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Question 1

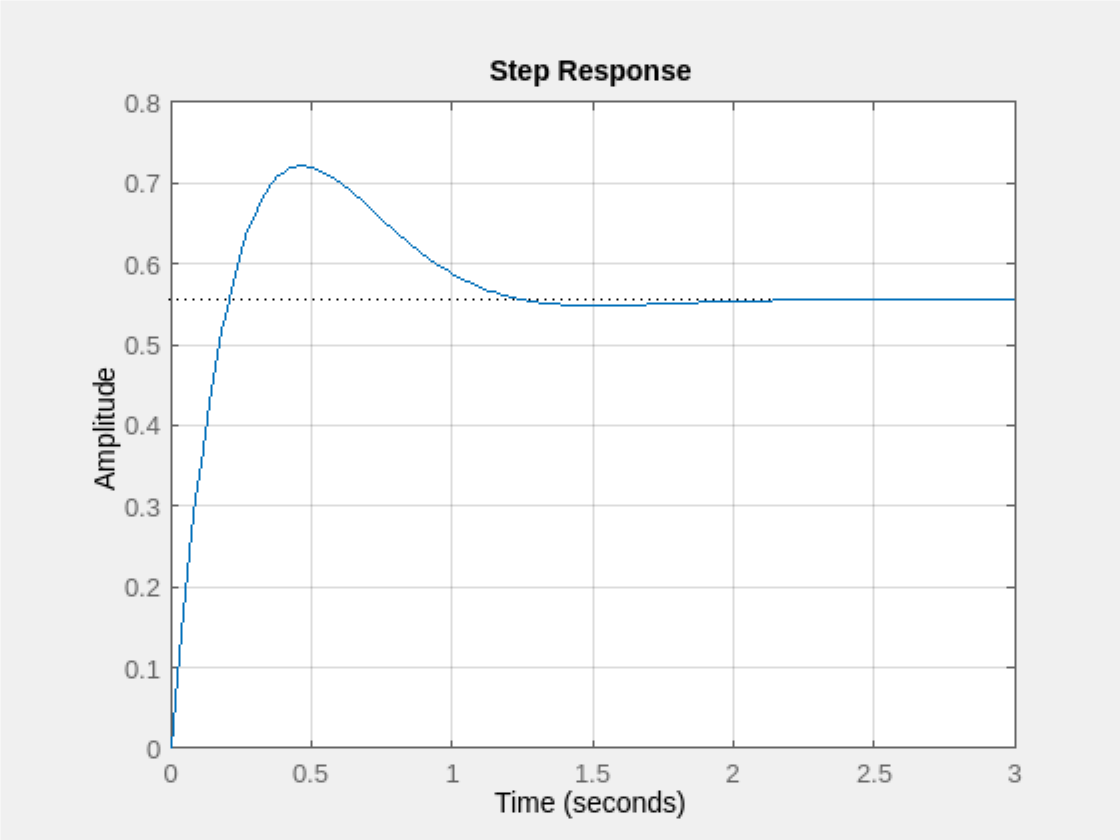
```
A = [[-2 1];[0 -3]];
B = [1;1];
C = [1 3];
D = 0;

% confirmation of controllability
Omega_c = [B A*B];

if rank(Omega_c) == min(size(A))
    fprintf('controllability matrix is full rank')
end
```

```
controllability matrix is full rank
```

Question 1 - Part 3

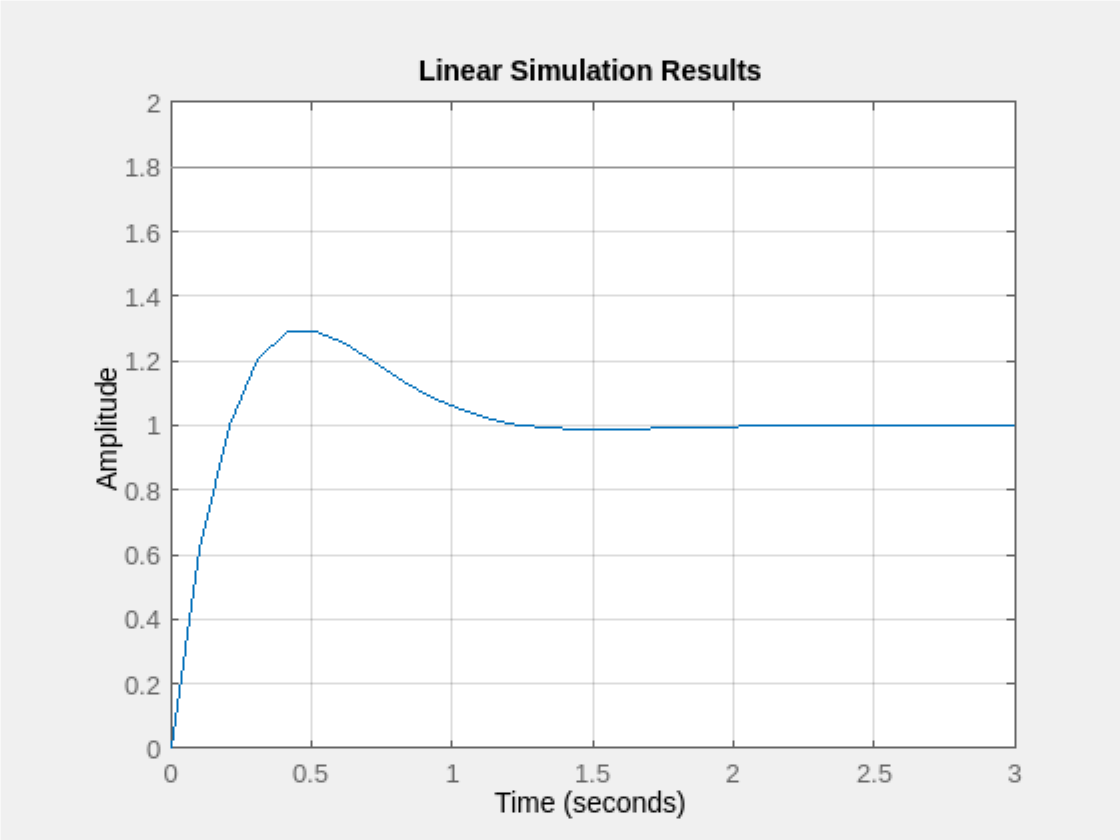
```
% introduce state feedback gain K

K = [5 -4];
closed_loop_eigenvalues = eig(A-B*K)

% simulate the step response on the closed loop system
A_closed = A - B*K;

sys_closed_loop = ss(A_closed, B, C, D);
step(sys_closed_loop)
grid on
```

```
closed_loop_eigenvalues =

  -3.0000 + 3.0000i
  -3.0000 - 3.0000i
```



Question 2

```
% simulate the step response on the closed loop system with an augmented
% output matrix

N_bar = inv(C*inv(-A+B*K)*B);

tmax = 3;
sample_time = 0.1;

% time period
t = linspace(0,tmax,tmax*(1/sample_time));

% psuedo "step" response (scaled by N_bar)
r = N_bar*ones(length(t),1);

% simulated
lsim(sys_closed_loop,r,t)
grid on
```

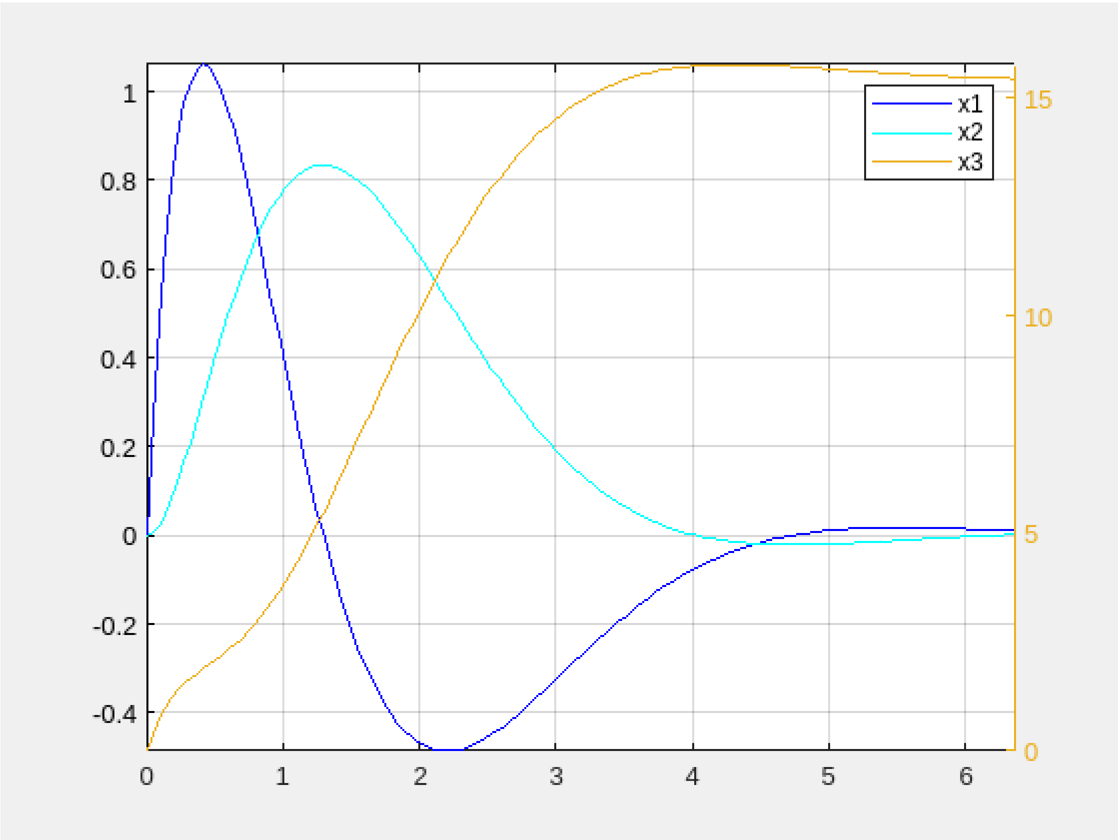


Question 4

```
% longitudinal helicopter dynamics

A = [[-0.4 0 -0.01];[1 0 0];[-1.4 9.8 -0.02]];
B = [6.3; 0; 9.8];
C = [0 0 1];
D = 0;

% controllability
if rank(ctrb(A,B)) == size(A,1)
    fprintf('longitudinal helicopter dynamics are controllable')
end


% system state space representation
helo_sys = ss(A,B,C,D);

% desired pole locations and required gain, K
p_desired = [-1 + 1i; -1 - 1i; -2];
K = acker(A,B,p_desired);

% check that the eigenvalues are what we want:
eigenvalues_closed_loop = eig(A-B*K)
helo_sys_closed = ss(A-B*K,B,C,D);
```

```
% simulate a step response, show all 3 output variables
[y, t, x] = step(helo_sys_closed);
yyaxis left
plot(t,x(:,1), 'b-');
hold on
plot(t,x(:,2), 'c-');
yyaxis right
plot(t,x(:,3));
axis tight
legend('x1', 'x2', 'x3')
grid on
```

```
longitudinal helicopter dynamics are controllable
eigenvalues_closed_loop =

  -2.0000 + 0.0000i
  -1.0000 + 1.0000i
  -1.0000 - 1.0000i
```



Question 5

```
A = [[4 -1];[-1 4]];
B = [1;1];
C = [1 1];
```

```
D = 0;

Omega_c = [B A*B]
sys = ss(A,B,C,D);

% check controllability
if rank(ctrb(A,B)) ~= size(A,1)
    fprintf('system is not fully controllable')
end

% controllability decomposition transformation matrix
Tinv = [[1 1];[1 2]];
T = inv(Tinv);

% determine "original" coordinate K via T
K_bar = [5 0];
K = K_bar*T;

% check eigenvalue of closed loop system
sys_closed = ss((A-B*K), B, C, D)
eig(sys_closed)
```

```
Omega_c =

     1     3
     1     3

system is not fully controllable
sys_closed =

  A = 
         x1   x2
   x1    -6    4
   x2   -11    9

  B = 
       u1
   x1   1
   x2   1

  C = 
       x1  x2
   y1   1   1

  D = 
       u1
   y1   0

Continuous-time state-space model.

ans =

    -2
```

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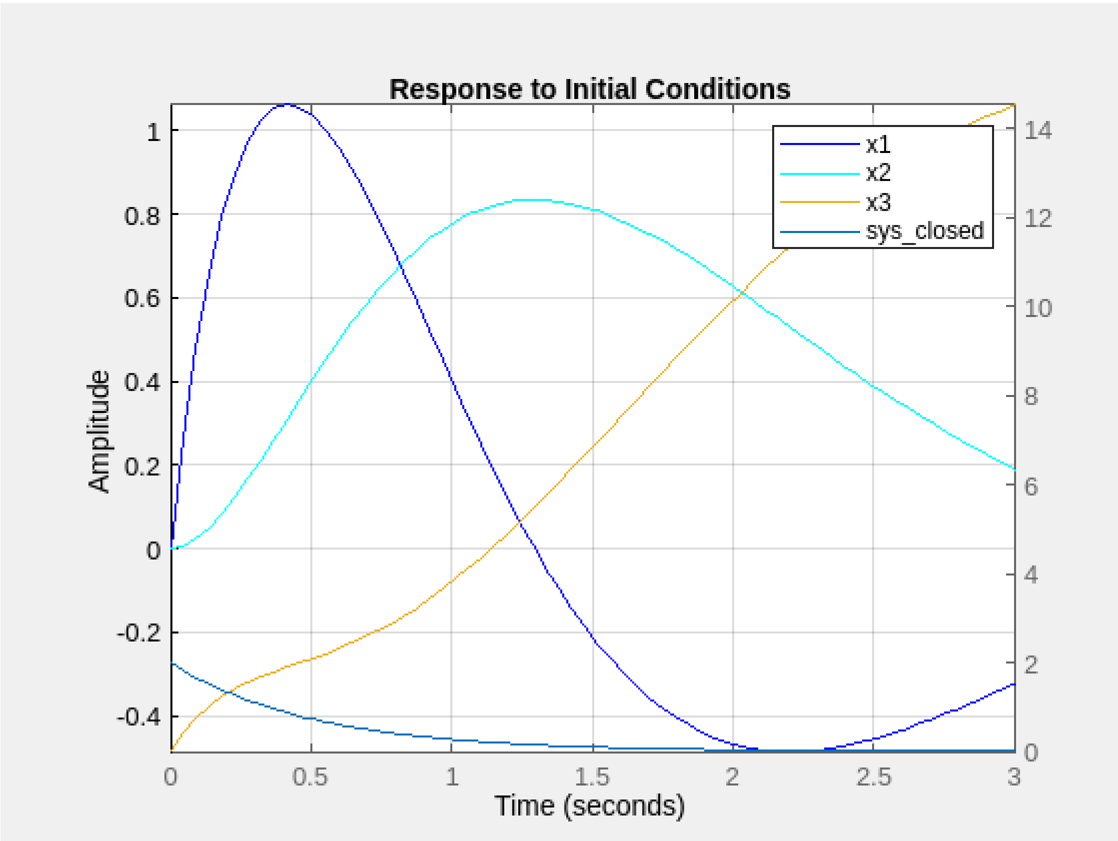
Question 5 Part 4 - IC Response

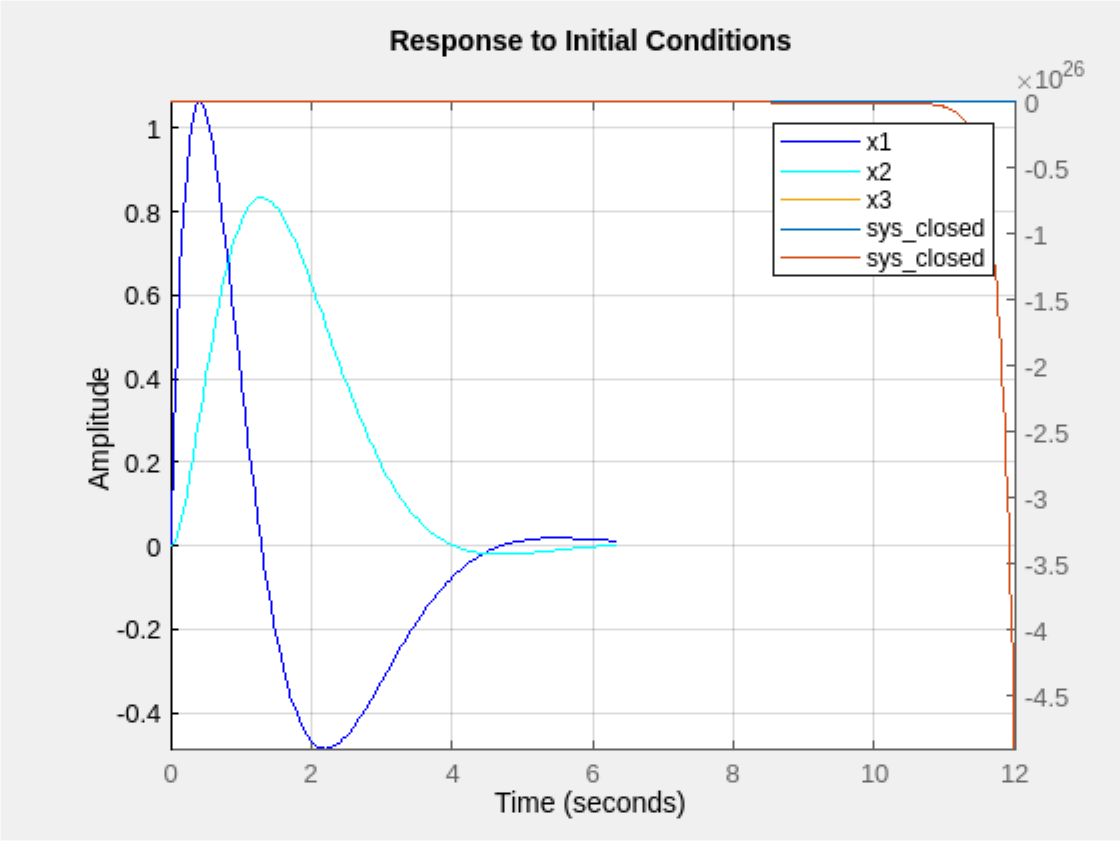
initial condition response 1

```
x_init_1 = [1;1];
initial(sys_closed, x_init_1)
grid on
```



initial condition response 2

```
x_init_2 = [1;-1];
initial(sys_closed, x_init_2)
grid on
```



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