HOMEWORK 1

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Section 12.1

24. Find an equation of the largest sphere with center $(5, 4, 9)$ that is contained in the first octant.

Since the circle must be located in the first octant, 4 is the maximum radius of the circle and still be located in the first octant.
$(x-5)^2 + (y-4)^2 + (z-9)^2 = 4^2 \Rightarrow (x-5)^2 + (y-4)^2 + (z-9)^2 = 16$

Section 12.2

28. What is the angle between the vector and the positive direction of the x-axis? $8\mathbf{i} + 6\mathbf{j}$

$\tan(\theta) = 6/8 \Rightarrow \theta = \arctan(3/4) \approx 36.87°$

Section 12.3

26. Find the values of x such that the angle between the vectors $\langle 2, 1, -1\rangle$ and $\langle 1, x, 0\rangle$ is $45°$.

$\langle 2, 1, -1\rangle \cdot \langle 1, x, 0\rangle = \sqrt{2^2 + 1 + 1} \cdot \sqrt{1 + x^2} \cdot cos(45) \rightarrow 2 + x = \sqrt{6} \cdot \sqrt{1 + x^2} \cdot (\sqrt{2}/2) \Rightarrow x^2 + 4x + 4 = 3(1 + x^2) \Rightarrow 2x^2 - 4x - 1 = 0$

$x = \frac{4 \pm \sqrt{16 - 4(2)(-1)}}{2a} \rightarrow x = 1 \pm \frac{\sqrt{6}}{2}$

Section 12.4

12. Find the vector not with determinants, but by using properties of the cross product? $(\mathbf{i} + \mathbf{j}) \times (\mathbf{i} - \mathbf{j})$

$\mathbf{i} \times (\mathbf{i} - \mathbf{j}) + \mathbf{j} \times (\mathbf{i} - \mathbf{j}) = \mathbf{i} \times \mathbf{i} - \mathbf{i} \times \mathbf{j} + \mathbf{j} \times \mathbf{i} - \mathbf{j} \times \mathbf{j} = 0 - \mathbf{k} + (-\mathbf{k}) + 0 = -2\mathbf{k} = -2\mathbf{k}$

Section 12.5

74. Find the distance between the given parallel planes. $6z = 4y - 2x$, $9z = 1 - 3x + 6y$

set $x = y = 0$, we get: $z = 0$ thus (0,0,0) is a point on the first line. $d = \frac{|1-0+0-0|}{\sqrt{3^2+6^2+9^2}} = \sqrt{14}/42$

Section 13.1

32. At what point does the helix $\mathbf{r}(t) = \langle \sin(t), \cos(t), t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

$(\sin^2(t) + \cos^2(t)) + t^2 = 5 \Rightarrow 1 + t^2 = 5 \Rightarrow t = \pm 2$

Point 1: $(\sin(2), \cos(2), 2)$ Point 2: $(\sin(-2), \cos(-2), -2)$

Section 13.2

16. Find the derivative of the vector function: $\mathbf{r}(t) = t\mathbf{a} \times (\mathbf{b} + t^2\mathbf{c})$

$\mathbf{r}'(t) = [\mathbf{a}] \times (\mathbf{b} + t\mathbf{c}) + [t\mathbf{a}] \times (0 + \mathbf{c}) = [\mathbf{a}] \times (\mathbf{b} + t\mathbf{c}) + [\mathbf{a}] \times (t\mathbf{c}) = [a] \times (\mathbf{b} + t\mathbf{c} + t\mathbf{c}) = \mathbf{a} \times (\mathbf{b} + 2t\mathbf{c}) = [a] \times (\mathbf{b} + t\mathbf{c} + t\mathbf{c})$

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