

# Data Structure

<http://smartlead.hallym.ac.kr>

**Instructor: Jin Kim**  
**010-6267-8189(033-248-2318)**  
**[jinkim@hallym.ac.kr](mailto:jinkim@hallym.ac.kr)**

**Office Hours**

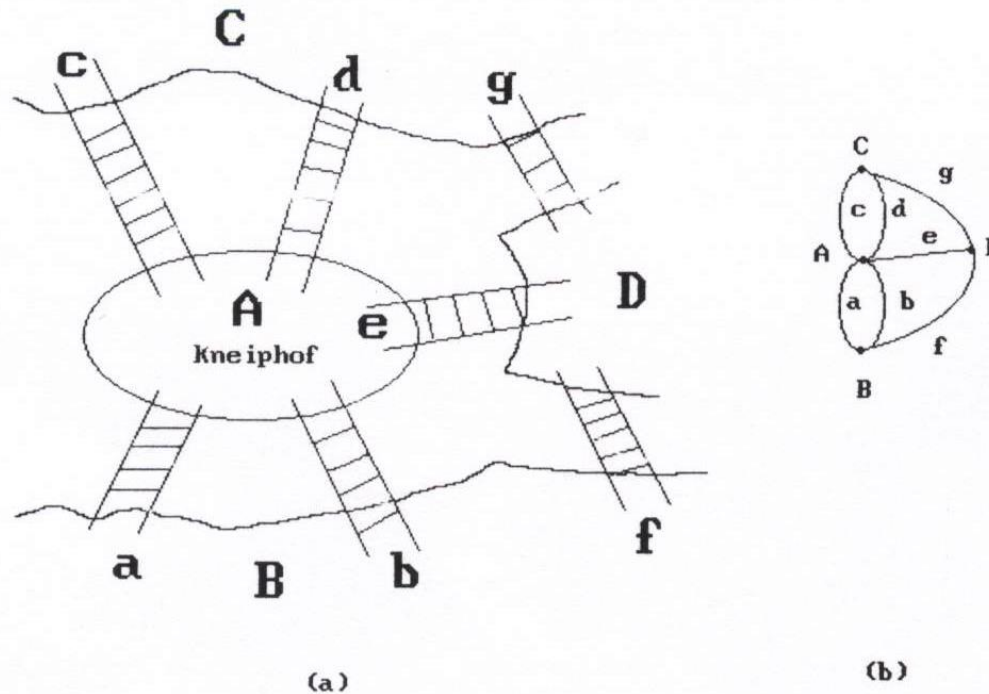
# 능름한 허스키



# The Graph ADT (1/13)

- ◆ Introduction

- ◆ A graph problem example: Königsberg bridge problem
- 



**Figure 6.1:** The bridges of Koenigsberg

# The Graph ADT (2/13)

## ◆ Definitions

- ◆ A **graph**  $G$  consists of two sets 그래프는
  - a finite, nonempty set of vertices  $V(G)$  정점의 집합과
  - a finite, possible empty set of edges  $E(G)$  간선의 집합 이다
- ◆  $G(V, E)$  represents a graph
- ◆ An **undirected graph**(무방향 그래프) is one in which the pair of vertices in an edge is unordered,  $(v_0, v_1) = (v_1, v_0)$
- ◆ A **directed graph**(유방향 그래프) is one in which each edge is a directed pair of vertices,  $\langle v_0, v_1 \rangle \neq \langle v_1, v_0 \rangle$

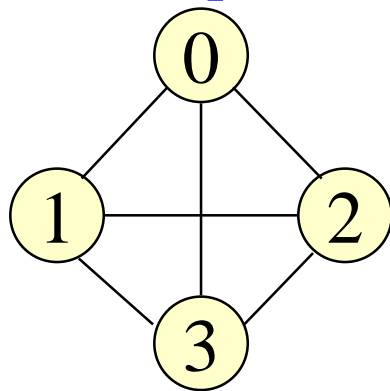
tail  $\longrightarrow$  head

# The Graph ADT (3/13)

## ◆ Examples for Graph

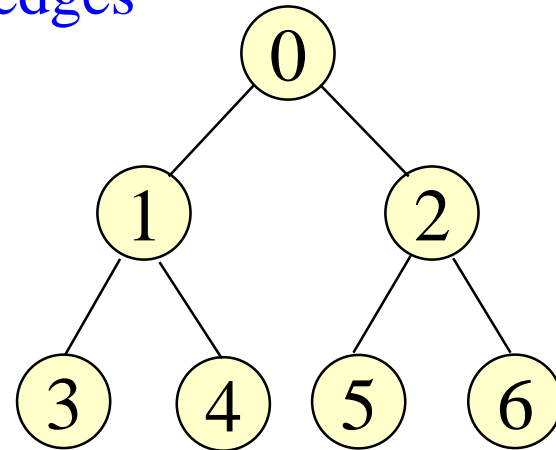
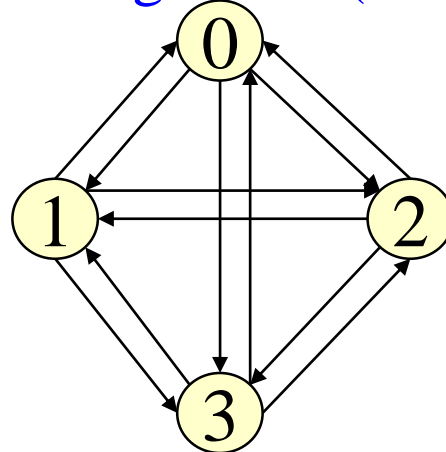
◆ complete undirected graph(완전 무방향 그래프):  $n(n-1)/2$  edges

◆ complete directed graph:  $n(n-1)$  edges



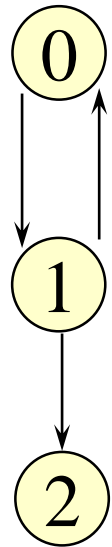
G1

complete graph



G2

incomplete graph



G3

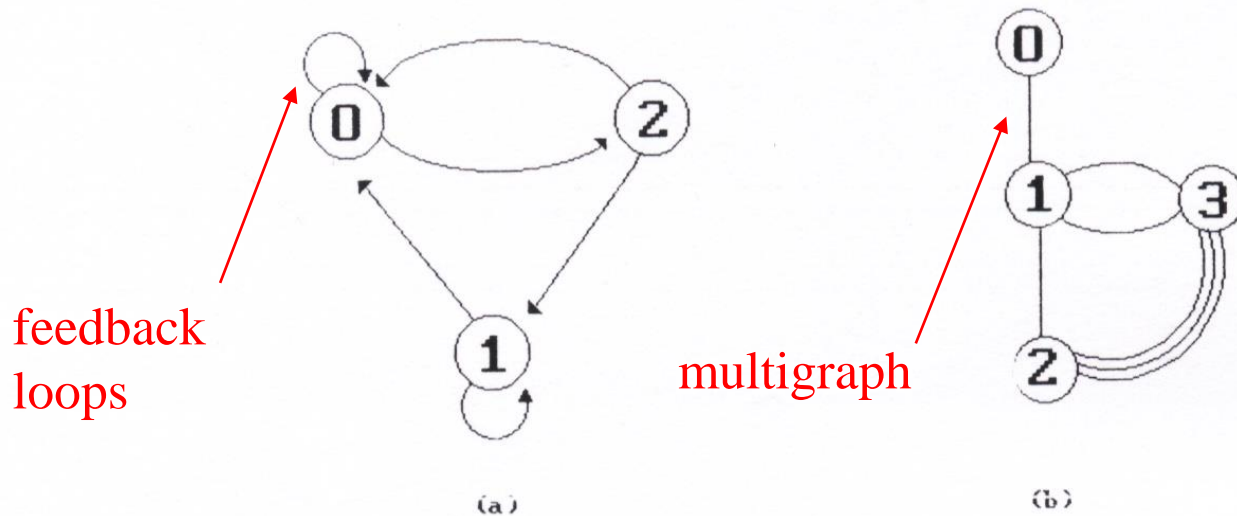
$G1 = \{V, E\}$ ,  $V(G1) = \{0, 1, 2, 3\}$   $E(G1) = \{(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)\}$

$G2 = \{V, E\}$ ,  $V(G2) = \{0, 1, 2, 3, 4, 5, 6\}$   $E(G2) = \{(0, 1), (0, 2), (1, 3), (1, 4), (2, 5), (2, 6)\}$

$G3 = \{V, E\}$ ,  $V(G3) = \{0, 1, 2\}$   $E(G3) = \{<0, 1>, <1, 0>, <1, 2>\}$

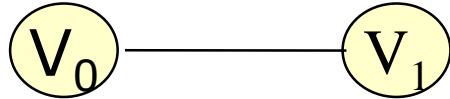
# The Graph ADT (4/13)

- ◆ Restrictions on graphs (그래프에서 제한)
  - ◆ A graph may **not have an edge** from a vertex,  $i$ , **back to itself**. Such edges are known as *self loops*(자기루프) 안됨
  - ◆ A graph **may not have multiple occurrences** of the same edge. If we remove this restriction, we obtain a data referred to as a *multigraph*(멀티그래프) 안됨

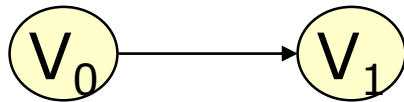


# The Graph ADT (5/13)

- ◆ Adjacent(인접한) and Incident(부속한)
- ◆ If  $(v_0, v_1)$  is an edge in an undirected graph,
  - ◆  $v_0$  and  $v_1$  are **adjacent**(정점들은 인접)
  - ◆ The edge  $(v_0, v_1)$  is **incident**(간선은 부속) on vertices  $v_0$  and  $v_1$

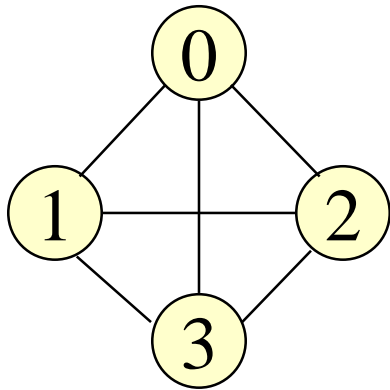


- ◆ If  $\langle v_0, v_1 \rangle$  is an edge in a directed graph
  - ◆  $v_0$  is **adjacent to**  $v_1$ , and  $v_1$  is **adjacent from**  $v_0$
  - ◆ The edge  $\langle v_0, v_1 \rangle$  is incident on  $v_0$  and  $v_1$

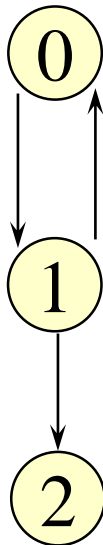


# The Graph ADT (6/13)

- ◆ A subgraph(서브그래프) of  $G$  is a graph  $G'$  such that  $V(G') \subseteq V(G)$  and  $E(G') \subseteq E(G)$ .



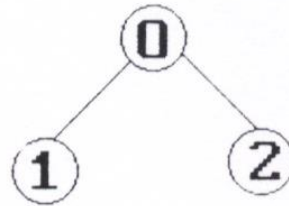
$G_1$



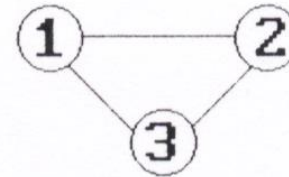
$G_3$



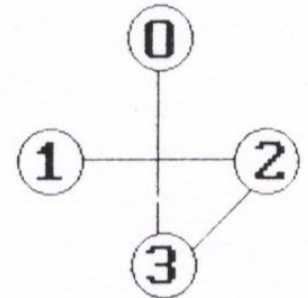
(i)



(ii)



(iii)



(iv)

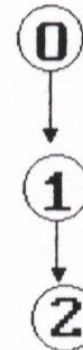
(a) Some of the subgraphs of  $G_1$



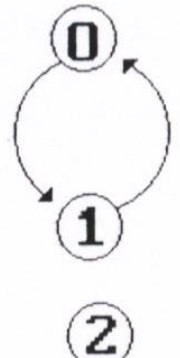
(i)



(ii)



(iii)



(iv)

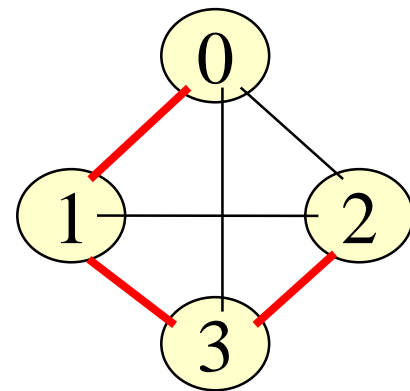
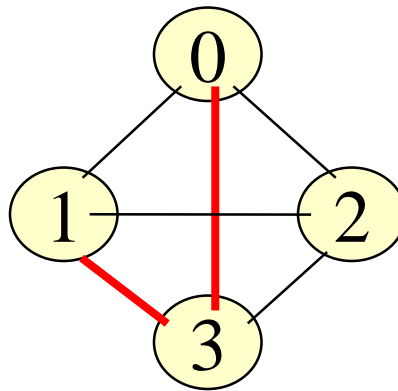
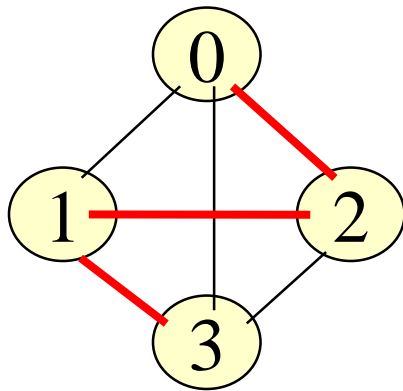
(b) Some of the subgraphs of  $G_3$



# The Graph ADT (7/13)

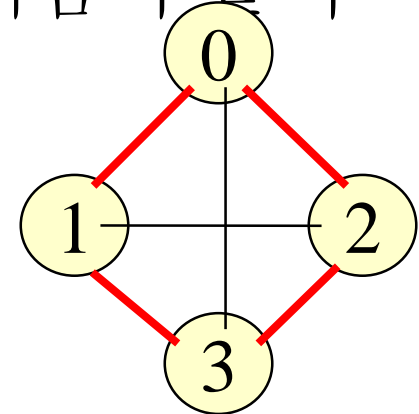
## ◆ Path(경로)

- ◆ A **path** from vertex  $v_p$  to vertex  $v_q$  in a graph  $G$ , is a sequence of vertices,  $v_p, v_{i1}, v_{i2}, \dots, v_{in}, v_q$ , such that  $(v_p, v_{i1}), (v_{i1}, v_{i2}), \dots, (v_{in}, v_q)$  are edges in an undirected graph.
  - A path such as  $(0, 2), (2, 1), (1, 3)$  is also written as  $0, 2, 1, 3$
- ◆ The **length of a path** (경로의 길이) is the number of edges on it



# The Graph ADT (8/13)

- ◆ Simple path and cycle(단순 경로와 사이클)
  - ◆ **simple path (simple directed path)**: a path in which all vertices, except possibly the first and the last, are distinct. (경로의 처음과 끝이 다름, 중간에 중복점 없음)
  - ◆ A **cycle** is a simple path in which the first and the last vertices are the same. (경로의 처음과 끝이 같음 → 사이클)



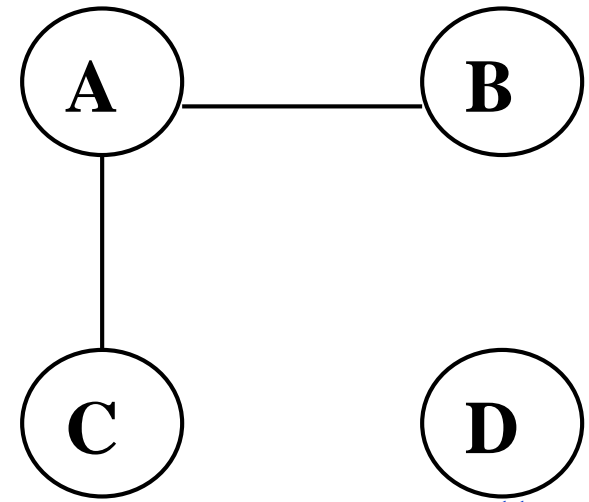
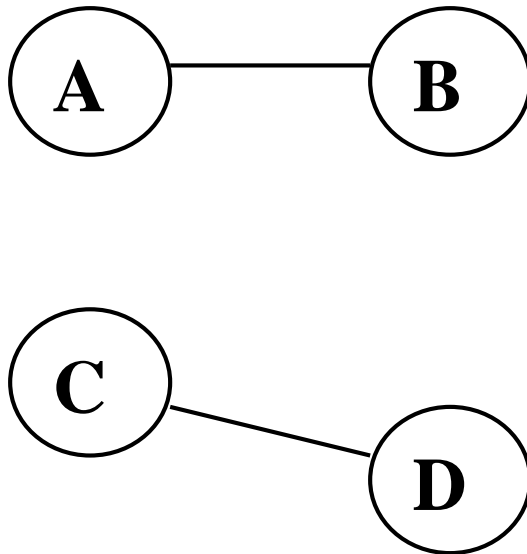
# Connected and Disconnected graphs

## 연결 그래프, 단절 그래프

**Connected graph(연결 그래프):** There is a path between each two vertices 어느 두 정점사이에도 경로존재

**Disconnected graph(단절 그래프) :** There are at least two vertices not connected by a path. 그렇지 않을 경우

**Examples of disconnected graphs(단절 그래프의 예):**

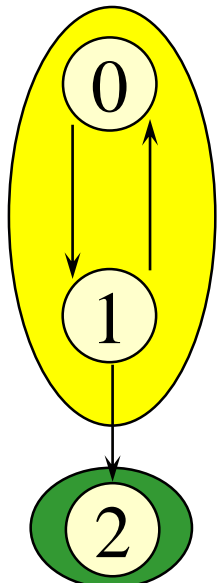


# The Graph ADT (9/13)

- ◆ Connected graph(연결 그래프)-무방향 그래프
  - ◆ In an undirected graph  $G$ , two vertices,  $v_0$  and  $v_1$ , are **connected** if there is a path in  $G$  from  $v_0$  to  $v_1$
  - ◆ An undirected graph is **connected** if, for every pair of distinct vertices  $v_i, v_j$ , there is a path from  $v_i$  to  $v_j$
  - ◆ 연결 그래프의 어느 두 정점을 선택해도, 그 두 정점 사이에 경로가 존재.
- ◆ Connected component(연결 요소)-무방향 그래프
  - ◆ A **connected component** of an undirected graph is a maximal connected subgraph. 연결 요소는 무방향 그래프에서 최대 연결 부분 그래프를 의미
  - ◆ A tree is a graph that is connected and acyclic (i.e, has no cycle).  
(트리는 그래프의 일종. 연결되었고 사이클이 존재하지 않음)
  - ◆ 매우 중요

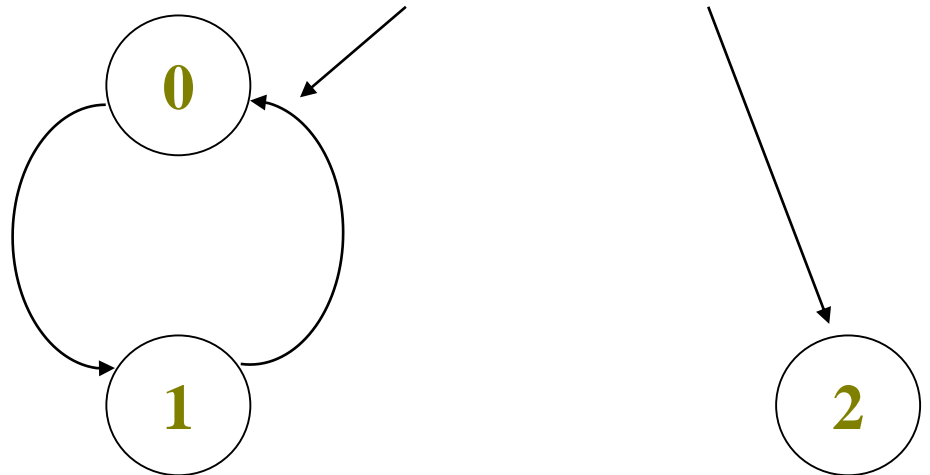
# The Graph ADT (10/13)

- ♦ Strongly Connected Component(강력연결요소)-**유**방향 그래프
  - ♦ 유방향그래프의 어느 두 정점을 선택해도 양방향으로 경로가 존재하면 강력연결그래프라고 함. A directed graph is **strongly connected** if there is a directed path from  $v_i$  to  $v_j$  and also from  $v_j$  to  $v_i$
  - ♦ A **strongly connected component** is a maximal subgraph that is strongly connected 강력연결요소는 강력하게 연결된



G3 not strongly connected

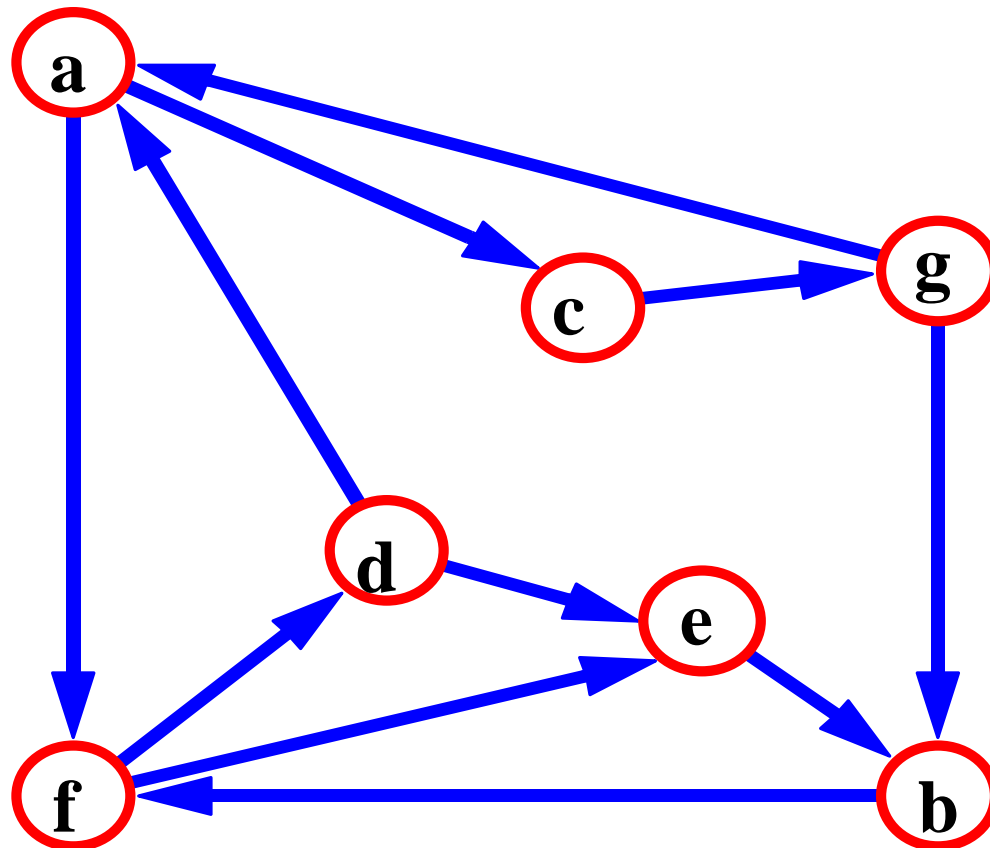
strongly connected component 강력연결요소  
(maximal strongly connected subgraph)



# Strongly Connected Directed graphs

## 강력연결그래프(유방향그래프)

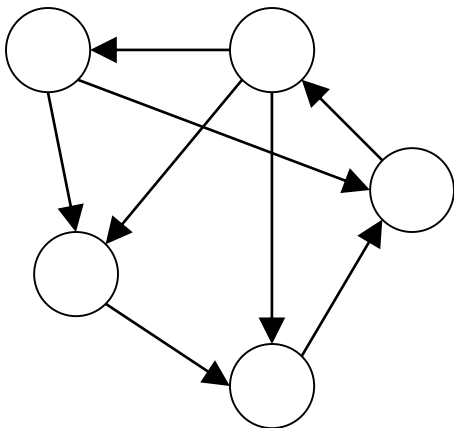
- ◆ Every pair of vertices are reachable from each other



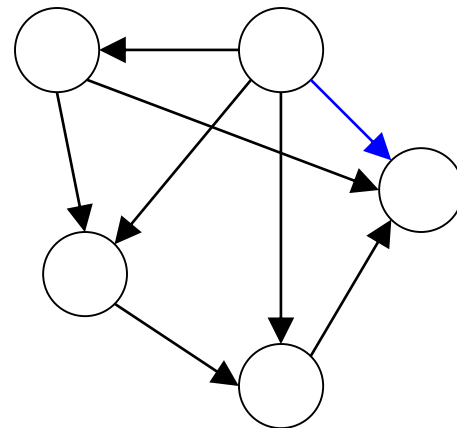
# Strongly-Connected 강력연결 그래프(유방향)

Graph  $G$  is *strongly connected* if, for every  $u$  and  $v$  in  $V$ , there is some path from  $u$  to  $v$  and some path from  $v$  to  $u$ .

Strongly  
Connected

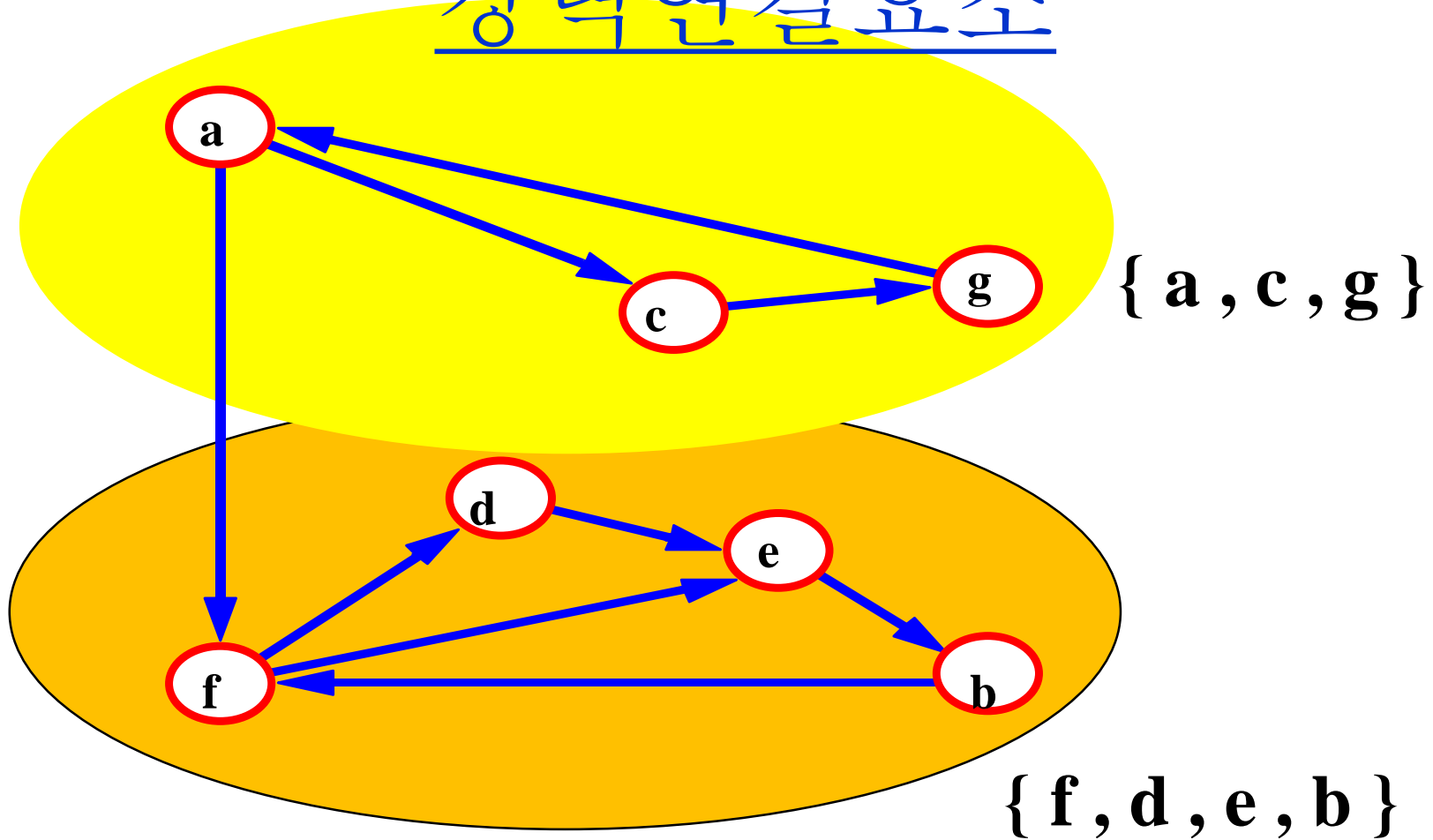


Not Strongly  
Connected



# Strongly Connected Components

강력연결요소





# The Graph ADT (11/13)

- ◆ Degree(차수)
  - ◆ The **degree** of a vertex is the number of edges incident to that vertex.
- ◆ For directed graph
  - ◆ **in-degree** ( $v$ )(진입차수) : the number of edges that have  $v$  as the head
  - ◆ **out-degree** ( $v$ )(진출차수) : the number of edges that have  $v$  as the tail
- ◆ If  $d_i$  is the degree of a vertex  $i$  in a graph  $G$  with  $n$  vertices and  $e$  edges, the number of edges is

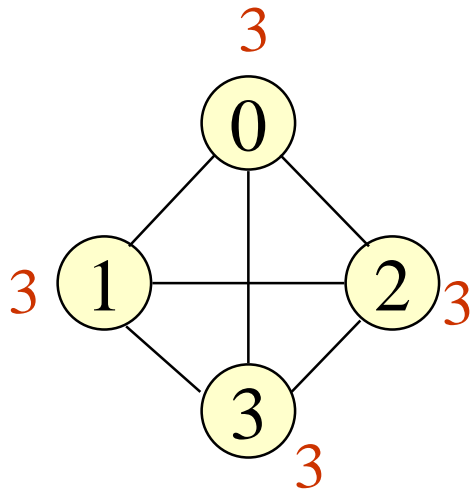
$$e = \left( \sum_{i=0}^{n-1} d_i \right) / 2$$

# The Graph ADT (12/13)

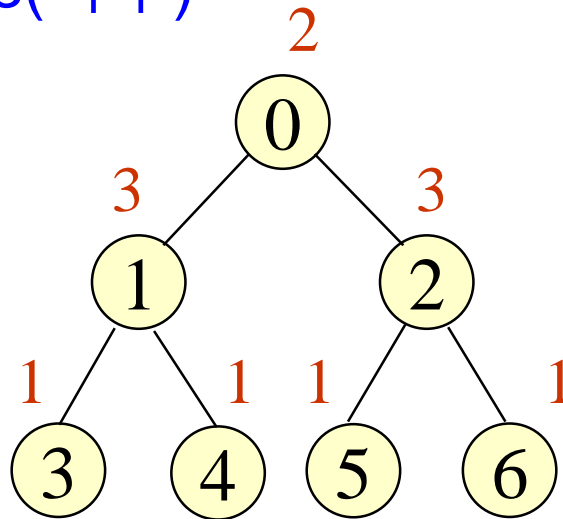
- ◆ We shall refer to a directed graph as a *digraph*. When we use the term *graph*, we assume that it is an undirected graph. 일반적으로 유방향그래프를 digraph라고도 함. 그래프라고 통칭할 때는 무방향그래프라 약속하자.

undirected graph(무방향)

Degree(차수)

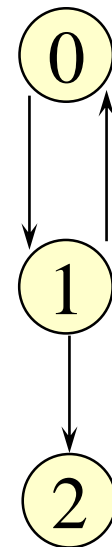


G1



G2

directed graph(유방향)  
in-degree & out-degree  
진입차수&진출차수



in:1, out: 1

in: 1, out: 2

in: 1, out: 0

G3

# The Graph ADT (13/13)

## ◆ Abstract Data Type *Graph*

### ADT Graph

데이터 : 공백이 아닌 정점(vertex)의 유한집합( $V$ )과 간선(edge)의 집합( $E$ ).  
여기서 간선은 두 정점의 쌍.

연산 내용 :  $G \in \text{Graph}$ ,  $u, v \in V$ ;

createG() ::= create an empty graph;

insertVertex( $G, v$ ) ::= insert vertex  $v$  into  $G$ ;

insertEdge( $G, u, v$ ) ::= insert edge  $(u, v)$  into  $G$ ;

deleteVertex( $G, v$ ) ::= delete vertex  $v$  and all edges incident on  $v$  from  $G$ ;

deleteEdge( $G, u, v$ ) ::= delete edge  $(u, v)$  from  $G$ ;

isEmpty( $G$ ) ::= **if**  $G$  has no vertex **then return** true, **else return** false;

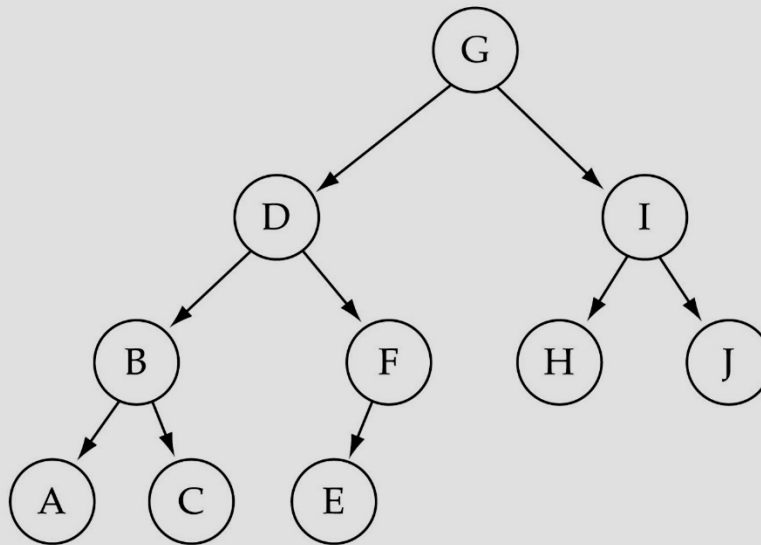
adjacent( $G, v$ ) ::= **return** set of all vertices adjacent to  $v$ ;

End Graph

# Trees vs graphs

- ◆ Trees are special cases of graphs!!(트리는 그래프의 특수한 종류)

(c) Graph3 is a directed graph.



$V(\text{Graph3}) = \{ A, B, C, D, E, F, G, H, I, J \}$

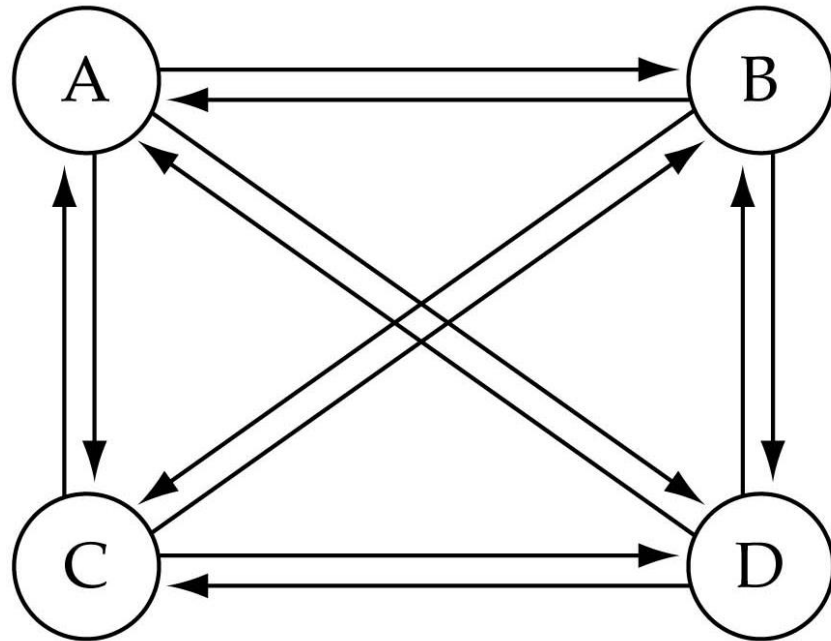
$E(\text{Graph3}) = \{ (G, D), (G, I), (D, B), (D, F), (I, H), (I, J), (B, A), (B, C), (F, E) \}$

# Graph terminology 그래프용어

- ◆ What is the number of edges in a complete directed graph(유방향완전연결그래프) with N vertices? 간선의 개수?

$$N * (N-1)$$

$$O(N^2)$$



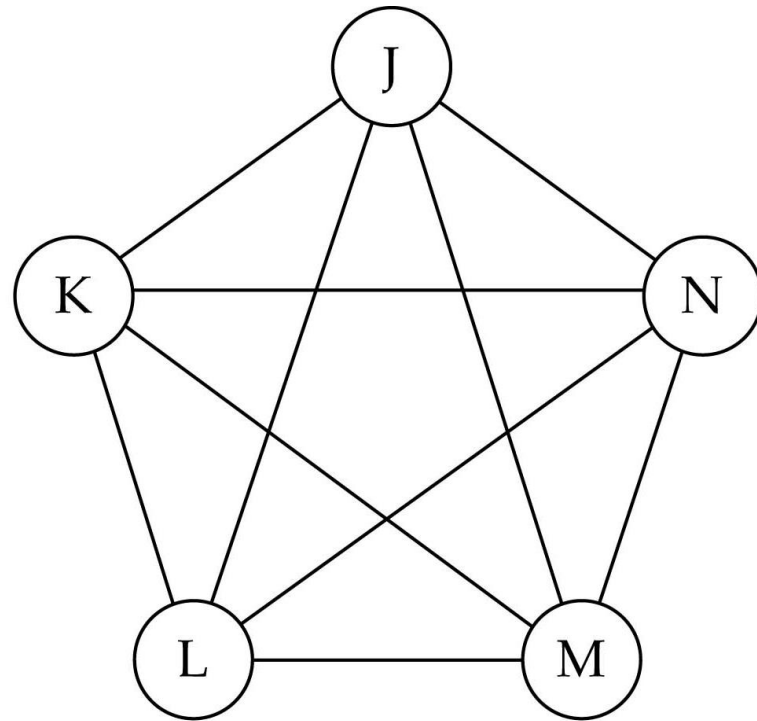
(a) Complete directed graph.

# Graph terminology(그래프용어)

- ◆ What is the number of edges in a complete undirected graph with N vertices? 무방향완전연결그래프

$$N * (N-1) / 2 (\text{최대간선의 개수})$$

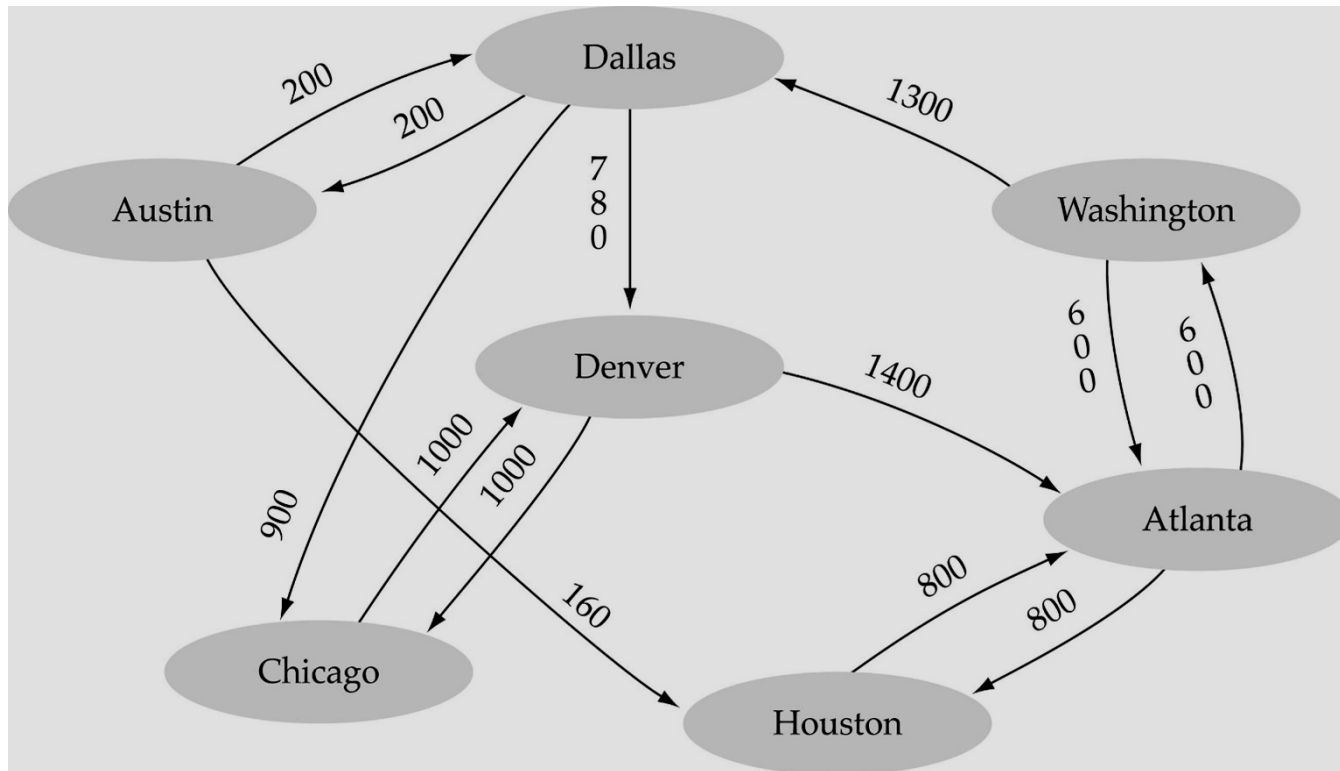
$$O(N^2)$$



(b) Complete undirected graph.

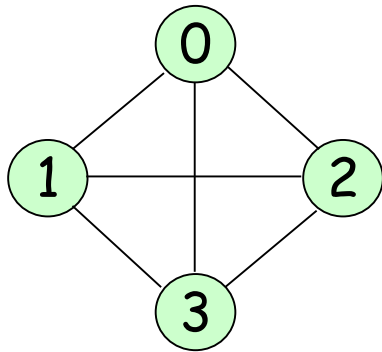
# Graph terminology (그래프용어)

- ◆ Weighted graph(가중치그래프): a graph in which each edge carries a value



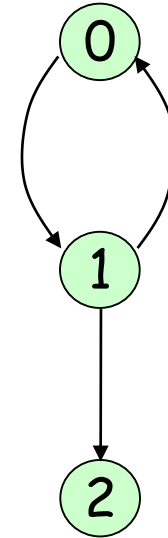
# Adjacency Matrix(인접행렬로표현)

- ◆ Adjacency Matrix is a two dimensional  $n \times n$  array.(인접행렬은 이차원배열)



$$\begin{array}{c} 0 \quad 1 \quad 2 \quad 3 \\ 0 \begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix} \\ 1 \begin{bmatrix} 1 & 0 & 1 & 1 \end{bmatrix} \\ 2 \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix} \\ 3 \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

(a)  $G_1$

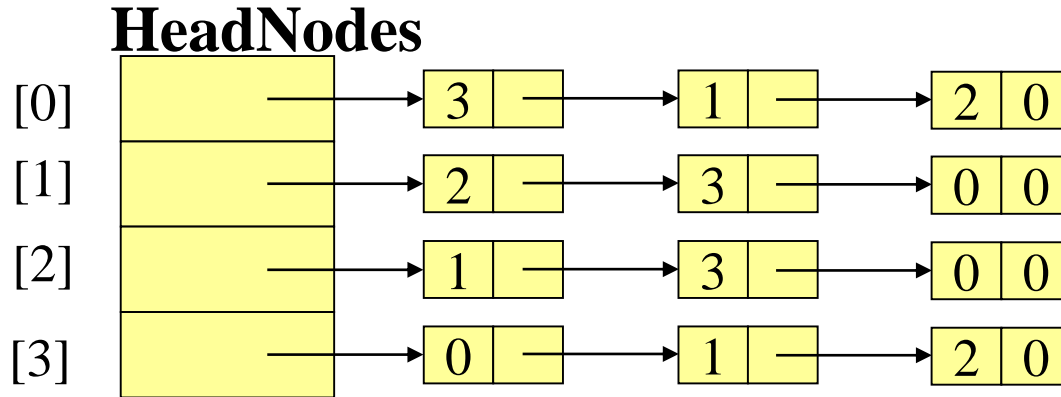


$$\begin{array}{c} 0 \quad 1 \quad 2 \\ 0 \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \\ 1 \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \\ 2 \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \end{array}$$

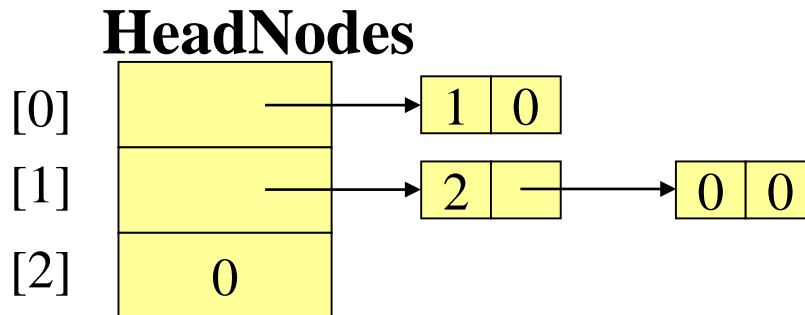
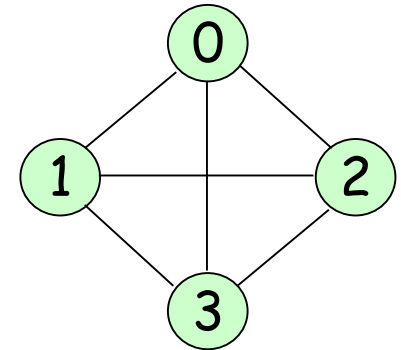
(b)  $G_3$



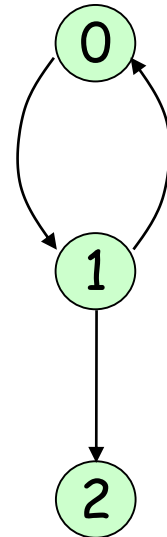
# Adjacency Lists(리스트로 표현)



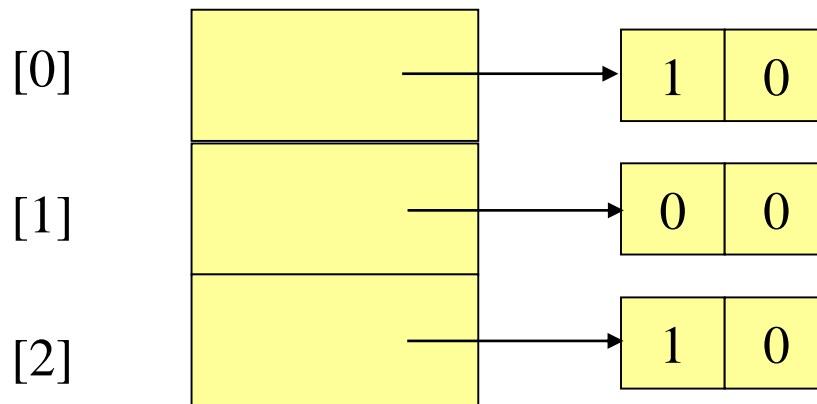
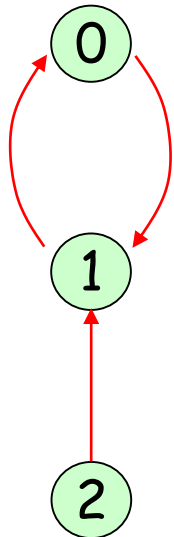
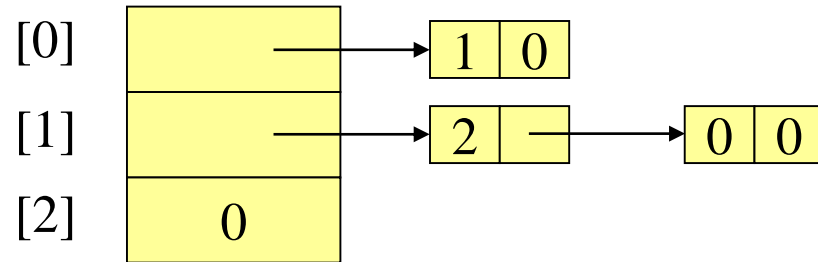
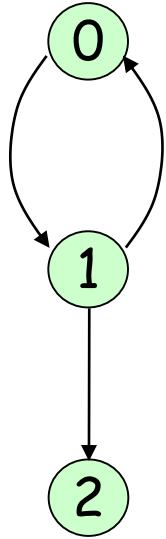
(a)  $G_1$



(b)  $G_3$

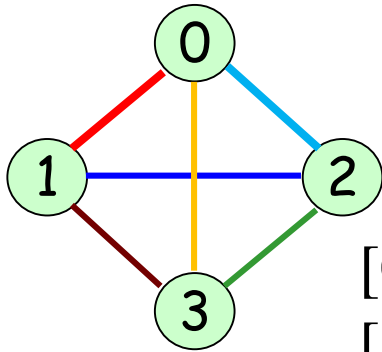


# Inverse Adjacency Lists(역인접리스트)

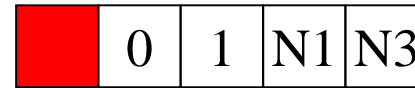
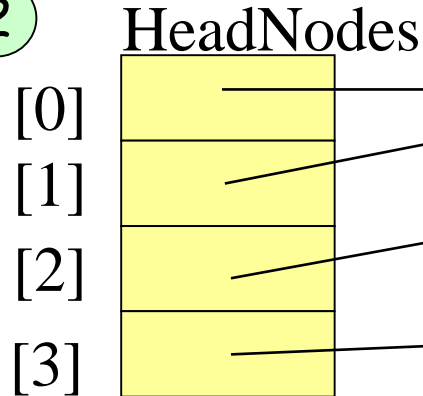


# Multilists(멀티리스트로 표현)

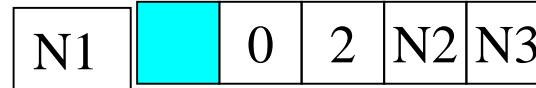
m	vertex1	vertex2	list1	list2
---	---------	---------	-------	-------



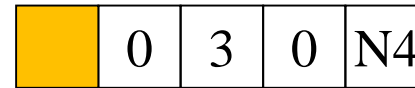
Six edges



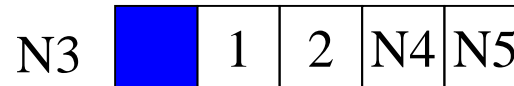
edge (0, 1)



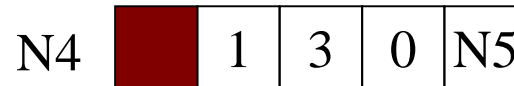
edge (0, 2)



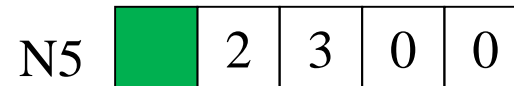
edge (0, 3)



edge (1, 2)



edge (1, 3)



edge (2, 3)

The lists are

Vertex 0: N0 -> N1 -> N2

Vertex 1: N0 -> N3 -> N4

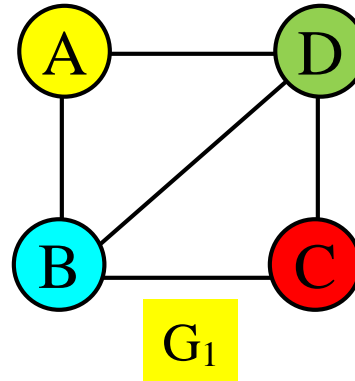
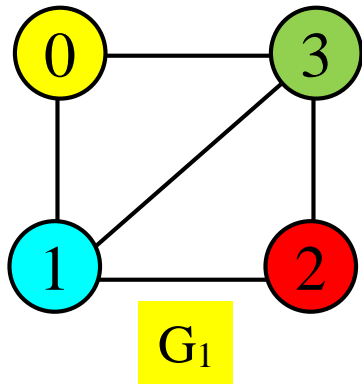
Vertex 2: N1 -> N3 -> N5

Vertex 3: N2 -> N4 -> N5

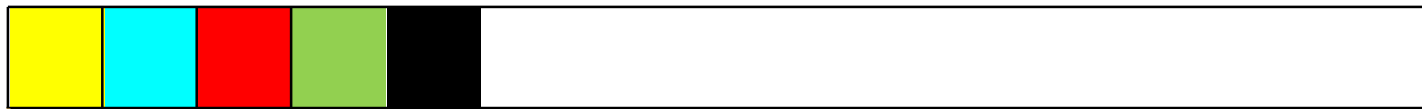
Six edges 여섯개 간선

# Adjacency Lists: Sequential Representation

## 인접리스트 : 선형표현



vertex [0] [1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13] [14]



리스트 시작과  
배열 크기 정보

간선 정보

vertex [0] [1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13] [14]

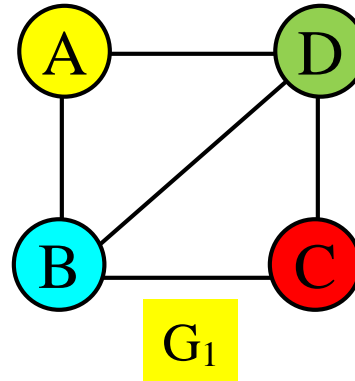
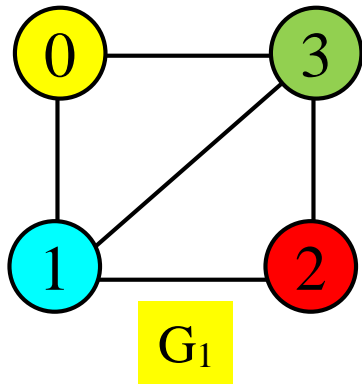


리스트 시작과  
배열 크기 정보

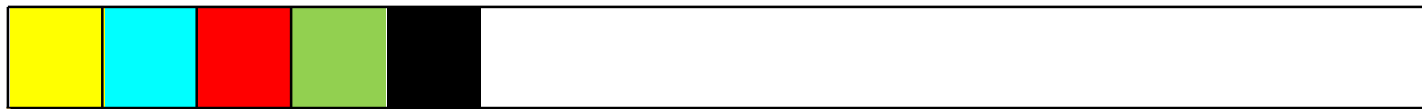
간선 정보

# Adjacency Lists: Sequential Representation

## 인접리스트 : 선형표현



vertex [0] [1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13] [14]



리스트 시작과  
배열 크기 정보

간선 정보

vertex [0] [1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13] [14]

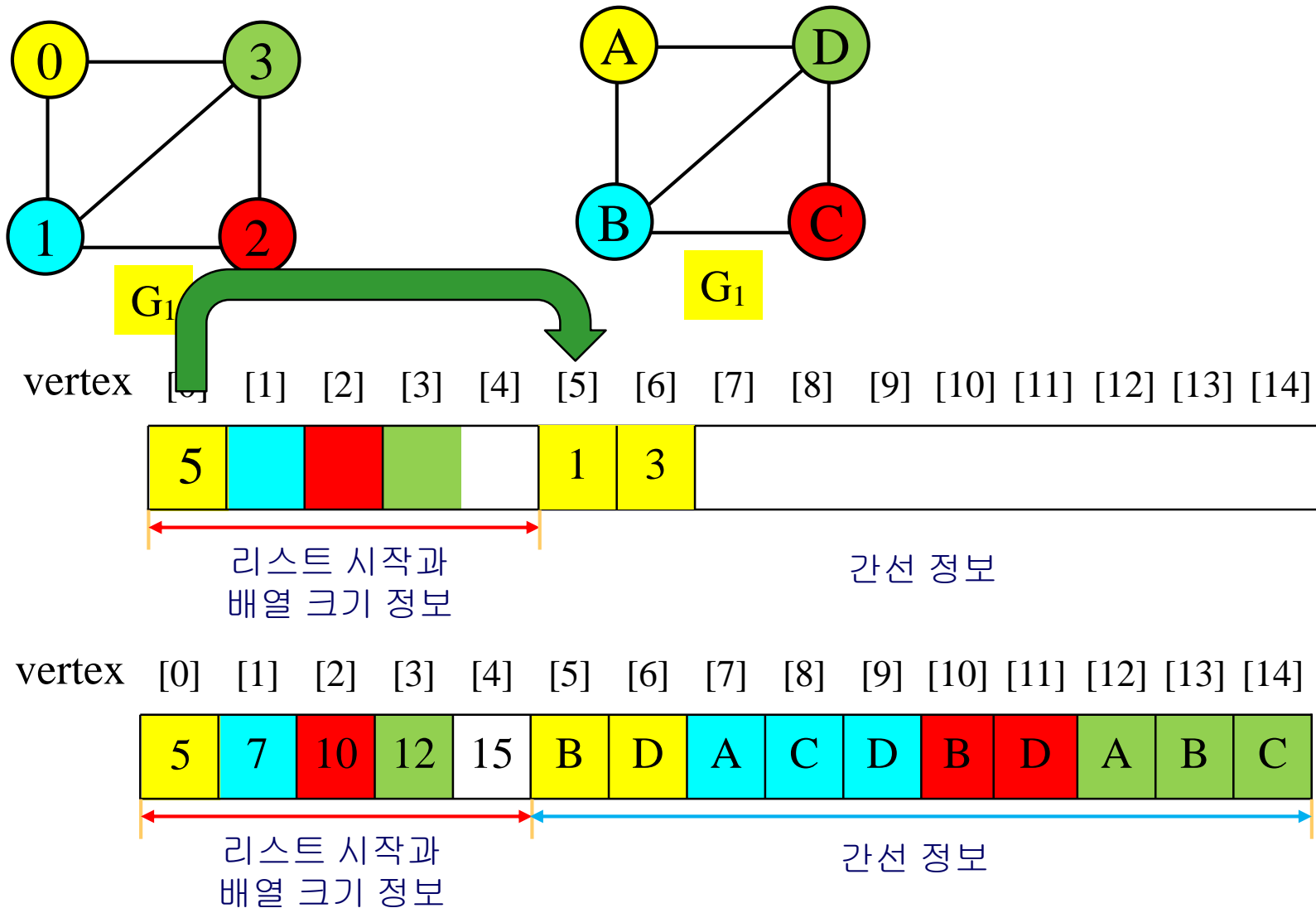


리스트 시작과  
배열 크기 정보

간선 정보

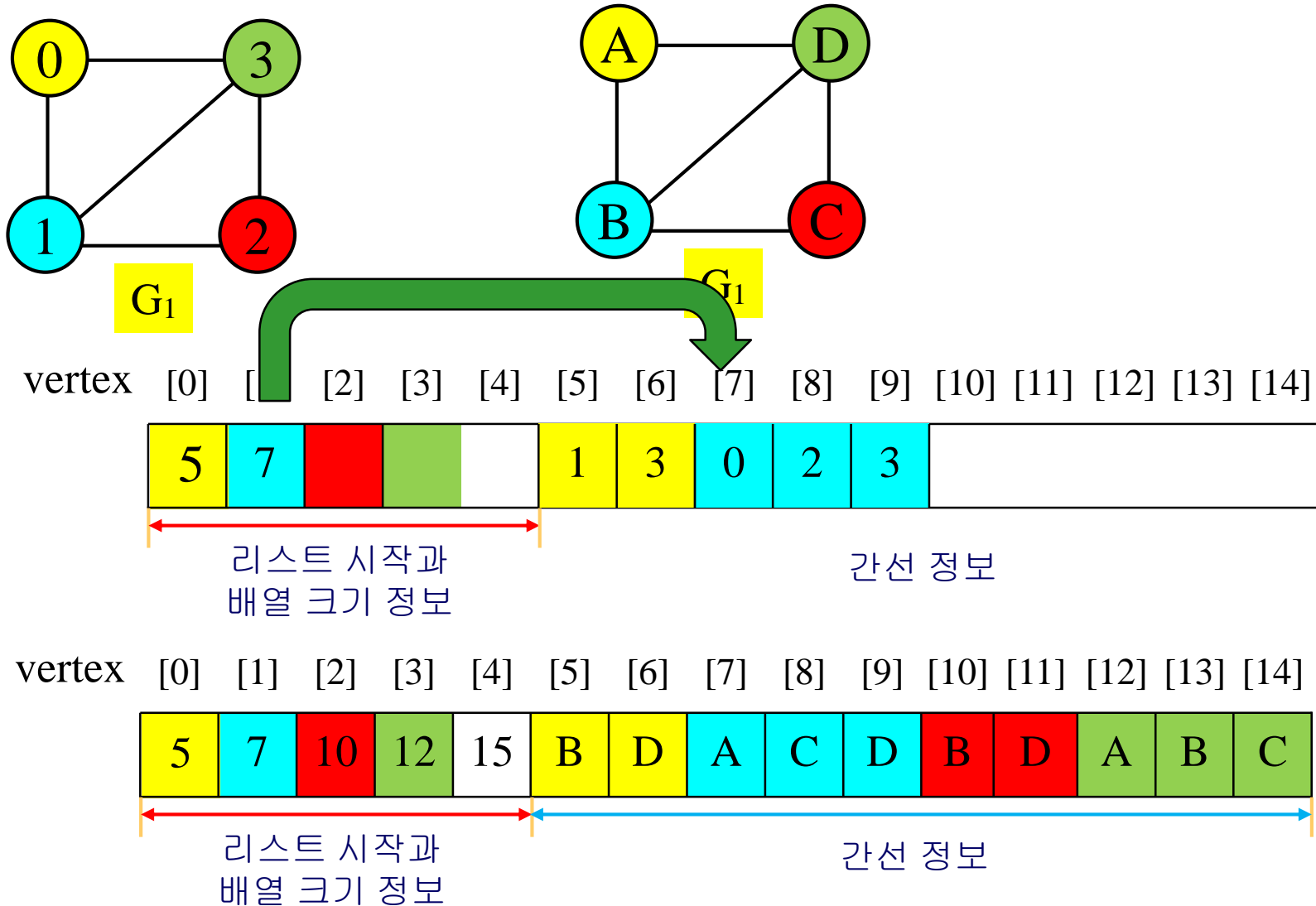
# Adjacency Lists: Sequential Representation

## 인접리스트 : 선형표현



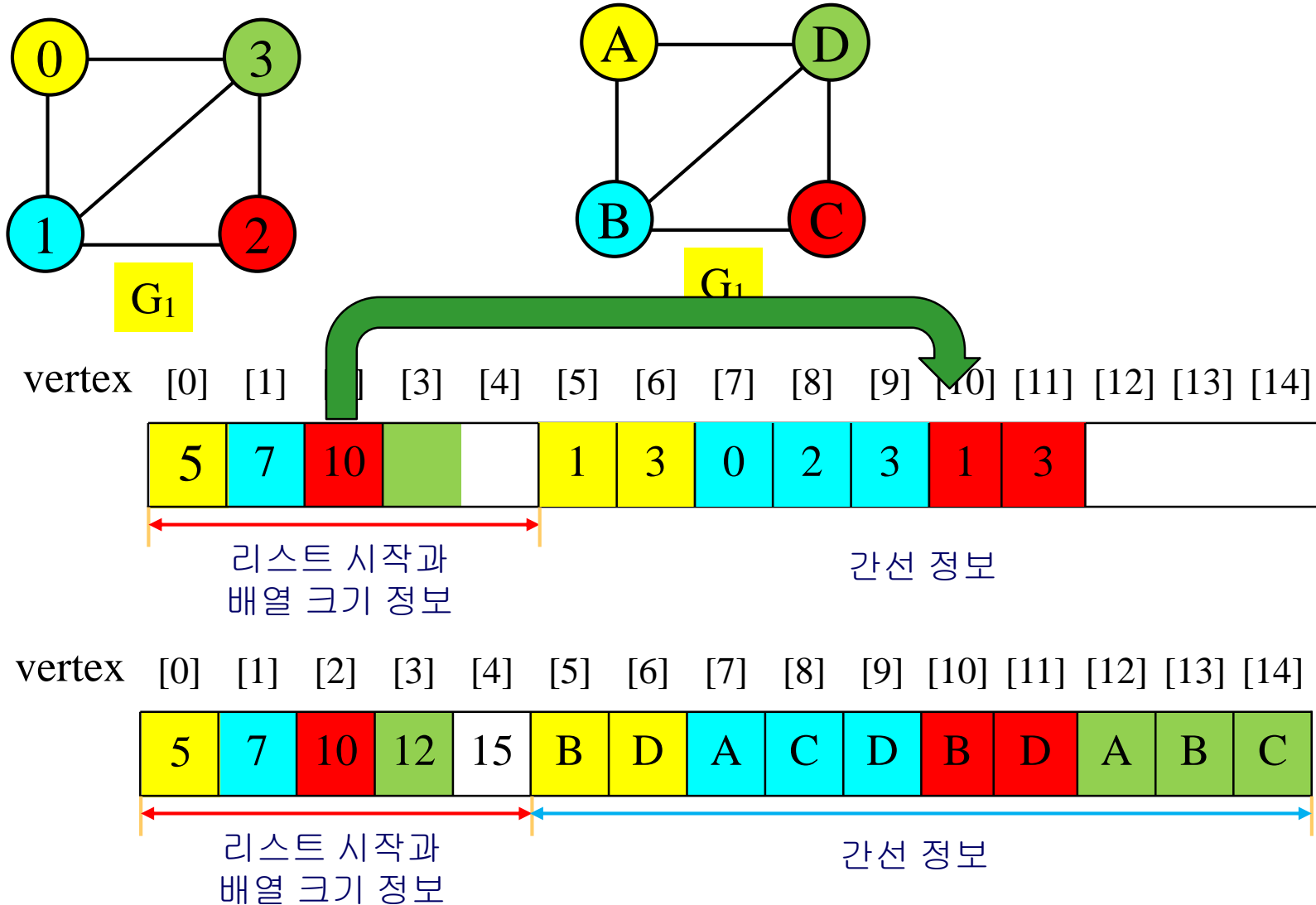
# Adjacency Lists: Sequential Representation

## 인접리스트 : 선형표현



# Adjacency Lists: Sequential Representation

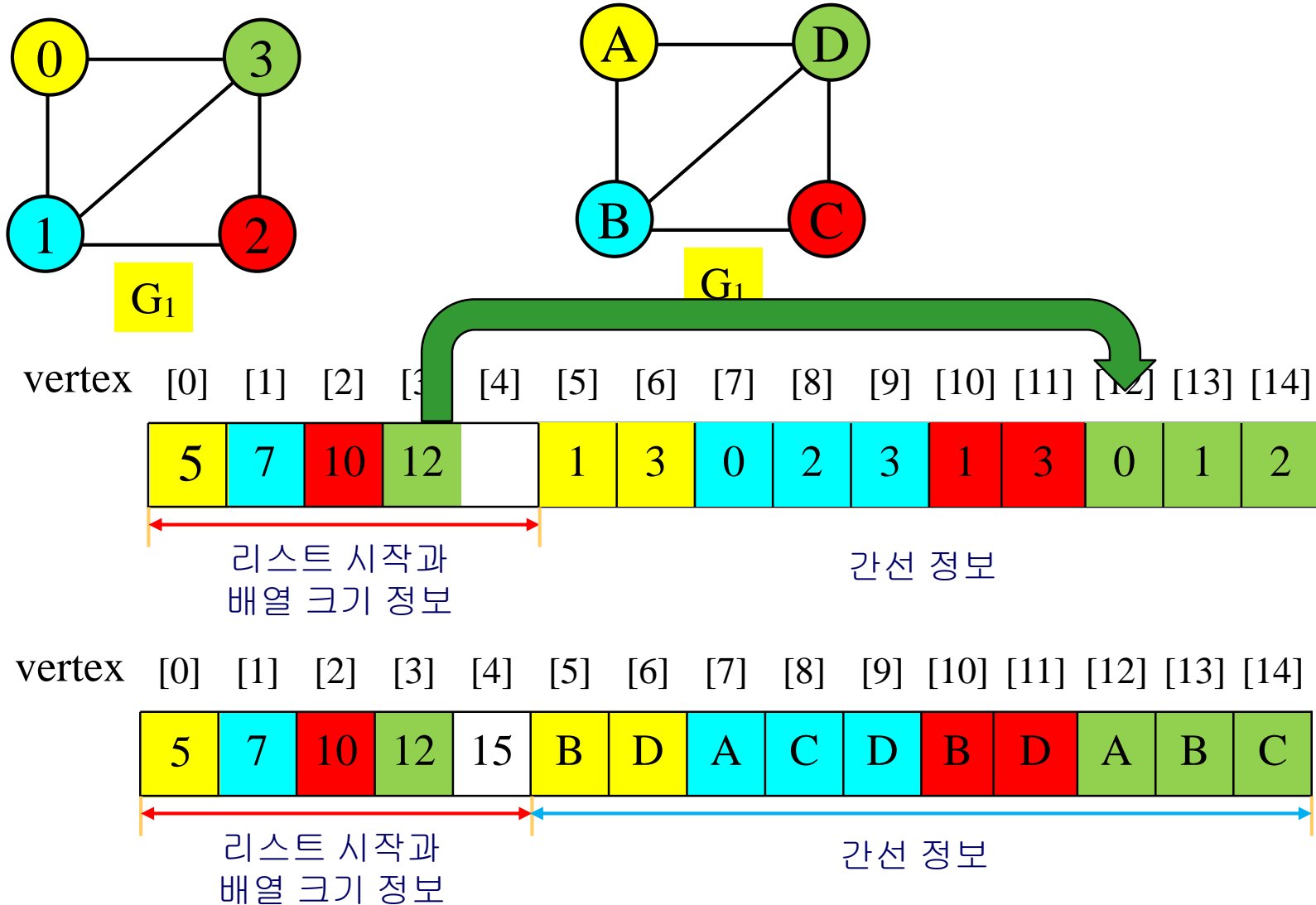
## 인접리스트 : 선형표현





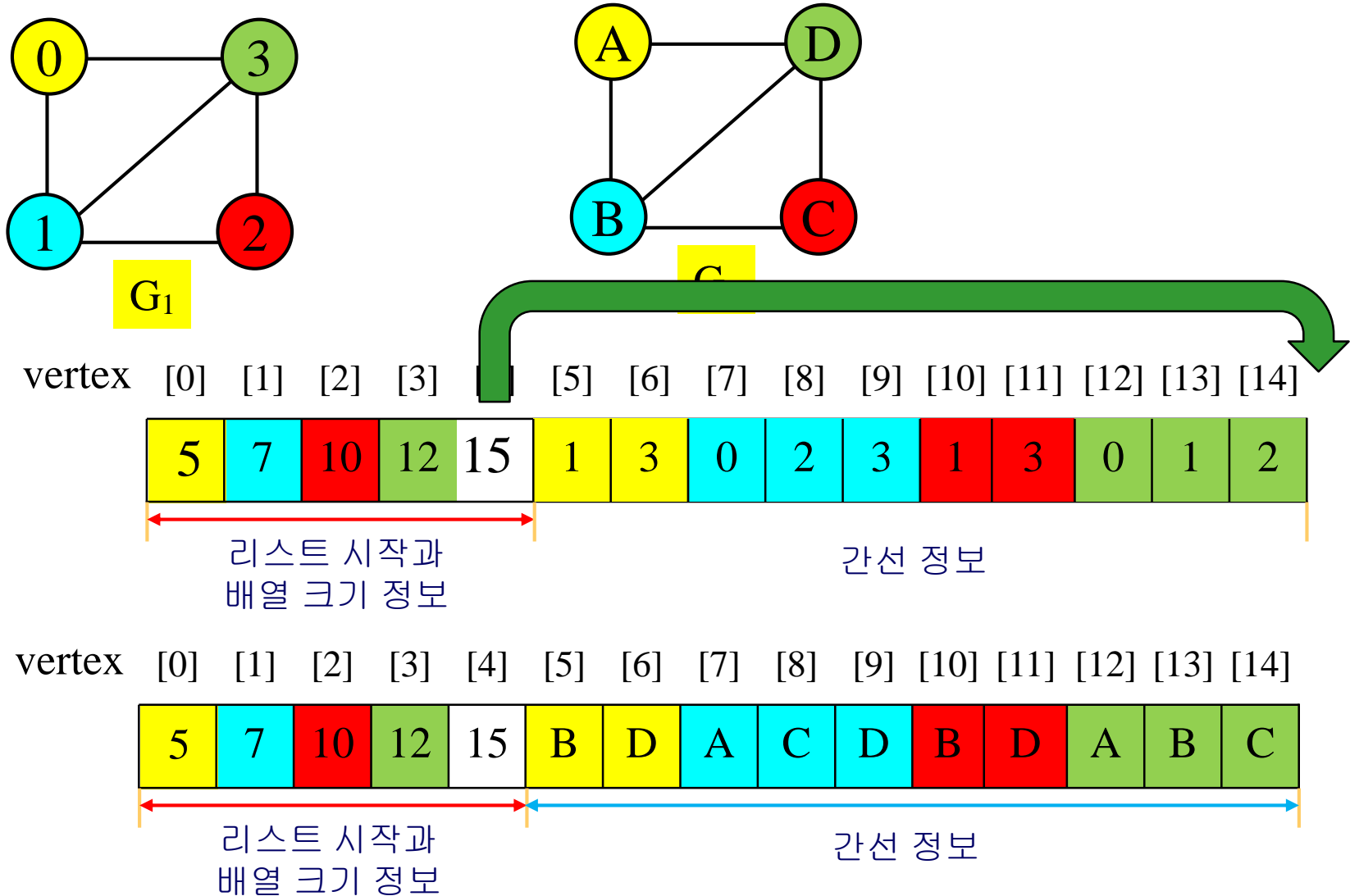
# Adjacency Lists: Sequential Representation

## 인접리스트 : 선형표현



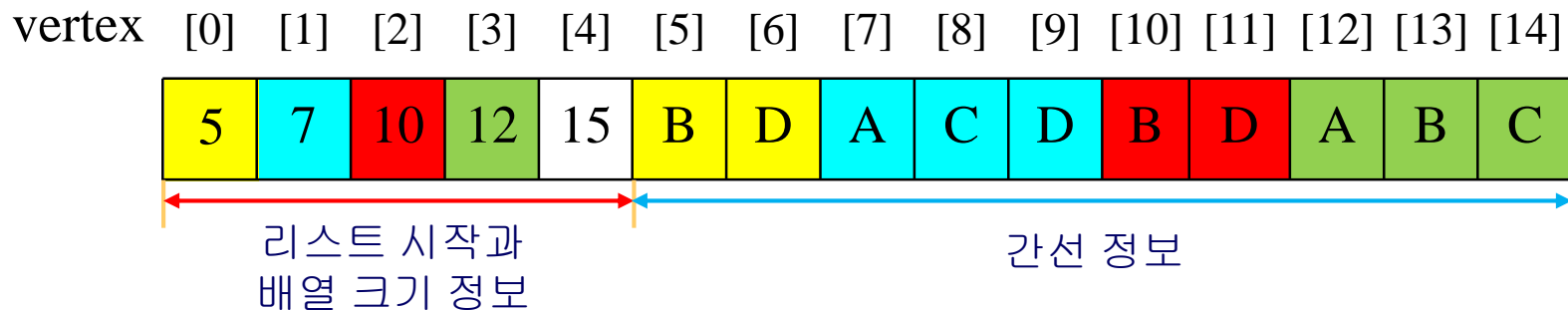
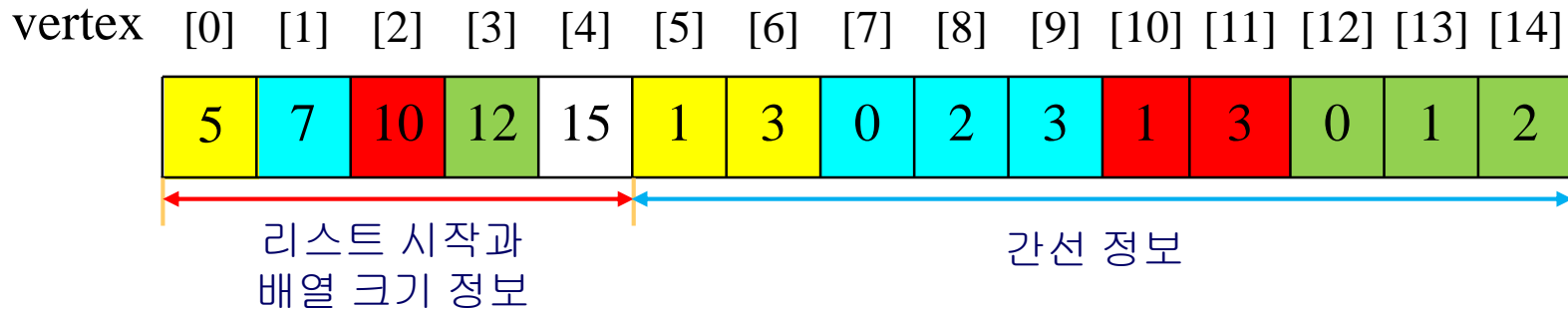
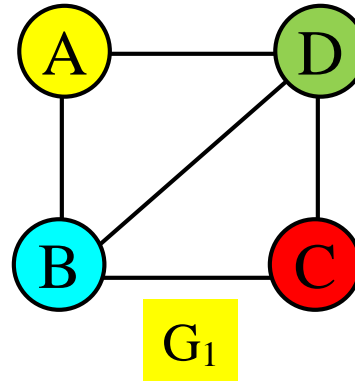
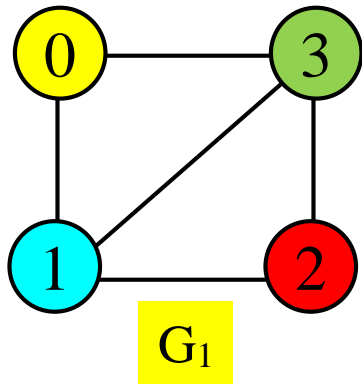
# Adjacency Lists: Sequential Representation

## 인접리스트 : 선형표현



# Adjacency Lists: Sequential Representation

## 인접리스트 : 선형표현



# Pros and Cons of Adjacency Matrices

## 인접행렬

### ◆ Pros: (장점)

- ◆ Simple to implement(구현하기 간단하다)
- ◆ Easy and fast to tell if a pair  $(i,j)$  is an edge: simply check if  $A[i][j]$  is 1 or 0 ((i,j)에 간선이 있는지 알기 쉽다)

### ◆ Cons: (단점)

- ◆ No matter how few edges the graph has, the matrix takes  $O(n^2)$  in memory (아무리 간선수가 적어도  $O(n^2)$ 개의 방이 필요)

# Pros and Cons of Adjacency Lists

## 인접리스트

- ◆ Pros:(장점, for)
  - ◆ Saves on space (memory): the representation takes as many memory words as there are nodes and edge.(공간이 절약된다)
- ◆ Cons: (단점, against)
  - ◆ It can take up to  $O(n)$  time to determine if a pair of nodes  $(i,j)$  is an edge: one would have to search the linked list  $L[i]$ , which takes time proportional to the length of  $L[i]$ . (i,j)사이에 간선이 있음을 알기위해 연결리스트탐색시간필요

# Adjacency matrix vs. adjacency list representation

## ♦ Adjacency matrix(인접행렬)

- ♦ Good for dense graphs(밀집그래프에 좋다) --  $|E| \sim O(|V|^2)$
- ♦ Memory requirements:  $O(|V| + |E|) = O(|V|^2)$
- ♦ Connectivity between two vertices can be tested quickly

## ♦ Adjacency list(인접리스트)

- ♦ Good for sparse graphs(희소그래프에 좋다) --  $|E| \sim O(|V|)$
- ♦ Memory requirements:  $O(|V| + |E|) = O(|V|)$
- ♦ Vertices adjacent to another vertex can be found quickly(이웃한정점을 알기 쉽다)

# Graph searching(그래프탐색)

## 순회방문을 생각

- ◆ Problem(문제): find a path between two nodes of the graph (e.g., Austin and Washington) 두 정점 사이에 경로가 존재할까?
- ◆ Methods: Depth-First-Search (DFS) or Breadth-First-Search (BFS) 깊이우선탐색, 너비우선탐색

# Overview

- ◆ Goal

- ◆ To systematically visit the nodes of a graph  
정점들을 효율적으로 방문하고 싶다.

- ◆ A tree is a directed, acyclic, graph  
(DAG)(트리는 유방향, 사이클없는 그래프)

- ◆ If the graph is a tree,(그래프가 트리이면)

- ◆ DFS is exhibited by preorder, postorder, and (for binary trees) inorder  
traversals(전위,중위,후위순회)깊이우선탐색은  
전위, 중위,후위순회로 탐색
  - ◆ BFS is exhibited by level-order  
traversal너비우선탐색은 (레벨순서방문)



# Depth-First-Search (DFS) 깊이 우선 탐색

- ◆ What is the idea behind DFS?
  - ◆ Travel as far as you can down a path(갈 수 있을 때까지 내려가자)
  - ◆ Back up *as little as possible* when you reach a "dead end" (i.e., next vertex has been "marked" or there is no next vertex)막다른 골목이면 표시하고 되돌아나오자.
- ◆ DFS can be implemented efficiently using a *stack*  
(깊이 우선 탐색은 스택 사용하면 효과적임)

# Depth-First-Search (DFS) (*cont.*)

- (1) 정점  $i$ 를 방문한다.
- (2) 정점  $i$ 에 인접한 정점 중에서 아직 방문하지 않은 정점이 있으면, 이 정점들을 모두 스택에 저장한다.
- (3) 스택에서 정점을 삭제하여 새로운  $i$ 를 설정하고, 단계 (1)을 수행한다.
- (4) 스택이 공백이 되면 연산을 종료한다.

$$\text{visited}[i] = \begin{cases} \text{true}, & \text{방문하였음} \\ \text{false}, & \text{방문하지 않았음} \end{cases}$$

# Depth First Search(스택 사용)

DFS(i)

// i 는 시작 정점

**for** (i←0; i<n; i ← i +1) **do** {

    visited[i] ← false;   // 모든 정점을 방문 안한 것으로 마크

}

createStack();   //방문할 정점을 저장하는 스택

push(Stack, i);   // 시작 정점 i 를 스택에 저장

**while** (not isEmpty(Stack)) **do** {   // 스택이 공백이 될 때까지 반복 처리

    j ← pop(Stack);

**if** (visited[j] = false) **then** {   //정점 j를 아직 방문하지 않았다면

        visit j;                       // 직접 j를 방문하고

        visited[j] ← true;           // 방문 한 것으로 마크

**for** (each k ∈ adjacency(j)) **do** {   // 정점 j에 인접한 정점 중에서

**if** (visited[k] = false) **then**   // 아직 방문하지 않은 정점들을

                push(Stack, k);               // 스택에 저장

        }

    }

}

end DFS()

# Breadth-First-Searching (BFS)너비우선탐색

- ◆ What is the idea behind BFS?
  - ◆ Look at all possible paths at the same depth before you go at a deeper level 같은 레벨의 모든 원소를 방문하고 다음레벨로가자.
  - ◆ Back up *as far as possible* when you reach a "dead end" (i.e., next vertex has been "marked" or there is no next vertex) 막다른 골목이면 표시하고 되돌아 나가자

# Breadth-First-Searching (BFS) (cont.)

- (1) 정점  $i$ 를 방문한다.
- (2) 정점  $i$ 에 인접한 정점 중에서 아직 방문하지 않은 정점이 있으면, 이 정점들을 모두 **큐**에 저장한다.
- (3) 큐에서 정점을 삭제하여 새로운  $i$ 를 설정하고, 단계 (1)을 수행한다.
- (4) 큐가 공백이 되면 연산을 종료한다.

# Breadth First Search(큐 사용)

BFS(i)

// i는 시작 정점

**for** (i←0; i<n; i ←i+1) **do** {

    visited[i] ← false;   // 모든 정점을 방문 안 한 것으로 마크

}

visited[i] ← true;

createQ();   // 방문할 정점을 저장하는 큐

enqueue(Q, i);

**while** (not isEmpty(Q)) **do** {

    j ← dequeue(Q);

**if** (visited[j] = false) **then** {

        visit j;

        visited[j] ← true;

    }

**for** (each k ∈ adjacency(j)) **do** {

**if** (visited[k] = false) **then** {

            enqueue(Q, k);

        }

    }

}

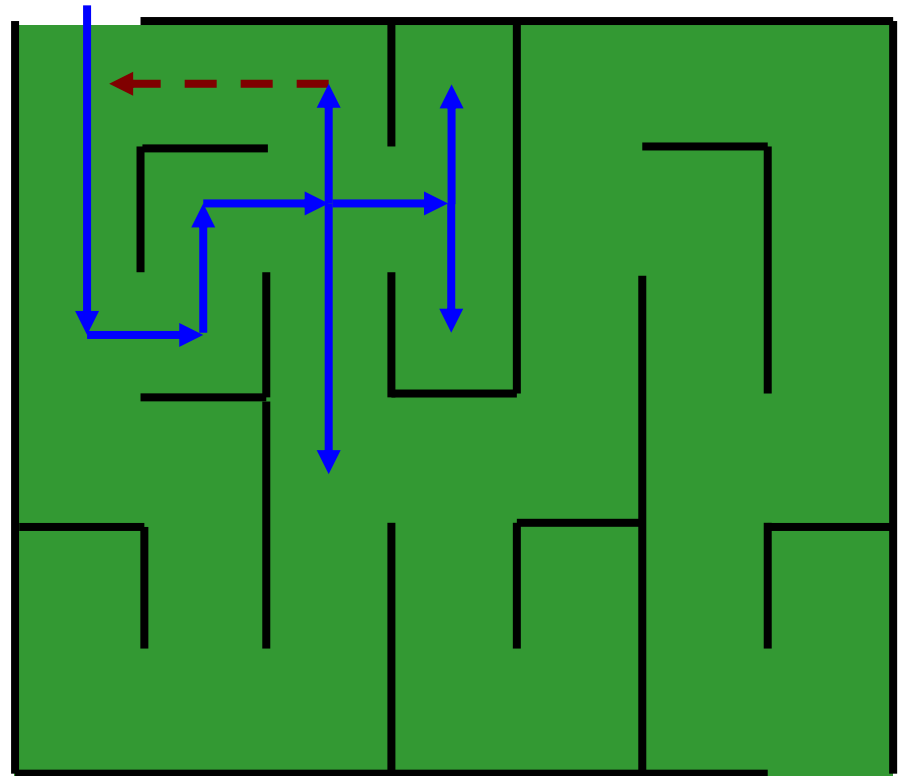
**end** BFS()

# DFS: Example1

# DFS and Maze Traversal

# 미로찾기는 깊이우선탐색

- ◆ The DFS algorithm is similar to a classic strategy for exploring a maze
  - ◆ We mark each intersection, corner and dead end (vertex) visited
  - ◆ We mark each corridor (edge ) traversed
  - ◆ We keep track of the path back to the entrance (start vertex) by means of a rope (recursion stack)





# Maze 미로

	0	1	2	3	4	5	6
0		1	1	1	1	1	1
1	1		1	1		1	1
2	1			1		1	1
3	1	1				1	1
4	1	1	1				1
5	1				1	1	1
6	1	1		1			
7	1					1	😊

maze

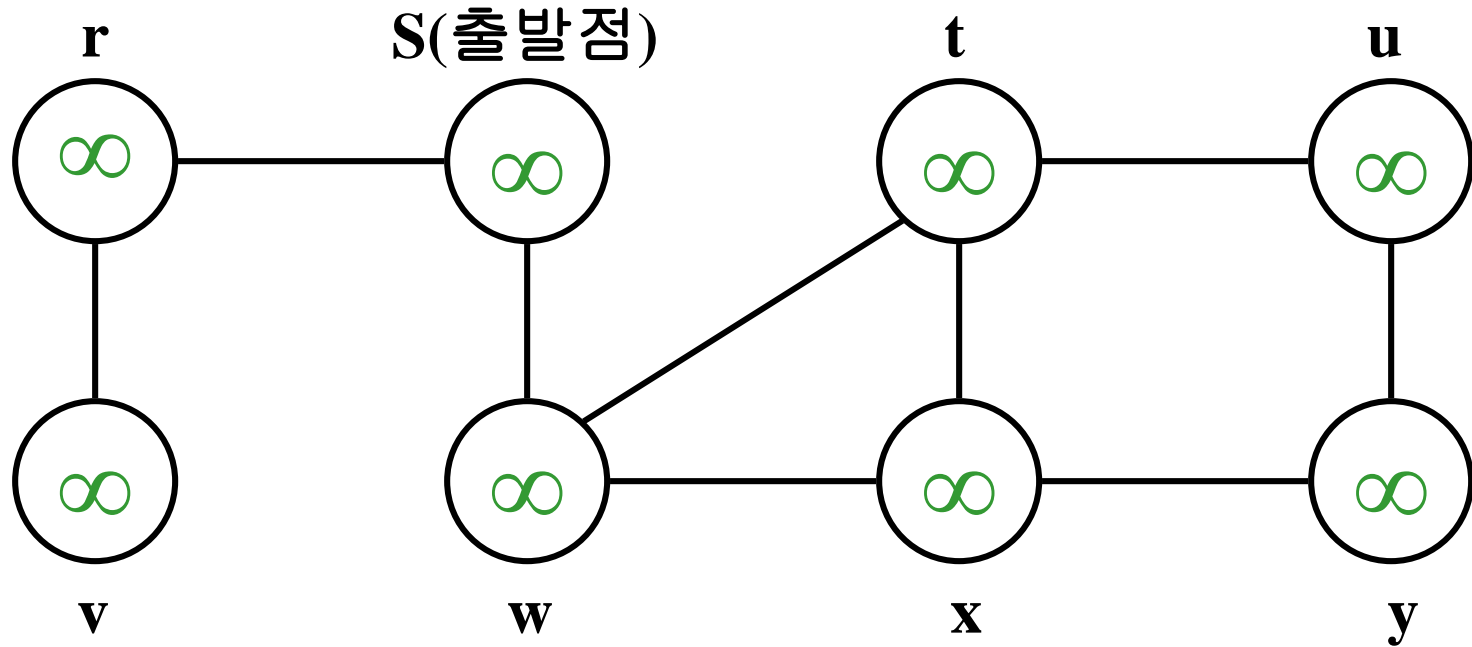
	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0

mark



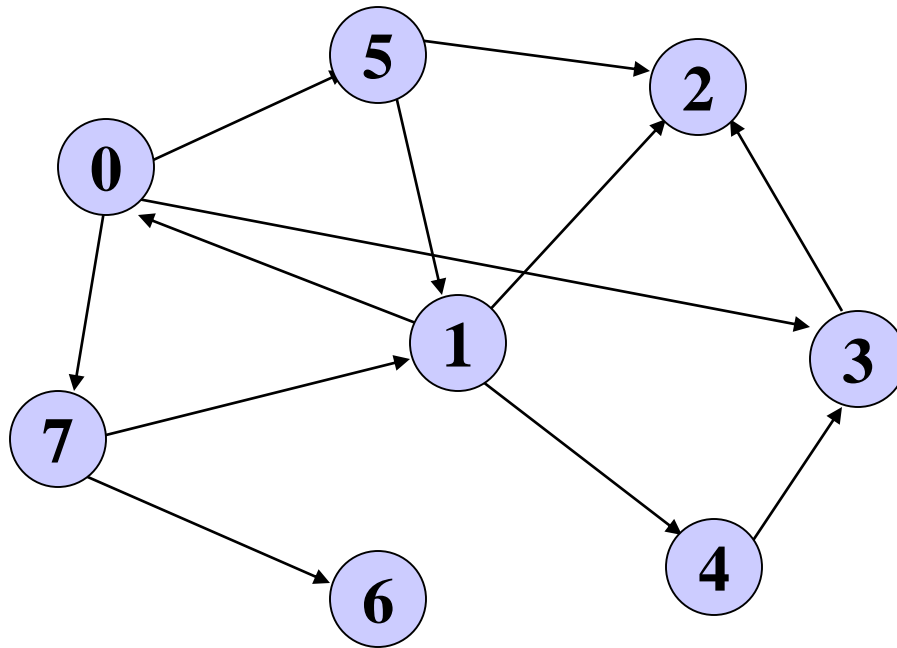
stack

# Depth-First Search: Example



## DFS: Example2

# Example

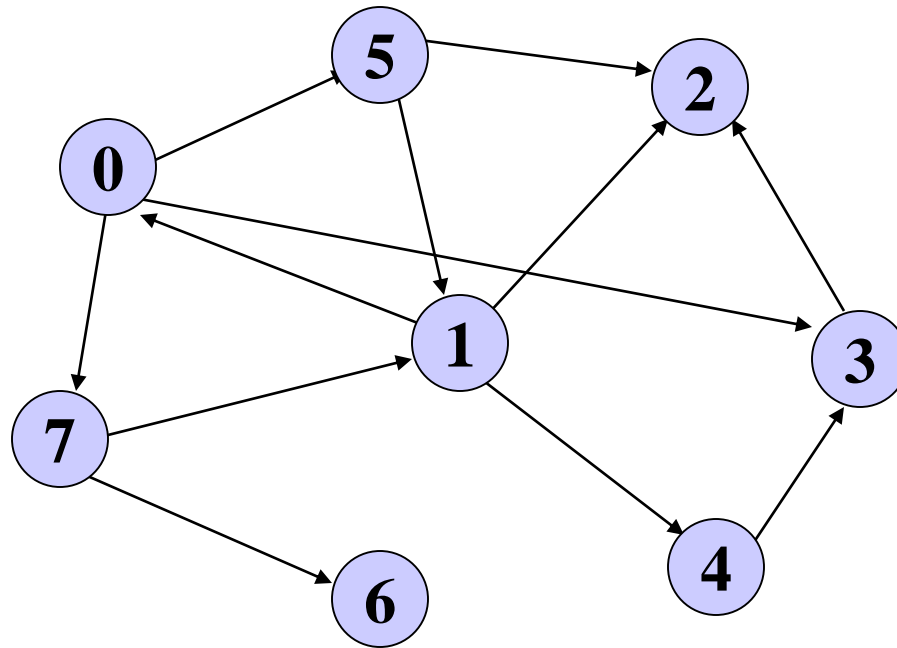


Policy: Visit adjacent nodes in increasing index order

이웃노드를 작은 순으로 방문

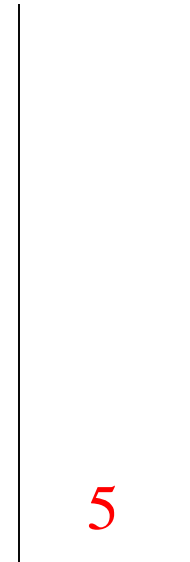
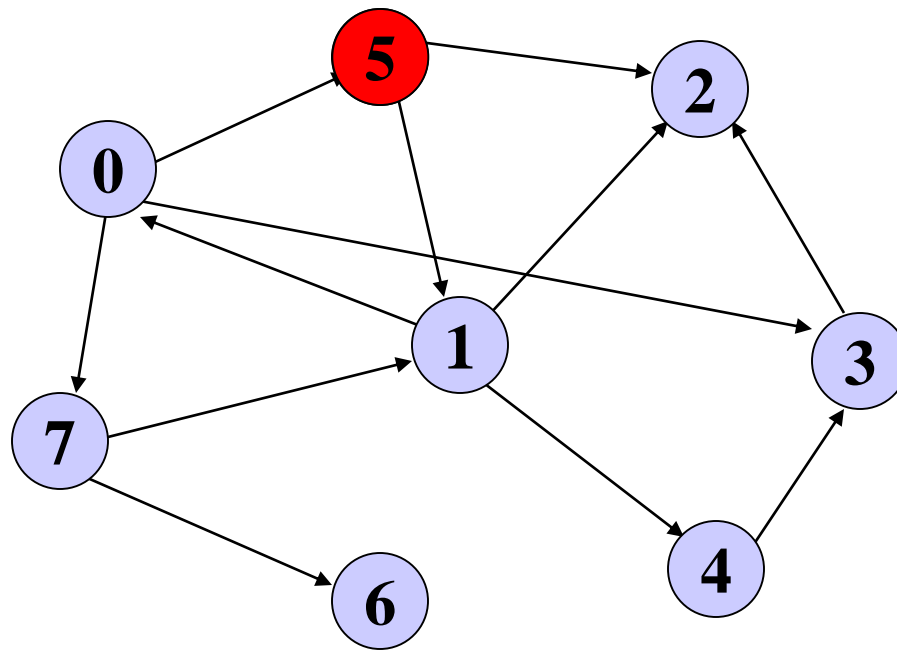
# Preorder DFS: Start with Node 5

노드 5 시작점



5 1 0 3 2 7 6 4

# Preorder DFS: Start with Node 5

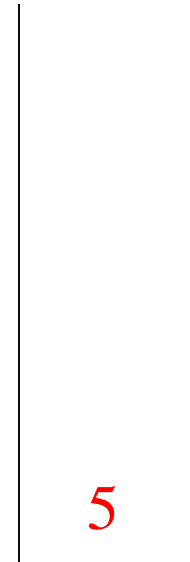
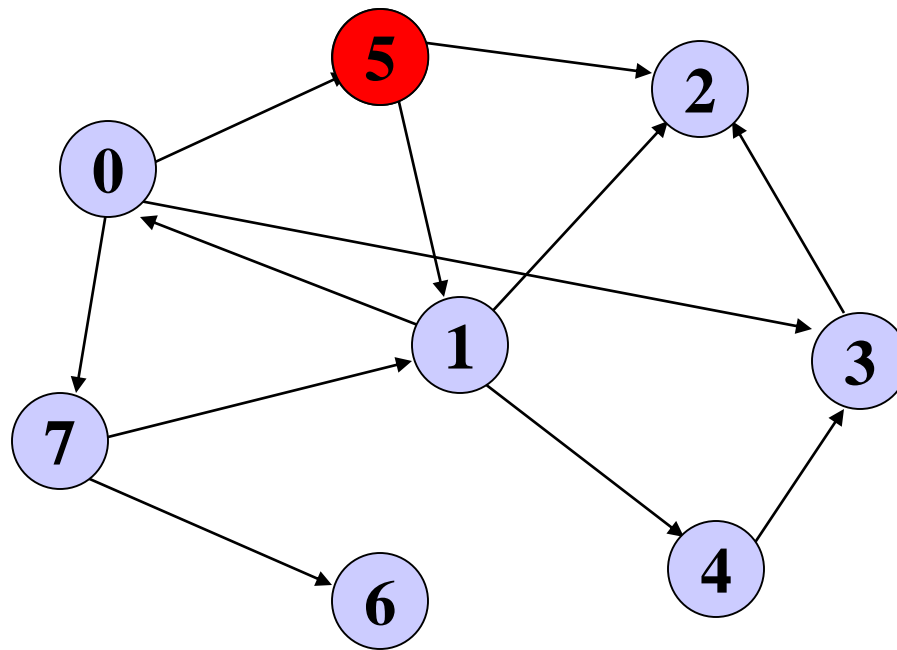


Pop/Visit/Mark 5

방문순서



# Preorder DFS: Start with Node 5

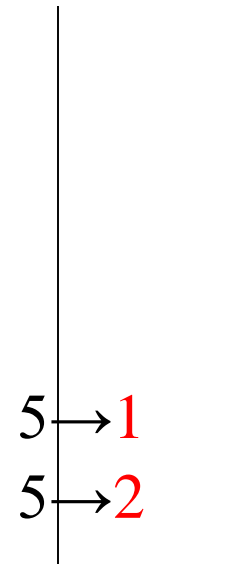
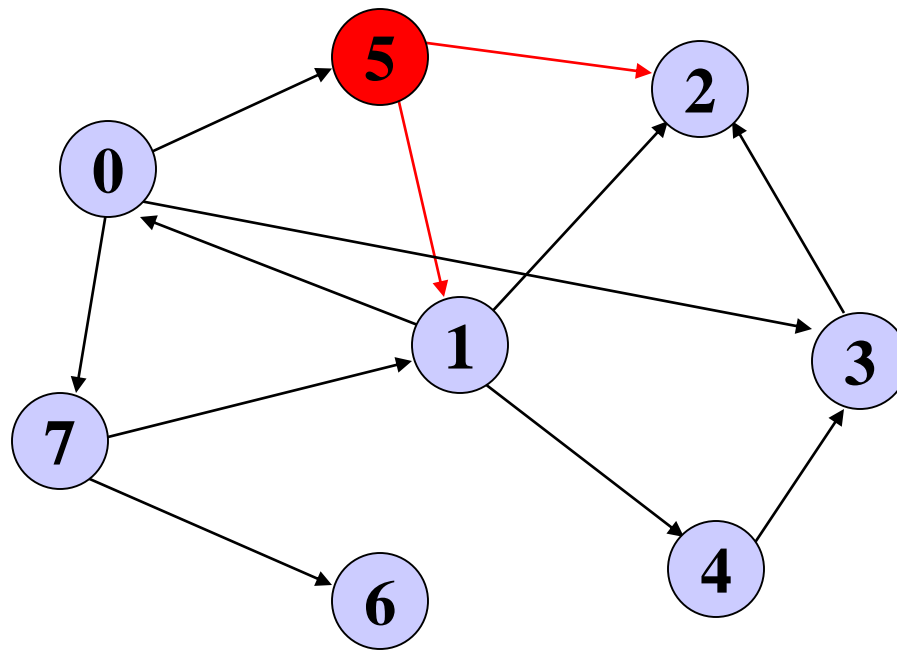


Pop/Visit/Mark 5

방문순서



# Preorder DFS: Start with Node 5

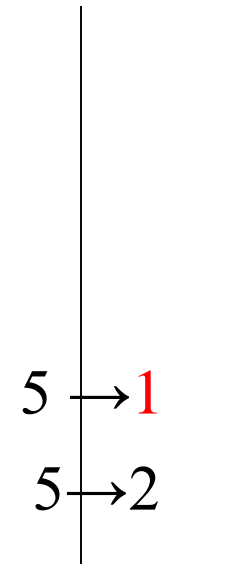
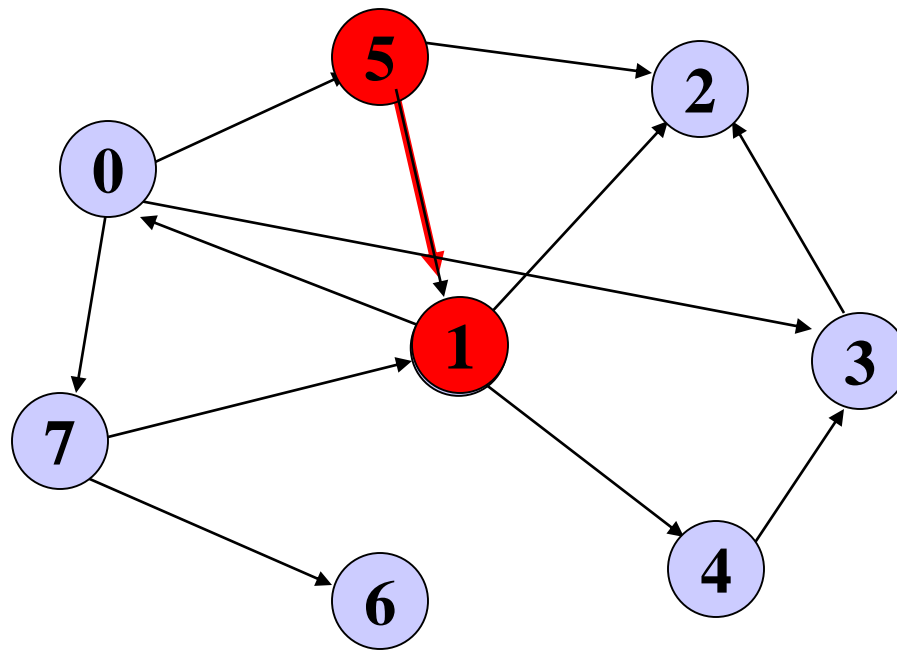


Push 2, Push 1

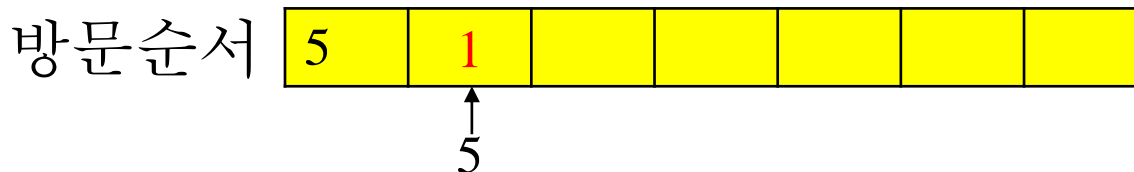




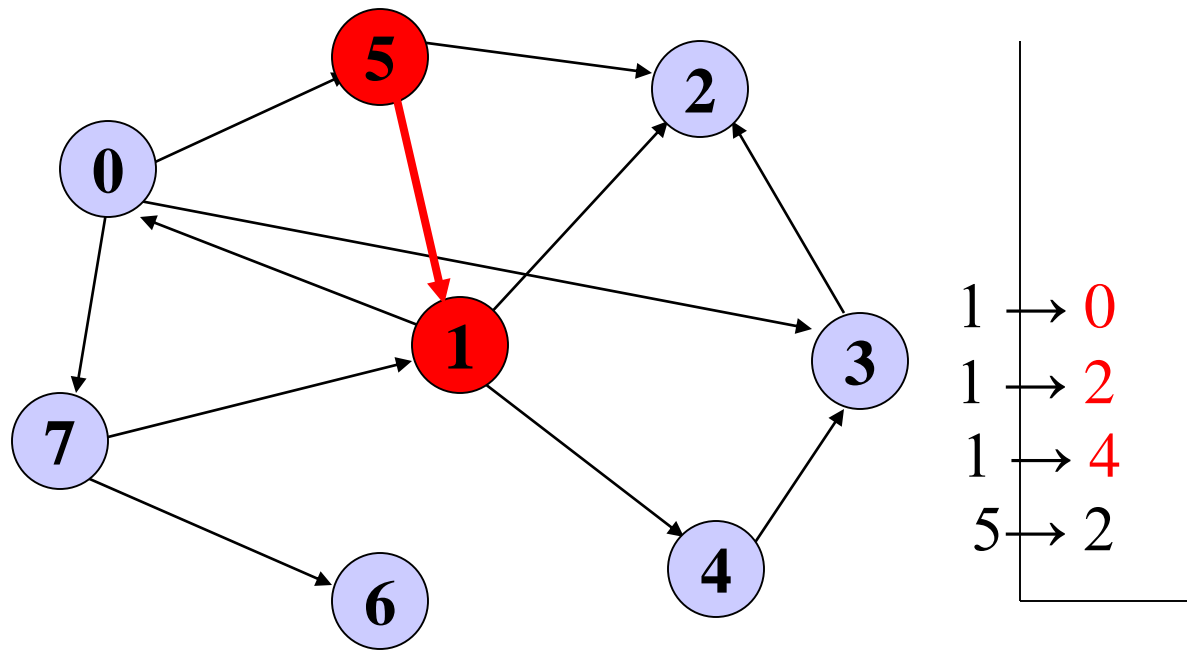
# Preorder DFS: Start with Node 5



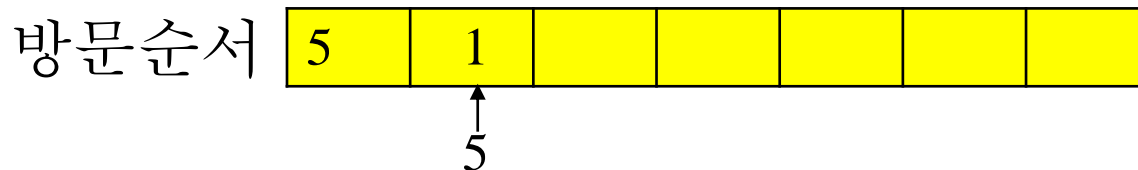
Pop/Visit/Mark 1



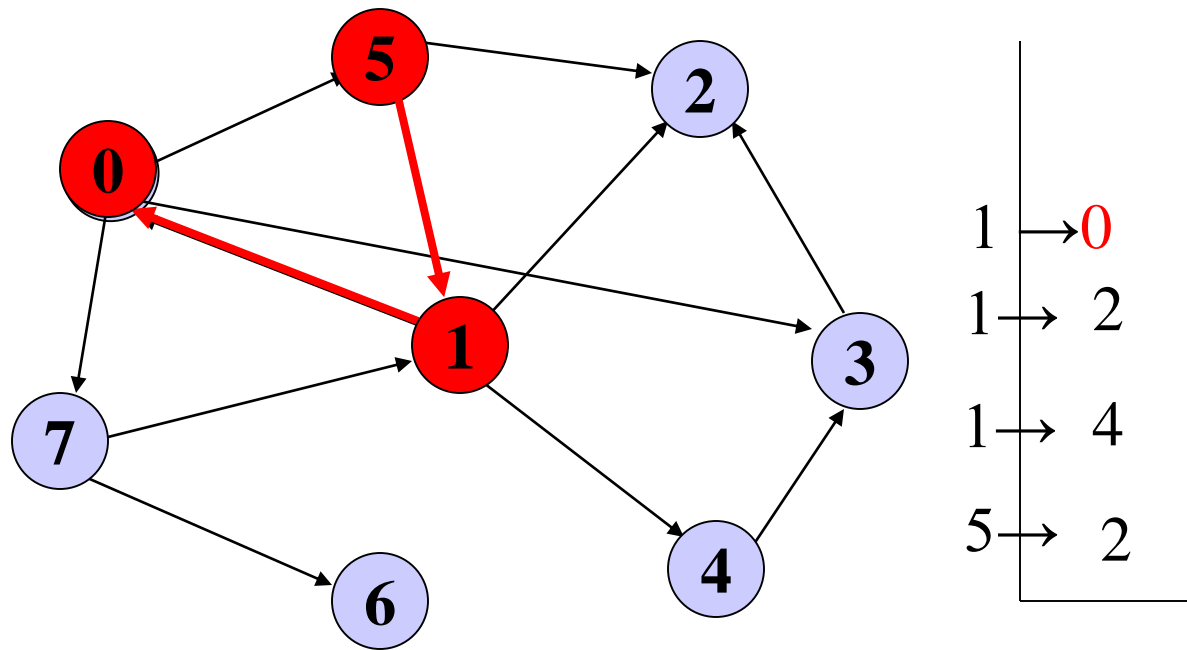
# Preorder DFS: Start with Node 5



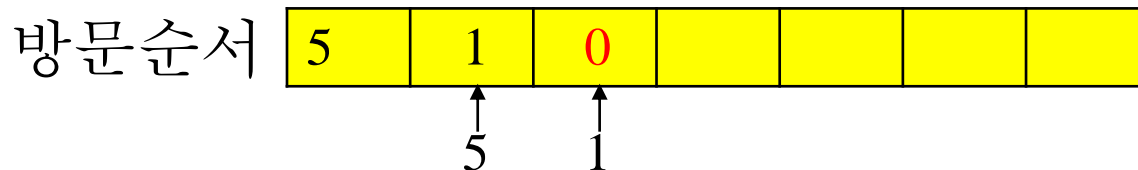
Push 4, Push 2,  
Push 0



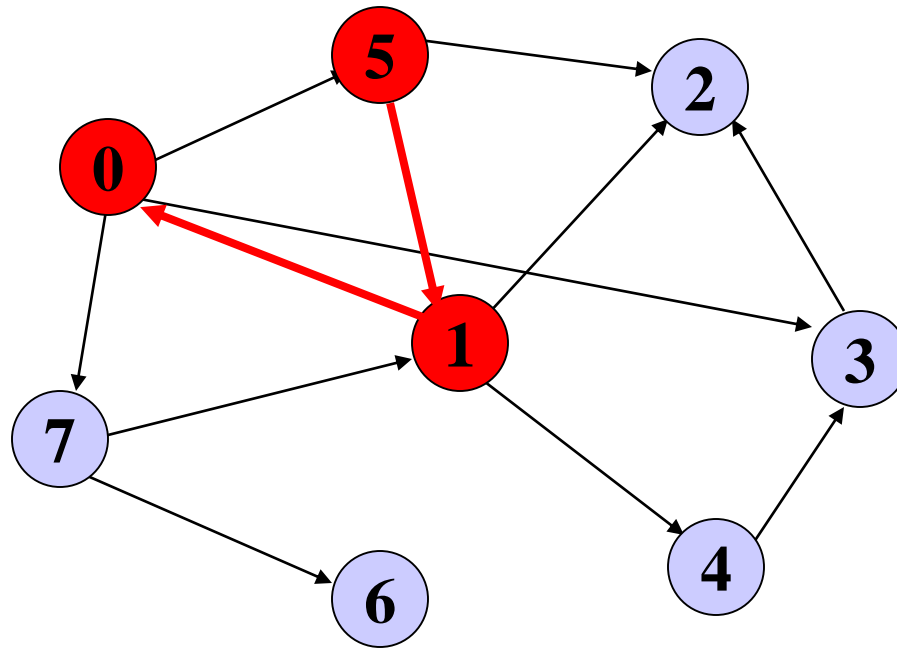
# Preorder DFS: Start with Node 5



Pop/Visit/Mark 0

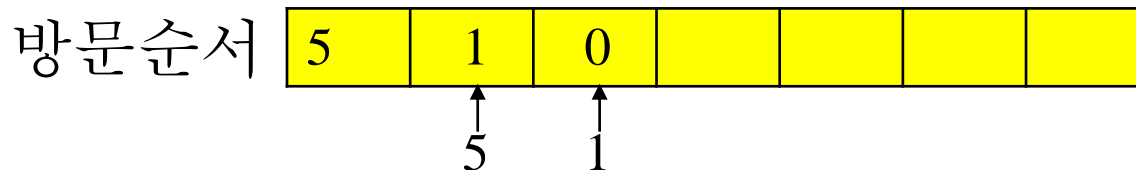


# Preorder DFS: Start with Node 5

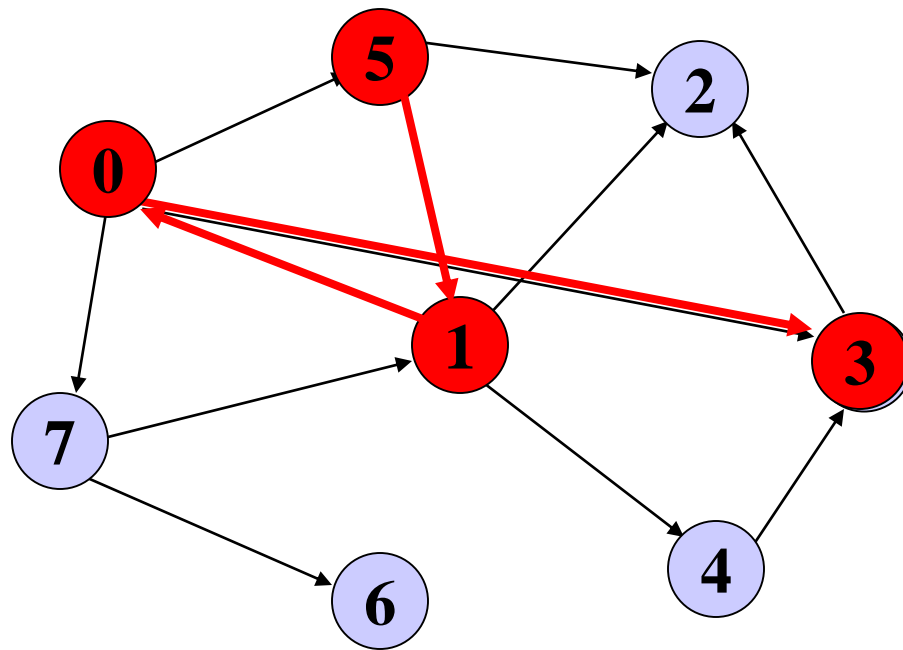


0	→	3
0	→	7
1	→	2
1	→	4
5	→	2

Push 7, Push 3



# Preorder DFS: Start with Node 5

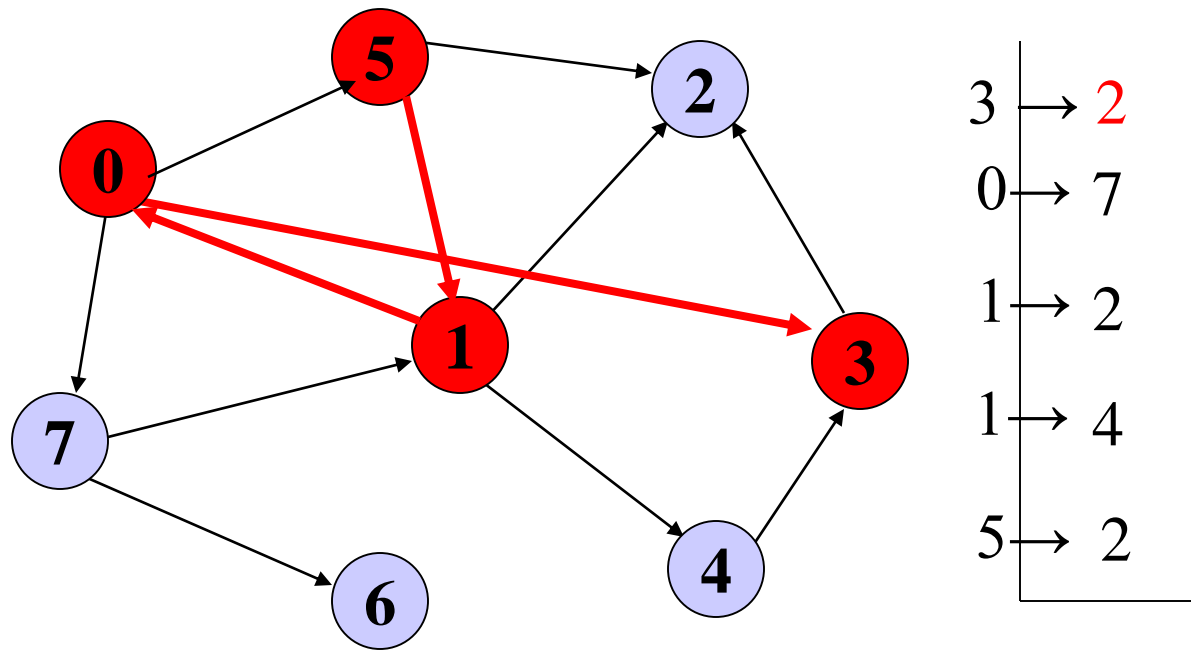


0	→	3
0	→	7
1	→	2
1	→	4
5	→	2

Pop/Visit/Mark 3

방문순서	5	1	0	3			
		↑ 5	↑ 1	↑ 0			

# Preorder DFS: Start with Node 5

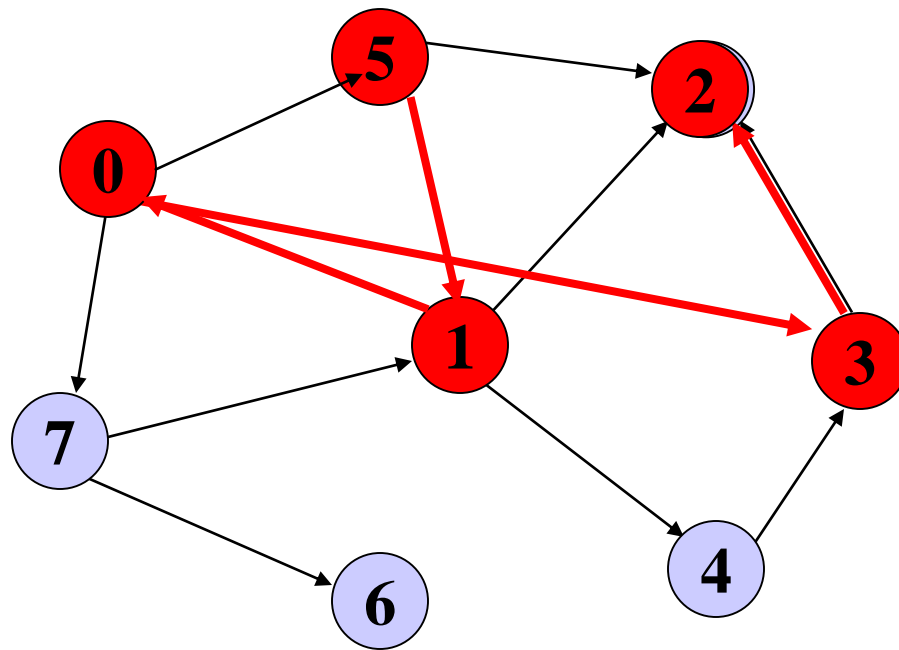


방문순서

5	1	0	3			
---	---	---	---	--	--	--

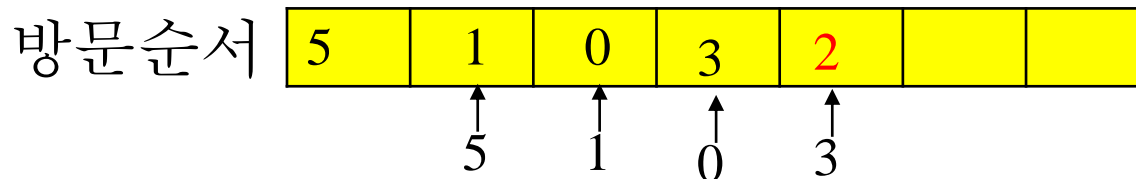
5      1      0

# Preorder DFS: Start with Node 5



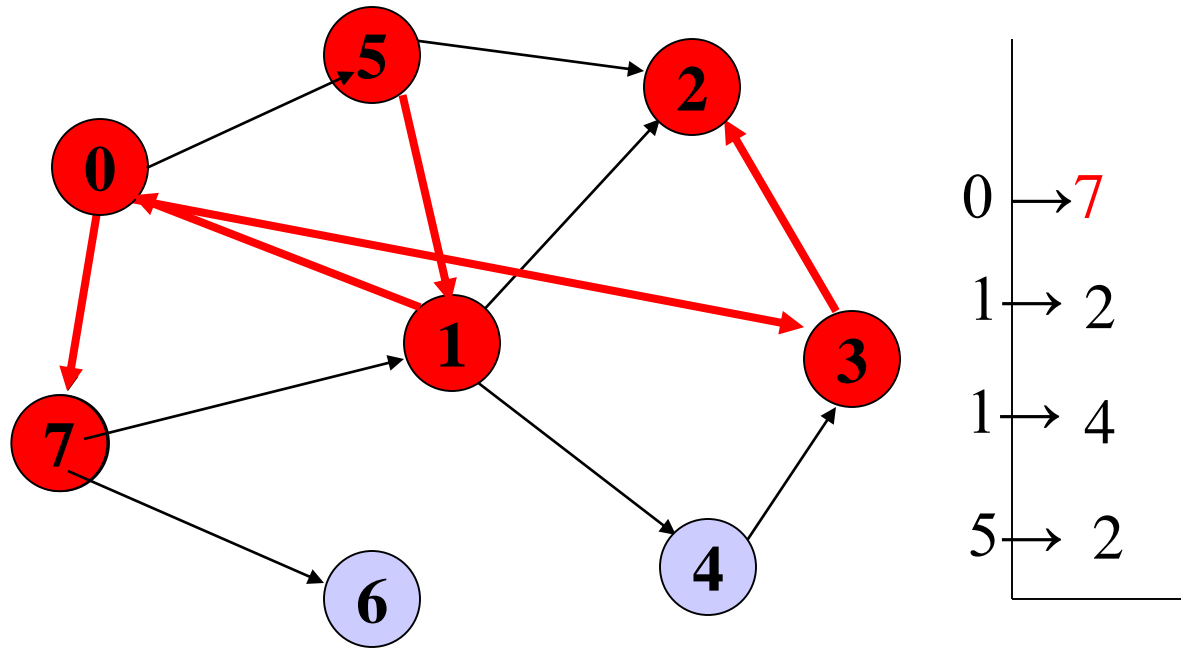
3	→	2
0	→	7
1	→	2
1	→	4
5	→	2

Pop/Mark/Visit 2

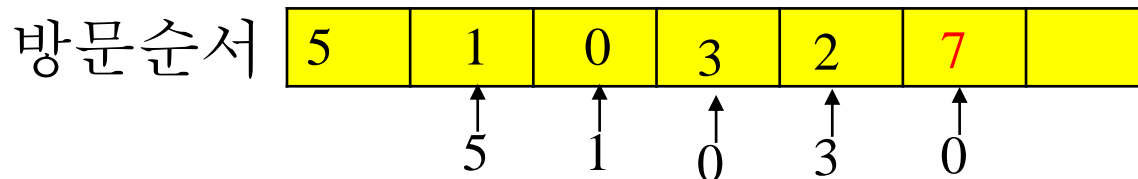


Push할  
이웃노드없음

# Preorder DFS: Start with Node 5

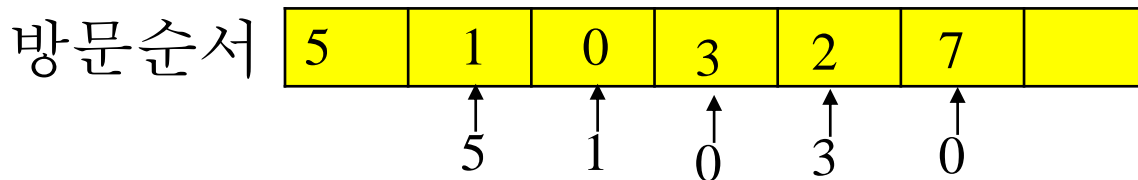
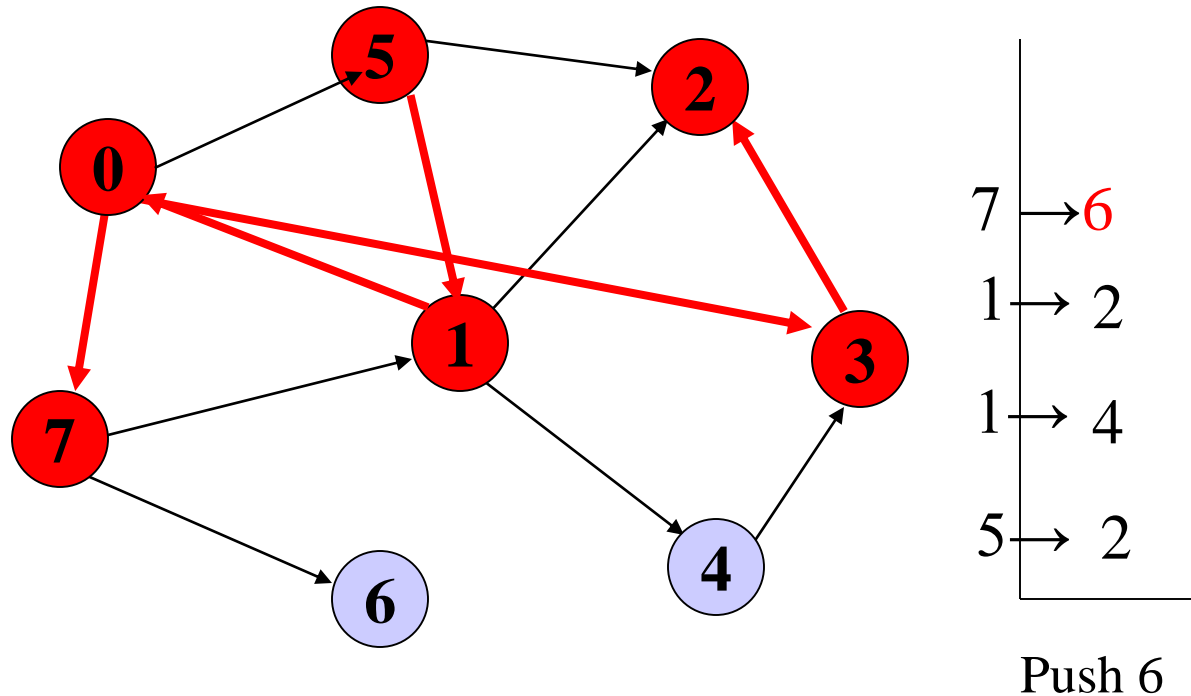


Pop/Mark/Visit 7

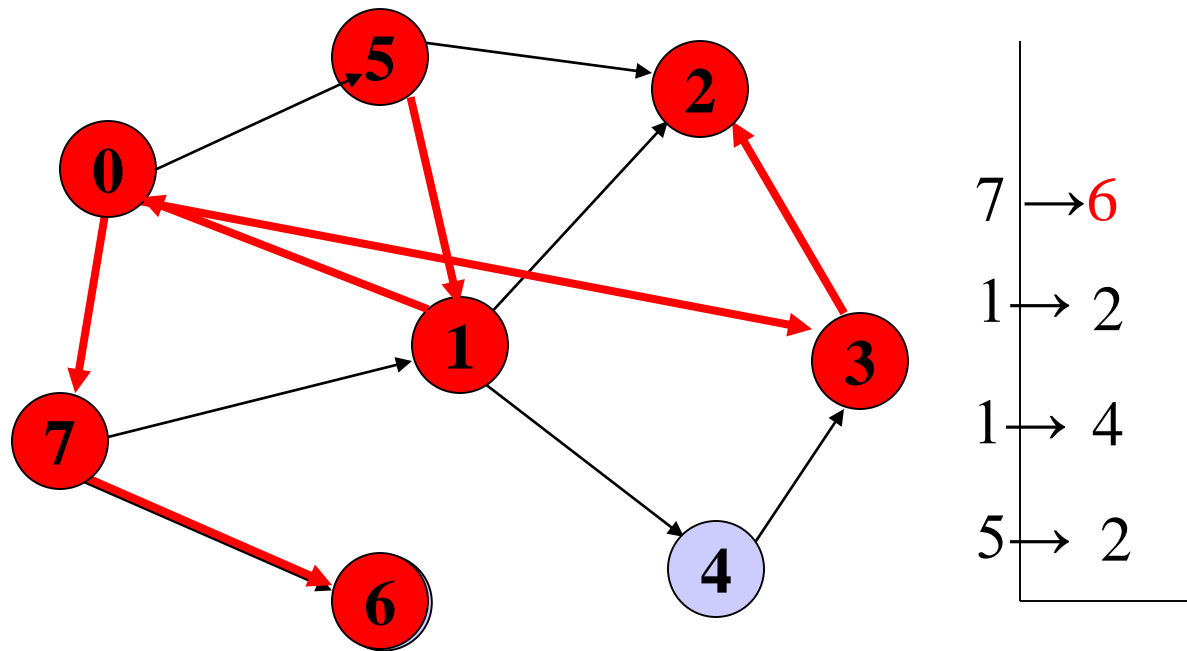




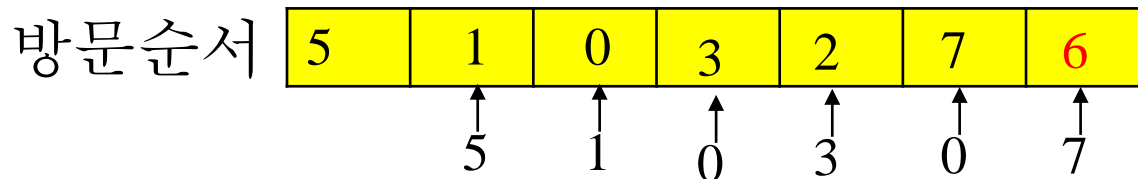
# Preorder DFS: Start with Node 5



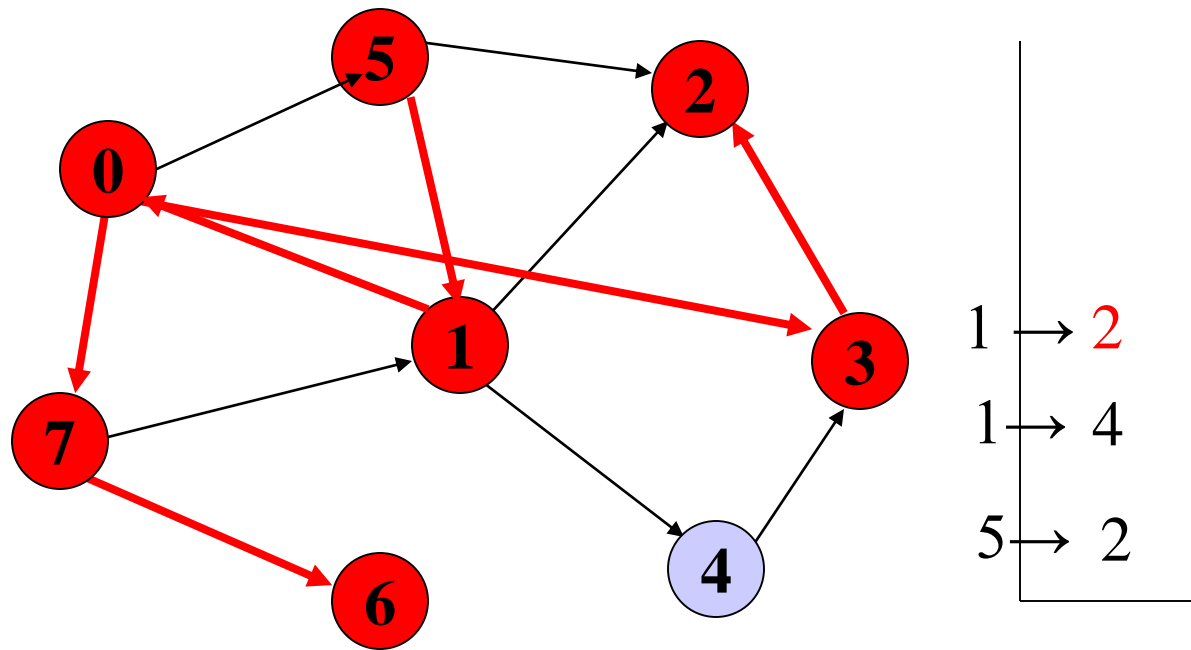
# Preorder DFS: Start with Node 5



Pop/Mark/Visit 6



# Preorder DFS: Start with Node 5

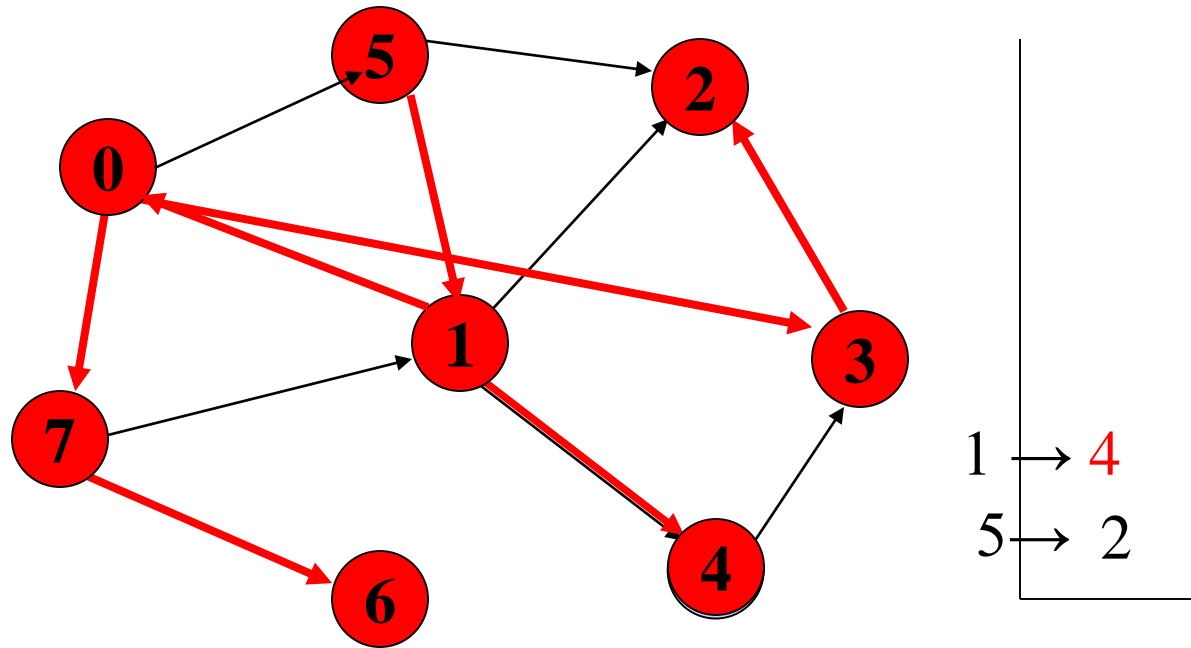


Pop (don't visit) 2

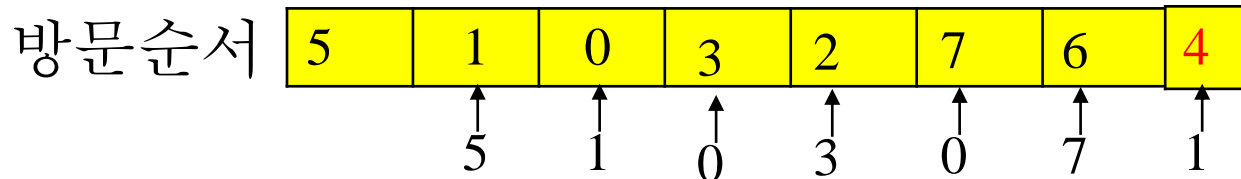
방문순서

5	1	0	3	2	7	6
	↑ 5	↑ 1	↑ 0	↑ 3	↑ 0	↑ 7

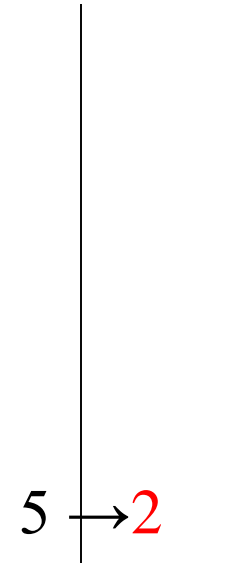
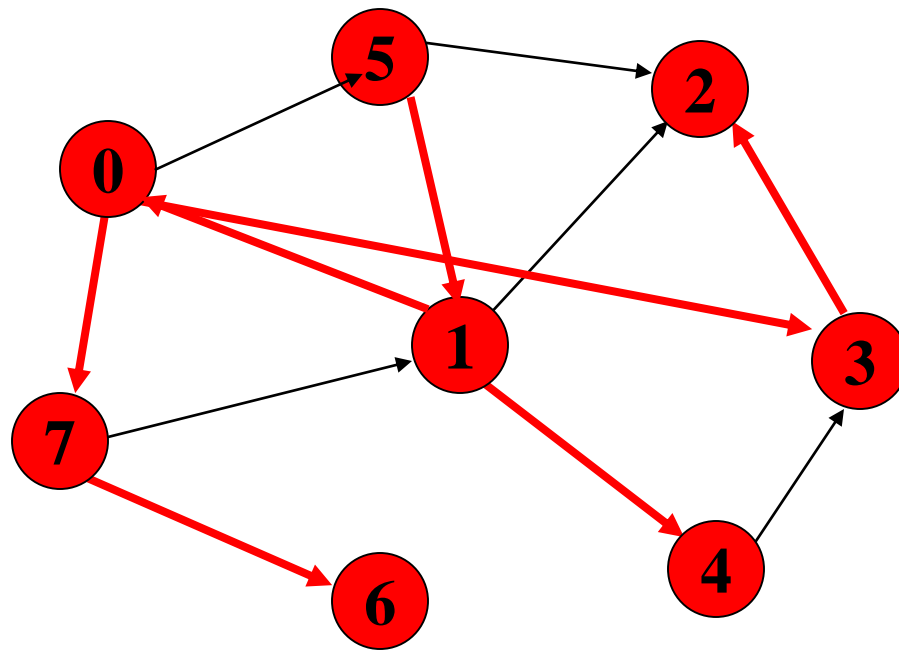
# Preorder DFS: Start with Node 5



Pop/Mark/Visit 4



# Preorder DFS: Start with Node 5

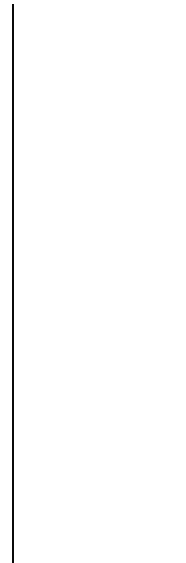
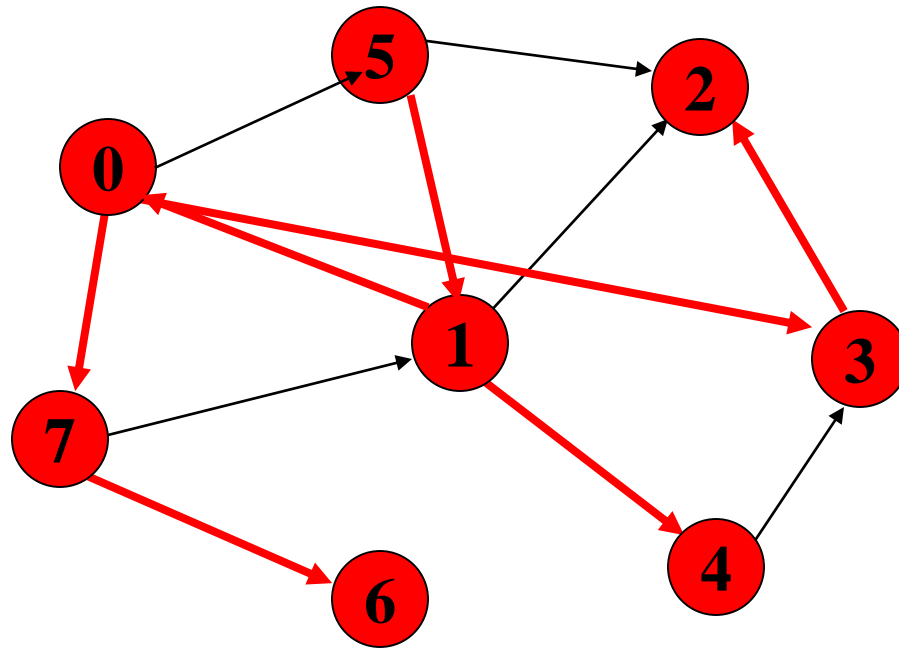


Pop (don't visit) 2

방문순서

5	1	0	3	2	7	6	4
	↑ 5	↑ 1	↑ 0	↑ 3	↑ 0	↑ 7	↑ 1

# Preorder DFS: Start with Node 5

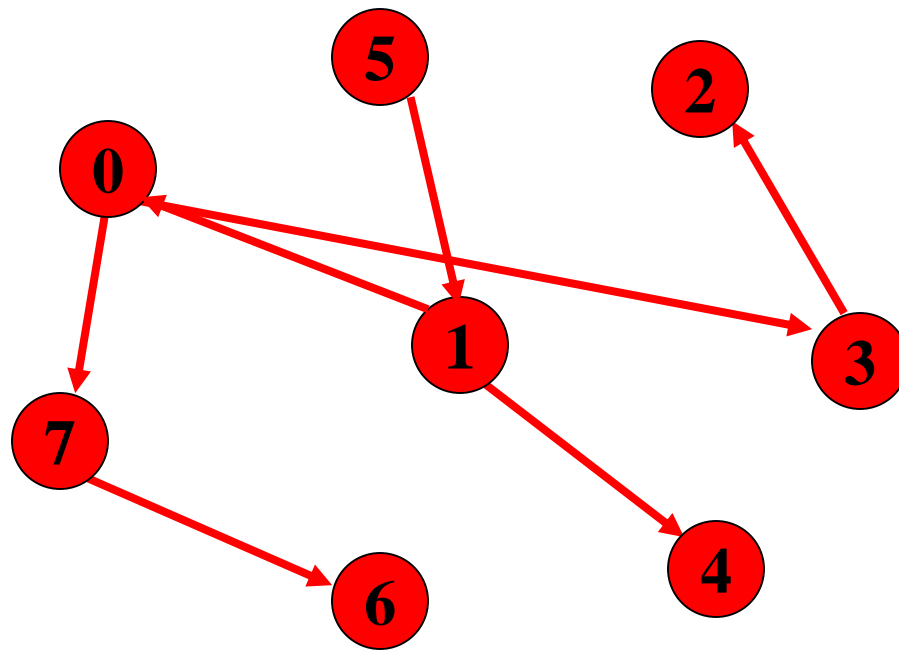


**Done**

5	1	0	3	2	7	6	4
	↑	↑	↑	↑	↑	↑	↑
	5	1	0	3	0	7	1

# Spanning Tree(간선트리)

DFS방식으로 그래프의 노드를 방문했을때 얻게되는  
트리를 간선트리라한다



5 1 0 3 2 7 6 4

5 1 0 3 2 7 6 4  
↑ ↑ ↑ ↑ ↑ ↑ ↑  
5 1 0 3 0 7 1

**BFS방식으로 노드순회를 마친 결과**

# Breadth-first Search

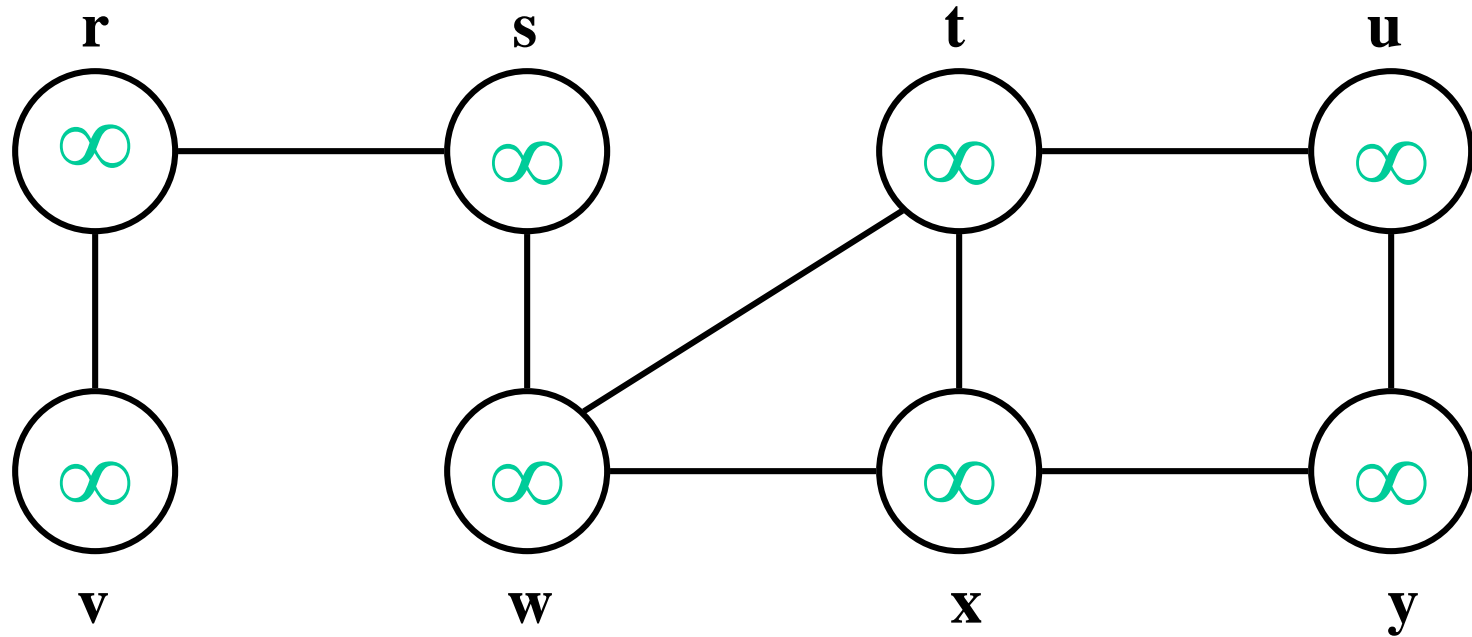
너비 우선 탐색

- Ripples in a pond 연못의 동심원을 생각.
- Visit designated node
- Then visited unvisited nodes a distance  $i$  away, where  $i = 1, 2, 3$ , etc.
- For nodes the same distance away, visit nodes in systematic manner (eg. increasing index order) 출발점에서 같은 거리만큼 떨어진 정점들을 모두 방문

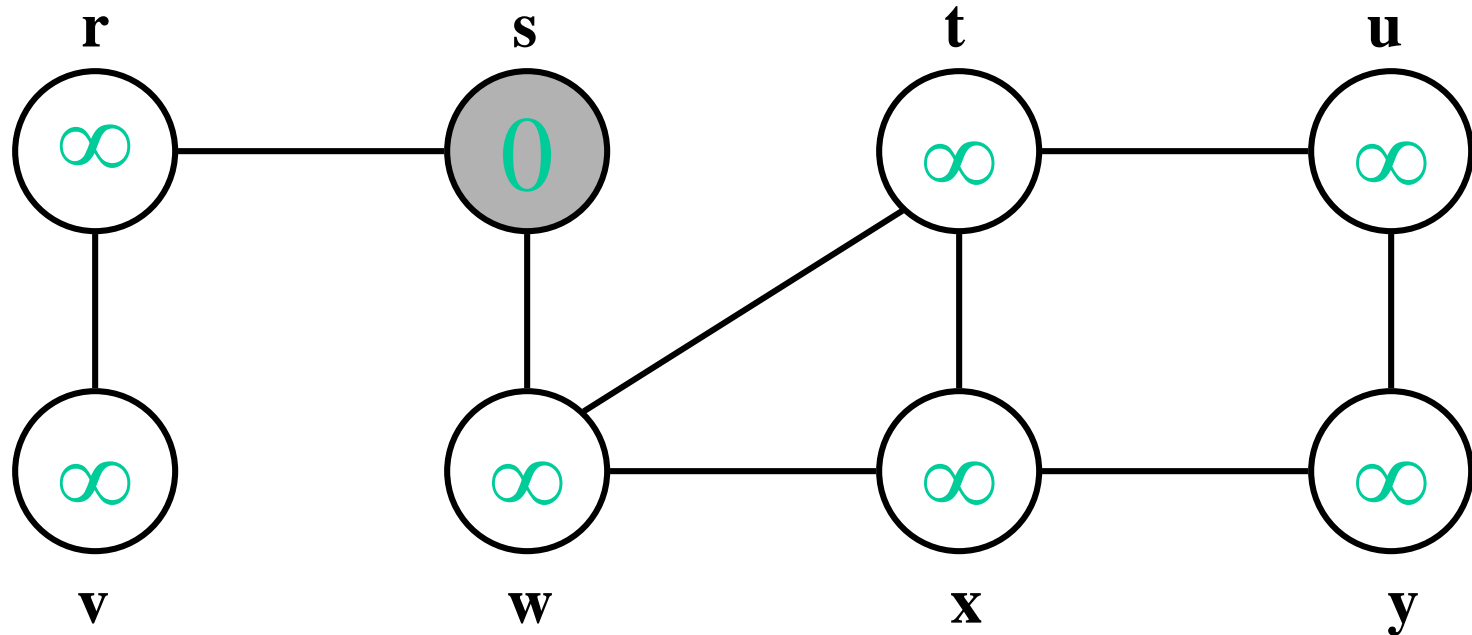


# BFS(너비우선탐색): Example1

# Breadth-First Search: Example

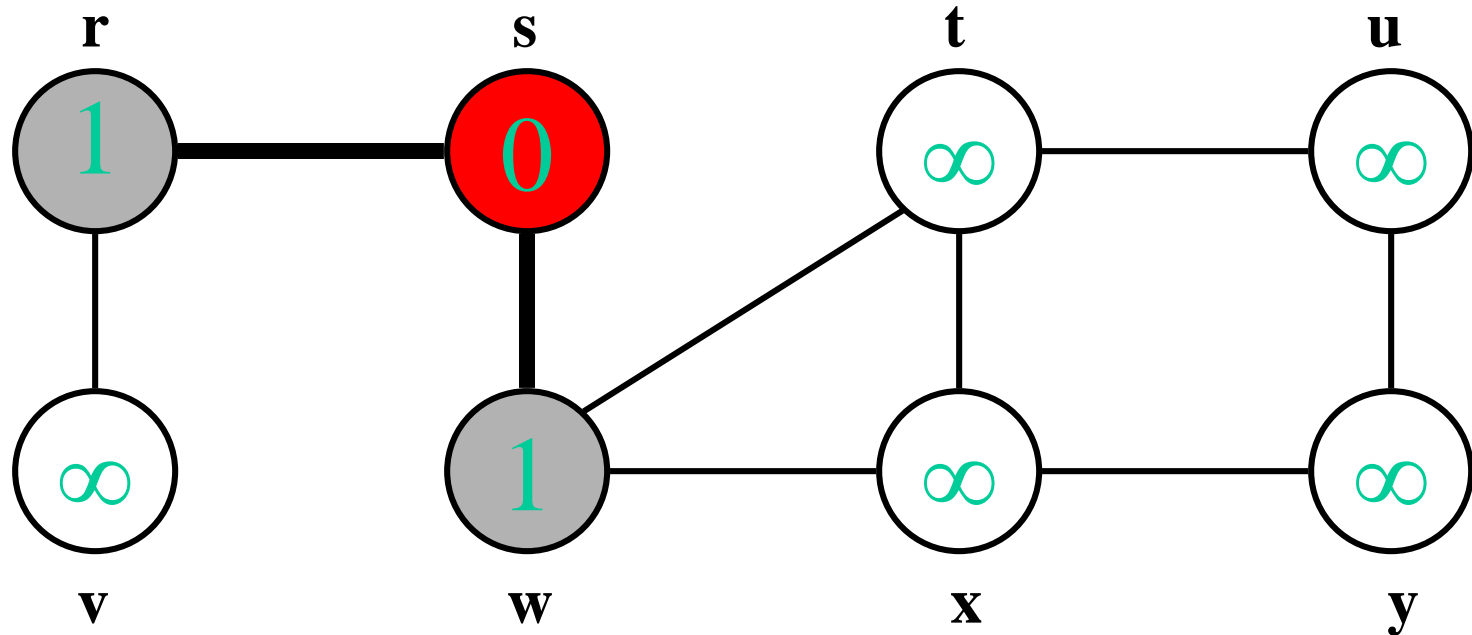


# Breadth-First Search: Example



**Q:** s

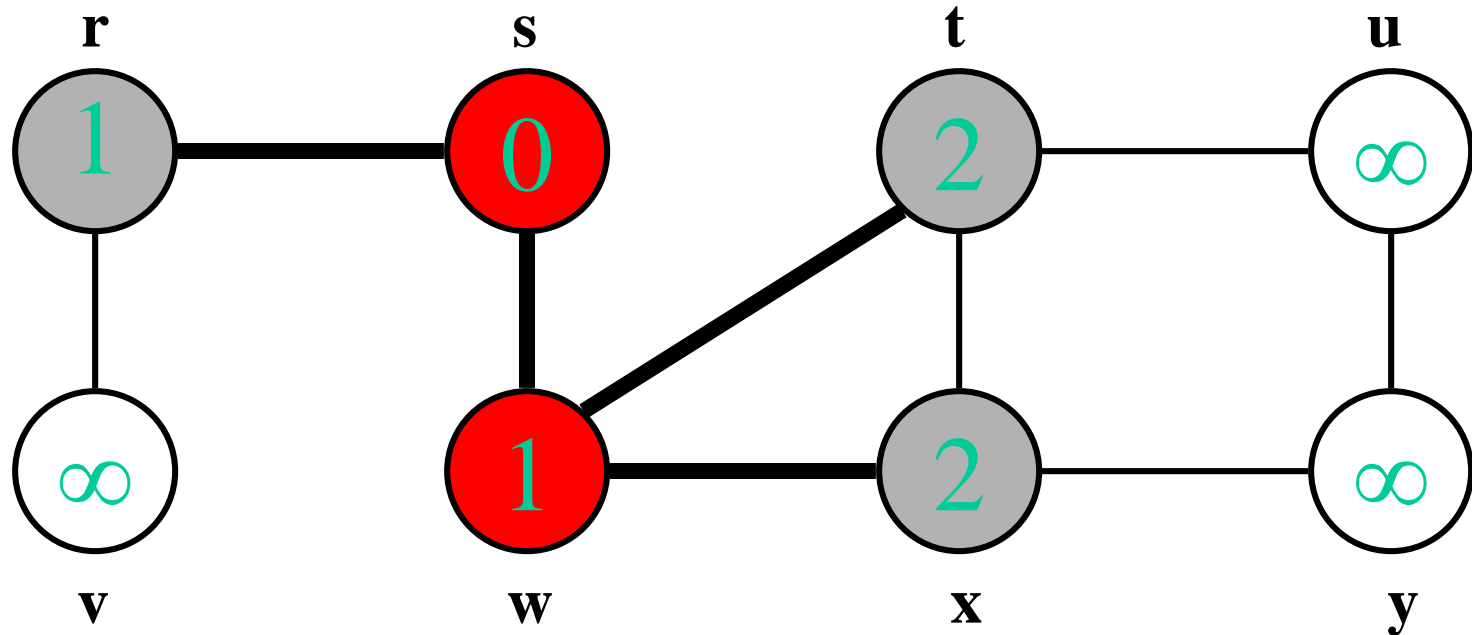
# Breadth-First Search: Example



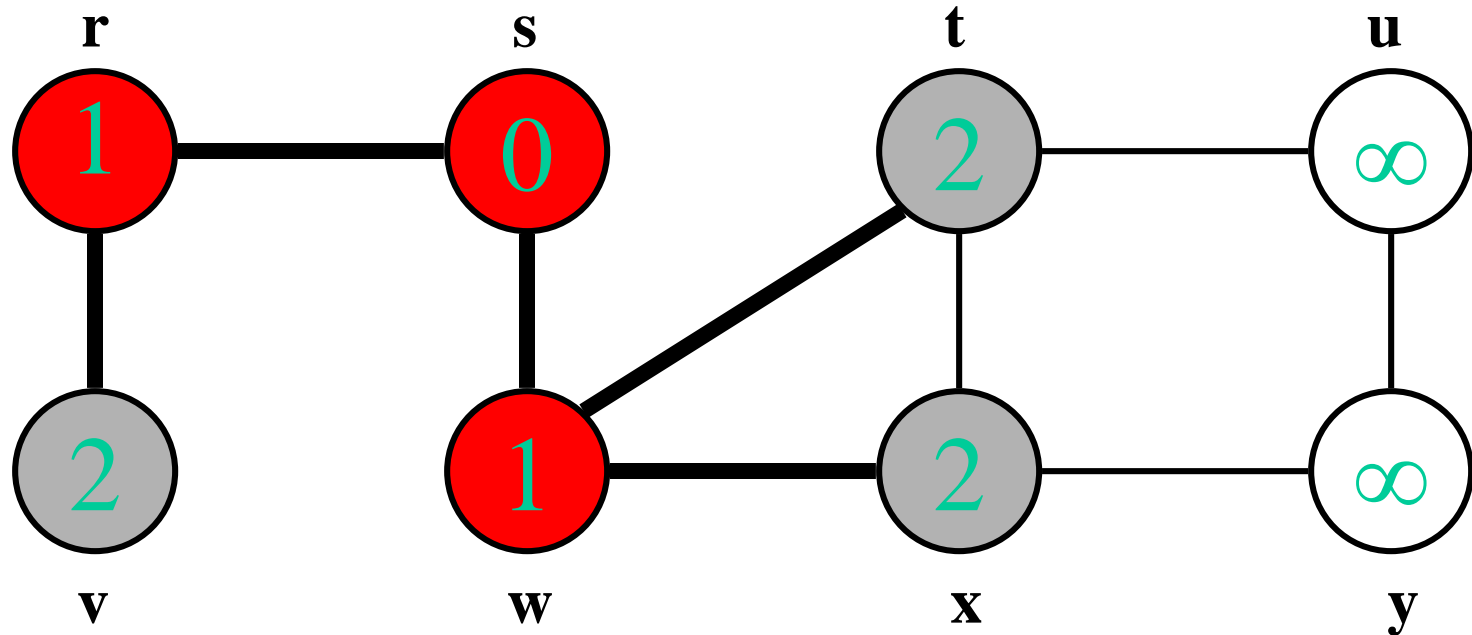
**Q:**

<b>w</b>	<b>r</b>
----------	----------

# Breadth-First Search: Example



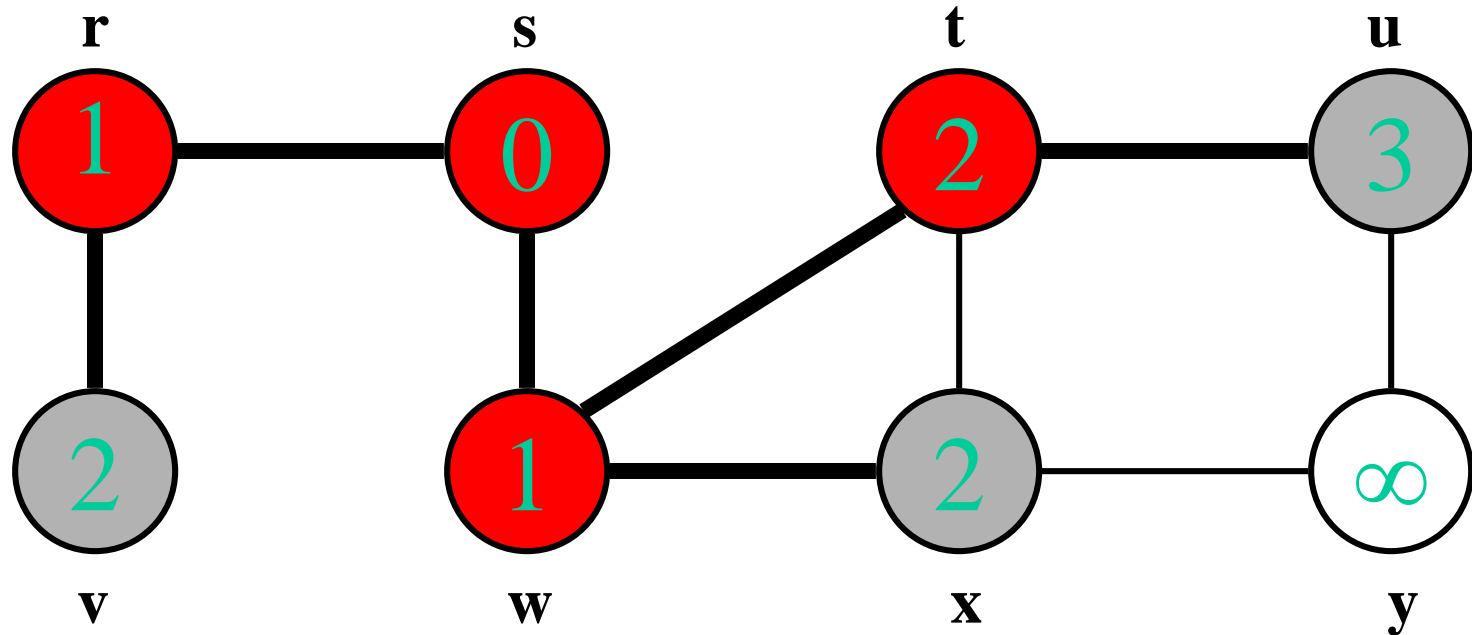
# Breadth-First Search: Example



Q: 

t	x	v
---	---	---

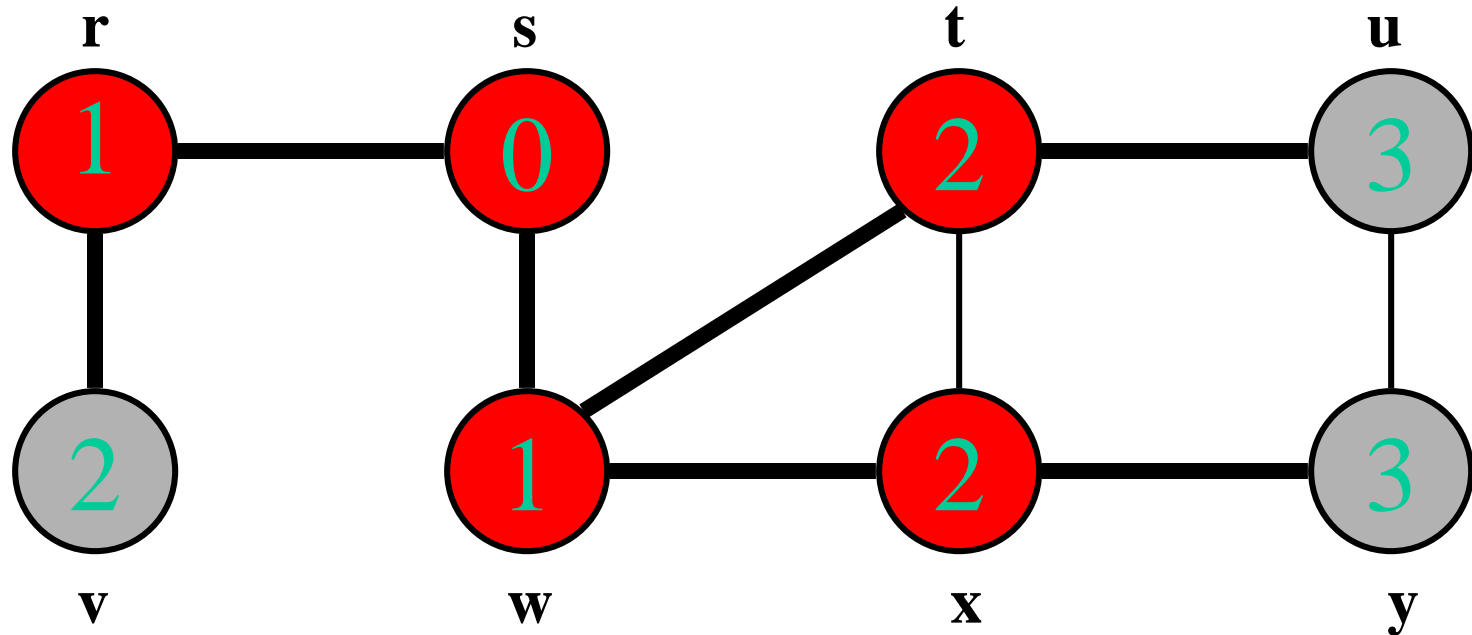
# Breadth-First Search: Example



Q: 

x	v	u
---	---	---

# Breadth-First Search: Example

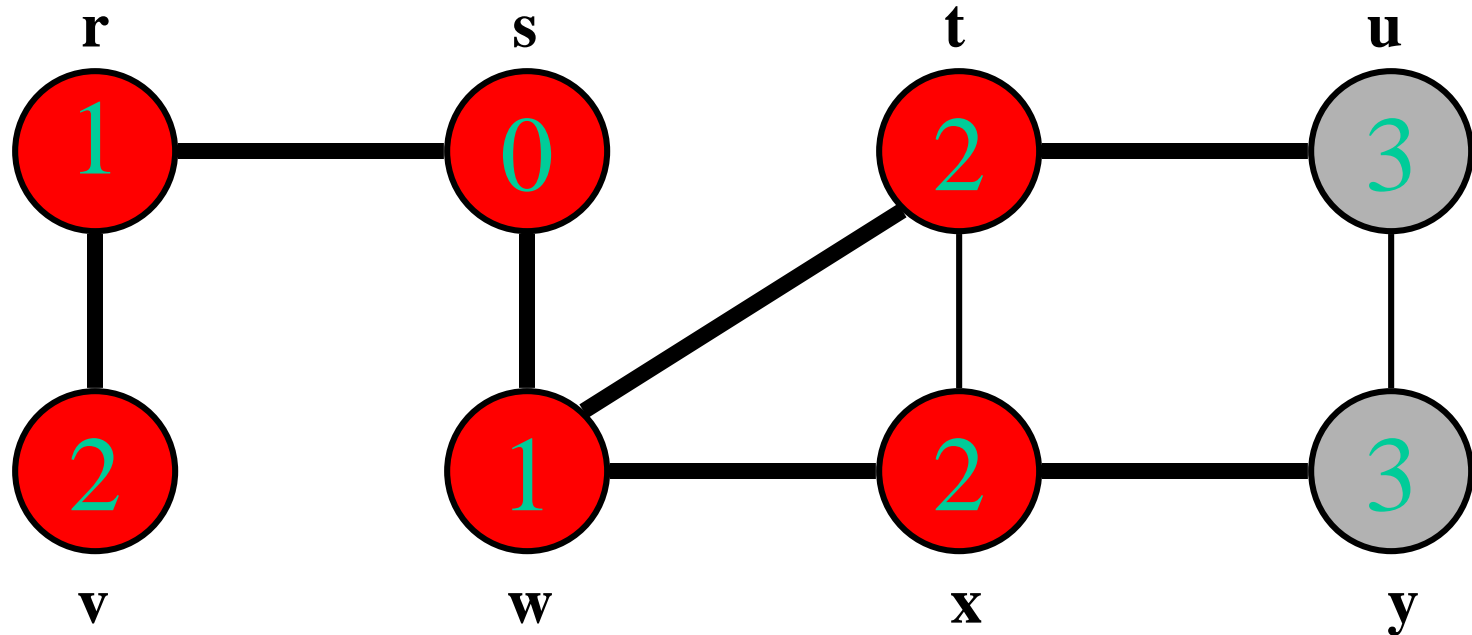


Q: 

v	u	y
---	---	---



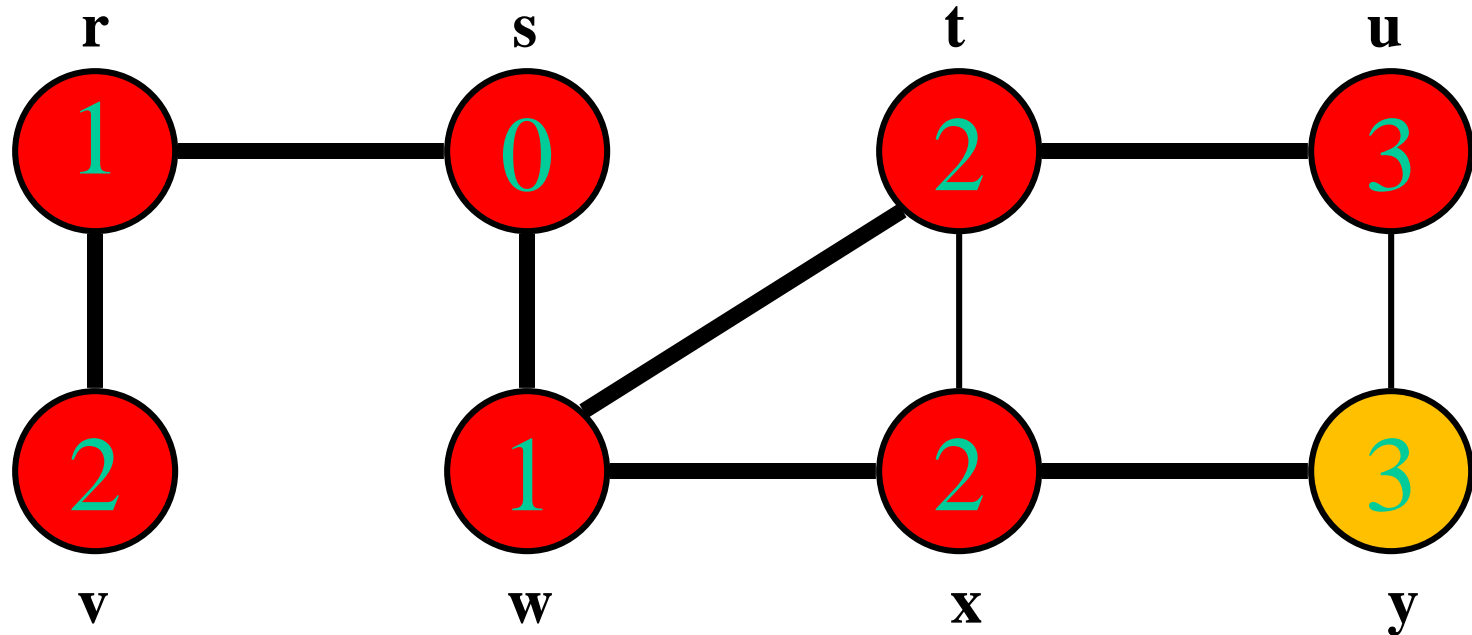
# Breadth-First Search: Example



Q: 

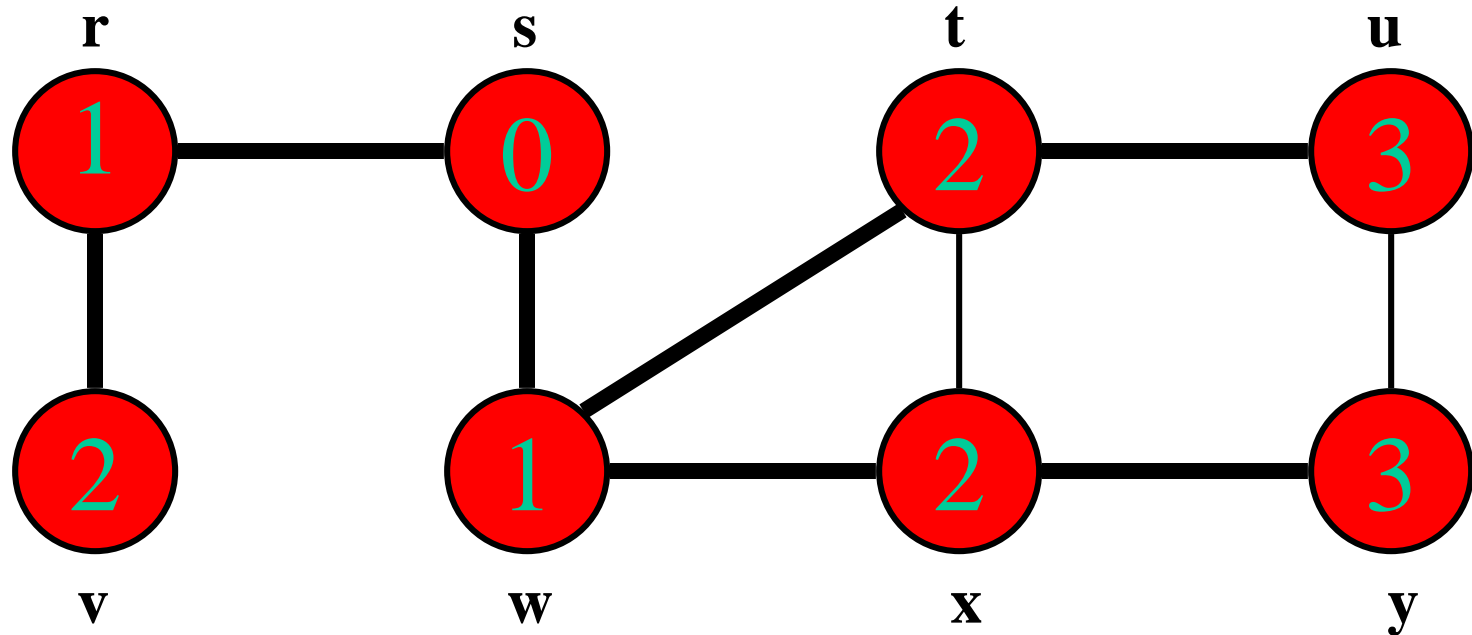
u	y
---	---

# Breadth-First Search: Example



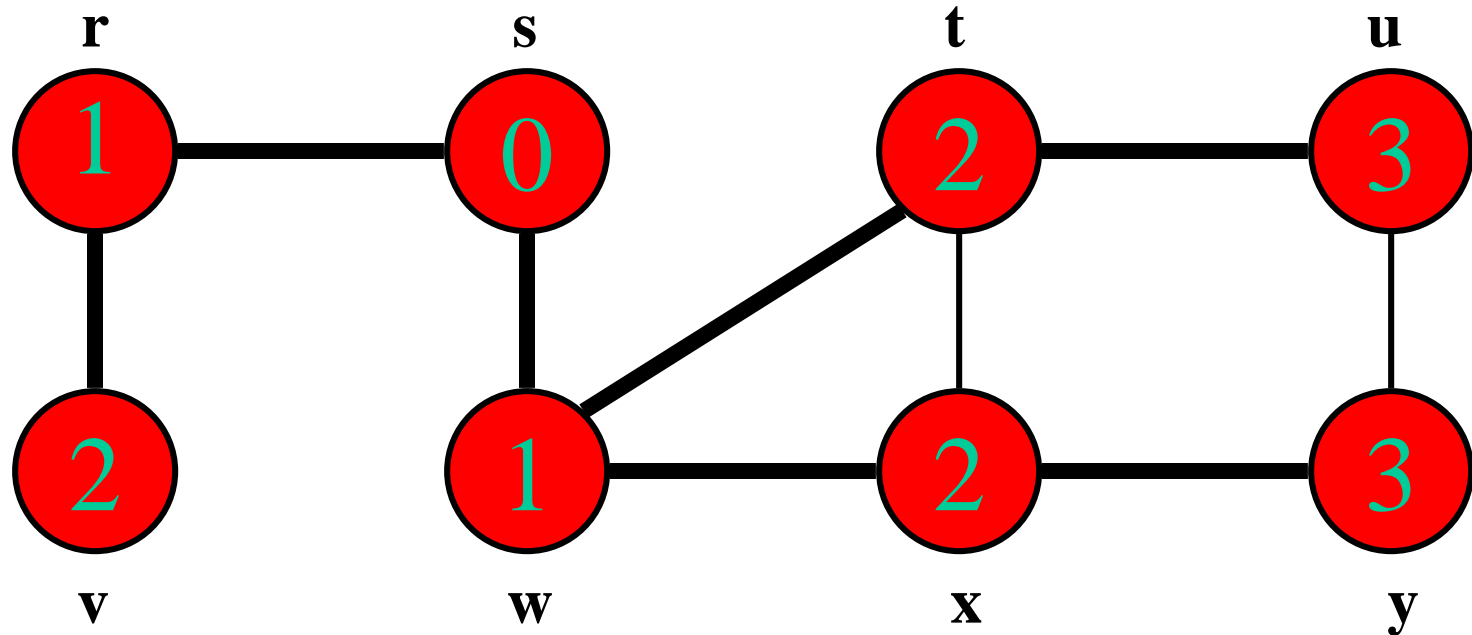
Q: y

# Breadth-First Search: Example



**Q:**  $\emptyset$

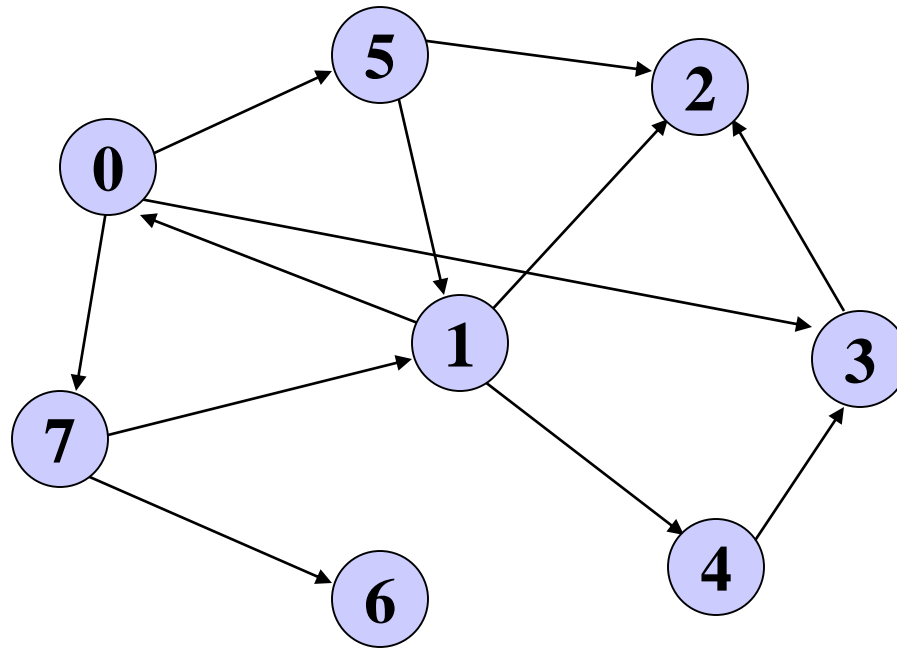
# Breadth-First Search: Example



**Q:**  $\emptyset$

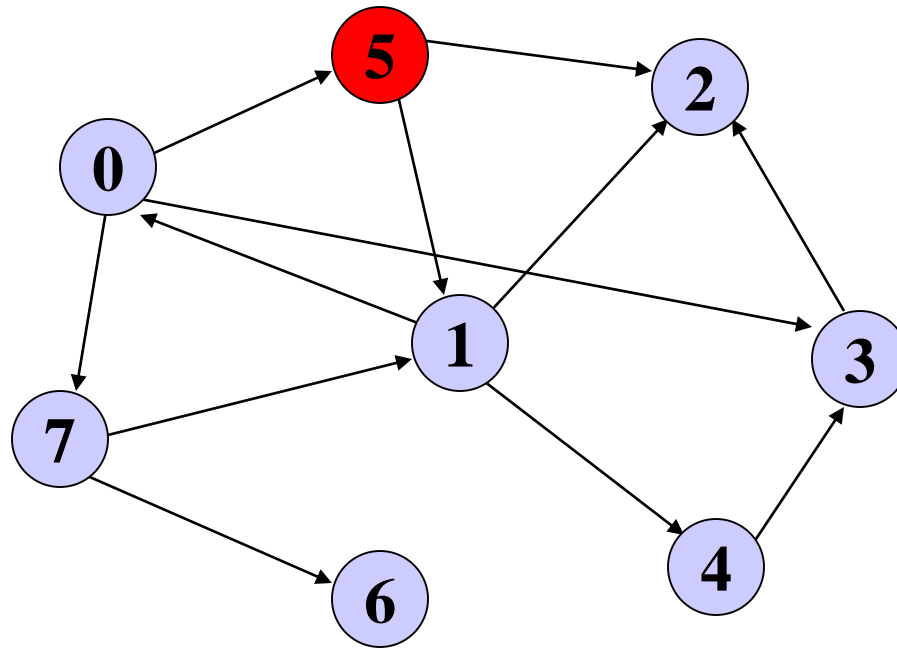
## BFS(너비우선탐색): Example2

# BFS: Start with Node 5



5 1 2 0 4 3 7 6

# BFS: Start with Node 5

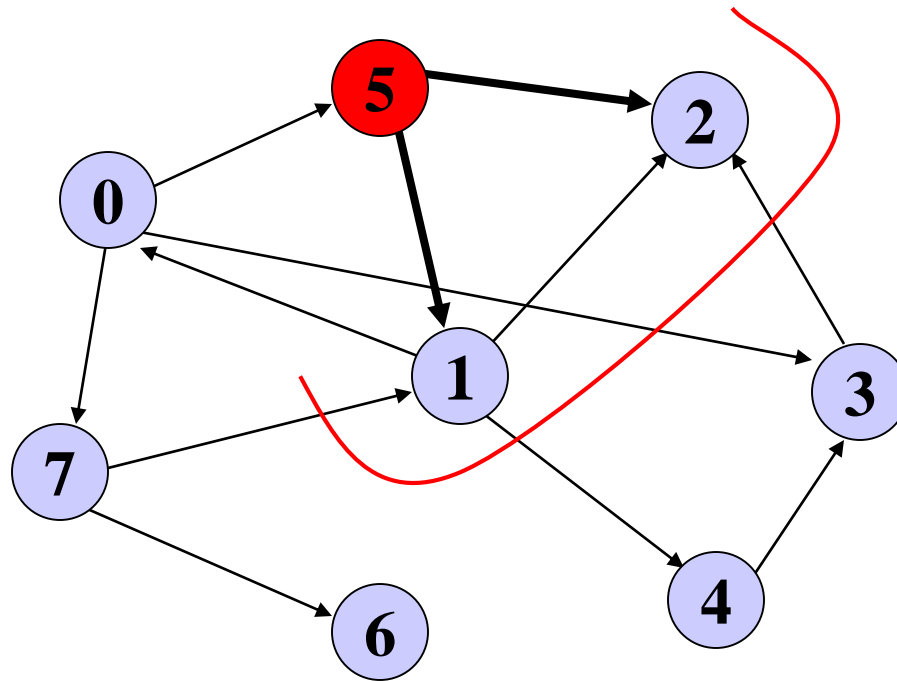


방문순서

5

Queue

# BFS: Node one-away



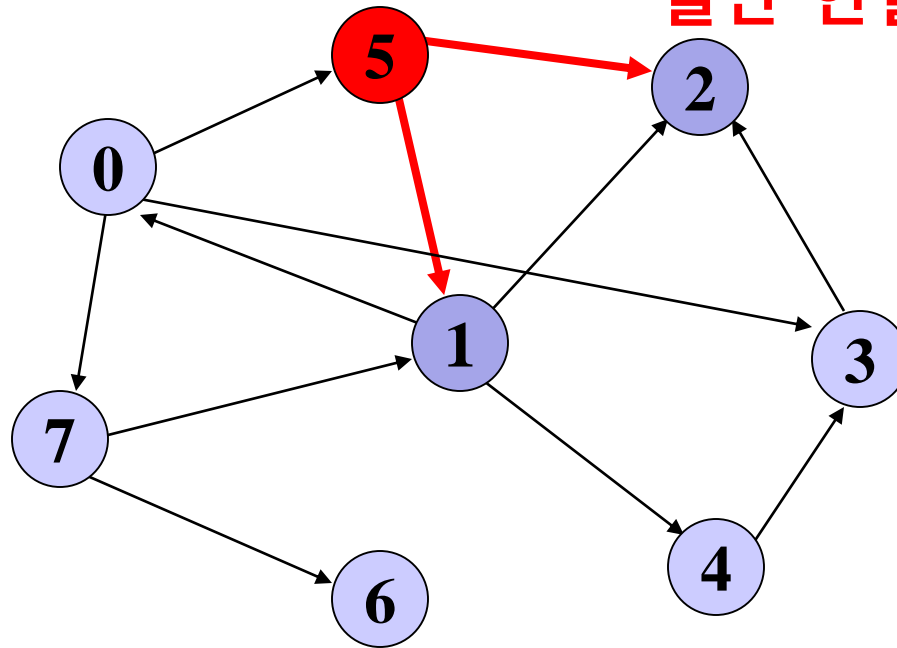
5

Queue



# BFS: Visit 1 and 2

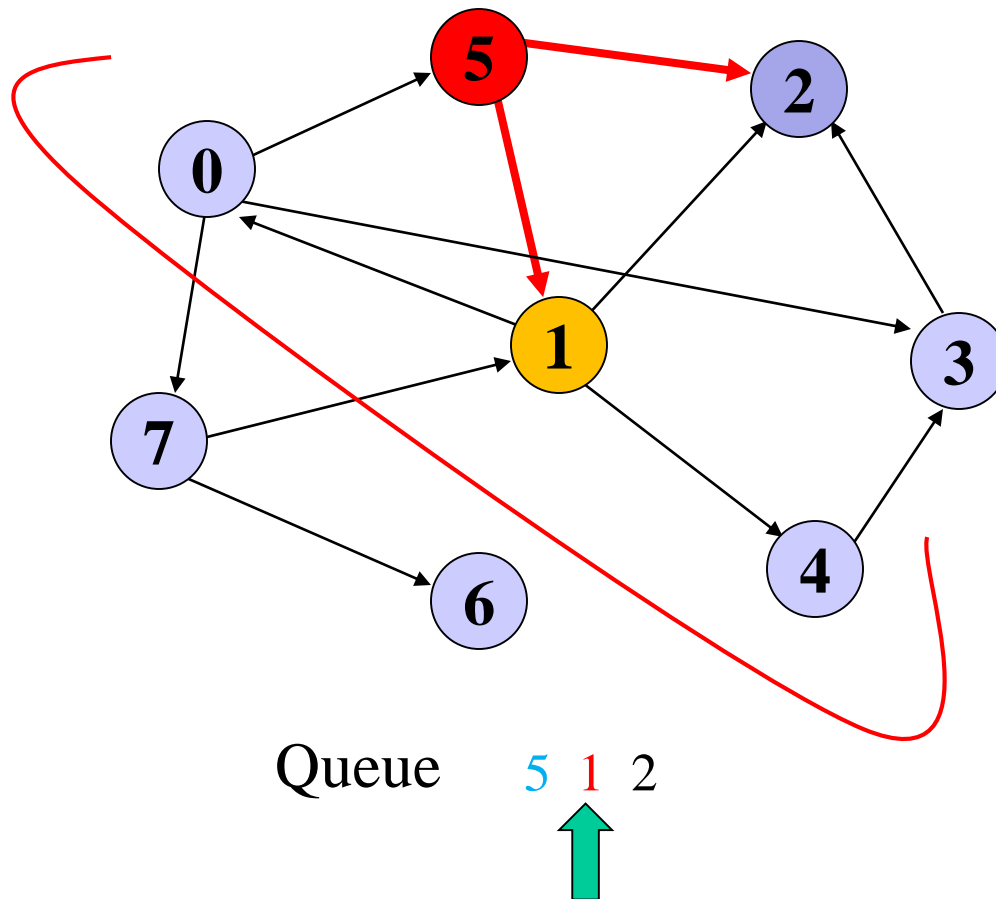
이웃노드 1,2를 빨강색으로  
일단 연결



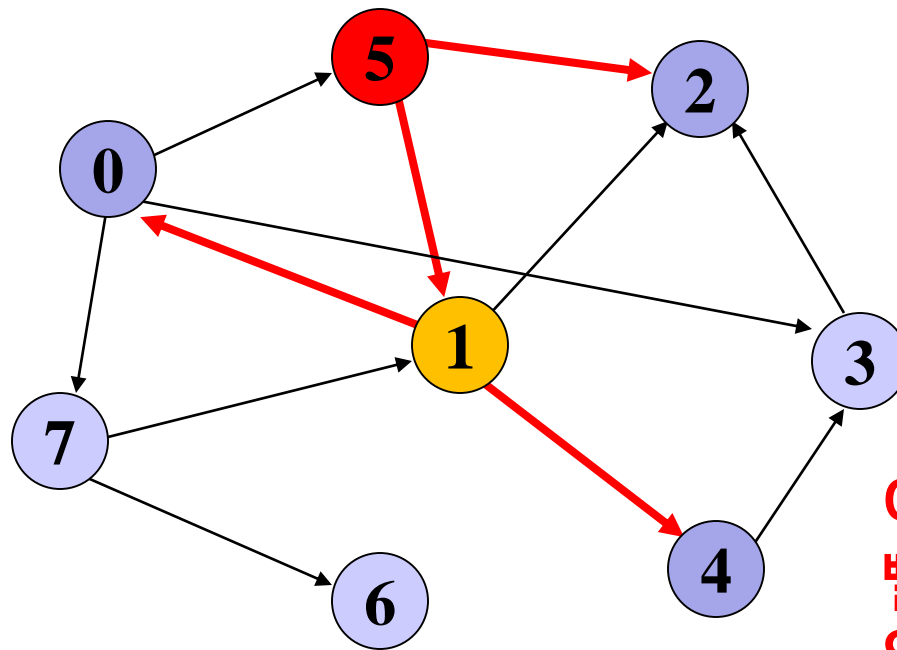
Queue    5 1 2  
          ↑

큐에 이웃인 1,2를 삽입  
그리고 dequeue

# BFS: Nodes two-away



# BFS: Visit 0 and 4

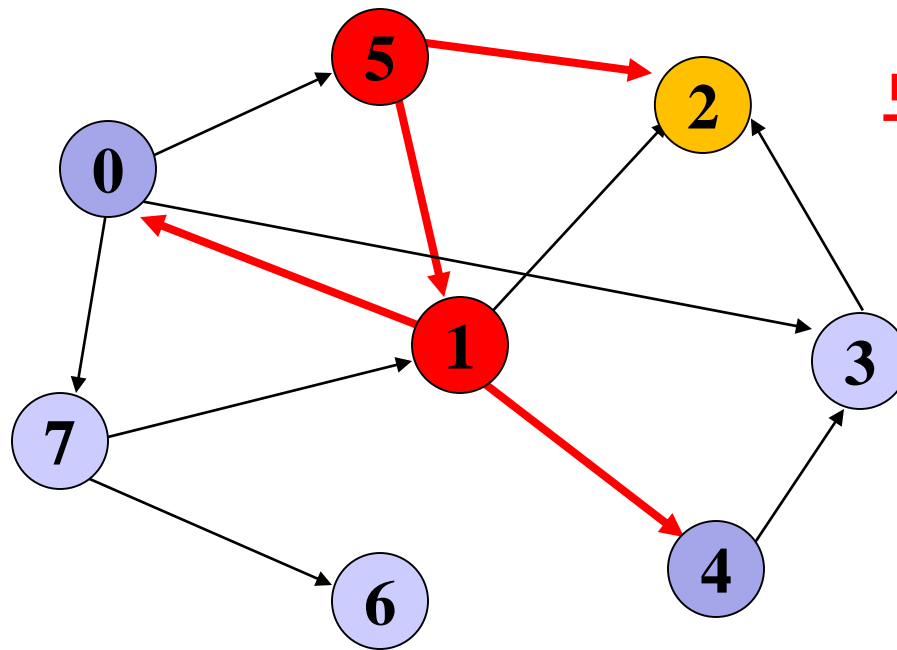


이웃노드 0,4를  
빨강색으로  
일단 연결

5 1 2 0 4  
↑

큐에 이웃인 0,4를 삽입  
그리고 dequeue

# BFS: Visit no node from 2



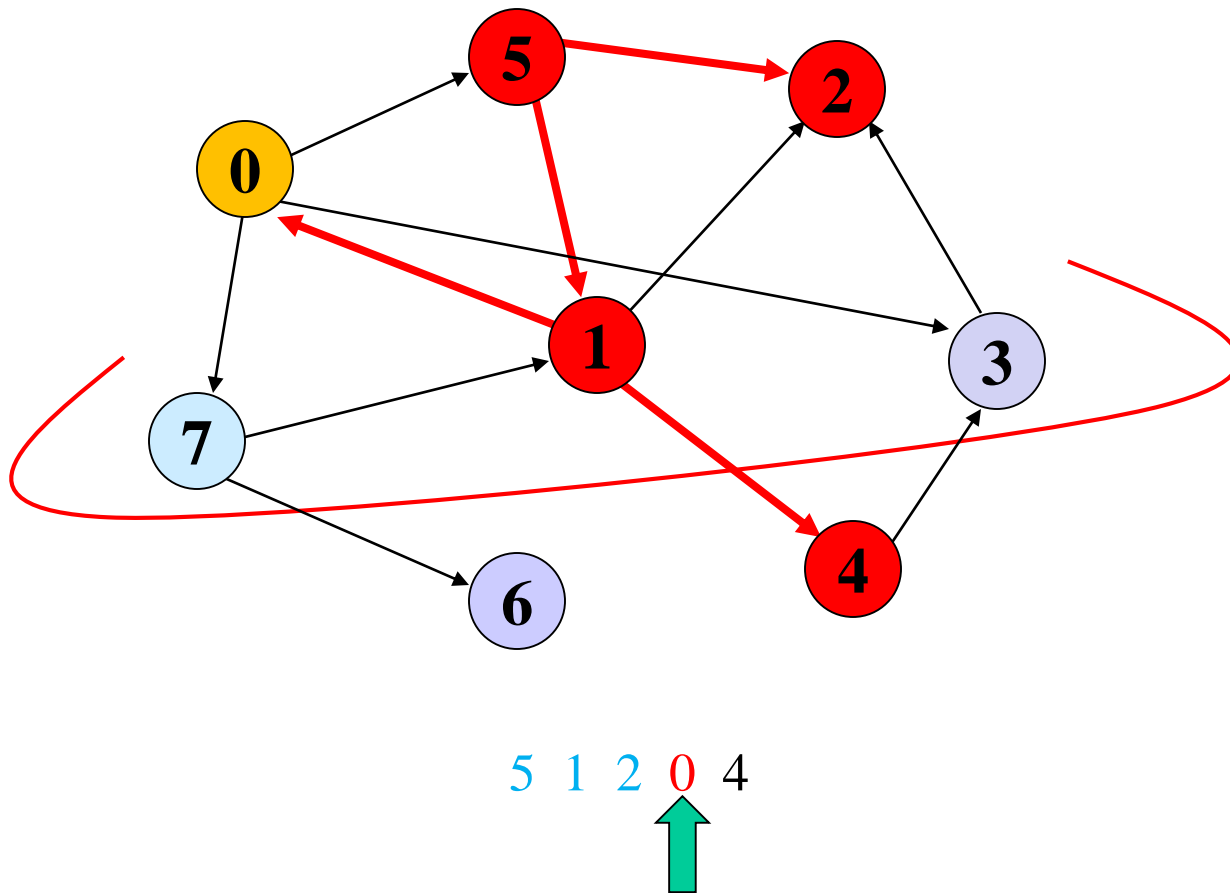
노드2에 이웃없음

5 1 2 0 4



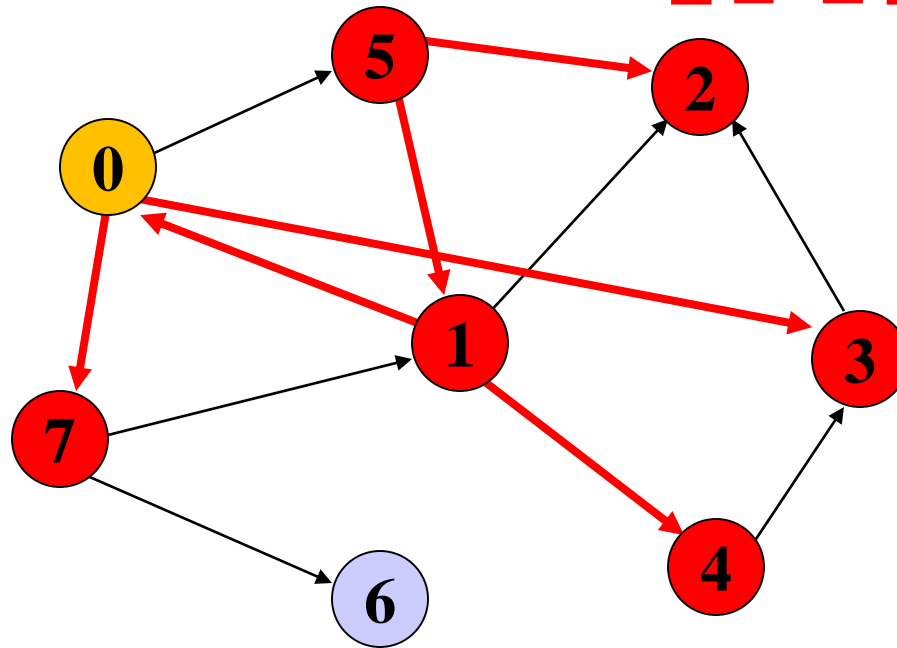
dequeue

# BFS: Nodes three-away



# BFS: Visit nodes 3 and 7

이웃노드 3,7을 빨강색으로  
일단 연결

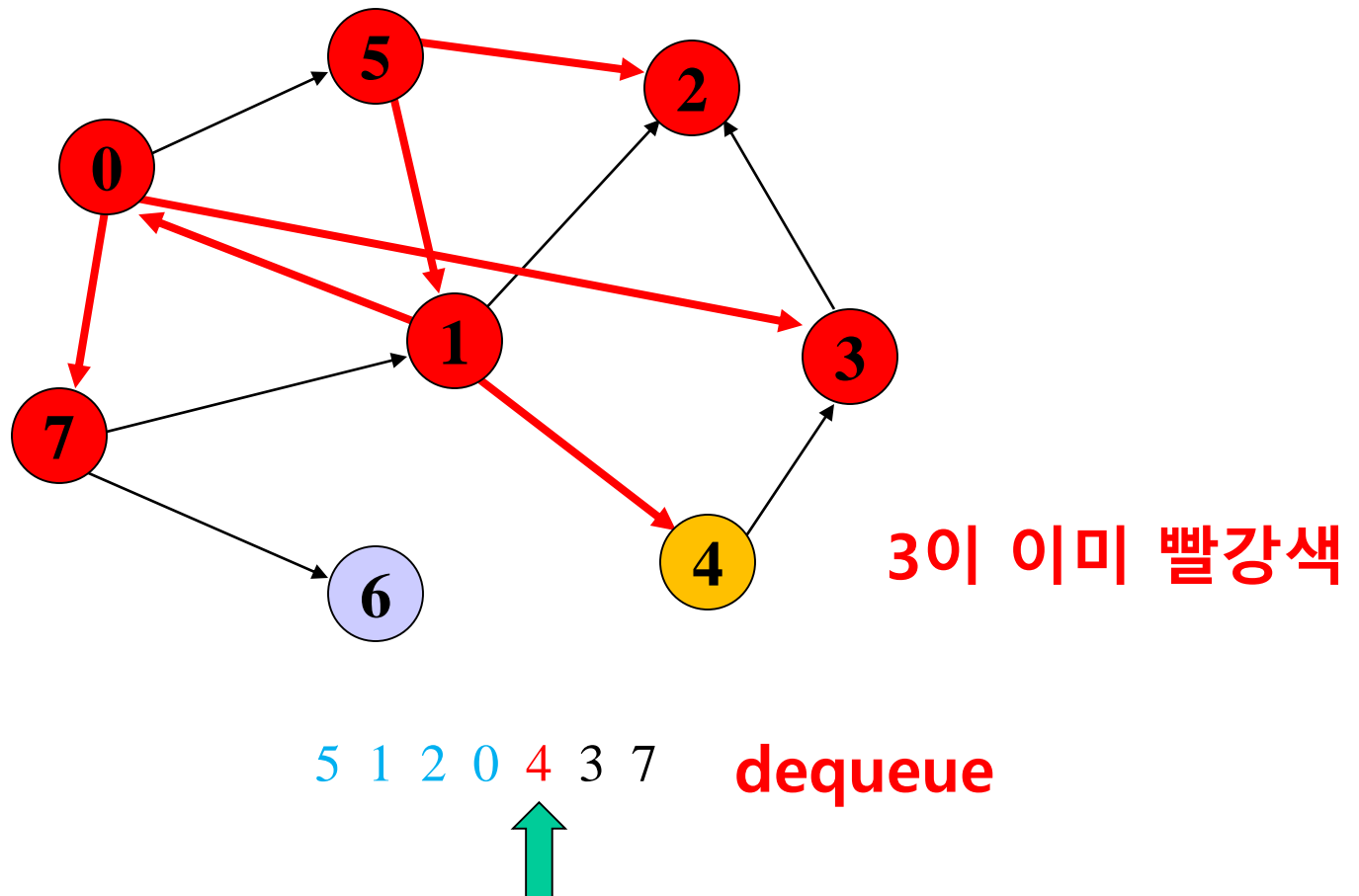


5 1 2 0 4 3 7

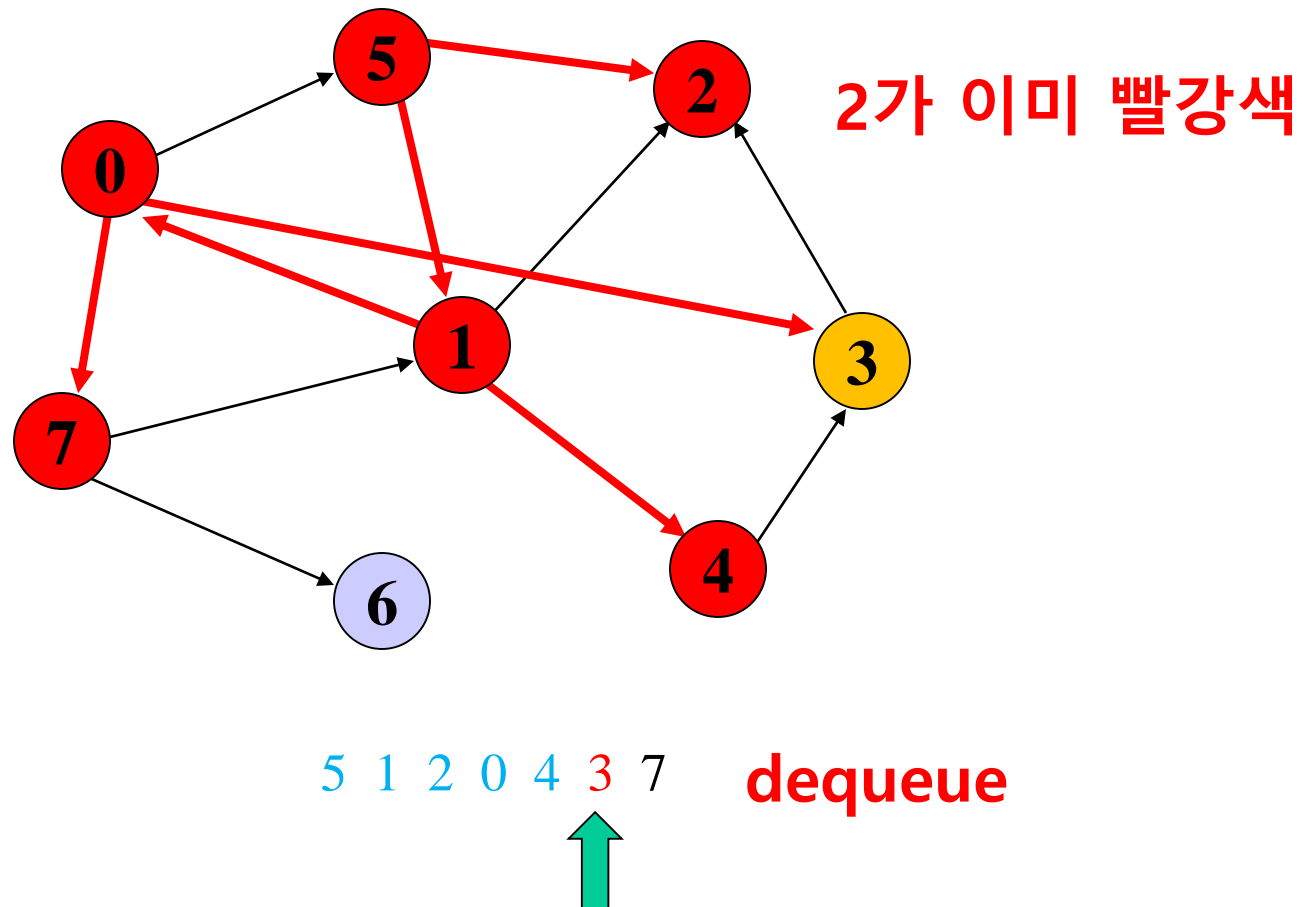


큐에 이웃인 3, 7을 삽입  
그리고 dequeue

# BFS: Visit no node from 4

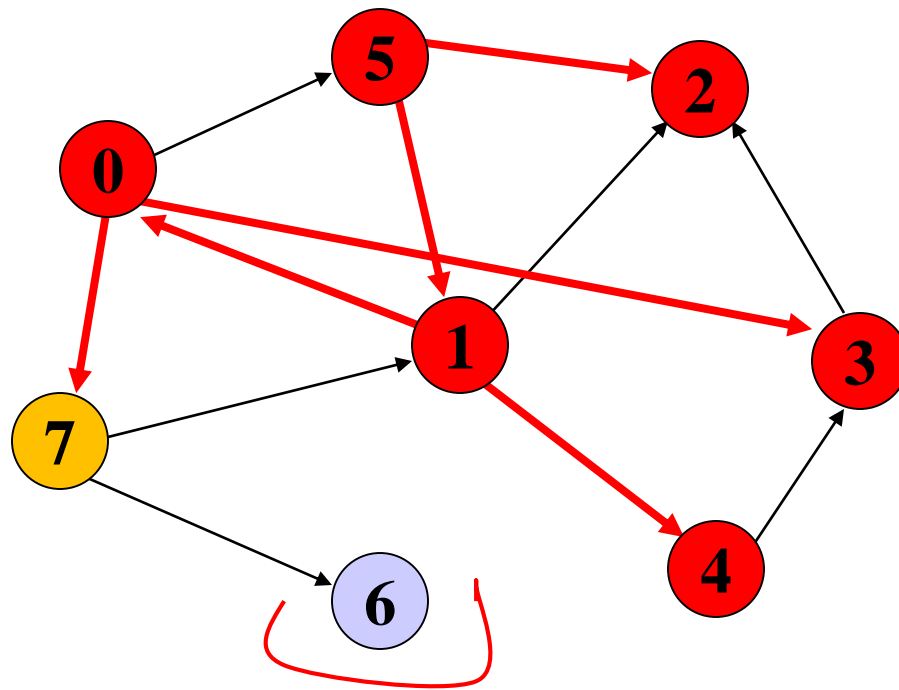


## BFS: Visit no nodes from 3





# BFS: Node four-away



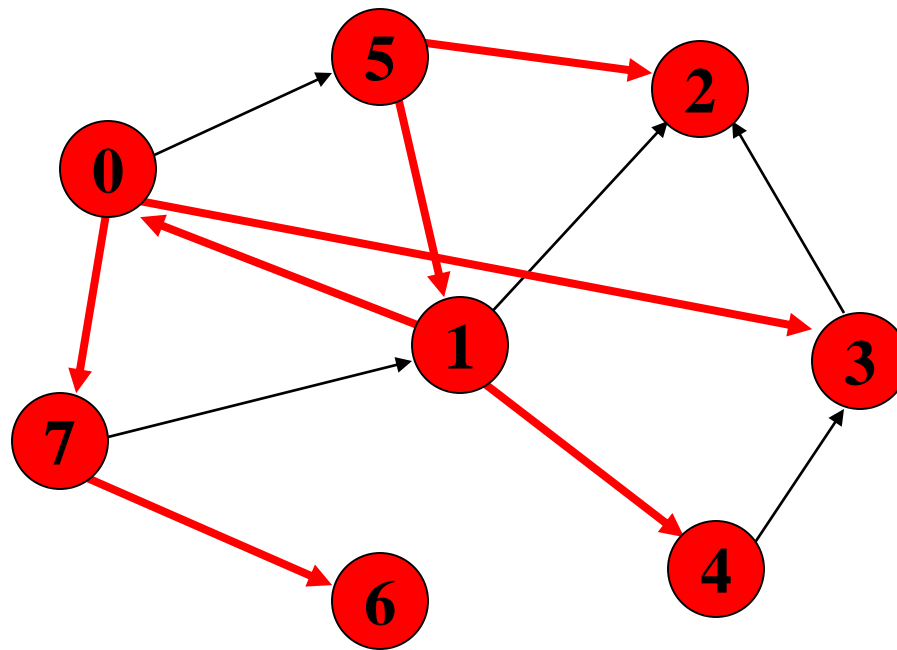
이웃노드 6를  
빨강색으로  
일단 연결

5 1 2 0 4 3 7 6



큐에 이웃인 6을 삽입  
그리고 dequeue

# BFS: Visit 6

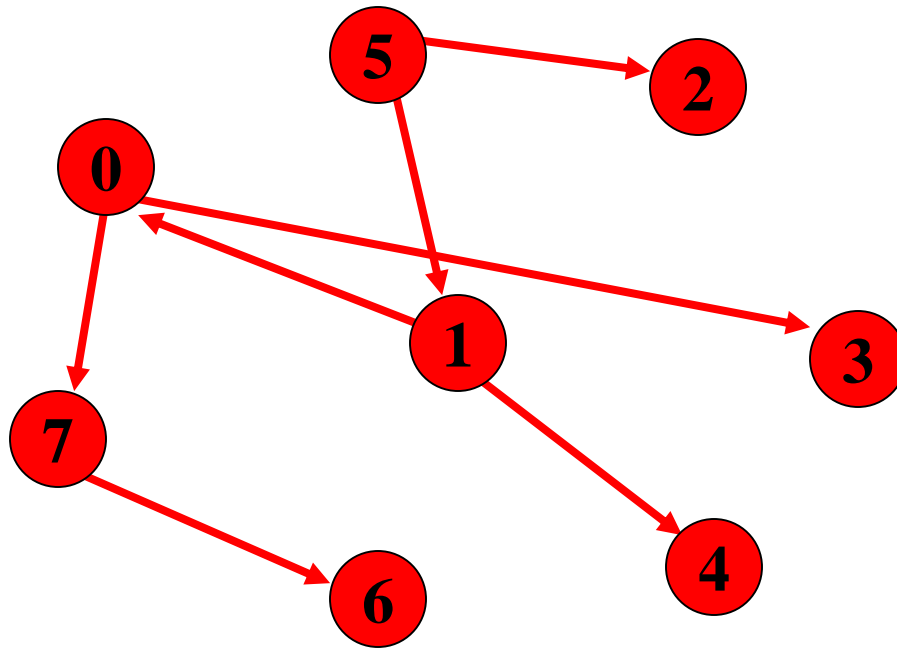


5 1 2 0 4 3 7 6



# Spanning Tree(간선트리)

BFS방식으로 그래프의 노드를 방문했을때 얻게되는  
트리를 간선트리라한다



5 1 2 0 4 3 7 6

**BFS방식으로 노드순회를 마친 결과**





감사합니다.