2018 IEEE Intelligent Vehicles Symposium (IV)
Changshu, Suzhou, China, June 26-30, 2018

A Game-Theoretical Approach to Driving Decision Making in Highway Scenarios

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***Abstract*— With the development of self-driving technology, the fundamental behaviors like car-following, lane change have been validated and tested in various kinds of scenarios. Currently one of the most challenging domain for self-driving is decision making under dynamic environments. For self-driving cars, it is essential to understand and estimate other vehicles' behavior and behave like a human driver to interact with other vehicles in the mean time. In this paper, a game theoretical approach is proposed to model the interaction of vehicles while considering the surrounding traffic situations. One of the novel move is that a neural network is applied to establish the payoff function in the game which is able to describe the interaction more precisely. The calibration method is then applied to estimate the parameters by using the Next Generation SIMulation (NGSIM) dataset. The experiments demonstrate the accuracy of the proposed method and the ability of making a cooperate decision in highway scenarios.**

I. Introduction

Recently, self-driving technology has been widely applied in the transportation and military. Highway is an important component of transportation system. However, the properties of long distance, high speed, and traffic congestion make highway driving is stressful and dangerous. Thus, the decision making in highway scenarios is one of the most challenging problem for self-driving cars.

The most common decision making methods are tackled by manual-defined rules corresponding to specific situations. A variety of solutions including finite state machines [1] and hierarchical state machines [2] are used to make decisions for self-driving cars. However, the methods of manual-defined rules are tailored for specific and simplified traffic scenarios without considering the uncertainty of drivers.

Partially Observable Markov Decision Process (POMDP) provides a mathematically framework of the decision making problem in dynamic, uncertain scenarios such as highway driving. Ulbrich [3] applied a POMDP to solve the decision making problem of self-driving cars. However, the method was time-consuming and can hardly applied in real-time. In order to solve this problem, Edwin [4] proposed an improved approach of POMDP named multi-policy decision making, which used a set of possible high-level policies to replace the continuous action space. However, these methods do not put the subject vehicle and the other vehicles in the same level when considering the dynamic interactions. They focus on the decision of the subject vehicle while ignoring the decisions of other vehicles.

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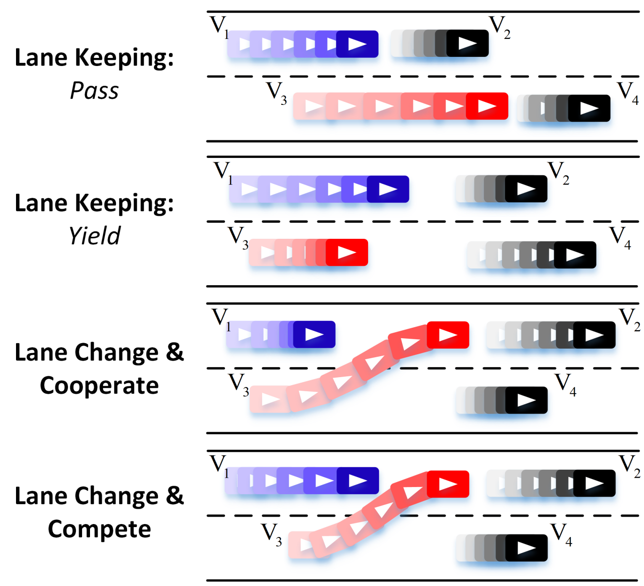


Fig. 1. Highway driving scenarios.

Other probabilistic methods were proposed to deal with the interactive merging scenarios [5], [6]. In addition, a social behavior generator was proposed to generate the lane change trajectory under interactions with the surrounding vehicles [7]. In order to model a more accurate interaction model between different drivers, the game theory is applied in this paper. Game theory constructs mathematical models to deal with the conflict and cooperation between decision makers. The drivers' decisions converge to the equilibrium of a set of strategies to maximize their rewards together in an independent game. Kita [8] first proposed a game theoretical model to describe the on-ramp merging behavior by using a discrete choice model. Talebpour [9] defined a two-type game of lane change behaviors according to the safety and speed gain for the drivers. Recently, Hesham [10] defined a model of merging maneuvers at freeway on-ramps. This model defined three strategies (change, wait, overtake) for the subject vehicle and two strategies (yield, block) for the lag vehicle. Based on the game theory, these methods considered the interactions among drivers and make a cooperate decision for each driver in the game. However, the most difficult part of the game theoretical methods is the formulation of the payoff function of the strategies for each game-player, because it is hardly to determine the effects of different factors to each player.

Many researches have demonstrated the effectiveness of artificial intelligence methods in car-following scenarios [11] and lane change scenarios [12]. Inspired by these work, a game theoretical based framework is proposed in this paper to model the interaction between human drivers. A two-person non-zero-sum non-cooperative game under complete information is presented to model the decision making behavior in highway driving. Moreover, an improved Gaussian Particle Swarm Optimization (GPSO) method is applied to calibrate the model. The main contribution of this paper is that we have built a neural-network based payoff model to describe the rewards of each driver and calibrate the model using real traffic data.

The remained paper is organized as follows. The framework of the proposed decision making method in highway driving is detailed in Section II. The traffic scenario extraction and model calibration are described in Section III. Experiments on model validation are carried out in Section IV. Conclusions and future works are presented in Section V.

II. Proposed Method

The interaction between the subject vehicle and the lag vehicle can be seen as a game, as shown in Fig. 1. The drivers of the subject vehicle and the lag vehicle choose their best strategy through the game. Generally, a game has three important factors: game players, strategies for each player and a payoff model representing the reward for different strategies combination of each player. A two-player, non-zero-sum, non-cooperative game under complete information is applied in this paper.

A. Players of Game

As shown in Fig 1, there are two players in this game: the driver of the subject vehicle (V_3, red) and the driver of the lag vehicle (V_1, blue), which is the closest following vehicle in the target lane. The meaning of non-zero-sum is that the sum of their payoffs in the game is not zero, because the payoffs of the two players are independent. Non-cooperative game means that players choose their strategies independently and no communication is established. It is assumed that the two players know the possible strategies of each other and the states of the surrounding vehicles. In another word, this game is a complete information game.

B. Strategies for Each Player

The subject vehicle has three pure strategies: (a) pass the leading vehicle(V_2, black) in the target lane; (b) yielded by the lag vehicle; (c) merge into the target lane. The lag vehicle has two corresponding strategies: cooperate or compete. Table I shows the structure of the game. P and Q denote the payoff of subject vehicle and lag vehicle, respectively.

Each player chooses one of the pure strategies to achieve the goal of the game. The selection of optimum strategy set has been a topic of interest since the introduction of game theory. In order to find the optimum strategies for the drivers of the subject vehicle and the lag vehicle, the

TABLE I
STRUCTURE OF LANE CHANGE GAME

Strategy		Lag Vehicle	
		b_1 (cooperate)	b_2 (compete)
Subject Vehicle	a_1 (pass)	(P_{11}, Q_{11})	(P_{12}, Q_{12})
	a_2 (yield)	(P_{21}, Q_{21})	(P_{22}, Q_{22})
	a_3 (lane change)	(P_{31}, Q_{31})	(P_{32}, Q_{32})

Nash equilibrium is considered. The Nash equilibrium is a solution concept of a non-cooperative game involving two or more players in which each player is assumed to know the equilibrium strategies of others, and no player has anything to gain by changing only his or her own strategy. The Nash equilibrium has two types: the pure strategies and the mixed strategies. The players select the pure strategies if Nash equilibrium of the pure strategies exists. It implies that one player maximizes his or her own reward considering that the opposite player also wants to maximize his or her reward. In this game, the driver of the subject vehicle has three possible strategies and the driver of the lag vehicle has two possible strategies, respectively: $S_1 = a_1, a_2, a_3$ or $S_2 = b_1, b_2$. This means that this game has six possible sets of strategies. The Nash equilibrium of each pure strategy set is defined as follows:

$$\begin{cases} P\left(a^*, b^*\right) \geq P\left(a, b^*\right) & \forall a \in \{a_1, a_2, a_3\} \\ Q\left(a^*, b^*\right) \geq Q\left(a^*, b\right) & \forall b \in \{b_1, b_2\} \end{cases} \tag{1}$$

where $P\left(a_i, b_j\right)$ and $Q\left(a_i, b_j\right)$ are equivalent to P_{ij} and Q_{ij} in Table I, (a^*, b^*) represents the Nash equilibrium strategy set for the drivers of the subject vehicle and the lag vehicle.

However, the Nash equilibrium of pure strategy set dose not always exist and the mixed strategy set can be used instead. A set of probabilities for each player's strategies are used to maximize his or her own payoff according to the set of probabilities selected by the opposite player. Thus, the Nash equilibrium of pure strategy is a special mixed strategy where the probability of a strategy is 1 for each player. In order to calculate the Nash equilibrium of mixed strategies, the MATLAB function "npg" developed by Chatterjee [13] is used to solve this two-player non-cooperative game in this paper.

C. Payoff Model

To the best of our knowledge, all the game theoretical approaches of driving decision making only used the fixed-form payoff functions to formulate the reward of each strategy set in past researches. One of the drawbacks of this assumption is that the fixed-form functions are unable to describe the effects of each factors precisely. The neural networks are universal function approximators and have been demonstrated effective in various domains of researches [14]. A neural network consists of three parts in general: an input layer, multiple hidden layers and an output layer. Vector-valued inputs are fed into the input layer and are manipulated by a set of linear transformation and nonlinear activations as they traverse the hidden layers to the output layer. In this paper, the neural network is used to formulate the payoff function. The interaction between the subject vehicle and the lag vehicle is influenced not only by their own states, but also by the states of the front vehicles in host lane and the target lane. Therefore, the inputs of the neural network are the states of these four vehicles and the outputs are the payoff values of different strategy sets.

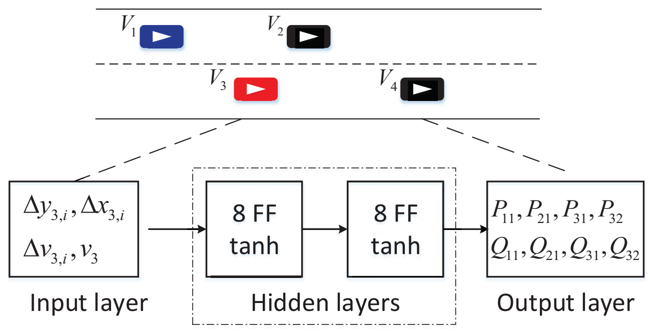


Fig. 2. Network structure of payoff model

The structure of the network used in this paper is shown in Fig. 2. It consists of two hidden feed-forward layers with 8 units. The hyperbolic tangent function (tanh) is used as the activation function of the hidden layers. The input layer $\mathbf{x}_{\text{in}}$ is denoted by:

$$\mathbf{x}_{\text{in}} = [\Delta v_{3,i}, \Delta y_{3,i}, \Delta x_{3,i}, v_3]\,, i = 1, 2, 4 \tag{2}$$

where $\Delta v_{3,i}$, $\Delta y_{3,i}$ and $\Delta x_{3,i}$ are respectively the relative speed, longitudinal gap and lateral gap between the subject vehicle and the other three surrounding vehicles, and v_3 is the speed of the subject vehicle.

In the lane keeping (pass or yield) scenarios, the lag vehicle's strategy is assumed not to affect the subject vehicle. The payoffs of the subject vehicle or the lag vehicle remain the same whatever strategy the lag vehicle selects in these scenarios, i.e.

$$P_{11} = P_{12},\ Q_{11} = Q_{12},\ P_{21} = P_{22},\ Q_{21} = Q_{22} \tag{3}$$

The output layer $\mathbf{x}_{\text{out}}$ is denoted by:

$$\mathbf{x}_{\text{out}} = [P_{11}, Q_{11}, P_{21}, Q_{21}, P_{13}, Q_{13}, P_{23}, Q_{23}] \tag{4}$$

After the neural network based payoff function is built, the real traffic data is then used to estimate the parameters in this model and validated in the next section.

III. Model Calibration

In this paper, the data used to calibrate and evaluate the proposed method is obtained from real traffic data. The detailed description of the dataset and the calibration method are presented in this section.

A. Scenario Extraction

This paper uses the public datasets of individual vehicle trajectories from NGSIM [15], a program funded by the US Federal Highway Administration. These trajectory data is thus far unique in the history of traffic research and provide a valuable basis for research into driving behavior on structured roads. All the experiments are performed on the I-80, showed in Fig. 3. The I80 dataset consists of three 15-minute periods: 4:00 p.m. to 4:15 p.m. (I-80-1), 5:00 p.m. to 5:15 p.m. (I-80-2), and 5:15 p.m. to 5:30 p.m. (I-80-3). These periods represent the buildup of congestion, or the transition between uncongested and congested conditions, and full congestion during the peak period.

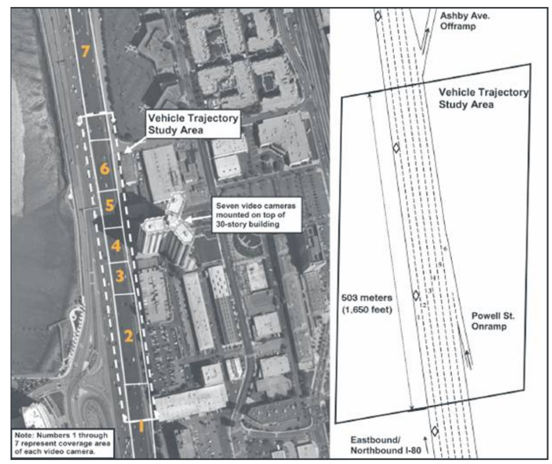


Fig. 3. I-80 scenario [15]

The segmented scenarios have the following properties:

- In each scenario, the subject vehicle and the lag vehicle remain the same.
- This work sets the relative longitudinal distance to 100 m, the relative lateral distance to 10 m and the relative speed to 0 for any of the surrounding vehicles that does not exist.
- A scenario ends when the subject vehicle crosses the lane marker, passes V_2, or yields to V_1.
- A new scenario restarts immediately after an ending of last scenario to ensure no gaps between driving scenarios.
- The segmented scenarios last at least two seconds to ensure a relatively complete lane change or lane keeping behavior.

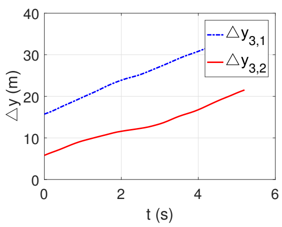
The summary of segmented sequences in the I-80 dataset is shown in Table II. The average duration of each scenario segmentation is about five seconds. The highly imbalanced data, i.e., much higher proportion of pass or yield scenarios than lane-changing scenarios, pose another significant challenge to the behavior recognition. However, it is consistent with daily driving.

There are two main difficult problems in the data scenario extraction. The first problem is the definition of the exact time at which a driver makes a decision (i.e., the time at

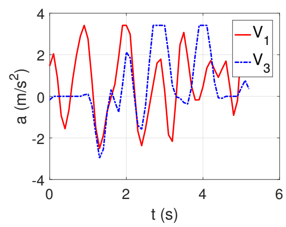
TABLE II
SCENARIOS SEGMENTATIONS

Dataset	$(a_1, -)$	$(a_2, -)$	(a_3, b_1)	(a_3, b_2)
I-80-1	1759	2897	105	122
I-80-2	1873	3743	99	91
I-80-3	1964	3944	124	92
Total	5596	10584	328	305

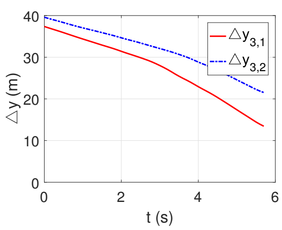
which the driver turns on the turn signal light in lane change scenario). In this work, it is assumed that the driver has made the decision 3 seconds before the end of the scenarios. And if a scenario's duration is less than 3 seconds, the start time of the scenario is regarded as the time of a driver making a decision.



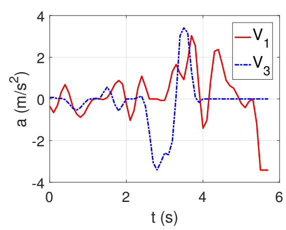
(a) Longitudinal distance



(b) Acceleration



(c) Longitudinal distance



(d) Acceleration

Fig. 4. Two indicators in different scenarios

The other problem is the classification of the strategy chosen by the lag vehicle in lane change scenarios. The general method uses average acceleration as a threshold. However, the acceleration changes frequently and has a large disturbance of measurement. Kang [10] used the longitudinal distance as the classification standard for strategy of the lag vehicle. In this work, we adopt this indicator to classify the lag vehicle's strategy. Examples of cooperate scenario and compete scenario are given in Fig. 4. The example of a cooperate scenario is given in Fig. 4(a) and Fig. 4(b), the longitudinal distance increases obviously when the subject vehicle changes the lane. The example of a compete scenario is shown in Fig. 4(c) and Fig. 4(d) where the longitudinal distance decreases when the subject vehicle changes the lane. However, it is hard to find a law of acceleration in different scenarios.

B. Calibration Approach

The method of Gaussian particle swarm optimization (GPSO) presented by Krohling [16] is used to estimate the parameters of the payoff model in this paper. GPSO has a strong ability of finding the global optimum, but its local search ability is weak [17]. In order to solve this problem, differential evolution (DE) is also used in our work. The calibration method is given in Algorithm 1 in details.

Algorithm 1: Model calibration with DE-GPSO [17]

Input: the structure of payoff model, marked data of different scenarios;
Output: the parameters of payoff model;

for *each particle* $i = 1, 2, \ldots, S$ **do**
  initialize the particle's position: x_i;
  initialize the particle's best known position to its initial position: $l_i \leftarrow x_i$;
Initialize the swarm's best known position: g;
k=0;
repeat
  for *each particle* $i = 1, 2, \ldots, S$ **do**
    Update the particle's position:
    $x_i \leftarrow x_i + abs(\alpha_1)(l_i - x_i) + abs(\alpha_2)(g - x_i)$, where α_1 and α_2 meet the Gaussian distribution;
    for *each training data* $n = 1, 2, \ldots, N$ **do**
      Calculate the payoff values through the network;
      Calculate the Nash equilibrium: $(p_{\mathrm{S},n}(1), p_{\mathrm{S},n}(2), p_{\mathrm{S},n}(3), p_{\mathrm{L},n}(1), p_{\mathrm{L},n}(2))$;
    Calculate the cost function: $J(x_i)$;
    if $J(x_i) < J(l_i)$ **then**
      Update the particle's best know position: $l_i \leftarrow x_i$;
    if $J(x_i) < J(g)$ **then**
      Update the swarm's best know position: $g \leftarrow x_i$;
  for *each parameter of network* $j = 1, 2, \ldots, P$ **do**
    Randomly select two particles x_m and x_n;
    $x_{tmp}(j) = g(j) + (x_m(j) - x_n(j))$;
    if $J(x_{tmp}) < J(g)$ **then**
      Update the swarm's best known position: $g \leftarrow g_{tmp}$;
  $k = k + 1$;
until $J < 0.1$ *or* $k > 1000$;

The One-hot encoding method is used to define the label of different sets of strategies (e.g. the label of subject vehicle in lane change scenarios is [0,0,1]). The error of the probabilities between the label and the output of our model is used as the optimization object which is denoted by:

$$J = \frac{1}{N} \sum_{n=1}^{N} \left(|t_{\mathrm{S},n} - p_{\mathrm{S},n}| + \alpha \, |t_{\mathrm{L},n} - p_{\mathrm{L},n}| \right) \tag{5}$$

$$\alpha = \begin{cases} 1, & \text{if the type of scenario is lane change} \\ 0, & \text{if the type of scenario is lane keeping} \end{cases} \tag{6}$$

where n is the index of the scenarios, N is the number of scenarios in the data set, $t_{\mathrm{S},n}$ is the label of the subject vehicle, $p_{\mathrm{S},n}$ is the possibility of the subject vehicle from our model, $t_{\mathrm{L},n}$ is the label of the lag vehicle, $p_{\mathrm{L},n}$ is the possibility of the subject vehicle from our model.

IV. Model Validation

In order to balance the number of different scenarios, 300 segmentations of each scenario type are selected in this paper. The 80% of the data (240 segmentations of each type) are used to calibrate the payoff function and the remaining 20% (60 segmentations of each type) are used to validate the model. The experiment results of the proposed method are listed in Table III. The prediction results of the pass and yield scenarios are accurate and there are several false alarm and missed detection in cut-in scenarios.

TABLE III
EXPERIMENT RESULTS

		Label			
		$(a_1,-)$	$(a_2,-)$	(a_3,b_1)	(a_3,b_2)
Prediction	$(a_1,-)$	55	2	3	8
	$(a_2,-)$	0	53	7	3
	(a_3,b_1)	2	0	39	9
	(a_3,b_2)	3	5	11	40

In order to evaluate the proposed model accurately, four quantitative metrics are used to evaluate the performance of the proposed model:

- Accuracy (ACC) is the fraction of correctly classified events out of all testing events. It is defined as

$$\mathrm{ACC} = \frac{\mathrm{TP}+\mathrm{TN}}{\mathrm{TP}+\mathrm{TN}+\mathrm{FP}+\mathrm{FN}}$$

  where TP means true positive, TN means true negative, FP means false positive (false alarm) and FN means false negative (missed detection) .
- Precision (PRE) is the fraction of events classified correctly out of all events predicted to be positive, i.e.

$$\mathrm{PRE} = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FP}}$$

- True Positive Rate (TPR),also named Recall, is the fraction of events classified correctly out of all true events, i.e.

$$\mathrm{TPR} = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FN}}$$

- F1 Score is the harmonic mean of the precision and the recall, i.e.

$$F_1 = 2 \times \frac{\mathrm{PRE}\times\mathrm{TPR}}{\mathrm{PRE}+\mathrm{TPR}}$$

The overall ACC of the proposed model is 0.7792. In order to evaluate the performance of the proposed model in each type of scenarios, the prediction can be regarded as a binary classification by treating the other three types' interaction as one type. The results of the metrics for each type of interaction are listed in Table IV.

TABLE IV
MODEL PERFORMANCE

	$(a_1,-)$	$(a_2,-)$	(a_3,b_1)	(a_3,b_2)
ACC	0.9250	0.9292	0.8667	0.8375
PRE	0.8088	0.8413	0.7800	0.6780
TPR	0.9167	0.8833	0.6500	0.6667
F_1	0.8594	0.8618	0.7091	0.6723

The performance of the proposed method in lane keeping scenarios is better than the performance in lane change scenarios. In the lane keeping scenarios, the proposed method has a good ability to classify the yield scenarios and pass scenarios. In the lane change scenarios, the prediction performance of the subject vehicle is better than the performance of the lag vehicle. Compared with the subject vehicle, the individual difference has more influence on the lag vehicle. Therefore, it is more difficult to predict the strategy of the lag vehicle.

Different game theoretical driving decision making methods have different strategies of vehicles. Therefore, this paper compares the performance of our method with that of the method proposed by Talebpour [9] under the same strategies of the subject vehicle (lane keeping and lane change). This method is evaluated by the four metrics mentioned above, which are listed in Table V.

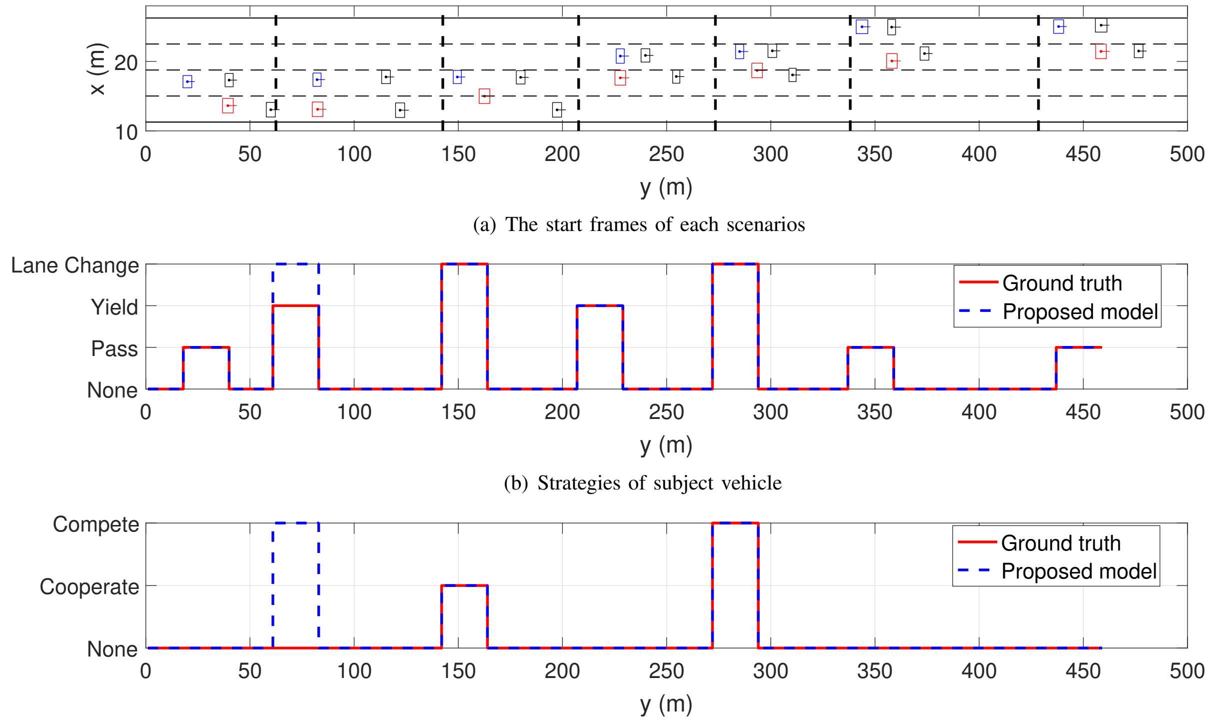
TABLE V
RESULTS COMPARISON

	ACC	PRE	TPR	F_1 Score
Our method	0.8708	0.9083	0.8250	0.8995
Talebpour [9]	0.5789	0.6614	0.5793	0.6176

The comparison shows that the proposed method has a better ability of decision making in highway scenarios. An application example of the proposed method with a continuous vehicle trajectory (subject vehicle ID: 820) is given in Fig. 5. The start frames of each segmented scenarios (also the end frames of the last scenarios) are shown in Fig. 5(a). Fig. 5(b) and Fig. 5(c) give the strategies of subject vehicle and lag vehicle, respectively. This example shows that the proposed method can make a correct decision in most of the highway scenarios.

V. Conclusion and Future Work

This paper proposes a driving decision making method in highway scenarios based on game theory and neural-network. A two-player, non-zero-sum, non-cooperative game under complete information is used to describe the interaction between two vehicles. The neural-network is used to build the payoff model. Compared with the fixed-rule based payoff model, neural-network based payoff model can describe the effects of each factor precisely and improve the ability of decision making. The model is calibrated by DE-GPSO method with the NGSIM dataset. Compared with another method [9], the performance of our method has been validated with multiple quantitative metrics.



(a) The start frames of each scenarios

(b) Strategies of subject vehicle

(c) Strategies of lag vehicle

Fig. 5. An example of the proposed lane-changing decision making method

In the future work, we want to extend the number of players and the strategy sets to consider the both sides of the subject vehicle and more complicated scenarios. In order to evaluate the performance of the proposed method under different traffic condition, we need to validate the method with the other data. Considered the development of self-driving technology, the decision making process should be regard as a cooperative game because the communication between vehicles is allowed.

Acknowledgment

This work was supported in part by the National Natural Science Foundation of China under Grant No. 61473209 and No. 61773291.

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