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Trajectory Planning with Shadow Trolleys for an Autonomous Vehicle on Bending Roads and Switchbacks

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Abstract— **In automated driving, the road geometry information such as waypoints is usually available from previously stored maps. In this paper, we present a scenario based Model Predictive Control (MPC) trajectory planning algorithm that consists of spatial planning with embedded temporal optimization that leverages waypoints information of the road. Our trajectory planning algorithm is structured such that the spatial and temporal planning is integrated so that both the longitudinal and lateral aspects, reflected in the shape and length of the planned trajectory, are dynamically changing to best negotiate the constraints from road curvature and surrounding vehicles.**

The concept of a vehicle connected to shadow trolleys traveling along the rails on the road is introduced. Given waypoints, a reference cubic spline can be constructed to define the rail for trolleys and form a curvilinear coordinate. This concept facilitates the description of vehicle motion, trajectories, and surroundings with respect to the trolley, which is especially convenient for a vehicle traveling on high curvature road and switchbacks. A temporal optimization of the trolley instances is first conducted, which in turn allows us to make a proper approximation and reduce an originally complex nonlinear spatial-temporal optimization problem into one that requires only Quadratic Programming (QP).

We present simulation results of various challenging scenarios on complex road geometry with multiple surrounding vehicles of varying behavior to demonstrate the effectiveness and efficiency of the proposed algorithm. Simulation results show reasonable, flexible, and safe maneuvers.

I. INTRODUCTION

The on-line path planning and control of AVs is a challenging problem since it requires the handling of nonlinearity of the vehicle and accomodate surrounding vehicles while achieving various tasks (e.g., maintaining desired speed, keeping comfort distance to the target vehicle, safely change the lane) in various scenarios on the public road.

One approach is to leverage curve interpolation techniques such as Clothoid curves [1], polynomial curves [2], Bezier curves [3], and spline curves [4]–[7]. The advantage of this approach is that the computational load is low since the curve can be designed by selecting only few control points as parameters. However, the approach does not guarantee the optimality of resulting trajectory. Also, due to the lack of vehicle model, trajectory could be infeasible. Moreover, its temporal planning is to execute the same spatial path with different speed profiles [8]. In other words, the resulting spatial/geometrical trajectory does not vary in response to possible change of surrounding enviriment in time.

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By virtue of the recent development of online optimization solvers [9]–[11], and modeling languages [12]–[14], the optimization based real-time control is gaining popularity [15]–[17].

The aim of this paper is to construct an MPC based trajectory planning algorithm that generates flexible trajectory by coupling spatial and temporal plannings. The proposed algorithm automatically decide the best lane to travel in the structured environment. In a structured environment where lanes are defined, trajectory planning algorithm needs to determine the optimal lane to stay. The optimal lane is, by its nature, an integer number and hence oftentimes formulated as Mixed Integer Problem (MIP) [18] or Nonlinear Problem (NP) [17], which are generally not favorable for real-time applications. These thwarting issues are addressed by introducing Scenario Based MPC where feasible trajectories are evaluated and the best one is chosen. In this approach, both spatial and temporal optimization problems are posed as QPs for computational efficiency. Moreover, the proposed trajectory planning algorithm can handle complex road geometry by creating spline curve that defines the curvilinear coordinate.

II. SYSTEM ARCHITECTURE AND SCOPE

We assume waypoints of the road are given to the path planning task, or specifically, trolley rail/ reference spline generator. The reference spline is the rail for our shadow trolley to travel on, and the curvilinear coordinate along the spline facilitates convenient description of vehicle motion relative to the trolley, overall vehicle trajectory, and surrounding obstacles.

Detailed discussion will be presented in the following subsections where spatial planner determines the path of AV and temporal planner generates speed profile. The online path planner is in the scenario based framework where feasible scenarios are considered. Depending on feasible scenarios, trajectory candidates are generated and the trajectory evaluator chooses the best trajectory among the candidates. Once the best trajectory is chosen, the trajectory followers execute the trajectory by generating control inputs such as steering wheel angle and throttle.

A. Reference Curve/Rail

In order to operate the vehicle, a human driver would try to understand the road geometry ahead. In the automated driving system, this corresponds to constructing a reference spline that represents the shape of the road. One of the techniques that have been successfully used to describe

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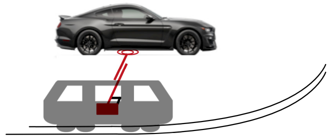


Fig. 1: Conceptually, AV is laterally connected to the shadow trolley.

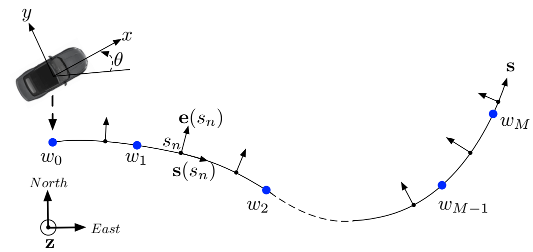


Fig. 2: Curvilinear coordinate that is defined by the spline curve

a complicated shape is to utilize parametric curves [19], [20]. Among them, the class of spline is used in many mobile robots and autonomous vehicle applications due to the simplicity of their construction and the accuracy of the evaluation [4], [5], [21], [22]. In this work, a cubic spline is of particular interest since it is the lowest order one that can guarantees continuity in curvatures of adjacent segments.

A waypoint is an (x, y) pair, where x and y are in an earth-fixed Cartesian coordinate. When multiple waypoints are considered, we use subscript as their index, e.g., $w_n = (w_{n,x}, w_{n,y})$. Suppose the nearest M waypoints, $\{w_1, w_2, \cdots, w_M\}$ are chosen and the current position of AV is projected to the road to construct the reference curve as w_0 in Fig. 2. Then the i^{th} segments of the cubic spline is expressed in the local frame. The reference curve is then the union of each spline segments $\mathbf{x}_r$ and $\mathbf{y}_r$in the local frame This approach has been described in [7] where the spline was used to describe the desired vehicle path. In this paper, the spline is used to construct the road reference curve frame only while leaving complete flexibility for possible vehicle paths.

B. Shadow Trolley

A shadow trolley is defined for the AV in the following way as illustrated in Fig. 1. An imaginary trolley is introduced. It travels on the reference curve/rail (e.g. center line of a lane), as the shadow reflection of our autonomous vehicle. While the shadow trolley's heading is always in the tangential direction of the rail, The relative distance and relative heading angle between the autonomous vehicle and its shadow trolley is not constrained.

C. Coordinates

In this paper, East-North-Up coordinate is chosen as global coordinates that define X and Y directions as East and North, respectively. The local Cartesian coordinate is attached to the vehicle's center of gravity where x and y directions are defined by vehicle's instantaneous longitudinal and lateral directions, respectively.

As AV starts to operate, local coordination must be updated to reconstruct the curvilinear coordinate. Let us define ${}_{j-1}T_j$ be the coordinate transformation from $j-1^{th}$ to the j^{th} local coordinate system. Then the Lie group transformation $SE(2)$ can be computed as

$${}_{j-1}T_j := \begin{bmatrix} \cos\delta_\theta & \sin\delta_\theta & -\delta_x \cos\delta_\theta - \delta_y \sin\delta_\theta \\ -\sin\delta_\theta & \cos\delta_\theta & \delta_x \sin\delta_\theta - \delta_y \cos\delta_\theta \\ 0 & 0 & 1 \end{bmatrix}, \quad (1)$$

where δ_x,δ_y, and δ_θ are increments of x, y and the orientation in each sample time, respectively. Then the transformation between j^{th} local to global coordinate, ${}_jT_{global}$ is performed as follows:

$${}_jT_{global} ={}_{j-1} T_{global} \times {}_{j-1}T_j{}^{-1}. \quad (2)$$

Then it is obvious to compute global to j^{th} local coordinate transformation as ${}_{global}T_j ={}_j T_{global}^{-1}$. Note that the index j is set to $j = 1$ as the autonomous driving mode is engaged and monotonically increased, and hence coordinate transformation between local coordinates are always defined.

In the j^{th} local coordinate, the vector of waypoints, W_j is a finite subset of entire waypoints in the map that is expressed as

$$W^j ={}_j T_{global} \times [w_m, w_{m+1}, \cdots, w_{m+M}], \quad (3)$$

where $w_m = [X_m, Y_m, 1]^T$ is a way point in global coordinate, $M > 0$ is the number of waypoints required to model the road geometry.

Fig. 2 illustrate the curvilinear coordinate system. The longitudinal position s is defined by the traveled distance along the reference curve. Let $\mathbf{z}$ be a out-of-page direction vector, then the lateral direction vector $\mathbf{e}(s)$ is computed as

$$\mathbf{e}(s) = \mathbf{z} \times \mathbf{s}(s), \quad (4)$$

where $\mathbf{s}(s)$ is instantaneous longitudinal direction at a longitudinal position s.

The constant lateral deviation e in $\mathbf{e}(s)$ direction naturally yields the maneuver where the AV drives along the reference curve without explicit steering wheel input, this can be interpreted as a tandem/shadow trolley traveling along the track as in Fig. 1. When e is set to the lane width, it conveniently expresses the reference curve of the adjacent lane in the optimization formulation.

III. Spatial Planning Algorithm

A. Vehicle and Environment Modeling

The motivation of utilizing the curvilinear coordinate system is to conveniently describe the trajectory and the vehicle model. In the curvilinear system, when the lateral deviation is a constant, it naturally describes a lane and it simplifies the optimization formulation as discussed in the next section. We assume that the motion of both AV and target vehicles are governed by the unicycle model (5).

$$\dot{s}(t) = v(t)\cos\theta(t), \quad \dot{e}(t) = v(t)\cos\theta(t), \tag{5a}$$
$$\dot{\theta}(t) = \omega(t), \quad \dot{v}(t) = \alpha(t), \tag{5b}$$

where states s and e denote the longitudinal and lateral positions of the vehicle, θ is a heading angle with respect to the reference curve, v is the speed of the vehicle. The continuous vehicle model is discretized as follow.

$$z_{k+1} = z_k + f(z_k, u_k)T, \tag{6}$$

where $z_k = [s_k, e_k, \theta_k, v_k]^T$ is the state of the vehicle at time k, $f(z_k, u_k) = [v_k\cos\theta_k, v_k\sin\theta_k, \omega_k, \alpha_k]^T$ and T is a sampling time. The control input $u_k = [\alpha_k, \omega_k]^T$ consists of the linear acceleration α_k and angular velocity ω_k. Then the nonlinear discrete model (6) is linearized at operating points $\bar{p}_k = [\bar{\theta}_k, \bar{v}_k]^T$, where $[\bar{p}_1, \bar{p}_2, \cdots, \bar{p}_N]$ is a solution of (8a)-(8d) from the previous step. The set of operaing points are illustrated in Fig. 2 with transparent trolleys.

$$z_{k+1} = A_k z_k + Bu_k + C_k, \tag{7}$$

$$A_k = \begin{bmatrix} 1 & 0 & -\bar{v}_k\sin(\bar{\theta}_k)T & \cos(\bar{\theta}_k)T \\ 0 & 1 & \bar{v}_k\cos(\bar{\theta}_k)T & \sin(\bar{\theta}_k)T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & T \\ T & 0 \end{bmatrix},$$

$$C_k = \left[\bar{v}_k\sin(\bar{\theta}_k)\bar{\theta}_k T, \; -\bar{v}_k\cos(\bar{\theta}_k)\bar{\theta}_k T, \; \Delta\theta_k^{ref}, \; 0\right]^T.$$

Note that the term $\Delta\theta_k^{ref}$ in C_k is the change of tangential direction of reference curve (trolley rail) between k and $k+1$ steps.

B. MPC formulation

The proposed trajectory planning algorithm consists of spatial and temporal planner such that the spatial planner includes temporal planner. The optimization problem in each planning layers is posed as a QPs. Also, we leverage scenario based approach in spatial MPC layer. Let J be the number of such scenarios, then the AV considers feasible scenarios depending on road geometry.

We now present spatial MPC formulation. Let N be the prediction horizon. The spatial planning MPC formulation is posed as the following QP:

$$\min_{u \in U} \sum_{k=1}^{N} \left\{ J(e_k, e_{ref}^j, v_k, v_k^{ref}, \omega_k, \alpha_k) \right\} \tag{8a}$$
$$s.t. \quad z_{k+1} = A_k z_k + Bu_k + C_k, \tag{8b}$$
$$\alpha_{min} \le \alpha_k \le \alpha_{max}, \tag{8c}$$
$$\omega_{min} \le \omega_k \le \omega_{max}, \tag{8d}$$
$$0 \le v_k \le v_{max} \tag{8e}$$
$$k = 1, \ldots, N-1, \; j = 1, \ldots, J.$$

Let W be the width of the lane, then the lateral reference valus $e_{ref}^j \in e_{ref}$ are defined as follows:

$$e_{ref} = \{-W, 0, W\}, \tag{9}$$

As one can observe, we assume the maximum number of the lane is 3 and scenarios $J = 3$. However, this assumption can be easily relaxed by expanding (9) since the number of lanes is a generally small integer number, say less than 10. The equation (8b) is the constraint imposed by the vehicle kinematics. The matrices A_k, B_k, and C_k are updated in each prediction steps so that the motion of AV can be faithfully described in high curvature roads, including bending roads and switchbacks, breaking the low curvature limitation [23]. The constraints (8c) and (8d) are actuator (linear acceleration and turning rate) limits of the vehicle.

The objective function (8a) consists of several different terms to achieve the desired trajectory of the AV. Minimizing their sum drives the vehicle to its desired trajectory appropriately. The vector v^{ref} is obtained by solving temporal planning problem, which is discussed in the following section.

IV. Temporal planning algorithm

The goal of the temporal planning is to generate the speed profile such that the desired longitudinal position is achieved by smoothly regulating the speed. Such a speed profile is then fed to the spatial planning as v^{ref} in (8a). Since only the longitudinal direction is concerned, system state consists of longitudinal position and speed $z^t = [s^t, v^t]^T$ only. The superscript t is used to distinguish variables from spatial planning. The vehicle model is then represented as follows.

$$z_{k+1}^t = A^t z_k^t + B^t a_k, \tag{10}$$

where $A^t = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$, $B^t = [0, T]^T$. One can observe that (10) is a special case of (7) when $\bar{\theta}_k = 0$ and the state is reduced to $[s^t, v^t]^T$.

The temporal planning MPC formulation is posed as a following QP:

$$\min_{\alpha \in A} \sum_{k=1}^{N} \left\{ J(s_k^t, s_k^{ref}, \xi_k, \zeta_k, \alpha_k^t, \alpha_{k-1}^t) \right\} \tag{11a}$$
$$s.t. \quad z_{k+1}^t = A_k^t z_k + B_k^t \alpha_k^t, \tag{11b}$$
$$\alpha_{min} \le \alpha_k^t \le \alpha_{max}, \tag{11c}$$
$$s_k^{ref} - s_k^t \le \xi_k, \tag{11d}$$
$$v_k^t - v_k^{bound} \le \zeta_k, \tag{11e}$$
$$0 \le v_k^t \le v_{max}$$
$$k = 1, \ldots, N-1.$$

Two vectors $\xi = [\xi_1, \xi_2, \cdots, \xi_N]$ and $\zeta = [\zeta_1, \zeta_2, \cdots, \zeta_N]$ are slack variables. The purpose of slack variables is to relax the inequality constraints to avoid the numerically infeasible state for the solver of the optimization problem albeit at the expanse of the increased objective function value. By minimizing the the objective function (11a), the longitudinal positions smoothly track the vector s^{ref}. The algorithm that constructs the longitudinal reference position vector s^{ref}, v^{bound} is presented in the following section.

A. Longitudinal reference position

When other vehicles, a.k.a. target vehicles are present in front of AV, the desired longitudinal position of AV is determined by the expanded target vehicle model - the target vehicle is expended longitudinally in both front and back of the vehicle with the length of $v^{tgt}T^{gap} + S^{static}$, where v^{tgt} is the speed of the target vehicle and T^{gap} is the desired time gap between AV and the target vehicle. The static gap S^{static} is introduced to obtain the safe gap in case of $v^{tgt} = 0$. Let longitudinal position of the target vehicle at time k s_k^{tgt}. Then the desired longitudinal position is defined as $l_k = s_k^{tgt} - (v^{tgt}T^{gap} + S^{static})$.

The following algorithm describes the construction of s^{ref}. We assume that the desired acceleration when there are no target vehicles, a_{des}, desired time gap T^{gap}, and static gap S^{static} are predetermined.

Result: s^{ref}, v^{bound}
Data: v, α^{des}, v^{tgt}, T^{gap}, S^{static}
for $k \leftarrow 1$ **to** N **do**
  $v_{k+1}^t = \min(v_k^t + \alpha^{des}T, v^{tgt})$;
  $s_{k+1}^t = s_k^t + v_k^t T$;
end
if $PossibleCollision$ **then**
  $s_k^{ref} = l_k^{ref}$, $v_k^{bound} = v^{tgt}$;
else
  $s_k^{ref} = s_k^t$, $v_k^{bound} = v_k^t$;
end

Algorithm 1: Longitudinal reference position

The logical flow of the algorithm is as follows. First, based on AV's current position and speed, the future positions and speeds are propagated in time and longitudinal space, respectively. If the future position is collision-free, propagated states are set to reference states. Otherwise, if possible collision occurs at time k, desired position l_k and target vehicle speed v^{tgt} are set to s_k^{ref} and v_k^{bound}.

B. Speed Bound

We may want to adjust our speed bound, v^{bound}, according to the road geometry in addition to the leading target vehicle speed. In this case, we will feed in an adjusted target speed, v^{tgtadj}, in place of v^{tgt}, into the above algorithm described in Secion A. In the curvilinear coordinate, the lateral component of the acceleration is $v^2\kappa$, where κ is a curvature. The curvature is analytically computed as in [24].

Therefore, given average curvature κ_{avg} of M waypoints from (3) and maximum lateral acceleration as $\bar{a}_{lat}$ and the speed limit as $\bar{v}_{max}$ then the v^{tgt_adj} is adjusted as follows:

$$v^{tgtadj} = \min(\sqrt{\bar{a}_{lat}/\kappa_{avg}}, \bar{v}_{max}, v^{tgt}). \tag{12}$$

V. Trajectory Evaluation

Depending on feasible scenarios, J number of trajectories are created by the online planning algorithm. The trajectory evaluator is responsible to select the best trajectory among them. The best trajectory is chosen by the following rule.

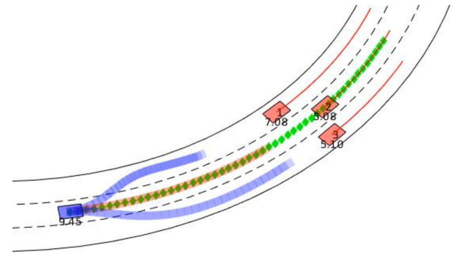


Fig. 3: A screen capture of simulation. Trajectories for each lane choice are different, reflecting the different positions and velocities of the corresponding target vehicles.

$$j^* = \arg \min_{j \in \{1, \cdots, J\}} w_p P_j^p(\sigma) + w_u P_j^u + w_s P_j^s + w_h P_j^h(j^0) \tag{13}$$

The cost function of (13) consists of four terms, with ws represent weights for each terms. $P_j^p(\sigma)$ is a penalty of the proximity to the target vehicles. Let Q be a total number of surrounding vehicles. The penalty is computed as a sum of Gaussian distribution in lateral distance to each target vehicles, indexed by q.

$$P_j^p(\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \sum_{q=1}^{Q} \sum_{k=1}^{N} \exp \frac{(e_k^j - e_k^q)^2}{2\sigma^2} \tag{14}$$

P_j^u is the lateral control effort to generate j^{th} trajectory, which is a sum of all lateral control inputs, ω_k in (8d). This term prevents AV from unnecessary lane change. P_j^s is the penalty for speed archived of j^{th} trajectory at the end of the horizon, $P_j^s = (v^{tgt} - v_N^j)^2$. Finally, P_j^h is the penalty for changing the index of the best trajectory from the previous decision, j^0. The penalty is computed as $(j - j^0)^2$.

VI. Simulation Results

The MPC formulations (8a)-(8d) and (11a)- (11e) can be solved with QP solver via the toolbox [12] in Matlab™. We present several simulation results that is focused on challenging situations.

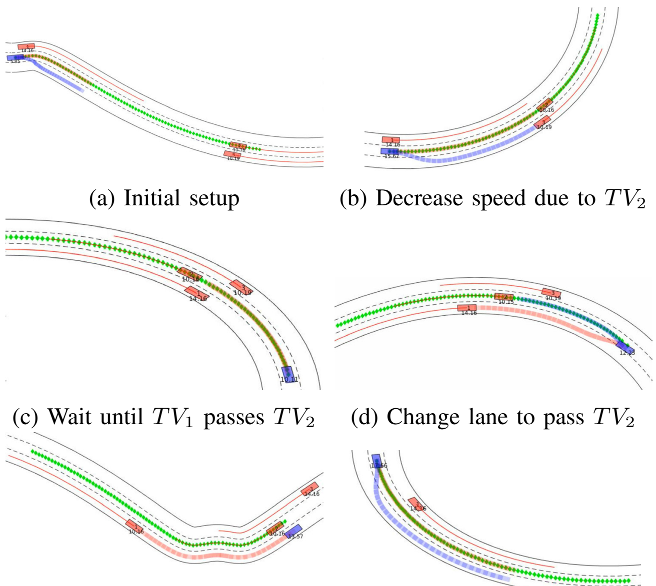
Fig. 3 shows simulation setup. Blue and red rectangles represent the AV and target vehicles. Values below vehicles are their speeds in m/s unit.

Among three trajectory candidates, the best trajectory that is chosen by (13) is colored in red, others are colored in blue. Red curves attached to target vehicles represents their expected trajectories. Green marks in the center lane are waypoints and green curves that connect marks are the reference curve.

A. Simulation No.1

Fig. 4a shows initial set up of the scenario. The speed of TV_1 is set to $15m/s$, both TV_2 and TV_3 are set to $10m/s$, and the initial speed of AV is $5m/s$. TV_1 is in the first lane,

TV_2 and TV_3 are in second and third lanes, respectively. TV_2 and TV_3 are ahead of TV_1 and AV. Since the speed of TV_2 is greater than the speed of AV and the distance is large enough, the AV increases the speed. As AV approaches to the TV_2, it decreases the speed. Due to TV_1, the penalty of trajectory for lane change left is high enough (13) and hence it is discarded (Fig. 4b). The AV waits until TV_1 passes TV_2 and TV_3. As TV_1 passes, the distance between AV and TV_1 is large and hence the trajectory of lane change left becomes the best trajectory. Therefore, the AV decided to make a lane change left (4d). The AV reduces the speed as the κ_{avg} increased (4e). Lastly, the AV increases the speed as κ_{avg} decreased and changes lane to pass TV_1 (Fig. 4f).



(a) Initial setup (b) Decrease speed due to TV_2

(c) Wait until TV_1 passes TV_2 (d) Change lane to pass TV_2

(e) Decrease speed due to curvature and TV_1 (f) Change lane to pass TV_1

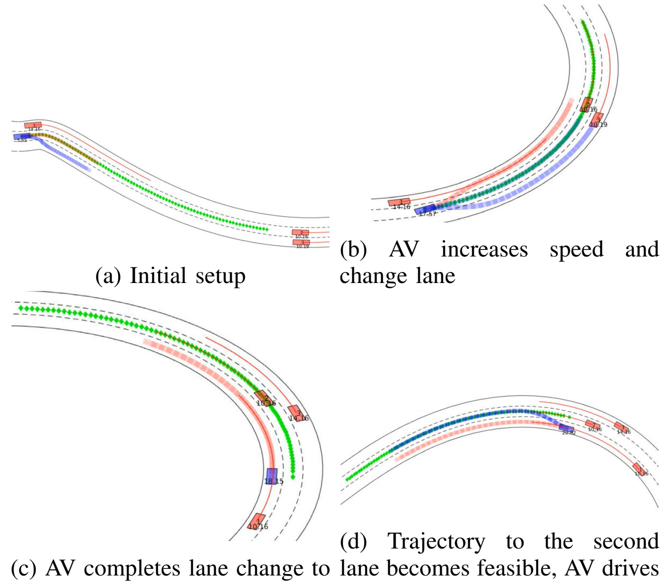
Fig. 4: Consecutive screen captures of the second scenario on the test track No.2

B. Simulation No.2

In this scenario, initial setup is identical to the first scenario except the initial longitudinal position of TV_2 and TV_3 are farther (compare Fig. 4a and Fig. 5a). In this set up, rather than wait until TV_1 passes TV_2, the AV decided to increase the speed to cut in front of TV_1 (compare Fig. 4b and Fig. 5b). The slightly larger distance between TV_1 and TV_2 allows the left lane change as a feasible scenario and corresponding trajectory candidate is chosen as the best trajectory as it has low P^s (13),i.e., it can achieve desired speed by changing lane without sacrificing the speed.

C. Simulation No.3

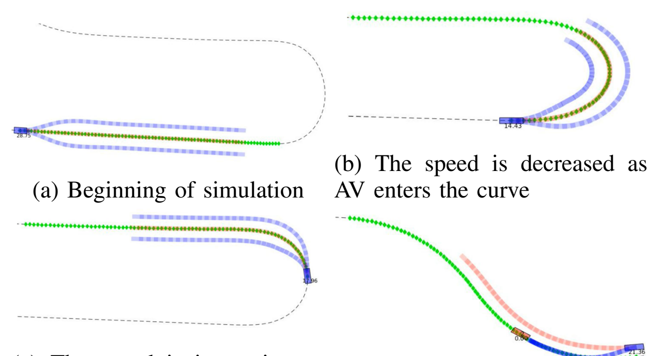
In this scenario, initial speed set to $30 m/s$. The AV decreases the speed as it enters a hair-pin turn (Fig. 6a and Fig. 6b). As the AV exits the curve, it starts to increase the speed (Fig. 6c). Then AV changed to the right most lane due to the obstacle/stopped vehicle in the middle lane (Fig. 6d).



(a) Initial setup (b) AV increases speed and change lane

(c) AV completes lane change to the left lane by cutting in (d) Trajectory to the second lane becomes feasible, AV drives freely

Fig. 5: Consecutive screen captures of the second scenario



(a) Beginning of simulation (b) The speed is decreased as AV enters the curve

(c) The speed is increasing as the AV exits the curve (d) AV changed to right lane

Fig. 6: Consecutive screen captures of the third scenario on the test track No.3

VII. Conclusion

In this paper, we present an MPC based online trajectory planning algorithm for autonomous vehicles in structured environments. The proposed planning algorithm generates trajectory candidates by solving spatial and temporal optimization problem for each feasible scenarios. The integration of spatial and temporal optimization problem brings flexibility to the shape of trajectories, without increasing much complexity. The best candidate is chosen according to the trajectory evaluation rule and executed in a receding horizon scheme. A curvilinear coordinate that utilizes a cubic spline is introduced. The first benefit of such coordinate is that it provides convenient expression of lateral position and therefore, the optimization problem can be posed as a favorable form, QP. Another benefit is that it provides robustness to the waypoints by creating spline curves that provide continuity in curvature.

We provide simulation results in various scenarios in a range of road geometries, including bending roads and switchbacks. The results show that the presented trajectory algorithm generates safe and comfortable trajectories for AV.

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