Table of Contents

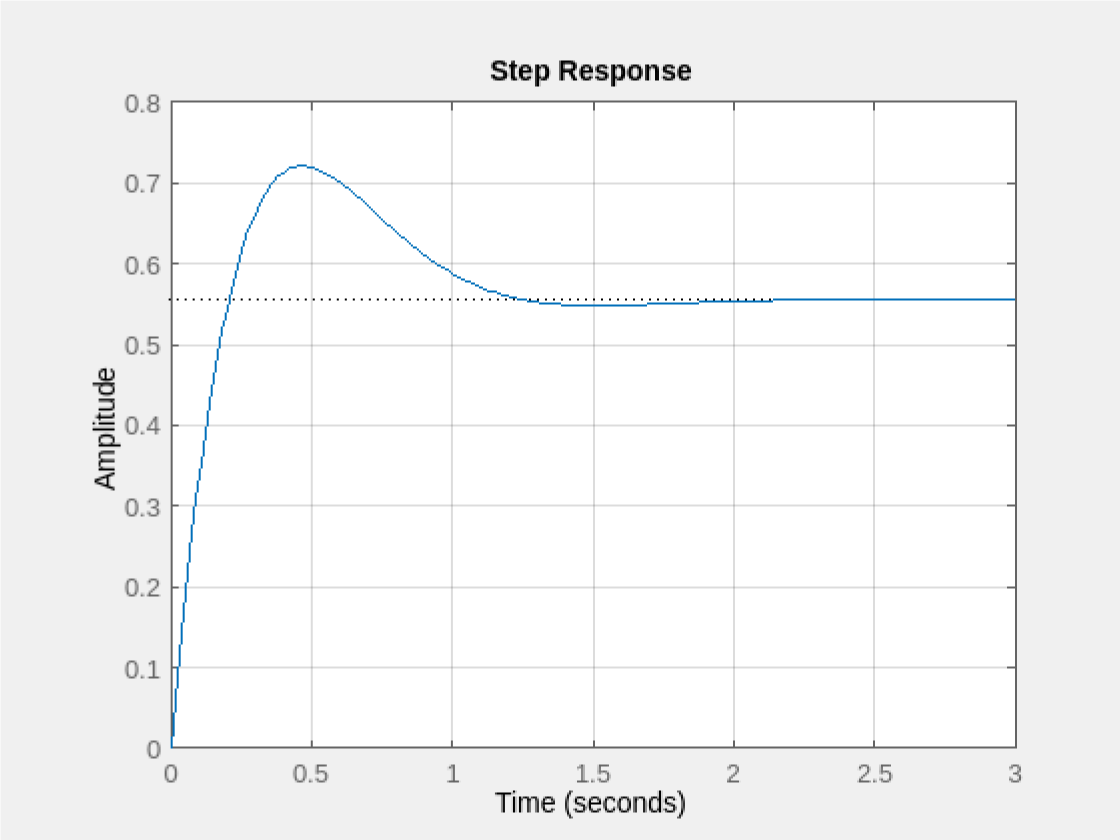
1. State Feedback Control

```
A = [[-2 1];[0 -3]];
B = [1;1];
C = [1 3];
D = 0;

% confirmation of controllability
Omega_c = [B A*B];

if rank(Omega_c) == min(size(A))
    fprintf('controllability matrix is full rank')
end

% introduce state feedback gain K

K = [5 -4];
closed_loop_eigenvalues = eig(A-B*K);
```

```
controllability matrix is full rank
```

1.3 Step Response

```
A_closed = A - B*K;

sys_closed_loop = ss(A_closed, B, C, D);
step(sys_closed_loop)
grid on
```

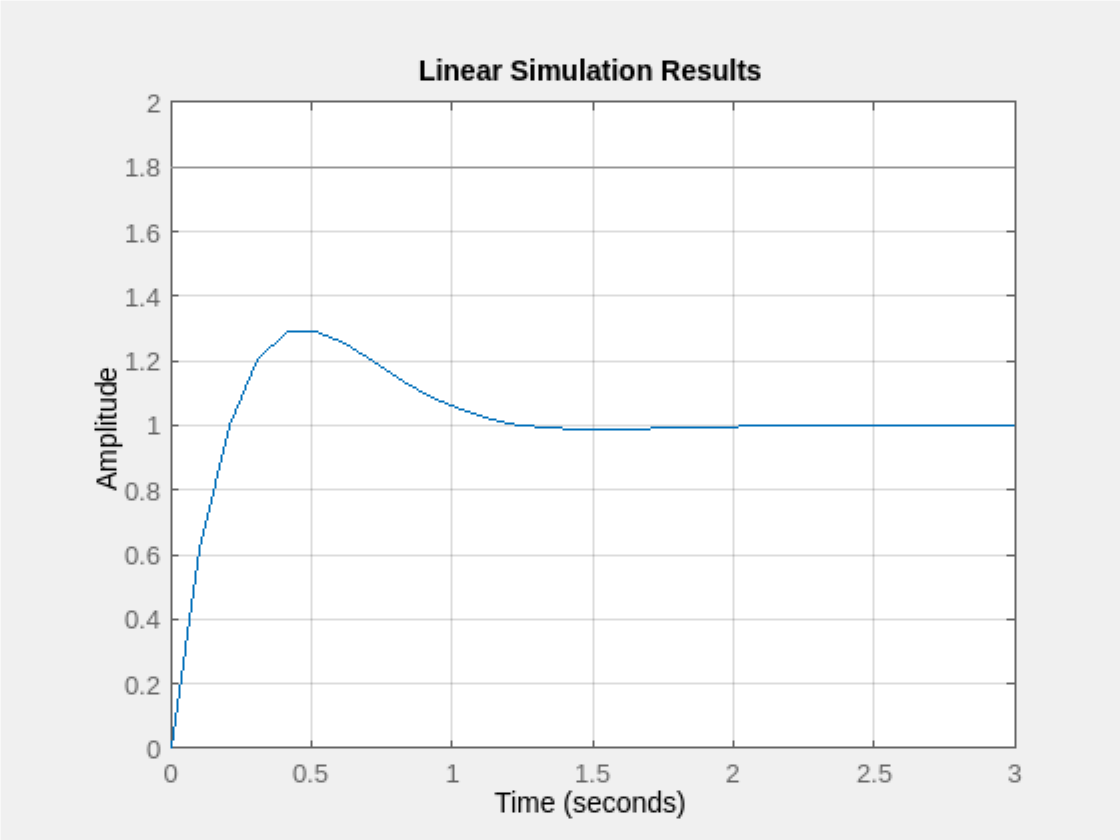


2. Reference Scaling

```
N_bar = inv(C*inv(-A+B*K)*B);
```

2.2 Reference Scaling Step Response

```
tmax = 3;
sample_time = 0.1;

% time period
t = linspace(0,tmax,tmax*(1/sample_time));

% psuedo "step" response (scaled by N_bar)
r = N_bar*ones(length(t),1);

% simulated response with scaled reference input
lsim(sys_closed_loop,r,t)
grid on
hold off
```



3. Transfer Function Equivalency

```
% This question does not need Matlab, just including this section for
% consistency/completeness.
```

4. Longitudinal Helicopter Dynamics

```
A = [[-0.4 0 -0.01];[1 0 0];[-1.4 9.8 -0.02]];
B = [6.3; 0; 9.8];
C = [0 0 1];
D = 0;
```

4.1 Controllability

```
if rank(ctrb(A,B)) == size(A,1)
    fprintf('longitudinal helicopter dynamics are controllable')
end
```

longitudinal helicopter dynamics are controllable

4.2 State Feedback Control Design

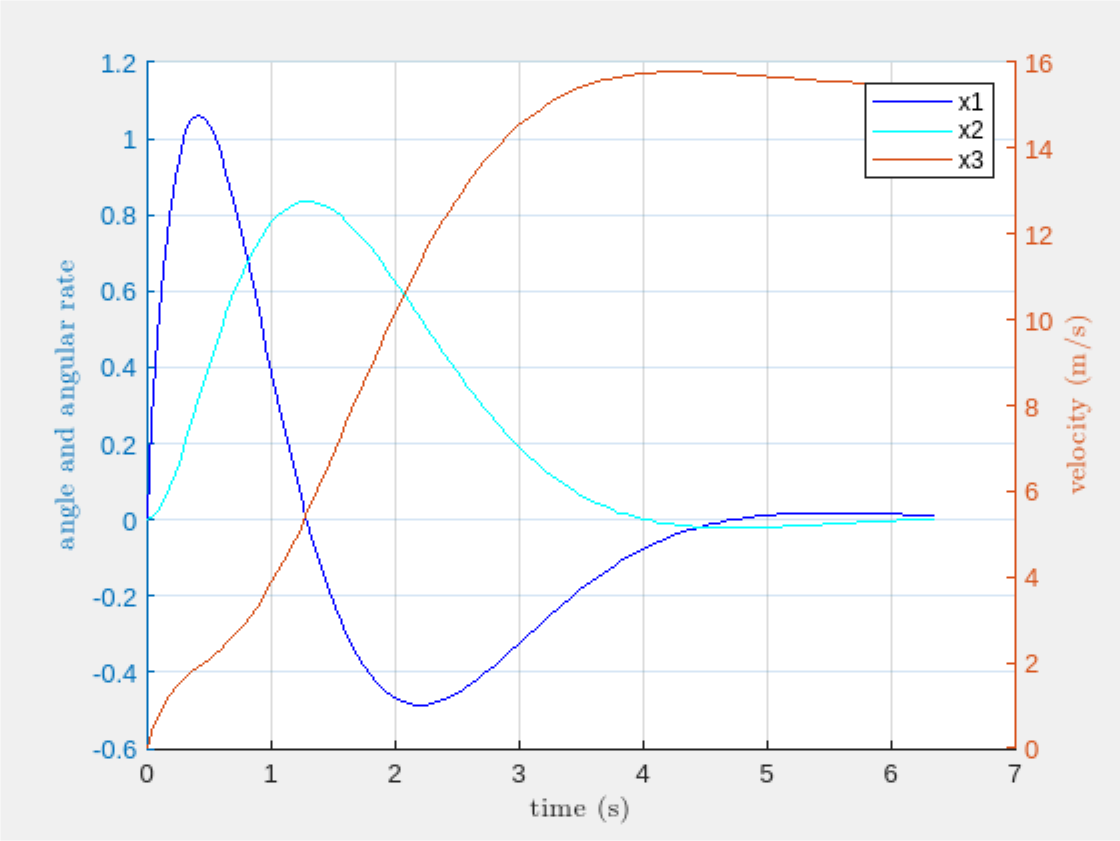
```
% system state space representation
```

```
helo_sys = ss(A,B,C,D);

% desired pole locations and required gain, K
p_desired = [-1 + 1i; -1 - 1i; -2];
K = acker(A,B,p_desired);

% check that the eigenvalues are what we want:
eigenvalues_closed_loop = eig(A-B*K)
helo_sys_closed = ss(A-B*K,B,C,D);
```

```
eigenvalues_closed_loop =

  -2.0000 + 0.0000i
  -1.0000 + 1.0000i
  -1.0000 - 1.0000i
```

4.3 Step Response Simulation

```
% simulate a step response, showing all 3 output variables
[y, t, x] = step(helo_sys_closed);

clf
hold on
xlabel('time (s)', 'interpreter', 'latex')

yyaxis left
ylabel('angle and angular rate', 'interpreter', 'latex')
plot(t,x(:,1), 'b-');
plot(t,x(:,2), 'c-');

yyaxis right
ylabel('velocity (m/s)', 'interpreter', 'latex')
plot(t,x(:,3))
grid on
legend('x1', 'x2', 'x3')
hold off
```



5. Controllability Decomposition

```
A = [[4 -1];[-1 4]];
B = [1;1];
C = [1 1];
D = 0;

sys = ss(A,B,C,D);
```

5.1 Controllability And Controllabiltiy Decom-position

```
% check controllability
if rank(ctrb(A,B)) ~= size(A,1)
    fprintf('system is not fully controllable')
end

% controllability decomposition transformation matrix
Tinv = [[1 1];[1 2]];
T = inv(Tinv);
```

system is not fully controllable

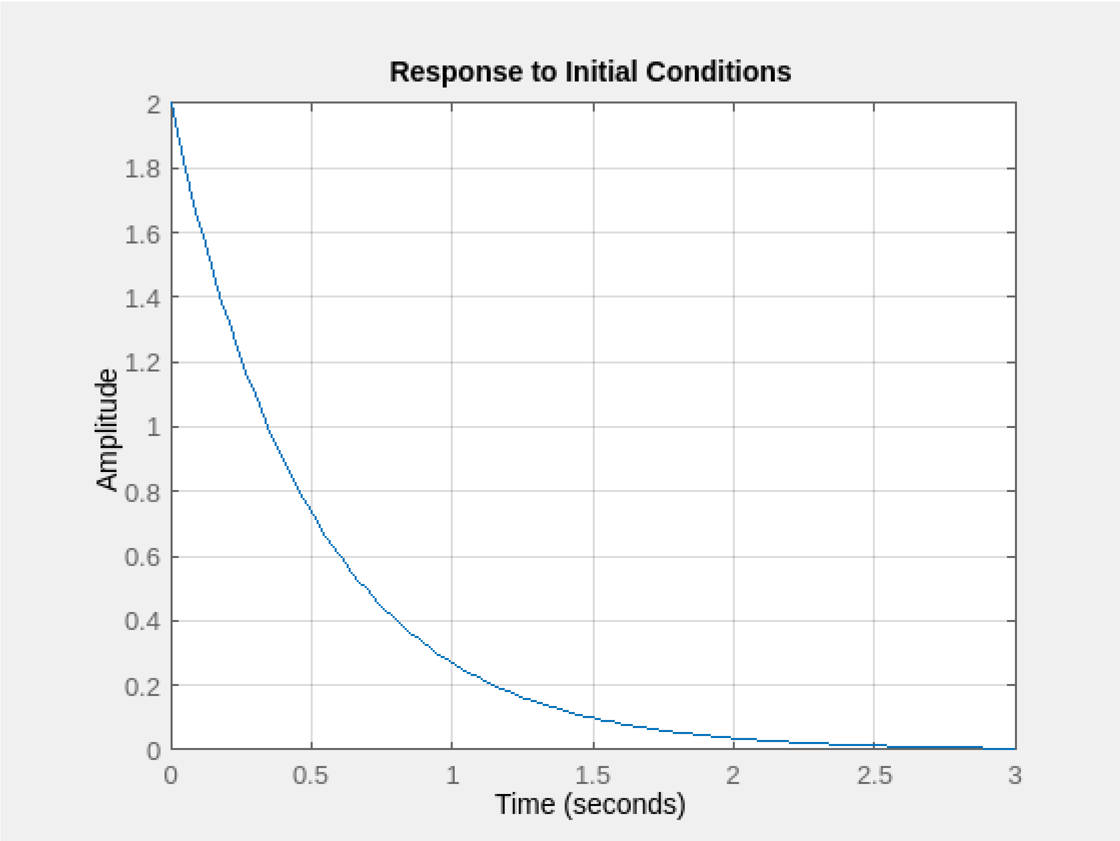
5.3 Control Design

```
% determine "original" coordinate K via T
K_bar = [5 0];
K = K_bar*T;

% check eigenvalue of closed loop system
sys_closed = ss((A-B*K), B, C, D);
[V,D] = eig((A-B*K));
```

5.4 IC Responses

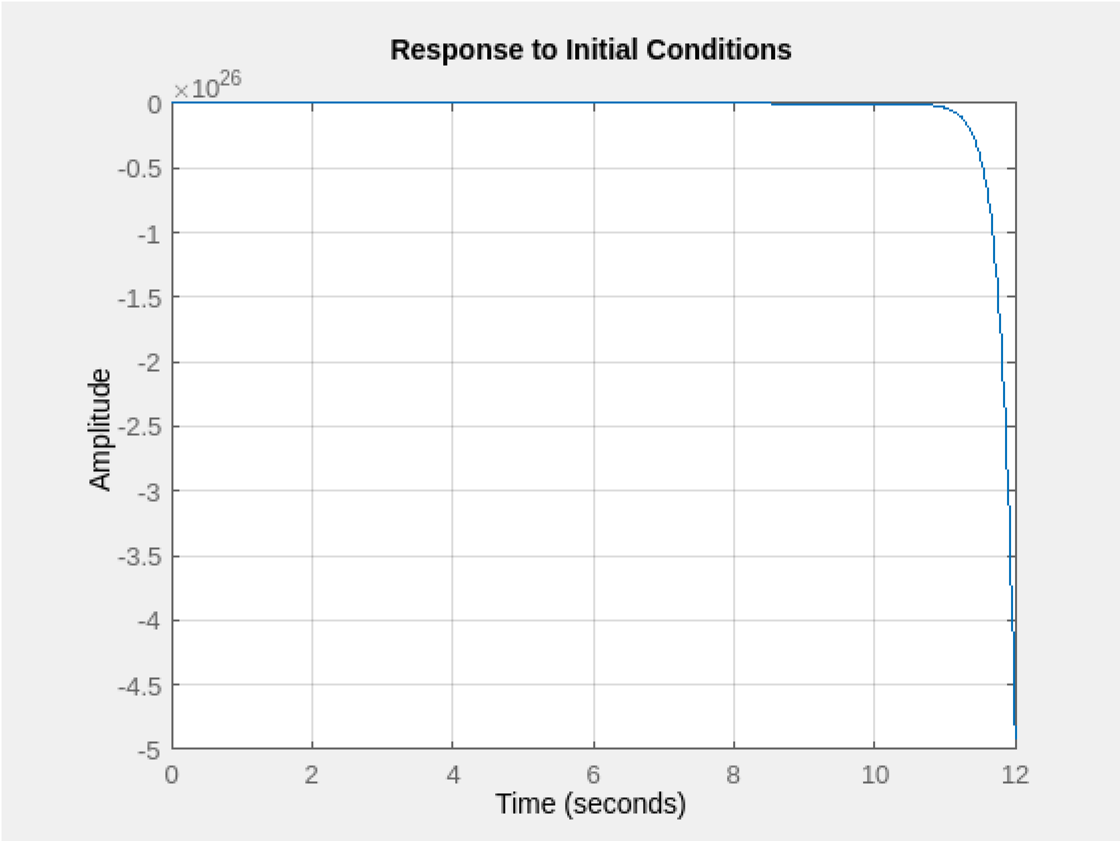
5.4.1 IC Response 1

```
x_init_1 = [1;1];
initial(sys_closed, x_init_1)
grid on
```



5.4.2 IC Response 2

```
x_init_2 = [1;-1];
initial(sys_closed, x_init_2)
grid on
```



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