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A Speed Guide Model for Collision Avoidance in Non-Signalized Intersections Based on Reduplicate Game Theory

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***Abstract*— In consideration of the convenience of drivers' operations, a velocity guide driving model for collision avoidance in non-signalized intersection is proposed. Based on reduplicate game theory, a profit function consists of safety, rapidity, controlling indicators is redefined. Pareto optimality is adopted to obtain the strategies which maximize the profit for the whole game. Simulation results show that the model is effective and gives a more comfortable vehicle-cross process.**

Keywords—non-signalized intersections; game theroy; collision avoidance; ITS;V2X;

I. INTRODUCTION

Road traffic death rate is always an important index to the road safety level of a country, where the number of deaths caused in the non-signalized intersection accounts for a significant proportion [1]. Road crossing leads vehicles to the risk of collision with vehicles in other routes and this requires drivers to pay more attention to the surroundings. Driver factors are still the main causes to the intersection accidents. Therefore, a good collision avoidance algorithm for driver guidance in the intersection is urgently needed.

In recent years, the utilization of V2X (Vehicles to Everything) communication technique has developed greatly in both academia and industry. V2X utilizes wireless communication to achieve real-time vehicle and vehicle, infrastructure information exchange. In this way, drivers are able to know more details about the surroundings, so as to effectively improve the traffic safety level and the operation efficiency [2].

With the support of V2X, many models have been proposed to solve the collision problem in the intersection. One of the well-known models is the Gap acceptance model. It extends the idea of gap acceptance theory to determine individual vehicles to go or stop in the intersection [3]. However, it is only suitable for intersections consists of roads with clear priority. Some scholars proposed models based on game theory. Mohammed et al. set up a strategy set as {acceleration, deceleration and uniform speed}, and define the profits table of all combinations of strategies, so that the vehicles can easily take the strategies of highest profit according to the profits table [4].

Xiaoming Liu and Shuhui Zheng set the profit function consisted of safety indicator and rapidity indicator. This model gives drivers guidance concerning acceleration in the small time scale and improves the efficiency of the objective vehicles approaching to the intersection [5].

Then, Zhuo Yang et al. further consider the effects to the physical and mental comfort made by the magnitude of acceleration. They add a comfort indicator to the profit function to restrict the change rate of the acceleration [6].

However, taking acceleration as strategies is still hard for human drivers to accept for the following reasons. First, the strategies are given to the drivers every moment, but human needs time to recognize and act. Therefore, it is too rough for human to cooperate. Second, human prefers a stable and certain guidance for a longer time. The strategies of acceleration are only for the current moment and vary frequently. It is easy to arouse human's puzzles.

In this study, a modified collision avoidance model which takes velocity as strategies will be proposed. Velocity is a common physical quantity for human and it is easy to maintain and achieve.

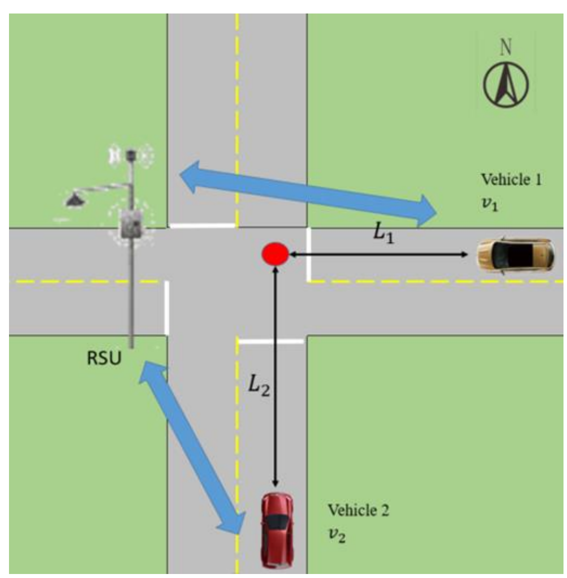

Fig. 1. The illustration of research situation

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In the following part of the paper, Section II is about the background of the research. The steps of the proposed model will be expressed in detail in Section III. Section IV shows the simulation results and analyses. Section V concludes the entire research.

II. PROBLEM PRESENTED

This research studies a potential two-vehicle collision situation in an isolated non-signalized intersection, regardless of non-motor vehicles and pedestrians (see Fig.1). Vehicle 1 and vehicle 2 are in different roads and their routes cross at the conflict point. Two vehicles approach to the junction with current velocities v_1, v_2, and the distances from the conflict point L_1, L_2. The drivers of two vehicles desire to accelerate and decelerate the vehicles at the values of acceleration a_1, a_2. Moreover, two vehicles and the intersection are necessarily equipped with V2X communication systems to meet the demand of information sharing.

The process from the situation described above to the situation while both vehicles have departed the intersection is called vehicle-cross process. This process is a finitely reduplicate dynamic game, which is a game with same structure repetitions until both vehicles get out of dangerous state. The game is made up of three essential elements, players, strategies and profits, where the profits are decided by the strategies that players take. The key idea of game theory is to find such a set of strategies which maximize the profits. In this research, the game elements are described as follows,

$$\text{Players} = \{\text{vehicle1}, \text{vehicle2}\} \tag{1}$$

$$\text{strategies} = \{\text{velocities} \mid 0 \le \text{velocities} \le v_{max}\} \tag{2}$$

$$\text{profits} = \{P_1(S_1, S_2), P_2(S_1, S_2)\} \tag{3}$$

Where S_1, S_2 denote vehicle 1's strategy and vehicle 2's strategy; $P_1(S_1, S_2)$ denotes vehicle 1's profits function of strategies.

III. MODEL PROPOSAL

Collision avoidance model is divided into 2 tasks. The first task is to judge the safety state of two vehicles and decide whether to trigger the system. The Second task is the core of this paper, utilizing the reduplicate game theory to find the best set of strategies for both vehicles and lead them out of danger gradually.

A. Judgment to start system

1) Collision criterion

There are some indicators used to estimate the criticality of traffic situation. One of the most widely used indicators is TTC (Time to Collision). Its definition is the time left before collision with both vehicles continuing on the same course and at the initial speeds. However, when TTC judges the situation in danger, two vehicles are close to the collision point and speed guidance is meaningless. PET (Post-Encroachment Time) is another useful indicator, which is measured by taking the time between two vehicles pass the same location. For evaluating PET in every moment, the form of TAdv (Time Advantage) is set up as [7],

$$\text{TAdv(t)} = \left| \frac{d_i(t)}{v_i(t)} - \frac{d_j(t)}{v_j(t)} \right| < T_M \tag{4}$$

Where $d_i(t)$ and $v_i(t)$ respectively denote the current distance to the conflict point and the current velocity of vehicle i. T_M denotes the time threshold of safety.

This research sets (4) as the collision criterion to reach.

2) Timing criterion

Drivers tend to believe the vision feelings. When a vehicle is still far away from the intersection, the driver will not follow the guidance the system gives. Therefore, this research utilizes the idea of safety distance as the timing criterion to meet.

Safety distance includes safety brake distance and safety warning distance [8]. Safety brake distance is the distance from conflict point that a vehicle slows down to zero at the current speed with maximum braking acceleration. The form is given as,

$$L_b = \frac{v^2}{2a_{max}} + \text{v} \cdot t_{sys} \tag{5}$$

Where v denotes the current speed of the vehicle. a_{max} denotes the maximum value of acceleration, which is decided by the vehicle characteristics; t_{sys} denotes the time delayed, which includes the reaction time of the driver and the transfer time from the brake pedal to the vehicle's transmission system.

In order to ensure that the system's guidance has enough time for a vehicle to achieve before it enters its safety break distance, the form of safety warning distance is set up as,

$$L_w = \left(\frac{v^2}{2a_{decend}}\right) + L_b \tag{6}$$

Where a_{decend} is the acceleration of deceleration. In general, a_{decend} is between the value of -2 and -0.9 m/s^2.

If L, the distance to the conflict point of a vehicle, is less than its safety brake distance while the safety of the situation is still in dangerous state, the risk of collision is still very high despite with speed guidance, so that the vehicle needs to brake down with it maximum deceleration. While the distance to the conflict point of a vehicle is larger than the safety warning distance, the driver does not feel the danger. This will make the driver reject following the guidance easily. Therefore, it is proper to set the expression form of timing criterion as,

$$L_w \ge \text{L} > L_b \tag{7}$$

B. *Game for strategies*

1) *Expression of profit function*

If the situation meets (4) and (7), the system works. Based on the game theory, the key to find the best set of strategies is to determine the form of the profit function. In collision avoidance models, the profit functions always include safety and rapidity indicators. They are two common indicators which are always used together to counterbalance each other. Moreover, the controlling indicator is also considered in this model to make drivers willing to follow the system's instructions.

a) *Safety indicator*

With vehicles status parameters $v_1, v_2, L_1, L_2, a_1, a_2$ and the strategies v_{sug1}, v_{sug2}, the idea of safety indicator is the value of TAdv if the velocities of vehicles altered to the strategies. The form is given as,

$$\Delta T^i = \left| \begin{array}{c} \left(\frac{v_{sug1}{}^i - v_1{}^i}{a_1} + \frac{L_1{}^i - S_1{}^i}{v_{sug1}{}^i}\right) - \\ \left(\frac{v_{sug2}{}^i - v_2{}^i}{a_2} + \frac{L_2{}^i - S_2{}^i}{v_{sug2}{}^i}\right) \end{array} \right|, \tag{8}$$

$v_{min} \le v_{sug1}{}^i, v_{sug2}{}^i \le v_{max}$, $\mathrm{i} = 1,2, \dots, \mathrm{N}$,

$L_j{}^i \ge S_j{}^i, \mathrm{j} = 1,2$

Where superscript i denotes the i th cycle of the game. v_{min}, v_{max} are the minimum velocity and maximum velocity the strategies v_{sug1}, v_{sug2} can be; $S_j{}^i$ is the distance vehicle j runs for altering its velocity from current velocity v_j to v_{sugj}, whose form is given as,

$$S_j{}^i = v_j{}^i \cdot \left(\frac{v_{sugj}{}^i - v_j{}^i}{a_i}\right) + \frac{1}{2} a_j \cdot \left(\frac{v_{sugj}{}^i - v_j{}^i}{a_i}\right)^2 \tag{9}$$

b) *Rapidity indicator*

Drivers usually hope to cross the intersection as fast as possible, because being fast means spending less time. Therefore, the rapidity indicator form is given as,

$$\Delta V_j^i = \left(\frac{v_{sugj}{}^i - v_j{}^i}{a_i}\right) + \frac{L_j{}^i - S_j{}^i}{v_{suai}{}^i}, \tag{10}$$

$j = 1,2; i = 1,2, \dots, N$

The form is similar to the form of safety indicator with the same parameters of vehicle j.

c) *Controlling indicator*

A large variation between the former strategy and current strategy would make drivers hard to operate and accept. Therefore, to improve the stability of the strategies, the expression of the controlling indicator is given as,

$$\Delta C^i = \left| v_{sugj}{}^i - v_{sugj}{}^{i-1} \right| \tag{11}$$

$j = 1,2; i = 2,3, \dots, N$

The controlling indicator ΔC^i is described as the variation between the strategies of the objective vehicle at i-1 th and i th cycles of the game. The smaller ΔC^i means the better stability of strategies. Introducing the controlling indicator to the model is expected to make the system not only to find and maintain the best strategies since the beginning, but also to minimize changes in the strategies according to the current situation.

2) *Standardization of indicators*

For the different units of the safety, rapidity and controlling indicators, this part describes the operations to standardize the effects each indicator makes to the profits.

Safety and controlling indicators are converted by the bell model whose expression is given as [9],

$$f(\mathrm{x}; \mathrm{a}, \mathrm{b}, \mathrm{c}) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}} \tag{12}$$

Where x is the original value. a denotes the conversion ratio of the difference value between x and c, which represents the acceptable range of x. b denotes the nonlinear variation degree of the transition function at the boundary. A larger b helps distinguishing whether x satisfies the requirements. c denotes the intermediate value. Hoping that the strategies will make safety indicator to be between 3-6 seconds and controlling indicator be even smaller, we take a=1.5, b=4, c=5 for safety indicator and a=1,b=4,c=0 for controlling indicator .

To standardize the rapidity indicator, Min-Max Normalization [10] is adopted,

$$\mathrm{g}(\mathrm{x}, \mathrm{a}, \mathrm{b}) = \frac{0-1}{(b-a)}(\mathrm{x} - \mathrm{a}) \tag{13}$$

Where x is the original value; a and b denote the minimum value and maximum value respectively.

At this point, the profit function can be expressed as follows,

$$f_j^i = \alpha_j f[\Delta T_j^i] + \beta_j g[\Delta V_j^i] + \gamma_j f[\Delta C_j^i], \mathrm{i} = 1,2, \dots, N \tag{14}$$

Where $f_j{}^i$ is vehicle j's profit function in the i th cycle of the game; $f[\Delta T_j^i], g[\Delta V_j^i], f[\Delta C_j^i]$ respectively denote safety indicator, rapidity indicator and controlling indicator after standardization; α, β, γ are their weight coefficients, whose values depend on the driving behaviors.

3) *Selection of strategies*

In game theory, cooperative game and non-cooperative game are two branches distinguished by cooperation or rejection of players. Pareto optimality is often used to solve the cooperative game, while Nash equilibrium is used to solve the non-cooperative game. Nash equilibrium makes each vehicle achieves its own best profit at every moment, which is against to keep the strategies stable, while Pareto optimality pursues a set of strategies which maximize the total profit of the game. Therefore, Pareto optimality is chosen to seek the best strategies for the vehicle-cross process directly [6]. Its form is given as,

$$F^i = argmax \sum_{j=1,2} w_j f_j^{\,i}(\boldsymbol{v}_{sug}^{\boldsymbol{i}}) \tag{15}$$

Where $\boldsymbol{F^i}$ is the optimal total profit in i th cycle of the game. $\boldsymbol{w_j}$ is the weight of vehicle j, which reflects the driving priority of vehicle j. $f_j^{\,i}(v_{sug}^i)$ is vehicle j's profit when the strategy set is $\boldsymbol{v}_{sug}^{\boldsymbol{i}}$.

Considering the inaccuracy existing in the drivers' operations, the precision of strategies is inappropriate to be high and it is better to give guidance of a velocity interval. Generally, it makes sense to set the precision of strategies as 5km/hr and the range of interval as ±2km/s. The maximum speeds of urban roads are usually set less than 100km/hr. Then, the selection amount of strategies is greatly reduced to less than about 400. The optimal solution can be found quickly even by enumerating method.

Integrating above tasks, following shows the steps of the collision avoidance model.

i Judgment of safety state

Vehicles approaching to the intersection constantly transmit their information to the RSU (Road Side Unit). According to the information of positions and speeds, the system would calculate the value of TAdv.

ii Choice of driving mode

Both vehicles will be in free driving mode if the situation is in the two conditions. One condition is that the value of TAdv is greater than the threshold, another condition is that even the value is less than the threshold, but they are not entering into their safety warning distances. If the value of TAdv is less than the threshold and at least one vehicle has been into its safety warning distance, the model will jump into step iii. If at least one vehicle is in the safety brake distance when the value of TAdv still does not reach safety standard, the model will enter the brake driving mode.

iii The game of strategies

The strategies of velocity will be obtained by Pareto optimality and provided to the drivers.

iv Driving operation

With system's instructions, drivers can choose to cooperate or not. If cooperate, then drivers will alter the

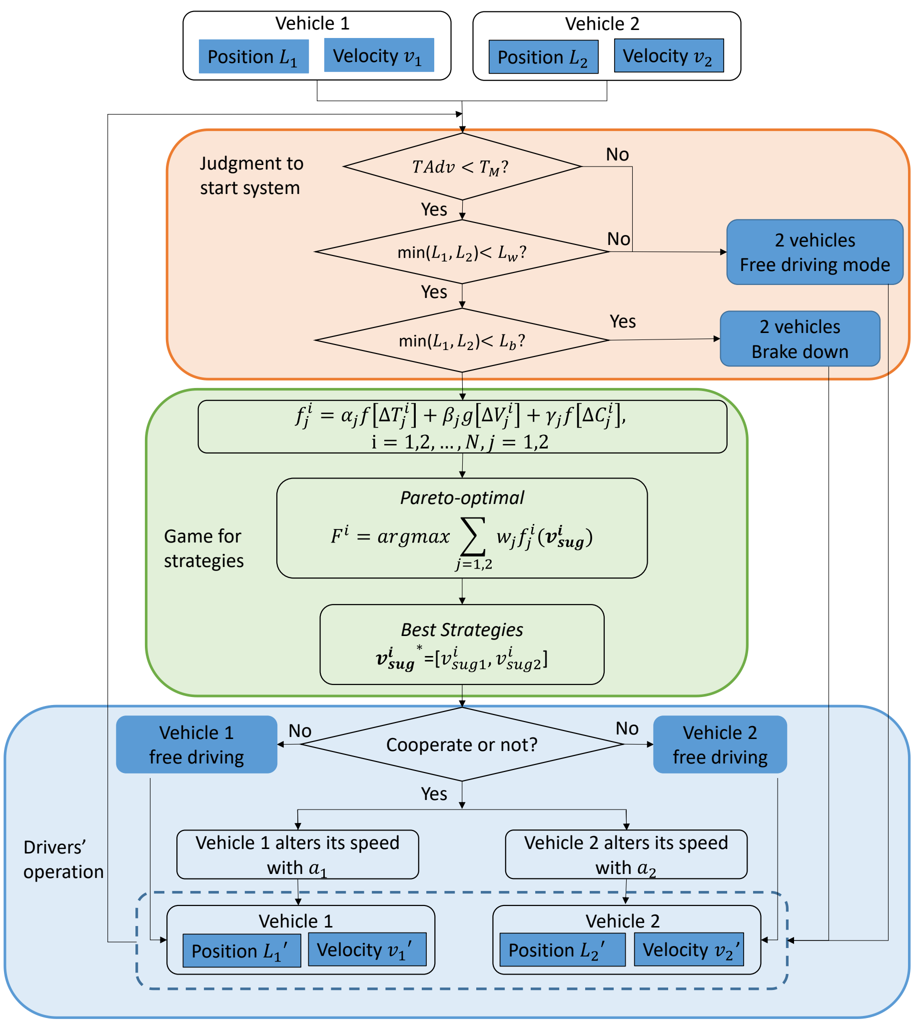

Fig. 2. Flow diagram of the model.

velocities of vehicles at their desired accelerations. Otherwise, the vehicles will continue running at their initial velocities.

The flow diagram of the model is shown in Fig.2.

IV. SIMULATION AND RESULTS

A. Expression of the driving behaviors

The collision avoidance model is designed to provide guidance for drivers to control the vehicles. Therefore, driving behaviors play important roles in the vehicle-cross process. Driving behaviors are shown in the degree of cooperation δ, driving acceleration a_{driver}, and the weight coefficients of indicators α,β,γ mentioned in section III.

Generally speaking, an aggressive driver will choose greater values of a_{driver} and β, while a conservative driver focuses on safety and smooth driving. Following shows the expression form of driving behaviors in different situation for simulation of the model.

During the guidance, the driving behaviors can be described as,

$$\text{if rand} < \delta, a_k = \begin{cases} a_{driver} + randn, & v_k < v_m - c \\ \pm \quad 0.5randn, & v_m - c < v_k < v_m + c \\ a_{driver} + randn, & v_k > v_m + c \end{cases}$$

$$\text{if rand} \geq \delta, a_k = \pm 0.5randn \qquad (16)$$

Where the value of δ is between [0,1]. The consideration of δ represents that the drivers have rights to reject following instructions. A greater δ is good for the stability of guidance velocity because the velocity of the vehicle is in line with the system's expectation. v_k, a_k denote the velocity and acceleration of the vehicle at k moment. a_{driver} denotes the value of acceleration the driver is used to take. v_m denotes the velocity of guidance. *c* denotes the range of desired velocity interval. *randn* denotes a normal random value, reflecting the uncertainty of human behaviors.

During free driving, drivers prefer to keep constant speed. The expression of driving behaviors is given as,

$$a_k = \pm 0.5randn \qquad (17)$$

B. Parameters initialization

This paper considers 2 initial conditions for simulation.

In condition 1, Vehicle 1 comes to the junction with initial velocity $v_1 = 60km/hr$, initial distance to conflict point $d_1 = 200m$, while Vehicle 2 comes from another direction with $v_2 = 50km/hr$, $d_2 = 150m$. The time interval is 0.2s. Both vehicles have equal rights to cross the intersection with weights $w_1 = w_2 = 0.5$. The precision of the strategies is set up as $5km/hr$. The range of recommended velocities is $\pm 2\text{km/hr}$. Considering most driving conditions, Both vehicles' parameters of driving behaviors are intuitively set up as δ=0.8, $a_{driver} = 2m/s^2$, [α,β,γ]=[0.4 ,0.3 ,0.3].

In condition 2, to compare the influence degree of the driving behaviors on the test results, the driver of vehicle 1 is set up as an aggressive driver. Vehicle 1's parameters of driving behaviors are set up as [α,β,γ]=[0.1 ,0.6, 0.3], $a_{driver1} = 3m/s^2$, while the remaining is as same as condition 1.

C. Results and Analyses

Fig.3 and Fig.4 show the simulation results of condition 1 and condition 2 respectively. In condition 1, the value of TAdv at the beginning is less than the threshold (Fig.3(a)). However, the system does not work until one of the vehicles enters its safety warning distance. At t=2s, the system starts providing recommended velocities and the curves of guidance are shown in Fig.3(b). The guidance plans to make vehicle 1 decelerate twice to increase the time between two vehicles arriving at the conflict point.

The velocity curves and the acceleration curves are showed in Fig.3(c) and Fig.3(d). Both kinds of curves are fully decided by the drivers. For most of the time, the acceleration curves maintain at $0m/s^2$, which provides smooth driving processes. At t=2s, the driver of vehicle 1 notices the recommended velocity given as 50km/hr and

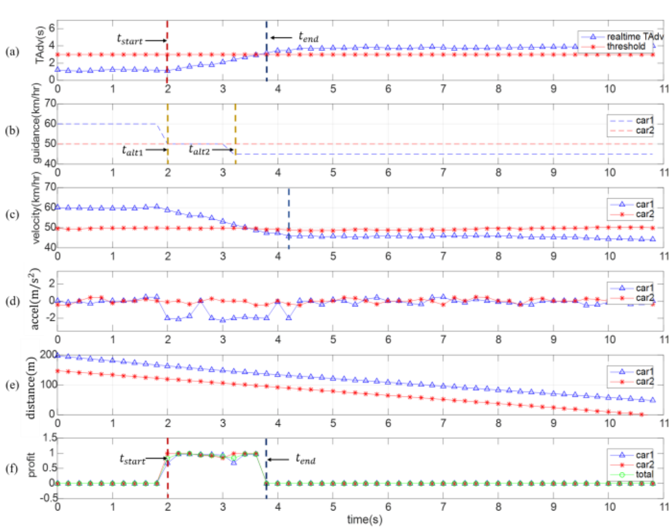

Fig. 3. Simulation results of condition 1

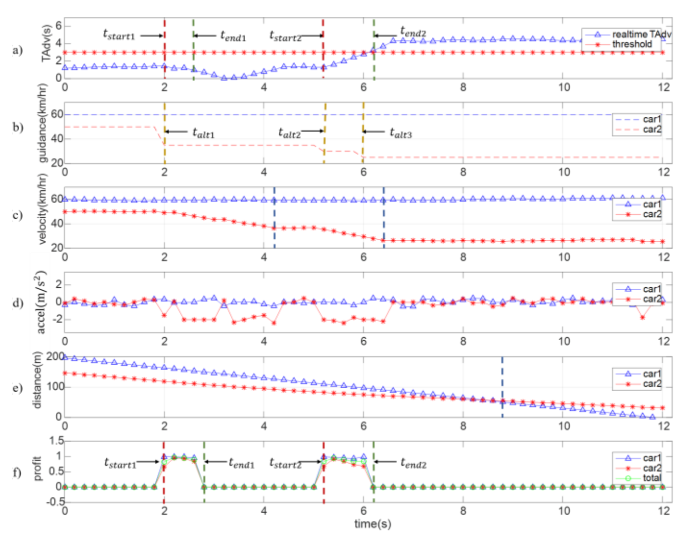

Fig. 4. Simulation results of condition 2

decelerates with acceleration of about -2 m/s^2 until the velocity of vehicle 1 approximates the recommended velocity. At t=3.2s, the driver of vehicle 1 again notices the recommended velocity changed to 45km/hr and decelerates until t=4.2s. Then, both vehicles continue driving with altered velocities. Before both vehicles get into their safety brake distances, the value of TAdv has already been larger than the threshold, which means the situation has been far from danger, so that they can cross the intersection without brake down.

The whole vehicle-cross process of condition 1 takes about 11s. In the process, the recommended velocity curves are stable, which makes drivers easy to accept. Operating vehicle by drivers themselves makes the driving process more comfortable without abruptly acceleration or deceleration.

In condition 2, Fig.4(f) shows that there are 2 working sections in the whole vehicle-cross process. Working section 1 starts working at t=2s, and the recommended velocity of vehicle 2 is given as 35 km/hr, while the system makes vehicle 1 maintain its speed (Fig.4(b)). However, the working section ends quickly at t=2.8, which is due to the safety warning distance of vehicle 2 decreases to be less than its distance to collision point as the velocity falls.

At t=5.2s, working section 2 begins, and the recommended velocity of vehicle 2 is changed to 30 km/hr. At t=6s, the recommended velocity of vehicle 2 is changed again to 25 km/hr. Finally at t=6.2s, the value of TAdv is larger than the threshold, which ends working section 2. Fig.4(a) shows that this section effectively makes the situation far from danger.

The whole vehicle-cross process of condition 2 takes about 12s. The last value of TAdv maintains at 5, which indicates that the safety indicator makes strong impact to the profit function. Because of the vehicle 1's setting of [α,β,γ], the system tends to unilaterally decelerate vehicle 2's velocity to achieve raising the value of TAdv and maintaining vehicle 1's velocity high at the same time. Fig.4(c) and Fig.4(d) clearly show that the velocity curves and the acceleration curves are following the guidance curves shown in Fig.4(b), and all of them are stable in the whole process.

V. CONCLUSION

In this paper, a collision avoidance algorithm suitable for human drivers' operations is presented. The judgment of safety and the selection of the strategies are considered in the model. Based on the reduplicate game theory, velocity is taken as strategies and redefines the expression of safety, rapidity, controlling indicators in the profit function. Pareto-optimal method is adopted to find such a set of strategies which maximize the total profit for the whole situation. Finally, the model is simulated in two initial conditions for comparison, one of which is with general parameters of driving behaviors, and the other is with extreme parameters of driving behaviors. From the analyses of the simulation results, the effectiveness of this model is indicated.

Compared to the predecessors' work, the advantages of this research are listed below.

1) Safety distance is taken into consideration

The concept of safety distance is introduced to the model as a criterion together with TAdv to trigger the system. In this way, drivers' crisis consciousness is aroused, which improves driver's cooperation degree to the system.

2) Strategies become easier to operate

The performance of taking acceleration as strategies is very dependent on the rapidity and accuracy of action. However, only machine can afford such work. Therefore, taking velocity as strategies is more suitable for human drivers, which can not only make drivers clearly know the coming situation, but also tolerate the uncertainty of driving behaviors.

3) It is suitable for both automatic driving and human driving

With stable guidance of velocity, drivers can choose their desired value of acceleration to alter the velocity of vehicles. Drivers tend not to accelerate or decelerates abruptly unless emergencies happen. Automatic driving works with constant parameters, so it is certain to have performance better than human driving with verified parameters. Hence, the model shows its wide practicality.

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