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A MINIMUM SWEPT PATH CONTROL STRATEGY FOR REVERSING ARTICULATED VEHICLES

Xuanzuo Liu[1] and David Cebon[1]

Abstract— **This paper presents a new control strategy called Minimum Swept Path Control (MSPC) for reversing articulated heavy goods vehicles. It improves on previous Path Following Control (PFC) methods by minimising large excursions of the tractor unit. A preview distance is integrated into the MSPC algorithm to predict future vehicle states and feed them back into the control system to compensate state errors. The relationship between the length of preview distance and maximum lateral offsets is investigated, which can be used to optimise the preview distance. The relationship between maximum lateral offsets and the corresponding weights in the control cost function is also established in this preliminary study.**

Index Terms— **Articulated Vehicles, Autonomous Reversing, Minimum Swept Path Control, Path Following Control, Linear Quadratic Regulator Method.**

I. INTRODUCTION

In recent years, there has been increasing pressure to reduce carbon footprints and fuel consumption in the road freight industry. Long Combination Vehicles (LCVs), with multiple articulation points provide an important route to improving fuel efficiency [1], [2], offering 18-32% decrease in fuel consumption in comparison with conventional articulated vehicles [3]. Reversing LCVs into small parking bays or interchanging trailers are common tasks for drivers. However, unlike forward driving, reversing of LCVs is unstable, with non-holonomic characteristics. Moving obstacles such as cars, workers or trollies in the vicinity of the LCVs exacerbate the difficulty. Furthermore, professional and experienced drivers capable of performing those tasks are rare and very sought after, as described by M. Kjell [4]. Hence, it is important to design assistive controllers for drivers reversing multiple-trailer combination vehicles.

Previous control strategies used to assist reversing, such as Path Following Control (PFC), developed by Rimmer [5]–[8], have been aimed at reducing lateral offsets between the rearmost axle of LCVs and a specified path. However, those strategies can cause large excursions of the other vehicle units, especially the tractor, thus increasing the overall swept path width substantially, as illustrated in the upper part of Figure 1. A new control method called Minimum Swept Path Control (MSPC) is proposed in this paper, as shown in the lower part of Figure 1. State Feedback Control (SFC) and a 'look-ahead' distance are integrated into the MSPC algorithm, enabling the relationship between maximum lateral offsets of the front axle of the tractor and the rear axle of the last trailer to be varied through tuning the weights in the control cost function. Because a tractor-semitrailer combination is widely used around the world, a simulation for this case has been considered for simplicity, but the theory can be extended to a tractor with any number of trailers.

[1]Xuanzuo Liu and David Cebon are with the Department of Engineering, University of Cambridge, Trumpington Street, Cambridge, CB2 1PZ, U.K. (Corresponding author is David Cebon)
e-mail: xzl20@cam.ac.uk; dc@eng.cam.ac.uk

II. RESEARCH APPROACH

A nonlinear model of a standard UK tractor-semitrailer vehicle was created. Due to lorry loading regulations [9], many trailers require axle-groups of 2 or 3 axles at the rear. This can cause lateral scrubbing of tyres in tight corners. However, this effect was not considered in this preliminary study, because the scrubbing action is symmetric between left and right sides and does not have a strong effect on the path [5]. Consequently, an equivalent, single trailer axle vehicle was used in the initial simulations. The approach proposed by Winkler was used to calculate the distance from the fifth wheel to the point of zero lateral velocity, which can be considered to be the position of an equivalent axle to replace multiple-axle groups [10]. Based on the concept of an 'equivalent axle' and the single-track 'bicycle' vehicle dynamics model [11], [12], a multiply articulated vehicle with an arbitrary number of trailers and any number of axles was developed. Because articulated vehicles typically reverse at very low speeds, the 'equivalent' (single axle) tyres stay in the linear performance regime. Linearised tyre models were therefore used for controller development. For a lane change manoeuvre, the nonlinear vehicle dynamics (introduced by large articulation and steering angles) was linearised about the equilibrium state of a straight line. The longitudinal velocity of the tractor unit was defined to be a constant in this study.

To achieve the main objective of MSPC, a linear controller was devised. A cost function was developed, with weights (W_{ra} and W_{fa}) applied to the lateral offsets of the rear axle of the last trailer (y_{ra}) and the front axle of the tractor unit (y_{fa}) respectively. The cost function J was defined as follows:

$$J = \int_0^\infty (W_{ra} y_{ra}^2 + W_{fa} y_{fa}^2 + \delta^2)\, dt, \tag{1}$$

where δ is the steering angle.

The MSPC approach is to control the articulation angles, heading angle and lateral offset of the rear axle of the last trailer by feeding back those states into the system, while implementing a preview distance to predict future vehicle motion to minimise those state errors. A Linear Quadratic Regulator (LQR) method [13] is used to tune the linearised

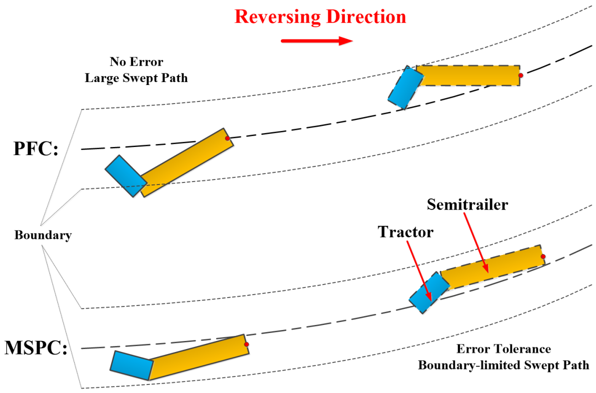

Fig. 1. Comparison between PFC and MSPC.

controller. By adjusting the weights to penalise the axle lateral offsets, the emphasis placed on each axle's lateral error versus the steering angle δ is varied. The equation of the system control input, steering angle δ, can be defined as follows:

$$\delta = \delta_d + K_{y_{ra}} y_{ra} + K_{y_{fa}} y_{fa} + \sum_{i=1}^{n} K_{\Gamma_i}(\Gamma_{id} - \Gamma_i), \quad (2)$$

where δ_d and Γ_{id} are the desired steering and articulation angle respectively, and Γ_i denotes the real-time articulation angle; y_{fa} and y_{ra} are the lateral offsets of the front axle of the tractor and the rear axle of the last vehicle unit; $K_{y_{fa}}$, $K_{y_{ra}}$ and K_{Γ_i} are the corresponding gains for the front and rear lateral offsets, and articulation angle error.

The system state vector ($\boldsymbol{Z}$) of the linearised vehicle model and its derivative ($\dot{\boldsymbol{Z}}$) can be defined as follows:

$$\boldsymbol{Z} = [v_1 \ \Omega_1 \ \Gamma_1 \ \cdots \ \Gamma_i \ \cdots \ \Gamma_n \ \dot{\Gamma}_1 \ \cdots \ \dot{\Gamma}_i \ \cdots \ \dot{\Gamma}_n \ y_{fa} \ y_{ra}]^T \quad (3)$$

$$\dot{\boldsymbol{Z}} = [\dot{v}_1 \ \dot{\Omega}_1 \ \dot{\Gamma}_1 \ \cdots \ \dot{\Gamma}_i \ \cdots \ \dot{\Gamma}_n \ \ddot{\Gamma}_1 \ \cdots \ \ddot{\Gamma}_i \ \cdots \ \ddot{\Gamma}_n \ \dot{y}_{fa} \ \dot{y}_{ra}]^T \quad (4)$$

where v_1 and Ω_1 denote the lateral and yaw velocities of the Centre of Mass (C.o.M) of the tractor, and Γ_i and $\dot{\Gamma}_i$ are the articulation angle and the corresponding rate between the i^{th} and $(i+1)^{th}$ vehicle units ($i \in [1, n]$, and n is the total number of trailers).

The linearised system can be defined in a state space form:

$$\dot{\boldsymbol{Z}} = [\boldsymbol{A}]\boldsymbol{Z} + [\boldsymbol{B}]\delta, \quad (5)$$

where $[\boldsymbol{A}]$ and $[\boldsymbol{B}]$ are the system and input matrices respectively.

Then, equation (1) can be rearranged in a quadratic form by using $\boldsymbol{Z}$:

$$J = \int_0^\infty (\boldsymbol{Z}^T[\boldsymbol{Q}]\boldsymbol{Z} + \delta^T[\boldsymbol{R}]\delta)\, dt, \quad (6)$$

where $[\boldsymbol{Q}] \geq \boldsymbol{0}$ and $[\boldsymbol{R}] > \boldsymbol{0}$ can penalise the state errors and the input respectively. In this study, $[\boldsymbol{R}] = \boldsymbol{1}$ and $[\boldsymbol{Q}]$ can be decomposed into a quadratic form:

$$[\boldsymbol{Q}] = [\boldsymbol{C}]^T[\boldsymbol{C}], \quad (7)$$

where $[\boldsymbol{C}] = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \sqrt{W_{fa}} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \sqrt{W_{ra}} \end{bmatrix}$.

As the nonlinear system was linearised around a straight line, all desired states in equation (2) become zero. The equation (2) can be represented in terms of $\boldsymbol{Z}$:

$$\delta = -[\boldsymbol{K}]\boldsymbol{Z}, \quad (8)$$

where $[\boldsymbol{K}] = \begin{bmatrix} 0 \ 0 \ K_{\Gamma_1} \ \cdots \ K_{\Gamma_i} \ \cdots \ K_{\Gamma_n} \ 0 \ \cdots \ 0 \ -K_{y_{fa}} \ -K_{y_{ra}} \end{bmatrix}$.

To find the optimal $[\boldsymbol{K}]$, the associated algebraic Riccati equation can be established for the linear quadratic optimisation problem [14], [15]:

$$[\boldsymbol{A}]^T[\boldsymbol{S}] + [\boldsymbol{S}][\boldsymbol{A}] - [\boldsymbol{S}][\boldsymbol{B}][\boldsymbol{R}]^{-1}[\boldsymbol{B}]^T[\boldsymbol{S}] + [\boldsymbol{Q}] = \boldsymbol{0}, \quad (9)$$

where $[\boldsymbol{S}]$ is the solution returned by the LQR approach and can be used to calculate the optimal gains further.

$$[\boldsymbol{K}_{LQR}] = [\boldsymbol{R}]^{-1}[\boldsymbol{B}]^T[\boldsymbol{S}], \quad (10)$$

As the MSPC algorithm also guarantees a relatively high accuracy of path following like PFC strategies, the original input equation can be transformed into an equivalent PFC form as follows:

$$\delta = \delta_d + K'_{y_{ra}} y_{ra} + K_{\theta_{ra}}(\theta_P - \theta_{ra}) + \sum_{i=1}^{n} K'_{\Gamma_i}(\Gamma_{id} - \Gamma_i), \quad (11)$$

where θ_P and θ_{ra} are the heading angles of the desired path and the rear axle of the last vehicle unit; $K'_{y_{ra}}$, $K_{\theta_{ra}}$ and K'_{Γ_i} are the corresponding gains for the rear lateral offset, heading angle and articulation angle error.

The equivalent state vector $\boldsymbol{Z}_{eq}$ can be defined as follows:

$$\boldsymbol{Z}_{eq} = [v_1 \ \Omega_1 \ \Gamma_1 \ \cdots \ \Gamma_i \ \cdots \ \Gamma_n \ \dot{\Gamma}_1 \ \cdots \ \dot{\Gamma}_i \ \cdots \ \dot{\Gamma}_n \ \theta_{ra} \ y_{ra}]^T \quad (12)$$

Likewise, equation (11) can be represented by using $\boldsymbol{Z}_{eq}$:

$$\delta = -[\boldsymbol{K}_{eq}]\boldsymbol{Z}_{eq}, \quad (13)$$

where $[\boldsymbol{K}_{eq}] = \begin{bmatrix} 0 \ 0 \ K'_{\Gamma_1} \ \cdots \ K'_{\Gamma_i} \ \cdots \ K'_{\Gamma_n} \ 0 \ \cdots \ 0 \ K_{\theta_{ra}} \ -K'_{y_{ra}} \end{bmatrix}$.

The relationship between $\boldsymbol{Z}$ and $\boldsymbol{Z}_{eq}$ is established by using a transformation matrix as follows:

$$\boldsymbol{Z}_{eq} = [\boldsymbol{T}]\boldsymbol{Z}, \quad (14)$$

where $[\boldsymbol{T}]$ is purely based on the vehicle geometry, which converts the lateral offset of the front axle of the tractor y_{fa} into the heading of the rear trailer θ_{ra}.

Substituting equation (14) into (8) generates the equivalent optimal gains.

$$[\boldsymbol{K}_{eq_{LQR}}] = [\boldsymbol{K}_{LQR}][\boldsymbol{T}]^{-1} \quad (15)$$

The partial gain matrix $[\boldsymbol{K}_{eq}]$ can be extracted from $[\boldsymbol{K}_{eq_{LQR}}]$ by neglecting some insignificant gains, which cannot affect the system stability.

Overall, $[\boldsymbol{K}_{eq}]$ is the desirable gain matrix implemented in the Simulink controller.

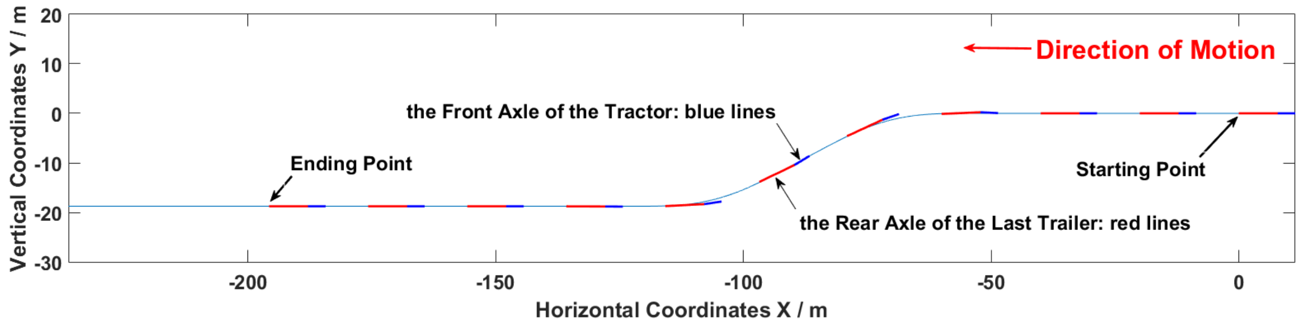

Fig. 2. Lane Change Manoeuvre.

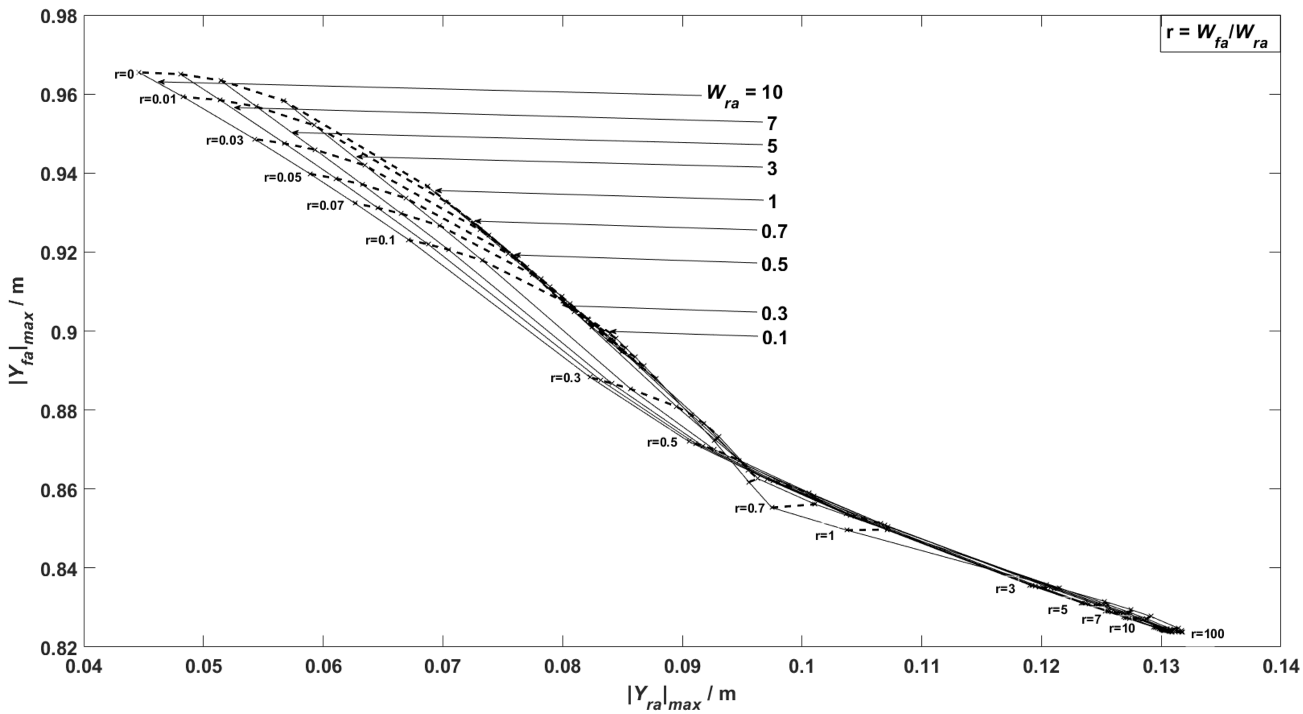

Fig. 3. Conflict Diagram of Constant W_{ra}.

TABLE I

LINEAR TYRE MODEL PARAMETERS

Parameter	Symbol	Value	Units	Source
Tractor front axle lateral stiffness coefficient	$K_{lat_{fa0}}$	-3.683e5	N/rad	Calculated based on [17]
Tractor rear axle lateral stiffness coefficient	$K_{lat_{ra0}}$	-3.338e5	N/rad	
Semitrailer rear axle lateral stiffness coefficient	$K_{lat_{ra1}}$	-4.119e5	N/rad	

III. Simulation Results and Discussion

The tyre model, vehicle configuration and simulation parameters are shown in Tables I to III. The longitudinal velocity of the tractor was defined to be -1 m/s, and the negative sign represents driving backwards. A target path for the simulation is depicted in Figure 2, representing a lane change manoeuvre. The relationship between maximum lateral offsets and weights is shown by plotting the maximum excursion of the front axle in the lane change manoeuvre against the maximum excursion of the rear axle for MSPC as a conflict diagram. A reasonable range for the weights is from 0 to 10, but both cannot be zero at the same time, because the LQR method requires semi-positive matrices [16]. It is noted that when $W_{fa} = 0$, the MSPC controller is similar to the Path Following Control (PFC) approach devised by Rimmer [5], [6] (apart from some differences in handling the 'look-ahead' distance) as only one weight plays a role in reversing. In Figure 3, each solid curve represents a varying W_{fa} as W_{ra} is fixed. This diagram shows that increasing one of the weights penalises the other response. For example, increasing W_{ra} reduces $|Y_{ra}|_{max}$, but increases

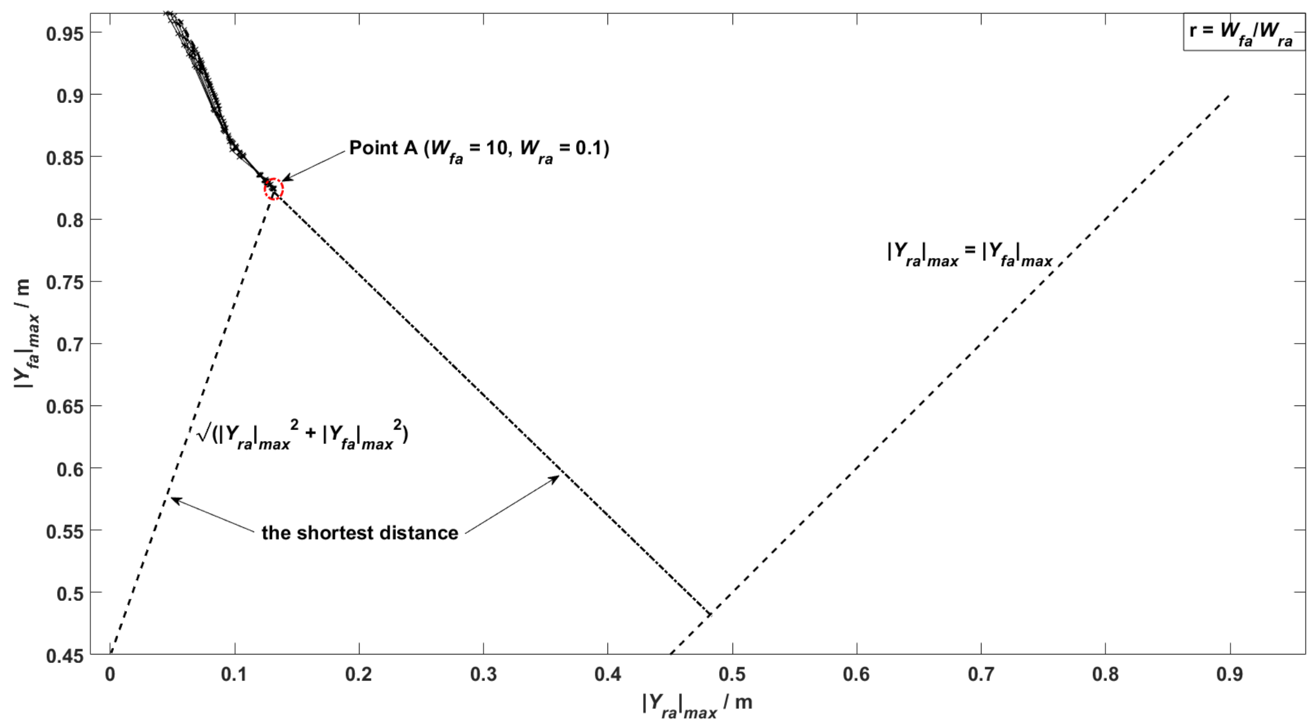

Fig. 4. Weight Selection Criteria.

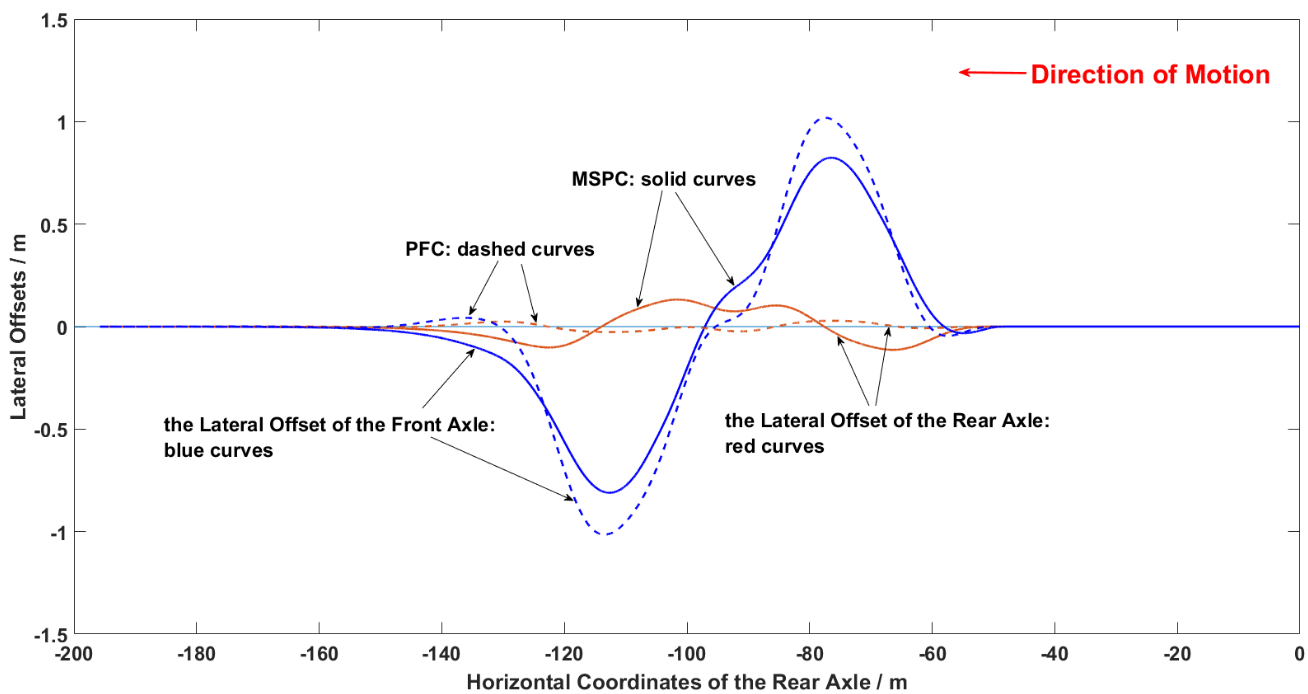

Fig. 5. Lateral Offset Comparison between PFC and MSPC.

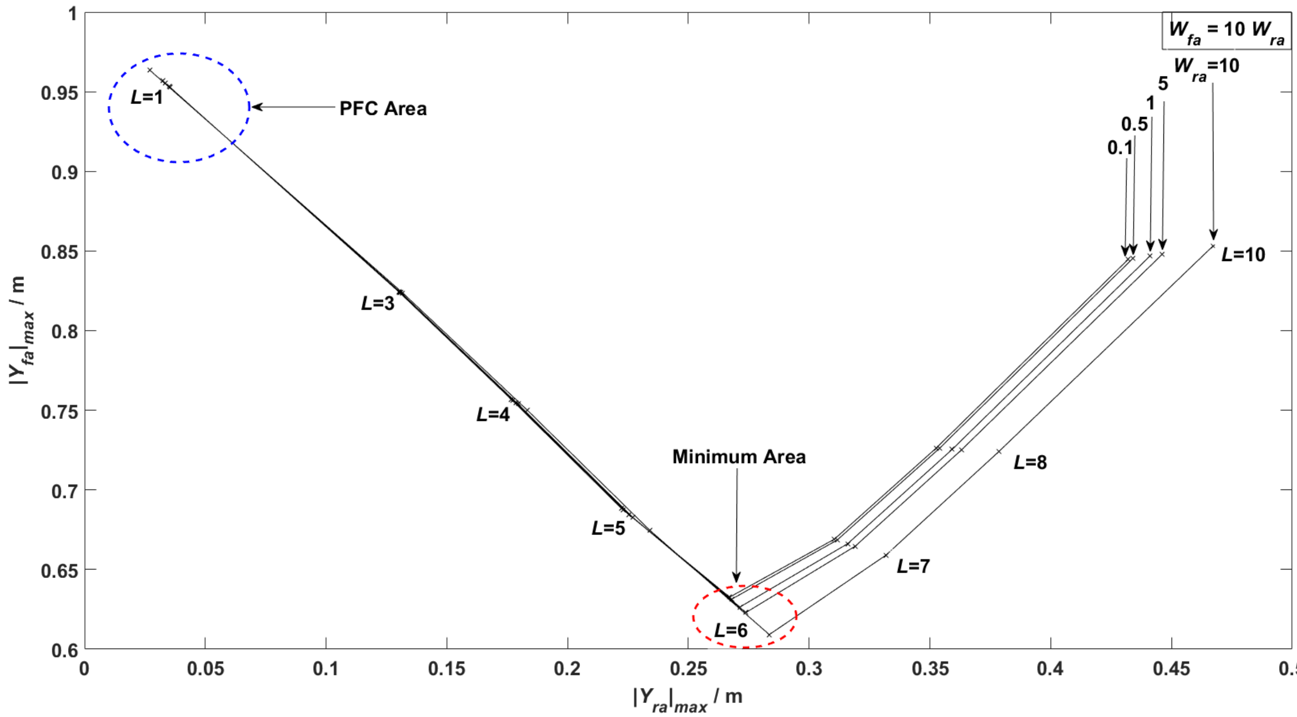

Fig. 6. the Effect of Preview Distance.

TABLE II
TRACTOR UNIT PARAMETERS

Parameter	Symbol	Value	Units	Source
Front axle to centre of gravity	a_0	1.125	m	[5]
Rear axle to centre of gravity	b_0	2.575	m	
Rear axle to hitch	c_0	-0.16	m	
Number of axles	n_a	2		
Mass	m_0	6988	kg	[17]
Yaw Inertia	I_{z0}	42147	$\mathrm{kg\,m^2}$	
Equivalent wheelbase	l_{eff0}	3.7	m	[5]

TABLE III
SEMI-TRAILER PARAMETERS

Parameter	Symbol	Value	Units	Source
Front axle to centre of gravity	a_1	6	m	[5]
Equivalent wheelbase	l_{eff_1}	7.85	m	
Equivalent rear axle to centre of gravity	b_1	1.85	m	Calculated
Mass	m_1	8800	kg	[5]
Yaw Inertia	I_{z_1}	156860	$\mathrm{kg\,m^2}$	[18]

$|Y_{fa}|_{max}$. Likewise, decreasing W_{fa} gives the same effect, decreasing the excursions of the rear axle, but increasing those of the front axle. The conflict diagram reveals a general trend of the ratio $r = W_{fa}/W_{ra}$ as well. Contours of constant r are displayed as dashed curves in Figure 2.

One way to select optimal weights might be to try to balance the maximum lateral offsets of the front and rear axles. Consequently, the conflict diagram is replotted on larger scales in Figure 4 with the line $|Y_{fa}|_{max} = |Y_{ra}|_{max}$ added. It is apparent that no solutions can achieve equal excursions at front and rear. Another strategy would be to minimise the least error norm $\|error\|_2 = \sqrt{|Y_{fa}|^2_{max} + |Y_{ra}|^2_{max}}$ - the shortest distance from the origin to the curves in the conflict diagram, as shown in Figure 4. The error norm is minimised at the largest values of W_{fa}, *i.e.* when the largest penalty is applied to excursions of the front axle. Overall, taking the two criteria into account, the set of $W_{fa} = 10$ and $W_{ra} = 0.1$ (Point A in Figure 4) gives the best result, resulting in $|Y_{fa}|_{max} = 0.824$ metres and $|Y_{ra}|_{max} = 0.132$ metres.

The lateral offsets of both axles during the lane change manoeuvre for an MSPC controller (Point A in Figure 4) and the corresponding pure PFC controller (only $W_{ra} = 0.1$) are shown in Figure 5. It can be seen that the MSPC strategy reduces the front axle's maximum lateral offset (solid curve) by approximately 20% compared with the PFC strategy (dashed curve). However, this is achieved at the expense of increasing the path error of the rear axle. The MSPC algorithm generates a significantly larger lateral offset on the rear axle, than the PFC algorithm.

The relationship between the 'look-ahead' distance L and the maximum lateral offset is also investigated by varying the length of distance for each fixed weighting set, as shown in Figure 6. As for the PFC algorithm, the optimal preview distance is short, dependent on the time delays of the controller. With respect to the tractor-semitrailer case in this simulation, the preview distance is tuned within the PFC area, as shown in Figure 6. However, for the MSPC algorithm, relaxing the constraint on the rear axle and choosing a relatively long distance can reduce the maximum lateral offset to a great degree. Different weighting sets reach their own minimum values by implementing the same preview distance. The optimised 'look-ahead' distance for the MSPC is dependent on the vehicle geometry and parameters.

IV. Conclusions and Future Work

A Minimum Swept Path Controller (MSPC) was designed to improve autonomous reversing of a tractor-semitrailer. A relationship between the lateral offsets of both axles and the corresponding controller weights was found. The weighting placed on one axle decreases the maximum lateral offset of that axle, but at the expense of the maximum lateral offset of the other axle. A relationship between the preview distance and the maximum lateral offset was also found. The preview distance plays a key role in reducing the maximum lateral offset and the optimal length is dependent on the vehicle parameters. This approach cannot only guarantee the accuracy of following a desired path in reverse, but also can reduce the lateral offsets of the front axle of the tractor unit significantly. In this case, the MSPC method improves the performance of PFC controllers.

The MSPC method will be implemented on a full-sized tractor and semitrailer to test its performance soon.

V. Acknowledgements

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References

[1] C. Hulne, "The Carbon Plan: Delivering our low carbon future, Energy and Climate Change", U.K., 2011.

[2] A. M. C. Odhams, R. L. Roebuck, Y. J. Lee, S. W. Hunt, and D. Cebon, "Factors influencing the energy consumption of road freight transport", *Proceedings of the IMechE*, vol. 224, no. 9, pp. 1995-2010, Sep. 2010.

[3] J. Woodrooffe and L. Ash, "Economic Efficiency of Long Combination Transport Vehicles in Alberta", Woodrooffe and Associates, Canada, 2001.

[4] "The Logistics Report 2017", Freight Transport Association, 2017. (http://www.fta.co.uk/about/logistics-report-2017.html).

[5] A. J. Rimmer, "Autonomous reversing of multiply-articulated heavy vehicles", Department of Engineering, University of Cambridge, Dissertation submitted for the degree of Doctor of Philosophy, 2014.

[6] A. J. Rimmer and D. Cebon, "Theory and Practice of Reversing Control on Multiply-Articulated Vehicles", *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 230, no. 7, pp. 899-913, Aug. 2015.

[7] A. J. Rimmer and D. Cebon, "Planning Collision-Free Trajectories for Reversing Multiply-Articulated Vehicles", *IEEE Transactions on Intelligent Transportation Systems*, vol. 17, no. 7, pp. 1998-2007, Jul. 2016.

[8] D. Cebon and A. J. Rimmer, "Implementation of Reversing Control on a Doubly Articulated Vehicle", *Journal of Dynamic Systems, Measurement and Control*, vol. 139, no. 6.061011, Nov. 2016.

[9] "A Simplified Guide to Lorry Types and Weights", Department for Transport, U.K., 2003.

[10] C. B. Winkler, "Simplified Analysis of the Steady-State Turning of Complex Vehicles", *Vehicle System Dynamics*, vol. 29, no. 3, pp. 141-180, 19980301.

[11] J. R. Ellis, *Vehicle Dynamics*, Hardcover-Bookbarn International, Century, 1969.

[12] T. D. Gillespie, *Fundamentals of Vehicle Dynamics*, Warrendale, Pennsylvania: Society of Automotive Engineers, 1992.

[13] B. D. O. Anderson and J. B. Moore, *Linear optimal control*, Englewood Cliffs, NJ: Prentice-Hall, 1971.

[14] A. K. Madhusudhanan, M. Corno, B. Bonsen and E. Holweg, "Solving Algebraic Riccati Equation Real Time for Integrated Vehicle Dynamics Control", *Proceedings of the 2012 American Control Conference*, pp. 3593-3598, Montreal, Canada, June 27-29, 2012.

[15] A. Laub, "A Schur method for solving algebraic Riccati equations", *in IEEE Transactions on Automatic Control*, vol. 24, no. 6, pp. 913-921, Dec. 1979.

[16] K. Huibert and S. Raphael, *Linear Optimal Control Systems*, First Edition. Wiley-Interscience, 1972.

[17] B. A. Jujnovich, "Active steering of a tractor-semi-trailer", Department of Engineering, University of Cambridge, Dissertation submitted for the degree of Doctor of Philosophy, 2006.

[18] Cheng C, "Enhancing safety of actively-steered articulated vehicles", Department of Engineering, University of Cambridge, Dissertation submitted for the degree of Doctor of Philosophy, 2009.