Data Driven Modeling and Control

MAE 298

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Homework 2

Problem 1 (20 points):
Use the Singular Value Decomposition (SVD) to find the least-squares solution to Ax = b

$$A = \begin{bmatrix} 0.3536 & 0 & 0.25 & 0.25 \\ 0 & -1.4142 & -1 & -1 \\ 0.6124 & 0 & 0.433 & -0.433 \end{bmatrix}$$

Using the following steps:

a. Find an orthonormal basis V_1 for $\mathcal{R}(A^*) = \mathcal{R}(A^*A)$
b. Find V_2 to span the $\mathcal{N}(A)$ and complete the orthonormal basis.
c. Find the singular values of A = the square roots of the eigenvalues of A^*A. Double-check that

$$A^*A = [V_1 \quad V_2]\begin{bmatrix} S^2 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} V_1^* \\ V_2^* \end{bmatrix}$$

d. Compute $U_1 = AV_1S^{-1}$ and verify that it is orthonormal.
e. Choose U_2 to complete the orthonormal basis U

You may use MATLAB for computation if you wish. Find x for $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Problem 2 (10 points):
Choose a picture of your choice and convert it to grey scale using MATLAB. Compress the picture using Eckart-Young theorem, with

a. $r = 5$
b. $r = 30$
c. The smallest rank that the difference between the original picture and the compressed picture are negligible.
d. Compare the storage required to store pictures in a, b, and c versus the original picture.

Submit your code and plot the pictures next to each other for comparison.

Problem 3 (20 points):
Open the HW2_p3 folder. There is a file named “CylinderFlow.mat”, which contains flow field over a cylinder, sampled in time domain with fixed Δt and reshaped into column vectors.

a. Visualize the flow field for $t = 1, 10, 20, 50, 100$ and provide the flow field plots. You can use the files “DMD_Cylinder.m” and “plotCylinder.m” that are provided in the same folder.
b. Complete the file “DMD.m”, which implements the DMD algorithm.
c. Compute the values of λ, Φ and b for $r = 5, 20$ and compare the values of λ and b.

d. Compute $x(10)$, $x(20)$, $x(50)$ and $x(100)$ using the dominant eigen values and eigen vectors found above, once with $r = 5$ and once with $r = 20$. Compare the predicted values of x (from DMD) and the actual values (from data) through plots and computing the RMSE of the error. Explain the observation and the trends that you see.

Problem 4 (20 points):
Read an article titled "Physics-informed dynamic mode decomposition" and write a 1-page report that includes the motivation for the methodology, formulation, and an example. You'll receive 20 points extra if you provide the code and data for your example.

Problem 5 (20 points):
The folder "HW2-P5" contains the MATLAB files for Sparse Identification of Dynamical System (SYNDy). The data file "SYS1.mat" and SYS2.mat" are sampled states of two different 3rd order dynamical systems, both sampled at 100 Hz rate. The "SYNDy_HW.m" is your main file to work with.

a. Load "SYS1.mat" and assume that somehow and **magically** (☺) you have access to the true system dynamics to compute $\dot{x}$ (see Lorenz.m, the code for differentiation is provided) and identify the nonlinear system dynamics. Adjust the sparsification knob (lambda) and report 3 identified systems with different lambdas and explain your observation.
b. Repeat part a but for "SYS2.mat". The system dynamics for this data is given in "dynamic_p5.m" file.
c. Now instead of assuming that you know the system dynamics in advance, use data to compute $\dot{x}$ and repeat part a and part b. Explain your observation.
d. Now let's add some small noise to the data and use the numerical differentiation method. Repeat part a and b and explain your observation. The amount of noise to be added to each dataset is given in the MATLAB file.

Problem 6 (10 points):
Visit "pysindy package" here https://pysindy.readthedocs.io/en/latest/api/pysindy.html,
Make yourself familiar with the package and read different examples. See the example "Part 2a: Differentiate the data with method other than finite differences!". Explain the method that is used to deal with noisy data and provide a short code for the method (either in Python or MATLAB).

10 extra points for applying the method to problem 5 part d and providing results and code.