Semantic Analysis

The role of semantic analysis is to assign meaning

- "It smells fishy."
- Lexical analysis
  - Tokenizes "It", "smells", "fishy", "."
  - Determines noun, verb, adjective, punctuation token types
- Syntax analysis
  - Parses the grammatical structure of the sentence
- Semantic analysis

The role of semantic analysis is to assign meaning

- "It smells fishy."

- Lexical analysis
  - Tokenizes "It", "smells", "fishy", "."
  - Determines noun, verb, adjective, punctuation token types

- Syntax analysis
  - Parses the grammatical structure of the sentence

- Semantic analysis
  - Assigns meaning to the words "It", "smells", "fishy"
  - Flags error if the sentence does not make sense

Semantic Analysis = Binding + Type Checking

- "I don't wanna eat that sushi."
  "It smells fishy."
  - "It": the sushi
  - "smells": feels to my nose
  - "fishy": that the sushi has gone bad
- "The professor says that the exam is going to be easy."
  "It smells fishy."
  - "It": the situation
  - "smells": feels to my sixth sense
  - "fishy": that it is highly suspicious

Semantic Analysis = Binding + Type Checking

- "I don't wanna eat that sushi."
  "It smells fishy."
  - "It": the sushi
  - "smells": feels to my nose
  - "fishy": that the sushi has gone bad
- "The professor says that the exam is going to be easy."
  "It smells fishy."
  - "It": the situation
  - "smells": feels to my sixth sense
  - "fishy": that it is highly suspicious
- Semantic analysis consists of two tasks
  - **Binding**: associating a pronoun to an object
  - **Type checking**: inferring meaning based on type of object

Semantic Analysis = Binding + Type Checking

- Semantic analysis performs binding
  - Done by traversing parse tree produced by syntax analysis
  - Declarations are stored in data structure called **symbol table**
  - Uses are bound to entries in the symbol table

- Semantic analysis performs type checking
  - Infers what "$a + b$" means:
    - If *a* and *b* are ints, integer add and return int
    - If *a* and *b* are floats, FP add and return float
    - If *a* and *b* are strings, concatenate and return string
  - Infers what "$x.foo()$" means:
    - If object *x* is a reference of class *A*, call to foo() in A
    - If object *x* is a reference of class *B*, call to foo() in B
  - Infers what "$a[i][j]$" means:
    - Offset from *a* based on array type and dimensions

Semantic analysis also performs semantic checks

- All symbol uses have corresponding declarations
- All symbols defined only once
  - Where symbols can be variables, methods, classes
  - Declaration: provides type information for a symbol
  - Definition: allocates a symbol in program memory
- All statements do not violate type rules
  - Operators (+, -, *, /, =, >, <, ==, ...) have legal parameters
  - Method calls have correct numbers of legal parameters
  - Private methods are not called by external classes
  - ...

Symbol Binding

What is symbol binding?

"Matching symbol **declarations** with **uses**"

- If there are multiple declarations, which one is matched?

What is symbol binding?

"Matching symbol **declarations** with **uses**"

- If there are multiple declarations, which one is matched?

```
void foo()
{
  char x;
  ...
  {
      int x;
      ...
  }
  x = x + 1;
}
```

What is symbol binding?

"Matching symbol **declarations** with **uses**"

- If there are multiple declarations, which one is matched?

```
void foo()
{
  char x;
  ...
  {
     int x;
  }
  x = x + 1;
}
```

?

?

Scope

- Binding: associating a symbol use to its declaration
  - Which variable (or function) an identifier is referring to
- Scope: section of program where a declaration is valid
  - Uses in the scope of declaration are bound to it
- Some implications of scopes
  - A symbol may have different bindings in different scopes
  - Scopes for the same symbol never overlap
    - there is always exactly one binding per symbol use
- Two types: static scope and dynamic scope

Static Scope

- Static Scope: scope expressed in program text
  - Also called Lexical Scope
  - C/C++, Java, JavaScript, Python

- Rule: bind to the closest enclosing declaration

```
void foo()
{
  char x;
  ...
  {
     int x;
     ...
  }
  x = x + 1;
}
```

Static Scope

- Static Scope: scope expressed in program text
  - Also called Lexical Scope
  - C/C++, Java, JavaScript, Python

- Rule: bind to the closest enclosing declaration

```
void foo()
{
   char x;
   ...
   {
      int x;
      ...
   }
   x = x + 1;
}
```

Dynamic Scope

- Dynamic Scope: bindings formed during code execution
  - LISP, Scheme, Perl
- Rule: bind to the most recent declaration during execution

```
void foo()
{
   (1) char x;
   (2) if (...) {
   (3)     int x;
   (4)      ...
         }
   (5) x = x + 1;
}
```

Dynamic Scope

- Dynamic Scope: bindings formed during code execution
  - LISP, Scheme, Perl

- Rule: bind to the most recent declaration during execution

```
void foo()
{
   (1) char x;
   (2) if (...) {
   (3)     int x;
   (4)      ...
        }
   (5) x = x + 1;
}
```

- Which *x*'s declaration is the closest?
  - Execution (a): ...(1)...(2)...(5)
  - Execution (b): ...(1)...(2)...(3)...(4)...(5)

Static vs. Dynamic Scoping

- Most languages that started with dynamic scoping (LISP, Scheme, Perl) added static scoping afterwards
- Why?
  - It is easier for human beings to understand
    - Bindings are visible in code without tracing execution
  - It is easier for compilers to understand
    - Compiler can determine bindings at compile time
    - Compiler can translate identifier to a single memory location
    - Results in generation of efficient code
  - With dynamic scoping...
    - There may be multiple possible bindings for a variable
    - Impossible to determine bindings at compile time
    - All bindings have to be done at execution time (Typically with the help of a hash table)

Symbol Table

Symbol Table

- **Symbol Table**: A compiler data structure that tracks information about all identifiers (symbols) in a program
  - Maps symbol uses to declarations given a scope
  - Needs to provide bindings according to the current scope
- Usually discarded after generating the binary code
  - All symbols are mapped to memory locations already
  - For debugging, symbols may be included in binary
    - To map memory locations back to symbols for debuggers
    - For GCC or Clang, add “-g” flag to include symbol tables

Maintaining Symbol Table

- Basic idea:
  int x; ... void foo() { int x; ... x=x+1; } ... x=x+1 ...
  - In *foo*, add *x* to table, overriding any previous declarations
  - After *foo*, remove *x* and restore old declaration if any
- Operations

enter_scope()	start a new nested scope
exit_scope()	exit current scope
find_symbol(x)	find declaration of *x*
add_symbol(x)	add declaration of *x* to symbol table

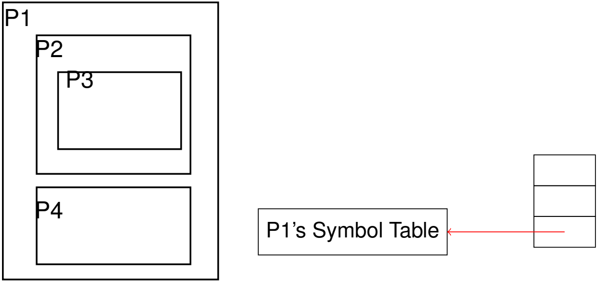
Adding Scope Information to the Symbol Table

- To handle multiple scopes in a program,
  - ➢ (Conceptually) need an individual table for each scope
  - ➢ Symbols added to the table may not be deleted just because you exited a scope

```
class X { ... void f1() {...} ... }
class Y { ... void f2() {...} ... }
class Z { ... void f3() {
    X v;
    v.f1();
} ... }
```

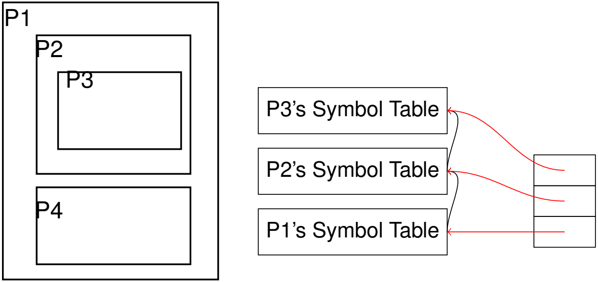
  - ➢ Without deleting symbols, how are scoping rules enforced?
    - ☞ Keep a list of all scopes in the entire program
    - ☞ Keep a stack of active scopes at a given point

Symbol Table with Multiple Scopes



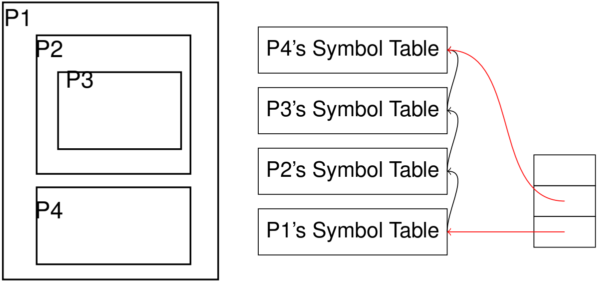
- For nested scopes,
  - Search from top of the active symbol table stack
  - Remove pointer to symbol table when exiting its scope

Symbol Table with Multiple Scopes



- For nested scopes,
  - Search from top of the active symbol table stack
  - Remove pointer to symbol table when exiting its scope

Symbol Table with Multiple Scopes



- For nested scopes,
  - Search from top of the active symbol table stack
  - Remove pointer to symbol table when exiting its scope

What Information is Stored in the Symbol Table

- Entry in Symbol Table:

string	kind	attributes

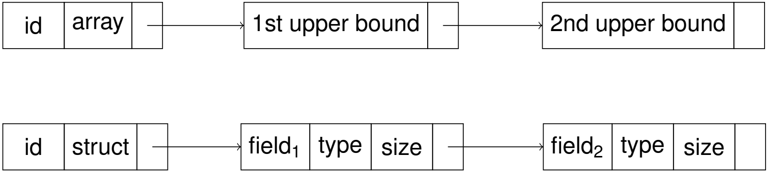
  - String — the name of identifier
  - Kind — variable, parameter, function, class, ...
- Attributes vary with the kind of symbol
  - variable $\rightarrow$ type, address in memory
  - function $\rightarrow$ return type, parameter types, address
- Vary with the language
  - Fortran's array $\rightarrow$ type, dimension, dimension size
    real A(5) /* dimension required for static allocation */
  - C's array $\rightarrow$ type, dimension, optional dimension size
    char A[5]; /* statically sized array */
    char A[]="hello"; /* dynamically sized to fit content */

Symbol Table Attribute List

- Type information might be arbitrarily complicated
  - In C:

```
struct {
        int a[10];
        char b;
        float c;
      }
```

- Store all relevant attributes in an attribute list

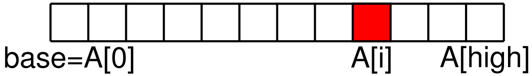


Example application of Type to an operator: Array index operator

Addressing Array Elements

int A[0..high];

A[i] ++;



- width — width of element type
- base — address of the first
- high — upper bound of subscript

- Addressing an array element:

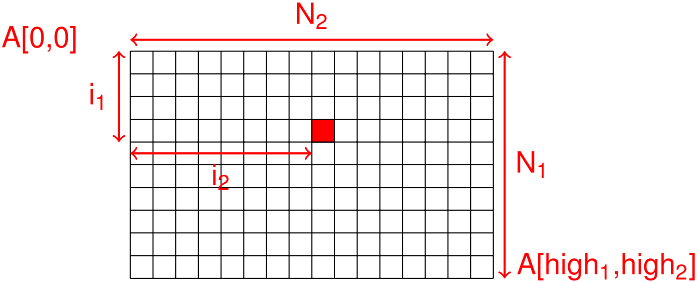
address(A[i]) = base + i * width
offset(A[i]) = i * width

Multi-dimensional Arrays

- Layout n-dimension items in 1-dimension memory

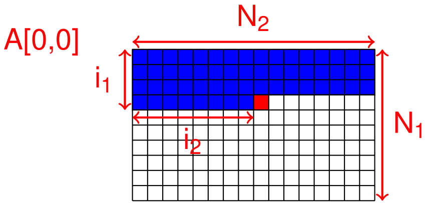
  int $A[N_1][N_2]$; /* int $A[0..high_1][0..high_2]$; */

  $A[i_1][i_2]$ ++;



Row Major

Row major — store row by row

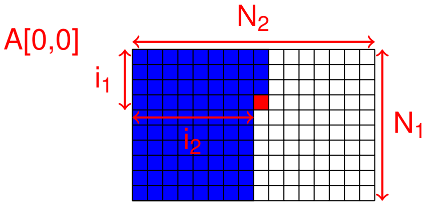


- Offset inclues all the "blue" items before $A[i_1,i_2]$

$$\text{offset}(A[i_1,i_2]) = (i_1 * N_2 + i_2) * \text{width}$$
$$= i_1 * N_2 * \text{width} + i_2 * \text{width}$$
$$= \text{offset}(A[i_1]) * N_2 + i_2 * \text{width}$$

Column Major

Column major — store column by column



- Offset inclues all the "blue" items before $A[i_1,i_2]$

$offset(A[i_1,i_2]) = (i_2 * N_1 + i_1)*width$
$= i_2 * N_1 * width + i_1 * width$
$= i_2*N_1*width + offset(A[i_1])$

Generalized Row/Column Major

- Let A_k = offset(A[i_1, i_2, ..., i_k]). Then,

- Row major (C/C++, C#, Objective-C)
  1-dimension: $A_1 = i_1$*width
  2-dimension: $A_2 = (i_1*N_2+i_2)$*width = $A_1*N_2+i_2$*width
  3-dimension: $A_3 = (i_1*N_2*N_3+i_2*N_3+i_3)$*width = $A_2*N_3+i_3$*width
  k-dimension: $A_k = A_{k-1}*N_k + i_k$*width
  ☞**Type** needs to provide $N_2 ... N_k$ and width for offset

- Column major (Fortran, Matlab, R)
  1-dimension: $A_1 = i_1$*width
  2-dimension: $A_2 = (i_2*N_1 + i_1)$*width = i_2*N_1*width + A_1
  3-dimension: $A_3 = ((i_3*N_2+i_2)*N_1+i_1)$*width = $i_3*N_2*N_1$*width + A_2
  k-dimension: $A_k = i_k*N_{k-1}*N_{k-2}*...*N_1$*width+$A_{k-1}$
  ☞**Type** needs to provide $N_1 ... N_{k-1}$ and width for offset

C's implementation

- C uses row major

```
int fun1(int p[ ][100])
{
...
   int a[100][100];
   a[i1][i2] = p[i1][i2] + 1;
}
```

Why is p[][100] allowed?

Why is a[][100] not allowed?

C's implementation

- C uses row major

```
int fun1(int p[ ][100])
{
...
  int a[100][100];
  a[i1][i2] = p[i1][i2] + 1;
}
```

Why is p[][100] allowed?

- The info is enough to compute $p[i_1][i_2]$'s address
- $A_2 = (i_1 * N_2 + i_2) * width$ (N_1 is not required)

Why is a[][100] not allowed?

- The info is not enough to allocate space for the array

Type Checking

What, When, and Why

- What?
  - Type: a set of values + a set of operations on values
  - Type Checking: Verifying and enforcing type consistency
    - Only legal values are assigned to a type
    - Only legal operations are performed on a type
- When?
  - Static Type Checking: Type checking at compile-time
  - Dynamic Type Checking: Type checking at execution time
- Static type checking is more desirable. Why?
  - Better to fail at compile time than during deployment
  - Dynamic type checking can impact runtime performance
  - Check dynamically only when static checking is infeasible
    - E.g. Java array bounds checks
    - E.g. Type checks to verify C++/Java downcasting

Static vs. Dynamic Typing

- Statically typed: C/C++, Java ☜Our discussion
  - Types are explicitly declared or can be inferred from code
    int x; /* type of x is int */
  - Better compiler error detection due to static type checks
  - Efficient code since dynamic type checks are not needed

- Dynamically typed: Python, JavaScript, PHP
  - Type is a runtime property decided only during execution
    var x; /* type of x is undecided */
    x = 42; /* type of x is int */
    x = "forty two"; /* type of x is now string */
    /* Type of x changes depending on the value it holds */
  - Static type checking and error reporting is impossible
  - More memory since every variable now needs a "type tag"
  - Inefficient code due to dynamic checks on type tags

Rules of Inference

- What are *rules of inference*?
  - Inference rules have the form
    if **Precondition** is true, then **Conclusion** is true
  - Below concise notation used to express above statement

$$\frac{\textbf{Precondition}}{\textbf{Conclusion}}$$

  - In the context of type checking:
    if expressions E1, E2 have certain types (Precondition), expression E3 is legal and has a certain type (Conclusion)
- Type checking via inference
  - Start from variable types and constant types
  - Repeatedly apply rules until entire program is inferred legal

Notation for Inference Rules

- By tradition inference rules are written as

$$\frac{\textbf{Precondition}_1\textbf{, ..., Precondition}_n}{\textbf{Conclusion}}$$

  - The precondition/conclusion has the form **“e:T”**
- Meaning
  - If **Precondition**$_1$ and ... and **Precondition**$_n$ are true, then **Conclusion** is true.
  - **“e:T” indicates “e is of type T”**
  - Example: rule-of-inference for add operation

$$\frac{\begin{array}{c}\mathbf{e_1: int}\\ \mathbf{e_2: int}\end{array}}{\mathbf{e_1+e_2:int}}$$

Rule: If e_1, e_2 are ints then e_1+e_2 is legal and is an int

Two Simple Rules

[Constant]

$$\frac{i \text{ is an integer}}{i: \text{int}}$$

[Add operation]

$$\frac{e_1: \text{int} \quad e_2: \text{int}}{e_1 + e_2 : \text{int}}$$

- Example: given “10 is an integer” and “20 is an integer”, is the expression “10+20” legal? Then, what is the type?

$$\frac{\dfrac{10 \text{ is an integer}}{10: \text{int}} \qquad \dfrac{20 \text{ is an integer}}{20: \text{int}}}{10+20: \text{int}}$$

- This type of reasoning can be applied to the entire program

More Rules

[New]

$$\frac{}{\textbf{new T: T}}$$

[Not]

$$\frac{\textbf{e: Boolean}}{\textbf{not e: Boolean}}$$

- However,

  [Var?]

  $$\frac{\textbf{x is an identifier}}{\textbf{x: ?}}$$

  - the expression itself insufficient to determine type
  - **solution:** provide context for this expression

Type Environment

- A *type environment* gives type info for free variables
  - A variable is *free* if not declared inside the expression
  - It is a function mapping Symbols to Types
    - Set of declarations active at the current scope
    - Conceptual representation of a symbol table

Type Environment Notation

Let O be a function from Symbols to Types,

the sentence **O e:T**

is read as "under the assumption of environment O, expression e has type T"

$$\frac{\textbf{i is an intger}}{\textbf{O i: int}} \qquad \frac{\textbf{O e1: int} \quad \textbf{O e2: int}}{\textbf{O e1+e2: int}} \qquad \frac{\textbf{O(x) == T}}{\textbf{O x: T}}$$

– "if i is an integer, expression i is an int in any environment"

– "if e1 and e2 are ints in O, expression e1+e2 is int in O"

– "if variable x is mapped to int in O, expression x is int in O"

Declaration Rule

[Declaration w/o initialization]

$$\frac{O[T_0/x]\ e_1 : T_1}{O\ \text{let } x : T_0 \text{ in } e_1 : T_1}$$

$O[T_0/x]$: environment O modified so that it return T_0 on argument x and behaves as O on all other arguments:

$O[T_0/x](x) = T_0$

$O[T_0/x](y) = O(y)$ when $x \neq y$

- Translation: "If expression e_1 is type T_1 when x is mapped to type T_0 in the current environment, expression e_1 is type T_1 when x is declared to be T_0 in the current environment"

Declaration Rule with Initialization

[Declaration with initialization (initial try)]

$$\frac{\begin{array}{c}\mathbf{O}\ \mathbf{e}_0 : \mathbf{T}_0 \\ \mathbf{O}[\mathbf{T}_0/\mathbf{x}]\ \mathbf{e}_1 : \mathbf{T}_1\end{array}}{\mathbf{O}\ \mathbf{let}\ \mathbf{x}: \mathbf{T}_0 \leftarrow \mathbf{e}_0\ \mathbf{in}\ \mathbf{e}_1 : \mathbf{T}_1}$$

- The rule is too strict (i.e. correct but not complete)

  Example

  class C inherits P ...

  let x:P ← new C in ...

  ☞ the above rule does not allow this code

Subtype

- A subtype is a relation $\leq$ on classes
  - $X \leq X$
  - if X inherits from Y, then $X \leq Y$
  - if $X \leq Y$ and $Y \leq Z$, then $X \leq Z$

- An improvement of our previous rule

  [Declaration with initialization]

$$\frac{\begin{array}{c} \mathbf{O}\ \mathbf{e}_0 : \mathbf{T} \\ \mathbf{T} \leq \mathbf{T}_0 \\ \mathbf{O}[\mathbf{T}_0/\mathbf{x}]\ \mathbf{e}_1 : \mathbf{T}_1 \end{array}}{\mathbf{O}\ \mathbf{let}\ \mathbf{x} : \mathbf{T}_0 \leftarrow \mathbf{e}_0\ \mathbf{in}\ \mathbf{e}_1 : \mathbf{T}_1}$$

  - Both versions of declaration rules are correct
  - The improved version checks more programs

Wrong Declaration Rule (case 1)

- Consider a hypothetical let rule

  [Wrong Declaration with initialization (case 1)]

$$\frac{\begin{array}{c} O\ e_0 : T \\ T \leq T_0 \\ O\ e_1 : T_1 \end{array}}{O\ \text{let}\ x : T_0 \leftarrow e_0\ \text{in}\ e_1 : T_1}$$

  - How is it different from the the correct rule?
  - The following good program does not pass check

    let x: int ← 0 in x+1

Wrong Declaration Rule (case 2)

- Consider a hypothetical let rule

  [Wrong Declaration with initialization (case 2)]

$$\frac{\begin{array}{c} O \vdash e_0 : T \\ T_0 \leq T \\ O[T_0/x] \vdash e_1 : T_1 \end{array}}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

  - How is it different from the the correct rule?
  - The following bad program passes the check

    class B inherits A { only_in_B() { ... } }
    let x: B ← new A in x.only_in_B()

Assignment

- A correct but too strict rule

  [Assignment]

$$\frac{\begin{array}{c} O(id) = T_0 \\ O \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O \vdash id \leftarrow e_1 : T_0}$$

  - The rule does not allow the below code

```
class C inherits P { only_in_C() { ... } }
let x:C in
let y:P in
x ← y ← new C
x.only_in_C()
```

Assignment

- An improved rule

[Assignment]

$$\frac{\begin{array}{c} O(id) = T_0 \\ O \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O \vdash id \leftarrow e_1 : T_1}$$

  - The rule now does allow the below code

```
class C inherits P { only_in_C() { ... } }
let x:C in
let y:P in
x ← y ← new C
x.only_in_C()
```

If-then-else

- Let's say semantics of "if e_0 then e_1 else e_2" is:
  - Returns the value of either e_1 or e_2, depending on e_0.

- What is the type of the above expression?
  - The type is either e_1's type or e_2's type.
  - Best compiler can do is to assign a super type of e_1 and e_2.

- Least upper bound (LUB): the super type of two types
  - Z = lub(X,Y) — Z is the least upper bound of X and Y *iff*
    - $X \leq Z \wedge Y \leq Z$; Z is an upper bound
    - $X \leq W \wedge Y \leq W \Longrightarrow Z \leq W$; Z is least among all upper bounds

If-then-else

[If-then-else]

$$\frac{\mathbf{O}\ e_0 : \mathbf{Bool} \quad \mathbf{O}\ e_1 : T_1 \quad \mathbf{O}\ e_2 : T_2}{\mathbf{O}\ \textbf{if}\ e_0\ \textbf{then}\ e_1\ \textbf{else}\ e_2\ \textbf{fi}: \mathrm{lub}(T_1, T_2)}$$

- The rule allows the below code

  let x:float, y:int, z:float in

  x ← if (...) then y else z

  /* Assuming lub(int, float) = float */

Discussion

- Type rules have to be carefully constructed, or
  - The type system becomes unsound
    (bad programs are accepted as well typed)
  - The type system becomes unusable
    (good programs are rejected as badly typed)

Discussion

- Type rules have to be carefully constructed, or
  - The type system becomes unsound (bad programs are accepted as well typed)
  - The type system becomes unusable (good programs are rejected as badly typed)
- What is a "good" program anyway?
  - Good program: a program where all operations on all values are type consistent **at runtime**

Discussion

- Type rules have to be carefully constructed, or
  - The type system becomes unsound (bad programs are accepted as well typed)
  - The type system becomes unusable (good programs are rejected as badly typed)
- What is a "good" program anyway?
  - Good program: a program where all operations on all values are type consistent **at runtime**
- All runtime behavior not expressed in a static type system
  - At below is a good program rejected by the type system
    obj $\leftarrow$ if (x > y) then new Child else new Parent
    if (x > y) then obj.only_in_Child()
  - LUB type makes a choice of soundness over usability

Designing a Good Type Checking System

- A good type system achieves two opposing goals:
  - Prevents **false negative** type errors, that is, runtime errors that are missed by type checking
  - Minimizes **false positive** type errors, that is, type errors that do not cause runtime errors
- A good type system should allow the following code:

```
class Parent {
  Parent clone() { return new this.getClass(); }
}
class Child inherits Parent { ... }
  void main() {
    // Error! Assignment of parent to child reference.
    Child c ← (new Child).clone();
  }
}
```

What Went Wrong?

- What is (new Child).clone()'s type?
  - Dynamic type — Child
  - Static type — Parent

  - Type system is not able to express runtime types precisely
  - This makes inheriting clone() not very useful
    - clone() needs redefinition to return correct type anyway

- A "SELF_TYPE" would be useful in these situations.

SELF_TYPE expresses runtime types precisely

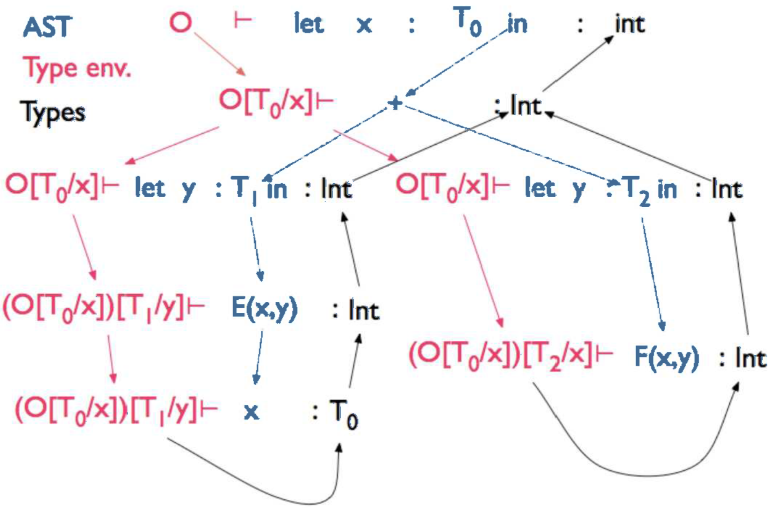
- What is SELF_TYPE?
  - clone() returns “self” instead of “Parent” type
  - Self can be Parent or any subclass of Parent
- SELF_TYPE is a static type
  - Type reflects precise runtime behavior for each class
  - Type violations can still be detected at compile time
- In practice
  - Python, Rust, Scala: language support for self types
  - C++: can emulate using C++ templates
  - Java: can emulate to a lesser degree using Java generics

Can Static Type Checking ever be Perfect?

- Many examples where "good" programs are disallowed
  - Reason for elaborate type systems like generics
  - Why programmers must sometimes typecast anyway

- Solution? Can't have your cake and eat it too.
  - Dynamic typing: values have types, variables do not
    - \+ Allows all runtime behaviors that are type consistent
    - \- Type errors occur at runtime rather than compile time
    - ☞ Best used for fast prototyping (scripting languages)
  - Static typing: variables have declared (or inferred) types
    - \+ Type errors can be caught at compile time
    - \- Effort needed to express "good" programs using type system
    - ☞ Best used when reliability is important

Implementing Type Checking on AST



Error Recovery

- Compiler must recover from type errors like syntax errors
  - Or else, below code results in multiple cascading errors
    let y: int $\leftarrow$ x+2 in y+3
    - Reports error "x is undefined"
    - Reports error "Type of x+2 is undefined"
    - Reports error "Type of let y: int $\leftarrow$ x+2 in y+3 is undefined"
    - ...

- Solution: introduce no-type for ill-typed expressions
  - It is compatible with all types $\rightarrow$ no cascading errors
  - Report only the place where no-type is generated

Syntax Directed Definitions (SDDs)

SDD: Definitions of attributes and rules

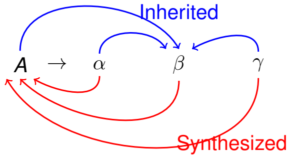
- Syntax Directed Definitions (SDD):
  1. Set of **attributes** attached to each grammar symbol
  2. Set of **semantic rules** attached to each production
  - Semantic rules define values of attributes

- Attribute Grammar:
  - An SDD where rules depend only on other attributes (i.e. An SDD that does not rely on any side-effects)
  - Think of it as a "grammar" for semantic analysis

- Example: let's say we want to define type checking
  - SDD can have semantic rules to access a symbol table
  - Attribute grammar must transmit type info through attributes

Synthesized vs. Inherited Attributes

- Semantic rule:



SDD has rule of the form for each CFG production

$b = f(c_1, c_2, ..., c_n)$

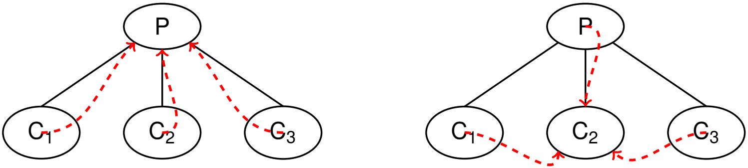
either

1. If b is a synthesized attribute of A,
   c_1 $(1 \leq i \leq n)$ are attributes of grammar symbols of its Right Hand Side (RHS); or
2. If b is an inherited attribute of one of the symbols of RHS,
   c_i's are attribute of A and/or other symbols on the RHS

Synthesized vs. Inherited Attributes

- **Synthesized attributes:** computed from children nodes
  - P.synthesized_attr = f(C_1.attr, C_2.attr, C_3.attr)

- **Inherited attributes:** computed from sibling/parent nodes
  - C_3.inherited_attr = f(P_1.attr, C_1.attr, C_3.attr)



Synthesized attribute | Inherited attribute

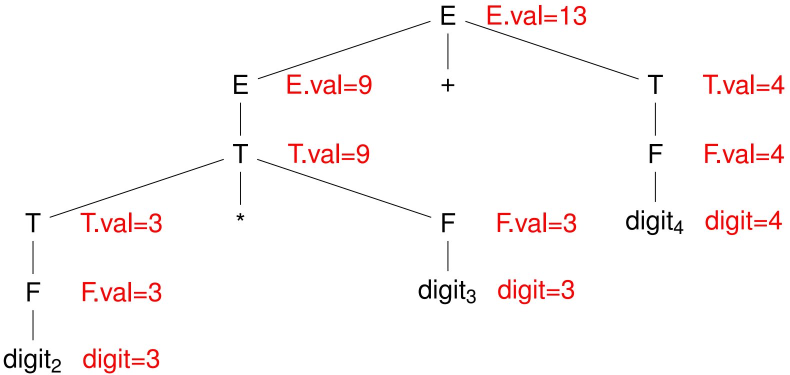
Synthesized Attribute Example

- Example
  - Each non-terminal symbol is associated with **val** attribute
  - The **val** attribute is computed soley from children attributes

[Grammar Rules]	[Semantic Rules]
$L \rightarrow E$	print(E.val)
$E \rightarrow E_1 + T$	$E.val = E_1.val + T.val$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T_1 * F$	$T.val = T_1.val * F.val$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow digit$	$F.val = digit.lexval$

Synthesized Attribute Example: Attribute Parse Tree

- **Attribute parse tree**: Parse tree decorated with attributes



Inherited Attribute Example

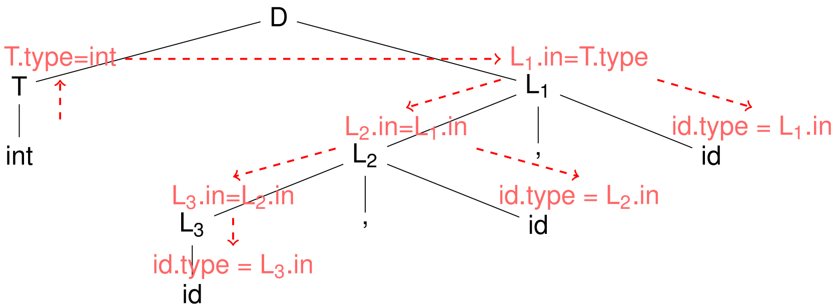
- Example:
  - T.type: synthesized attribute
  - L.in: inherited attribute
  - id.type: inherited attribute

[Grammar Rules]	[Semantic Rules]
$D \rightarrow T\ L$	L.in = T.type
$T \rightarrow$ int	T.type = integer
$T \rightarrow$ real	T.type = real
$L \rightarrow L_1$, id	L_1.in = L.in, id.type = L.in
$L \rightarrow$ id	id.type = L.in

- Why is L.in an inherited attribute?
  - L.in is computed from a sibling T.type
  - L_1.in is computed from a parent L.in

Inherited Attribute Example: Attribute Parse Tree

- Red arrows denote dependencies between attributes
- Arrows for inherited attributes go sideways or downwards
- Arrows for synthesized attributes go upwards



SDD Implementation

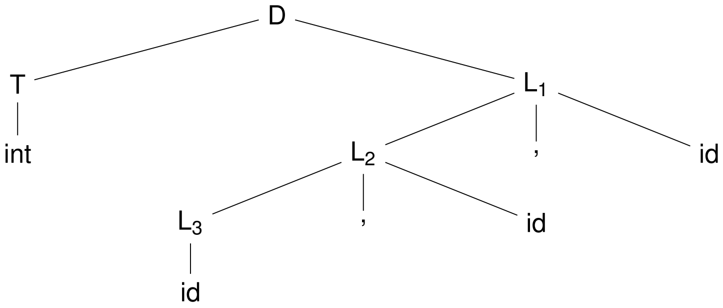
SDD Implementation using Parse Trees

- Assumes a previous parse stage
  - Input: a parse tree with no attribute annotations
  - Output: an attribute parse tree

- Goal: compute attribute values from leaf token values
  - Traverse in some order, apply semantic rules at each node
  - Traversal order must consider attribute dependencies

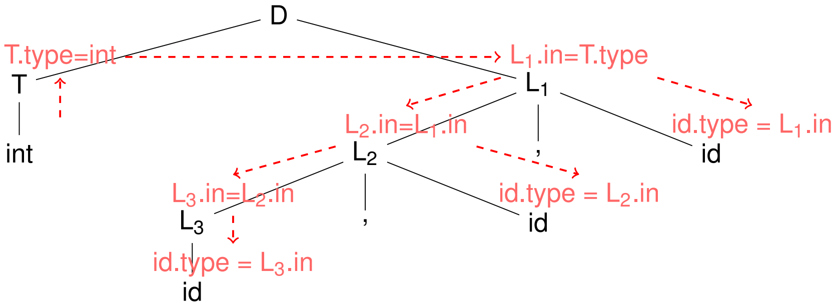
Dependency Graph

- Directed graph where edges are attribute dependencies
  - "To" attribute is computed base on "from" attribute
  - Must be **acyclic** such that there exists "a" traversal order



Dependency Graph

- Directed graph where edges are attribute dependencies
  - "To" attribute is computed base on "from" attribute
  - Must be **acyclic** such that there exists "a" traversal order



SDD Implementation using SDT

- Syntax Directed Translation (SDT)
  - Applying semantic rules as part of syntax analysis (parsing)
  - Does NOT assume a pre-existing parse tree
  - Done through **semantic actions** embedded in grammar
- Semantic action:
  - Code between curly braces embedded into RHS
  - Executed "at that point" in the RHS
    - Top-down: Right after previous symbol has been consumed
    - Bottom-up: After previous symbol has been pushed to stack (when the 'dot' reaches the semantic action)
  - Example of building a parse tree:
    - Program : Program IDNum ; Classes
      { $$=makeTree(ProgramOp, $2, $4); }
  - $2 and $4 are indices into the parse stack
    - RHS is currently at top of stack waiting to be reduced
    - $2 is attribute value for IDNum and $4 is Classes

Syntax Directed Translation Scheme (SDTS)

- Syntax Directed Translation Scheme (SDTS)
  - A "scheme" or plan to perform SDT
  - A grammar specification embedded with semantic actions
  - Depends on choice of top-down or bottom-up parser
- Example:

$$
\begin{aligned}
E &\rightarrow T \quad R \\
R &\rightarrow + \quad T \quad R \\
R &\rightarrow - \quad T \quad R \\
R &\rightarrow \varepsilon \\
T &\rightarrow (\; E \;) \\
T &\rightarrow \text{num}
\end{aligned}
$$

- Both inherited and synthesized attributes are used
  - T — synthesized attribute T.val
  - R — inherited attribute R.i
    synthesized attribute R.s
  - E — synthesized attribute E.val

Syntax Directed Translation Scheme (SDTS)

Evaluating attributes using SDTS

$E \rightarrow T$ {R.i=T.val} R {E.val=R.s}

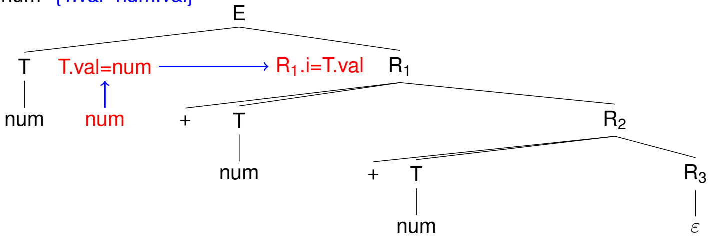
$R \rightarrow + \; T$ {R_1.i=R.i+T.val} R_1 {R.s=R_1.s}

$R \rightarrow - \; T$ {R_1.i=R.i-T.val} R_1 {R.s=R_1.s}

$R \rightarrow \varepsilon$ {R.s=R.i}

$T \rightarrow (\; E \;)$ {T.val=E.val}

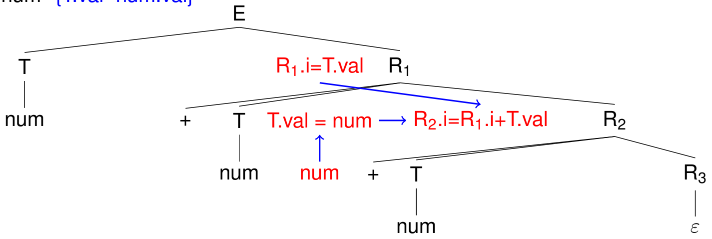
$T \rightarrow$ num {T.val=num.val}



Syntax Directed Translation Scheme (SDTS)

- Evaluating attributes using SDTS

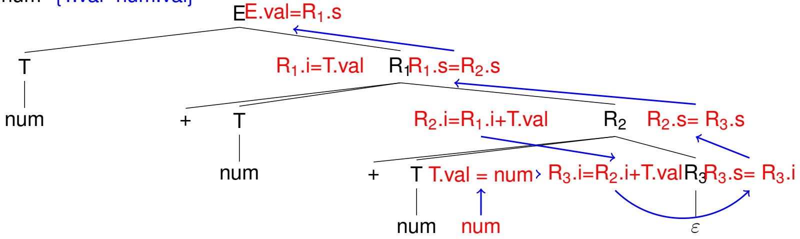
$E \rightarrow T$ {R.i=T.val} R {E.val=R.s}
$R \rightarrow +\ T$ {R_1.i=R.i+T.val} R_1 {R.s=R_1.s}
$R \rightarrow -\ T$ {R_1.i=R.i-T.val} R_1 {R.s=R_1.s}
$R \rightarrow \varepsilon$ {R.s=R.i}
$T \rightarrow (E)$ {T.val=E.val}
$T \rightarrow$ num {T.val=num.val}



Syntax Directed Translation Scheme (SDTS)

- Evaluating attributes using SDTS

E $\rightarrow$ T {R.i=T.val} R {E.val=R.s}
R $\rightarrow$ + T {R_1.i=R.i+T.val} R_1 {R.s=R_1.s}
R $\rightarrow$ - T {R_1.i=R.i-T.val} R_1 {R.s=R_1.s}
R $\rightarrow$ ε {R.s=R.i}
T $\rightarrow$ (E) {T.val=E.val}
T $\rightarrow$ num {T.val=num.val}



What are the dependencies allowed in SDTS?

- Parse trees: dependencies only required to be acyclic

- What is required of dependencies for SDTS?
  - Different parsing schemes see nodes in different orders
    - Top-down parsing — LL(k) parsing
    - Bottom-up parsing — LR(k) parsing
  - What if dependency node has not been seen yet?

- For certain classes of SDDs, using SDTS is feasible
  - If dependencies of SDD are amenable to parse order
  - This class of SDDs are called L-Attributed Grammars

Left-Attributed Grammar

- An SDD is L-attributed if each of its attributes is either:
  - a synthesized attribute of A in A$\rightarrow$ $X_1 ... X_n$,

    or

  - an inherited attribute of X_j in A$\rightarrow$ $X_1 ... X_n$ that
    - depends on attributes of siblings to its left i.e. $X_1 ... X_{j-1}$
    - and/or depends on parent A

Left-Attributed Grammar

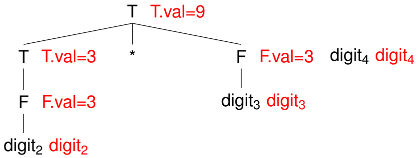
- An L-Attributed grammar
  - may have synthesized attributes
  - may have inherited attributes but only from left sibling attributes or inherited attributes of the parent
- Evaluation order
  - Left-to-right depth-first traversal of the parse tree
    - Order for both top-down and bottom-up parsers
  - Evaluate inherited attributes while going down the tree
  - Evaluate synthesized attributes while going up the tree
- Can be evaluated using SDTS w/o parse tree

Syntax Directed Translation Scheme (SDTS)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is natural and easy to evaluate synthesized attributes



parsing stack:

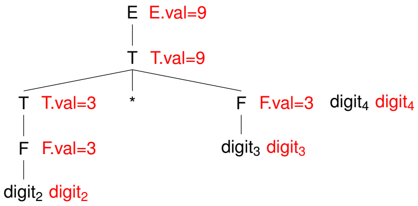
$S_?$	T	T.val=9
$S_?$	$	-

(state) (symbol) (attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is natural and easy to evaluate synthesized attributes



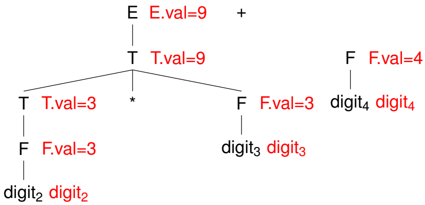
parsing stack:

(state)	(symbol)	(attribute)
$S_?$	E	E.val=9
$S_?$	$	-

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is natural and easy to evaluate synthesized attributes



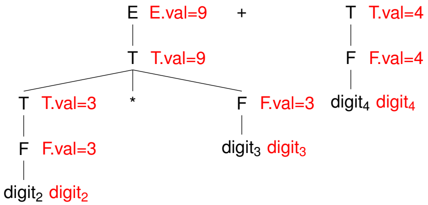
parsing stack:

$S_?$	F	F.val=4
$S_?$	+	-
$S_?$	E	E.val=9
$S_?$	$	-
(state)	(symbol)	(attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is natural and easy to evaluate synthesized attributes



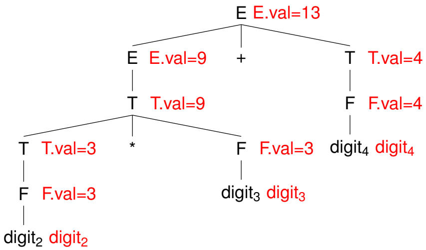
parsing stack:

(state)	(symbol)	(attribute)
$S_?$	T	T.val=4
$S_?$	+	-
$S_?$	E	E.val=9
$S_?$	$	-

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is natural and easy to evaluate synthesized attributes



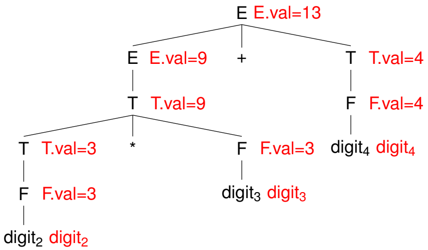
parsing stack:

$S_?$	E	E.val=13
$S_?$	\$	-
(state)	(symbol)	(attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is natural and easy to evaluate synthesized attributes



parsing stack:

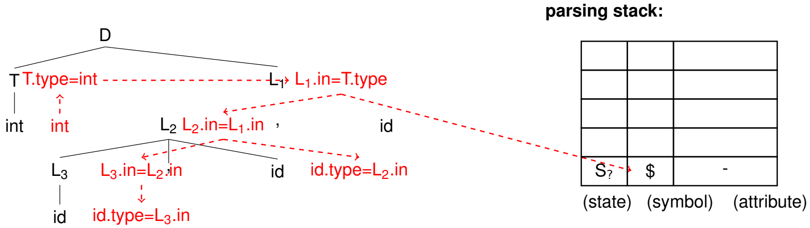
(state)	(symbol)	(attribute)
$S_?$	E	E.val=13
$S_?$	\$	-

- Grammars with only synthesized attributes are called **S-Attributed Grammars**

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is **not natural** to evaluate inherited attributes



Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is **not natural** to evaluate inherited attributes

int , id

, id

id

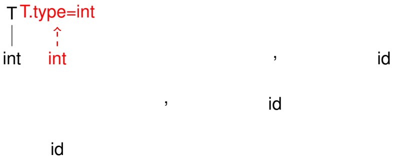
parsing stack:

$S_?$	$	-
(state)	(symbol)	(attribute)

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is **not natural** to evaluate inherited attributes



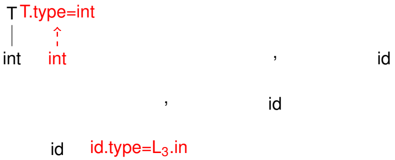
parsing stack:

(state)	(symbol)	(attribute)
$S_?$	T	T.type=int
$S_?$	\$	-

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is **not natural** to evaluate inherited attributes



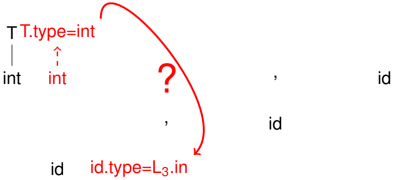
parsing stack:

(state)	(symbol)	(attribute)
$S_?$	id	**id.type=L_3.in**
$S_?$	T	T.type=int
$S_?$	$	-

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is **not natural** to evaluate inherited attributes



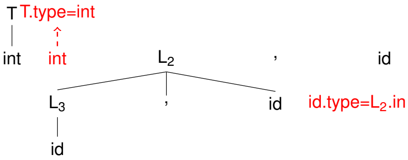
parsing stack:

(state)	(symbol)	(attribute)
$S_?$	id	**id.type=L_3.in**
$S_?$	T	T.type=int
$S_?$	$	-

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is **not natural** to evaluate inherited attributes



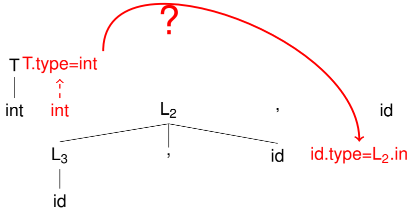
parsing stack:

(state)	(symbol)	(attribute)
$S_?$	id	**id.type=L_2.in**
$S_?$	,	
$S_?$	L_3	L_3.in=L_2.in
$S_?$	T	T.type=int
$S_?$	\$	-

Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing),

- it is **not natural** to evaluate inherited attributes



parsing stack:

$S_?$	id	**id.type=L_2.in**
$S_?$	,	
$S_?$	L_3	L_3.in=L_2.in
$S_?$	T	T.type=int
$S_?$	$	-
(state)	(symbol)	(attribute)

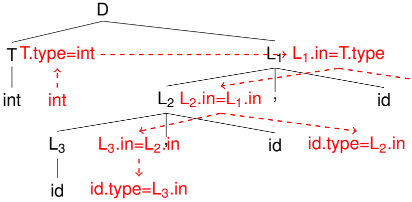
Evaluating Inherited Attributes using LR

- **Claim:** Given an L-Attributed grammar, inherited attributes needed for the computation are already on the stack

- Recall: What is an L-Attributed grammar?
  - May have synthesized attributes
  - May have inherited attributes but only from:
    - **Left** sibling attributes
    - Parent attribute

- Finding inherited attributes on the stack
  - Left siblings: previously reduced, so already on the stack
  - Parent: not yet reduced, but left siblings of the parent used to compute the parent attribute are on the stack

$D \rightarrow T \quad L$
$T \rightarrow$ int {T.type=int}
$T \rightarrow$ real {T.type=real}
$L \rightarrow L$, id {id.type=stack[top-3].type}
$L \rightarrow$ id {id.type=stack[top-1].type}



parsing stack:

$S_?$	\$	-
(state)	(symbol)	(attribute)

$D \rightarrow T \quad L$
$T \rightarrow$ int {T.type=int}
$T \rightarrow$ real {T.type=real}
$L \rightarrow L$, id {id.type=stack[top-3].type}
$L \rightarrow$ id {id.type=stack[top-1].type}

parsing stack:

$S_?$	$	-
(state)	(symbol)	(attribute)

int , id

, id

id

D → T L
T → int {T.type=int}
T → real {T.type=real}
L → L , id {id.type=stack[top-3].type}
L → id {id.type=stack[top-1].type}

T T.type=int

int int , id

, id

id

parsing stack:

$S_?$	T	T.type=int
$S_?$	$	-
(state)	(symbol)	(attribute)

$D \rightarrow T \quad L$

$T \rightarrow$ int {T.type=int}

$T \rightarrow$ real {T.type=real}

$L \rightarrow L \;,\; \text{id}$ {id.type=stack[top-3].type}

$L \rightarrow \text{id}$ {id.type=stack[top-1].type}

T T.type=int

int int   ,   id

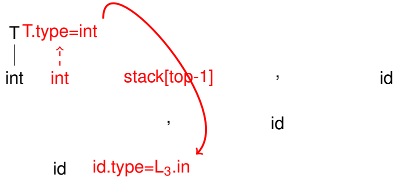
,   id

id   id.type=L_3.in

parsing stack:

$S_?$	id	**id.type=stack[top-1]**
$S_?$	T	T.type=int
$S_?$	$	-
(state)	(symbol)	(attribute)

D $\rightarrow$ T L
T $\rightarrow$ int {T.type=int}
T $\rightarrow$ real {T.type=real}
L $\rightarrow$ L , id {id.type=stack[top-3].type}
L $\rightarrow$ id {id.type=stack[top-1].type}



parsing stack:

(state)	(symbol)	(attribute)
$S_?$	id	**id.type=stack[top-1]**
$S_?$	T	T.type=int
$S_?$	$	-

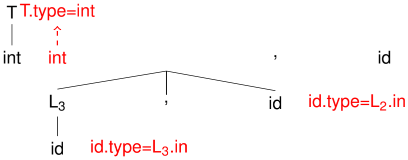
D $\rightarrow$ T L

T $\rightarrow$ int {T.type=int}

T $\rightarrow$ real {T.type=real}

L $\rightarrow$ L , id {id.type=stack[top-3].type}

L $\rightarrow$ id {id.type=stack[top-1].type}



parsing stack:

(state)	(symbol)	(attribute)
$S_?$	id	**id.type=stack[top-3]**
$S_?$	,	
$S_?$	L_3	L_3.in=int
$S_?$	T	T.type=int
$S_?$	$	-

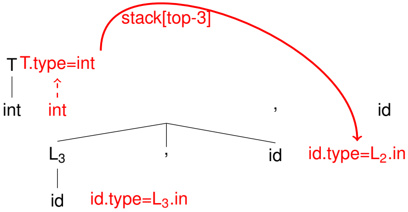
D $\rightarrow$ T L

T $\rightarrow$ int {T.type=int}

T $\rightarrow$ real {T.type=real}

L $\rightarrow$ L , id {id.type=stack[top-3].type}

L $\rightarrow$ id {id.type=stack[top-1].type}



parsing stack:

$S_?$	id	**id.type=stack[top-3]**
$S_?$	,	
$S_?$	L_3	L_3.in=int
$S_?$	T	T.type=int
$S_?$	$	-

(state) (symbol) (attribute)

Marker

- Given the following SDD, where $|\alpha|$!= $|\beta|$

  A$\rightarrow$ X α Y | X β Y

  Y$\rightarrow$ γ {... = f(X.s)}

- Problem: cannot generate stack location for X.s since X is at different relative stack locations from Y
- Solution: introduce *markers* M_1 and M_2 that are at the same relative stack locations from Y

  A$\rightarrow$ X α M_1 Y | X β M_2 Y

  Y$\rightarrow$ γ {... = f(M_{12}.s)}

  $M_1$$\rightarrow$ ε {M_1.s = X.s}

  $M_2$$\rightarrow$ ε {M_2.s = X.s}

  (M_{12} = the stack location of M_1 or M_2, which are identical)

- A marker intuitively marks a stack location that is equidistant from the reduced non-terminal

Example

- When is a marker necessary and how is it added?

Example 1:

S $\rightarrow$ a A { C.i = A.s } C
S $\rightarrow$ b A B { C.i = A.s } C
C $\rightarrow$ c { C.s = f(C.i) }

Solution:

S $\rightarrow$ a A { C.i = A.s } C
S $\rightarrow$ b A B { M.i=A.s } M { C.i = M.s } C
C $\rightarrow$ c { C.s = f(C.i) }
M $\rightarrow$ ε { M.s = M.i }

That is:

S $\rightarrow$ a A C
S $\rightarrow$ b A B M C
C $\rightarrow$ c { C.s = f(stack[top-1]) }
M $\rightarrow$ ε { M.s = stack[top-2] }

When and how to add a marker

1. Identify the stack offset(s) to find the desired attribute
2. If stack offsets are different, add a marker
3. Add marker where it would result in uniform stack offsets

Example:

S → a A B C E D
S → b A F B C F D
C → c {/* C.s = f(A.s) */}
D → d {/* D.s = f(B.s) */}

Answer

S $\rightarrow$ a A B C E D
S $\rightarrow$ b A D M B C F D
C $\rightarrow$ c {/* C.s = f(stack[top-2]) */}
D $\rightarrow$ d {/* D.s = f(stack[top-3]) */}
M $\rightarrow$ ε {/* M.s = f(stack[top-2]) */}

- Regarding C.s, from stack[top-2], and stack[top-3]
  .... add a Marker
- Regarding D.s, always from stack[top-2]
  ... no need to add

- How about Top-Down Parsing?

Translation Scheme for Top-Down Parsing

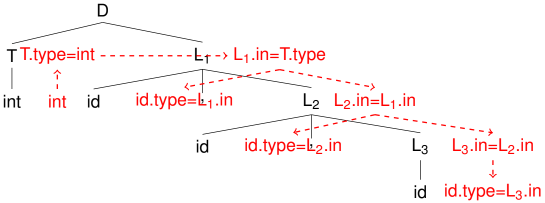
- Recursive Descent Parsers: Straightforward
  - Synthesized Attribute
    - Say function for non-terminal returns synthesized attribute
    - Compute attribute from children function call return values
  - Inherited Attribute
    - Pass as argument to function call for inheriting non-terminal
    - Left sibling attributes: left sibling calls already complete
    - Parent attributes: passed in as arguments to parent function

- How about table-driven LL parsers?

Translation Scheme for LL Parsing

- it is natural to evaluate inherited attributes

D $\rightarrow$ T {L.in=T.type} L
T $\rightarrow$ int {T.type=int}
T $\rightarrow$ real {T.type=real}
L $\rightarrow$ {id.type=L.in} id , {L_1.in=L.in} L_1
L $\rightarrow$ {id.type=L.in} id



parsing stack:

(symbol)	(attribute)

Translation Scheme for LL Parsing

- it is natural to evaluate inherited attributes

D $\rightarrow$ T {L.in=T.type} L
T $\rightarrow$ int {T.type=int}
T $\rightarrow$ real {T.type=real}
L $\rightarrow$ {id.type=L.in} id , {L_1.in=L.in} L_1
L $\rightarrow$ {id.type=L.in} id

D

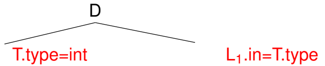
parsing stack:

(symbol)	(attribute)
D	

Translation Scheme for LL Parsing

- it is natural to evaluate inherited attributes

$D \rightarrow T$ {L.in=T.type} L
$T \rightarrow$ int {T.type=int}
$T \rightarrow$ real {T.type=real}
$L \rightarrow$ {id.type=L.in} id , {L_1.in=L.in} L_1
$L \rightarrow$ {id.type=L.in} id



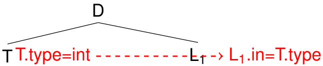
parsing stack:

(symbol)	(attribute)
T	T.type=int
	{L_1.in=T.type}
L	L_1.in=???

Translation Scheme for LL Parsing

- it is natural to evaluate inherited attributes

D $\rightarrow$ T {L.in=T.type} L
T $\rightarrow$ int {T.type=int}
T $\rightarrow$ real {T.type=real}
L $\rightarrow$ {id.type=L.in} id , {L_1.in=L.in} L_1
L $\rightarrow$ {id.type=L.in} id



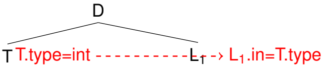
parsing stack:

	{L_1.in=int}
L	L_1.in=???
(symbol)	(attribute)

Translation Scheme for LL Parsing

- it is natural to evaluate inherited attributes

$D \rightarrow T$ {L.in=T.type} L
$T \rightarrow$ int {T.type=int}
$T \rightarrow$ real {T.type=real}
$L \rightarrow$ {id.type=L.in} id , {L_1.in=L.in} L_1
$L \rightarrow$ {id.type=L.in} id



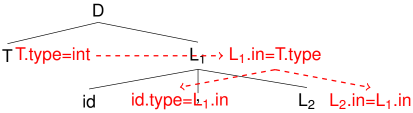
parsing stack:

(symbol)	(attribute)
	L_1.in=int

Translation Scheme for LL Parsing

- it is natural to evaluate inherited attributes

$D \rightarrow T$ {L.in=T.type} L
$T \rightarrow$ int {T.type=int}
$T \rightarrow$ real {T.type=real}
$L \rightarrow$ {id.type=L.in} id , {L_1.in=L.in} L_1
$L \rightarrow$ {id.type=L.in} id



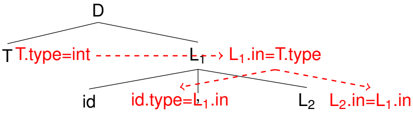
parsing stack:

(symbol)	(attribute)
	{id.type=L_1.in}
id	id.type=???
,	
	{L_2.in=L_1.in}
L_2	L_2.in=???

Translation Scheme for LL Parsing

- it is natural to evaluate inherited attributes

D → T {L.in=T.type} L
T → int {T.type=int}
T → real {T.type=real}
L → {id.type=L.in} id , {L_1.in=L.in} L_1
L → {id.type=L.in} id



parsing stack:

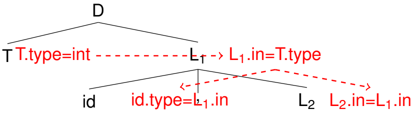
	{id.type=int}
id	id.type=???
,	
	{L_2.in=int}
L_2	L_2.in=???

(symbol) (attribute)

Translation Scheme for LL Parsing

- it is natural to evaluate inherited attributes

$D \rightarrow T$ {L.in=T.type} L
$T \rightarrow$ int {T.type=int}
$T \rightarrow$ real {T.type=real}
$L \rightarrow$ {id.type=L.in} id , {L_1.in=L.in} L_1
$L \rightarrow$ {id.type=L.in} id



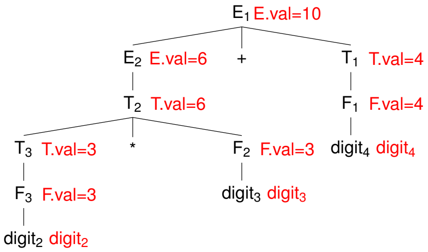
parsing stack:

(symbol)	(attribute)
	{id.type=int}
id	id.type=???
,	
	{L_2.in=int}
L_2	L_2.in=???

- Semantic actions on the stack are called **action-records.**

Translation Scheme for LL Parsing

- it is **not natural** to evaluate synthesized attributes



parsing stack:

(symbol) (attribute)

Translation Scheme for LL Parsing

- it is **not natural** to evaluate synthesized attributes

E_1

parsing stack:

E_1	
(symbol)	(attribute)

Translation Scheme for LL Parsing

- it is **not natural** to evaluate synthesized attributes

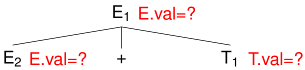
E_1 E.val=?

parsing stack:

E_1	E_1.val=?
(symbol)	(attribute)

Translation Scheme for LL Parsing

- it is **not natural** to evaluate synthesized attributes

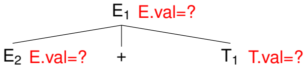


parsing stack:

(symbol)	(attribute)
E_2	E_2.val=?
+	
T_1	T_1.val=?

Translation Scheme for LL Parsing

- it is **not natural** to evaluate synthesized attributes



parsing stack:

E_2	E_2.val=?
+	
T_1	T_1.val=?
(symbol)	(attribute)

Translation Scheme for LL Parsing

- it is **not natural** to evaluate synthesized attributes

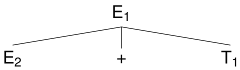
E_1

parsing stack:

E_1	
E_1.val	???
(symbol)	(attribute)

Translation Scheme for LL Parsing

- it is **not natural** to evaluate synthesized attributes

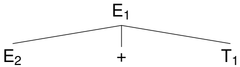


parsing stack:

(symbol)	(attribute)
E_2	
E_2.val	???
+	
T_1	
T_1.val	???
E_1.val	E_2.val + T_1.val

Translation Scheme for LL Parsing

- it is **not natural** to evaluate synthesized attributes



parsing stack:

E_2	
E_2.val	???
+	
T_1	
T_1.val	???
E_1.val	E_2.val + T_1.val
(symbol)	(attribute)

- Synthesized attributes on the stack: **synthesize-records.** (Inserted below non-terminal with synthesized attribute)
- In synthesize-record E_1.val = E_2.val + T_1.val, E_2.val and T_1.val are place holders for pending values. (Updated when records E_2.val and T_1.val are popped.)