Final Q2

```
A = [[-3/2 0 0 0];
     [0 -2/3 1/3 0];
     [0 1/3 -2/3 0];
     [0 0 0 -1]];
B = [1/2;1/3;1/3;0];
C = [0 2/3 2/3 -1];
D = -1/3;

% state space representation
sys=ss(A,B,C,D);

% identify the basis of each:
ctrb_reduced = rref(ctrb(A,B)); % -> pivot columns identify basis
obsv_reduced = rref(obsv(A,C)); % -> pivot rows identify basis

% throwback to homework 1
fundamental_subspaces(obsv(A,C))
% probably has some bug, I am ignoring the clearly erroneous result ([0 1 0
% 0]')

% confirm [1 0 0 0]^T resides in R
reachable_basis = [1/2 -3/4; 1/3 -1/9; 1/3 -1/9 ; 0 0];
human_reachable_basis = [reachable_basis(:,1)*3 reachable_basis(:,2)*9]
rref([human_reachable_basis [1;0;0;0]])

% transformation matrix to Kalman Decomposition
T = [[0 1 0 0];[1 0 0 1];[1 0 0 -1];[0 0 1 0]];
Tinv = inv(T);

% kalman decomposition
A_tilde = Tinv*A*T
B_tilde = Tinv*B
C_tilde = C*T
D_tilde = D
```

```
E_r =

     0     1     1     0
     0     0     0     1
     0     0     0     0
     0     0     0     0


row_space =

     0     0
     1     0
     1     0
     0     1
```

```
column_space =

    0.6667   -1.0000
   -0.2222    1.0000
    0.0741   -1.0000
   -0.0247    1.0000


left_null_space =

     1     0
     0     1
   -13     4
   -12     3


right_null_space_numerical =

    0.9911   -0.1329
   -0.0940   -0.7008
    0.0940    0.7008
   -0.0000   -0.0000


right_null_space =

     1     0     0
     0     1    -1
     0     0     1
     0     0     0


ans =

    'something is off, the column subspace dimensions do not agree with the
 matrix dimensions'


ans =

    0.6667   -1.0000
   -0.2222    1.0000
    0.0741   -1.0000
   -0.0247    1.0000


human_reachable_basis =

    1.5000   -6.7500
    1.0000   -1.0000
    1.0000   -1.0000
         0         0
```

```
ans =

    1.0000         0   -0.1905
         0    1.0000   -0.1905
         0         0         0
         0         0         0


A_tilde =

   -0.3333         0         0         0
         0   -1.5000         0         0
         0         0   -1.0000         0
         0         0         0   -1.0000


B_tilde =

    0.3333
    0.5000
         0
         0


C_tilde =

    1.3333         0   -1.0000         0


D_tilde =

   -0.3333
```

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