A Proofs of Propositions

Proof of Proposition 1. Before jumping into the details of the proposition, we first prove the posterior updating result (3.3). The definition indicates

$$f^{\theta}(x|s) \propto \left[\frac{f(x|s)}{f(x)}\right]^{\theta} = \left(\frac{\alpha}{\alpha+\beta}\right)^{\frac{\theta}{2}} \tag{A.1}$$

$$\times \exp\left(-\frac{\theta}{2}\left\{(\alpha+\beta)\left(x-\frac{\alpha z+\beta s}{\alpha+\beta}\right)^2-\alpha(x-z)^2\right\}\right) \tag{A.2}$$

where $f(\cdot)$ denotes the density function. Since

$$(A.3) = \exp\left\{-\frac{\alpha+\beta(1+\theta)}{2}\left(x-\frac{\alpha z+\beta(1+\theta)s}{\alpha+\beta(1+\theta)}\right)^2\right\}, \tag{A.3}$$

we obtain the posterior distribution (3.3).

Substituting $\widehat{s} = \widehat{x} + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\widehat{x})$ (3.2) into (3.4), we have

$$1-\Phi\left(\sqrt{\alpha+\beta(1+\theta)}\cdot\frac{\sqrt{\beta}(1+\theta)\Phi^{-1}(\widehat{x})+\alpha(z-\widehat{x})}{\alpha+\beta(1+\theta)}\right) = c$$

$$\iff \Phi^{-1}(1-c) = \frac{\sqrt{\beta}(1+\theta)\Phi^{-1}(\widehat{x})+\alpha(z-\widehat{x})}{\sqrt{\alpha+\beta(1+\theta)}}$$

$$\iff \sqrt{1+\frac{\alpha}{\beta(1+\theta)}}\cdot\frac{\Phi^{-1}(1-c)}{\sqrt{1+\theta}}+\frac{\alpha(\widehat{x}-z)}{\sqrt{\beta}(1+\theta)} = \Phi^{-1}(\widehat{x}) \tag{A.4}$$

Since $\min_{x\in(0,1)}\frac{d}{dx}\Phi^{-1}(x) = \min_{x\in(0,1)}\frac{1}{\phi(\Phi^{-1}(x))} = \sqrt{2\pi}$, the equilibrium is in monotone strategies and is unique if and only if

$$\frac{\alpha}{\sqrt{\beta}(1+\theta)} \leq \sqrt{2\pi}$$

$$\iff \alpha \leq (1+\theta)\sqrt{2\pi\beta}$$

$$\iff \theta \geq \frac{\alpha}{\sqrt{2\pi\beta}} - 1.$$

It only remains to show that the unique monotone equilibrium is the only equilibrium that survives iterated deletion of strictly dominated strategies. Let $h(s)$ denote the threshold that an agent finds it optimal to follow when all other agents use a threshold s. Note that there is a 1-to-1 mapping between the thresholds $\widehat{x}$ that solve (3.2) and the fixed points of h. The monotone equilibria are defined by the fixed points of h. Construct two sequences $\{\underline{s}_j\}_{j=0}^{\infty}$ and $\{\overline{s}_j\}_{j=0}^{\infty}$ by $\underline{s}_0 = -\infty, \underline{s}_j = h(\underline{s}_{j-1})$ and $\overline{s}_0 = \infty, \overline{s}_j = h(\overline{s}_{j-1})$. Each sequence is increasing or decreasing with upper or lower bound. Thus, they both converge to some $\underline{s}$ and $\overline{x}$. By continuity of h, these points must be a fixed point of h, which is $\widehat{s}$. Since the only strategies that survives after j rounds of iterated deletion of dominated strategies are functions k such that $k(s) = 1$ for all $s \leq \underline{s}_j$ and $k(s) = 0$ for all $s > \overline{s}_j$, the only strategy that survives in the limit is the unique monotone equilibrium. $\square$

Proof of Proposition 2. As $\beta \to \infty$ with $\alpha < \infty$, (LHS) of (A.4) with $\theta = 0$ becomes $\Phi^{-1}(1-c)$ and (RHS) of (A.4) with $\theta = 0$ remains the same. Thus, we obtain $x_\infty = 1-c$ since Φ^{-1} is monotone. Similarly, (LHS) of (A.4) becomes $\Phi^{-1}(1-c)/\sqrt{1+\theta}$ and (RHS) of (A.4) remains the same. Thus, we obtain $x_\infty = \Phi\left(\frac{\Phi^{-1}(1-c)}{\sqrt{1+\theta}}\right)$. $\square$

Proof of Corollary 1. Since $\Phi(\cdot)$ is an monotone increasing function and $\Phi^{-1}(\cdot)$ undergoes a sign change at $c = \frac{1}{2}$, the order of the two equilibrium thresholds flips at $c = \frac{1}{2}$. $\square$

B Simulation Results

To investigate how effective the comparative statics analysis is for finite values of β, I perform numerical analysis on how the equilibrium threshold changes as I vary the value of c. Since the equilibrium thresholds do not have analytical solutions, I implement simulation exercises with $\alpha = 0.3$ and $\beta = 0.5$ so that they satisfy the uniqueness condition of Proposition 1 and 2. For the other parameters, let $z = 0$ for computational convenience and $\theta = 0.5$ following the estimation results using survey expectation data in Bordalo et al. (2020).

As expected by Proposition 2 and Corollary 1, equilibrium thresholds differ for Bayesian global games and diagnostic global games. As I change the value of c from 0 to 1, this difference in thresholds becomes narrower, at some point around $c \in [0.5, 0.6]$ their order is reversed, and the gap becomes larger thereafter. Here, I present 3 cases where $c = 0.1, 0.6, 0.9$ which illustrate this property well in Figure 2, 3, and 4, where equilibrium thresholds are the intersections of the inverse of the normal distribution and red lines. In addition, I also implement sensitivity analysis with respect to α and β when $c = 0.1, 0.6, 0.9$.

Figure 1: Equilibrium thresholds when $c = 0.1$

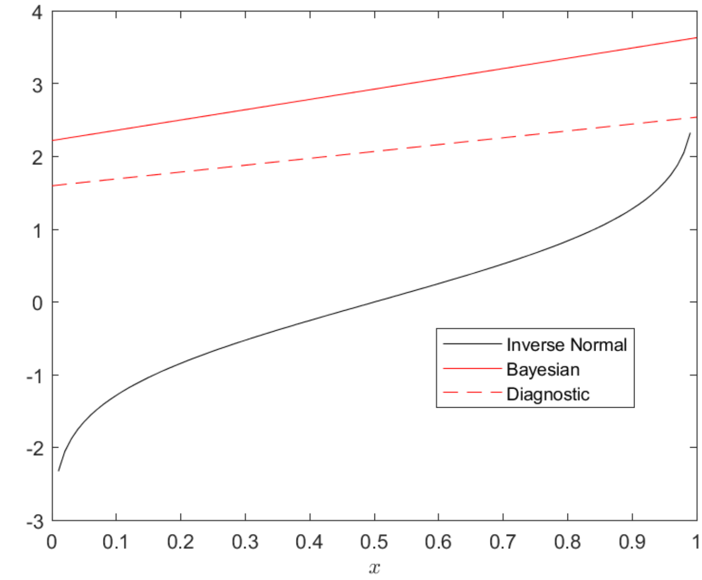


Figure 2: Equilibrium thresholds when $c = 0.6$

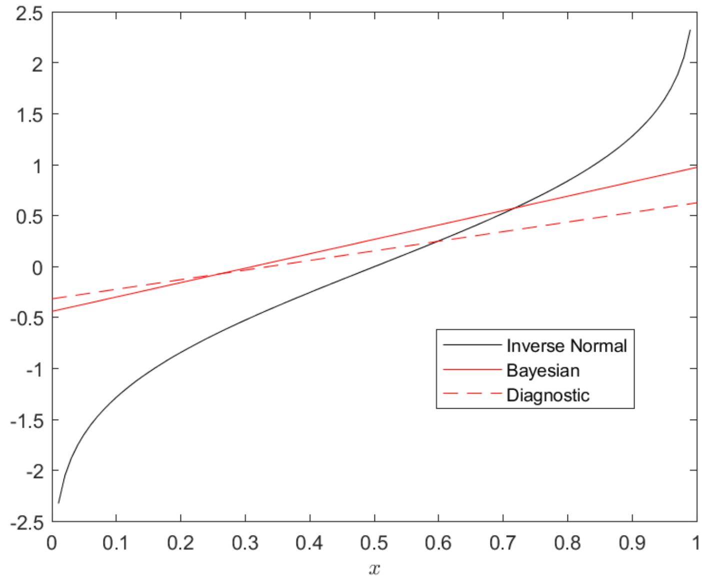


Figure 3: Equilibrium thresholds when $c = 0.9$

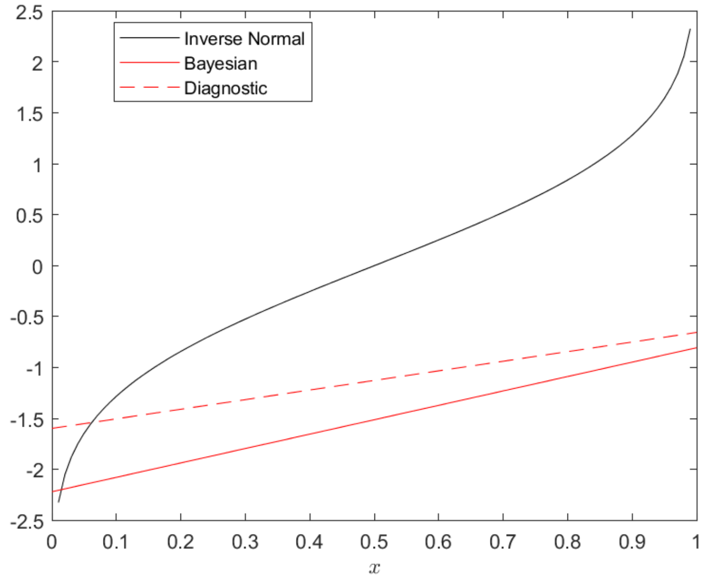
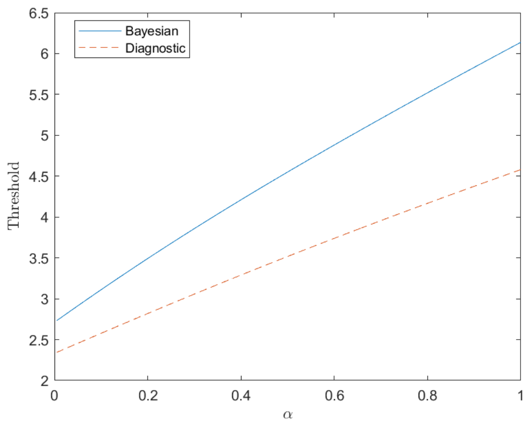
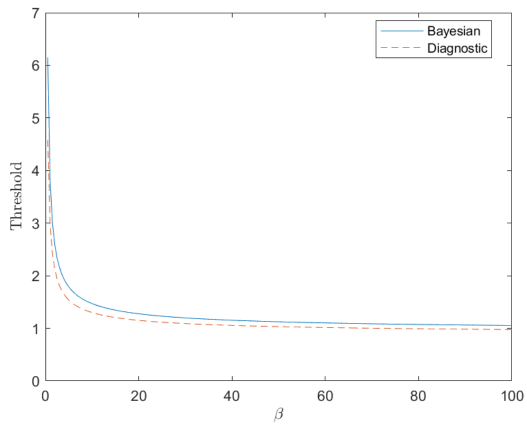


Figure 4: Sensitivity analysis when $c = 0.1$

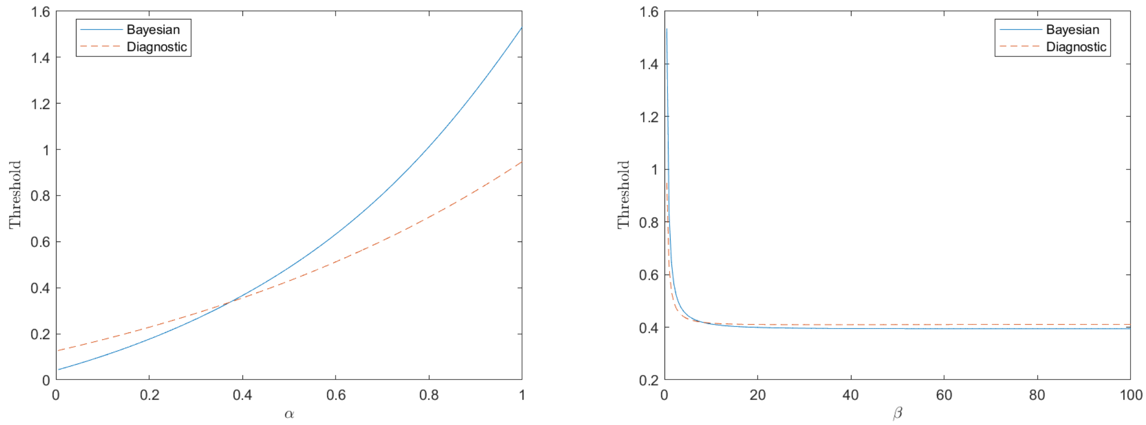


(a) α



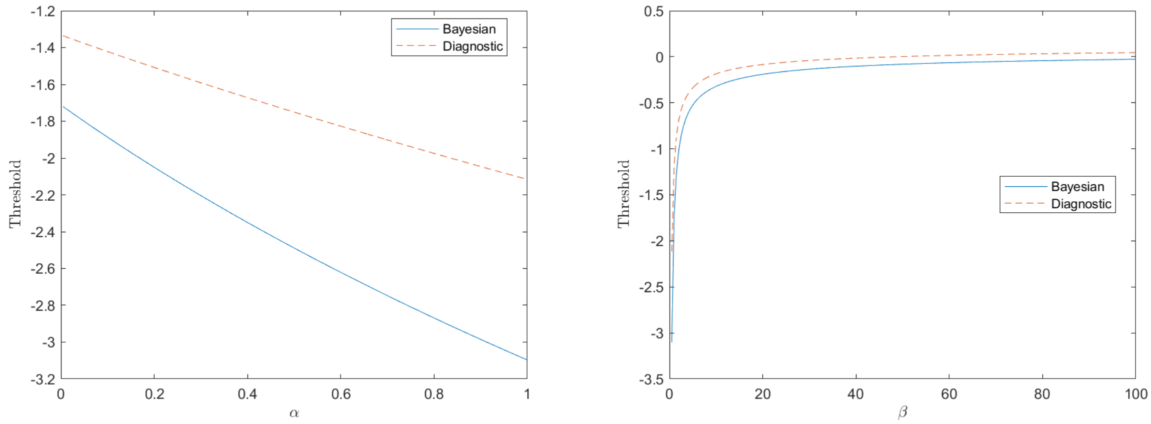
(b) β

Figure 5: Sensitivity analysis when $c = 0.6$



(a) α (b) β

Figure 6: Sensitivity analysis when $c = 0.9$



(a) α (b) β

References

[1] Bordalo, P., Gennaioli, N., Ma, Y., and Shleifer, A. (2020). Overreaction in Macroeconomic Expectations. *American Economic Review*, 110(9), 2748-82.