

# Intelligent vehicle sideslip angle estimation considering measurement signals delay

Wei Liu, Lu Xiong, Xin Xia and Zhuoping Yu

**Abstract**—Considering vehicle sideslip angle estimation difficulty under severe driving conditions with dynamic model based method due to vehicle nonlinear characteristic and parameter uncertainty, a novel kinematic model based method is proposed with the fusion of intelligent vehicle sensors. The state space models of the vehicle yaw angle and the roll angle are constructed based on the IMU and the lateral arrangement of the dual-GPS. In order to reduce the weight of the previously estimated value, an adaptive fading Kalman filtering algorithm is adopted to improve the filtering effect on the yaw and roll angle. A nonlinear adaptive observer is constructed to estimate the vehicle sideslip angle with the integration of the road line information from the camera, velocity from the GPS and acceleration/ angular velocity from the IMU. Furthermore, compared with the IMU, the information obtained from the GPS and the camera can't be utilized directly as large measurement delay. Thus, an observer-predictor is developed with multi-sensor fusion to handle the measurement delay problem. Finally, the proposed algorithm is validated through co-simulation under different maneuvers.

## I. INTRODUCTION

There is no doubt that vehicle safety is of great importance all the time [1]. And the vehicle lateral stability has drawn much attention during the past few years. As is known to us all, the vehicle sideslip angle which is the angle between the direction of the vehicle and the vehicle longitudinal axis is an important signal reflecting vehicle lateral stability. Thus, compared with yaw rate tracking control solely, the yaw rate and sideslip joint control architecture can effectively improve the vehicle lateral stability especially under severe driving conditions [2-4]. The vehicle sideslip angle can be measured through optical sensor and RT3000 directly, but it is too expensive to be used in mass-production vehicles. As a result, researchers worldwide have carried out comprehensive research in estimating vehicle sideslip angle. Existing studies of vehicle sideslip angle estimation could be classified into three categories: kinematic model based method, dynamic model based method, and fusion method.

The kinematic model based method, which didn't rely on the vehicle model and the tire model, was based on the on-board sensors such as IMU, wheel speed sensor and steering wheel angle sensor. In [5], sideslip angle estimation algorithm was conducted by direct integration with the use of

the acceleration sensor and yaw rate sensor. It's a fact that kinematic model based method depended on the sensor information strictly. Long time integration of the acceleration signal directly without the consideration of the error calibration would degrade estimation results as there were drift and noise for low-cost IMU.

Compared with the kinematic model based method, the dynamic model based method showed high accuracy in linear region, and it relied less on sensors signal. In [4], an arctangent model was adopted to describe the nonlinear characteristic of the lateral tire force and the tire slip angle. Based on a nonlinear single-track vehicle model, a Luenberger observer was constructed to realize sideslip angle estimation. To adaptive tire-road friction, an adaptive sideslip angle high-gain observer which based on input-output feedback linearization theory was put forward in [6]. In [7], the bounded Jacobian observer based on nonlinear vehicle model was designed, and the observer gain could be calculated by LMI toolbox. However, dynamic model based method is sensitive to the model parameter variation such as vehicle mass, road friction, what's more, it's difficult to ensure the accuracy of the dynamic model under severe driving conditions.

In order to make up for the respective disadvantages of the kinematic model based method and the dynamic model based method, some researchers designed fusion method by combining with above two methods [8], [9]. The kinematic model based method and the dynamic model based method were integrated from the frequency domain in [8]. A variable structure extended Kalman filter combined with direction integration was established to estimate vehicle sideslip angle, and a damping term was adopted to eliminate the accumulated error of low-cost sensors in [9]. With the rapid development of intelligent vehicles for both academia and industry [10], some researchers have involved intelligent vehicle sensors such as the GPS and camera into vehicle sideslip angle estimation [11-13]. In [11], a two-antenna GPS receiver integrated with INS was used to take into account the effect of road side slope and vehicle roll on sideslip estimation, but the effect of GPS measurement delay was ignored. A vehicle kinematic relationship was established to estimate vehicle sideslip angle with two GPS receivers installed at the front and rear ends of the vehicle in [12], and a "measurement shifting" technique was adopted to handle GPS measurement delay. In [13], as measurement delay from camera was relative large for signal processing and transmission, a system model integrating camera with traditional bicycle-model was designed by the delay-augmented state space equation to estimate vehicle sideslip angle estimation, and simulations and experiments

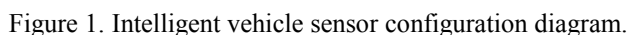
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To further enhance the accuracy of the vehicle sideslip angle estimation, this paper aims to design a kinematic based method with the fusion of IMU/GPS/camera, which could avoid imprecise modeling of tire and vehicle models under extreme conditions. The work lies in the following aspects: (1) Vehicle yaw angle and roll angle estimation algorithms are modelled by using an adaptive Kalman filter with fading factors; (2) A nonlinear observer is proposed to estimate the vehicle sideslip angle; (3) With the consideration of signals' measurement delay of the GPS and the camera, an observer-predictor framework is employed to take full use of the slow information source.

## II. SENSORS CONFIGURATION AND SYSTEM MODEL

As is shown in Figure 1, the intelligent vehicle is equipped with IMU, dual-GPS and camera. The IMU could measure the information of vehicle triaxial acceleration and triaxial angular velocity in the body frame. The dual-GPS is perpendicular to the longitudinal axis of the vehicle. The vehicle roll angle could be obtained when the GPS measurement is updated. Furthermore, the GPS can also get the vehicle velocity of north/east direction and the vehicle yaw angle. The yaw angle is referred to as the angle between the longitudinal axis of vehicle coordinate system and that of the geodetic coordinate system. In addition, the lateral distance between the preview point and the road right line could be obtained by the camera.



As the measurement signal of the low-cost IMU contains drift and noise, it needs to be filtered to improve the measurement accuracy. Assuming that the drift is a constant value, and the noise is the Gauss white noise, the relationship between the real value and the measurement value of the yaw angle can be given by the following equation.

Where  $\dot{\psi}_m$  is the yaw rate measured by the IMU,  $\dot{\psi}$  is the real value of the yaw rate, and  $\dot{\psi}_{bias}$  is the constant drift of the measurement signal.

Where  $\psi$  is the yaw angle,  $\hat{\beta}$  is the sideslip angle which will be estimated in the following section, and  $\varphi_{gps}$  is the course angle measured by the GPS.

Where  $w_\psi$  is the Gauss white noise of the measurement signal, and  $v_\phi$  is the yaw angle measurement noise from the GPS.

Where  $\dot{\phi}_m$  is the roll rate measured by the IMU,  $\dot{\phi}$  is the real value of the roll rate, and  $\dot{\phi}_{bias}$  is the constant drift of the measurement signal.

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Where  $w_\phi$  is the Gauss white noise of the measurement signal, and  $v_\phi$  is the roll angle measurement noise from the GPS.

As for the linear systems (3) and (5), the Kalman filter is adopted to estimate the vehicle roll angle and yaw angle. Considering the fast time-varying characteristics of the vehicle under severe driving conditions, the current observed data should be fully utilized. In this paper, an adaptive fading Kalman filter is adopted to reduce the weight of the past information. Thus, the effect of the estimation algorithm would be improved. The estimation algorithm is as follows [14].

$$\begin{aligned} X_{k+1|k} &= \Phi_{k+1} X_{k|k} + H \cdot u_k \\ P_{k+1|k} &= \lambda_{k+1} \Phi_{k+1} P_k \Phi_{k+1}^T + Q_k \\ K_{k+1} &= P_{k+1|k} C^T (C P_{k+1|k} C^T + R_{k+1})^{-1} \\ \hat{X}_{k+1|k+1} &= X_{k+1|k} + K_{k+1} (Y_{k+1} - C \hat{X}_{k+1|k}) \\ P_{k+1|k+1} &= (I - K_{k+1} C) P_{k+1|k} \end{aligned} \quad (6)$$

Where  $\lambda_{k+1}$  is the adaptive fading factor which needs to be designed,  $Q_k$  is the covariance of system noise, and  $R_{k+1}$  is the covariance of observation noise.

The predicted residual can be given by the following equation.

$$v_k = y_k - C \cdot x_{k|k-1} \quad (7)$$

When the system becomes stable,  $v_k$  satisfies normal distribution, which can be described as

$$v_k \sim N(0, C P_{k|k-1} C^T + R_k) \quad (8)$$

Thus, the following equation formula satisfies the chi-square distribution.

$$\gamma_k = v_k^T [C(\lambda_k \Phi_{k|k-1} P_k \Phi_{k|k-1}^T + Q_{k-1}) C^T + R_k]^{-1} v_k \sim \chi^2(m) \quad (9)$$

The optimal filtering time is verified by the following criteria.

$$\varsigma = \frac{\gamma_k}{\varepsilon} = \begin{cases} \geq 1 & \text{Optimal} \\ < 1 & \text{Not optimal} \end{cases} \quad (10)$$

Assuming that  $m=1$  and confidence level is 99%, we can obtain that  $\varepsilon$  is 6.64 by looking up the table.

At epoch  $k$ ,  $A_k$  and  $B_k$  can be represented as follows.

$$\begin{aligned} A_k &= \Phi_{k|k-1} P_{k-1} \Phi_{k|k-1}^T \\ B_k &= C Q_{k-1} C^T + R_k \end{aligned} \quad (11)$$

When  $\frac{[v_i(k)]^2}{C_{ii} A_{ii}(k) \varepsilon} - \frac{B_{ii}(k)}{A_{ii}(k)} > 0$ , ( $i=1, 2, 3 \dots$ ), the adaptive fading factor can be expressed as.

$$\lambda_i = \max(1, \sqrt{\frac{[v_i(k)]^2}{C_{ii} A_{ii}(k) \varepsilon} - \frac{B_{ii}(k)}{A_{ii}(k)}}) \quad (12)$$

Where  $A_{ii}(k)$ ,  $B_{ii}(k)$  and  $C_{ii}$  are the diagonal element of the  $i$ th row. In order to avoid the filtering effect becoming worse, the following formula should be satisfied at the same time.

$$\lambda_i = \min(1.5, \sqrt{\frac{[v_i(k)]^2}{C_{ii} A_{ii}(k) \varepsilon} - \frac{B_{ii}(k)}{A_{ii}(k)}}) \quad (13)$$

Then, by combining (12) and (13), the adaptive fading factor can be obtained.

When  $\frac{[v_i(k)]^2}{C_{ii} A_{ii}(k) \varepsilon} - \frac{B_{ii}(k)}{A_{ii}(k)} \leq 0$ , ( $i=1, 2, 3 \dots$ ) or the corresponding state is not observable, the value of the adaptive fading factor is as follows.

$$\lambda_i = 1 \quad (14)$$

### C. Sideslip angle estimation with compensation of roll angle

In most cases, the acceleration signals measured by the IMU also contain the gravity component caused by the roll and pitch motion of the vehicle in addition to the drift and noise from the sensor itself. As the influence of the lateral acceleration signal quality on the sideslip angle estimation is usually much greater than that of the longitudinal acceleration signal, proper measures should be taken to compensate the influence of the roll angle. The relationship between the acceleration measurement value and the real value can be written as follows.

$$a_{y,m} = a_y + g \cdot \sin \phi \cdot \cos \theta + b_{a_y} + w_{a_y} \quad (15)$$

Where  $a_{y,m}$  is the vehicle lateral acceleration measurement value,  $\theta$  is the vehicle pitch angle which is neglected in this paper,  $b_{a_y}$  is the constant drift value of the IMU, and  $w_{a_y}$  is measurement noise; therefore, equation (15) can be simplified as follows.

$$a_{y,c} = a_{y,m} - g \cdot \sin \hat{\phi} - b_{a_y} \quad (16)$$

Where the roll angle is estimated by the above session.

The relationship between the vehicle lateral velocity and the lateral distance obtained by the camera can be expressed as follows.

$$\dot{y}_f = v_y \cos \psi + v_x \sin \psi + \dot{\psi} \cdot l_{cf} \cdot \cos \psi \quad (17)$$

By integrating the IMU, GPS and camera, the state space equation of vehicle longitudinal velocity and lateral velocity can be obtained as follows [15].

$$\begin{bmatrix} \dot{y}_f \\ \dot{v}_x \\ \dot{v}_y \end{bmatrix} = \begin{bmatrix} 0 & \sin \psi & \cos \psi \\ 0 & 0 & \dot{\psi} \\ 0 & -\dot{\psi} & 0 \end{bmatrix} \begin{bmatrix} y_f \\ v_x \\ v_y \end{bmatrix} + \begin{bmatrix} \dot{\psi} \cos \psi & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_{cf} \\ a_{x,m} - b_x \\ a_{y,c} \end{bmatrix} \quad (18)$$

$$\begin{bmatrix} y_f \\ v_e \\ v_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & -\sin \psi \\ 0 & \sin \psi & \cos \psi \end{bmatrix} \begin{bmatrix} y_f \\ v_x \\ v_y \end{bmatrix}$$

The feedback factor is designed by the Lyapunov theory.

### III. MULTI-SENSOR FUSION FRAMEWORK

Although the measurement signal of IMU includes measurement drift and noise as well as the signal-to-noise ratio is relatively low, it shows good real-time performance. On the contrary, the signal-to-noise ratio of the GPS is relatively high, but since the original signal needs to be processed and transmitted, measurement delay would take place, which is much larger than the control period of the vehicle that cannot be ignored. The delay time of the GPS and camera is expressed as  $\tau$  while  $T$  denotes the control period of the vehicle. Thus, at time  $t$ , the vehicle state information obtained from the GPS and camera is at time  $(t - \tau)$ . To solve this problem, an observer-predictor is adopted [16].

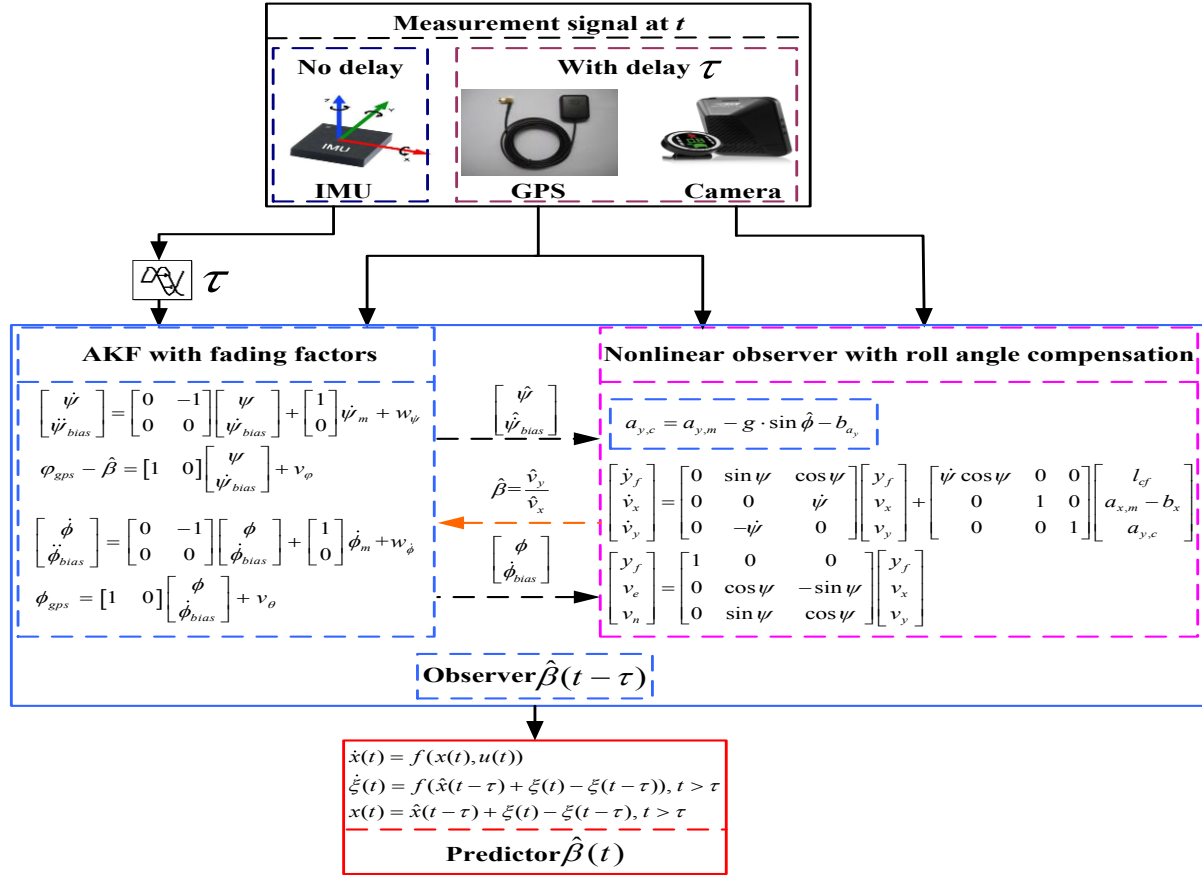


Figure 2. Fusion estimation algorithm block diagram.

For the nonlinear system,

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)) \\ \dot{\xi}(t) &= f(\hat{x}(t - \tau) + \xi(t) - (t - \tau)), t > \tau \\ x(t) &= \hat{x}(t - \tau) + \xi(t) - (t - \tau), t > \tau \end{aligned} \quad (19)$$

Where  $\xi(t)$  belongs to intermediate variable, and  $u(t)$  denotes the current control input of the system. The state of  $\hat{x}(t - \tau)$  can be estimated by the above session, then the current state  $\hat{x}(t)$  could be predicted by the equation (19).

It can also be proved that the convergence property of the algorithm at time  $t$  is consistent with the convergence characteristic of the time  $(t - \tau)$ .

The fusion estimation algorithm is shown in Figure 2. First, the signal from the IMU is delay by  $\tau$ , thus the information from the IMU, GPS and camera can be consistent in time domain of  $(t - \tau)$ . Then, the adaptive fading Kalman filter is used to estimate the yaw angle, roll angle, yaw rate drift and roll rate drift of the vehicle at time  $(t - \tau)$ , which are used as the input information of the sideslip angle estimator. At the same time, the sideslip angle which is as the input information of the yaw angle estimator is obtained by the sideslip angle

estimator at time  $(t - \tau)$ . As a result, the interconnected observer is constructed. An observer-predictor is developed to predict the current vehicle sideslip angle finally.

Assuming that the two adjacent sampling interval of the GPS and the camera is  $K$  times that of the vehicle control period of  $T$  ( $K$  is a positive integer which is larger than 1), and the GPS and camera signals are updated at time  $t_0$ . During the period of  $t_0$  to  $t_0 + KT$ , it needs to modify the observer. This research proposes to keep the updated signal for  $s$  seconds, which can be calibrated by the simulation. On the contrary, when  $(t_0 + s) < t < (t_0 + KT)$ , we consider that there is no updated signals from the GPS and camera.

#### IV. PERFORMANCE EVALUATION

##### A. Simulation Settings and Scenarios

In this paper, the simulation scenarios are based on the Matlab/Carsim&PreScan co-simulation platform, where PreScan can be used for designing and evaluating intelligent vehicle system that equipped with GPS and camera. To simply the problem, we assume that the sampling period of the GPS is equal to that of the camera. Some key parameters of the intelligent vehicle are presented in Table 1.

TABLE 1. KEKY PARAMETERS OF THE INTELLIGENT VEHICLE.

Parameter	Value	Unit
Measurement delay of GPS	200	ms
Measurement delay of camera	200	ms
Sampling period of GPS	200	ms
Sampling period of camera	200	ms
Distance between COG and Preview point	10	m

##### B. Simulation Results

Double-lane change and slalom test are carried out at a velocity of 90 km/h, and the road surface friction coefficient is 0.8. The characteristics of the vehicle lateral acceleration are showed in Figure 3 and Figure 5. The peak value of the vehicle lateral acceleration is close to 0.6g, which means that the vehicle is working under severe driving condition considering the road coefficient. As can be seen from Figure 4 and Figure 6, the proposed observer can estimate the vehicle sideslip well in general. However, when  $t=3s$  in Figure 4 and  $t=4s$  in Figure 6, the estimated error is relatively large, the reason of which is that the error would integrates as no feedback of the measurement signal during the adjacent sampling time of the GPS and camera.

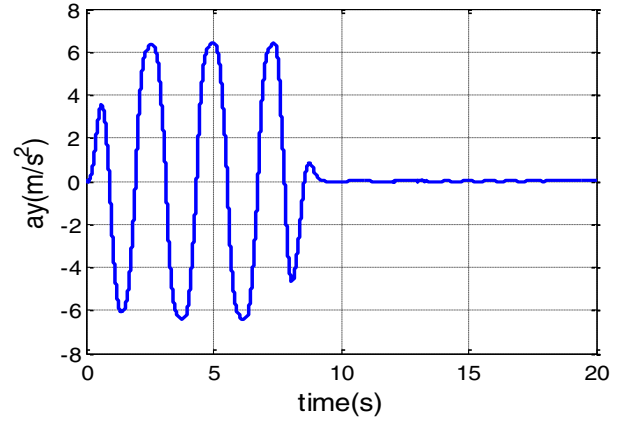


Figure 3. Lateral acceleration of slalom simulation.

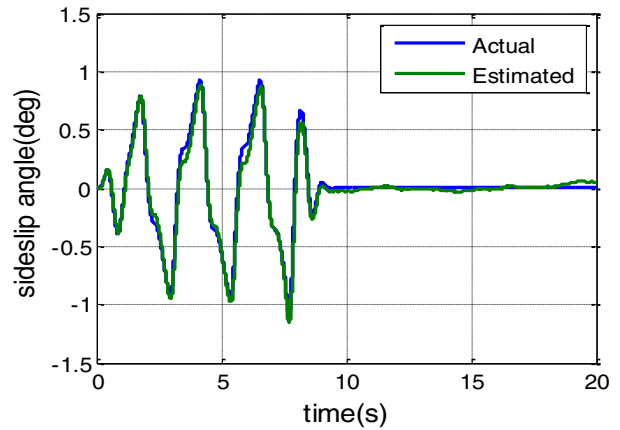


Figure 4. Sideslip angle value of slalom simulation.

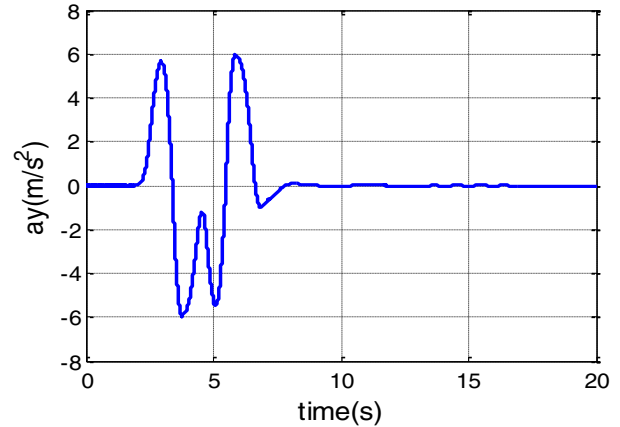


Figure 5. Lateral acceleration of double-lane change simulation.

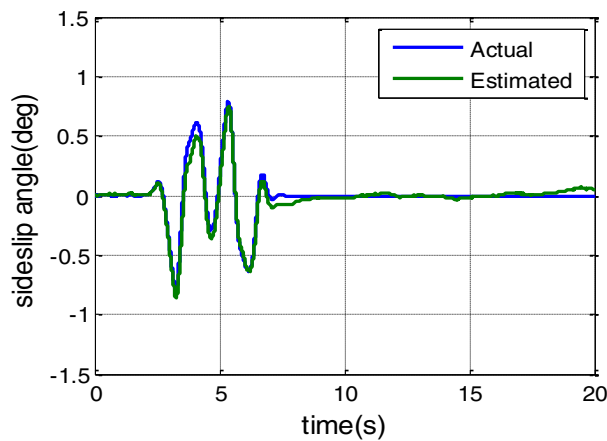


Figure 6. Sideslip angle result of double-lane change simulation.

## V. CONCLUSION AND FUTURE WORKS

In this paper, a novel intelligent vehicle sideslip angle estimation method considering measurement delay was proposed. Based on the IMU characteristics, the vehicle yaw angle and roll angle were estimated using an adaptive Kalman filter with fading factors, and the measured value of acceleration signals were calibrated as well. With the fusion of GPS/camera/IMU, a nonlinear observer with roll angle compensation was established to obtain sideslip angle. Furthermore, compared with the IMU signal, as there was large measurement delay for the GPS and camera signal which could not be neglected, an observer-predictor algorithm was employed to enhance practical effectiveness. Finally, simulation results validated the effectiveness of the proposed method. The results in this research will help to provide a basis for intelligent vehicle control.

Further work can be carried out in the following area: vehicle sideslip angle estimation algorithm for inconsistency delay time between camera and GPS will be modelled; fault tolerant estimation algorithm for the unreliable information of GPS and camera will be developed; the effectiveness and robustness of the algorithm will be verified by real vehicle test.

## REFERENCES

- [1] C. Lv, J. Zhang, Y. Li and Y. Yuan, "Directional-stability-aware brake blending control synthesis for over-actuated electric vehicles during straight-line deceleration," *Mechatronics*, 38, 121-131, 2016.
- [2] W. Liu, L. Xiong, B. Leng, H. Meng, and R. Zhang, "Vehicle Stability Criterion Research Based on Phase Plane Method," *WCX<sup>TM</sup> 17: SAE World Congress Experience* 2017.
- [3] X. Xia, L. Xiong, Y. Hou and Z. Yu, "Vehicle stability control based on driver's emergency alignment intention recognition," *International Journal of Automotive Technology*, 18(6), 993-1006, 2017.
- [4] L. Xiong, Z. Yu, Y. Wang, C. Yang, and Y. Meng, "Vehicle dynamics control of four in-wheel motor drive electric vehicle using gain scheduling based on tyre cornering stiffness estimation," *Vehicle System Dynamics*, 50(6), 831-846, 2012.
- [5] C. Matthew, J. Timothy, "Real-time state estimation of vehicle handling dynamics using an adaptive kalman filter," *5th International Symposium on Advanced Vehicle Control*, 183-188, 1998.
- [6] X. Gao, Z. Yu, J. Neubeck, and J. Wiedemann, "Sideslip angle estimation based on input-output linearisation with tire-road friction adaptation," *Vehicle System Dynamics*, 48(2), 217-234, 2010.
- [7] G. Phomchoeng, R. Rajamani, and D. Piyabongkarn, "Nonlinear observer for bounded jacobian systems, with applications to automotive slip angle estimation," *IEEE Transactions on Automatic Control*, 56(5), 1163-1170, 2011.
- [8] D. Piyabongkarn, R. Rajamani, J. Grogg, J. Lew, "Development and experimental evaluation of a slip angle estimator for vehicle stability control," *IEEE Transactions on Control Systems Technology*, 17(1), 78-88, 2009.
- [9] L. Li, G. Jia, X. Ran, J. Song, K. Wu, "A variable structure extended kalman filter for vehicle sideslip angle estimation on a low friction road," *Vehicle System Dynamics*, 52(2), 280-308, 2014.
- [10] C. Lv, D. Cao, Y. Zhao, D. Auger, M. Sullman, H. Wang, et al, "Analysis of autopilot disengagements occurring during autonomous vehicle testing," *IEEE/CAA Journal of Automatica Sinica*, 5(1), 58-68, 2017.
- [11] D. Bevely, J. Ryu, J. Gerdes, "Integrating INS Sensors With GPS Measurements for Continuous Estimation of Vehicle Sideslip, Roll, and Tire Cornering Stiffness," *IEEE Transactions on Intelligent Transportation Systems*, 7(4):483-493, 2006.
- [12] J. Yoon, H. Peng, "A Cost-Effective Sideslip Estimation Method Using Velocity Measurements From Two GPS Receivers," *IEEE Transactions on Vehicular Technology*, 63(6), 2589-2599, 2014.
- [13] Y. Wang, B. Nguyen, H. Fujimoto, and Y. Hori, "Vision-based integrated lateral control system for electric vehicles considering
- [14] Y. Geng, J. Wang, "Adaptive estimation of multiple fading factors in Kalman filter for navigation applications," *Gps Solutions*, 12(4):273-279, 2008.
- [15] W. Liu, L. Xiong, X. Xia, R. Zhang, "Sideslip angle estimation based on camera and GPS for autonomous vehicle," *The 25th IAVSD Symposium on Dynamics of Vehicles on Roads and Tracks*, 2018.
- [16] L. Khosravian, O. Trumpf, and O. Mahony, "State estimation for nonlinear systems with delayed output measurements," *IEEE Conference on Decision and Control*, (pp.6330-6335), 2015.