Table of Contents


```
clearvars; close all; clc
```

1 - Implications of the Observability Gramian

1.1 - Observability Gramian

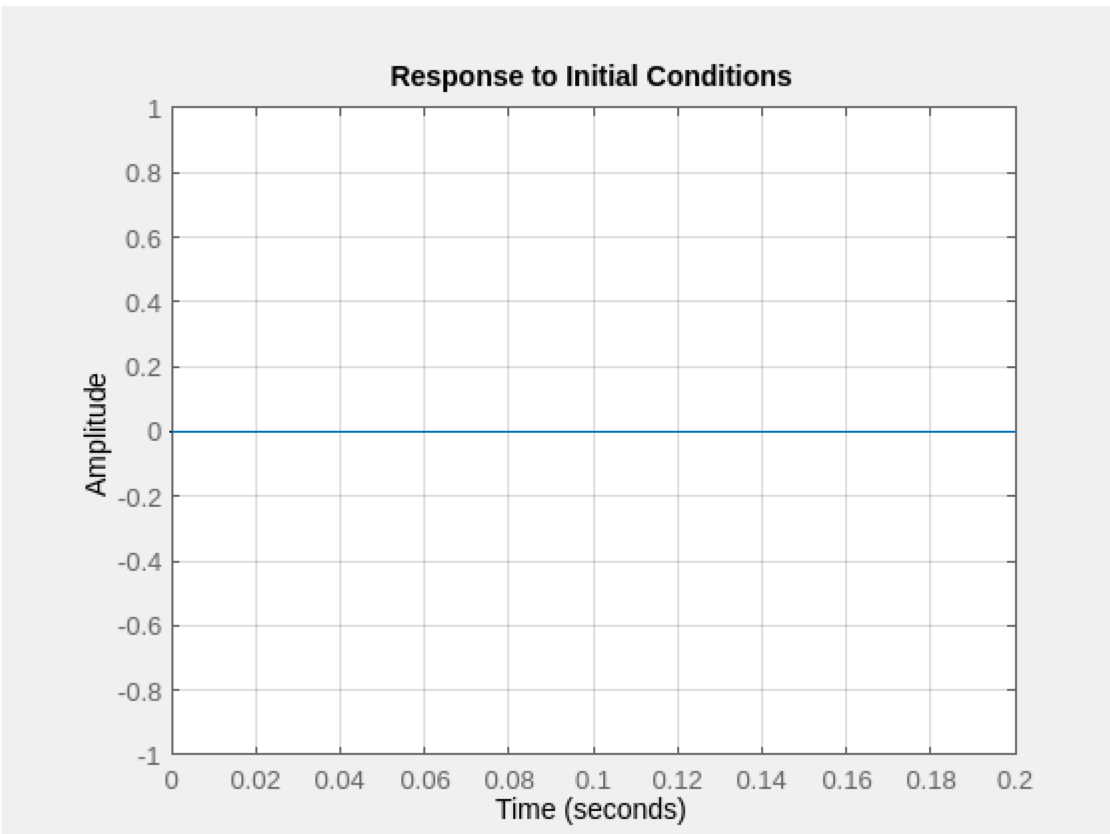
```
A1 = [[-1 0 0];[0 -1 0];[0 0 -2]];
B1 = [1;0;0];
C1 = [1 1 0];
D1 = 0;

sys1 = ss(A1,B1,C1,D1);
observability_gramian1 = gram(sys1, 'o')

[V_o, D_o] = eig(observability_gramian1)
```

```
observability_gramian1 =

    0.5000    0.5000         0
    0.5000    0.5000         0
         0         0         0


V_o =

   -0.7071         0    0.7071
    0.7071         0    0.7071
         0    1.0000         0
```

```
D_o =

     0     0     0
     0     0     0
     0     0     1
```

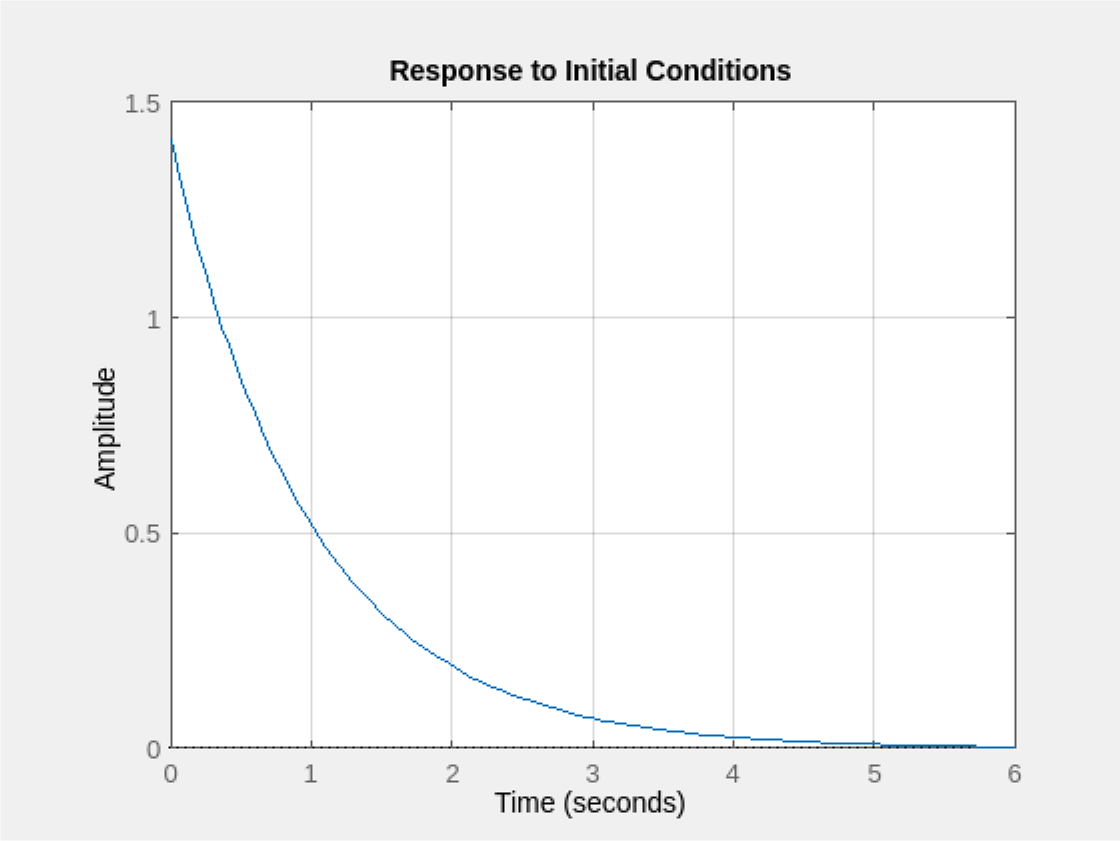
1.2 - Zero Observability Initial Condition

```
% zero output initial condition response
figure(12)
h12 = initialplot(sys1, [0 0 3.141519e7]);
h12.setoptions('Grid', 'on')
```



1.3 - Unit Initial Condition Maximization

```
%  max output initial condition response
figure(13)
h13 = initialplot(sys1, [1/sqrt(2) 1/sqrt(2) 0]);
h13.setoptions('Grid', 'on')
```



2 - Stability

2.1 - Lyapnuov Stability (Lyapunov Method)

```
A2 = [[1 2 3];[0 5 6];[0 8 9]];
B2 = [1;1;1];
C2 = [0 1 0];
D2 = 0;

% perform lyapunov method stability analysis
P = lyap(A2',eye(3))
```

```
P =

   -0.5000   -0.1667    0.2500
   -0.1667    2.2143   -1.4048
    0.2500   -1.4048    0.7976
```

2.2 - Lyapunov Stability (Eigenvalues)

```
A2 = [[1 2 3];[0 5 6];[0 8 9]];
eig(A2)
```

```
ans =

    1.0000
   -0.2111
   14.2111
```

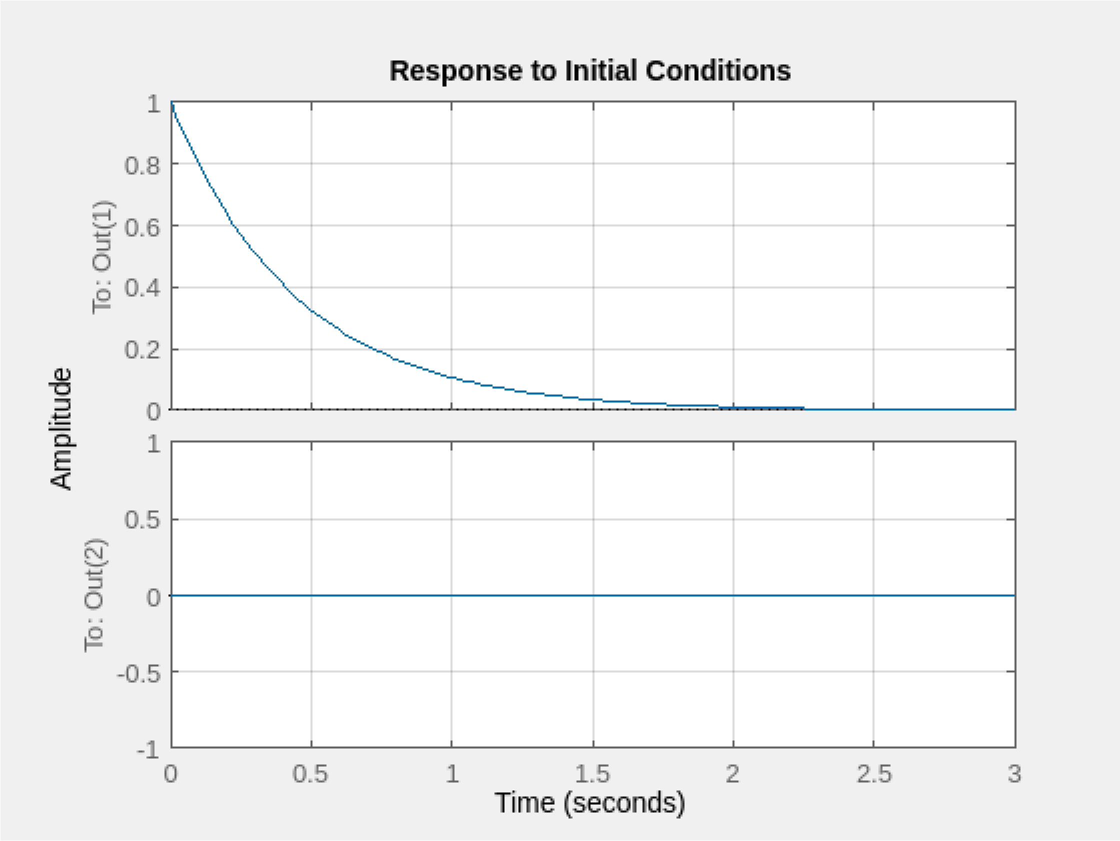
3 - LQR (Infinite Horizon)

3.1 - Definition

3.2 - Design

3.3 - Simulation

```
A3 = [[-1 0];[0 -1]];
B3 = [[2 0];[0 2]];
C3 = [1 1];
Ceye = eye(2);
D3 = 0;

% hand derived optimal gain K
K3 = [[0.618 0];[0 0.618]];
eig(A3-B3*K3);

% simulation of the closed loop system
sys3_cl = ss(A3-B3*K3, B3, Ceye, D3);
initialplot(sys3_cl, [1;0])
grid on
```



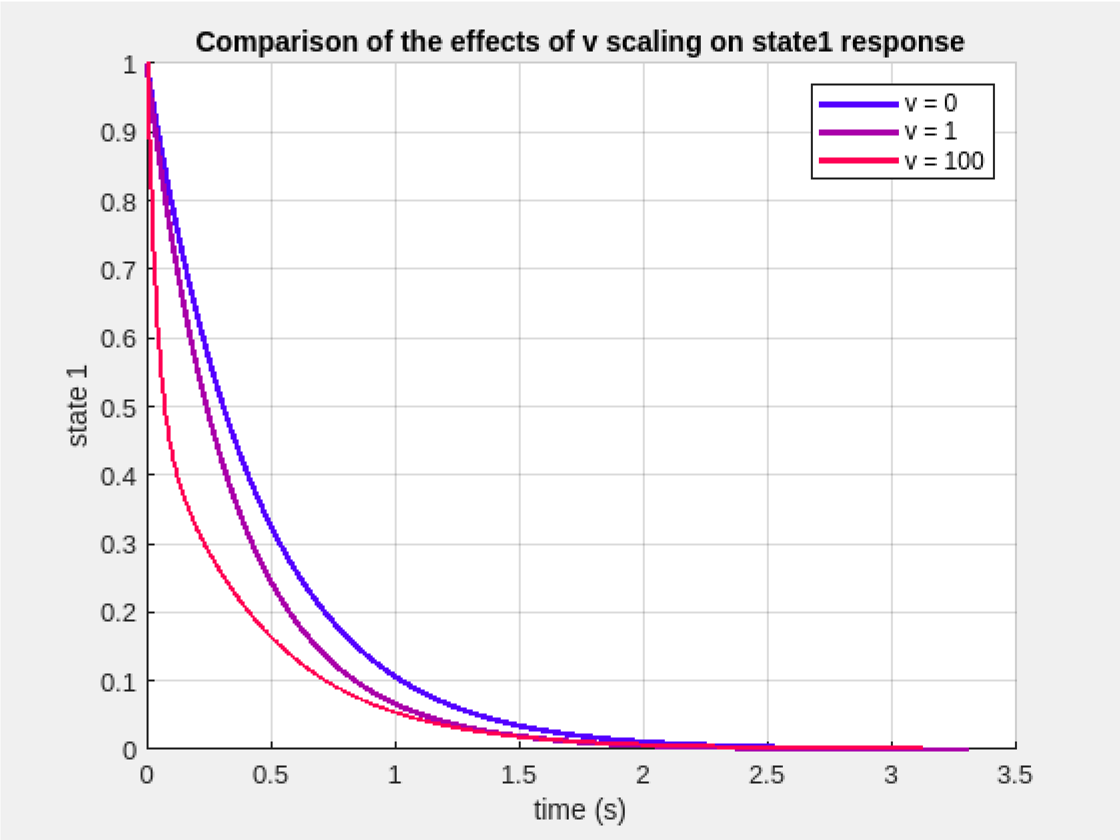
3.4 - Comparison

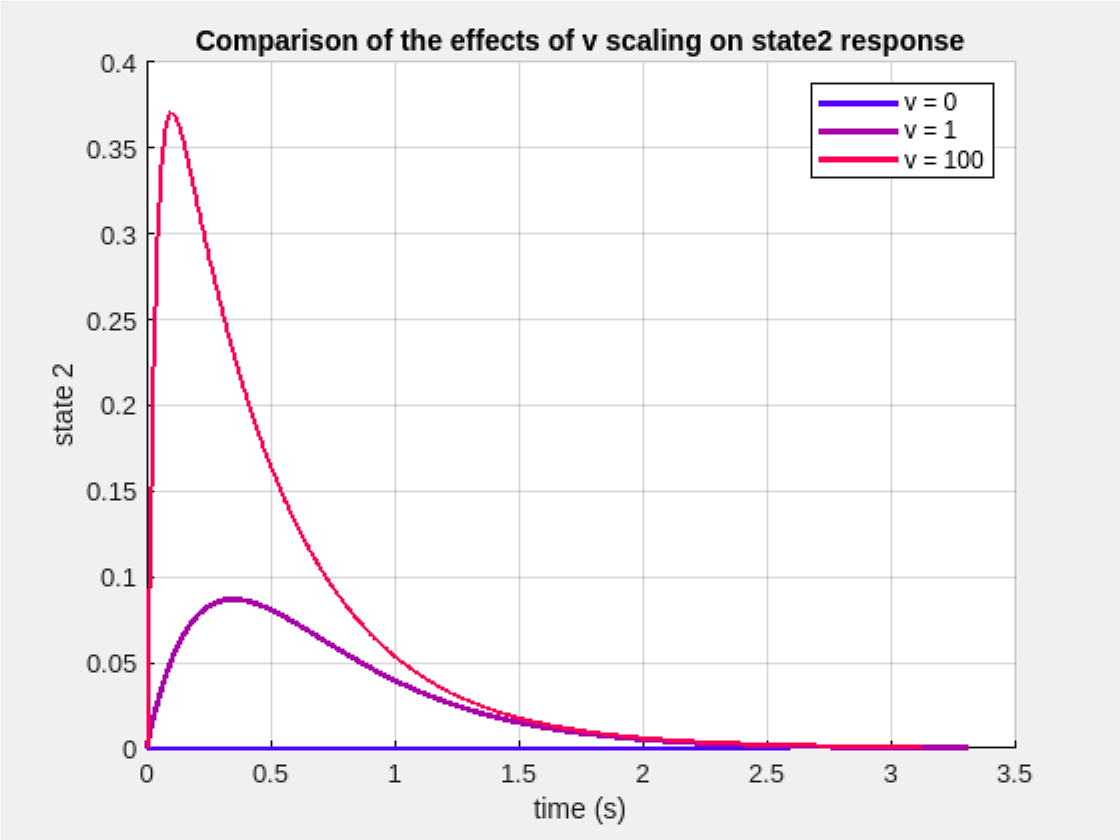
```
R = [[1 0];[0 1]];
v_vec = [0 1 100];

for i = 1:size(v_vec,2)
    vi=v_vec(i);                          % select entry of v
    sprintf('for v = %d', vi)
    Qi = [[(1+vi) -vi];[-vi (1+vi)]]      % calculate Q
    [Ki,~,~] = lqr(A3,B3,Qi,R, [0])       % derive optimal gain K for Q and R
    sys3_cl_i = ss(A3-B3*Ki, B3, Ceye, D3);

    [Y,T,X] = initial(sys3_cl_i, [1;0]);% simulate response

    % plotting
    figure(31)
    hold on;
    grid on;
    plot(T,X(:,1), 'LineWidth', 2, 'DisplayName', sprintf('v = %d',
 v_vec(i)), 'Color', [i/size(v_vec,2),0,(1+1/size(v_vec,2))-i/size(v_vec,2)]);
    legend('-DynamicLegend');
    legend('show');
    xlabel('time (s)');
    ylabel('state 1');
    title('Comparison of the effects of v scaling on state1 response');
```

```
    figure(32)
    hold on;
    grid on;
    plot(T,X(:,2), 'LineWidth', 2, 'DisplayName', sprintf('v = %d',
 v_vec(i)), 'Color', [i/size(v_vec,2),0,(1+1/size(v_vec,2))-i/size(v_vec,2)]);
    legend('-DynamicLegend');
    legend('show');
    xlabel('time (s)');
    ylabel('state 2');
    title('Comparison of the effects of v scaling on state2 response')
end
figure(31)
hold off;
figure(32)
hold off;
```

```
ans =

    'for v = 0'


Qi =

     1     0
     0     1


Ki =

    0.6180         0
         0    0.6180


ans =

    'for v = 1'


Qi =

     2    -1
    -1     2


Ki =

    0.9604   -0.3424
   -0.3424    0.9604


ans =
```

```
   'for v = 100'


Qi =

   101   -100
  -100    101


Ki =

    7.1521    -6.5341
   -6.5341     7.1521
```





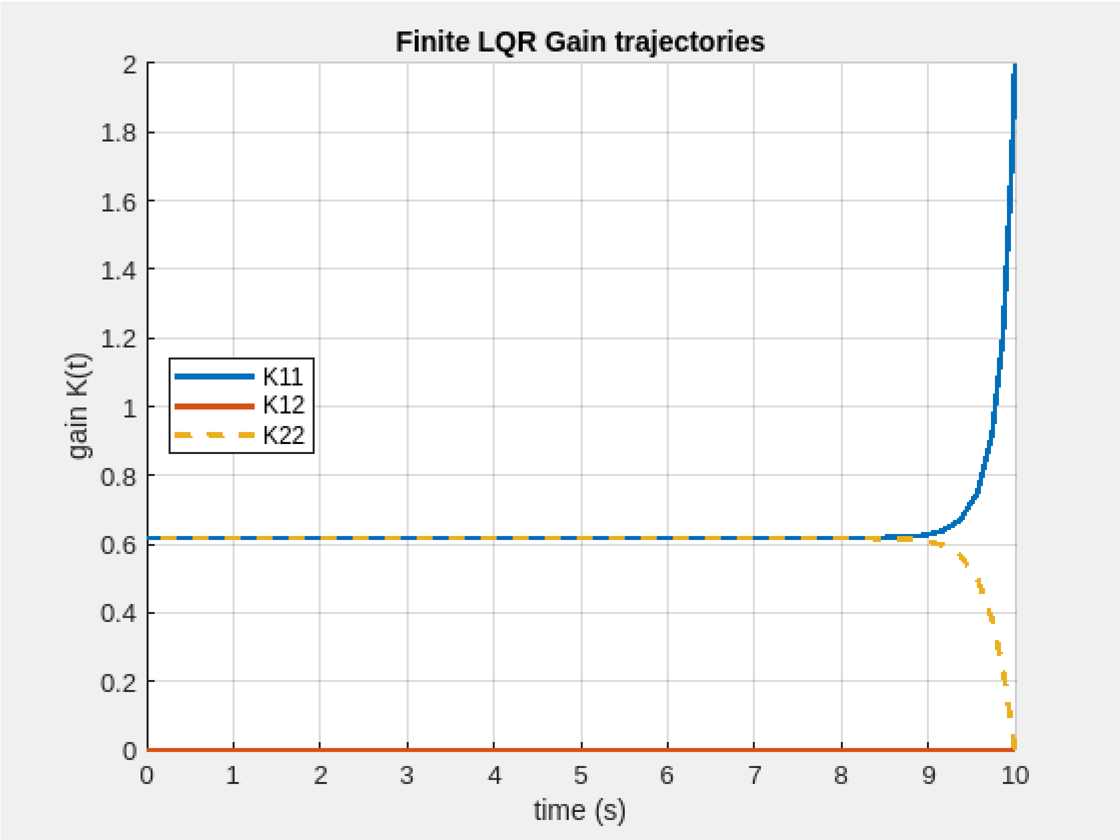
4 - LQR (Finite Horizon)

4.1 - Forward Ricatti Equation

4.2 - Optimal Gain Trajectories

```
tf = 10;
R = eye(2);
B4 = [[2 0];[0 2]];

% simulate the corresponding simulink model
simOut = sim('MAE272_HW5_Q4');
y = simOut.get('yout');
t = simOut.get('tout');

p11_bar = y{1}.Values.Data;
p12_bar = y{2}.Values.Data;
p22_bar = y{3}.Values.Data;

% reverse time (now we go forwards)
t = tf - t;

P = [[p11_bar p12_bar];[p12_bar p22_bar]];
```

```
% Multiply P by R^{-1}*B^T = 2I here
K11 = 2*p11_bar;
K12 = 2*p12_bar;
K22 = 2*p22_bar;

K = [[K11 K12];[K12 K22]];
figure(4)
grid on
hold on
plot(t,K11, 'LineWidth', 2, 'LineStyle', '-', 'DisplayName', 'K11')
plot(t,K12, 'LineWidth', 2, 'DisplayName', 'K12')
plot(t,K22, 'LineWidth', 2, 'LineStyle', '--', 'DisplayName', 'K22')
xlabel('time (s)')
ylabel('gain K(t)')
title('Finite LQR Gain trajectories')
legend('Location', 'west')
```



Published with MATLAB® R2023a