

Compiler Optimization

Compiler optimizations transform code

- ❑ Code optimization transforms code to equivalent code
 - ... but with better performance
- ❑ The code transformation can involve either
 - **Replacing** code with more efficient code
 - **Deleting** redundant code
 - **Moving** code to a position where it is more efficient
 - **Inserting** new code to improve performance

The four categories of code transformations

- ❑ Replacing code (e.g. **strength reduction**)

$A=2*a;$ \equiv $A=a\ll 1;$

- ❑ Deleting code (e.g. **dead code elimination**)

$A=2; A=y;$ \equiv $A=y;$

- ❑ Moving code (e.g. **loop invariant code motion**)

`for (i = 0; i < 100; i++) { sum += i + x * y; }`

\equiv

$t = x * y;$

`for (i = 0; i < 100; i++) { sum += i + t; }`

- ❑ Inserting code (e.g. **data prefetching**)

`for (p = head; p != NULL; p = p->next)`
`{ /* do work on node p */ }`

\equiv

`for (p = head; p != NULL; p = p->next)`
`{ prefetch(p->next); /* do work on node p */ }`

Compiler optimization categories according to range

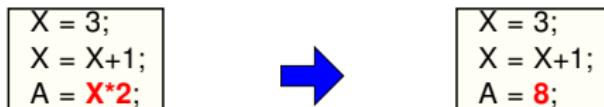
- ❑ How much code does the compiler view while optimizing?
 - The wider the view, the more powerful the optimization
- ❑ Axis 1: optimize across control flow?
 - **Local optimization**: optimizes only within straight line code
 - **Global optimization**: optimizes across control flow
(if,for,...)
- ❑ Axis 2: optimize across function calls?
 - **Intra-procedural optimization**: only within function
 - **Inter-procedural optimization**: across function calls
- ❑ The two axes are orthogonal (any combination is possible)

Local vs. Global Constant Propagation

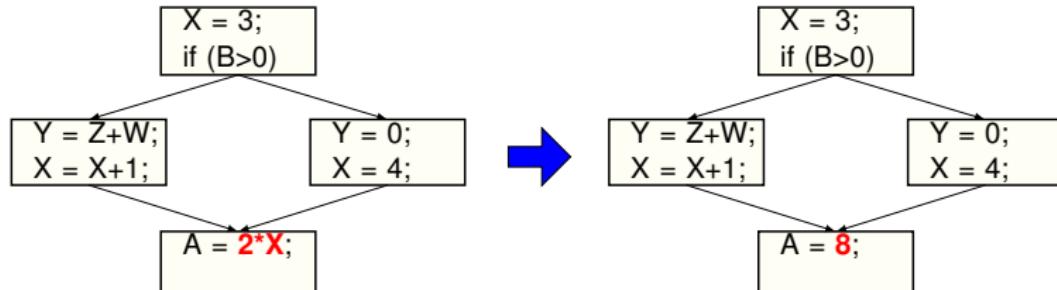
❑ Constant propagation

- Optimization: if $x = y \text{ op } z$ and y and z are constants then compute at compile time and replace

❑ Local Constant Propagation



❑ Global Constant Propagation



- Additional **control flow analysis** is needed to enable this.

Intra- vs. Inter-procedural Constant Propagation

❑ Intra-procedural Constant Propagation

```
X = 3;  
X = X+1;  
A = X*2;
```



```
X = 3;  
X = X+1;  
A = 8;
```

- Above is a local intra-procedural constant propagation.

❑ Inter-procedural Constant Propagation

```
X = 3;  
foo();  
A = X*2;
```

```
void foo() {  
    // No update to X.  
}
```



```
X = 3;  
foo();  
A = 6;
```

```
void foo() {  
    // No update to X.  
}
```

- Above is a local inter-procedural constant propagation.
- The fact that global variable X is not updated must be propagated from callee to caller function to enable this.

Control Flow Analysis

Basic Block

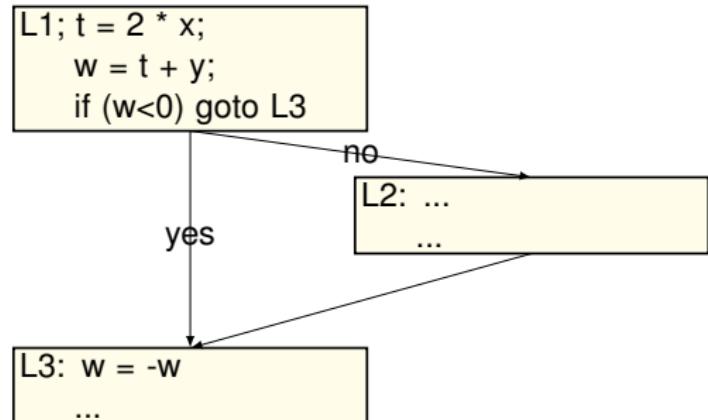
- ❑ A function body is composed of one or more **basic blocks**.
- ❑ **Basic block:** a maximal sequence of instructions that either execute all together or none at all. Meaning:
 - There are no jumps into the middle of the basic block
 - There are no jumps out of the middle of the block
- ❑ That means:
 - No instruction other than the first is a jump target
 - No instruction other than the last is a jump or branch
- ❑ Either all instructions in basic block execute or none
 - Smallest unit of execution in control flow analysis
 - Hence the descriptor "basic" in the name

Control Flow Graph

- ❑ A **Control Flow Graph (CFG)** is a directed graph in which
 - Nodes are basic blocks
 - Edges represent flows of execution between basic blocks
- ❑ CFGs are widely used to represent a program for analysis
- ❑ CFGs are especially essential for global optimizations

Control Flow Graph Example

```
L1; t = 2 * x;  
    w = t + y;  
    if (w<0) goto L3  
L2: ...  
...  
L3: w = -w  
...
```



Construction of CFG

- ❑ Step 1: partition code into basic blocks
 - Identify **leader** instructions, where a leader is either:
 - the first instruction of a program, or
 - the target of any jump/branch, or
 - an instruction immediately following a jump/branch
 - Create a basic block out of each leader instruction
 - Expand basic block by adding subsequent instructions
(Stopping when the next leader instruction is encountered)

- ❑ Step 2: add edge between two basic blocks B1 and B2 if
 - there exist a jump/branch from B1 to B2, or
 - B2 follows B1, and B1 does not end with unconditional jump

Example

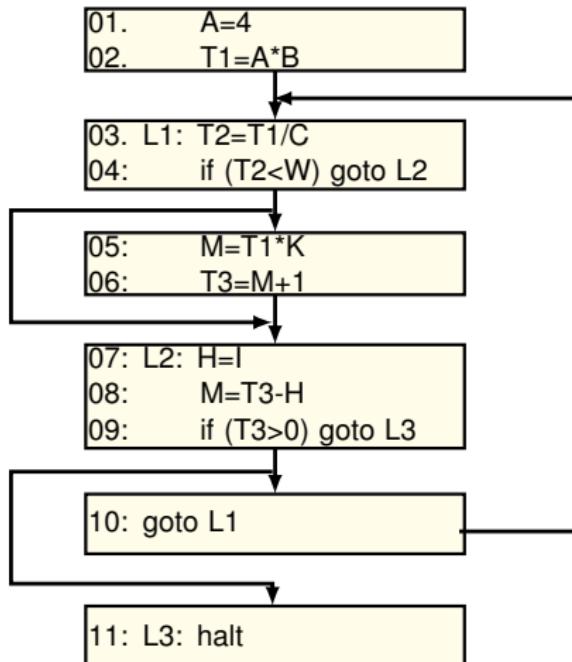
```
01.    A=4
02.    T1=A*B
03. L1: T2=T1/C
04:    if (T2<W) goto L2
05:    M=T1*K
06:    T3=M+1
07: L2: H=I
08:    M=T3-H
09:    if (T3>0) goto L3
10:   goto L1
11: L3: halt
```

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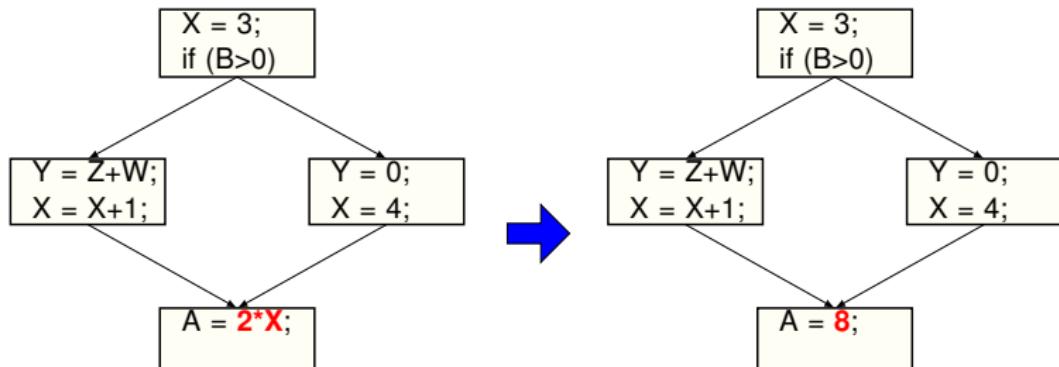
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Data Flow Analysis

Global Optimizations

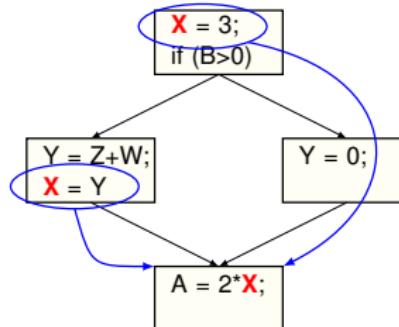
- ❑ Extends optimizations across control flows, i.e. CFG
- ❑ Like in this example Global Constant Propagation (GCP):



- ❑ How do we know it is OK to globally propagate constants?

Correctness criteria for GCP

- ❑ There are situations that prohibit GCP:



- ❑ To replace X by a constant C **correctly**, we must know
 - **Along all paths**, the last assignment to X is " $X = C$ "
- ❑ Paths may go through loops and/or branches
 - When two paths **meet**, need to make **conservative** choice

Global Optimizations need to be Conservative

- ❑ Many compiler optimizations depend on knowing some property X at a particular point in program execution
 - Need to prove at that point property X holds along all paths
- ❑ To ensure correctness, optimization must be **conservative**
 - An optimization is enabled only when X is definitely true
 - If not sure, be conservative and say **don't know**
 - **Don't know** typically disables the optimization

Dataflow Analysis Framework

❑ **Dataflow analysis:** discovering properties about values at each statement of the program

- E.g. discovering a value is constant before a statement
- Done by observing the flow of data through the CFG

❑ **Dataflow analysis framework:**

- A framework for implementing various dataflow analyses
- 4 parameters defining analysis is passed into framework:

$$\{ \mathbf{D}, \mathbf{V}, \wedge: (\mathbf{V}, \mathbf{V}) \rightarrow \mathbf{V}, \mathbf{F}: \mathbf{V} \rightarrow \mathbf{V} \}$$

- **D:** direction of dataflow (forward or backward)
- **V:** domain of values denoting property
- **\wedge :** **meet operator** that merges values when paths meet
- **F:** **flow propagation function** that propagates values through a basic block

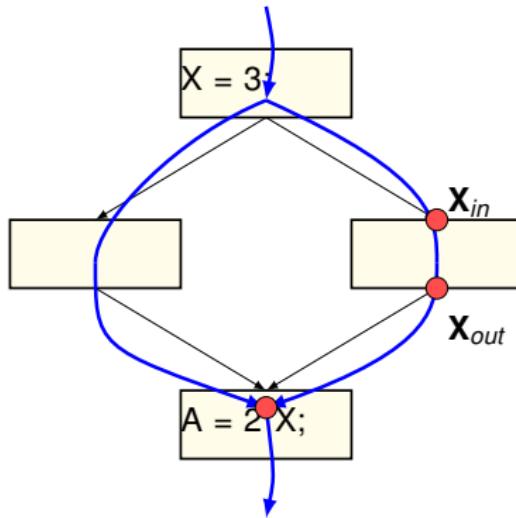
Global Constant Propagation

Global Constant Propagation (GCP)

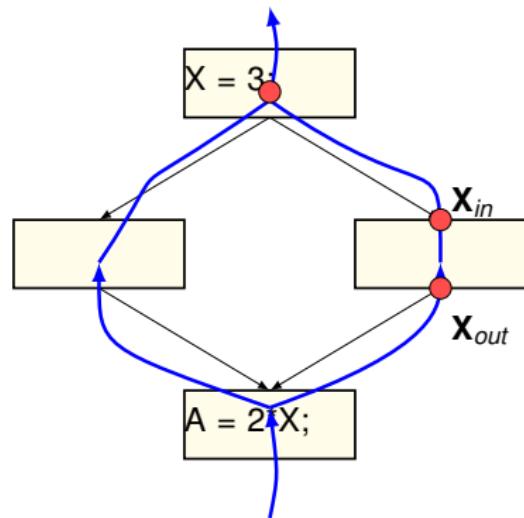
- ❑ Let's use **GCP** to study dataflow analysis framework
- ❑ We will define each component one by one for GCP
 - **D**: direction of dataflow for constant property
 - **V**: domain of values denoting constant property
 - **\wedge** : **meet operator** that merges values when paths meet
 - **F**: **flow propagation function** for GCP

Direction D for GCP

- ❑ Is GCP a forward or backward analysis?



Forward Analysis



Backward Analysis

- ❑ Forward, since "constantness" of a variable flows forward to subsequent instructions starting from assignment

Dataflow property V for GCP

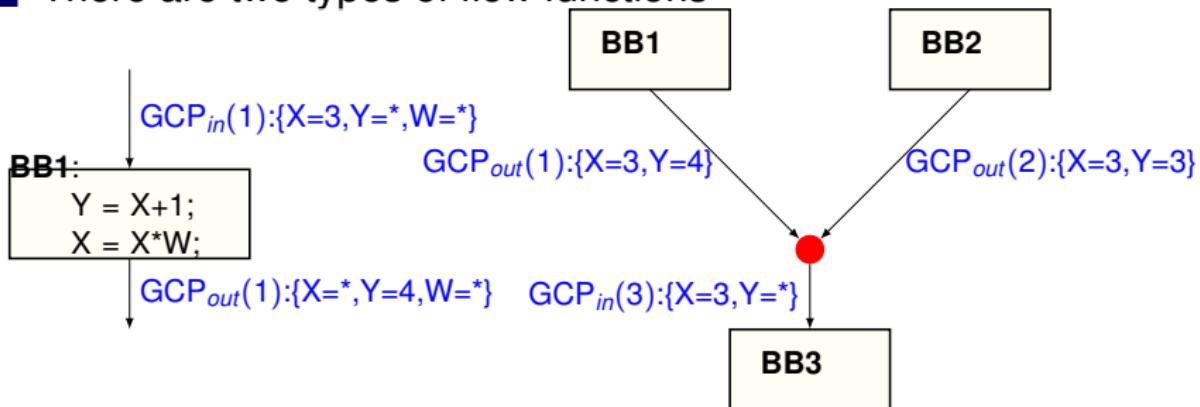
- ❑ V is a map of variables to values, where a value is:
(in the case where value is an int type)

```
# /* not defined yet */  
..., -1, 0, 1, ... /* a constant */  
* /* not a constant */
```

- ❑ **GCP(*i*)**: GCP dataflow property of basic block *i*
 - **GCP_{in}(*i*)**: at the entry of basic block *i*
 - **GCP_{out}(*i*)**: at the exit of basic block *i*
- ❑ **GCP(*i*)[X]**: value mapped to variable X in GCP(*i*)
- ❑ Example: given $\text{GCP}_{\text{in}}(1) = \{\text{X}=1, \text{Y}=\#, \text{Z}=\ast\}$
 - $\text{GCP}_{\text{in}}(1)[\text{X}] = 1, \text{GCP}_{\text{in}}(1)[\text{Y}] = \#, \text{GCP}_{\text{in}}(1)[\text{Z}] = \ast$

Dataflow Equations for GCP

- ❑ There are two types of flow functions



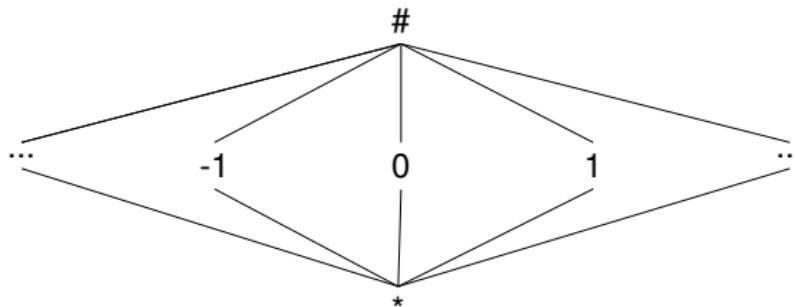
- Flow transfer function $F: V \rightarrow V$
 - Computes data flow across statements
 - If statement assigns X , update $GCP_{out}(i)[X]$ accordingly
- Meet operator $\wedge: (V, V) \rightarrow V$
 - Computes data flow at control flow merges
 - Merge property from two paths using the meet operator

Flow Transfer Function F for GCP

- ❑ Treat each statement as basic block i to apply F
- ❑ If statement is not an assignment, $GCP_{out}(i) = GCP_{in}(i)$
- ❑ If statement is of the form $X = Y + Z$,
 - If $GCP_{in}(i)[Y]$ and $GCP_{in}(i)[Z]$ are both constants,
 $GCP_{out}(i)[X] = GCP_{in}(i)[Y] + GCP_{in}(i)[Z]$
 - Else if either $GCP_{in}(i)[Y]$ or $GCP_{in}(i)[Z]$ is $*$,
 $GCP_{out}(i)[X] = *$
 - Else if either $GCP_{in}(i)[Y]$ or $GCP_{in}(i)[Z]$ is $\#$,
 $GCP_{out}(i)[X] = \#$

Meet operator \wedge for variable values

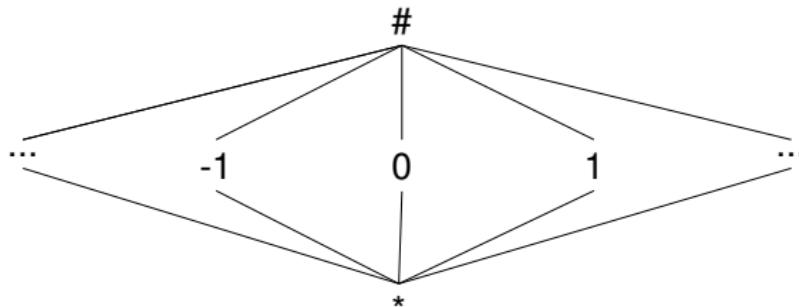
- ❑ Given basic block 1 and 2 merge into basic block 3,
 $GCP_{in}(3) = GCP_{out}(1) \wedge GCP_{out}(2)$
 - Where \wedge is applied to each variable X in $GCP_{in}(3)$:
 $GCP_{in}(3) = GCP_{out}(1)[X] \wedge GCP_{out}(2)[X]$
- ❑ Meet operator \wedge is given by this **semi-lattice**:
 - $a \wedge b = \text{greatest lower bound (glb)}$ in the below graph



- # is called the **top** value denoted as \top
- * is called the **bottom** value denoted as \perp

Meet operator \wedge for variable values

- ❑ Some results of meets \wedge given by this **semi-lattice**:

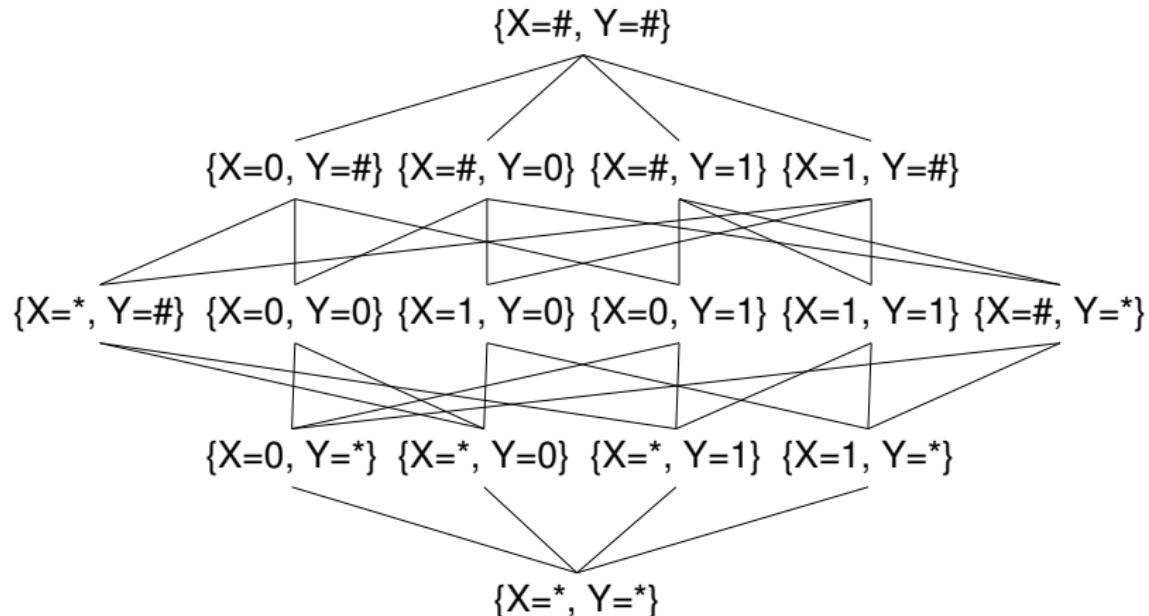


- $\# \wedge 1 \equiv \text{glb}(\#, 1) \equiv 1$
 - Meet of undefined value and a constant $\rightarrow x$ is that constant
- $0 \wedge 1 \equiv \text{glb}(0, 1) \equiv *$
 - Meet on different constants $\rightarrow x$ is no longer constant
- $* \wedge 1 \equiv \text{glb}(*, 1) \equiv *$
 - Meet of not a constant and a constant $\rightarrow x$ is not constant

- ❑ Greatest lower bound finds the maximal conservative value

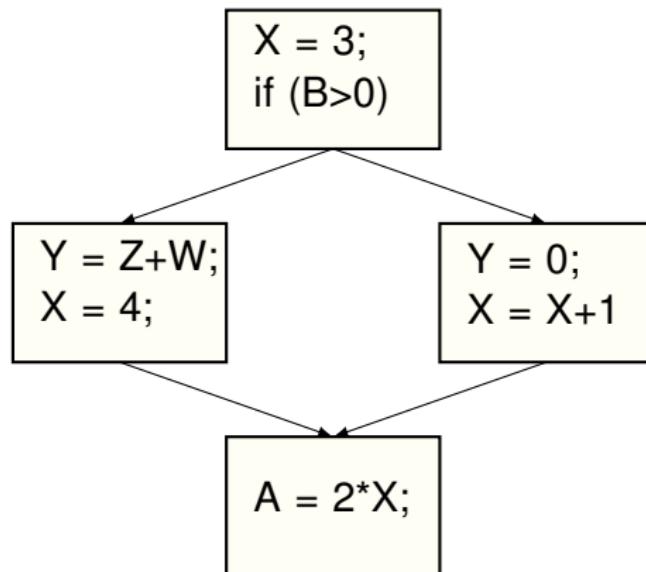
Meet operator \wedge for GCP values

- ❑ The \wedge operator for GCP values also forms a **semi-lattice**.
- ❑ Two variable example (values limited to 0, 1 for brevity):

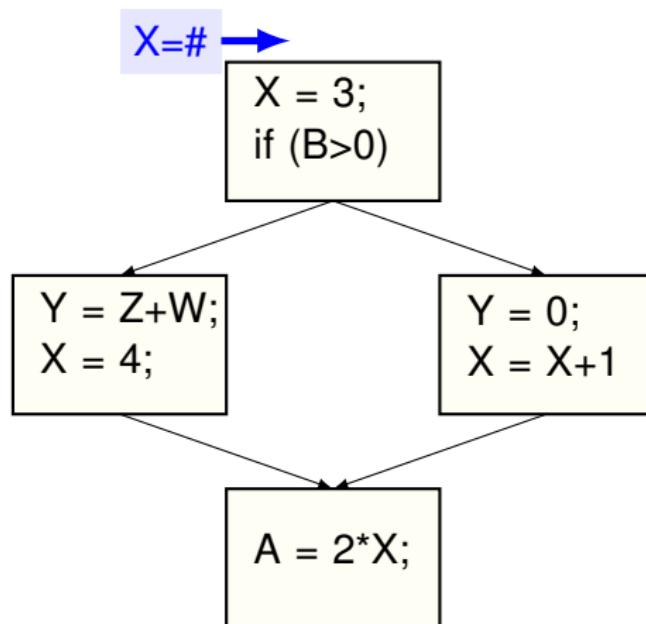


$$\Rightarrow \{X=0, Y=0\} \wedge \{X=1, Y=\#\} \equiv \{X=*, Y=0\}$$

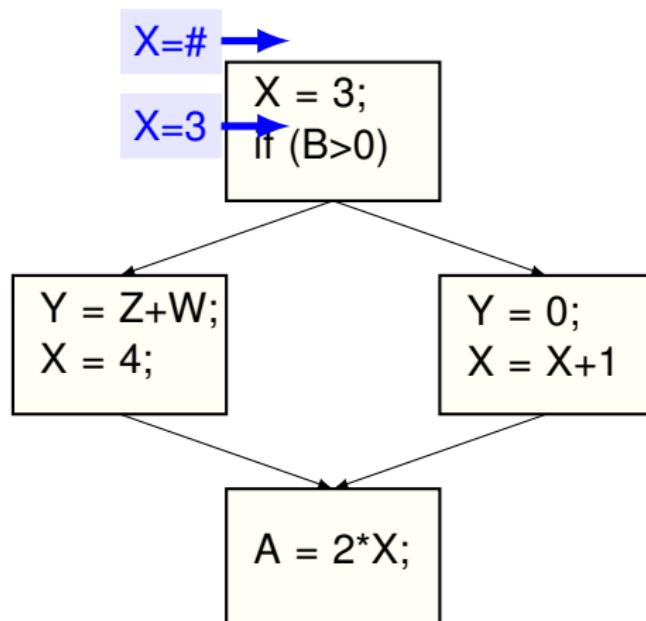
GCP Propagation without loops



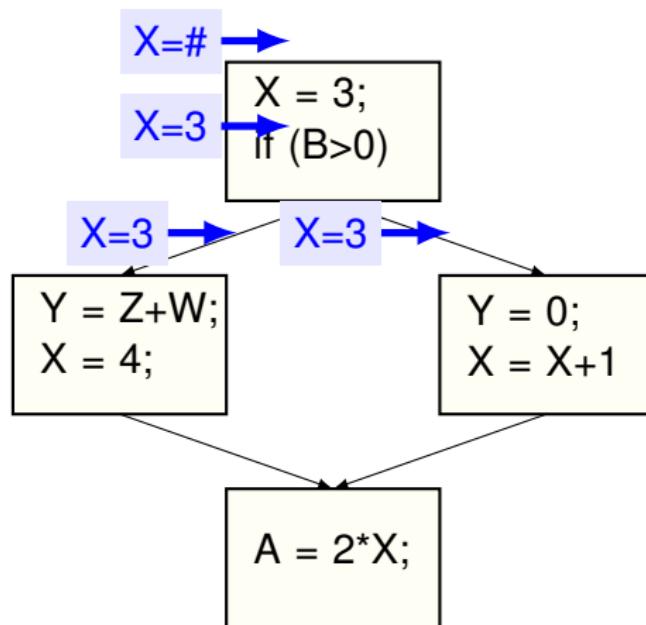
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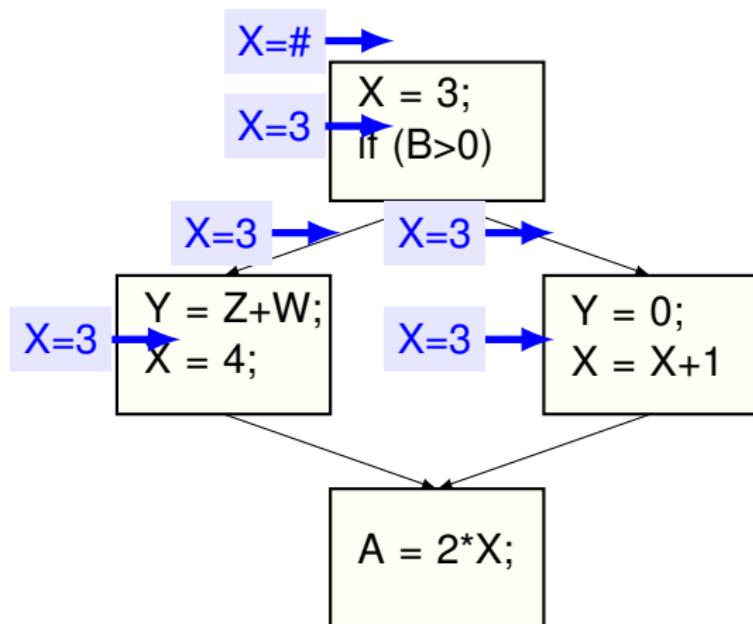
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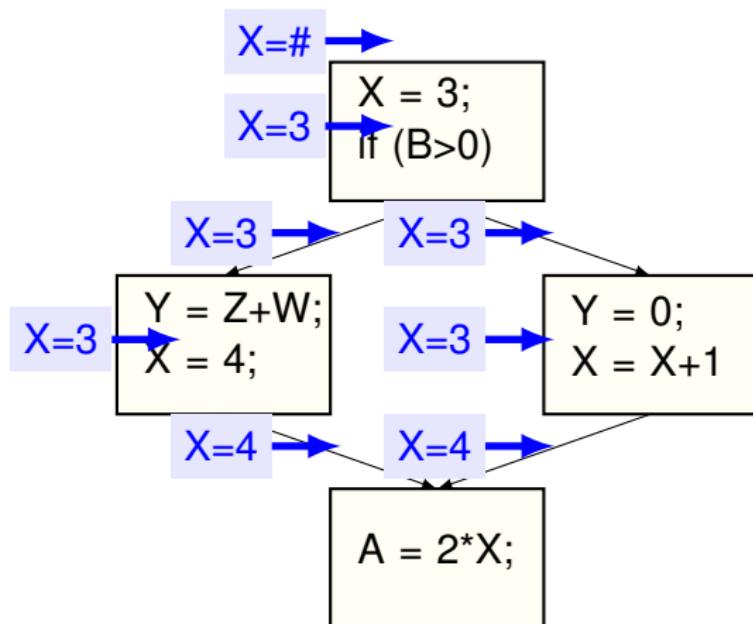
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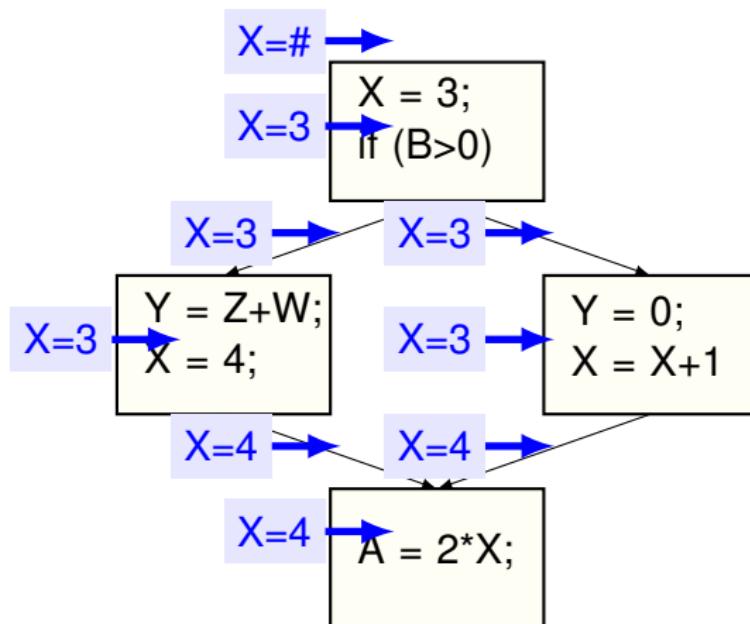
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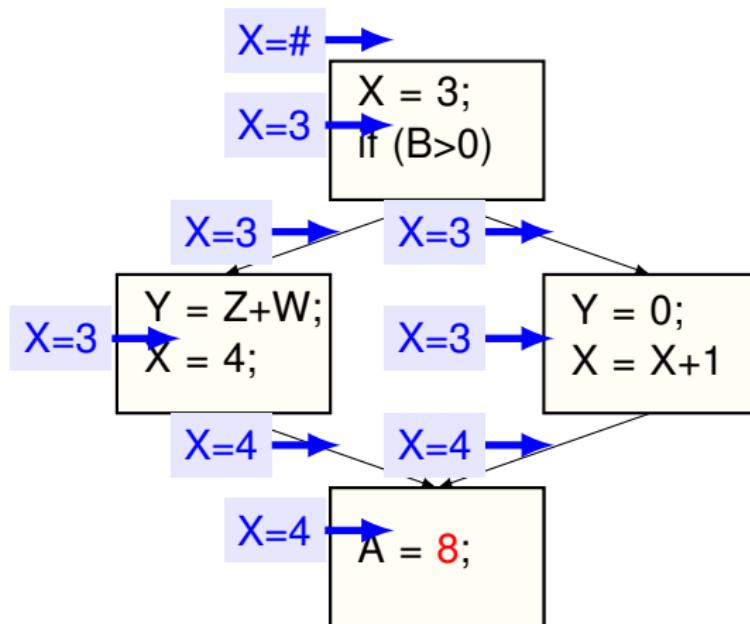
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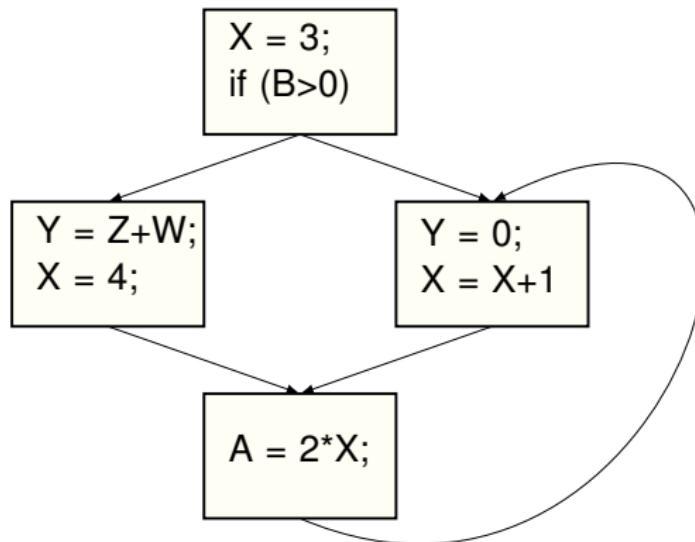


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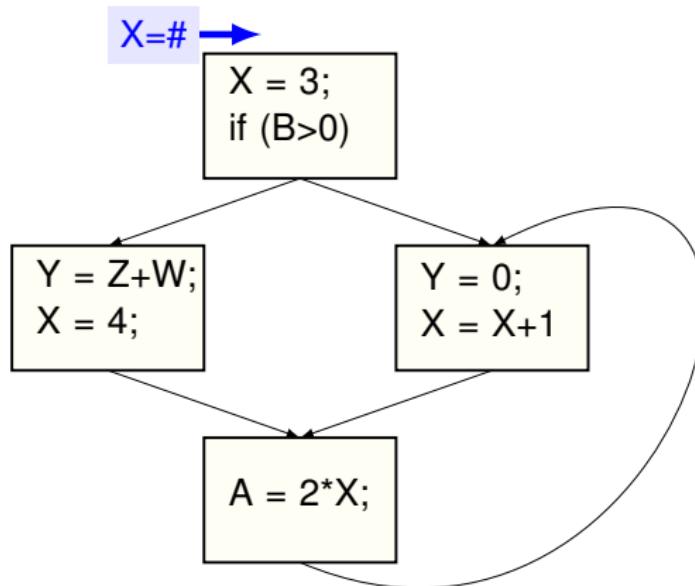
GCP Propagation with loops

- ❑ Iterate until there are no changes to values
 - This is called the **maximum fixed point** solution



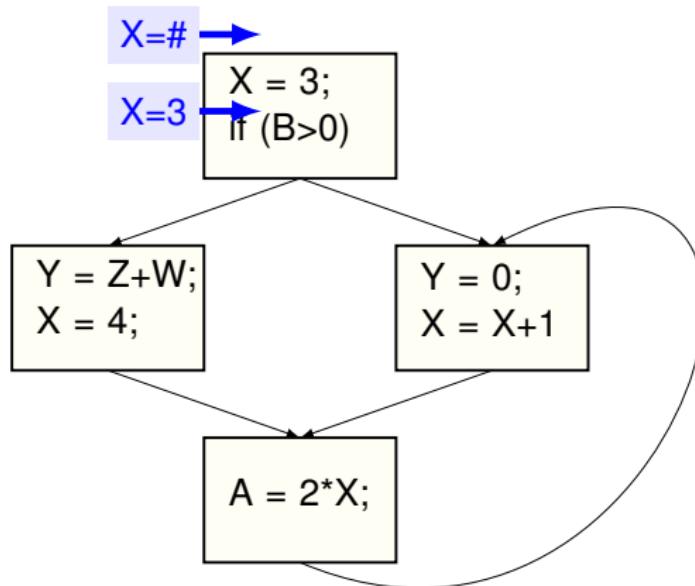
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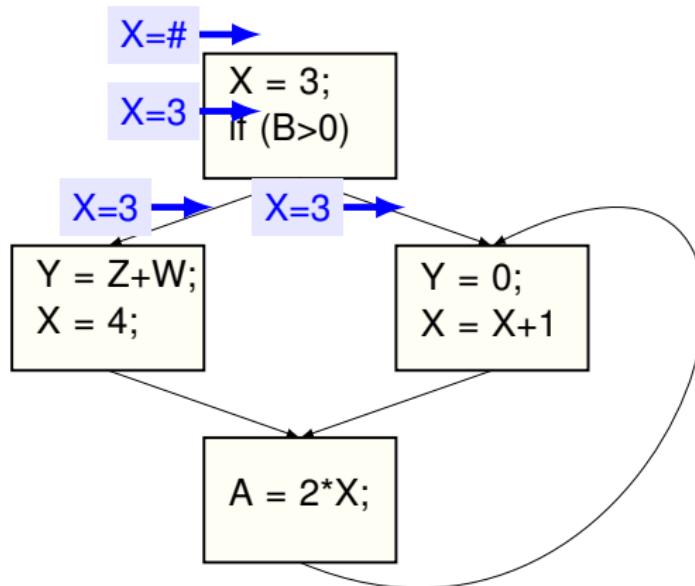
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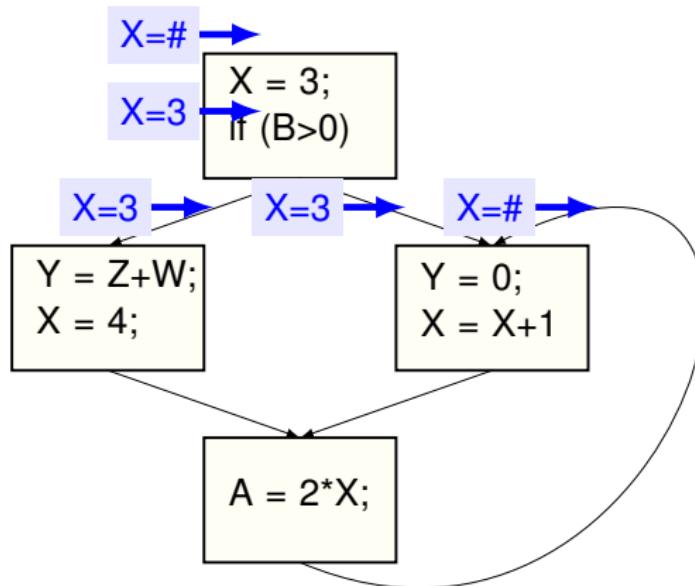
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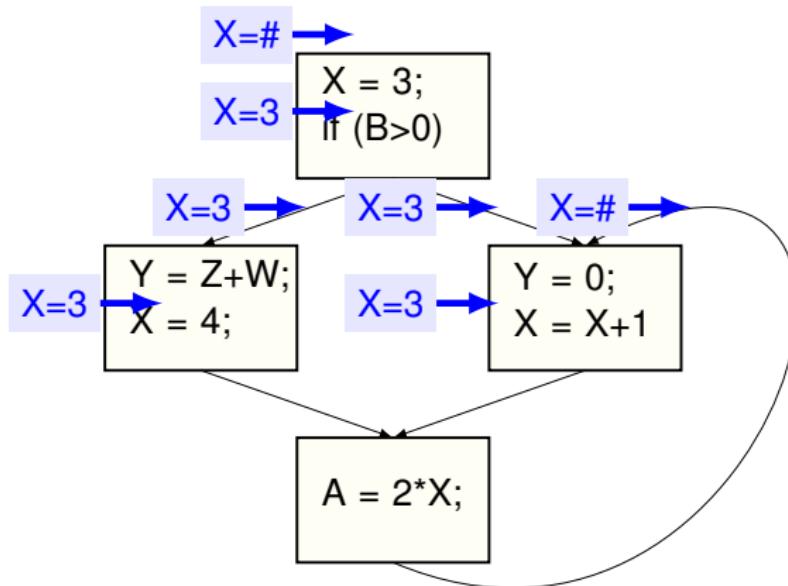
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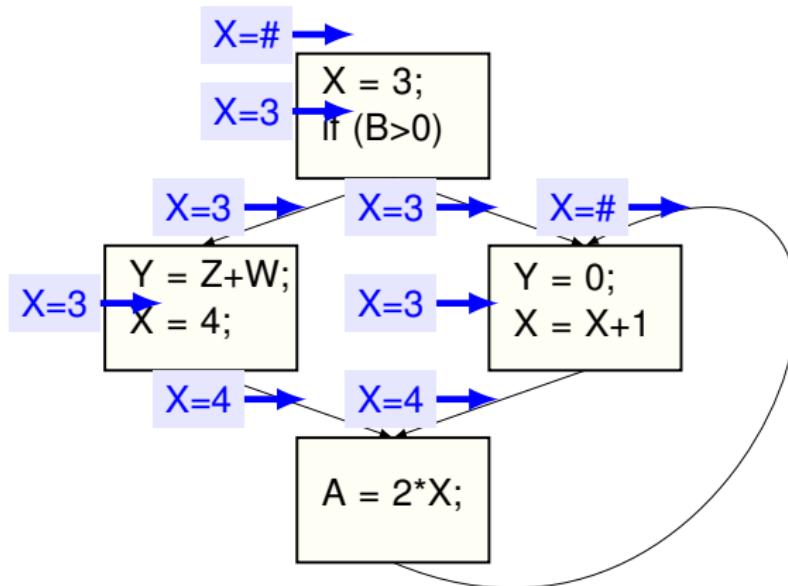
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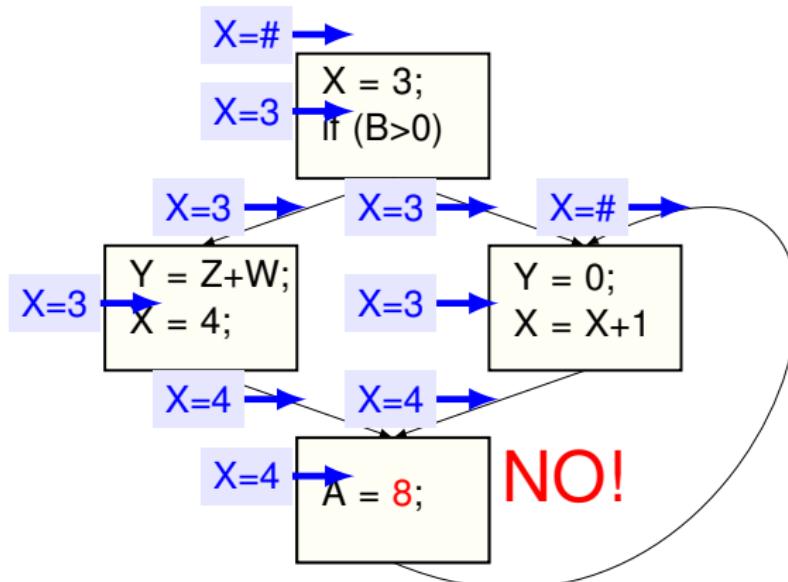
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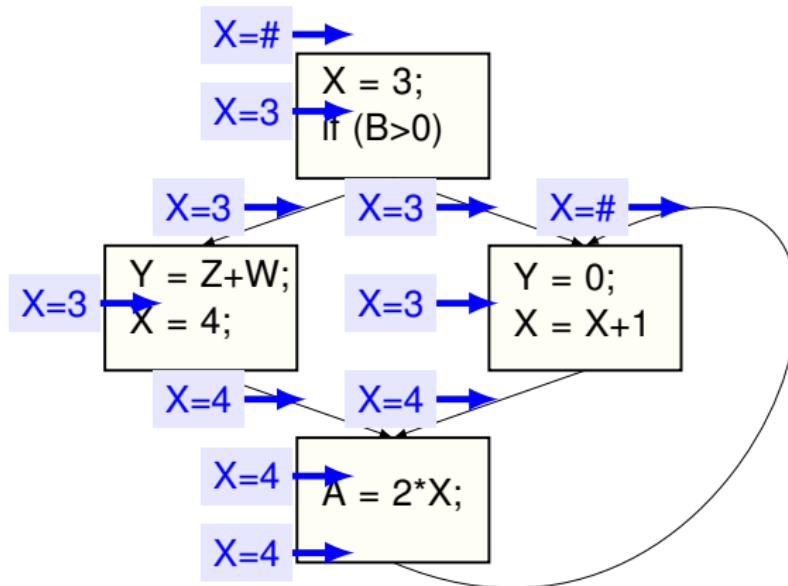
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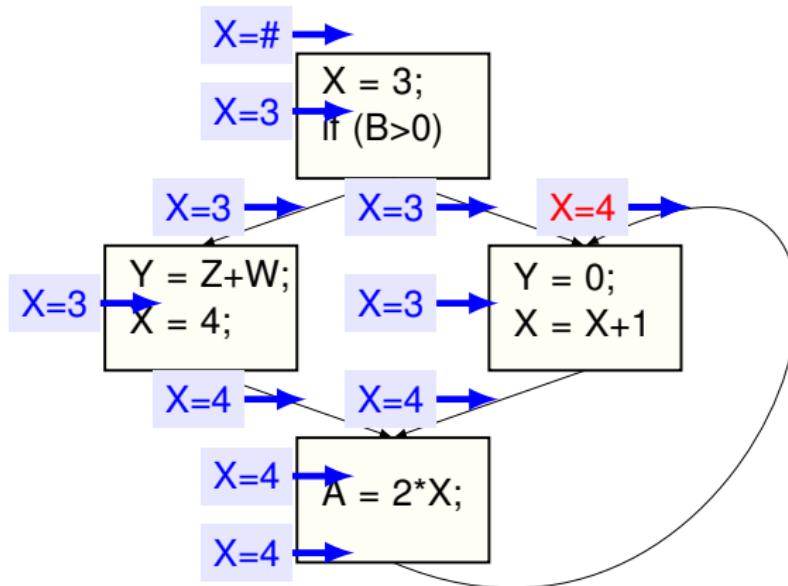
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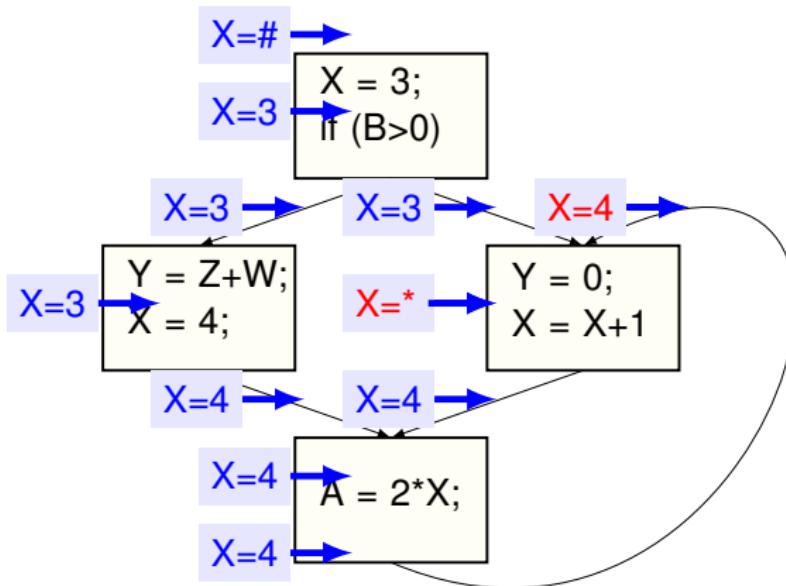
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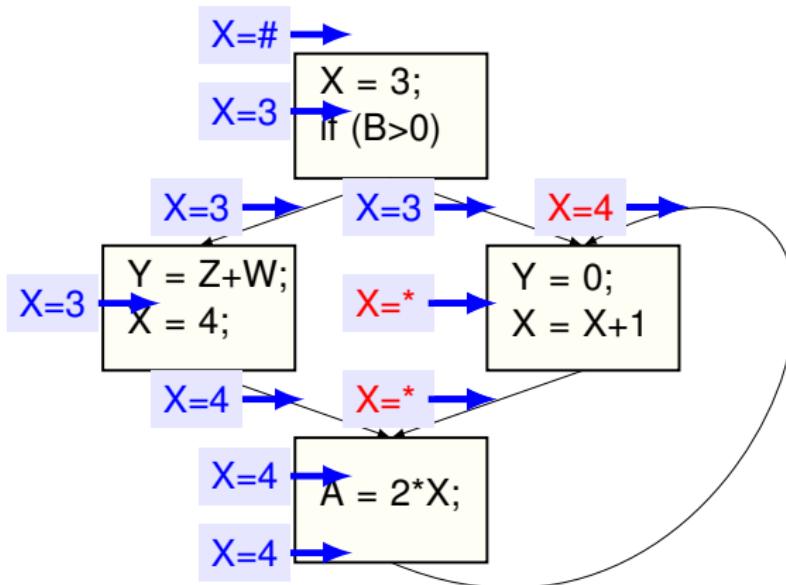
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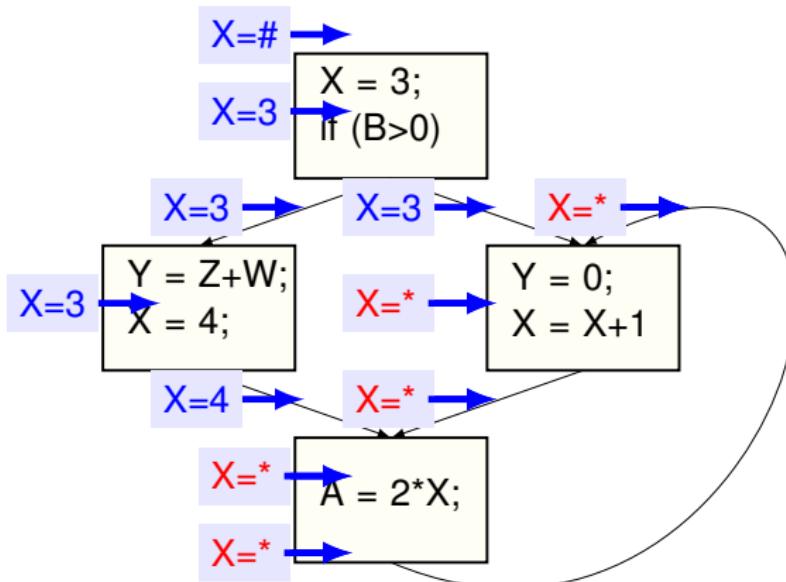
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Worklist Algorithm for Iterative Dataflow Analysis

- ❑ The **Maximum Fixedpoint (MFP)** solution is:
 - Maximum: All values optimistically initialized to \top values
 - Fixedpoint: Values are propagated until no changes occur
- ❑ MFP is efficiently computed using **worklist** algorithm:

Worklist = all nodes in CFG

while Worklist is not empty:

n = remove a node from Worklist

$OUT[n] = transfer_function(IN[n])$

 if $OUT[n]$ changed:

 for each successor s of n :

$IN[s] = \wedge (OUT[p] \text{ for } p \text{ in } \text{preds}(s))$

 if $IN[s]$ changed:

 add s to Worklist

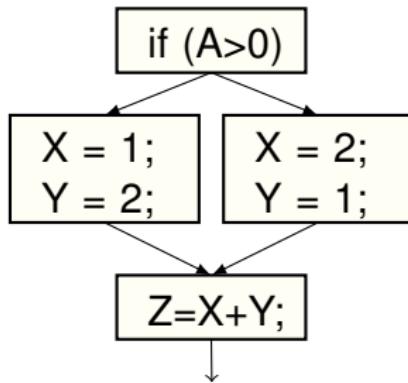
Time Complexity of Worklist Algorithm

- ❑ Termination: **Greatest lower bound** ensures termination
 - Values start from the top \top value
 - Values can only flow downward in the semi-lattice
 - Values are guaranteed to reach a fixedpoint in finite steps
- ❑ Time complexity: $O(d \times (N + E))$
 - d = the height of the semi-lattice
 - N = the number of nodes in CFG
 - E = the number of edges in CFG
- ❑ Why?
 1. A node enters worklist only when value changes.
 2. An edge is processed only when value changes.
 3. A value can change at most d times.

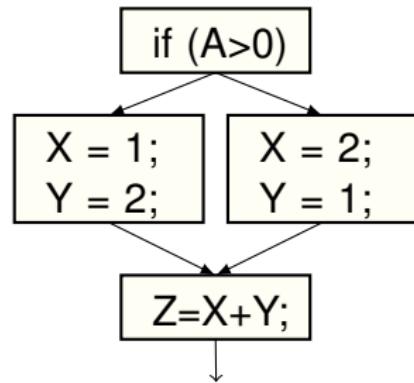
Maximum Fixedpoint \leq Meet-Over-Paths Solution

- ❑ How accurate is the maximal fixedpoint solution?

Maximum Fixedpoint (MFP)



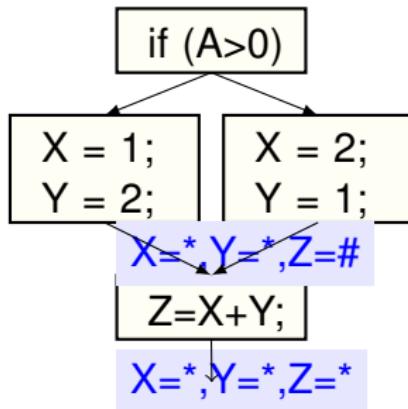
Meet-Over-Paths (MOP)



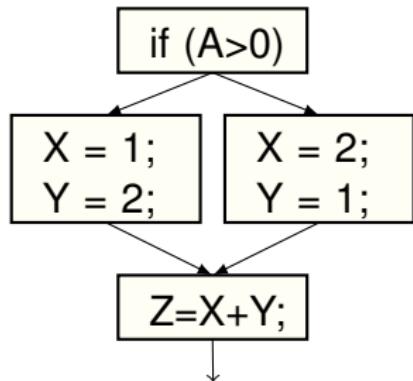
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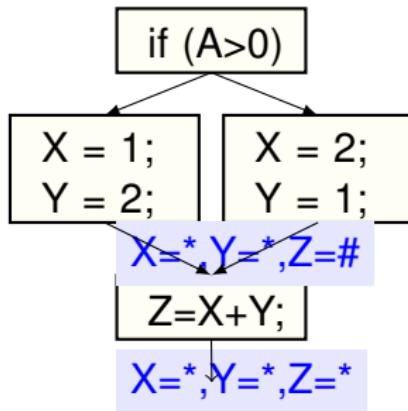
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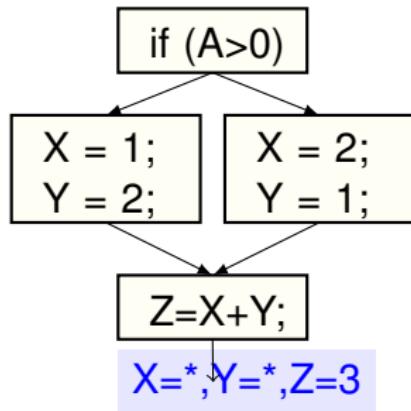
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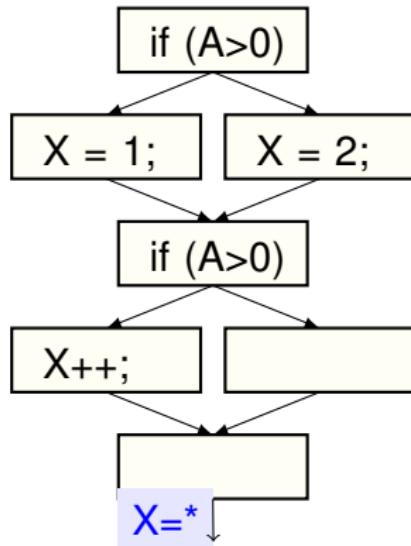
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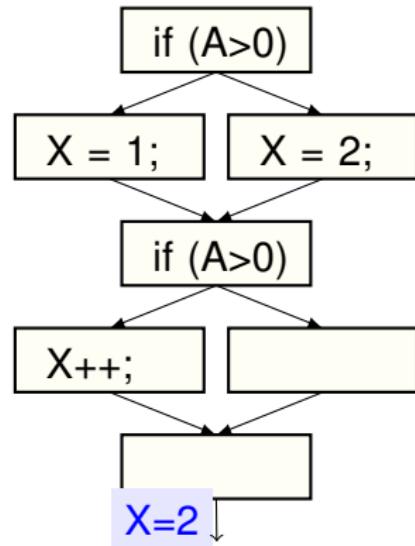
- ❑ For MOP, $OUT[b] = \wedge (OUT[b]$ for all paths to b)
 - Computing $OUT[b]$ where b is last basic block in example:
 $OUT[b] = \{X=1, Y=2, Z=3\} \wedge \{X=2, Y=1, Z=3\} = \{X=*, Y=*, Z=3\}$
- ❑ MFP \leq MOP (MFP is less precise)

Meet-Over-Paths \leq Ideal Solution

Meet-Over-Paths (MOP)



Ideal Solution



- ❑ For Ideal, $OUT[b] = \wedge (OUT[b] \text{ for all } \textit{feasible} \text{ paths to } b)$
 - $\text{ideal} = \{X=2\} \wedge \{X=2\} = \{X=2\}$
 - $MOP = \{X=1\} \wedge \{X=2\} \wedge \{X=2\} \wedge \{X=3\} = \{X=^*\} \leq \text{Ideal}$

MFP is Safe but Conservative

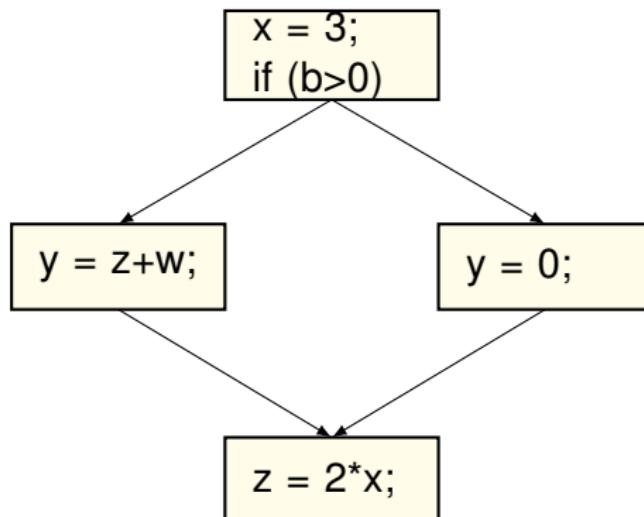
- In short, for the GCP dataflow analysis:
 $\text{Maximum Fixedpoint} \leq \text{Meet-Over-Paths} \leq \text{Ideal}$
- This is both good and bad.
 - Good** : MFP \leq Ideal means all GCP optimizations are safe.
 - Bad** : MFP \leq Ideal also means optimizations are conservative.

- MOP \leq Ideal is obvious, but is MFP \leq MOP true?
- MFP \leq MOP because GCP is a **monotone framework**.
 - In a monotone framework, $f(x \wedge y) \leq f(x) \wedge f(y)$
(Read textbook 9.4.4 for a proof that GCP is monotone.)
- Sometimes MFP = MOP, called **distributive frameworks**.
 - In a distributive framework, $f(x \wedge y) = f(x) \wedge f(y)$
(Liveness Analysis, which we'll learn next, is an example.)

Liveness Analysis

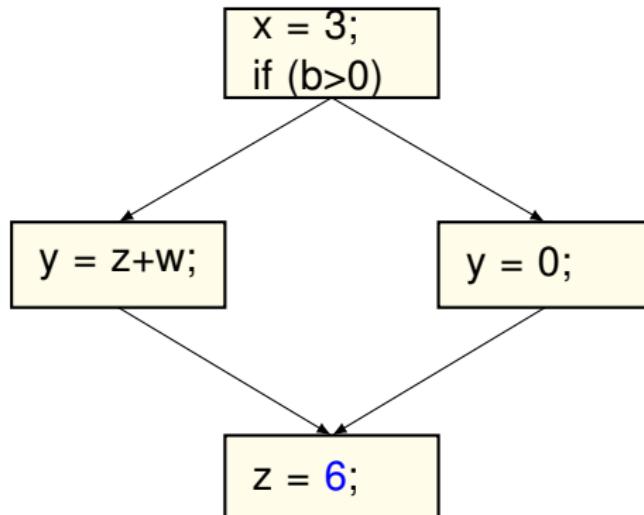
Another Analysis: Liveness Analysis

- ❑ After GCP, we would like to eliminate the dead code



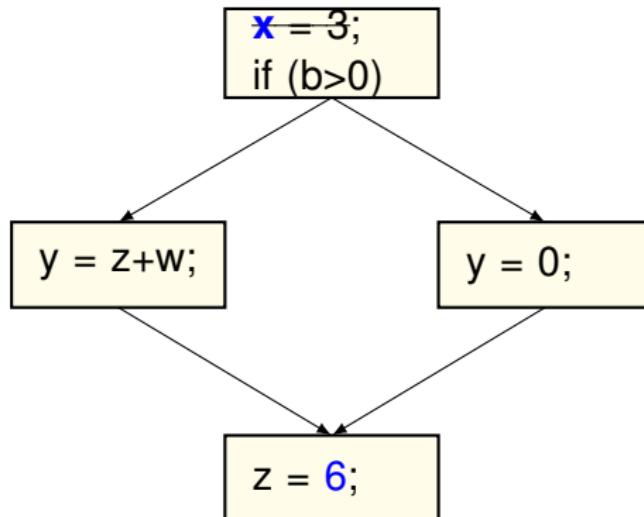
Another Analysis: Liveness Analysis

- ❑ After GCP, we would like to eliminate the dead code



Another Analysis: Liveness Analysis

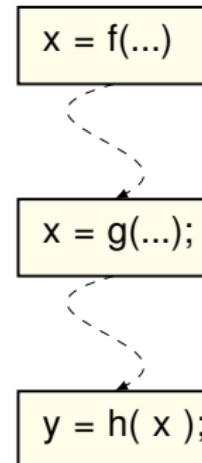
- ❑ After GCP, we would like to eliminate the dead code



Live/Dead Statement

- A **dead statement** assigns a value that is not used later
- Otherwise, it is a **live statement**

In the example,
the 1st statement is dead,
the 2nd statement is live



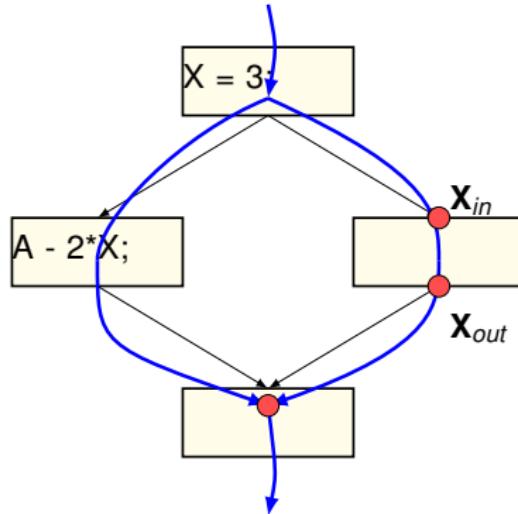
- Assuming inter-procedural analysis says $f(\dots)$ is internally free of assignments used later (e.g. global variables).

Global Liveness Analysis (GLA)

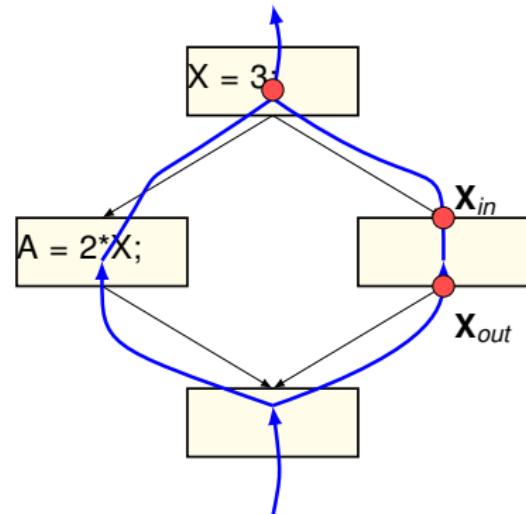
- ❑ Again, let's use the dataflow analysis framework
- ❑ Here are the 4 components of the framework
 - **D**: direction of dataflow for liveness property
 - **V**: domain of values denoting liveness property
 - **\wedge** : **meet operator** that merges values when paths meet
 - **F**: **flow propagation function** for liveness
- ❑ **V**: Liveness property is the set of live variables
 - $\{\}, \{a\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \dots$
- ❑ **\wedge** : Meet operator for Liveness Analysis is a union
 - Variable x is live if x is live along at least one path

Direction D for GLA

- ❑ Is Liveness a forward or backward analysis?



Forward Analysis

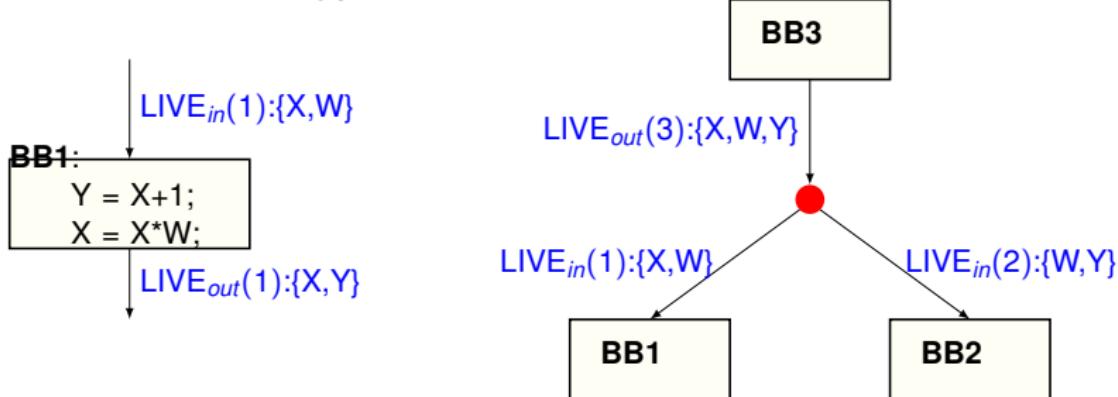


Backward Analysis

- ❑ Backward, since liveness of a variable flows backward to preceding definitions starting from use

Dataflow Equations for GLA

- There are two types of flow functions



- Flow transfer function $F: V \rightarrow V$
 - Now F computes P_{in} from P_{out} since it is backward analysis
 - Remove variable definitions, add variable uses to live set
- Meet operator $\wedge: (V, V) \rightarrow V$
 - Merge values from two paths using the previous semi-lattice
 - $LIVE_{out}(i) = \cup LIVE_{in}(k)$ where k is successor of i

Flow Transfer Function F for GLA

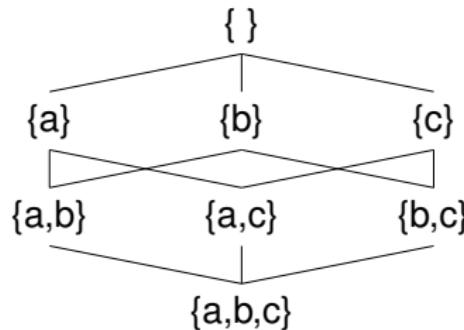
❑ F for Global Liveness Analysis (GLA)

$$\text{LIVE}_{in}(i) = (\text{LIVE}_{out}(i) - \text{DEF}(i)) \cup \text{USE}(i)$$

where $\text{DEF}(i)$ is the set of defined variables in basic block i
 $\text{USE}(i)$ is the set of used variables in basic block i

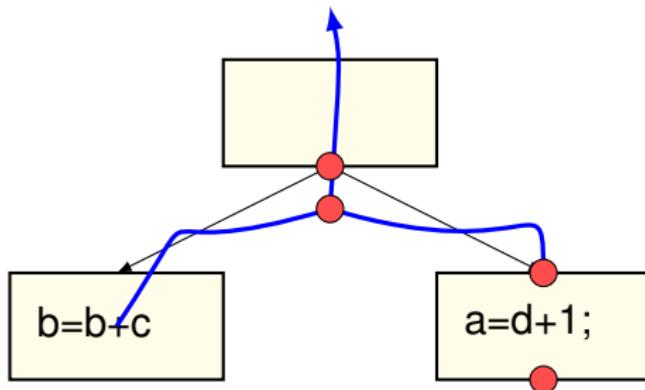
Meet operator \wedge for GLA

- ❑ Meet operator \wedge is given by this **semi-lattice**:

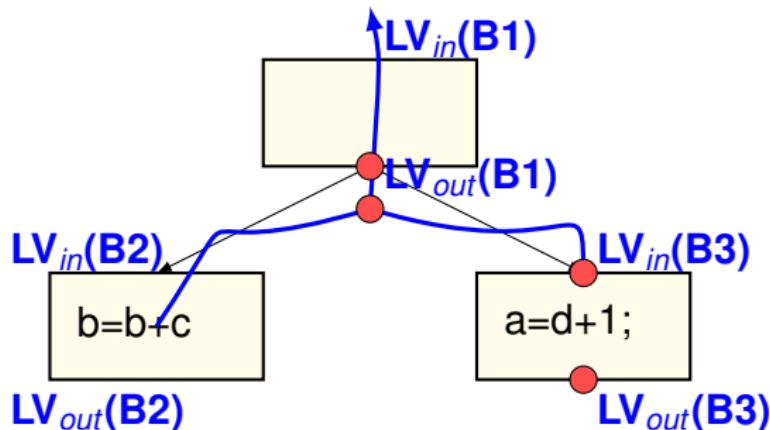


- $\{a\} \wedge \{b\} = \text{glb}(\{a\}, \{b\}) = \{a,b\}$
- $\{b\} \wedge \{a,c\} = \text{glb}(\{b\}, \{a,c\}) = \{a,b,c\}$
- The semi-lattice expresses the union relationship

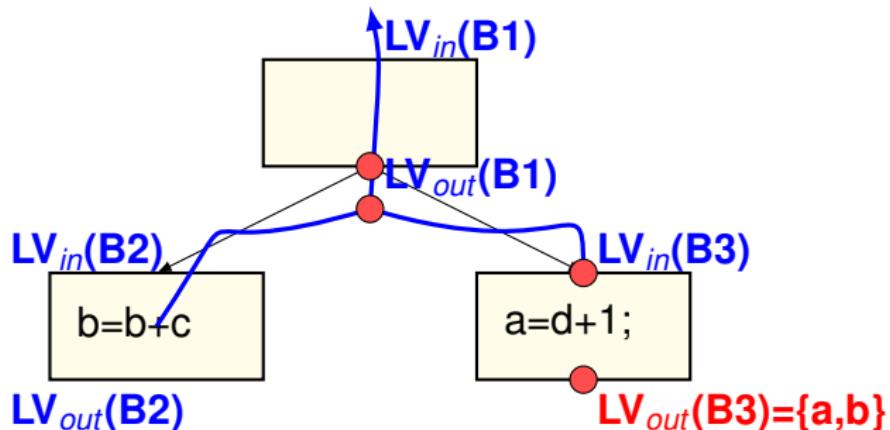
Liveness Example



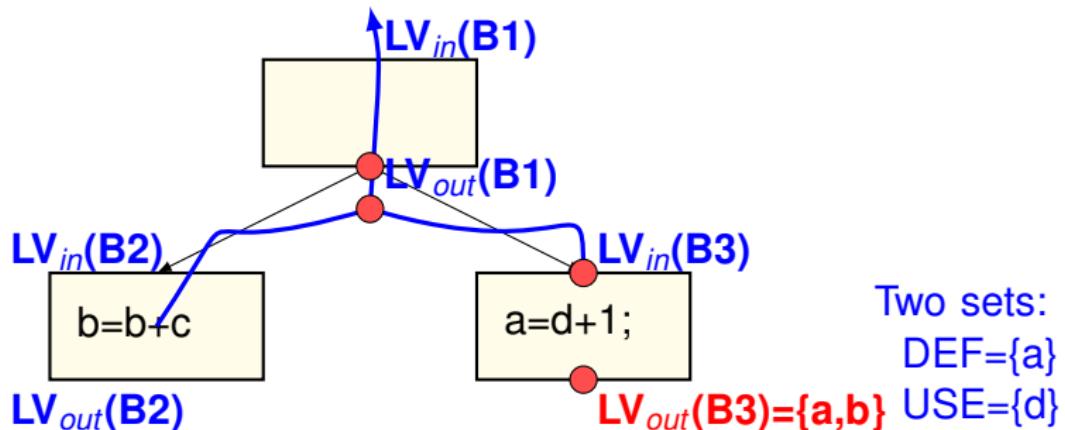
Liveness Example



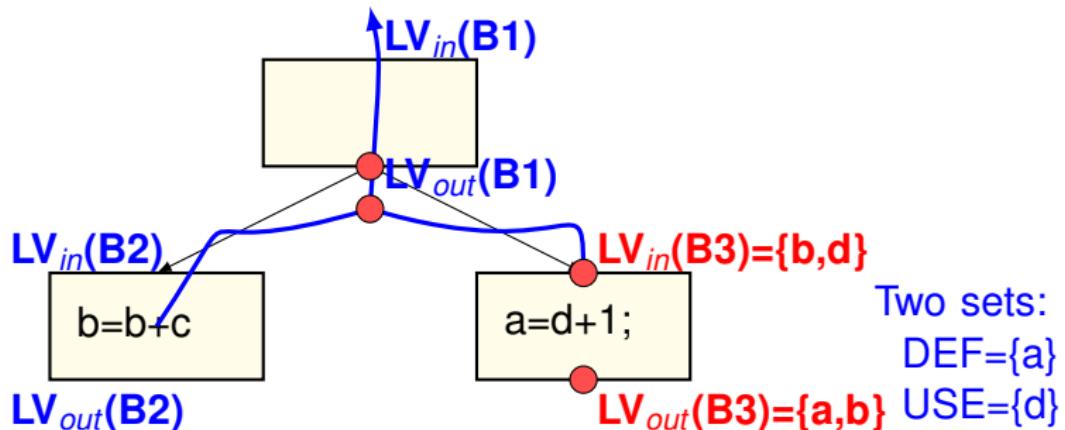
Liveness Example



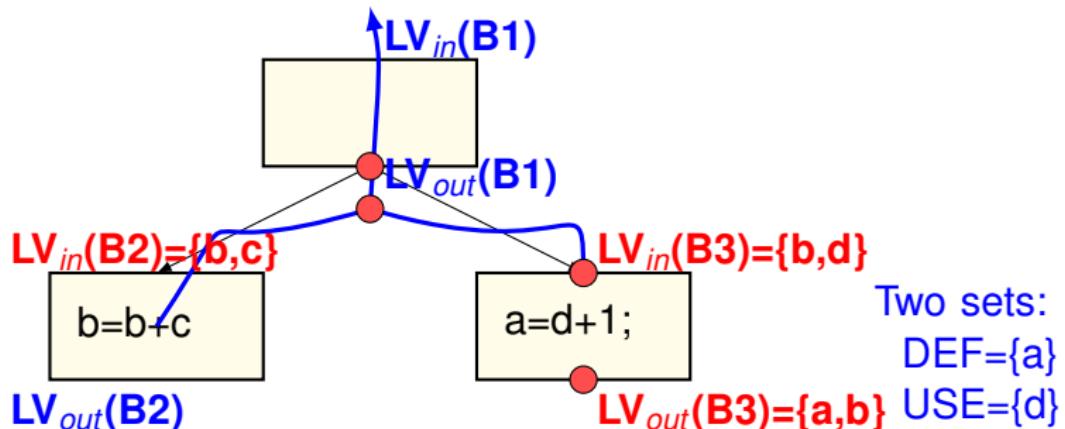
Liveness Example



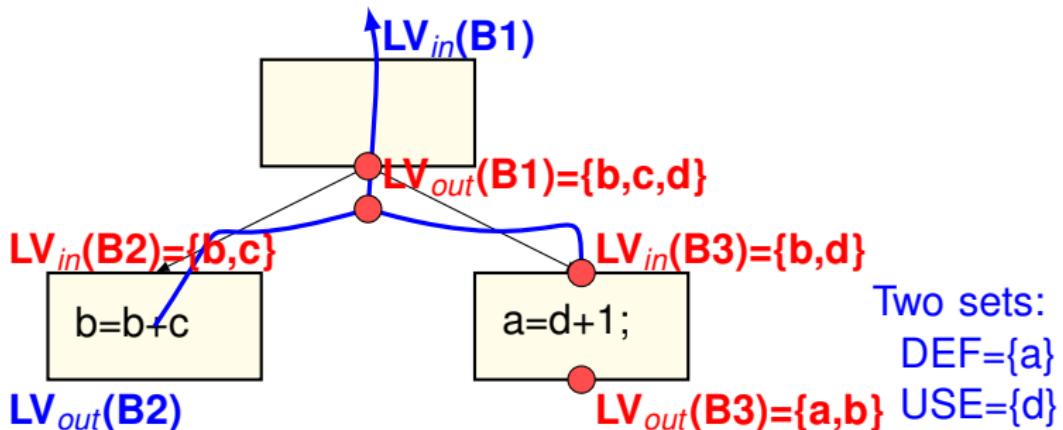
Liveness Example



Liveness Example



Liveness Example



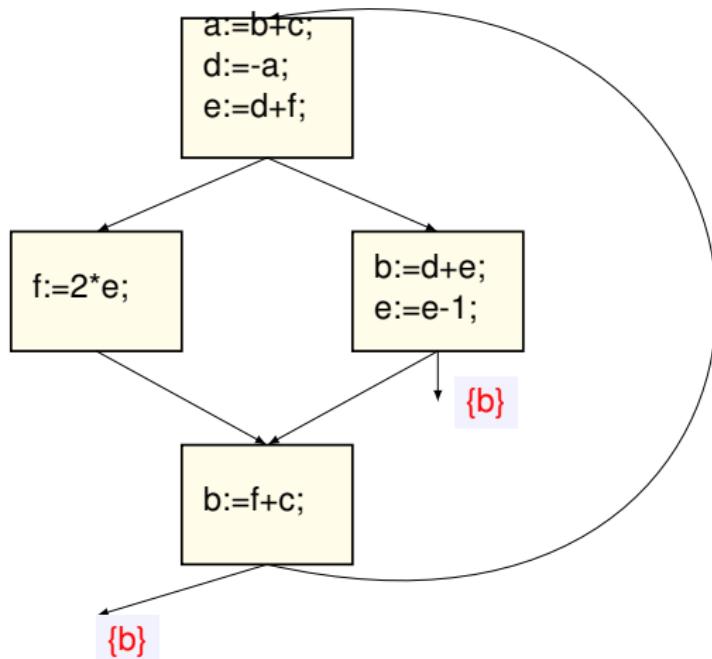
Applications of Global Liveness Analysis

- ❑ Global Dead Code Elimination is based on GLA
 - A statement $x = \dots$ is dead code if x not used
 - Dead statements can be deleted from the program

- ❑ Register allocation is also based on GLA
 - **Register allocation:** assigning variables to registers
 - If two variables are simultaneously live at any point
 - they cannot be allocated to the same register
 - and this is called **interference**.
 - If there are insufficient CPU registers to hold variables
 - some variables must be stored in stack memory
 - and this is called **register spilling**.
 - Register spilling typically leads to worse performance.
 - Leads to extra load/store instructions to access memory

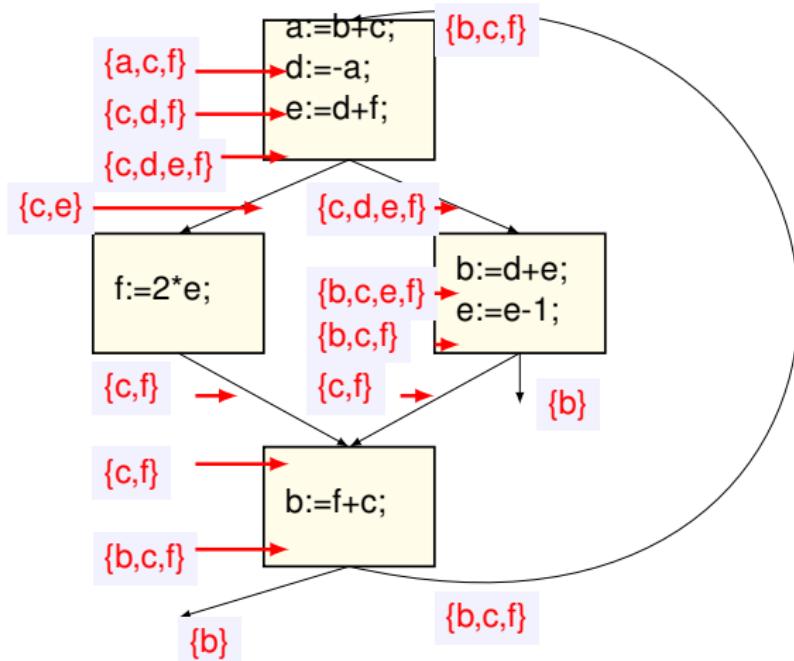
Register Allocation: Compute Register Interference

- At each point P, compute live variables and interference



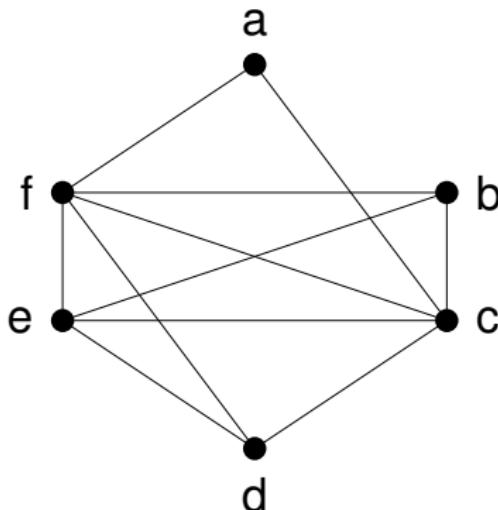
Register Allocation: Compute Register Interference

- At each point P, compute live variables and interference



Register Allocation: Register Interference Graph

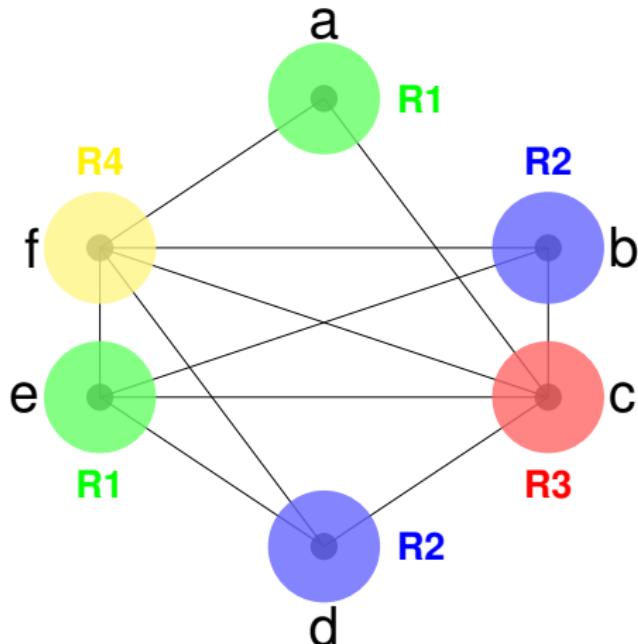
- ❑ Construct **Register Interference Graph (RIG)** such that
 - Nodes represent variables
 - Edges between variables represent interference



- ❑ Two variables can be allocated in same register if no edge
- ❑ Otherwise, they cannot be allocated in the same register

Register Allocation: Allocation using Graph Coloring

- ❑ Each color represents a CPU register
 - There are 4 colors in the coloring result
 - No register spilling occurs with 4 or more CPU registers



Summary of Dataflow Analysis

- ❑ A dataflow analysis framework is defined as:
 $\{ D, V, \wedge: (V, V) \rightarrow V, F: V \rightarrow V \}$
 - **D**: direction of dataflow
 - **V**: domain of values denoting property
 - **\wedge** : **meet operator** that merges values when paths meet
 - **F**: **flow propagation function** within a basic block
- ❑ Other analyses can be expressed using this framework:
 - Loop Invariant Code Motion (LICM)
 - Common Subexpression Elimination (CSE)
 - Partial Redundancy Elimination (PRE)
- ❑ Please refer to the textbook on how these are formulated.

The END !