Observer-based Cooperative Adaptive Cruise Control of Vehicular Platoons with Random Network Access

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Abstract—This paper investigates cooperative adaptive cruise control (CACC) of vehicles, focusing on the effect of medium access control (MAC) protocol and the inaccurate acceleration measurement. A Markov chain is used to describe the randomness in vehicular network access under a MAC protocol; a reduced-order observer is proposed to estimate the relative acceleration of neighboring vehicles. Based on stochastic system techniques, a series of sufficient conditions are given in the form of backward recursive Riccati Difference Equations (RDE) to guarantee the stable tracking error. Numerical simulations are given to verify the effectiveness of the proposed approach.

I. INTRODUCTION

The growing freight transport and private cars cause significant congestion on many highways and urban areas [1]-[2]. Improving the traffic efficiency and capacity of the existing transportation network has been attracting significant attention from government, academia and industry since 1990s. Intelligent vehicle highway systems (IVHSs), or called automated highway systems (AHSs), is regarded as a promising solution to the traffic congestion issue [3]-[4], since, by enabling vehicle-road cooperative control, it can result in greatly increased traffic capacity, largely reduced adverse environmental effect, and a safer and more comfortable driving experience [5]-[6].

A prominent feature in IVHSs/AHSs is that vehicles in the same lane can be automatically organized into platoons with very small inter-vehicle space/time headway [7]. Letting cars to run close to one another can not only improve road capacity but also save fuel consumption due to reduction in the aerodynamic drag [8]-[9]. The technique supporting vehicle platooning is known as cooperative adaptive cruise control (CACC), which enabled by the use of wireless communications for V2V (vehicle-to-vehicle) or even V2I communications. By nature, vehicles in a CACC system are dynamically coupled, hence, the spacing and velocity errors of

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a vehicle may affect those behind or even amplify as they propagate along the string of vehicles. The unstable CACC system may lead to instable vehicular string [10]-[11], and, in turn, an uncomfortable ride experience and even rear-ending accidents [11]. Therefore, the most key control objective for the CACC system is to guarantee the asymptotical stable tracking error (e.g., spacing error, velocity error and acceleration error).

Here we are interested in how the MAC (medium access control) protocol would affect vehicle platooning. The MAC protocol in an IEEE 802.11p-based vehicular ad hoc network (VANET) usually uses a CSMA/CA (carrier-sense multiple access with collision avoidance) protocol to resolve communications conflicts among vehicles. The CSMA/CA belongs to the stochastic medium access protocol and has been descried by the Markov possess [22]. What is also interested in is the effect of inaccurate measurement on CACC control. Most of the existing results generally assume that the spacing, velocity and acceleration can be precisely obtained. However this assumption is not reasonable in practical CACC systems, since some sensors might produce inaccurate measurements. Due to causes like low battery power, severe weather conditions (e.g., rain, snow and sandstorm), interference of signals (see, e.g., [13]) and continuous background changing [13]. This is true at least for accelerometers currently available in the market. As reported in [14] and [15], accelerometers cannot maintain the orientation in long term deployment and calibrating an accelerometer is usually quite difficult. In addition, accelerometers inevitably have biased or drifted outputs due to variations in temperature, mechanical stresses/pressure and humidity changes. One effective way to cope with the problem of inaccurate or uncertain measurements is to estimate them based on reliably available information [16] and [17].

This paper investigates vehicle platoon control focusing on the effect of random MAC protocol and unmeasureable acceleration. It is assumed that the acceleration measurement of the vehicles is inaccurate, therefore we use a reduced-order observer to estimate the acceleration, which is based on the position and velocity information of itself and the preceding vehicle (transmitted via a vehicular communications network). The stochastic effect of the MAC protocol for each vehicle is modeled as a Markov chain. The contributions of this paper are lists as follows, 1) A novel platoon control method is proposed based on the reduced-order observer of relative acceleration between neighboring vehicles. 2) A set of backward recursive Riccati difference equations (RDEs) are derived as the sufficient conditions to guarantee the stable tracking error of the CACC system. More important, the tracking error of the CACC system satisfies the defined

performance requirement to reject the disturbance causing by the acceleration of the preceding vehicle.

Notation: Throughout the paper, R^n denotes the n-dimensional Euclidean space. For symmetric matrices P and Q, P < Q ($P \le Q$) means that matrix P - Q is negative definite (negative semi-definite). $prob\{\cdot\}$ stands for the probability of event " \cdot ". $E\{x\}$ and $E\{x \mid y\}$ respectively denote the expectation of the stochastic variable x and expectation of x conditioned on y. I is an identity matrix of appropriate dimension. $\|\cdot\|$ denotes the Euclidean norm. H^T represent the transpose of matrix H respectively.

II. PROBLEM FORMULATION

A. CACC Modeling

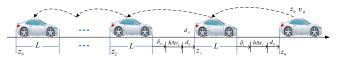


Figure 1. CACC system

Consider a platoon of N+1 vehicles running in a horizontal road segment. Denote by z_r , v_r and a_r the r-th (r=0,1,...,N) vehicle's position, velocity and acceleration, with r=0 standing for the leading vehicle and the others being the followers. Each following vehicle can measure its position and velocity by the onboard sensor and receive the position and velocity of its preceding vehicle from the vehicular ad hoc network (VANET). Due to capacity limitation of the wireless communication channels in the VANETs, it is assumed that each time at most one vehicle can have access to the network. The communication conflicts are resolved by the MAC protocol based on backoff mechanism.

The dynamics of vehicle r, r=1, 2, ..., N, is described by the following differential equations [18]:

$$\dot{z}_r = v_r
\dot{v}_r = a_r
\dot{a}_r(t) = -a_r(t)/\zeta + u_r(t)/\zeta$$
(1)

where ς denotes the engine time constant time, $u_r(t)$ is the commanded acceleration for follower vehicle r.

Sampling system (1) with period h and a zero-order hold, gives the following discrete-time state space representation of vehicle r,

$$s_{r}(k+1) = A_{1}s_{r}(k) + B_{1}u_{r}(k)$$
 (2)

where

$$s_r(k) = \begin{bmatrix} z_r(k) \\ v_r(k) \\ a_r(k) \end{bmatrix}, A_1 = \begin{bmatrix} 1 & h & 0 \\ 0 & 1 & h \\ 0 & 0 & b_2 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ b_1 \end{bmatrix},$$

with $b_1 = 1 - e^{-h/\varsigma}$ and $b_2 = e^{-h/\varsigma}$.

Define the spacing error, velocity error and acceleration error, respectively, as

$$\delta_{r}(k) = z_{r-1}(k) - z_{r}(k) - L - h_{v}v_{r}(k) - d_{0}$$

$$\Delta v_{r}(k) = v_{r-1}(k) - v_{r}(k)$$

$$\Delta a_{r}(k) = a_{r-1}(k) - a_{r}(k)$$
(3)

where $\delta_0 = 0$. h_v is the time gap, d_0 is a given minimum distance, and L is the length of vehicle. Note that here, as in [19], we use a hybrid spacing policy.

B. Modeling Backoff Effect in Vehicular MAC Protocol

In the IEEE 802.11p-based VANET, a CSMA/CA protocol is used on resolve the communication conflicts. When two or more vehicles request to have access to the channel, each vehicle will wait for a random backoff time in the range (0, w-1), where w is called the contention window. It is well known that the network access status of a vehicle features a Markov chain with conditional collision probability [20]. Use a binary function $\rho_{\kappa}(k) \in \{0,1\}$ to denote the channel access status of vehicle r at time k. When $\rho_{r}(k) = 1$, the position and velocity of vehicle r-1 is transmitted to vehicle r and it can then decide its control action based on the spacing error and velocity error. Otherwise $\rho_{i}(k) = 0$, vehicle r is not accessing the communication channel and it will consider the spacing error and velocity error to be zero. convenience, use matrix we $H_{\rho}(k) = diag\{\rho_1(k),...,\rho_N(k)\}$ to describe the channel access status of the N following vehicles. Under the capacity limitation, it is clear that if the i-th following vehicle is allowed to access the wireless network ($\rho_i(k)=1$), then communication matrix $H_{o}(k) = H_{i}$, where matrix H_{i} is a diagonal matrix with the i-th diagonal element being '1' and '0'. For other elements example, $H_{o}(k) = H_{1} = diag\{1, 0, ..., 0\}$ if $\rho_{1}(k)=1$ $H_{\rho}(k) = H_{\gamma} = diag\{0,1,...,0\}$ if $\rho_{\gamma}(k) = 1$. According to [21], the communication matrix $H_{\rho}(k)$ is a Markov chain taking matrix values in a finite set $H = \{H_1, ..., H_N\}$ with transmission probability matrix $II = [\pi_{ii}]$, where $\pi_{ii} = prob\{H_{o}(k+1) = H_{i} \mid H_{o}(k) = H_{i}\}, i, j = 1,...,N$. For simplifying the forthcoming discussion, we use $\tau(k)$ as the

 $\pi_{ij} = prob\{H_{\rho}(k+1) = H_{j} \mid H_{\rho}(k) = H_{i}\}, i, j = 1,...,N$. For simplifying the forthcoming discussion, we use $\tau(k)$ as the indicator of the Markov process, e.g., $\tau(k) = i$ if $H_{\rho}(k) = H_{i}$, i = 1,...,N.

C. Relative Acceleration Observer

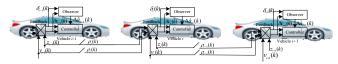


Figure 2. Observer-based controller for CACC system

The observer-based control block diagram for CACC is given in Fig. 2, in which CACC controller is based on the spacing error, velocity error and acceleration error information. Unlike the easily available position and velocity information, the acceleration is not precisely and reliably measureable.

Here, we use a reduced-order observer to estimate the relative acceleration on the basis of the available information of relative spacing and velocity errors. To this end, we first rewrite the dynamics of the relative spacing, velocity and acceleration errors in (2) as below

$$\begin{cases}
\Delta a_r(k+1) = b_2 \Delta a_r(k) - b_1 u_r(k) + b_2 \theta_r(k) \\
\delta_r(k+1) \\
\Delta v_r(k+1)
\end{cases} = A_{11} \begin{bmatrix} \delta_r(k) \\ \Delta v_r(k) \end{bmatrix} + A_{12} \Delta a_r(k) + D' \varpi_r(k)$$
(4)

where

$$A_{11} = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix}$$
, $A_{12} = \begin{bmatrix} h_v \\ h \end{bmatrix}$ and $D' = \begin{bmatrix} -hh_v \\ 0 \end{bmatrix}$,

 $\varpi_r(k) = a_{r-1}(k)$ and $\vartheta_r(k) = u_{r-1}(k)$ are the acceleration and control input of preceding vehicle, respectively, which are treated as disturbances here.

We introduce the following reduced-order observer to estimate the relative acceleration error

$$\begin{cases}
\Delta \hat{a}_{r}(k+1) = b_{2}\Delta \hat{a}_{r}(k) - b_{1}u_{r}(k) + \rho_{r}(k)C'F' \left[\begin{bmatrix} \delta_{r}(k) \\ \Delta v_{r}(k) \end{bmatrix} - \begin{bmatrix} \hat{\delta}_{r}(k) \\ \Delta \hat{v}_{r}(k) \end{bmatrix} \right] \\
\left[\begin{bmatrix} \hat{\delta}_{r}(k+1) \\ \Delta \hat{v}_{r}(k+1) \end{bmatrix} = A_{11} \left[\begin{bmatrix} \hat{\delta}_{r}(k) \\ \Delta \hat{v}_{r}(k) \end{bmatrix} + A_{12}\Delta \hat{a}_{r}(k) \right]
\end{cases} (5)$$

where $\Delta \hat{a}_r(k)$ is the estimated relative acceleration error, $\hat{\delta}_r(k+1)$ and $\Delta \hat{v}_r(k+1)$ are the estimated spacing error and velocity error computed using the estimated acceleration error $\Delta \hat{a}_r(k)$, $C' = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$, $F' \in \mathbb{R}^{3 \times 2}$ is the given observer gain.

The observer-based CACC controller is then given in the following form, whose design will be discussed later

$$u_r(k) = k_i^p \delta_r(k) + k_i^v \Delta v_r(k) + k_i^a \Delta \hat{a}_r(k) . \tag{6}$$

where k_i^p , k_i^v and k_i^a are the given controller gain for the CACC system.

D. Model Transformation and the Objective

Denote by

$$x_{r}(k) = \begin{bmatrix} y_{r}(k) \\ \Delta a_{r}(k) \end{bmatrix}, \quad \hat{x}_{r}(k) = \begin{bmatrix} \hat{y}_{r}(k) \\ \Delta \hat{a}_{r}(k) \end{bmatrix}, \quad y_{r}(k) = \begin{bmatrix} \delta_{r}(k) \\ \Delta v_{r}(k) \end{bmatrix},$$

$$\hat{y}_{r}(k) = \begin{bmatrix} \hat{\delta}_{r}(k) \\ \Delta \hat{v}_{r}(k) \end{bmatrix} \text{ and } \bar{x}_{r}(k) = \begin{bmatrix} y_{r}(k) \\ \Delta \hat{a}_{r}(k) \end{bmatrix}$$

respectively as the state, the estimated state, measurement output, the estimated output and feedback state. Then the dynamics of CACC system in terms of tracking errors defined in (3) can be rewritten as

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + D\varpi(k) \\ y(k) = Cx(k) + D\varpi(k) \end{cases}$$
 (7

with the following observer-based controller

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + Bu(k) + FH_{\rho}(k)(y(k) - \hat{y}(k)) \\ \hat{y}(k) = C\hat{x}(k) \\ u(k) = K\overline{x}(k) \end{cases}$$
(8)

where

(4)
$$x(k) = \begin{bmatrix} x_1(k) \\ \vdots \\ x_N(k) \end{bmatrix}, \overline{x}(k) = \begin{bmatrix} \overline{x}_1(k) \\ \vdots \\ \overline{x}_N(k) \end{bmatrix}, y(k) = \begin{bmatrix} y_1(k) \\ \vdots \\ y_N(k) \end{bmatrix},$$

 $\varpi(k) = [\varpi_1(k) \cdots \varpi_N(k)]^T$ is the preceding vehicle acceleration disturbance, $u(k) = [u_1^T(k) \cdots u_N^T(k)]^T$ is the control input, and

and are
$$A = \begin{bmatrix} A' & & & \\ & \ddots & & \\ & & A' \end{bmatrix}, A' = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, A_{21} = \begin{bmatrix} 0 & 0 \end{bmatrix},$$
to $A_{22} = b_2, B = \begin{bmatrix} -B_1 & & \\ B_1 & -B_1 & & \\ & \ddots & \ddots & \\ & & B_1 & -B_1 \end{bmatrix}, C = \begin{bmatrix} C_1 & & \\ & \ddots & \\ & & C_1 \end{bmatrix},$
or, $C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} D_1 & & \\ & \ddots & \\ & & D_1 \end{bmatrix}, D_1 = \begin{bmatrix} D' \\ 0 \end{bmatrix},$
and for $C_1 = \begin{bmatrix} K'_{\tau(k)}(k) & & & \\ K'_{\tau(k)}(k) & & & \\ & & \ddots & & \\ & & & & & \end{bmatrix}, K' = \begin{bmatrix} K_i^p & k_i^p & k_i^a \end{bmatrix},$
the $C_1 = \begin{bmatrix} F' & & \\ & \ddots & & \\ & & & & & \end{bmatrix}$

Define $e(k) = x(k) - \hat{x}(k)$ as the estimation error. Then, the dynamics of the estimation error can be obtained from (7) and (8) as follows

$$e(k+1) = (A - FH_{\bullet}(k)C)e(k) + D\varpi(k)$$
(9)

Introducing an augmented vector $\xi(k) = [x^T(k), e^T(k)]^T$, we can then have the following augmented CACC system

$$\xi(k+1) = \overline{A}_{\tau(k)}(k)\xi(k) + \overline{D}(k)\varpi(k)$$
 (10)

here we use the fact that $\overline{x}(k) = x(k) - \overline{C}e(k)$, and

$$\overline{A}_{r(k)}(k) = \begin{bmatrix} A + BK & -BK\overline{C} \\ 0 & A - FH_{\rho}(k)C \end{bmatrix},$$

$$\overline{C} = \begin{bmatrix} C' \\ \ddots \\ C' \end{bmatrix}, \ \overline{D}(k) = \begin{bmatrix} D \\ FH_{\rho}(k) \end{bmatrix}.$$

The objective of this paper is to design the observer-based CACC controller so that the following criteria are met:

1). Individual vehicle stability: the entire platoon of vehicles (i.e., system (7)) is asymptotically stable.

2). Disturbance rejection performance: under zero initial condition, the tracking error $\bar{x}(k)$ of the CACC system satisfies the following performance requirement

$$J = \mathbb{E}\{\sum_{k=0}^{N-1} \|\bar{x}(k)\|^2 - \gamma^2 \|\boldsymbol{\varpi}(k)\|^2\} - \gamma^2 \boldsymbol{\xi}^T(0) W \boldsymbol{\xi}(0) \le 0, \quad (11)$$

for $\forall (\xi(0), \varpi(0)) \neq 0$, where $\gamma > 0$ is a given scalar, and W is a given positive definite matrix.

III. OBSERVER-BASED CONTROLLER DESIGN FOR CACC

In order to reject the disturbance caused by the preceding vehicle's acceleration, we analyze the performance requirement defined in (11) for CACC system in this subsection. It is assumed that the disturbance attenuation level $\gamma > 0$ and the positive definite matrix W are to be given. Define the following forward difference equation

$$\Delta V_{\tau(k)}(k) = \mathbb{E}\{\xi^{T}(k+1)P_{\tau(k+1)}(k+1)\xi(k+1) - \xi^{T}(k)P_{\tau}(k)\xi(k) \mid i = \tau(k)\}$$
(12)

where $P_i(k)$ (i = 1,...,n , $0 \le k < N$) is a family of non-negative definite matrices.

Along the state trajectory (10), $\Delta V_{r(k)}(k)$ can be rewritten

$$\begin{split} &\Delta V_{\tau(k)}(k) = \mathbb{E}\{[\overline{A}_i(k)\xi(k) + \overline{D}(k)\varpi(k)]^T P_{\tau(k+1)}(k+1)[\overline{A}_i(k)\xi(k) + \overline{D}(k)\varpi(k)] - \xi^T(k)P_i(k)\xi(k) | i = \tau(k)\} \\ &= \mathbb{E}\{\xi(k)^T [\overline{A}_i^T(k)\hat{P}_i(k+1)\overline{A}_i(k) - P_i(k)]\xi(k) + \\ &+ 2\xi(k)^T \overline{A}_i^T(k)\hat{P}_i(k+1)\overline{D}(k)\varpi(k) + \\ &+ \varpi^T(k)\overline{D}^T(k)\hat{P}_i(k+1)\overline{D}(k)\varpi(k) | i = \tau(k)\} \end{split}$$
 where $\hat{P}_i(k+1) = \sum_{i=1}^n \pi_i P_i(k+1)$.

Adding the following zero term to both sides of (13)

$$\|\bar{x}(k)\|^2 - \gamma^2 \|\varpi(k)\|^2 - (\|\bar{x}(k)\|^2 - \gamma^2 \|\varpi(k)\|^2)$$

and taking the mathematical expectation, we have

$$\Delta V_{\tau(k)}(k) = \mathbb{E}\{\gamma^2 \| \boldsymbol{\varpi}(k) \|^2 + \xi(k)^T [\boldsymbol{R}^T(k)\boldsymbol{R}(k) + \overline{\boldsymbol{A}}_i^T(k)\hat{\boldsymbol{P}}_i(k+1)\overline{\boldsymbol{A}}_i(k) - \boldsymbol{P}_i(k)]\boldsymbol{\xi}(k) - \| \overline{\boldsymbol{x}}(k) \|^2 + 2\xi(k)^T \overline{\boldsymbol{A}}_i^T(k)\hat{\boldsymbol{P}}_i(k+1)\boldsymbol{\delta}\overline{\boldsymbol{D}}(k)\boldsymbol{\varpi}(k) - \boldsymbol{\varpi}^T(k)(\gamma^2 I - \overline{\boldsymbol{D}}^T(k)\hat{\boldsymbol{P}}_i(k+1)\overline{\boldsymbol{D}}(k))\boldsymbol{\varpi}(k) | i = \tau(k)\}$$
where $\boldsymbol{R}(k) = [I - \overline{\boldsymbol{C}}]^T$.

Applying the completing squares method results in $\Delta V_{_{\tau(k)}}(k)$ =

$$\begin{split} &\mathbb{E}\{\xi(k)^{T}[R^{T}(k)R(k)+\overline{A}_{i}^{T}(k)\hat{P}_{i}(k+1)\overline{A}_{i}(k)-P_{i}(k)]\xi(k)\\ &+(\varpi^{*}(k))^{T}\Delta_{i}(k)\varpi^{*}(k)-\mathbb{E}\{\left\|\overline{x}(k)\right\|^{2}-\gamma^{2}\left\|\varpi(k)\right\|^{2}\}-\\ &(\varpi(k)-\varpi^{*}(k))^{T}\Delta_{i}(k)(\varpi(k)-\varpi^{*}(k))\mid i=\tau(k)\}\\ &\text{where }\Delta_{i}(k)=\gamma^{2}I-\overline{D}^{T}(k)\hat{P}_{i}(k+1)\overline{D}(k) \text{ and }\\ &\varpi^{*}(k)=\Delta_{\tau(k)}^{-1}(k)\overline{D}^{T}(k)\hat{P}_{i}(k+1)\overline{A}_{\tau(k)}(k)\xi(k)\,. \end{split}$$

Set the following backward recursive RDE for P(k) as

$$P_{i}(k) = \overline{A}_{i}^{T}(k)\hat{P}_{i}(k+1)\overline{A}_{i}(k) + R^{T}(k)R(k) + \overline{A}_{i}^{T}(k)\hat{P}_{i}(k+1)\overline{D}(k)\Delta_{i}^{-1}(k)\overline{D}^{T}(k)\hat{P}_{i}(k+1)\overline{A}_{i}(k)$$

$$(17)$$

which leads $\Delta V_{\tau(k)}(k)$ equivalent to

$$\Delta V_{\tau(k)}(k) =$$

$$\mathbb{E}\{(\boldsymbol{\varpi}(k) - \boldsymbol{\varpi}^*(k))^T \boldsymbol{\Delta}_i(k)(\boldsymbol{\varpi}(k) - \boldsymbol{\varpi}^*(k)) \mid i = \tau(k)\} - \mathbb{E}\{\|\bar{\boldsymbol{x}}(k)\|^2 - \gamma^2 \|\boldsymbol{\varpi}(k)\|^2\}$$
(18)

Taking the sum on both sides of (12) and (18) from k=0 to k=N-1, we have

$$\mathbb{E}\{\xi^{T}(N)P_{\tau(N)}(N)\xi(N) - \xi^{T}(0)P_{\tau(0)}(0)\xi(0)\} \\
= \mathbb{E}\{-\sum_{k=0}^{N-1} (\varpi(k) - \varpi^{*}(k))^{T} \Delta_{\tau(k)}(k)(\varpi(k) - \varpi^{*}(k)) - \sum_{k=0}^{N-1} (\|\bar{x}(k)\|^{2} - \gamma^{2} \|\varpi(k)\|^{2})\} \tag{19}$$

Since $P_i(N) = 0$, if $\Delta_i(k)$ and $P_i(0)$ in (19) satisfy the following conditions

$$\begin{cases} \Delta_{i}(k) = \gamma^{2} I - \overline{D}^{T}(k) \hat{P}_{i}(k+1) \overline{D}(k) > 0\\ P_{i}(0) \leq \gamma^{2} W \end{cases}$$
 (20)

then it is clear that

$$J = \mathbb{E}\{-\sum_{k=0}^{N-1} (\varpi(k) - \varpi^*(k))^T \Delta_{\tau(k)}(k) (\varpi(k) - \varpi^*(k)) - \xi^T(0) (P_i(0) - \gamma^2 W) \xi(0)\} \le 0$$
(21)

which means the performance requirement is guaranteed for CACC.

IV. SIMULATIONS STUDY

We first through a numerical example to show how we can apply the observer-based controller for a six-vehicle CACC system with the Markov chain based stochastic MAC protocol. In the simulation, the Markov transition probability matrix $II = [\pi_{ij}]$, i, j = 1, 2, ..., 6 for the communication matrix $H_a(k)$ is given as follows

(15)
$$II = \begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.1 & 0.2 & 0.1 \\ 0.25 & 0.15 & 0.1 & 0.2 & 0.15 & 0.15 \\ 0.2 & 0.1 & 0.2 & 0.1 & 0.3 & 0.1 \\ 0.18 & 0.12 & 0.2 & 0.15 & 0.05 & 0.3 \\ 0.24 & 0.16 & 0.1 & 0.25 & 0.15 & 0.1 \\ 0.12 & 0.21 & 0.37 & 0.1 & 0.1 & 0.1 \end{bmatrix}.$$

Then the random network access of the six-vehicle platoon can be described by the Markov chain with transition probability matrix *II*. In the simulation, the random network (16) access status is shown in Fig. 3.

In the simulation, the length of the vehicle, the desired vehicle spacing are chosen as L=10m, $\delta_d=25m$, the engine time constant is $\varsigma=0.25$, the sampling period for all vehicles is h=1s, respectively. Other parameters needed for controller design is given as $\gamma=2.25$, $\varepsilon_1=0.47$, $\varepsilon_2=0.65$ and W=I. In the proposed observer-based CACC controller, the

controller and observer gain is given to be $K' = \begin{bmatrix} 5 & 9 & 11 \end{bmatrix}$

and
$$F' = \begin{bmatrix} 3 & 5 \\ 4 & 7 \\ 2 & 9 \end{bmatrix}$$
, respectively.

Figure 3. Random network access status of the six-vehicle platoon system stimulated by a Markov chain

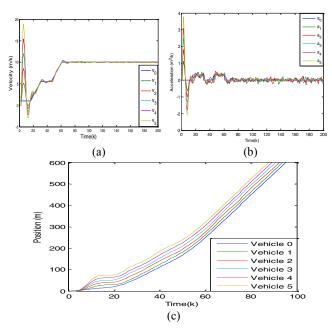


Figure 4. Six-vehicles in CACC system with the observer-based controller and random network access. (a) Velocity. (b) Acceleration. (c) Position.

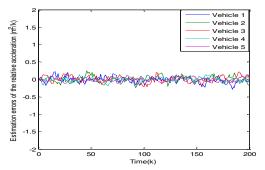


Figure 5. Estimation errors of the relative acceleration.

With the obtained observer-based controller and the random network access shown in Fig. 3, the velocity,

acceleration and position of a six-vehicle platoon is shown in Fig. 4. The estimation errors of relative acceleration of the vehicles are given in Fig. 5. From those figures, it is clear that the proposed observer-based controller can guarantee the string stability of the platoon subjected to unmeasurable acceleration and random network access in VANET.

Next we are going to show the effect of the inaccurate acceleration measurement on the platoon performance. In the simulation, we adopt the following model to describe the acceleration measurement inaccuracy

$$a_{x}^{f}(k) = \theta_{r}a_{r}(k) \tag{22}$$

where θ_r is a coefficient denoting the acceleration inaccuracy. Assume that each following vehicle r receives the inaccurate acceleration measurement $a_{r-1}^f(k)$ from its preceding vehicle via the communications network. Therefore, the feedback controller of vehicle r has the following form:

$$u_r(k) = k_i^p \delta_r(k) + k_i^v \Delta v_r(k) + k_i^v \Delta a_r^f(k)$$

where $\Delta a_{r}^{f}(k) = a_{r-1}^{f}(k) - a_{r}^{f}(k)$.

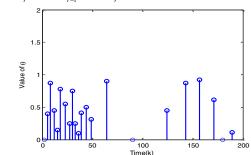


Figure 6. Accelerator inaccuracy measurement status.

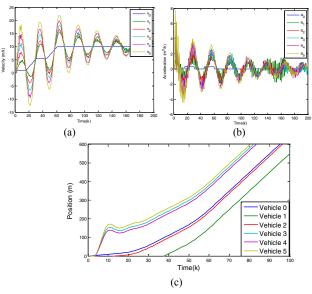


Figure 7. Six-vehicles in CACC system with inaccurate acceleration measurement using in the feedback controller and random network access. (a)

Velocity (b) Acceleration (c) Position..

In the simulation, we suppose that all the vehicles are subject the same acceleration inaccuracy, namely, $\theta_r = \theta$. We use a random process to produce the acceleration

inaccuracy coefficient over time interval [0, 200], which is shown in Fig. 6. The simulation results of the position, velocity, acceleration and frequency responses of a six-vehicle platoon with inaccurate acceleration measurement are shown Fig. 7. It can be clearly seen from Fig. 7 that due to the inaccurate acceleration measurement, the vehicular platoon cannot maintain string stability. And serious fluctuations in the velocity and acceleration are witnessed (which may lead to uncomfortable driving experience or even rear-end accidents). This shows that our method is advantageous.

V. CONCLUSION

In this paper, an observer-based controller is proposed to the platoon to solve random network access and unmeasurable acceleration information. According to those obtained *RDE*, the sufficient conditions are given to guarantee the stability of the vehicle platoon and reject the disturbance generated by the preceding vehicle's acceleration. The effectiveness of the proposed method is illustrated by the numerical simulations. In the further research, we will extend the proposed framework in this paper for the vehicles platoon with the uncertain dynamic behavior (e.g., the mass of passenger vehicles or the engine time constant).

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