Compiler Optimization

Compiler optimizations transform code

- Code optimization transforms code to equivalent code
  - ... but with better performance

- The code transformation can involve either
  - **Replacing** code with more efficient code
  - **Deleting** redundant code
  - **Moving** code to a position where it is more efficient
  - **Inserting** new code to improve performance

The four categories of code transformations

- Replacing code (e.g. **strength reduction**)

  A=2*a; ≡ A=a«1;

- Deleting code (e.g. **dead code elimination**)

  A=2; A=y; ≡ A=y;

- Moving code (e.g. **loop invariant code motion**)

```
for (i = 0; i < 100; i++) { sum += i + x * y; }
≡
t = x * y;
for (i = 0; i < 100; i++) { sum += i + t; }
```

- Inserting code (e.g. **data prefetching**)

```
for (p = head; p != NULL; p = p->next)
{ /* do work on node p */ }
≡
for (p = head; p != NULL; p = p->next)
{ prefetch(p->next); /* do work on node p */ }
```

Compiler optimization categories according to range

- How much code does the compiler view while optimizing?
  - The wider the view, the more powerful the optimization
- Axis 1: optimize across control flow?
  - **Local optimization**: optimizes only within straight line code
  - **Global optimization**: optimizes across control flow (if,for,...)
- Axis 2: optimize across function calls?
  - **Intra-procedural optimization**: only within function
  - **Inter-procedural optimization**: across function calls
- The two axes are orthogonal (any combination is possible)

Local vs. Global Constant Propagation

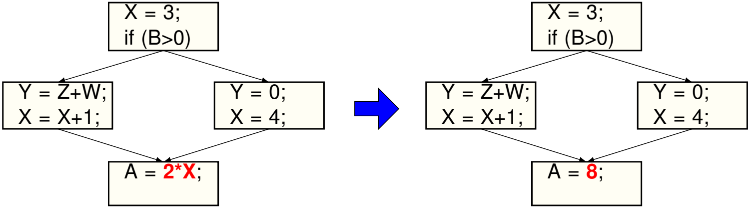
- **Constant propagation**
  - Optimization: if **x= y op z** and y and z are constants then compute at compile time and replace
- Local Constant Propagation

```
X = 3;
X = X+1;
A = X*2;
```

→

```
X = 3;
X = X+1;
A = 8;
```

- Global Constant Propagation



  - Additional **control flow analysis** is needed to enable this.

Intra- vs. Inter-procedural Constant Propagation

Intra-procedural Constant Propagation

```
X = 3;
X = X+1;
A = X*2;
```

⇒

```
X = 3;
X = X+1;
A = 8;
```

- Above is a local intra-procedural constant propagation.

Inter-procedural Constant Propagation

```
X = 3;
foo();
A = X*2;
```

```
void foo() {
   // No update to X.
}
```

⇒

```
X = 3;
foo();
A = 6;
```

```
void foo() {
   // No update to X.
}
```

- Above is a local inter-procedural constant propagation.
- The fact that global variable X is not updated must be propagated from callee to caller function to enable this.

Control Flow Analysis

Basic Block

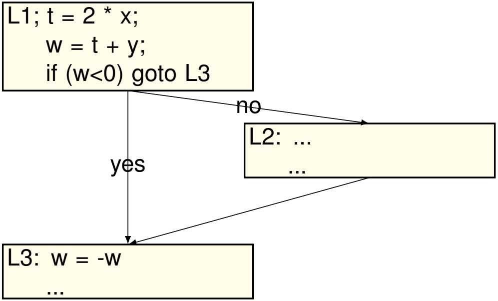
- A function body is composed of one or more **basic blocks.**
- **Basic block**: a maximal sequence of instructions that either execute all together or none at all. Meaning:
  - There are no jumps into the middle of the basic block
  - There are no jumps out of the middle of the block
- That means:
  - No instruction other than the first is a jump target
  - No instruction other than the last is a jump or branch
- Either all instructions in basic block execute or none
  - Smallest unit of execution in control flow analysis
  - Hence the descriptor "basic" in the name

Control Flow Graph

- A **Control Flow Graph (CFG)** is a directed graph in which
  - Nodes are basic blocks
  - Edges represent flows of execution between basic blocks
- CFGs are widely used to represent a program for analysis
- CFGs are especially essential for global optimizations

Control Flow Graph Example

```
L1; t = 2 * x;
    w = t + y;
    if (w<0) goto L3
L2: ...
    ...
L3: w = -w
    ...
```



Construction of CFG

- Step 1: partition code into basic blocks
  - Identify **leader** instructions, where a leader is either:
    - the first instruction of a program, or
    - the target of any jump/branch, or
    - an instruction immediately following a jump/branch
  - Create a basic block out of each leader instruction
  - Expand basic block by adding subsequent instructions (Stopping when the next leader instruction is encountered)
- Step 2: add edge between two basic blocks B1 and B2 if
  - there exist a jump/branch from B1 to B2, or
  - B2 follows B1, and B1 does not end with unconditional jump

Example

```
01.       A=4
02.       T1=A*B
03.  L1:  T2=T1/C
04:       if (T2<W) goto L2
05:       M=T1*K
06:       T3=M+1
07:  L2:  H=I
08:       M=T3-H
09:       if (T3>0) goto L3
10:  goto L1
11:  L3:  halt
```

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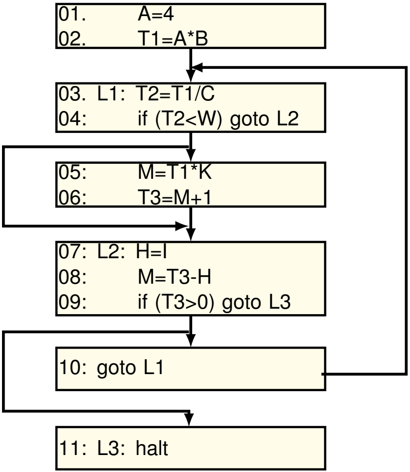
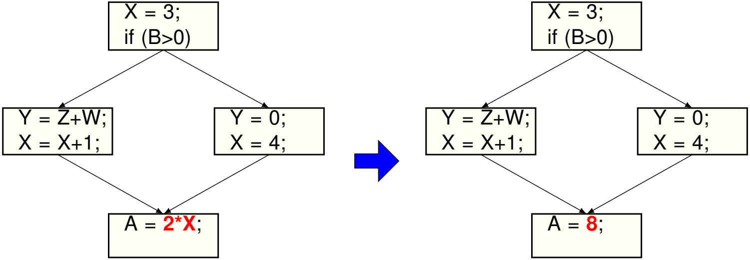


Data Flow Analysis

Global Optimizations

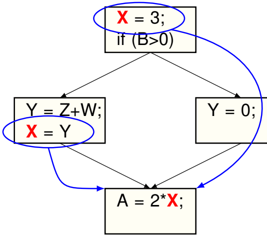
- Extends optimizations across control flows, i.e. CFG
- Like in this example Global Constant Propagation (GCP):



- How do we know it is OK to globally propagate constants?

Correctness criteria for GCP

- There are situations that prohibit GCP:



- To replace X by a constant C **correctly**, we must know
  - **Along all paths**, the last assignment to X is “X = C”

- Paths may go through loops and/or branches
  - When two paths **meet**, need to make **conservative** choice

Global Optimizations need to be Conservative

- Many compiler optimizations depend on knowing some property X at a particular point in program execution
  - Need to prove at that point property X holds along all paths

- To ensure correctness, optimization must be **conservative**
  - An optimization is enabled only when X is definitely true
  - If not sure, be conservative and say **don't know**
  - **Don't know** typically disables the optimization

Dataflow Analysis Framework

- **Dataflow analysis**: discovering properties about values at each statement of the program
  - E.g. discovering a value is constant before a statement
  - Done by observing the flow of data through the CFG

- **Dataflow analysis framework**:
  - A framework for implementing various dataflow analyses
  - 4 parameters defining analysis is passed into framework:

$$\{ \mathbf{D}, \mathbf{V}, \wedge: (V, V) \rightarrow V, \mathbf{F}: V \rightarrow V \}$$

- **D**: direction of dataflow (forward or backward)
- **V**: domain of values denoting property
- $\wedge$: **meet operator** that merges values when paths meet
- **F**: **flow propagation function** that propagates values through a basic block

Global Constant Propagation

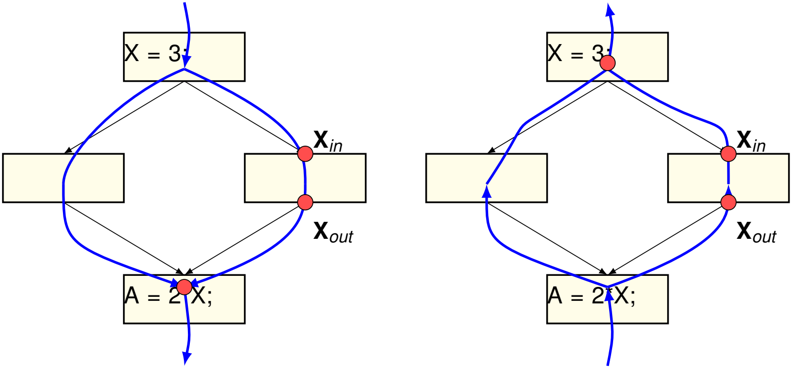
Global Constant Propagation (GCP)

- Let's use **GCP** to study dataflow analysis framework

- We will define each component one by one for GCP
  - **D**: direction of dataflow for constant property
  - **V**: domain of values denoting constant property
  - $\wedge$: **meet operator** that merges values when paths meet
  - **F**: **flow propagation function** for GCP

Direction D for GCP

- Is GCP a forward or backward analysis?



Forward Analsysis **Backward Analsysis**

- Forward, since "constantness" of a variable flows forward to subsequent instructions starting from assignment

Dataflow property V for GCP

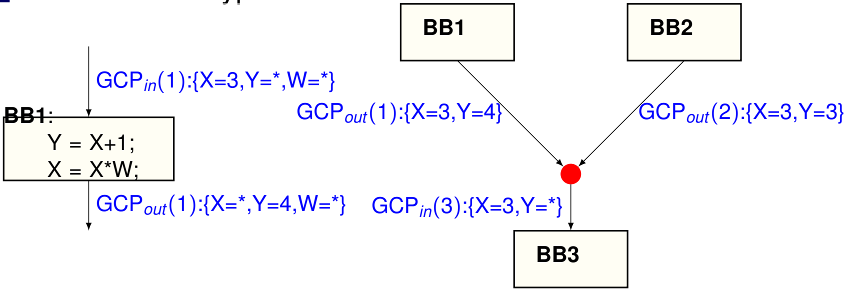
- V is a map of variables to values, where a value is:
  (in the case where value is an int type)
  - # /* not defined yet */
  - ..., -1, 0, 1, ... /* a constant */
  - * /* not a constant */

- **GCP(i)**: GCP dataflow property of basic block **i**
  - **GCP_{in}(i)**: at the entry of basic block **i**
  - **GCP_{out}(i)**: at the exit of basic block **i**
- **GCP(i)[X]**: value mapped to variable X in GCP(i)

- Example: given $GCP_{in}(1)$ = {X=1, Y=#, Z=*}
  - $GCP_{in}(1)$[X] = 1, $GCP_{in}(1)$[Y]=#, $GCP_{in}(1)$[Z]=*

Dataflow Equations for GCP

- There are two types of flow functions



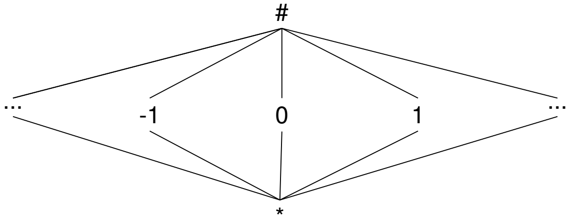
- Flow transfer function F: V $\rightarrow$ V
  - Computes data flow across statements
  - If statement assigns X, update GCP$_{out}$(i)[X] accordingly
- Meet operator $\wedge$: (V, V) $\rightarrow$ V
  - Computes data flow at control flow merges
  - Merge property from two paths using the meet operator

Flow Transfer Function F for GCP

- Treat each statement as basic block i to apply F
- If statement is not an assignment, $GCP_{out}(i) = GCP_{in}(i)$
- If statement is of the form X = Y + Z,
  - If $GCP_{in}(i)[Y]$ and $GCP_{in}(i)[Z]$ are both constants, $GCP_{out}(i)[X] = GCP_{in}(i)[Y] + GCP_{in}(i)[Z]$
  - Else if either $GCP_{in}(i)[Y]$ or $GCP_{in}(i)[Z]$ is *, $GCP_{out}(i)[X] = *$
  - Else if either $GCP_{in}(i)[Y]$ or $GCP_{in}(i)[Z]$ is #, $GCP_{out}(i)[X] = \#$

Meet operator $\wedge$ for variable values

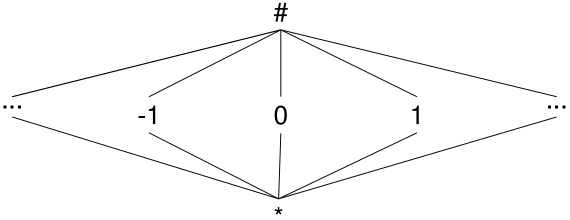
- Given basic block 1 and 2 merge into basic block 3, $GCP_{in}(3) = GCP_{out}(1) \wedge GCP_{out}(2)$
  - Where $\wedge$ is applied to each variable X in $GCP_{in}(3)$: $GCP_{in}(3) = GCP_{out}(1)[X] \wedge GCP_{out}(2)[X]$
- Meet operator $\wedge$ is given by this **semi-lattice**:
  - a $\wedge$ b = greatest lower bound (glb) in the below graph



  - # is called the **top** value denoted as $\top$
  - * is called the **bottom** value denoted as $\bot$

Meet operator $\wedge$ for variable values

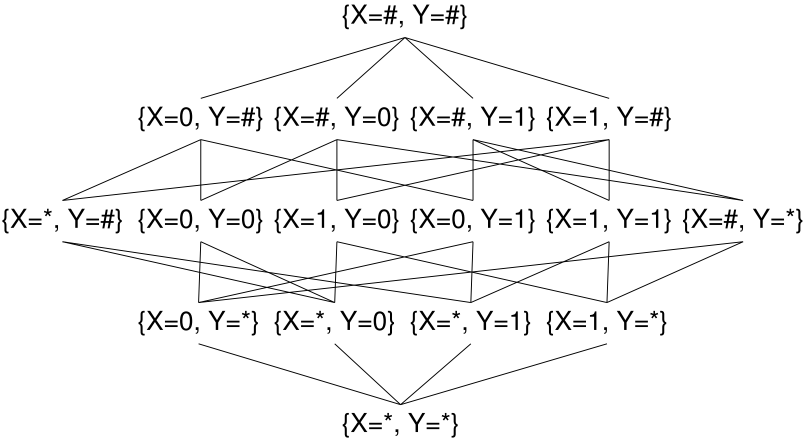
- Some results of meets $\wedge$ given by this **semi-lattice**:



  - $\# \wedge 1 \equiv \text{glb}(\#, 1) \equiv 1$
    - Meet of undefined value and a constant $\rightarrow$ x is that constant
  - $0 \wedge 1 \equiv \text{glb}(0, 1) \equiv *$
    - Meet on different constants $\rightarrow$ x is no longer constant
  - $* \wedge 1 \equiv \text{glb}(*, 1) \equiv *$
    - Meet of not a constant and a constant $\rightarrow$ x is not constant
- Greatest lower bound finds the maximal conservative value

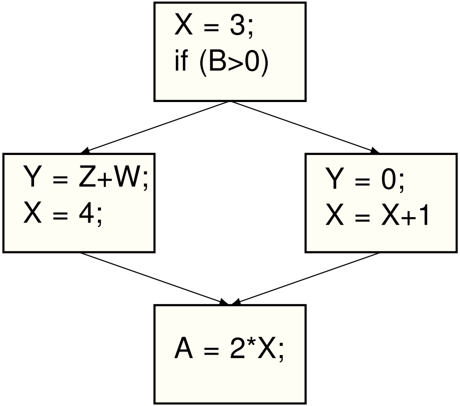
Meet operator $\wedge$ for GCP values

- The $\wedge$ operator for GCP values also forms a **semi-lattice.**
- Two variable example (values limited to 0, 1 for brevity):



➢ {X=0, Y=0} $\wedge$ {X=1, Y=#} $\equiv$ {X=*, Y=0}

GCP Propagation without loops



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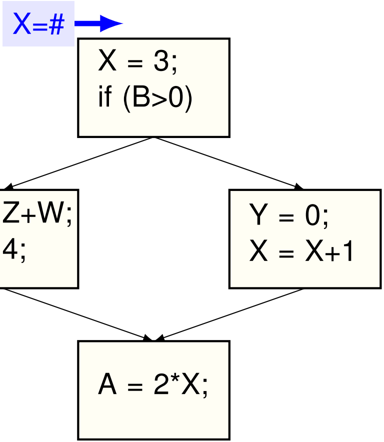

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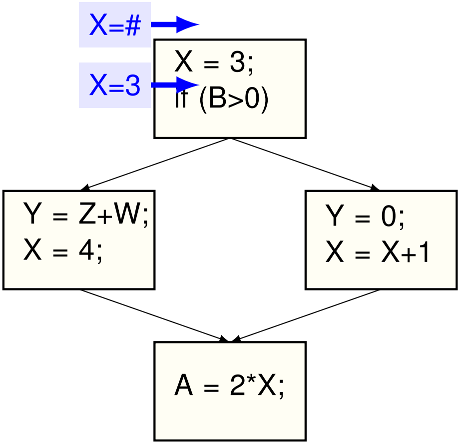

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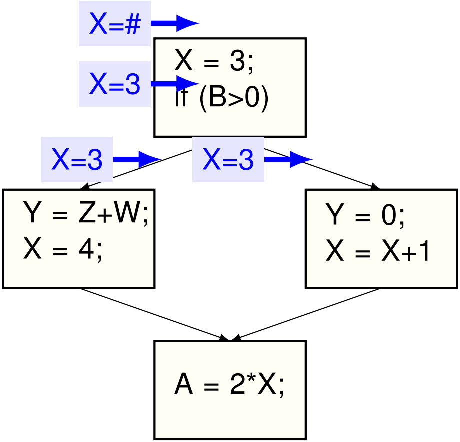

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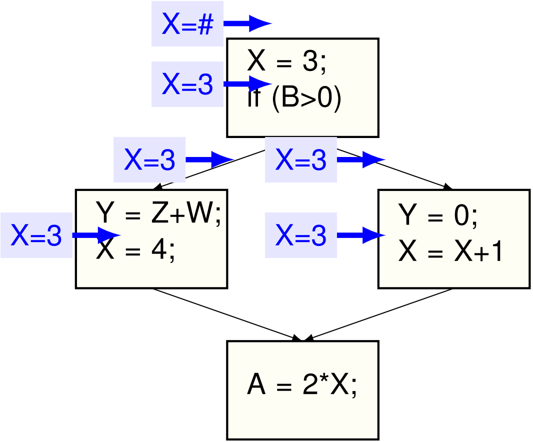

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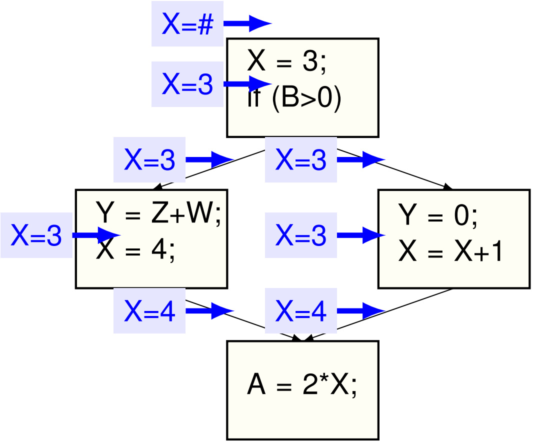

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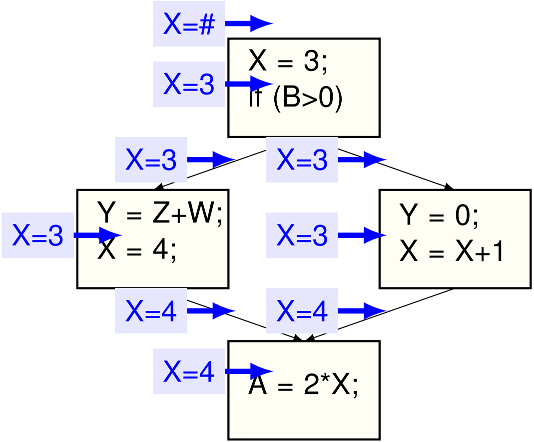

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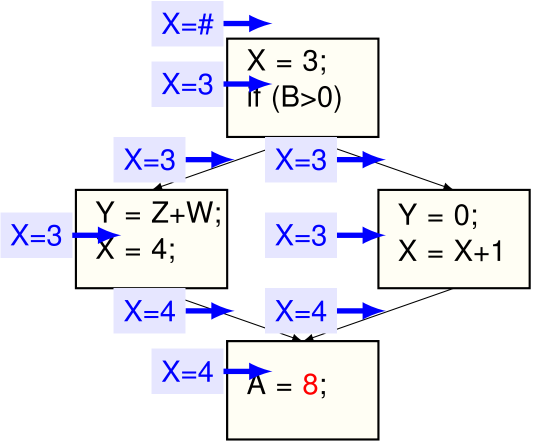
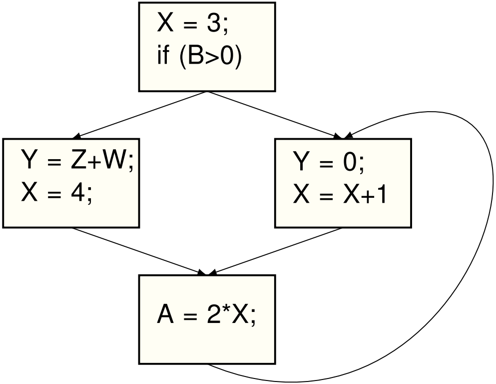


GCP Propagation with loops

- Iterate until there are no changes to values
  - This is called the **maximum fixed point** solution



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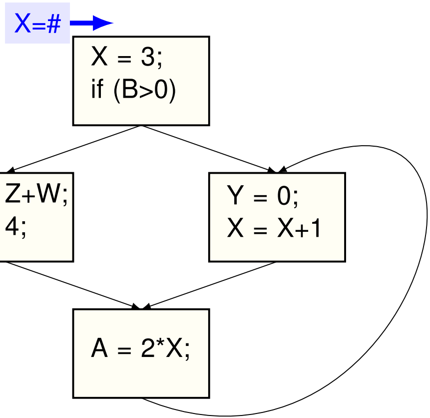

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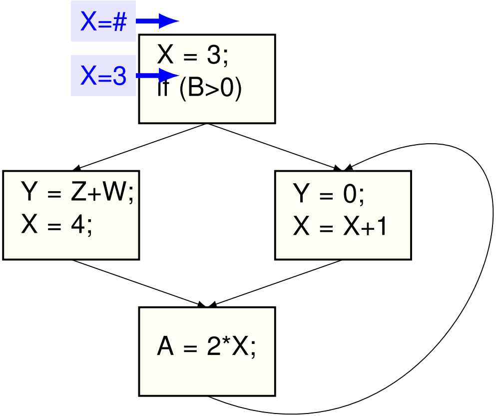

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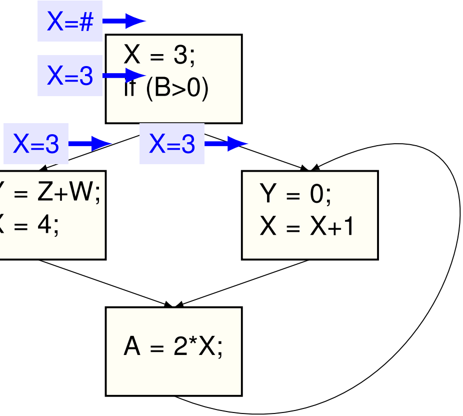

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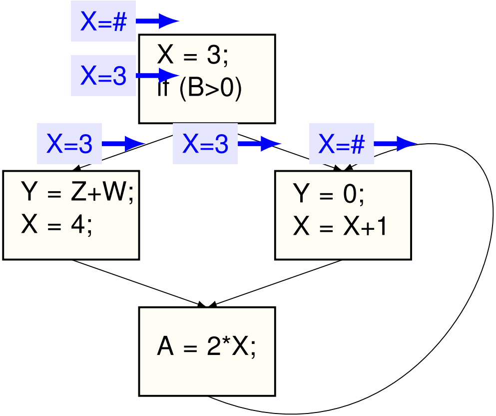

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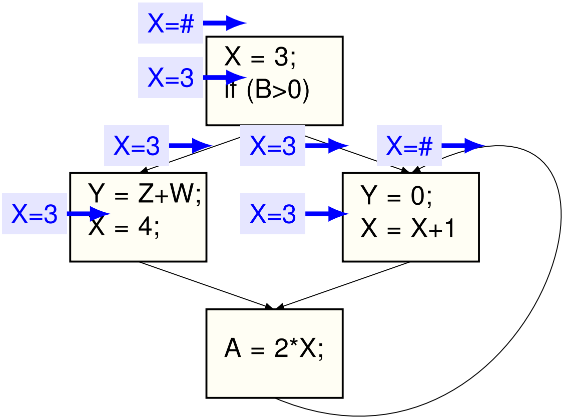

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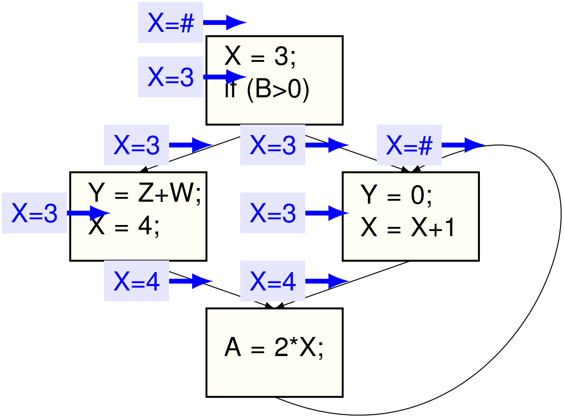

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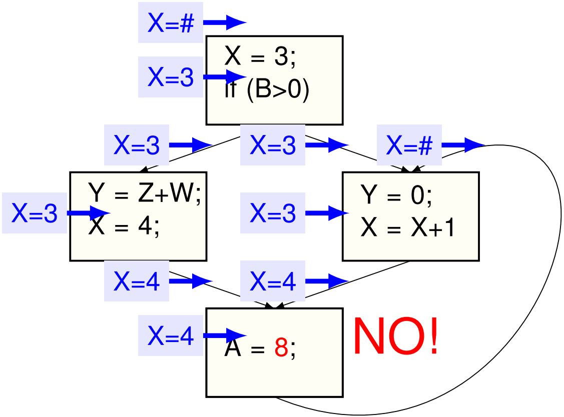

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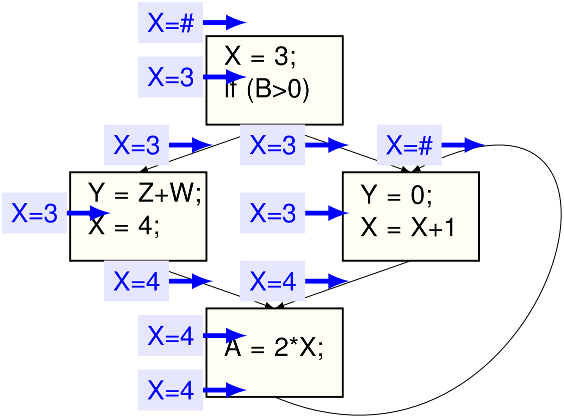

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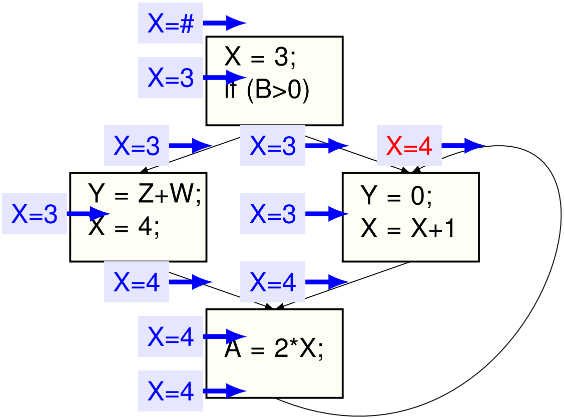

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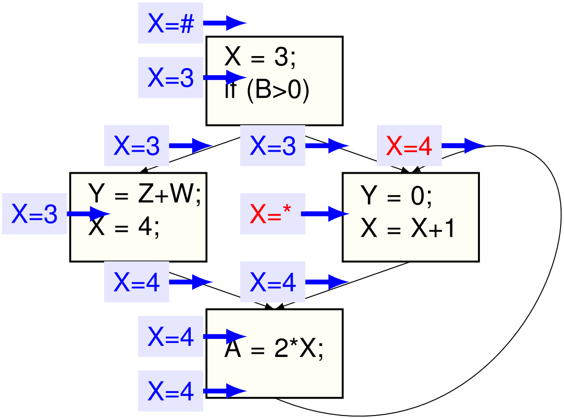

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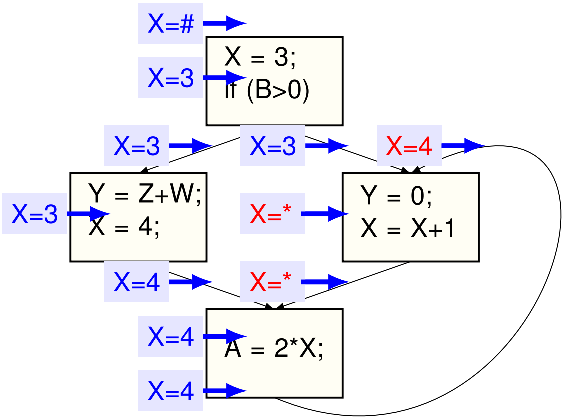

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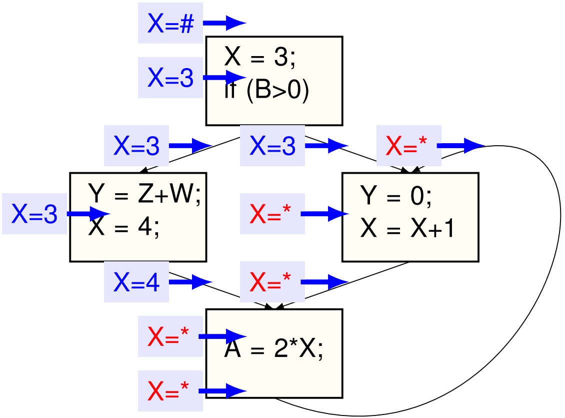


Worklist Algorithm for Iterative Dataflow Analysis

- The **Maximum Fixedpoint (MFP)** solution is:
  - Maximum: All values optimistically initialized to $\top$ values
  - Fixedpoint: Values are propagated until no changes occur
- MFP is efficiently computed using **worklist** algorithm:

```
Worklist = all nodes in CFG
while Worklist is not empty:
    n = remove a node from Worklist
    OUT[n] = transfer_function(IN[n])
    if OUT[n] changed:
        for each successor s of n:
            IN[s] = ∧ (OUT[p] for p in preds(s))
            if IN[s] changed:
                add s to Worklist
```

Time Complexity of Worklist Algorithm

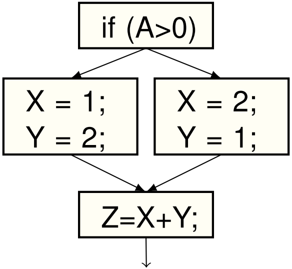
- Termination: **Greatest lower bound** ensures termination
  - Values start from the top $\top$ value
  - Values can only flow downward in the semi-lattice
  - Values are guaranteed to reach a fixedpoint in finite steps

- Time complexity: $O(d \times (N + E))$
  - d = the height of the semi-lattice
  - N = the number of nodes in CFG
  - E = the number of edges in CFG
- Why?
  1. A node enters worklist only when value changes.
  2. An edge is processed only when value changes.
  3. A value can change at most d times.

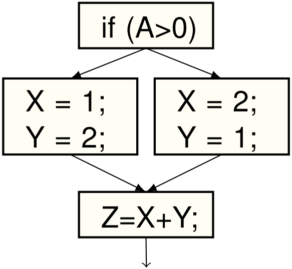
Maximum Fixedpoint $\leq$ Meet-Over-Paths Solution

- How accurate is the maximal fixedpoint solution?

Maximum Fixedpoint (MFP)



Meet-Over-Paths (MOP)



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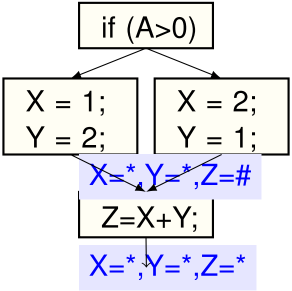

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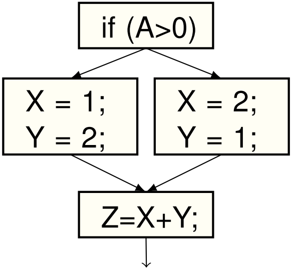

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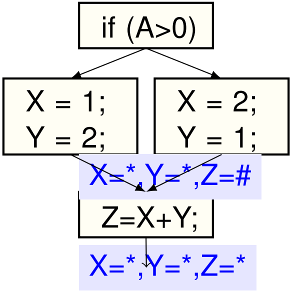
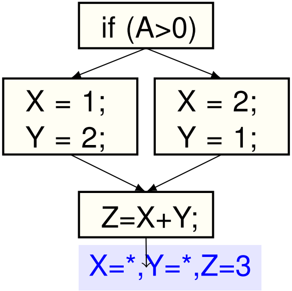


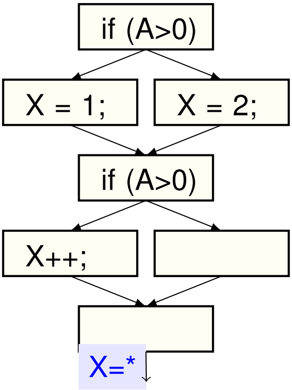
Meet-Over-Paths (MOP)



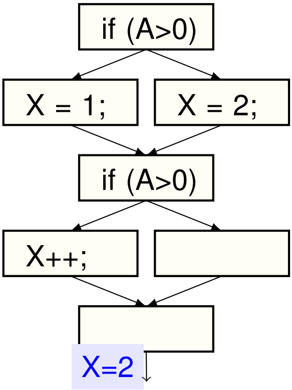
- For MOP, OUT[b] = $\wedge$ (OUT[b] for all paths to b)
  - Computing OUT[b] where b is last basic block in example: OUT[b] = {X=1,Y=2,Z=3} $\wedge$ {X=2,Y=1,Z=3} = {X=*,Y=*,Z=3}
- MFP $\leq$ MOP (MFP is less precise)

Meet-Over-Paths $\leq$ Ideal Solution

Meet-Over-Paths (MOP)



Ideal Solution



- For Ideal, OUT[b] = $\wedge$ (OUT[b] for all *feasible* paths to b)
  - ideal = {X=2} $\wedge$ {X=2} = {X=2}
  - MOP = {X=1} $\wedge$ {X=2} $\wedge$ {X=2} $\wedge$ {X=3} = {X=*} $\leq$ Ideal

MFP is Safe but Conservative

- In short, for the GCP dataflow analysis:

  Maximum Fixedpoint $\leq$ Meet-Over-Paths $\leq$ Ideal
- This is both good and bad.

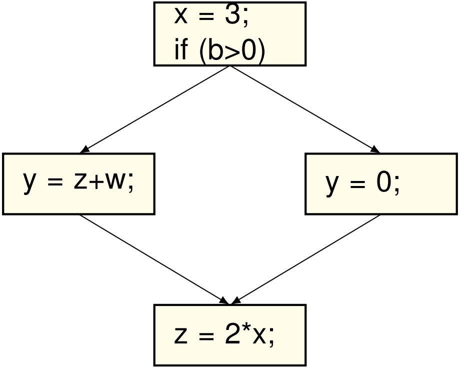
  Good : MFP $\leq$ Ideal means all GCP optimizations are safe.
  Bad : MFP $\leq$ Ideal also means optimizations are conservative.

- MOP $\leq$ Ideal is obvious, but is MFP $\leq$ MOP true?
- MFP $\leq$ MOP because GCP is a **monotone framework.**
  - In a monotone framework, $f(x \wedge y) \leq f(x) \wedge f(y)$
    (Read textbook 9.4.4 for a proof that GCP is monotone.)
- Sometimes MFP = MOP, called **distributive frameworks.**
  - In a distributive framework, $f(x \wedge y) = f(x) \wedge f(y)$
    (Liveness Analysis, which we'll learn next, is an example.)

Liveness Analysis

Another Analysis: Liveness Analysis

- After GCP, we would like to eliminate the dead code



Another Analysis: Liveness Analysis

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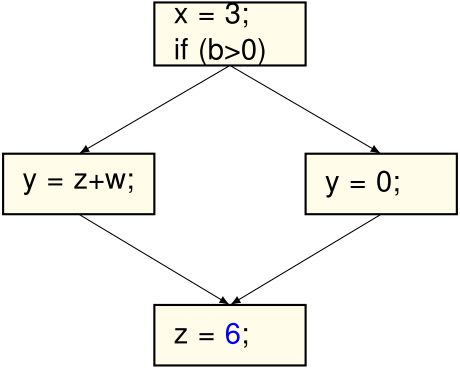

Another Analysis: Liveness Analysis

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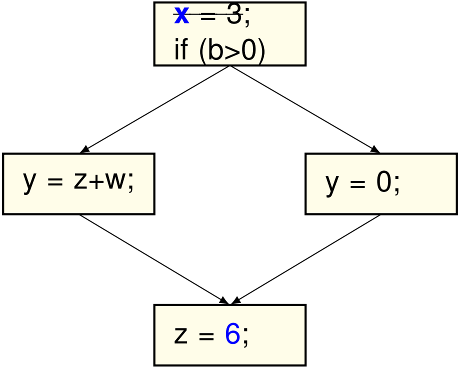
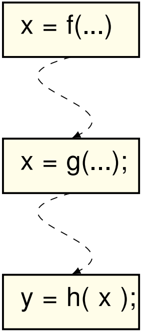


Live/Dead Statment

- A **dead statement** assigns a value that is not used later
- Otherwise, it is a **live statement**

In the example,
the 1st statement is dead,
the 2nd statement is live



➢ Assuming inter-procedural analysis says f(...) is internally free of assignments used later (e.g. global variables).

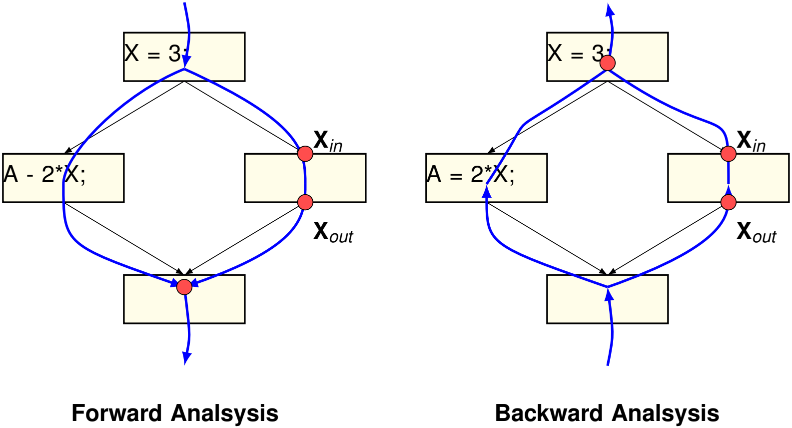
Global Liveness Analysis (GLA)

- Again, let's use the dataflow analysis framework
- Here are the 4 components of the framework
  - **D**: direction of dataflow for liveness property
  - **V**: domain of values denoting liveness property
  - $\wedge$: **meet operator** that merges values when paths meet
  - **F**: **flow propagation function** for liveness

- **V**: Liveness property is the set of live variables
  - {}, {a}, {a, b}, {b, c}, {a, b, c}, ...
- $\wedge$: Meet operator for Liveness Analysis is a union
  - Variable x is live if x is live along at least one path

Direction D for GLA

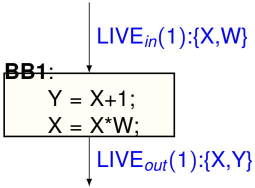
- Is Liveness a forward or backward analysis?

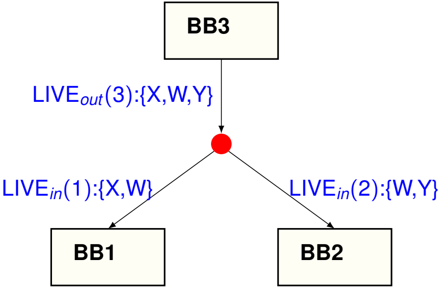


- Backward, since liveness of a variable flows backward to preceding definitions starting from use

Dataflow Equations for GLA

- There are two types of flow functions





- Flow transfer function F: V $\rightarrow$ V
  - Now F computes P_{in} from P_{out} since it is backward analysis
  - Remove variable definitions, add variable uses to live set
- Meet operator $\wedge$: (V, V) $\rightarrow$ V
  - $\mathbf{LIVE_{out}(i) = \cup\ LIVE_{in}(k)}$ where **k** is successor of **i**

Flow Transfer Function F for GLA

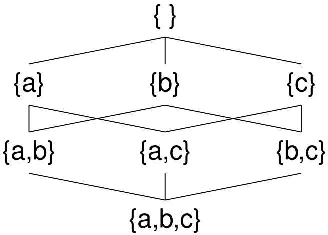
- F for Global Liveness Analysis (GLA)

$$\mathbf{LIVE}_{in}\mathbf{(i) = (LIVE}_{out}\mathbf{(i) - DEF(i))} \cup \mathbf{USE(i)}$$

where DEF(i) is the set of defined variables in basic block i
USE(i) is the set of used variables in basic block i

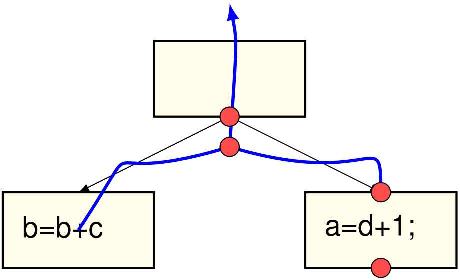
Meet operator $\wedge$ for GLA

- Meet operator $\wedge$ is given by this **semi-lattice**:

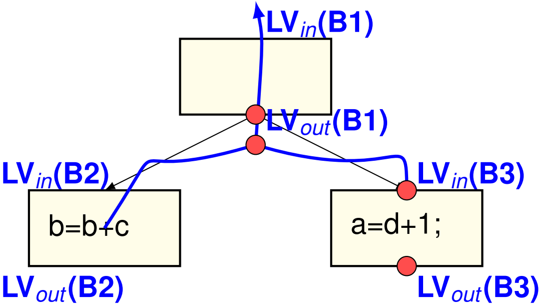


  - {a} $\wedge$ {b} = glb({a}, {b}) = {a,b}
  - {b} $\wedge$ {a,c} = glb({b}, {a,c}) = {a,b,c}
  - The semi-lattice expresses the union relationship

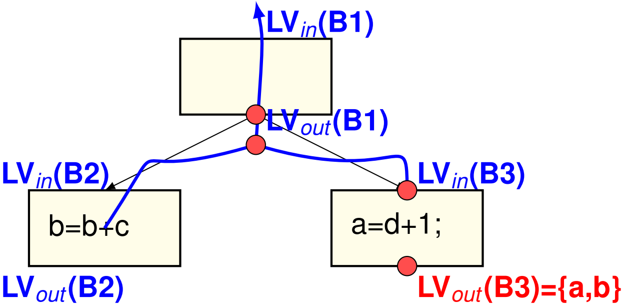
Liveness Example



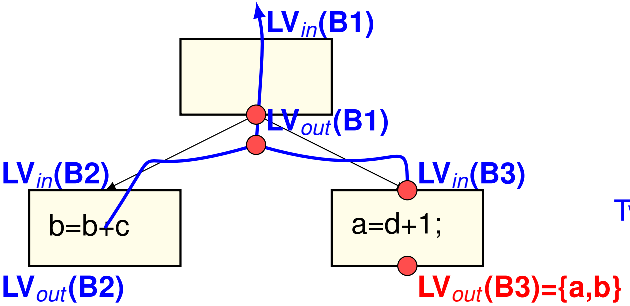
Liveness Example



Liveness Example

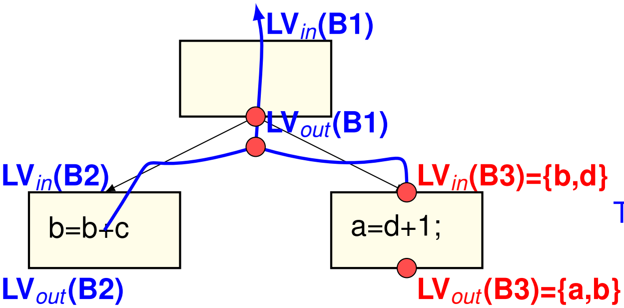


Liveness Example



Two sets:
DEF={a}
USE={d}

Liveness Example



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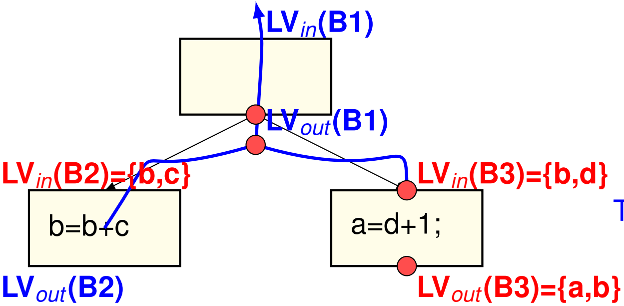
Liveness Example



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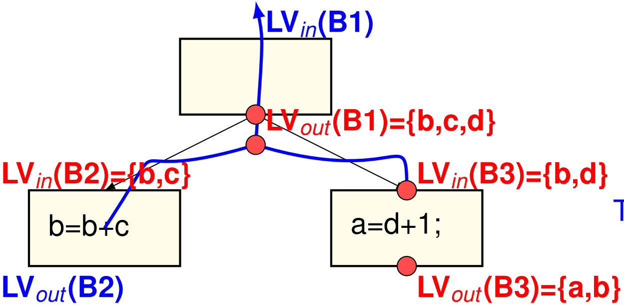
DEF={a}
USE={d}

Liveness Example



Two sets:
DEF={a}
USE={d}

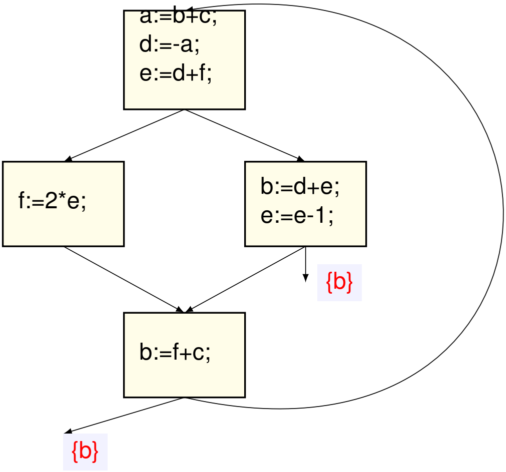
Applications of Global Liveness Analysis

- Global Dead Code Elimination is based on GLA
  - A statement x = ... is dead code if x not used
  - Dead statements can be deleted from the program

- **Register allocation** is also based on GLA
  - **Register allocation**: assigning variables to registers
  - If two variables are simultaneously live at any point
    $\rightarrow$ they cannot be allocated to the same register
    $\rightarrow$ and this is called **interference**.
  - If there are insufficient CPU registers to hold variables
    $\rightarrow$ some variables must be stored in stack memory
    $\rightarrow$ and this is called **register spilling**.
  - Register spilling typically leads to worse performance.
    $\rightarrow$ Leads to extra load/store instructions to access memory

Register Allocation: Compute Register Interference

- At each point P, compute live variables and interference



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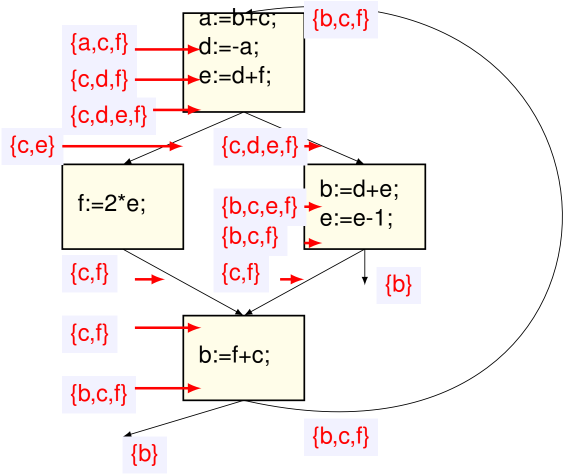
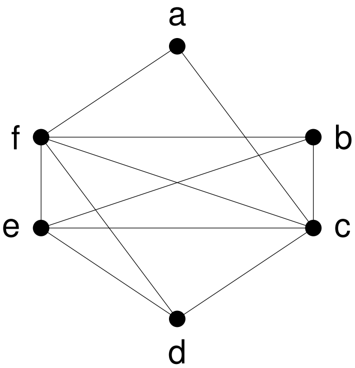


Register Allocation: Register Interference Graph

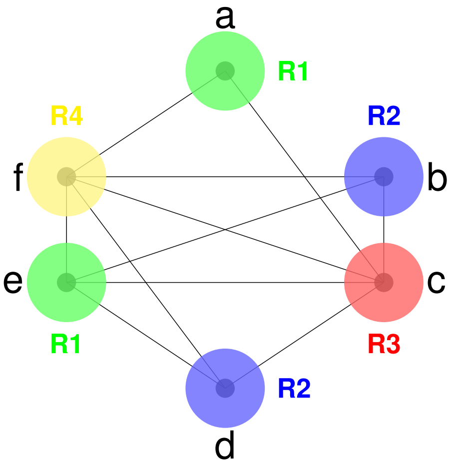
- Construct **Register Interference Graph (RIG)** such that
  - Nodes represent variables
  - Edges between variables represent interference



- Two variables can be allocated in same register if no edge
- Otherwise, they cannot be allocated in the same register

Register Allocation: Allocation using Graph Coloring

- Each color represents a CPU register
  - There are 4 colors in the coloring result
  - No register spilling occurs with 4 or more CPU registers



Summary of Dataflow Analysis

- A dataflow analysis framework is defined as:
  $\{$ **D**, **V**, $\wedge$: (V, V) $\rightarrow$ V, **F**: V $\rightarrow$ V $\}$
  - **D**: direction of dataflow
  - **V**: domain of values denoting property
  - $\wedge$: **meet operator** that merges values when paths meet
  - **F**: **flow propagation function** within a basic block

- Other analyses can be expressed using this framework:
  - Loop Invariant Code Motion (LICM)
  - Common Subexpression Elimination (CSE)
  - Partial Redundancy Elimination (PRE)

- Please refer to the textbook on how these are formulated.

The END !