2018 IEEE Intelligent Vehicles Symposium (IV)
Changshu, Suzhou, China, June 26-30, 2018

The Mathematical Modeling of the Two-Echelon Ground Vehicle and Its Mounted Unmanned Aerial Vehicle Cooperated Routing Problem

Zhihao Luo[1, *], Zhong Liu[1], Jianmai Shi[1], Qi Wang[1], Tianren Zhou[1], Yao Liu[1]

Abstract— **In this paper, we presents a novel Two-Echelon Ground Vehicle and Its Mounted Unmanned Aerial Vehicle Cooperated Routing Problem (2E-GUCRP), which consists of optimizing the route of both ground vehicle (GV) and its mounted Unmanned Aerial Vehicle(UAV) in the context of Intelligence, Surveillance and Reconnaissance(ISR) mission. The UAV is launched from the ground vehicle and automatically flies to the designated target to accomplish the ISR mission. Meanwhile, the ground vehicle is synchronized to charge or change the UAV's battery on the designated landing points based on the UAV's battery life. The objective is to design efficient ground vehicle and UAV routes to minimize the total mission time while meeting the operational constraints. The experimental results show that the model proposed in this paper is correct, but the existing commercial software cannot solve the large-scale problem with an acceptable time.**

I. Introduction

With the miniaturization and intellectualization of unmanned aerial vehicles (UAVs), a new working model is emerging, where the UAV is mounted on the ground vehicle to complete the cooperative task. In recent years, such model has caught increasingly attention from both academia and industry, driving the development of practical applications.

One typical extensive study on this model is the application in the field of logistics transportation. Murray and Chu [1] analyzed a variant of Travelling Salesman Problem (TSP) t with single ground vehicle and single UAV called the flying sidekick TSP (FSTSP), where the UAV and ground vehicle start and end at a depot, the ground vehicle (GV) travels on the road network, and the UAV can deliver on its own. At the beginning of 2017, UPS began testing practical applications for this novel working model. In its first test, UPS successfully employed a UAV mounted in a ground vehicle to deliver a shipment to a customer's home [2]. In 2016 International Consumer Electronics Show (CES), DJI-Innovations and Ford proposed a novel mode for wild search, where the F-150 carried a UAV and conducted the search for multiple targets in the wilderness cooperatively [3]. In addition to the extensions of specific tasks, UAV can assist in automatic control of ground vehicles as well. Coincidentally, a vehicle manufacturer RINSPEED developed a new conceptual vehicle in the CES as mentioned above [4]. A special electric quadrotor UAV is mounted in their new conceptual vehicle, aiming at achieving the monitoring task and expanding the scope of vision for real-time vehicle control.

The emergence of this new cooperated mode is inevitably posing urgent requirement for the corresponding cooperative routing method to simultaneously optimize both the ground vehicle and the UAV route. However, the existing vehicle routing and UAV routing methods seldom consider the cooperated situation. To our knowledge, the general ideas to address such problem are to directly take ground vehicle and UAV as the different layers of routing, and then describe the process with a classic two-echelon routing problem (2E-RP) in the field of transportation. The 2E-RP includes two types of fundamental problems, in which a manager should determine depot locations and determine the first level route for distribution of products to open depot together and the second level routes to deliver goods to customers [5]. To solve the routing problem of ground vehicle and its mounted UAV with the 2E-RP model, the choice of the temporary stopping point of the ground vehicle is considered as the choice of the depot, and the path of the UAV is considered as the path in the second level. Manyam et.al [6] applied this model to give a solution to the ISR mission of ground vehicle and its mounted UAV. This model is inefficient and inapposite for UPS or DJI as the ground vehicle stays still when the UAV performs its tasks. In view of above, this paper focuses on the Two-Echelon Ground Vehicle and its Mounted UAV Cooperated Routing Problem (2E-GUCRP) and its solutions.

Unlike FSTSP, this mathematical model concentrates on the Intelligence, Surveillance and Reconnaissance (ISR) mission, where UAV is capable to serve multiple targets in one flight. When UAV collaborates with the ground vehicle, the ground vehicle travels on a road network near the reconnaissance area and keeps launching and recycling UAV, while UAV automatically flies to the target point to collect the target information and then returns to the vehicle before the battery runs out. Ground vehicles can carry UAV to patrol more extensive reconnaissance areas and make up the shortage of UAV's endurance. Though the GV's routes are restricted on partial road networks or flat hard ground, the involvement of UAVs enables the cooperative transportation system to rapidly explore ground vehicle unreachable areas and perceive more accurate target information.

1 Science and Technology on Information System Engineering Laboratory, National University of Defense Technology, Changsha 410073, China; liuzhong@nudt.edu.cn (Z. Liu.); jianmaishi@nudt.edu.cn (J.S.); wangqi15@nudt.edu.cn (Q. W);995848273@qq.com (T. Zhou); liuyao13@nudt.edu.cn (Y. Liu)
*Corresponding Author: Mobile Phone: +86-151-1106-6043, e-mail: luozhihao15@nudt.edu.cn (Z. Luo)

As shown in Figure 1, the GV mounts the UAV, starts from a depot, takes a trip on the road network and then travels back to the previous depot or another depot. There exist a number of optional stopping nodes (parking lots) in the road network, at which they are available for the GV to stop and allow the UAV to take off or/and land. A set of predetermined targets are located outside the road, which can only be visited by the UAV. The mission is to minimize the time of the UAV route with its endurance concerned and select appropriate rendezvous nodes to launch or recycle the UAV. Thus, this problem involves three critical decisions: planning the GV's route, determining where to stop the GV, and planning the UAV's route for visiting targets during each flight.

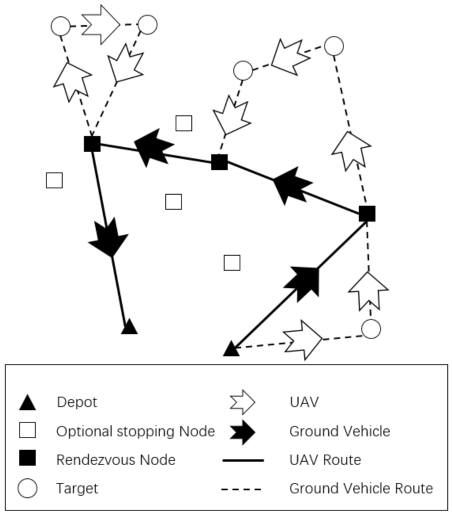

Figure 1. Schematic Diagram of the 2E-GUCRP

In the Section 2, previous studies on relative problems are reviewed and classified. Three meta-problems are constructed according to the shape of the route. After distinguishing levels of routing, we compare the differences between previous literatures and this paper. Meanwhile, we would discuss the necessity of some assumptions and the realistic application scenarios as well.

In the Section 3, the characteristics of the cooperated work model and related challenges in the routing progress are further analyzed. Vehicle-mounted UAV routing imposes requirements on accurate synchronization: In the time dimension, the UAV subjects to the time limit of its endurance and needs to return to the ground vehicle to charge or change its battery in time. In the spatial dimension, the location of the next rendezvous node should not only be close to the area of undetected target, but also ensure that the UAV can arrive within the limits of endurance after the current reconnaissance mission. In view of the characteristics of this two-echelon cooperative routing problem, a mixed integer programming (MIP) model is established. Complex aspects including different modalities of cooperation, synchronization and endurance constraints are considered in the proposed MIP model.

The contributions of the paper are in two folds: (i) to summarize and categorize the previous literatures according to the meta-questions we proposed, (ii) to construct a MIP model of the 2E-GUCRP.

II. Literature Review

According to the choice of starting point and destination in the routing process, the classic routing problem can be divided into the following three meta-problems:

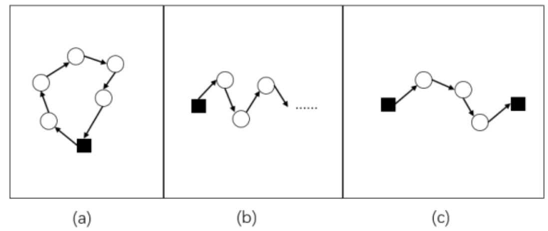

Figure 2. Three Classic Routing Problem

(i) The circulation routing problem: as shown in Figure 2(a), this type of problem assumes the vehicle to start from a fixed base station and return to the base station after all the target points have been visited/served.

(ii) The radiation routing problem: as shown in Figure 2(b), this type of problem also assumes the vehicle to start from a fixed base station, but does not need to return the base station after visiting all the target points, and there is no limit to the destination of the route, which means this route can end at any target point.

(iii) The segment routing problem: as shown in Figure 2(c), this type of problem assumes the vehicle to start from a fixed base station and return to another designated base station after all the target points have been visited/served.

The circulation routing problems are the most widely studied problems. The Travelling Salesman Problem (TSP) and Vehicle Routing Problem (VRP) are both typical circulation problems. The most significant difference between TSP and VRP is that it takes the constraints of the endurance (or capability) of the routing process in VRP into considerations, which constrains the number of points per trip that can access. The radiation routing problems are also commonly seen in the transaction field. More common open questions include Open Travelling Salesman Problem (OTSP) and Open Vehicle Routing Problem (OVRP). They are practical variants of the basic circulation problems in some special situations.

The segment routing problem is the combination of the circulation routing problem and the radiation routing problem. It obtains the characteristics of designation and linearity. There are several researches about the segment routing problem in academia. In fact, if we add a virtual edge from the end point to the starting point, the segment routing problem is transformed into a special TSP. Thus, it does not make much sense to study the problem separately. However, we could use such problems to categorize existing literature.

Manyam et.al [6] applied a typical 2E-RP model to solve the problem of the ISR mission performed with a GV and its mounted UAV. As mentioned above, the GV needs to stay where it is until the UAV is recycled in this model. From the perspective of meta-problem, the first level in their problem is a classical TSP, which is mentioned as a typical circulation problem, and the second layer of the model is a classical TSP as well. Thus, their problems can be classified as a single-circulation-single-circulation problem.

Agatz et.al [7] and Ha et.al [8, 9] constructed a Traveling Salesman Problem with Drone (TSP-D) problem to address the routing problem of UAV delivery in city logistics. In their problems, the ground vehicle carries a UAV to complete the delivery task of multiple target points. All target points can be served by both GV and UAV. However, every time the UAV starts from the vehicle, it can only serve one target point, and then immediately return to the vehicle. It limits the UAV route must form a "triangle" with GV routing, whose three sides respectively is: UAV flies to target, flies back to the vehicle and the GV from the launching point moved to the recycle rendezvous node. Thus, their problems can be classified as a single-circulation-single-segment problem.

Similar to TSP-D, Dorling et.al [10] and Wang et.al [11] proposed a Vehicle Routing Problem with Drone (VRP-D) problem. Unlike the TSP-D, the first level in VRP-D is constructed as a specialized VRP, which means the constraints of the endurance (or capability) have been considered in their formulations. Thus, their problems can be classified as a multi-circulation-single-segment problem.

The FSTSP that Murray and Chu [1] adopted is similar to the TSP-D in the second level of route. However, in each segment, the GV is able to serve multiple costumers while UAV performs its own delivery task. It extends the “triangle routes” to the “polygon routes” and makes itself closer to the reality. Thus, their problems can also be classified as a single-circulation-single-segment problem.

Halil Savuran and Murat Karakaya [12, 13] proposed a largest coverage problem of a vehicle-mounted UAV (Mobile Depot VRP, MoDVRP). In their case, the vehicle moves along a straight line, on both sides of the vehicle's path, with a large number of UAV target points. The UAV starts from the vehicle and, within the endurance allowed, visits as many target points as possible before returning to the vehicle. The GV’s route is fixed on a straight line, and one of the critical issues is to determine the rendezvous node on the line. This problem is a combination of a specialized radiation routing problem and a partial segment problem. Thus, their problems can be classified as a partial-radiation-partial-segment problem.

Our previous research [14] constructed a partial -circulation- single-segment problem for ISR mission. In that paper, a hypothesis of rendezvous nodes limits the application scenarios. We assumed that there exist sufficient rendezvous nodes in the road network, and that the GV can always find a rendezvous node in the following trip for recycling the UAV after it launches the UAV from some nodes, which means that each road arc traversed by the GV corresponds to a flight route of the UAV. As the research goes further, we notice such hypothesis conflicts with universal applications. Hence, we relaxed this hypothesis in this paper and proposed two different special variant problems based on realistic application scenarios. The analysis of these two special scenarios would be further elaborated in the next section.

Based on the analysis on the literature review, we can capture the differences of models in previous studies. In this paper, we focus on the 2E-GUCRP. Since the GV only selects some rendezvous nodes among optional stopping nodes on the road network, and the starting node and end node of the GV route can be different, the first level of the 2E-GUCRP is reduced as a partial-segment problem. Combined with the UAV's flight route a typical segment routing, this problem is classified as a partial-segment-single-segment problem.

TABLE I. LITERATURE REVIEW

Literature	1st -Level	2nd -Level	Notes
Manyam et.al[6]	1-circulation	1-circulation	the GV stays still when the UAV performs its tasks.
Agatz et.al[7]	1-circulation	1- segment	“triangle routes”
Ha et.al [8]			
Ha et.al [9]			
Dorling et.al[10]	multi-circulation	1- segment	
Wang et.al[11]			
Murray and Chu[1]	1-circulation	1- segment	“polygon routes”
Halil Savuran and Murat Karakaya[12]	partial -radiation	partial -segment	The GV route is fixed on a straight line
Halil Savuran and Murat Karakaya[13]			
Authors previous research[14]	partial -circulation	1- segment	Has a hypothesis of rendezvous nodes
This paper	partial -segment	1- segment	Section 3

III. MATHEMATICAL MODEL

In this section, we would first detail the application background of the problem and then establish a reasonable MIP model with some practical hypotheses. Finally, the correctness of the MIP model would be demonstrated by solving a randomly generated instance with some commercial software.

A. The Discussion of Hypotheses

Describing the task execution of a GV and its mounted UAV is a complicated process. To facilitate modeling, we introduce a series of hypotheses to simplify and abstract this problem. Some details are as follows:

1) General hypothesis

With the improvement of the capacity and endurance, it is plausible for UAV to carry out multiple missions during a single flight. The assumption that a UAV can only access one point at a time is not in line with current development. At the same time, in order to facilitate the modeling and analysis, we might as well abstract the task area into a target point without considering the specific task execution process. **H1: the UAV's task execution area and the optional temporary stop points of vehicles are abstracted as points and a UAV can serve one or more targets during once flight.**

2) The discussion of starting points and end points

In most of the previous literature, there was a base for deploying vehicles and UAVs, all of which started from the base and returned to the exactly base after completing the mission. Generally, it is relatively rare to model an open problem in practice. Because in the vast situations, the vehicle is not a disposable consumable. With the recycle of the vehicle, it can be convenient for the unified dispatching management and greatly reduce its costs. There are two main situations, where the vehicle is not required to return to the base. In the first case, the vehicle is rented and does not need to be returned together. In the second case, the subject of the problem is a disposable consumable, such as missiles.

In this problem, the GV and UAV are neither rented or disposable. Nevertheless, it is also necessary to study the routing problems with different starting points and end points. The main considerations are summarized below:

From the practical point of view, this model has wider application prospect. In the civil field, when there are sufficient base stations that can be managed uniformly, in order to improve the efficiency of the task, the vehicles are able to return to another base. In the military field, it is relatively difficult to build a frontier base, and the adoption of the model that returns to a fixed base will increase the risk of exposing its own forces, deployment and command centers. In actual combat operations, the areas of supply and maintenance tend to differ from the initial assembly areas.

From a quick solution perspective, this model provides an approach to deconstruct large-scale problems. It is acknowledged that vehicle routing problem is a NP-hard problem. With the expansion of the problem scale, the growth of solution space is much bigger than the scale of the problem. Therefore, it will inevitably lead to a sharp increase in the time of solving the problem. A better approach is to deconstruct the large-scale tasks. In this method, the large-scale task is divided into several sub-tasks according to its temporal and spatial characteristics. The solution of large-scale tasks tend to be quickly and effectively completed by solving the sub-tasks parallelly.

In summary, **H2: During the task execution, the GV starts from a specified starting depot and chooses partial rendezvous nodes among the optional stopping nodes to complete the launching and recycling work of the UAV. After all the missions ends, the GV travels to another designated depot.**

3) The special cases of segments

Obviously, when a UAV completes its mission, moving the GV to another rendezvous node to recycle and resupply the UAV can improve the efficiency of the whole missions. Specifically, the GV can choose a closer rendezvous node to reduce the recycling time or the distance to the next target point. In some extreme cases, however, such strategy may be ineffective. In the following sections we will describe two of these extreme cases and build the appropriate hypotheses.

a) Empty Edge

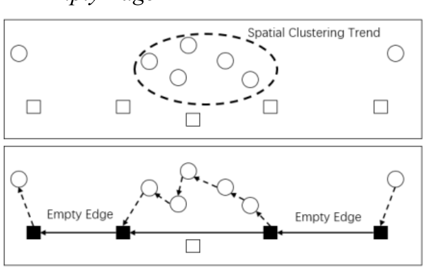

Figure 3. Empty Edge

This scenario arises when there is a spatial clustering trend between the target points. Specifically, as shown in Figure 3, the spatial clustering trend means that for some target points they are close to each other and far away from other target points. In practical scenarios, spatial clustering trends are quite common. For example, when delivering goods in cities, the distribution of customers is mainly concentrated in various residential areas. Customers in the same residential area are close to each other and are far from customers in other residential areas. Therefore, it is necessary to include this scenario in modeling. **H3: The UAV does not need to take off immediately after landing on the GV to complete the supply and maintenance work, and can take off again as the GV travels to the next rendezvous node.**

b) Holding Strategy

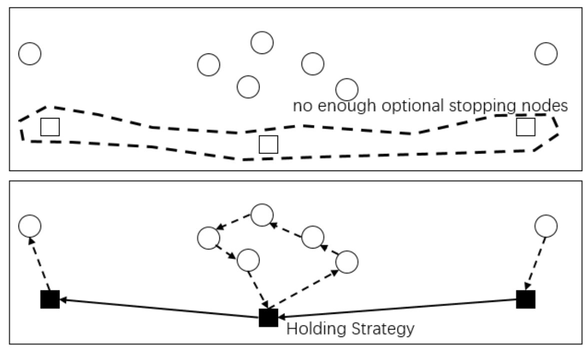

Figure 4. Holding Strategy

This scenario arises when there are not enough optional stopping nodes. Specifically, as shown in Figure 4, under the constraint of the endurance, the GV is allowed to stay in place and wait for the UAV to return in order to ensure the existence of a feasible solution. In practical scenarios, this scenario is also acceptable, such as: in the central area of the city, restricted by air traffic control, there are very few areas where UAV can take off. Therefore, it is also necessary to cover this scenario in modeling. **H4: The GV can stay in place while the UAV performs its tasks, waiting for the UAV or heading for the next rendezvous node.**

4) Complementary hypothesis

The four hypotheses mentioned above describe the basic routing rules and two extended scenarios of GV and UAV. To better formulate our model, we add several related hypotheses here.

Existing technologies can fully support the autonomous task of a UAV in the context of a serious of given input. For example, in Amazon's Prime Air plan, they use a combination of GPS and image recognition technology for autonomous delivery [15]. In addition to Amazon, the United Parcel Service's on-board drone delivery, Google X lab's Project Wing, can also do the self-control process. **H5: The UAV can complete its own tasks autonomously, without the need for real-time control. Therefore, we do not consider the limitation of communication distance between GV and UAV.**

At this stage, small and medium-sized UAVs are mainly battery-powered. In order to perform tasks quickly and continuously, a good scheme is to prepare two batteries for alternative use. When the UAV performs its task, another battery is charged in the GV. In this scheme, the replacement time of the UAV's battery is negligible. As mentioned above, this paper focuses on application scenarios for ISR tasks. With the development of fast data transmission technology, the time to import reconnaissance data from UAV into GV is also negligible. **H6: The time of taking off, landing and replacing the battery can be ignored. After completing the task, the UAV can take off immediately to complete the follow-up tasks.**

To simplify the model, we supply the following hypotheses:

TABLE II. SUPPLIED HYPOTHESES

No.	Details
H7	The location and service time of the task target point is known as a fixed number.
H8	The UAV flies in a straight line at a constant speed during the flight. The impact of terrain and no-fly zones on UAV flight paths is not considered
H9	Once the drone arrives at the target point, it immediately begins to perform the corresponding task, regardless of waiting time
H10	All task target points should be served and only served once
H11	The UAV has a known maximum battery life, but it does not consider the GV's endurance
H12	The vehicle cannot access the target point by itself

B. The Discussion of Synchronization

Michael Drexl [16-18] offers a synchronic view of the problem of vehicle routing. He first proposed Vehicle Routing Problems with Multiple Synchronization Constraints (VRPMSs), and then through the summary of Vehicle Routing Problem with Trailers and Transshipments (VRPTT) literature to illustrate the influence of the concept of planning Problem. Based on his analysis of the synchronic analysis, this paper discusses the synchronic constraints of the task from the following three aspects.

1) Generalized synchronization

It is different from the previous multi-echelon vehicle routing problems that the routing of each echelon in 2E-GUCRP needs to be carried out in the same time period, while the traditional 2E-RP does not require the co-time of the two stages. In fact, in the actual application, the 2E-RP generally adopts the larger transport vehicles in the first echelon, and carries out the transportation of the goods at a lower frequency (once a month, once a week); In the second echelon, the transport of goods is usually carried out in a smaller transport vehicle with a higher frequency (multiple times a week, multiple times a day). Changing the second echelon of the route has less impact on the first phase. The generalized synchronization of the echelon qualifies a macro time period and requires that the tasks of GV and UAV are performed simultaneously.

2) Spatial-temporal synchronization

We need to consider how to ensure the synchronization between the GV and UAV in time and space, which means when the UAV arrives at a rendezvous node, the GV should arrive at the same rendezvous node before the UAV's landing. To simplify the calculation of total time in a segment, we have added a restriction that the GV must arrive before the UAV reaches the rendezvous node.

C. Mixed Integer Programming Model

TABLE III. NOTATIONS AND ITS IMPLICATIONS

Notations	
$\boldsymbol{G}$	The gragh of nodes and edges.
$\boldsymbol{V}$	The set of all nodes.
$\boldsymbol{V_s}$	The set of all optional stopping nodes
$\boldsymbol{V_t}$	The set of all target nodes.
$\boldsymbol{E}$	The set of all edge.
$\boldsymbol{E_1}$	The edge for GV routing
$\boldsymbol{E_2}$	The edge for UAV routing
$\boldsymbol{C_i}$	The serive time of node *i*.
$\boldsymbol{d_{ij}}$	The distance between node *i* and node *j*
$\boldsymbol{v_1}$	The average velocity of GV
$\boldsymbol{v_2}$	The average velocity of UAV
$\boldsymbol{\theta}$	The maximum enduarence of UAV
$\boldsymbol{M}$	A sufficiently large positive number

The mathematical formulation is defined on a graph $G = (V, E)$. V is the set of all nodes, where $\{0\}$ indicates that the vehicle's starting assembly area or base, $\{*\}$ indicates the destination of the vehicle to assemble a region or a base, and V is composed of the following independent set of sets: $V = \{0\} \cup \{*\} \cup V_s \cup V_t$. The $V_s = \{1,2, \dots, m\}$ represents the set of all optional stopping nodes and the $V_t = \{m+1, m+2, \dots, m+n\}$ represents the set of all target nodes. For each target $j \in V_t$, there is a corresponding service time C_j for the drone to complete the specified task. Since we assume that A is moving at A constant speed, the residual power of A can be converted directly to the remaining time. Since we assume that UAV is moving at a constant speed (H8), the residual power of UAV can be converted directly to the remaining time. We introduce the following variables:

$$x_{ij} = \begin{cases} 1 & if\ GV\ travels\ from\ node\ i\ to\ node\ j \\ 0 & otherwise \end{cases}$$

$$y_{ij} = \begin{cases} 1 & if\ UAV\ travels\ from\ node\ i\ to\ node\ j \\ 0 & otherwise \end{cases}$$

$$S_{ij} = \begin{cases} 1 & if\ target\ i\ is\ served\ from\ node\ j \\ 0 & otherwise \end{cases}$$

$$Q_i = the\ consumed\ capacity\ of\ battery\ in\ node\ i$$

$$T_i = the\ access\ order\ of\ rendezvous\ node\ i$$

$$t_i = the\ departure\ time\ of\ rendezvous\ node\ i$$

$$s_i = the\ standing\ time\ of\ rendezvous\ node\ i$$

Then the 2E-GUCRP can be formulated as the following mixed integer programming in following model:

Minimize:

$$t_* \quad (0)$$

Subject to:

$$\sum_{i\in\{0\}\cup V_s} x_{ij} = \sum_{i\in\{*\}\cup V_s} x_{ji} \le 1, j \in V_s \quad (1)$$
$$\sum_{i\in V_s} x_{i*} = \sum_{i\in V_s} x_{0i} = 1 \quad (2)$$
$$\sum_{i\in V_s} x_{*\mathrm{i}} = \sum_{i\in V_s} x_{i0} = 0 \quad (3)$$
$$N-1 \ge T_i - T_j + N \times x_{ij}, i \in \{0\}\cup V_s, j \in \{*\}\cup V_s \quad (4)$$
$$\sum_{i\in V} y_{ij} = \sum_{i\in V} y_{ji} = 1, j \in V_t \quad (5)$$
$$M \times (1-y_{ij}) \ge |C_j + d_{ij}/v_1 - Q_j| + |S_{ji} - 1|, i \in \{0\}\cup V_s, j \in V_t \quad (6)$$
$$M \times (2-y_{ij}-S_{ik}) \ge |Q_i + C_j + d_{ij}/v_1 - Q_j| + |S_{ik} - S_{jk}|, \ \ i \in V_t, j \in V_t, k \in \{0\}\cup V_s \quad (7)$$
$$M \times (1-y_{ij}) \ge (Q_i + d_{ij}/v_1 - \theta), i \in V_t, j \in \{*\}\cup V_s \quad (8)$$
$$M \times (2-y_{ij}-S_{ik}) \ge |x_{kj} - 1|, i \in V_t, j \in \{*\}\cup V_s, k \in \{0\}\cup V_s, j \ne k \quad (9)$$
$$M \times (3-y_{ij}-x_{kj}-S_{ik}) \ge (d_{kj}/v_2 - d_{ij}/v_1 - Q_i), i \in V_t, j \in \{*\}\cup V_s, k \in \{0\}\cup V_s \quad (10)$$
$$y_{ij} = 0, i \in \{0\}\cup\{*\}\cup V_s, j \in \{0\}\cup\{*\}\cup V_s \quad (11)$$
$$\sum_{i\in\{0\}\cup V_s} x_{ij} \le \sum_{i\in V_t} y_{ij} + \sum_{i\in V_t} y_{ji}, j \in \{*\}\cup V_s \quad (12)$$
$$M \times (2-y_{ij}-S_{ij}) \ge |s_j - Q_i - d_{ij}/v_2|, \ i \in V_t, j \in \{0\}\cup\{*\}\cup V_s \quad (13)$$
$$M \times (3-y_{ij}-x_{kj}-S_{ik}) \ge |t_j - t_k - d_{ij}/v_1 - Q_i - s_j|, i \in V_t, j \in \{*\}\cup V_s, k \in \{0\}\cup V_s \quad (14)$$
$$M \times (1-x_{ij}) \ge t_i - t_j + d_{ij}/v_1, i \in \{0\}\cup V_s, j \in \{*\}\cup V_s \quad (15)$$
$$M \times \sum_{i\in\{0\}\cup V_s} x_{ij} \ge t_j, \ j \in \{*\}\cup V_s \quad (16)$$
$$\sum_{j\in\{0\}\cup\{*\}\cup V_s} S_{ij} = 1, i \in V_t \quad (17)$$
$$0 \le Q_i \le \theta, \ i \in V \quad (18)$$
$$T_0 = 1 \quad (19)$$
$$t_0 = s_0 \quad (20)$$

Constraint (1) ensures that each optional stopping node can be accessed at most once and the indegree and outdegree is equal, except for the starting node and end node. The indegree of end node is equal to the outdegree of starting node, and they are both equal to 1. The outdegree of end node is equal to the indegree of starting node, and they are both equal to 0. Constraint (4) makes sure that there is no subloop in GV's route, and marks the accessing order of the rendezvous nodes. Constraint (5) guarantees that all target nodes are served once only.

Constraint (6) describes the taking off progress of UAV. When UAV takes off from node i and heads to the target node j, the S_{ji} should be equal to 1 and the consumed capacity of battery in node j should be equal to the sum of the duration of the flight and the duration of the service time in node j. Constraint (7) describes the flying progress between targets. When the UAV flies from target i to target j, the node i and node j should be in the same segment. Thus, if S_{ik} equals to 1 and y_{ij} equals to 1, then $S_{ik} = S_{jk}$. Under this circumstance, the consumed capacity of battery in node j should be equal to the sum of the duration of the flight, the consumed capacity of battery in node i and the duration of the service time in node j. Constraint (8) describes the landing progress. When the UAV lands at a rendezvous node j, the y_{ij} equals to 1. At this circumstance, the left-hand side of the inequality is zero. In order to meet the constraint, the following is true that $Q_i + d_{ij}/v_1 \le \theta$, which means that the entire flight process in each segment will not exceed the endurance limit. Constraint (6) ~ (8) ensures the continuity of UAV's flight in each segment, and assigns a value to Q through those constraints, thus it ensures that each flight can meet the endurance capability.

The constraint (9) ensures that if the UAV lands at a certain rendezvous node, then this rendezvous node must be on the GV's route, which connects the decision variable x_{ij} to y_{ij}. The constraint (10) limits that the GV must arrive at the designated landing rendezvous node before the UAV. Constraints (11) ensures that UAV does not fly on the road network. The constraint (12) guarantees that if the vehicle has accessed to an optional stopping node, this node must be a rendezvous node at which the UAV takes off or lands.

The constraint (13) is the calculation of the waiting time, considering the situation of the Holding Strategy in H4 where the vehicle stays in place while the UAV performs the task. On the basis of constraint (13), constraint (14) calculates the specific time of the GV leaving each rendezvous node. Constraint (15) is a complement to the previous constraint, ensuring the consistency of the time to leave the rendezvous node and the order of the rendezvous node.

Constraint (16) makes sure that the values of *t* in all the optional stopping nodes that are not accessed are equal to 0. Constraint (17) ensures that each UAV's target node is assigned to a certain segment. Constraint (18) limits the consumed capacity of battery. Constraint (19) initializes the value of *T* in the starting point. Constraint (20) ensures a special scenario where the UAV needs to start from the starting point and land at the starting point.

The goal of optimization is to minimize t_*. As we mentioned above, $\{*\}$ indicates the destination. Thus, the t_* indicates the total time of task completion. The minimization of the t_* is equivalent to minimizing the time of whole missions.

The proposed model is quite complicated, and the actual number of constraints would change in a power series growth with the scale of the problem. If we assume that the number of target nodes is *n*, the number of vehicle optional stopping nodes is N (including start and end), the problem contains, in fact, $3 \times N^2 \times n + 3 \times N^2 + N \times n^2 - 4 \times N \times n - n^2 + 5 \times n + 5$ constraints.

For example, constraint (1) contains *N-2* constraints because constraint (1) has to limit all the j in V_s. Constraint (2) contains two constraints, which is actually a combination of $\sum_{i \in V_s} x_{i*} = \sum_{i \in V_s} x_{0i}$ and $\sum_{i \in V_s} x_{0i} = 1$. We analyzed all the constraints and constructed the following tables in sequence.

TABLE IV. CONSTRAINT QUANTITY ANALYSIS

1	$N-2$	11	N^2
2	2	12	*N-1*
3	2	13	$n \times N$
4	$(N-1)^2$	14	$n \times (N-1)^2$
5	2×n	15	$(N-1)^2$
6	$(N-1) \times n$	16	*N-1*
7	$n^2 \times (N-1)$	17	*N*
8	$n \times (N-1)$	18	*1*
9	$n \times (N-1)^2 - n \times (N-2)$	19	*1*
10	$n \times (N-1)^2$	20	*1*
SUM:$3 \times N^2 \times n + 3 \times N^2 + N \times n^2 - 4 \times N \times n - n^2 + 5 \times n + 5$			

D. *Case Study*

In order to verify the correctness of this model, we have constructed a small-scale instance and solved it using the CPLEX software.

We randomly generated four targets and four optional stopping nodes in a 100×100 field, and the service time of target is randomly generated between 5 to 10 units. Their coordinates are as follows:

TABLE V. COORDINATES OF RANDOM CASE

Optional Stopping Node	X	Y	Target	X	Y	Service Time
1	3.23	8.86	1	45.27	9.58	9.87
2	80.20	66.45	2	27.89	17.03	6.98
3	69.64	38.02	3	72.64	27.85	7.29
4	2.74	97.98	4	42.94	47.11	8.09

Assuming that the maximum range of the UAV's endurance is 100 units, the speed of the UAV is 2 units per second and the speed of the GV is 1 unit per second.

We respectively mark the optional stopping nodes with red dots and the targets with small blue circles. We specify that the first point in optional stopping nodes is the starting point and the second point is the endpoint and mark them in big green and blue circle. Their positions are as follows:

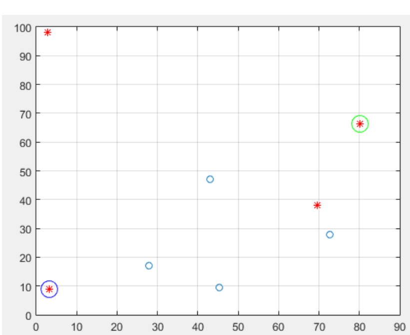

Figure 5. The position of small-scale instance

By using CPLEX software, we obtained a solution with the objective function value of 112.56 in 1.0 seconds. We respectively mark the UAV routes with red dashed lines and the GV routes with green lines. The solution is as follows:

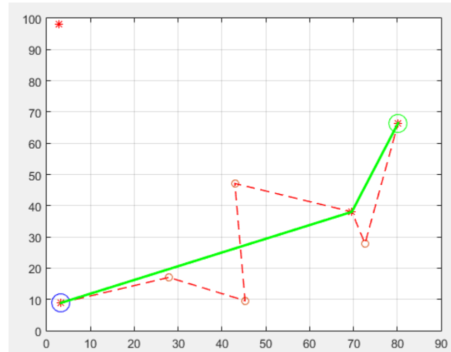

Figure 6. CPLEX solution

From the result, we found a good feasible solution through CPLEX software. However, according to our analysis on the scale of constraints, we cannot solve larger problems by using CPLEX software. In order to verify this hypothesis, we calculate the instances with only several differences in scale. Figure 7 presents the calculation time in each instances.

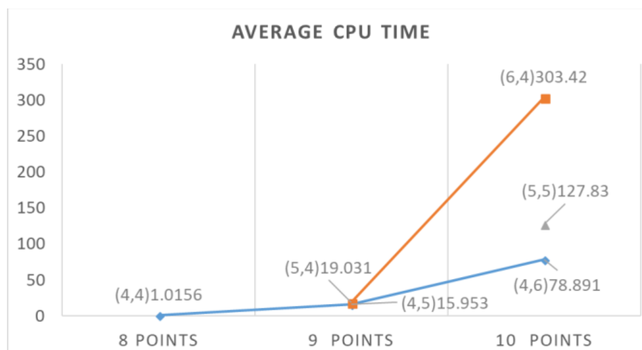

Figure 7. The Average Calculation Time in CPLEX

In this figure, the content in bracket before each data represents the size of the instances, in which the first number represents the number of targets and the second number represents the number of optional stopping nodes. The blue line indicates the trend of calculation time with the increase of the number of target points, while the red line indicates the trend of calculation time with the increase of the number of optional stops nodes. When the number of N and n is fixed, we can calculate quantity of constraints according to the conclusion of TABLE IV. The results are shown in the table below.

TABLE VI. THE GROWTH OF SCALE

The sum of points	Target	Optional Stopping Node	Constraint Quantity	CPU Time
8	4	4	249	1.0156
9	4	5	384	15.953
9	5	4	313	19.031
10	4	6	549	78.891
10	5	5	480	127.83
10	6	4	383	303.42

With the expansion of instance scale, the growth of computing time is very significant. We can learn from both the Figure and Table above that increasing the number of Target will significantly slow down the solution progress. This slowdown is not due to an increase in the number of constraints, but because it becomes more difficult to deal with the constraints associated with the targets. In fact, when setting 8 targets and optional stopping nodes, the solving time is more than 1 hour. Thus, it is impossible to use CPLEX software in a short time to solve a small problem of only 10 targets.

To solve this problem, we believe that the quick and feasible solution can be solved with some heuristic methods. Either a structured heuristic, such as the Clarke-Wright algorithm, or a generated heuristic, such as the Split heuristic [19-20], should be able to solve the problem. In future research, we will consider how to adjust these algorithms to solve this problem.

IV. Conclusion

In this paper, we summarize and reclassify the previous literatures through the analysis of the routes shape in different echelons. We elaborate a 2E-GUCRP after the systematic analyses and hypotheses. The general hypotheses and the extreme cases of segments are discussed with practical demands. Through the discussion of hypotheses, we present the MIP model of 2E-GUCRP and verify its correctness by the CPLEX software in small-scale instances. With the power level growth of the constrains, it is inefficient to solve the problem through the CPLEX.

Further research on the solution methodology may focus on neighborhood search algorithms. With various aspects in constraints, the removal and repair operators will be modified to accommodate the specified constrains. And more discussions about the practical application will emerge in future researches.

Acknowledgment

The research is supported by National Natural Science Foundation of China (no. 71771215 and no. 71471174)

References

[1] C. C. Murray and A. G. Chu, "The flying sidekick traveling salesman problem: Optimization of drone-assisted parcel delivery," Transportation Research Part C Emerging Technologies, vol. 54, pp. 86-109, 2015.

[2] 2017, UPS tests drone delivery system. Available: http://www.businessinsider.com/ups-tests-drone-delivery-system-2017-2

[3] 2016, DJI Kicks Off 2016 DJI Developer Challenge. Available: http://www.dji.com/newsroom/news/dji-kicks-off-2016-dji-developer-challenge

[4] 2016, Rinspeed demonstrates future possibilities and raises future questions with the self-driving "Σtos"concept car. Available: http://www.rinspeed.eu/aktuelles.php?aid=17

[5] J. W. Escobar, R. Linfati, and P. Toth, "A two-phase hybrid heuristic algorithm for the capacitated location-routing problem," Computers & Operations Research, vol. 40, pp. 70-79, 2013.

[6] S. G. Manyam, D. W. Casbeer, and K. Sundar, "Path planning for cooperative routing of air-ground vehicles," presented at the American Control Conference, 2016.

[7] N. Agatz, P. Bouman, and M. Schmidt, "Optimization Approaches for the Traveling Salesman Problem with Drone," Social Science Electronic Publishing, 2015.

[8] Q. M. Ha, Y. Deville, Q. D. Pham, and M. H. Hà, "Heuristic methods for the Traveling Salesman Problem with Drone," Computer Science, 2015.

[9] Q. M. Ha, Y. Deville, Q. D. Pham, and M. H. Hà, "On the Min-cost Traveling Salesman Problem with Drone," Computer Science, 2016.

[10] K. Dorling, J. Heinrichs, G. G. Messier, and S. Magierowski, "Vehicle Routing Problems for Drone Delivery," IEEE Transactions on Systems, Man, and Cybernetics: Systems, pp. 1-16, 2016.

[11] X. Wang, S. Poikonen, and B. Golden, "The vehicle routing problem with drones: several worst-case results," Optimization Letters, pp. 1-19, 2016.

[12] H. Savuran and M. Karakaya, "Route Optimization Method for Unmanned Air Vehicle Launched from a Carrier," Lecture Notes on Software Engineering, vol. 3, pp. 279-284, 2015.

[13] H. Savuran and M. Karakaya, "Efficient route planning for an unmanned air vehicle deployed on a moving carrier," Soft Computing, vol. 20, pp. 2905-2920, 2016.

[14] Z. Luo, Z. Liu, and J. Shi, "A Two-Echelon Cooperated Routing Problem for a Ground Vehicle and Its Carried Unmanned Aerial Vehicle," Sensors, vol. 17, 2017.

[15] C. Rose. (2013, Amazon's Jeff Bezos looks to the future. Available: http://www.cbsnews.com/news/amazons-jeff-bezos-looks-to-the-future/

[16] M. Drexl, "On Some Generalized Routing Problems," Rheinisch-Westf¨alischen Technischen Hochschule Aachen, 2007.

[17] M. Drexl, "Synchronization in Vehicle Routing---A Survey of VRPs with Multiple Synchronization Constraints," Transportation Science, vol. 46, pp. 297-316, 2012.

[18] M. Drexl, "Applications of the vehicle routing problem with trailers and transshipments," European Journal of Operational Research, vol. 227, pp. 275-283, 2013.

[19] C. Prins, "A simple and effective evolutionary algorithm for the vehicle routing problem," Computers & Operations Research, vol. 31, pp. 1985-2002, 2004.

[20] C. Prins, P. Lacomme, and C. Prodhon, "Order-first split-second methods for vehicle routing problems: A review," Transportation Research Part C Emerging Technologies, vol. 40, pp. 179-200, 2014.