarrow a$

$b \in \text{Follow}(A)$ and also $b \in \text{Follow}(B)$



- What is reduced when “a b” is seen? reduce to A or B?
 - Reduce-reduce conflict
- G is not SLR(1) but SLR(2)
 - We need 2 symbols of look ahead to look past b:
 - b c – reduce to A
 - b d – reduce to B
 - Possible to extend SLR(1) to k symbols of look ahead – allows larger class of CFGs to be parsed

SLR(k)

□ Extend SLR(1) definition to SLR(k) as follows

let $\alpha, \beta \in V^*$

- $\text{First}_k(\alpha) = \{ x \in V_T^* \mid (\alpha \Rightarrow *x\beta \text{ where } |x|=k) \text{ or } (\alpha \Rightarrow *x \text{ where } |x| \leq k) \}$
 - all k-symbol terminal prefixes of strings derivable from α
- $\text{Follow}_k(B) = \{ w \in V_T^* \mid S \Rightarrow *\alpha B\gamma \text{ and } w \in \text{First}_k(\gamma) \}$
 - all k symbol terminal strings that can follow B in some derivation

SLR(k) Parse Table

Let S be a state and lookahead $b \in V_T^*$ such that $|b| \leq k$

1. If $A \rightarrow \alpha. \in S$ and $b \in \text{Follow}_k(A)$ then
 - $\text{Action}(S, b)$ – reduce using production $A \rightarrow \alpha$,
2. If $D \rightarrow \alpha.a \gamma \in S$ and $a \in V_T$ and $b \in \text{First}_k(a \gamma \text{ Follow}_k(D))$
 - $\text{Action}(S, b) = \text{shift “a” and push state goto}(S, a)$

For $k = 1$, this definition reduces to SLR(1)

Reduce: Trivially true

Shift: $\text{First}_1(a \gamma \text{ Follow}_1(D)) = \{a\}$

$SLR(k-1) \subset SLR(k)$

□ Consider

$$S \rightarrow A b^{k-1} c \mid B b^{k-1} d$$

$$A \rightarrow a$$

$$B \rightarrow a$$

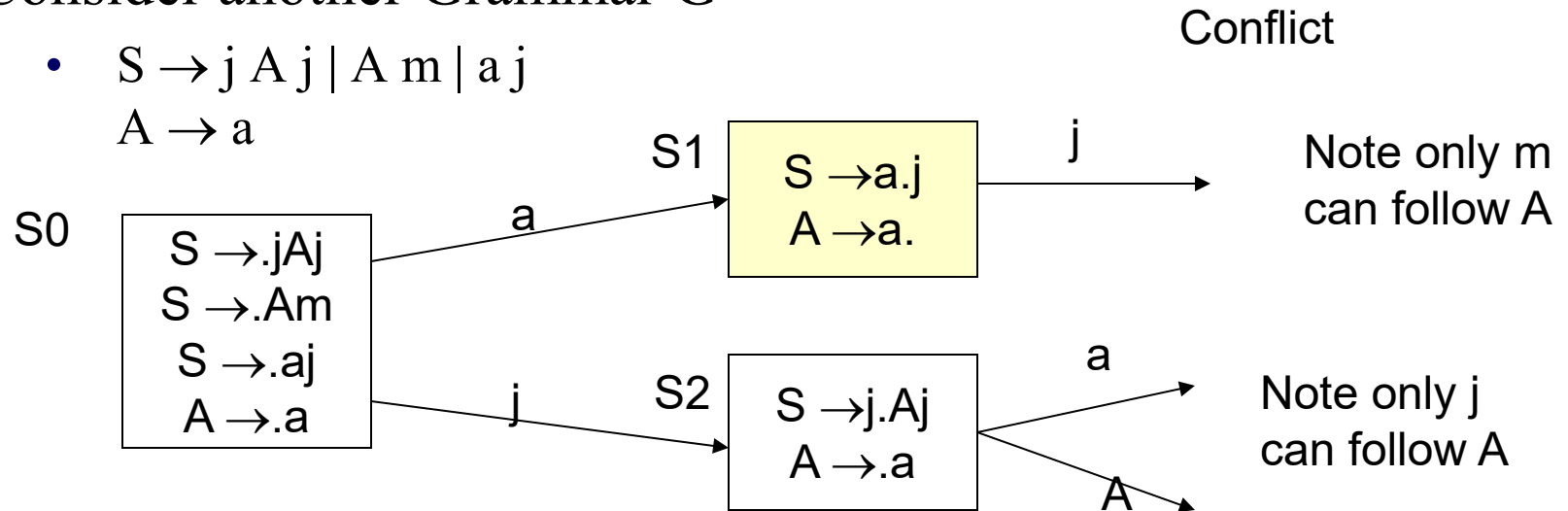
$SLR(k)$ not $SLR(k-1)$

- cannot decide what to reduce,
- reduce a to A or B depends the next k symbols
 $b^{k-1} c$ or $b^{k-1} d$

Non-SLR(k) for any k

□ Consider another Grammar G

- $S \rightarrow j A j \mid A m \mid a j$
 $A \rightarrow a$



$\text{Follow}(A) = \{j, m\}$

State S1: $[A \rightarrow a.]$ – reduce using this production (on j or m)

$[S \rightarrow a.j]$ – shift j \rightarrow shift-reduce conflict \rightarrow not SLR(1)

? SLR(k) for some $k > 1$?

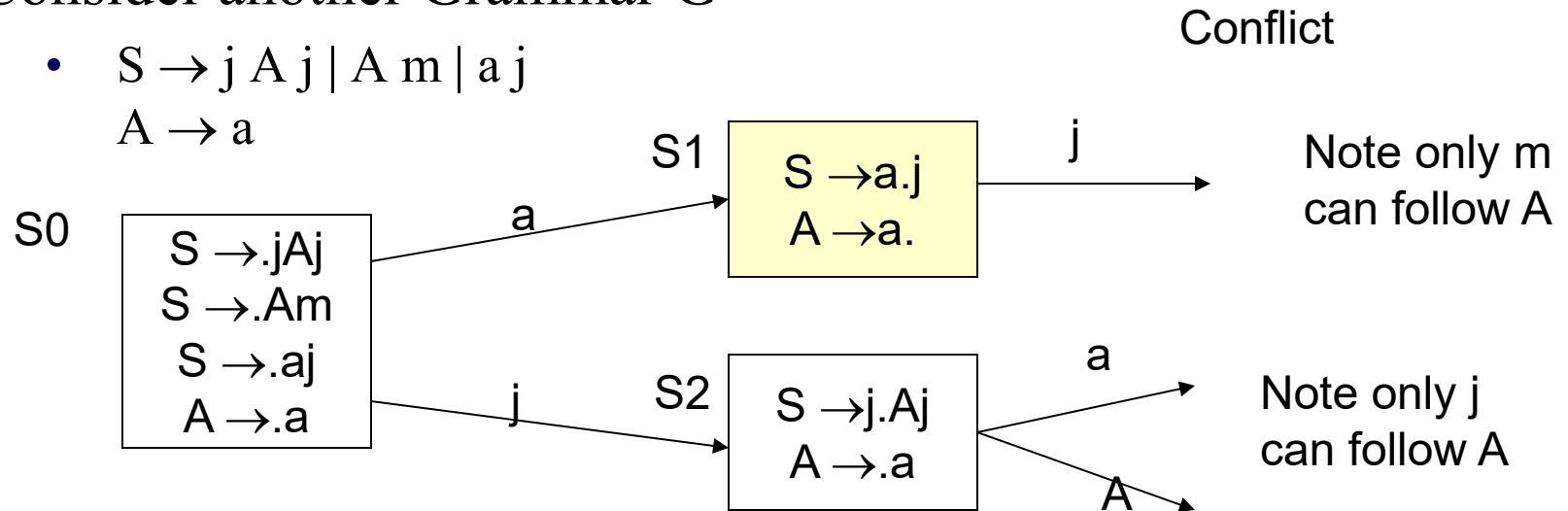
For reducing $A \rightarrow a.$: $\text{Follow}_k(A) = \text{First}_k(j\text{Follow}_k(S)) + \text{First}_k(m\text{Follow}_k(S)) = \{j\$, m\$, \}$,

For shifting $S \rightarrow a.j$: $\text{First}_k(j\text{Follow}_k(S)) = \{j\$, \}$ so not SLR(k) for any k !!!

Non-SLR(k) for any k

□ Consider another Grammar G

- $S \rightarrow j A j \mid A m \mid a j$
 $A \rightarrow a$



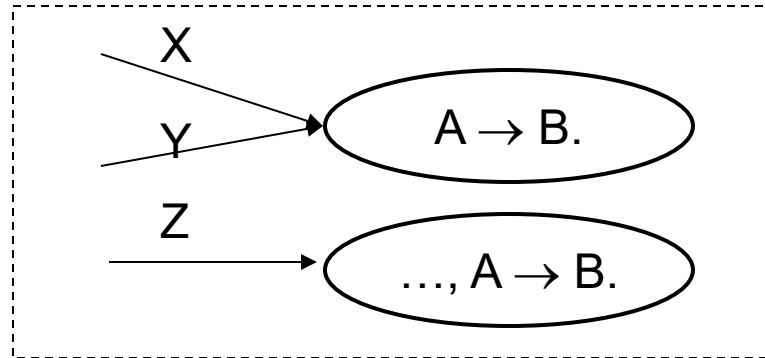
□ Problem: $\text{Follow}(A) = \{j, m\}$ is too imprecise

- In S1, we should reduce A only when lookahead is $\{m\}$
- The fact that $\{j\}$ can follow A in another context is irrelevant

□ Canonical LR:

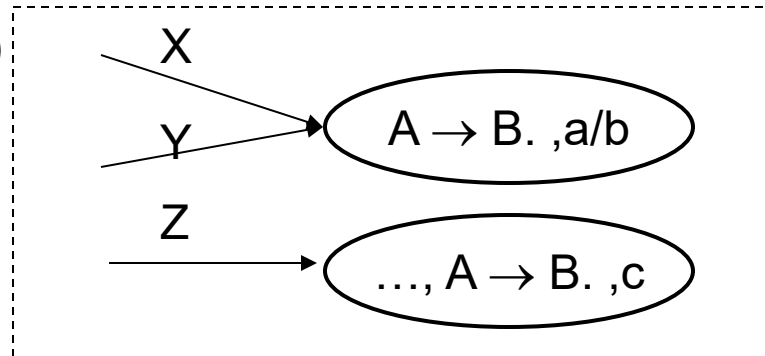
- Encode appropriate lookahead for the reduction of each LR item
- Appropriate lookahead is the follow set in the given context

SLR(1)



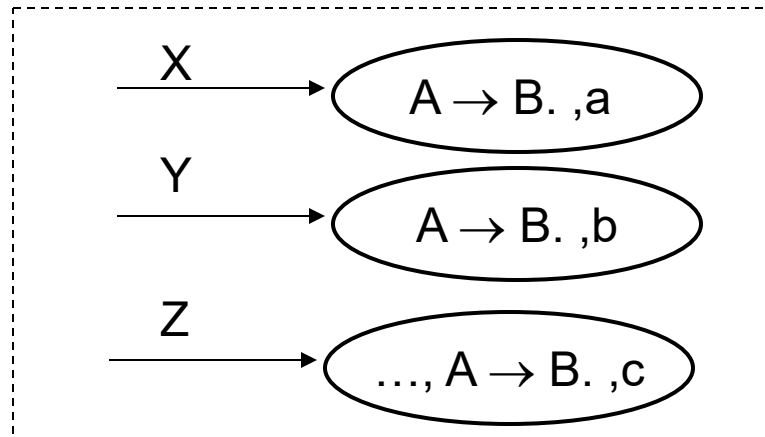
$\text{Follow}(A) = \{a, b, c\}$

LALR(1)



state merging

LR(1)



state splitting

Constructing Canonical LR

□ LR(1) item: LR item with one lookahead

- $[A \rightarrow \alpha.\beta, \mathbf{a}]$ where $A \rightarrow \alpha\beta$ is a production and \mathbf{a} is a terminal or $\$$
 - Meaning: Only terminal \mathbf{a} can follow A in this context
 - When we reach $[A \rightarrow \alpha\beta., \mathbf{a}]$, reduce A only if lookahead matches terminal \mathbf{a}
- $[A \rightarrow \alpha.\beta, \mathbf{a/b}]$: means both \mathbf{a} and \mathbf{b} can follow A in this context
 - $\{a, b\} \subseteq \text{Follow}(A)$, a more precise version of the follow set

□ LR(k) item: LR item with k lookahead

- $[A \rightarrow \alpha.\beta, \mathbf{a/b}]$: $a, b \in V_T^*$ such that $|a| \leq k, |b| \leq k$

Constructing Canonical LR

- Essentially the same as LR(0) items only adding lookahead

- Modify closure and goto function

- Changes for **closure** function

- Initialize lookahead:

- If $[A \rightarrow \alpha.B\beta, a]$ and $B \rightarrow \delta$,

- then $[B \rightarrow \cdot \delta, c] \in \text{closure}([A \rightarrow \alpha.B\beta, a])$, where $c \in \text{First}(\beta a)$

- Changes for **goto** function

- Carry over lookahead:

- if $[A \rightarrow \alpha.X\beta, a] \in I$, then $\text{goto}(I, X) = [A \rightarrow \alpha X \cdot \beta, a]$

Example

□ Grammar

$S' \rightarrow S$

$S \rightarrow CC$

$C \rightarrow eC \mid d$

□ S0: $\text{closure}(S' \rightarrow .S, \$)$

$[S' \rightarrow .S, \$]$

$[S \rightarrow .CC, \$]$

$\text{first}(\epsilon \$) = \{\$ \}$

$[C \rightarrow .eC, e/d]$

$\text{first}(C \$) = \{e, d\}$

$[C \rightarrow .d, e/d]$

$\text{first}(C \$) = \{e, d\}$

□ S1: $\text{goto}(S0, S) = \text{closure}(S' \rightarrow S., \$)$

$[S' \rightarrow S., \$]$

□ S2: $\text{goto}(S0, C) = \text{closure}(S \rightarrow C.C, \$)$

$[S \rightarrow C.C, \$]$

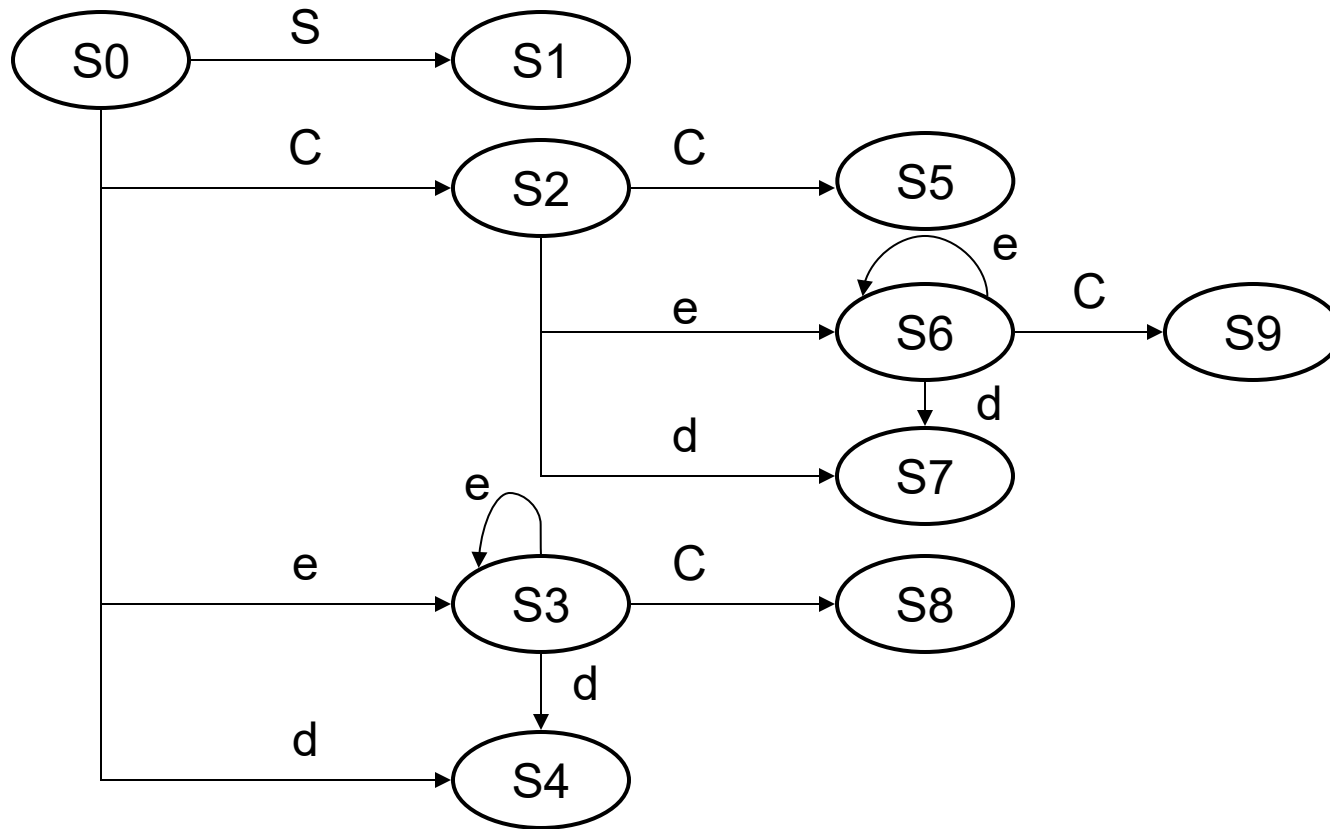
$[C \rightarrow .eC, \$]$

$\text{first}(\epsilon \$) = \{\$ \}$

$[C \rightarrow .d, \$]$

$\text{first}(\epsilon \$) = \{\$ \}$

- S3: $\text{goto}(S0, e) = \text{closure}(C \rightarrow e.C, e/d)$
 $[C \rightarrow e.C, e/d]$
 $[C \rightarrow .eC, e/d]$ $\text{first}(\epsilon e/d) = \{e, d\}$
 $[C \rightarrow .d, e/d]$ $\text{first}(\epsilon e/d) = \{e, d\}$
- S4: $\text{goto}(S0, d) = \text{closure}(C \rightarrow d., e/d)$
 $[C \rightarrow d., e/d]$
- S5: $\text{goto}(S2, C) = \text{closure}(S \rightarrow CC., \$)$
 $[S \rightarrow CC., \$]$
- S6: $\text{goto}(S2, e) = \text{closure}(C \rightarrow e.C, \$)$
 $[C \rightarrow e.C, \$]$
 $[C \rightarrow .eC, \$]$ $\text{first}(\epsilon \$) = \{\$ \}$
 $[C \rightarrow .d, \$]$ $\text{first}(\epsilon \$) = \{\$ \}$
- S7: $\text{goto}(S2, d) = \text{closure}(C \rightarrow d., \$)$
 $[C \rightarrow d., \$]$
- S8: $\text{goto}(S3, C) = \text{closure}(C \rightarrow eC., e/d)$
 $[C \rightarrow eC., e/d]$
- S9: $\text{goto}(S6, C) = \text{closure}(C \rightarrow eC., \$)$
 $[C \rightarrow eC., \$]$



Note S3, S6 are same except for lookahead (also true for S4, S7 and S8, S9)
In SLR(1) – one state represents both

Constructing Canonical LR Parse Table

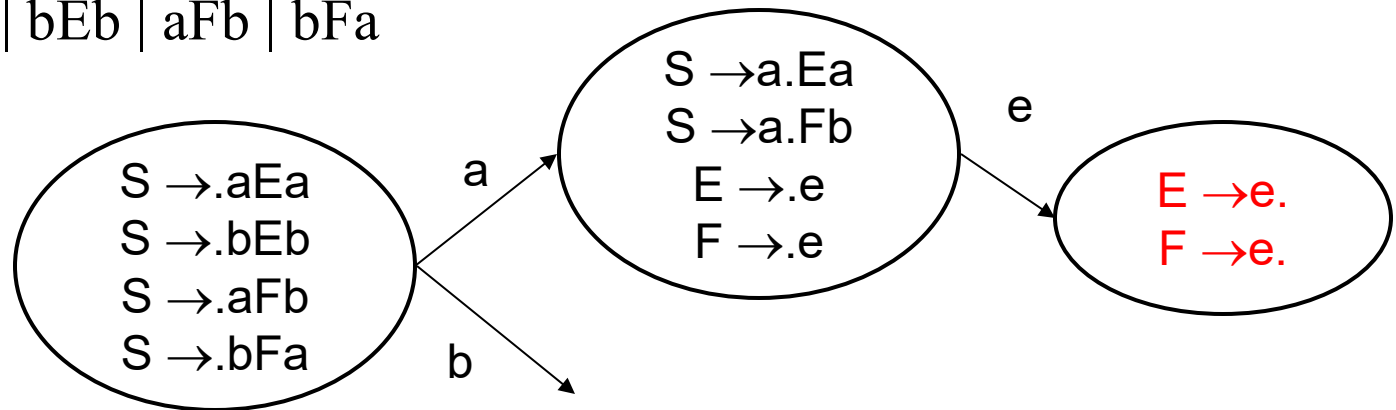
- ❑ Shifting: same as before
- ❑ Reducing:
 - Don't use follow set (too coarse grain)
 - Reduce only if input matches lookahead for item
- ❑ Action and GOTO
 1. if $[A \rightarrow \alpha \bullet a \beta, b] \in S_i$ and $\text{goto}(S_i, a) = S_j$,
Action[I,a] = s[Sj] – shift and goto state j if input matches a
Note: same as SLR
 2. if $[A \rightarrow \alpha \bullet, a] \in S_i$
Action[I,a] = r[R] – reduce R: $A \rightarrow \alpha$ if input matches a
Note: for SLR, reduced if input matches Follow(A)

□ Revisit SLR and LR

➤ $S \rightarrow aEa \mid bEb \mid aFb \mid bFa$

$E \rightarrow e$

$F \rightarrow e$



□ Not SLR(1): reduce/reduce conflict.

➤ $\text{Follow}(E) = \text{Follow}(F) = \{a, b\}$

□ LR(1): no conflict because state is split to account for context

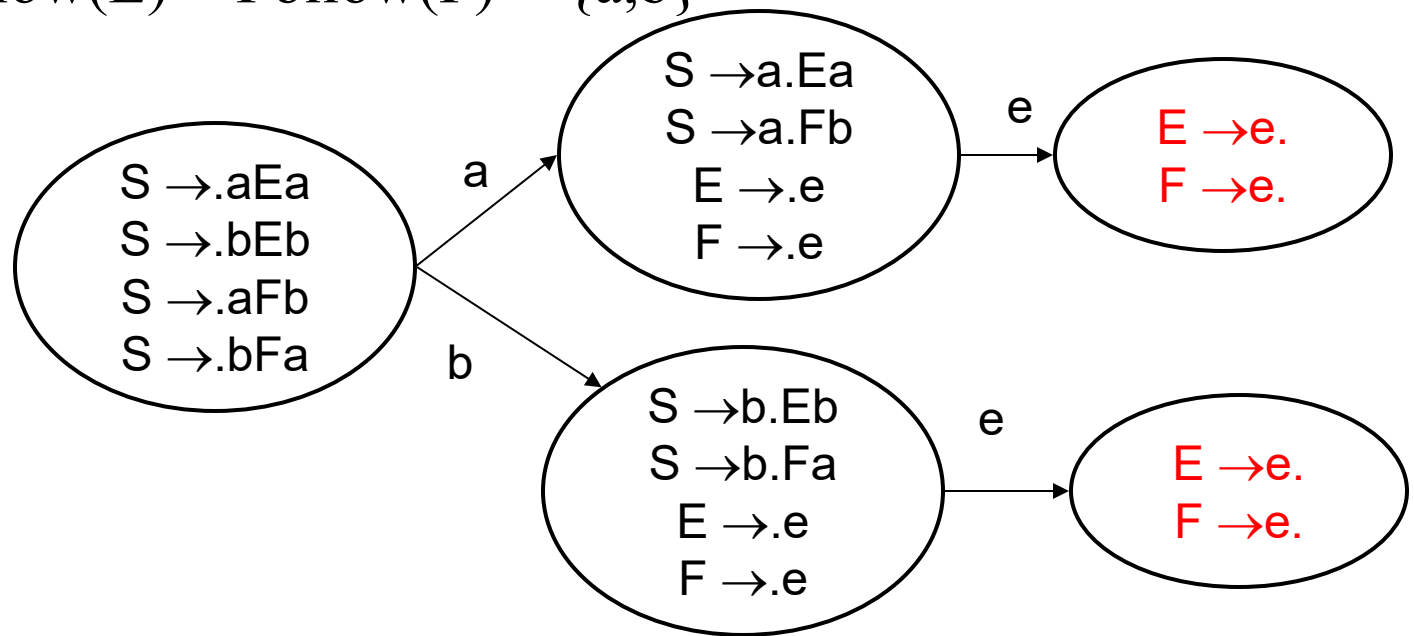
➤ $\text{Follow}(E) = \{a\}$ only if preceded by a

➤ $\text{Follow}(E) = \{b\}$ only if preceded by b

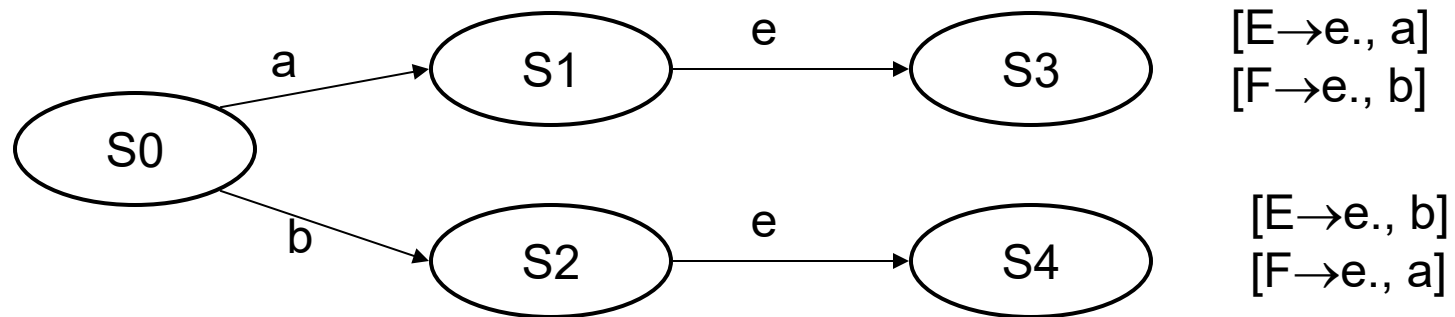
➤ $\text{Follow}(F) = \{a\}$ only if preceded by b

➤ $\text{Follow}(E) = \{b\}$ only if preceded by a

❑ SLR: $\text{Follow}(E) = \text{Follow}(F) = \{a, b\}$



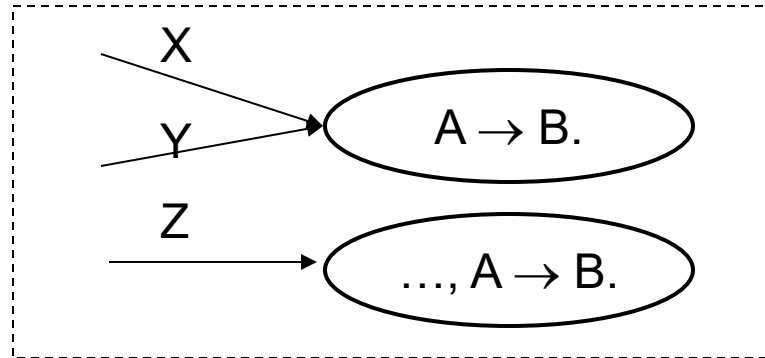
❑ LR: Follow sets more precise



SLR(1) and LR(1)

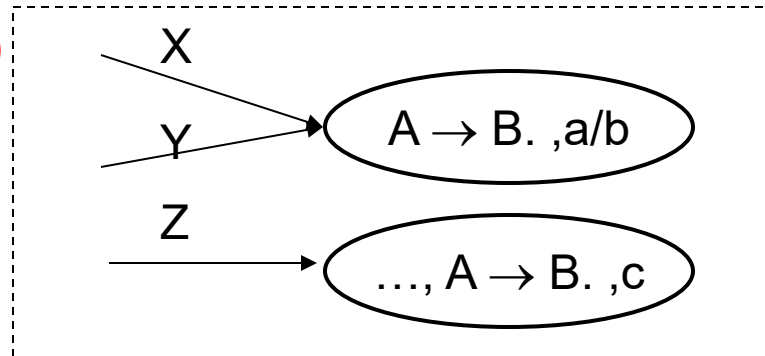
- ❑ LR(1) more powerful than SLR(1) – can parse more grammars
- ❑ But LR(1) may end up with many more states than SLR(1)
 - One LR(0) item may split up to many LR(1) items
(Potentially as many as the powerset of the entire alphabet)
- ❑ LALR(1) – compromise between LR(1) and SLR(1)
 - Constructed by merging LR(1) states with the same core
 - Ends up with same number of states as SLR(1)
 - But items still retain some lookahead info – still better than SLR(1)
 - Used in practice because most programming language syntactic structures can be represented by LALR (not true for SLR)

SLR(1)



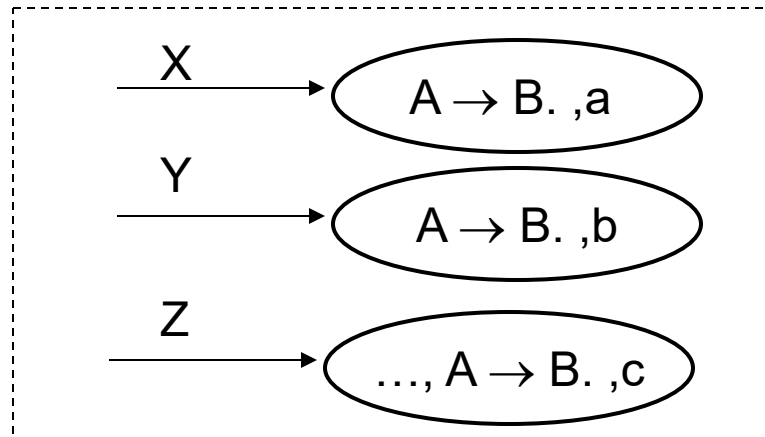
$\text{Follow}(A) = \{a, b, c\}$

LALR(1)

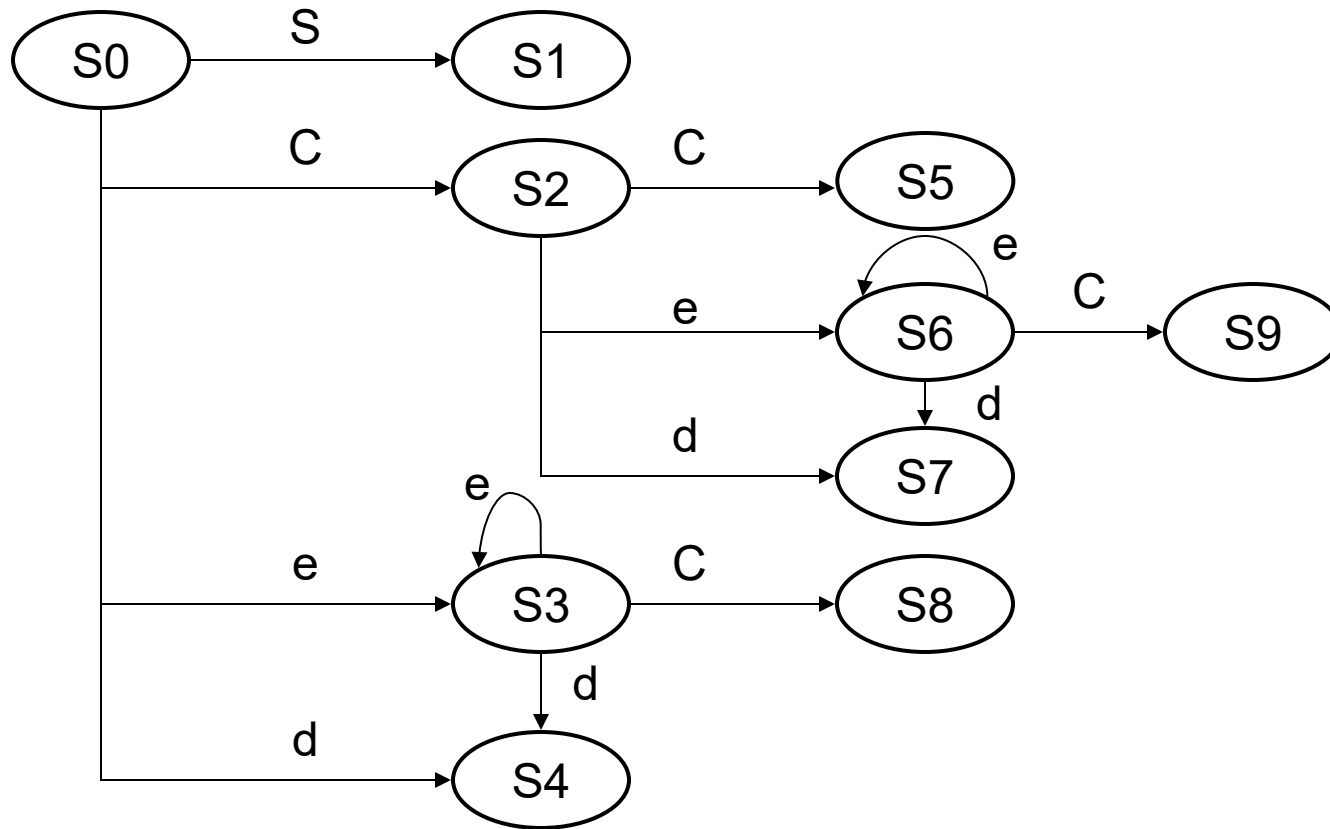


state merging

LR(1)



state splitting



Note S3 / S6, S4 / S7, S8 / S9 have same core (same except for lookahead).
In an SLR(1) parser, one state represents both states.

Example

□ Grammar

$$S' \rightarrow S$$
$$S \rightarrow CC$$
$$C \rightarrow eC \mid d$$

S3: goto(S0,e)=closure($C \rightarrow e.C$, e/d)

[$C \rightarrow e.C$, e/d]

[$C \rightarrow .eC$, e/d]

[$C \rightarrow .d$, e/d]

S4: goto(S0, d)=closure($C \rightarrow d.$, e/d)

[$C \rightarrow d.$, e/d]

S8: goto(S3, C)=closure($C \rightarrow eC.$, e/d)

[$C \rightarrow eC.$, e/d]

S6: goto(S2,e)=closure($C \rightarrow e.C$, \$)

[$C \rightarrow e.C$, \$]

[$C \rightarrow .eC$, \$]

[$C \rightarrow .d$, \$]

S7: goto(S2, d)=closure($C \rightarrow d.$, \$)

[$C \rightarrow d.$, \$]

S9: goto(S6,C)=closure($C \rightarrow eC.$, \$)

[$C \rightarrow eC.$, \$]

Note S3 / S6, S4 / S7, S8 / S9 have same core (same except for lookahead).
In an SLR(1) parser, one state represents both states.

Merging states

❑ Can merge S3 and S6

S3: goto(S0,e)=closure($C \rightarrow e.C$, e/d)

$[C \rightarrow e.C, e/d]$

$[C \rightarrow .eC, e/d]$

$[C \rightarrow .d, e/d]$

S6: goto(S2,e)=closure($C \rightarrow e.C$, \$)

$[C \rightarrow e.C, \$]$

$[C \rightarrow .eC, \$]$

$[C \rightarrow .d, \$]$

S36: $[C \rightarrow e.C, e/d/\$]$

$[C \rightarrow .eC, e/d/\$]$

$[C \rightarrow .d, e/d/\$]$

❑ Similarly

- S47: $[C \rightarrow d., e/d/\$]$
- S89: $[C \rightarrow eC., e/d/\$]$

Effect of Merging: Introduces conflicts

1. Merging of states can introduce conflicts

- cannot introduce shift-reduce conflicts
- can introduce reduce-reduce conflicts

□ Shift-reduce conflicts

Suppose S_{ij} : $[A \rightarrow \alpha., \mathbf{a}/b/c]$ reduce on input a
 $[B \rightarrow \beta.\mathbf{a}\delta, x/y/z]$ shift on input a
formed by merging S_i and S_j

Then, S_i : $[A \rightarrow \alpha., \text{lookahead}_i]$ S_j : $[A \rightarrow \alpha., \text{lookahead}_j]$
 $[B \rightarrow \beta.\mathbf{a}\delta, \dots]$ $[B \rightarrow \beta.\mathbf{a}\delta, \dots]$

And, either $\mathbf{a} \in \text{lookahead}_i$ or $\mathbf{a} \in \text{lookahead}_j$

☞ Conflict existed in the first place!

A reduce-reduce conflict due to merging

$S \rightarrow aEa \mid bEb \mid aFb \mid bFa$

$E \rightarrow e$

$F \rightarrow e$

S3: $[E \rightarrow e., a]$

$[F \rightarrow e., b]$

S4: $[E \rightarrow e., b]$

$[F \rightarrow e., a]$

After merging S34: $[E \rightarrow e., a/b]$

$[F \rightarrow e., a/b]$

- Both reductions are applied on lookahead a and b,
i.e. reduce-reduce conflict

Effect of Merging: Delays error detection

2. Detection of errors may be delayed

- On error, LALR parsers will not perform shifts beyond an LR parser but may perform more reductions before finding error

- Example: $S' \rightarrow S$ $S \rightarrow CC$

$C \rightarrow eC \mid d$

and input string eed\$

- Canonical LR: Parse Stack $S0 \ e \ S3 \ e \ S3 \ d \ S4$

State $S4$ on $\$$ input = error $S4: \{C \rightarrow d., e/d\}$

- LALR:

stack: $S0 \ e \ S_{36} \ e \ S_{36} \ d \ S_{47}$ \rightarrow state S_{47} input $\$$, reduce $C \rightarrow d$

stack: $S0 \ e \ S_{36} \ e \ S_{36} \ C \ S_{89}$ \rightarrow reduce $C \rightarrow eC$

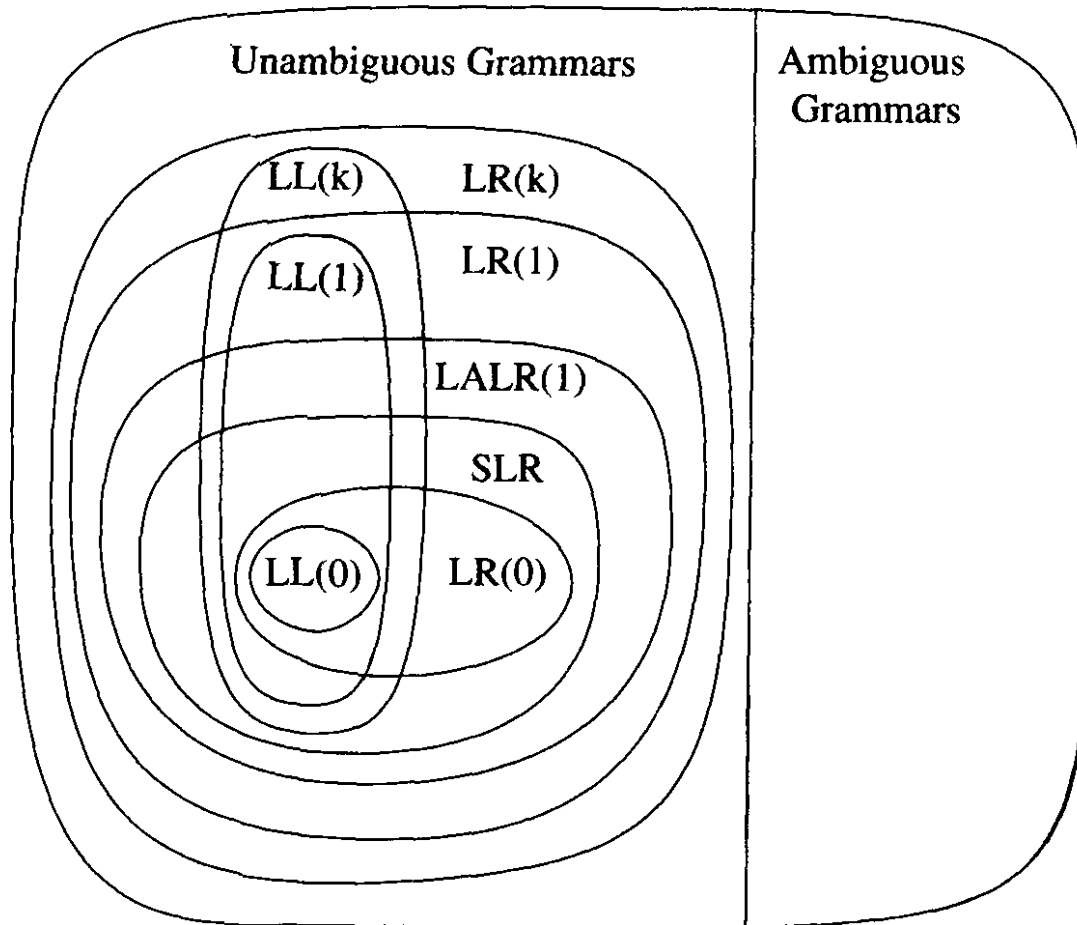
stack: $S0 \ e \ S_{36} \ C \ S_{89}$ \rightarrow reduce $C \rightarrow eC$

stack: $S0 \ C \ S2$ \rightarrow state $S2$ on input $\$$, error

Error Recovery

- ❑ To uncover multiple errors, parser must be able to recover from errors.
- ❑ Simple error recovery (by discarding offending code sequence)
 1. Decide on non-terminal A: candidate for discarding
 - Typically, an expression, statement, or block of code
 2. Continue to scan down the stack until a state S with a goto on a particular non-terminal A is found
 3. Discard input tokens until a token 'a' is found that can follow A
 - E.g. if A is a statement, then 'a' would be ';'.
 4. Push state Goto[a,A] on stack and continue parsing

A Hierarchy of Grammar Classes



$LALR(k) \subset LR(k)$

- ❑ $LR(k)$ is strictly more powerful compared to $LALR(k)$
 - $LALR$ merges states, which can introduce conflicts
- ❑ Unlike LL and LR , no formal definition on what is $LALR$
 - Definition by construction: if $LALR$ parser has no conflicts
 - Conflicts due to state merging are hard to define formally (Hence, they are unpredictable and hard to reason with)
- ❑ Nonetheless, $LALR(1)$ has become popular
 - YACC, Bison, etc.
 - Most programming languages have an $LALR(1)$ grammar
 - Reduce-reduce conflicts due to state merging are rare (conflicts are mostly due to ambiguity)

SLR(k) \subset LALR(k)

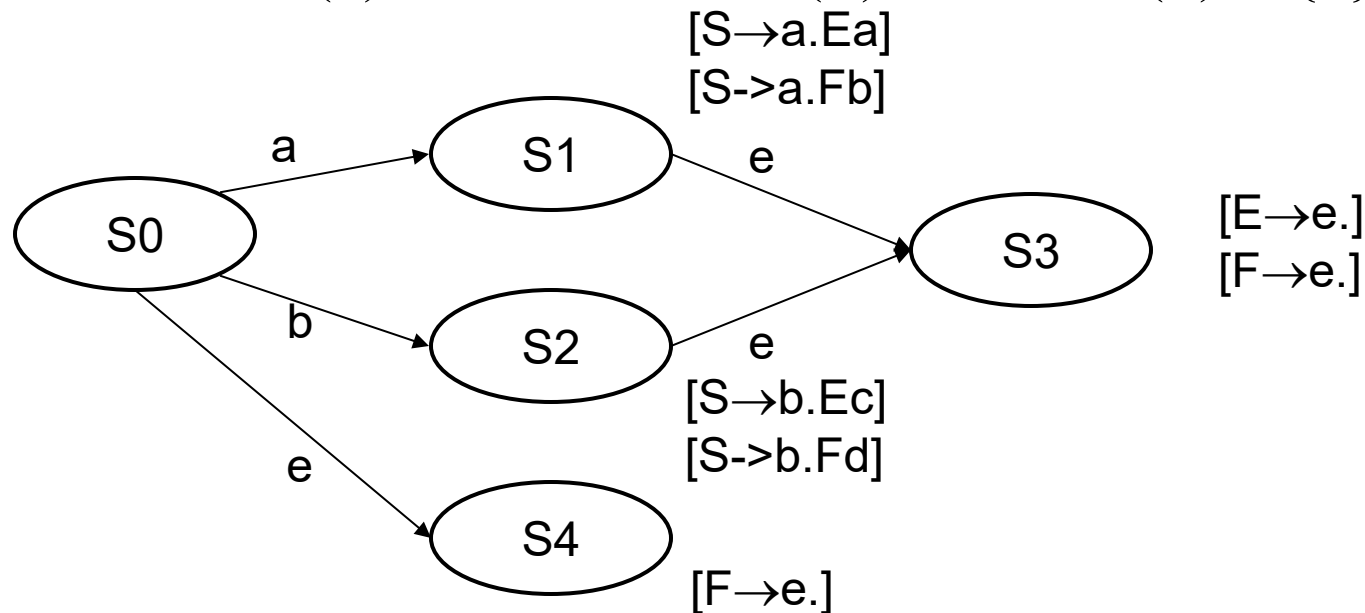
□ Let's consider this Grammar G:

➤ $S \rightarrow aEa \mid aFb \mid bEc \mid bFd \mid Fa$

$E \rightarrow e$

$F \rightarrow e$

□ It is non-SLR(1) because $\text{Follow}(E) \cap \text{Follow}(F) = \{a\}$



SLR(k) \subset LALR(k)

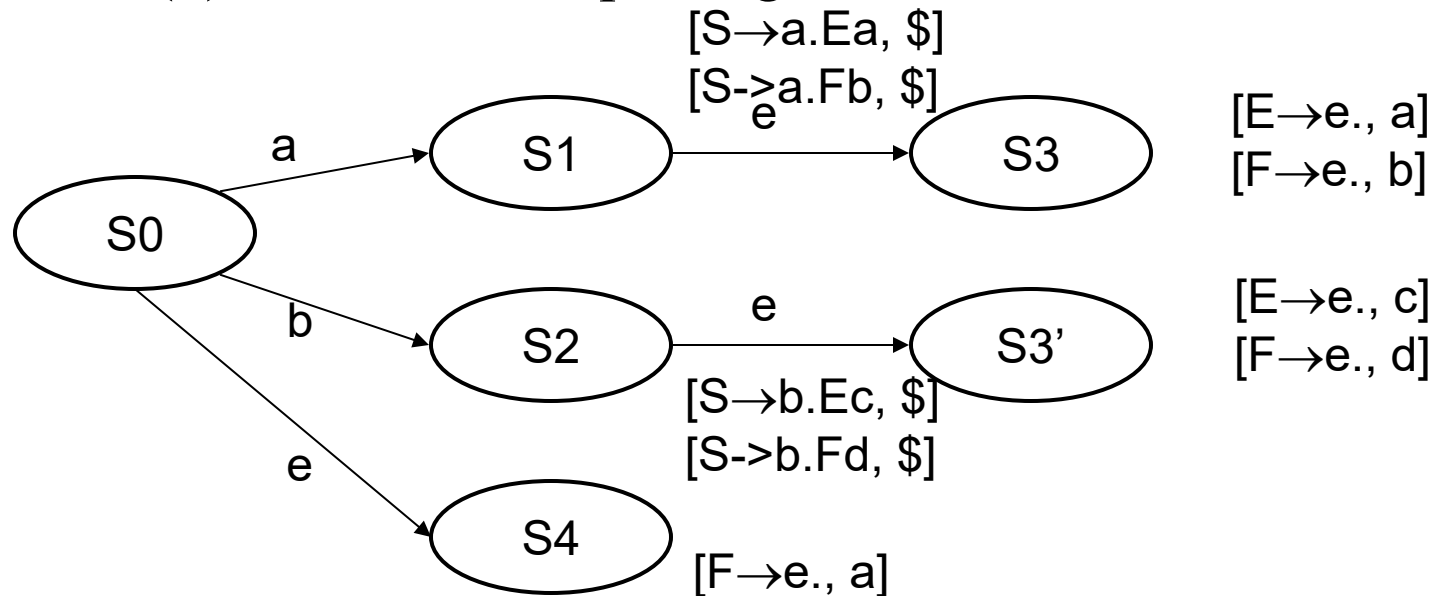
□ Let's consider this Grammar G:

➤ $S \rightarrow aEa \mid aFb \mid bEc \mid bFd \mid Fa$

$E \rightarrow e$

$F \rightarrow e$

□ But LR(1) thanks to S3 splitting to S3 and S3'



SLR(k) \subset LALR(k)

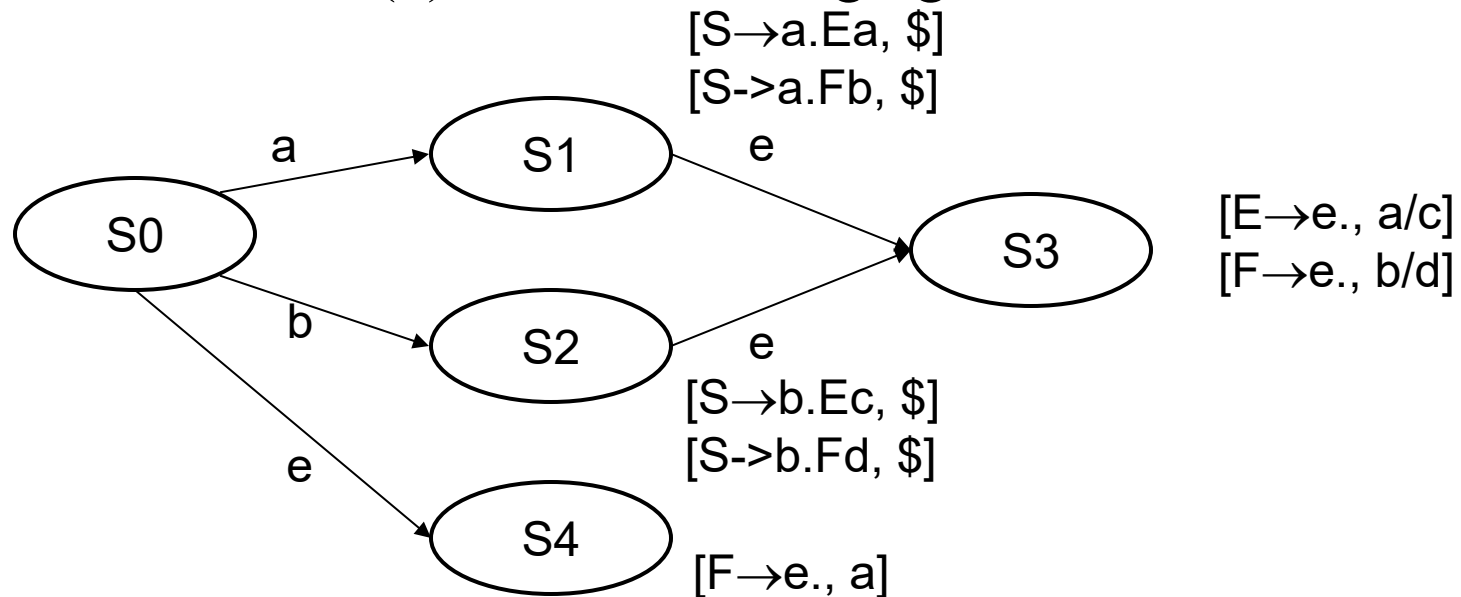
□ Let's consider this Grammar G:

➤ $S \rightarrow aEa \mid aFb \mid bEc \mid bFd \mid Fa$

$E \rightarrow e$

$F \rightarrow e$

□ And also LALR(1) even after merging back S3' and S3



$LL(k) \subset LR(k)$

- ❑ LL(k) parser, each expansion $A \rightarrow \alpha$ is decided on the basis of
 - Current non-terminal at the top of the stack
 - Which LHS to produce
 - k terminals of lookahead at *beginning* of RHS
 - Must guess which RHS by peeking at first few terminals of RHS
- ❑ LR(k) parser, each reduction $A \rightarrow \alpha \bullet$ is decided on the basis of
 - RHS at the top of the stack
 - Can postpone choice of RHS until entire RHS is seen
 - Common left factor is okay – waits until entire RHS is seen anyway
 - Left recursion is okay – does not impede forming RHS for reduction
 - k terminals of lookahead *beyond* RHS
 - Can decide on RHS after looking at entire RHS plus lookahead

LL(k) \neq SLR(k)

- ❑ Neither is strictly more powerful than the other
- ❑ Advantage of SLR: can delay decision until entire RHS seen
 - LL must decide RHS with a few symbols of lookahead
- ❑ Disadvantage of SLR: lookahead applied out of context
 - Consider grammar: $S \rightarrow Bb \mid Cc \mid aBc$, $B \rightarrow \varepsilon$, $C \rightarrow \varepsilon$
 - Initial state $S_0 = \{S \rightarrow . Bb \mid . Cc \mid . aBc, B \rightarrow ., C \rightarrow .\}$
 - For SLR(1), reduce-reduce conflict on $B \rightarrow .$ and $C \rightarrow .$
 - $\text{Follow}(B) = \{b, \textcolor{red}{c}\}$ and $\text{Follow}(C) = \{\textcolor{red}{c}\}$
 - For LL(1), no conflict
 - $\text{First}(Bb) = \{b\}$, $\text{First}(Cc) = \{c\}$, $\text{First}(aBC) = \{a\}$
- ❑ For the same reason, LL \neq LALR

$$LL(0) \subset LR(0) \equiv LALR(0) \equiv SLR(0)$$

□ $LR(0) \equiv LALR(0) \equiv SLR(0)$

- $LR(0) \equiv LALR(0) \equiv SLR(0)$ since lookahead is meaningless.
- If a state has a reduce item, there can be no other items.
(If there is, it will result in a conflict with the reduce action.)
- This makes grammars very restrictive and unusable.

□ $LL(0) \subset LR(0)$

- $LL(0)$ can only have one RHS per non-terminal to avoid conflict.
- $LR(0)$ can still have multiple RHSs per non-terminal.
- E.g. $S \rightarrow a \mid b$ is not $LL(0)$ but is $LR(0)$.

$L(\text{GLR}) \equiv L(\text{CFG})$

□ GLR: Generalized LR parser where $L(\text{GLR}) \equiv L(\text{CFG})$

➤ “Parsing Techniques. A Practical Guide.” by Grune et al. (2008)

<https://link.springer.com/book/10.1007/978-0-387-68954-8>

➤ An LR family parser that does the following on a conflict

1. Fork the parse stack and follow each action separately
2. If forked parse stack results in a parse error, discard it

➤ Uses any LR table (e.g. SLR, LALR, Canonical LR)

➤ GNU Bison: an implementation of GLR

<https://www.gnu.org/software/bison/>

$L(\text{GLL}) \equiv L(\text{CFG})$

❑ Is there a generalized LL parser that can parse all CFGs?

➤ Recall, LL parsers have trouble with left-recursion

❑ GLL: Generalize LL parser

➤ “GLL Parsing” by Scott et al. (2010)

<https://www.sciencedirect.com/science/article/pii/S1571066110001209>

➤ How does it deal with left-recursion?

- Idea similar to GLR: fork stack on every conflict due to left-recursion (And try out all numbers of left-recursion until parse is successful)
- Difference is, you can potentially end up with many more forked stacks
- Developed “Graph Structured Stack” to minimize stack memory

➤ GoGLL: an implementation of GLL

<https://github.com/goccmack/gogll>

Using Automatic Tools

-- YACC

Pitt, CS 1622

Using a Parser Generator

- ❑ YACC is an LALR(1) parser generator
 - YACC: Yet Another Compiler-Compiler
- ❑ YACC constructs an LALR(1) table and reports an error when a table entry is multiply defined
 - A shift and a reduce – reports shift/reduce conflict
 - Multiple reduces – reports reduce/reduce conflict
 - Most conflicts are due to ambiguous grammars
 - Must resolve conflicts
 - By specifying associativity or precedence rules
 - By modifying the grammar
 - YACC outputs detail about where the conflict occurred (by default, in the file “y.output”)

Shift/Reduce Conflicts

- Typically due to ambiguities in the grammar
- Classic example: the dangling else

$S \rightarrow \text{if } E \text{ then } S \mid \text{if } E \text{ then } S \text{ else } S \mid \text{OTHER}$

will have DFA state containing

$[S \rightarrow \text{if } E \text{ then } S. , \text{else}]$

$[S \rightarrow \text{if } E \text{ then } S. \text{ else } S , \text{else}]$

so on ‘else’ we can shift or reduce

- Default (YACC, bison, etc.) behavior is to shift
 - Default behavior is the correct one in this case
 - Better not to rely on this and remove ambiguity

More Shift/Reduce Conflicts

□ Consider the ambiguous grammar

$$E \rightarrow E + E \mid E * E \mid \text{int}$$

we will have the states containing

$$\begin{array}{ll} [E \rightarrow E * . E, +/*] & [E \rightarrow E * E . , +/*] \\ [E \rightarrow . E + E, +/*] & \xRightarrow{E} [E \rightarrow E . + E, +/*] \end{array}$$

...

...

Again we have a shift/reduce conflict on input +

- In this case, we need to reduce (* is higher than +)
- Easy (better) solution: declare precedence rules for * and +
- Hard solution: rewrite grammar to be unambiguous

More Shift/Reduce Conflicts

□ Declaring precedence and associativity in YACC

`%left '+' '-'`

`%left '*' '/'`

➤ Interpretation:

- `+`, `-`, `*`, `/` are left associative
- `+`, `-` have lower precedence compared to `*`, `/`
(associativity declarations are in the order of increasing precedence)
- Precedence of a candidate rule for reduction is the precedence of the last terminal in that rule (e.g. For `'E → E+E.'`, level is same as `'+'`)

➤ Resolve shift/reduce conflict with a shift if:

- No precedence declared for either rule or terminal
- Input terminal has higher precedence than the rule
- The precedence levels are the same and right associative

Use Precedence to Solve S/R Conflict

$$\begin{array}{ccc} [E \rightarrow E^* \cdot E, +/*] & & [E \rightarrow E^* E \cdot, +/*] \\ [E \rightarrow \cdot E + E, +/*] & \xRightarrow{E} & [E \rightarrow E \cdot + E, +/*] \\ \dots & & \dots \end{array}$$

- we will choose reduce because precedence of rule $E \rightarrow E^* E$ is higher than that of terminal $+$

$$\begin{array}{ccc} [E \rightarrow E + \cdot E, +/*] & & [E \rightarrow E + E \cdot, +/*] \\ [E \rightarrow \cdot E + E, +/*] & \xRightarrow{E} & [E \rightarrow E \cdot + E, +/*] \\ \dots & & \dots \end{array}$$

- we will choose reduce because $E \rightarrow E + E$ and $+$ have the same precedence and $+$ is left-associative

❑ Back to our dangling else example

$[S \rightarrow \text{if } E \text{ then } S. , \text{ else}]$

$[S \rightarrow \text{if } E \text{ then } S. \text{ else } S, \text{ else}]$

- Can also eliminate conflict by precedence declarations:
%nonassoc 'then'
%nonassoc 'else'
- Perhaps less intuitive compared to arithmetic precedence
- Use precedence only if it enhances readability of code

Reduce/Reduce Conflicts

- Usually due to ambiguity in the grammar

- Example: a sequence of identifiers

$$S \rightarrow \varepsilon \mid \text{id} \mid \text{id } S$$

There are two rightmost derivations for the string 'id'

$$S \Rightarrow \text{id}$$

$$S \Rightarrow \text{id } S \Rightarrow \text{id}$$

How does this ambiguous grammar confuse the parser?

Reduce/Reduce Conflicts

□ Consider the states

$[S' \rightarrow .S, \$]$

$[S \rightarrow ., \$]$

$[S \rightarrow .id, \$]$

$[S \rightarrow .id S, \$]$

\xRightarrow{id}

$[S \rightarrow id. , \$]$

$[S \rightarrow id.S, \$]$

$[S \rightarrow ., \$]$

$[S \rightarrow .id, \$]$

$[S \rightarrow .id S, \$]$

Reduce/reduce conflict on input “id\$”

$S' \Rightarrow S \Rightarrow id$

$S' \Rightarrow S \Rightarrow id S \Rightarrow id$

Remove ambiguity by rewriting the grammar: $S \rightarrow \varepsilon \mid id S$

Semantic Actions

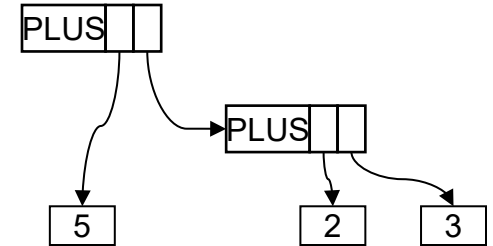
- ❑ Semantic actions are implemented for LR parsing
 - keep attributes on the semantic stack – parallel to the parse stack
 - on shift a , push attribute for a on semantic stack
 - on reduce $X \rightarrow \alpha$
 - pop attributes for α
 - compute attribute for X based on attributes for α
 - push it on the semantic stack
- ❑ Creating an AST
 - Bottom up
 - Create leaf node from attribute values of token(s) in RHS
 - Create internal node from subtree(s) passed on from RHS

Performing Semantic Actions

❑ Example 1: attribute is value of expression

$$E \rightarrow T + E \qquad \{\$ \$ = \$1 + \$2;\}$$
$$T \mid \{\beta = 1\}$$
$$T \rightarrow \text{int} * T \qquad \{\$ \$ = \$1 * \$2;\}$$
$$| \text{int} \quad \{ \$\$ = \$1; \}$$

consider the parsing of the string $3 * 5 + 8$



❑ Example 2: attribute is AST for expression

$$E \rightarrow \text{int} \quad \{\$ \$ = \text{mkleaf}(\$1);\}$$

E+E	{ \$\$ = mktree(plus, \$1, \$2); }
-----	------------------------------------

(E) $\{\$ = \$1;\}$

- a bottom-up evaluation of the ast attribute:

```
E.ast = mktree(plus, mkleaf(5),
               mktree(plus, mkleaf(2), mkleaf(3) ) )
```