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Optimization of Speed Profile and Energy Interaction at Stations for a Train Vehicle with On-board Energy Storage Device

Chaoxian Wu[1], Shaofeng Lu[1*], Fei Xue[1], Lin Jiang[2] and Jie Yang[3]

Abstract— **With the increasing application of railway transportation, energy consumption of railway transportation rises dramatically, which in turn undermines its sustainability. Optimization on train speed profile and use of regenerative energy is becoming a feasible and applicable approach to achieve an energy-efficient operation without changing existing infrastructures. Considering both dwelling at stations and running in the inter-station sections, the paper proposes an integrated optimization model for reducing net energy consumption from the viewpoint of energy interaction among train, substation and on-board energy storage device (ESD), based on which the optimal train speed profile is also found. The discharge/charge strategy of on-board ESD is explored and comparative case studies are given, under the assumption of higher efficiency from the on-board ESD, less net energy consumption can be achieved via energy interaction with substation. Through charging from substation the net energy consumption can be reduced, i.e. 0.2%, in the comparison.**

I. INTRODUCTION

With the increasing demand for railway transportation, the energy consumption of this transportation mode surges. As a result, researches and applications on regenerative energy management is now the hot spot in the field of railway study.

One way to absorb the regenerative braking energy is to use the energy storage devices (ESD) e.g. on-board ESD. Due to its high efficiency and absence of transmission loss, on-board ESD have already been applied in many railway systems such as Paris Tramway and LRV (Light Rail Vehicle) in Mannheim to reduce energy consumption [1]. The energy-saving potential of on-board ESD and its evident effect on reducing energy consumption have also been investigated in some papers [2]–[4]. In addition, Miyatake *et al.* proposed an optimization model on train speed trajectory considering on-board ESD [5]–[7]. In [8] and [9] , the optimal train speed control without catenary is studied and the strategy of quick charging of the on-board ESD at station is also discussed. However, in these papers, integrated optimization method considering energy interaction between substation, train and on-board ESD during both running in sections and dwelling at stations and the energy-saving potential depending on it are not given.

Optimization on train speed control is a traditional way to save traction energy and it is usually achieved through seeking optimal speed profile. One way to locate this optimal speed profile is to find the optimal speed at each switching point. Once the switching points are found, the whole speed profile can be obtained. The optimal train speed trajectory can be found by maximum principle based on optimal control theory [10]–[13]. The solution can be obtained through numerical algorithm by determining the cruising regimes and connecting two consecutive constant-speed regimes with traction, coasting or braking operation. Mathematical programming is also used in finding the optimal speed profile and [14] proposes a speed-based model to optimize the partial speed trajectory during speed-varying processes such as acceleration and decelerations for recovering more regenerative energy. The work is extended in [15] and it proposes a distance-based model which uses energy to replace the speed. Nevertheless, these researches did not take on-board ESD into account.

On-board ESD not only should be utilised when the train runs but also should be used freely when the train stops at the station as long as the reduction of energy consumption can be achieved. Therefore, from the viewpoint of energy flow, the paper proposes an integrated optimization model using Mixed Integer Linear Programming (MILP) to investigate the free energy interaction of on-board ESD at railway station as well as the optimal speed profile of a single train vehicle with on-board ESD. The regenerative braking energy stored in on-board ESD is given the freedom to interact with power system when the train stops at the station and the on-board ESD can charge or discharge according to different demand for achieving an energy-efficient operation of the whole system.

To demonstrate our points more clearly, the energy flow for a typical train vehicle in this research is shown in Figure 1. Both the energy from the substation and the energy stored in the on-board ESD are able to jointly supply the energy needed by the train for motoring. On the other hand, in this paper the substation is reversible, which means that it can accept regenerative braking energy. Thus, when the train motor conducts electrical braking operation, the regenerative braking energy can be absorbed by on-board ESD and fed back to reversible substation. Due to the fixed capacity of the on-board ESD and limited regenerative power, a fraction of

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[1]Chaoxian Wu, Shaofeng Lu, Fei Xue are with the Department of Electrical and Electronic Engineering, Xi'an Jiaotong-Liverpool University, China, 215123; Chaoxian.Wu@liverpool.ac.uk; Shaofeng.Lu@xjtlu.edu.cn; Fei.Xue@xjtlu.edu.cn

*Corresponding autor

[2]Lin Jiang is reader in the Department of Electrical Engineering and Electronics, University of Liverpool, UK; L.Jiang@liverpool.ac.uk

[3]Jie Yang is currently an Associate Professor with School of Electric Engineering, Jiangxi University of Science and Technology, Ganzhou, China; 15405993@qq.com

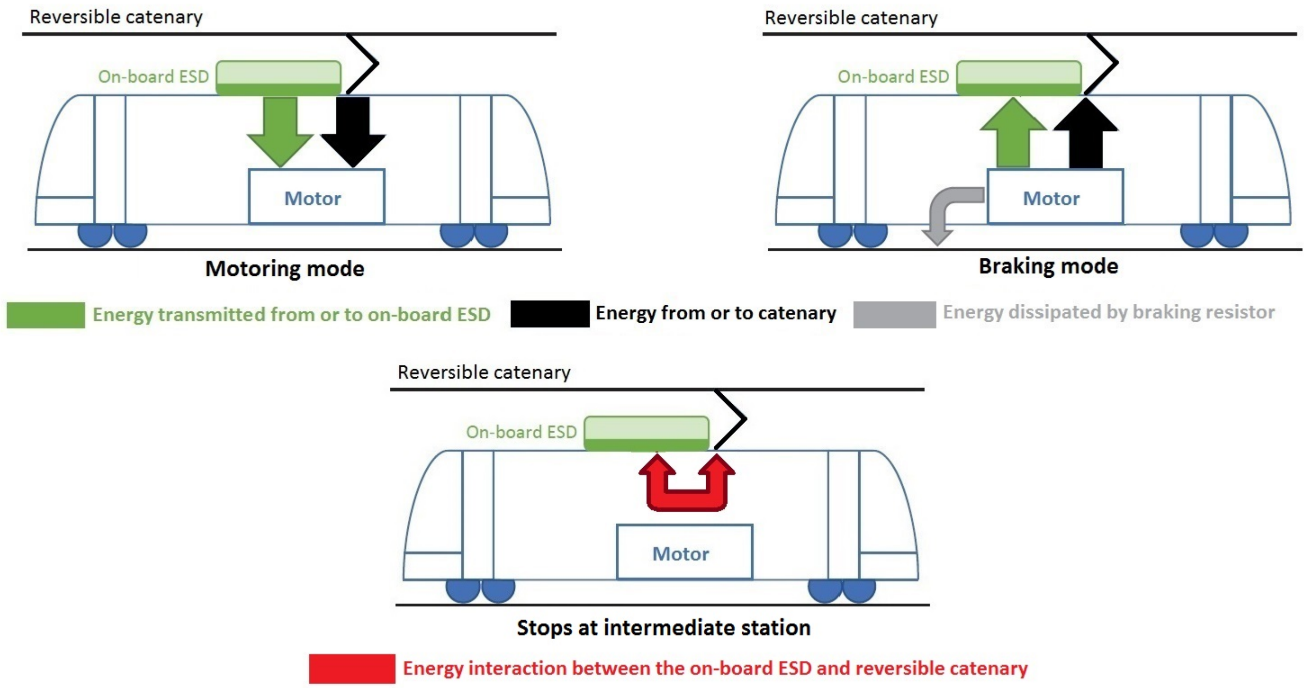

Fig. 1. Schematic of the energy flow for a typical train with on-board ESD

the regenerative energy would not be recovered and would be dissipated by the braking resistor. After the running, when the train stops at the station, the on-board ESD can discharge to or charge from the catenary to adjust its state of charge (SOC).

II. Mathematical Modelling

A. Distance-based Model and Piecewise Linear

In this paper, the train operation between two stations is discretized to simplify the calculation, the whole journey will be divided into several segments with the same distance Δd in **Distance-based Model** [15]. For distance D of each inter-station journey, the amount of the distance segments is $N = D/\Delta d$ and the train is assumed to do uniformly accelerated motion and have $\Delta d = (v_{i+1}^2 - v_i^2)/2a_i$, where a_i is the acceleration of each distance segment. For N distance segments, there will be $N+1$ speed points v_i, where $i = 1, 2, 3...N+1$, in our optimized solution.

In the research, the full train speed trajectory is studied and the diagram to illustrate the model is demonstrated in Figure 2. The grey hollow points are the candidate speeds at the endpoints of each distance segment and black solid ones are the found optimized solution. When the optimal speed points are found, the optimal speed profile can be obtained by linking them directly.

Piecewise linear is utilized to model these nonlinear constraints and this methodology can represent a nonlinear function with a series of nonnegative variables of special ordered sets type 2 (SOS2), among which only two adjacent ones can be greater than 0 with the total sum of all variables equal to 1 [16]. Therefore, the square and reciprocal of velocity equals to the combination of points on $f(v) = v^2$ and $f(v) = 1/v$ separately. In this method, δ (see Figure 2) is set to be the a common difference and it is a small number representing the linear sections over the piecewise linear valid range from v_{min} to v_{max} thus we have $K = (v_{max} - v_{min})/\delta, K \in \mathbb{N}$. The v^2 and v are expressed in (1) and (2) respectively.

$$v_i^2 = \sum_{k=1}^{K} (v_{min} + (k-1)\delta)^2 \cdot \alpha_i^k \tag{1}$$

$$v_i = \sum_{k=1}^{K} (v_{min} + (k-1)\delta) \cdot \alpha_i^k \tag{2}$$

where α_i^1 ,α_i^2 ,α_i^3 ...,α_i^K are variables of SOS2 mentioned above in each distance segment and they should satisfy the (3) and (4).

$$\sum_{k=1}^{K} \alpha_i^k = 1 \tag{3}$$

$$0 \leq \alpha_i^k \leq 1, k = 1, 2, 3...K \tag{4}$$

The average speed of each distance segment $v_{i,avg}$ can be calculated as:

$$v_{i,avg} = \frac{v_i + v_{i+1}}{2} \tag{5}$$

For connecting the v^2 and the $1/v$, another set of SOS2 has to be used so we have (6) and (7) where the β_i^k are variables of this set of SOS2 in each segment.

$$v_{i,avg} = \sum_{k=1}^{K} (v_{min} + (k-1)\delta) \cdot \beta_i^k \tag{6}$$

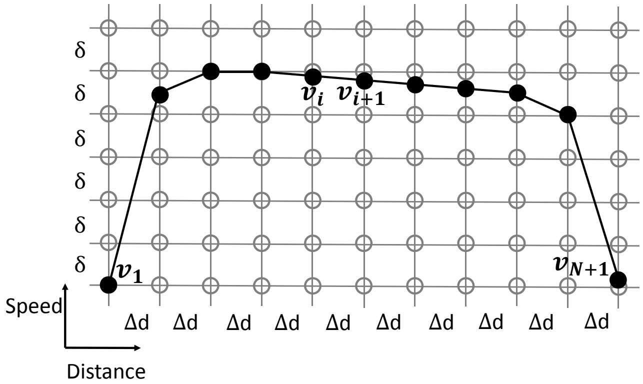

Fig. 2. Distance-based Model for full speed trajectory

$$\frac{1}{v_{i,avg}} = \sum_{k=1}^{K} \frac{\beta_i^k}{v_{min} + (k-1)\delta} \tag{7}$$

Similar as α_i^k, variables β_i^1 ,β_i^2 ,β_i^3 ...,β_i^K should satisfy the (8) and (9) as well.

$$\sum_{k=1}^{K} \beta_i^k = 1 \tag{8}$$

$$0 \leq \beta_i^k \leq 1, k = 1, 2, 3...K \tag{9}$$

B. Motion Analysis

During the journey, the drag force can be calculated according to Davis equation (10), where A, B and C are the Davis coefficients. (11) indicates that the acceleration of each distance segments should not exceed the maximum acceleration a_{max} allowed. The time interval Δt_i for each distance segment can be calculated using (12).

$$F_{i,drag} = A + Bv_{i,avg} + Cv_{i,avg}^2 \tag{10}$$

$$-a_{max} \leq \frac{v_{i+1}^2 - v_i^2}{2\Delta d} \leq a_{max} \tag{11}$$

$$\Delta t_i = \frac{\Delta d}{v_{i,avg}} \tag{12}$$

(13) is the constraint on journey time T and the inequality gives a flexibility to the train operation.

$$\sum_{i=1}^{N} \Delta t_i \leq T \tag{13}$$

The journey time can be automatically adjusted during the optimization process as long as it does not exceed the maximum allowed time.

C. Modelling the Inter-station Energy Flow

The model is established on the energy flow using linearization and the energy flow is transferred between suppliers (substation and on-board ESD) and train motor. In each distance segement, the train might either consume the energy from substation and on-board ESD or conduct electrical braking to regenerate the energy and send it back to substation and on-board ESD.

As shown in (14), $E_{i,m}$ is the energy consumed or regenerated by the train motor in the i^{th} distance segment. MILP is used [16] for distinguishing different operation modes. When $\lambda = 1$, the train is motoring, which the motor consumes both the energy from substation E_{i,sub_con} with efficiency η_{sub} and the energy from the on-board ESD $E_{i,dch}$ with efficiency η_{ESD}; when $\lambda = 0$, the train conducts regenerative braking and regenerative energy $E_{i,ch}$ would be recovered by on-board ESD with efficiency η_{ESD} while the other part of regenerative energy E_{i,sub_reg} would be sent back to the substation with efficiency η_{sub}. It is noted that the η_{sub} is to specify the total efficiency for motors with the driving system and energy transmission of the substation; similarly, η_{ESD} here is to specify the total efficiency for motors with the driving system and energy conversion from the on-board ESD. In general, η_{ESD} is greater than η_{sub} due to its absence of line losses and the higher transmission losses of the grid [17].

$$\begin{aligned} E_{i,m} &= \lambda(E_{i,sub\_con} \cdot \eta_{sub} + E_{i,dch} \cdot \eta_{ESD}) \\ &+ (1-\lambda)(\frac{-E_{i,sub\_reg}}{\eta_{sub}} + \frac{-E_{i,ch}}{\eta_{ESD}}) \end{aligned} \tag{14}$$

$$\lambda = 0 \ or \ 1 \tag{15}$$

To ensure that when there are E_{i,sub_con} and $E_{i,dch}$, there is no $E_{i,ch}$ and E_{i,sub_reg} existing at the same time, vice versa. Thus (16), (17), (18) and (19) need to be imposed.

$$0 \leq E_{i,sub\_con} \leq \lambda L_1 \tag{16}$$

$$0 \leq E_{i,dch} \leq \lambda L_1 \tag{17}$$

$$0 \leq E_{i,sub\_reg} \leq (1-\lambda) L_2 \tag{18}$$

$$0 \leq E_{i,ch} \leq (1-\lambda) L_2 \tag{19}$$

where L_1 and L_2 are two sufficiently large numbers.

Therefore, the transformation of the energy could be expressed as a constraint presented in (20), where M is the total mass of the train including the on-board ESD, g is the gravitational constant and Δh_i is the altitude difference between the current segment and the previous segment. When the train is in traction mode, $E_{i,m} \geq 0$, the electricity energy from both the substation and the on-board ESD are used by the motor and transformed into kinetic energy, heat and potential energy; when the train conducts electrical braking, $E_{i,m} \leq 0$, the kinetic energy is transformed into heat, potential energy and electrical energy which is recovered by the on-board ESD and substation.

$$E_{i,m} - \frac{1}{2}M(v_{i+1}^2 - v_i^2) - F_{i,drag}\Delta d - Mg\Delta h_i \geq 0 \tag{20}$$

In addition to the energy restriction, the power limit of the motor and on-board ESD should be added as the constraints respectively. For the motor, (21) and (22) is used to ensure the power that the motor supplies in both traction mode and braking mode does not exceed the maximum power $\overline{P_m}$.

$$E_{i,sub\_con} \cdot \eta_{sub} + E_{i,dch} \cdot \eta_{ESD} \leq \overline{P_m} \Delta t_i \tag{21}$$

$$\frac{E_{i,sub\_reg}}{\eta_{sub}} + \frac{E_{i,ch}}{\eta_{ESD}} \leq \overline{P_m}\Delta t_i \tag{22}$$

For the on-board ESD, the power for the input and output energy cannot exceed the maximum charge and discharge power.

$$E_{i,dch} \leq \overline{P_{ESD}}\Delta t_i \tag{23}$$

$$E_{i,ch} \leq \overline{P_{ESD}}\Delta t_i \tag{24}$$

When the velocity is high, the maximum force provided by the motor reaches its limit F_{max}. Therefore, the work done by the motor can be restricted by (25) and (26):

$$E_{i,sub\_con} \cdot \eta_{sub} + E_{i,dch} \cdot \eta_{ESD} \leq F_{max}\Delta d \tag{25}$$

$$\frac{E_{i,sub\_reg}}{\eta_{sub}} + \frac{E_{i,ch}}{\eta_{ESD}} \leq F_{max}\Delta d \tag{26}$$

To ensure a valid SOC of the on-board ESD between 0 and 100% during the journey, (27) has to be imposed as an extra constraint, where E_{ini} is the initial available energy in the on-board energy when the journey begins and E_{cap} is the capacity of the on-board ESD.

$$\begin{gathered} 0 \leq E_{ini} - \sum_{i=1}^{J} E_{i,dch} + \sum_{i=1}^{J} E_{i,ch} \leq E_{cap} \\ J = 1, 2, 3...N \end{gathered} \tag{27}$$

D. The Energy Interaction at Intermediate Station

When the train stops at the station, the dwell time is given for passengers' boarding and alighting movement. During this time, the on-board ESD is given the freedom to interact with reversible catenary, namely discharges to catenary or charge from catenary.

Therefore, for charge and discharge behaviour, two equations should be satisfied as shown in (28) and (29) where E_{s,sub_con} represents the energy consumed from substation and E_{s,sub_reg} means the energy received by substation; $E_{s,ch}$ is the energy received by on-board ESD and $E_{s,dch}$ is the energy discharged to substation; η_s is the transmission efficiency between on-board ESD and substation.

$$E_{s,sub\_con} \cdot \eta_s = E_{s,ch} \tag{28}$$

$$\frac{E_{s,sub\_reg}}{\eta_s} = E_{s,dch} \tag{29}$$

Similar as the method in last subsection, there is no discharge and charge behaviour at the same time thus γ is another integer variable introduced to guarantee this, see (30), (31), (32), (33) and (34).

$$0 \leq E_{s,sub\_con} \leq \gamma L_3 \tag{30}$$

$$0 \leq E_{s,ch} \leq \gamma L_3 \tag{31}$$

$$0 \leq E_{s,sub\_reg} \leq (1-\gamma) L_4 \tag{32}$$

$$0 \leq E_{s,dch} \leq (1-\gamma) L_4 \tag{33}$$

$$\gamma = 0 \ or \ 1 \tag{34}$$

where L_3 and L_4 are two sufficiently large numbers.

In the intermediate station, the charging/discharging time cannot exceed the dwell time thus the (35) and (36) should be imposed. The dwell time T_d is given in the paper and the charging/discharging power is set to be the maximum power of the on-board ESD.

$$0 \leq E_{s,ch} \leq \overline{P_{ESD}}T_d \tag{35}$$

$$0 \leq E_{s,dch} \leq \overline{P_{ESD}}T_d \tag{36}$$

E. Optimization on Energy Management

As mentioned in Section I, the research is conducted with the objective of achieving minimum net energy consumption. Thus, by summing the net energy consumption of each segment the net energy consumption of the running between two stations E_i can be obtained (37).

$$\begin{aligned} \sum_{i=1}^{N} E_i = \sum_{i=1}^{N} E_{i,sub\_con} + \sum_{i=1}^{N} E_{i,dch} \\ - \sum_{i=1}^{N} E_{i,sub\_reg} - \sum_{i=1}^{N} E_{i,ch} \end{aligned} \tag{37}$$

Due to the transmission loss, the net energy consumption at the station E_s resulted from the discharge/charge operation of on-board ESD can be expressed as (38).

$$E_s = E_{s,sub\_con} - E_{s,ch} + E_{s,dch} - E_{s,sub\_reg} \tag{38}$$

The **objective function** is shown in (39)

$$\begin{gathered} \min(\sum_{i=1}^{N} E_i + E_s) \\ \text{s.t. } (1) - (36) \end{gathered} \tag{39}$$

The model minimizes the energy consumption of traction, maximizes the regenerative braking energy and minimizes the extra energy consumption of charging/discharge behaviour at station, resulting in a minimization on the net energy consumption of the whole journey.

III. Results and Discussion

In this model, the parameters are set according to Table I. The track condition of the journey is shown in Figure 3 and it contains three stations and two sections with no gradient variation. D_1 is 1800 m and D_2 is 2200 m, which the whole track length is 4000 m. T_1 and T_2 is the journey time for inter-station section 1 and section 2 respectively. Thus, journey time from the initial station to terminal station is $T_1 + T_d + T_2 = 210s$ which is set to be fixed with T_d given as 30 s in the cases. The capacity of the on-board ESD is 30000 kJ and charging/discharging power of it at intermediate station is fixed to be the maximum value 500 kW.

Since the energy interaction at intermediate station is given the full freedom, it means that as long as this interaction can reduce the net energy consumption of the whole journey, the system will charge or discharge automatically. See case 1 in Figure 4, when the train stops at the station, there is a

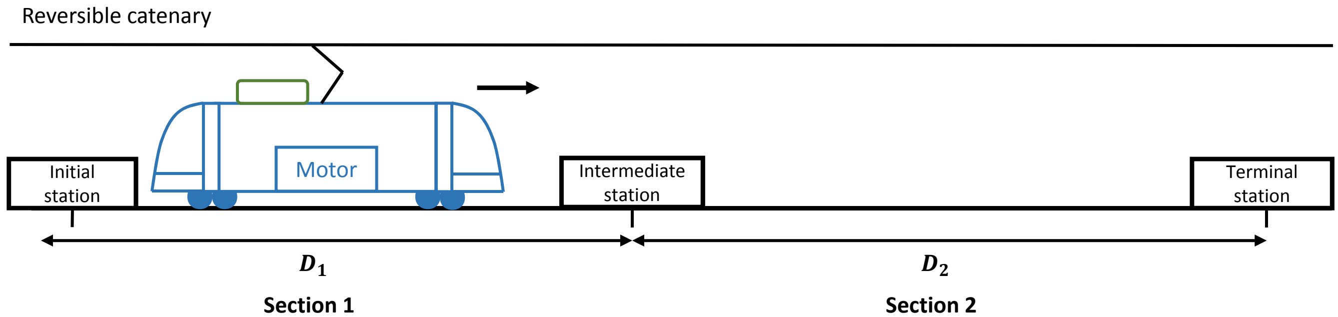

Fig. 3. Track condition for multiple stations

TABLE I
PARAMETERS IN A JOURNEY FOR A TYPICAL TRAIN VEHICLE WITH ON-BOARD ENERGY STORAGE DEVICE

M (t)	$\overline{P_m}$ (kW)	F_{max} (kN)	a_{max} (m/s^2)	V_{max} (m/s)	$\overline{P_{ESD}}$ (kW)	Capacity (kJ)
178	5000	200	1.2	45	500	30000

η_{sub}	η_{ESD}	η_s	Δd (m)	Davis coefficients		
				A	B	C
0.7	0.9	0.9	50	2.0895	0.0098	0.0065

gap between the final SOC of on-board ESD for the first section and the initial SOC of on-board ESD for the second section. It means that the on-board ESD charges from the substation and it raises its SOC from 62.4% to 80.3%. The time consumed on charging at station is 10.7 s which is less than the given dwell time T_d and the net energy consumption of this journey is 192.65 MJ.

For a clear demonstration on how this energy interaction at intermediate station reduces the net energy consumption of the whole journey, the result of the model without energy interaction at intermediate station is also shown in case 2 in Figure 4. The objective function of the model without energy interaction at stations is expressed in (40).

$$\min \sum_{i=1}^{N} E_i \quad \text{s.t. } (1) - (27) \tag{40}$$

The freedom of energy interaction between on-board ESD and substation is not given in case 2 and its net energy consumption of it is 193.05 MJ, which is 0.2% higher than that of case 1, showing that adjusting the SOC of on-board ESD at intermediate station can save more energy from the substation during the whole journey. Therefore, it can be observed that when we give this freedom of interaction between on-board ESD and substation, it can bring a more energy-efficient operation of the whole system though there is an extra energy consumption when on-board ESD charges at station.

In addition, the train speed profile of two cases are obtained and it sees changes with different behaviours of on-board ESD at intermediate station. We can see that when on-board ESD charges at station, the braking distance of

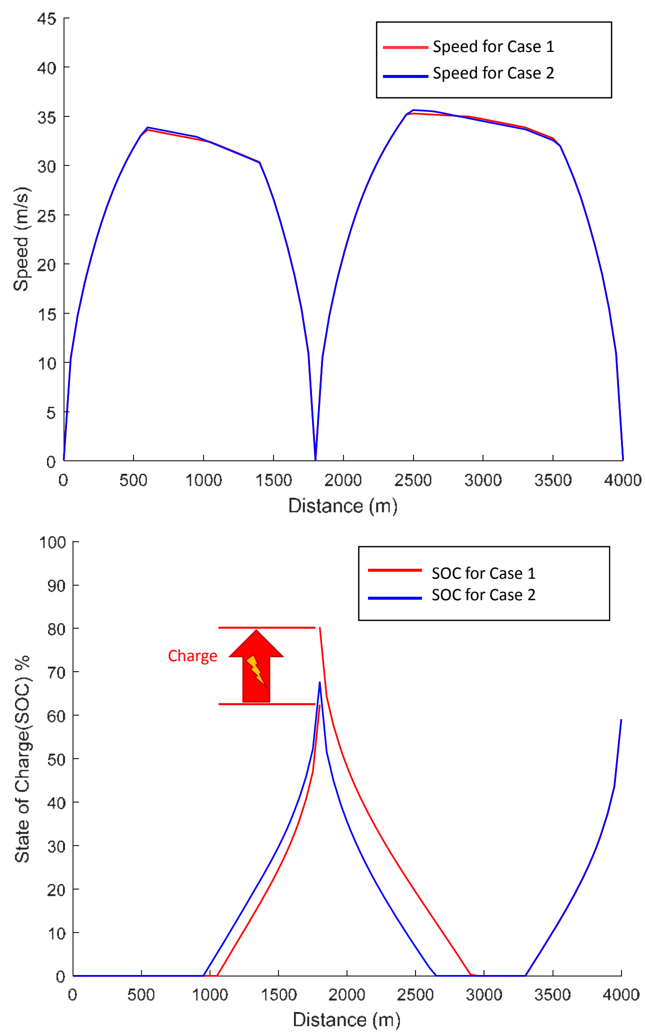

Fig. 4. Train speed profile and SOC of on-board ESD for two cases

train vehicle is shortened in the first section while its traction distance in the second section is lengthened.

The energy interaction at station results in a difference in speed trajectory, afterwards influences the energy flow from or to each component of the system, see Table II. From the table, different distributions of the energy consumption and energy regeneration are observed. By comparing two

TABLE II
ENERGY CONSUMPTION AND REGENERATION FOR TWO CASES

	Section 1		Station		Section 2	
Case	1	2	1	2	1	2
$\sum E_{i,sub\_con}$	153.00	155.15	/	/	146.55	149.75
$\sum E_{i,dch}$	0	0	/	/	24.09	20.28
$\sum E_{i,sub\_reg}$	-44.09	-43.90	/	/	-51.05	-50.23
$\sum E_{i,ch}$	-18.73	-20.28	/	/	-17.71	-17.73
$\sum E_{i}$	90.17	90.97	/	/	101.88	102.08
E_{s,sub_con}	/	/	5.96	0	/	/
$E_{s,ch}$	/	/	-5.36	0	/	/
E_{s,sub_reg}	/	/	0	0	/	/
$E_{s,dch}$	/	/	0	0	/	/
E_{s}	/	/	0.6	0	/	/
* All the values are in MJ and ”-” means the energy recovered						

cases, we can get that as for case 1, it tends to use less on-board ESD to absorb the braking energy in the first section since it can charge from the substation with high efficiency when dwells at the intermediate station to reduce energy consumption. It also uses more energy from on-board ESD in the traction mode of the second section. In contrast, more energy is from substation to support train's motoring in both first and second inter-station sections in case 2 and on-board ESD of it tends to recover more braking energy and avoid releasing too much energy when the train is conducting traction. Therefore, after the reconfiguration of the energy flow, the energy interaction at station can help achieve a more energy-saving operation for the whole system.

IV. CONCLUSION AND FUTURE WORK

Based on Mixed Integer Linear Programming (MILP), the optimal operation of a train vehicle with on-board energy storage device (ESD) is studied in this paper. From the viewpoint of energy interaction among train, substation and on-board ESD, the energy-efficient management on energy stored in on-board ESD and the optimal train speed control can be discussed in the optimization model. The results show that charging from the substation at intermediate station can reduce the net energy consumption of the whole journey from initial station to the terminal station, resulting from the corresponding adjustment of the speed profile and the discharge/charge strategy during the running since the on-board ESD can directly charge with higher transmission efficiency at stations rather than merely absorb the energy regenerated by motor in the sections.

This paper only considers the simple cases without complex situations such as the speed limits and gradient information. However, this could be easily achieved based on the current distance-based model since both of the speed limits and gradients can determined by using distance segments. Besides, as only 3 stations are involved in the cases, in the future the model can be extended to model the operation of a whole metro line with more stations. Along with that, the scheduling problem regarding to train operation with on-board ESD could be investigated. Since there is an assumption that energy transmission efficiency of stationary energy interaction is high and it is regarded as an input parameter, in the future the model can take the dynamic of the grid into account to enhance the practicality of the model. Based on the current model, the optimization on the energy interaction of other energy resources e.g. wayside ESD for train operation can also be explored.

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