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Trajectory optimization for autonomous vehicles on crossroads with mobile obstacles

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***Abstract*— Autonomous vehicles issues have emerged in the past years. Among the classic trajectory planning methods, the potential field method, which is mainly local, is purely reactive to the environment and creates rough trajectories. In this article the creation of a dynamic optimal minimum of the potential is proposed, using multi-criteria trajectory optimization, for unmanned terrestrial vehicles. Obstacles are mobile and with the anticipation of their movement, motion and path of this optimal dynamic minimum are jointly generated. The effects of adding this anticipation in the planning are illustrated, and the optimization is done using a Genetic Algorithm (GA), to improve the Potential Fields (PF) method. In the first two parts of this article, the problem is formulated and the different criteria are described. Then, a GA-PF method is proposed. Finally simulations of the potential field method and its combination with a genetic algorithm are presented.**

I. INTRODUCTION

In robotics, path planning means finding a path between two or more locations, this notion being purely spatial. Trajectory planning means finding a path with an attached motion. In this article, the final purpose is to plan an optimal trajectory for an autonomous car, called "Ego-Vehicle" (EGV) to distinguish it from other vehicles. The trajectory is then used by the EGV which can further adapt it while executing its mission in real-time, given the mobile environment.

In vehicle path planning, short or safe trajectories between obstacles [4] have been widely studied. Even though energetic or comfort aspects have been treated, whether in trajectory planning [12] or eco-driving strategies [13] [8], they are a more recent field of research. On the whole, energetic aspects have been treated as if they only come from the motion, *i.e.* from the longitudinal speed. But in an environment with mobile obstacles, the motion will impact the choice of a path too.

The contributions of this study are to include new criteria in the trajectory planning and optimization, such as energy or comfort, so as to optimize the overall strategy of the vehicle, its motion for a given path. It is highly interesting, for example, for crossing crossroads, or overtaking on the motorway. The work of [11] has been improved by transforming a static target in the potential field into a dynamic optimal target, and by adding a new way of considering mobile obstacles without recalculating all the potential field. It makes the algorithm faster and therefore more reactive. Moreover, the vehicle size is taken into account and a realistic crossroads scenario will be tested. Compared to usual works on Genetic Algorithm (GA) [5], trajectories given here are smoother, and adaptable. Potential Fields (PF) have been used in the IMS laboratory before [6], and the method is extended here. Compared to recent works on the PF method [3], the planning horizon is wider, with more anticipation, and the method can be applied to many different scenarios. For example, it could also be applied to UAVs (Unmanned Aerial Vehicles) [1], as the model considered here is holonomic.

In section II, the motion planning problem is formulated, including the environment shape and the criteria. Hypothesis on the obstacles are formulated there. Then, in section III motion planning with PF and GA are detailed. Finally, simulation results are presented on the crossroads scenario in section IV. Section V is the conclusion. All parameter values are given in section VI.

II. MOTION PLANNING PROBLEM FORMULATION

In the following subsections, the EGV is represented by a 2 dimensional point mass (M) model submitted to different forces. Its position, speed, acceleration and jerk are, respectively $\overrightarrow{p} = (x, y)$, $\overrightarrow{v} = (v_x, v_y)$, $\overrightarrow{a} = (a_x, a_y)$ and $\overrightarrow{\sigma} = (\sigma^{long}, \sigma^{lat})$, where x and y are the cartesian coordinates and $long$ and lat respectively stand for the longitudinal and lateral direction of the vehicle. As for the constraints, only the maximum admissible torque is considered here. Though, spline trajectories are considered and can be easily parametrized so as to respect maximum turning angles, which will be implemented in further studies, as well as more complex vehicle models.

A. Trajectory planning environment

As was explained in [11], a vehicle journey can be divided into three main categories:

- the long range path planning (from several hundred meters to kilometers long);
- the short range motion (a few centimeters to one meter);
- the middle range trajectory, handled by this trajectory planner (from one meter up to a few hundred meters).

For this planner, the choice of the third category makes sense: one hundred meters corresponds to the range of sensors in the car (camera, LIDAR, etc...) and to a characteristic distance formerly handled by the driver. For instance, a typical situation involving such a distance is: "On the motorway, the exit being two hundred meters further, should the car in front be overtaken or not?". This particular scenario implies that the vehicle in front is mobile.

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For all tests, the environment is composed of a starting point, a target point and a set of obstacles (see section IV for examples). Its size is $T_x \times T_y$. The EGV meets obstacles along its path, including pedestrians and other vehicles. Black areas are obstacles and grey (untouched) or red (touched) areas are "danger zones", or security zones around obstacles. Mobile obstacles are represented with an arrow as their motion direction.

B. Optimization criteria definition

Five criteria are to be minimized: a danger criterion, a trajectory length one, a time one, an energetic one, and a discomfort one. The driver might have preferences, particularly concerning the importance of the comfort and energetic criteria: he may want the trajectory to be more comfortable than fast.

In the following formulas, n_{interp} is the number of points used to generate the trajectory, x_l^{int} are the x-values of the interpolated trajectory, y_l^{int} its y-values, t_0 and t_f respectively the initial and final times. l is the index of the point in the interpolated trajectory (*i.e.* $l \in [0, n_{interp}]$). For computational purpose, the environment is discretized, and so is the potential field.

$n_{interp} \geq 2\sqrt{(x_f - x_0)^2 + (y_f - y_0)^2}$ is chosen big enough to respect Shannon's theorem on sampling, so that the interpolated trajectory can be considered continuous. b_1 and b_2 are two vehicle coefficients, depending on the motor parameters, speed constant or maximum torque, and on the wheel radius and transmission ratio (see [9]). v and a and σ are the EGV speed norm acceleration norm, and jerk. All criteria are presented in more details in [11].

- The danger criterion is defined as:

$$J_{danger} = \frac{1}{n_{interp}} \sum_l U_{tot}^{rep}(x_l^{int}, y_l^{int}), \qquad (1)$$

where U_{tot}^{rep} is the total repulsive potential field described in section III-A.

- The trajectory length criterion can be expressed as:

$$J_{length} = \sum_l \sqrt{(x_{l+1}^{int} - x_l^{int})^2 + (y_{l+1}^{int} - y_l^{int})^2}. \qquad (2)$$

- The time criterion expression is:

$$J_{time} = t_f - t_0. \qquad (3)$$

- The energetic criterion (or fuel cost criterion) expression is:

$$J_{energy} = \int_0^{t_f} b_1|u(t)|v(t) + b_2 u(t)^2 \mathrm{d}t, \qquad (4)$$

where $u(t) = \frac{\tau(t)}{\tau_{max}}$ is the fraction of the maximum torque τ_{max} provided by the motor at time t, τ being the instantaneous torque.

- The discomfort criterion expression is:

$$J_{dcomfort} = \frac{1}{n_{interp}} \sum_l (\sigma_l^{long} + \sigma_l^{lat}). \qquad (5)$$

A weight coefficient $\alpha_{criterion}$ is attributed to each criterion, and the global criterion is the pondered sum of those individual criteria. Then, the objective is to find a trajectory from the starting point to the target point minimizing the criterion:

$$J_{global} = \alpha_{danger} J_{danger} + \alpha_{length} J_{length} + \alpha_{time} J_{time} + \alpha_{energy} J_{energy} + \alpha_{dcomfort} J_{dcomfort}. \qquad (6)$$

For this study, a set of weight coefficients has been chosen and is given in section VI. The impact of these coefficients on the trajectory, and how to chose them, is being currently studied and will be presented in further articles. In a first approach, vehicle constraints are not considered, the vehicle being a holnomic sold with a size and a mass. For the optimization, the only constraints are boundary constraints, and linear constraints on the time variables (see section III-B). In section III-B, the volume of the EGV is considered to avoid obstacles, but it is still a mass point dynamically speaking, without non-holonomic constraints. Next section describes the Potential Field and Genetic Algorithm methods.

III. Method principles

The objective of the presented method is to control the vehicle using the control loop shown on Figure 1. The attractive potential field has been interpreted as a controller. β is the front wheel angle.

A. Potential Fields (PFs)

The concept of the PF method is to fill the EGV's virtual environment with an artificial PF in which the EGV is attracted toward a target point and repulsed away from obstacles (see [4]). This method is particularly seductive thanks to its reactivity and elegant mathematical theory. The PF method in this article is mainly inspired from the work of [2] and [7].

1) Repulsive Weyl potential field: The repulsive potential field U_k^{rep} of the k-th obstacle is calculated using a potential field defined by [10] :

- if $n_k \in [0, 2[\cup]2, +\infty[$ and $r \in [r_{min}, r_{max}]$:

$$U_k^{rep}(r) = \frac{r^{n_k-2} - r_{max}^{n_k-2}}{r_{min}^{n_k-2} - r_{max}^{n_k-2}}; \qquad (7)$$

- if $n_k = 2$ and $r \in [r_{min}, r_{max}]$:

$$U_k^{rep}(r) = \frac{\ln(r_{max}) - \ln(r)}{\ln(r_{max}) - \ln(r_{min})}; \qquad (8)$$

- if $r \notin [r_{min}, r_{max}]$:

$$U_k^{rep}(r) = 0, \qquad (9)$$

where r is the distance from the obstacle center, and r_{min} and r_{max} are two chosen values, r_{min} often being the obstacle radius, and r_{max} the maximum obstacle influence radius. n_k is a positive real number, called order of the obstacle, depending on the obstacle speed and nature.

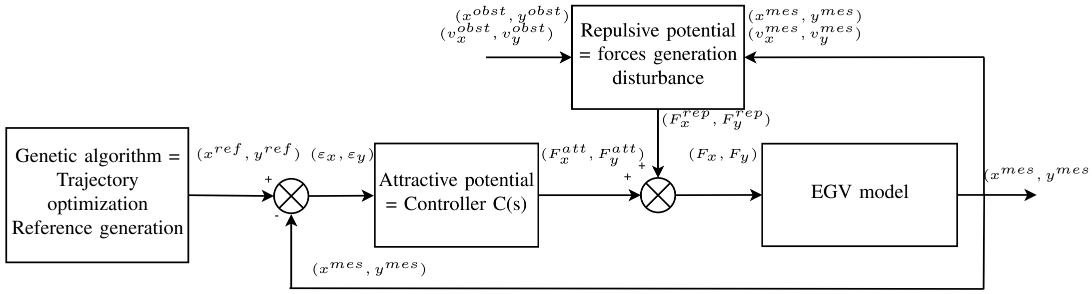


Fig. 1: EV control loop with attractive potential field

This definition leads to potential values between 0 and 1. Then, a threshold is generally fixed, defining the acceptable level of danger. With the order and the threshold, a danger zone around the obstacle is therefore defined. A representation of the repulsive potential field can be found in [11]. Considering the whole environment, and the cartesian coordinates x and y, the total repulsive potential field is :

$$U_{tot}^{rep}(x, y) = \max_k (U_k^{rep}(x, y)) \tag{10}$$

2) Attractive potential field: The attractive PF U^{att} is defined as a function of the relative position and velocity of the target with respect to the EGV. Accordingly, the virtual force is defined as the negative gradient of the potential in terms of both position and velocity rather than position only. Their expressions are:

$$U^{att}(\overrightarrow{p}, \overrightarrow{v}) = \alpha_p \|\overrightarrow{p}_{target} - \overrightarrow{p}\| + \alpha_v \|\overrightarrow{v}_{target} - \overrightarrow{v}\|, \tag{11}$$

where $\overrightarrow{p}$ and $\overrightarrow{v}$ are, respectively, the EGV position and speed, and $\overrightarrow{p}_{target}$ and $\overrightarrow{v}_{target}$ those of the target. α_p and α_v are weight coefficients discussed in previous studies [6], where they were attributed a value and a physical meaning. The corresponding force is:

$$\overrightarrow{F}^{att}(\overrightarrow{p}, \overrightarrow{v}) = -\nabla_p U^{att}(\overrightarrow{p}, \overrightarrow{v}) - \nabla_v U^{att}(\overrightarrow{p}, \overrightarrow{v}), \tag{12}$$

where

$$\nabla_w U^{att}(\overrightarrow{p}, \overrightarrow{v}) = \frac{\partial U^{att}(\overrightarrow{p}, \overrightarrow{v})}{\partial \overrightarrow{w}}. \tag{13}$$

In this expression, w can be either p or v.

3) Mobile obstacle potential field: In this article a new structure is given to the PF to deal with mobile obstacles. As the global PF is an aggregation of the different obstacle repulsive fields, it should be entirely recalculated when an obstacle moves. Here, a trajectory is generated, and for each point of the trajectory, each with a different associated time, a danger value must be calculated, meaning that for each point of the trajectory, the potential field should be recalculated. The method presented here is meant to avoid recalculating all the potential field in [10] for each time step, during the optimization. See section III-B for the algorithm complexity details.

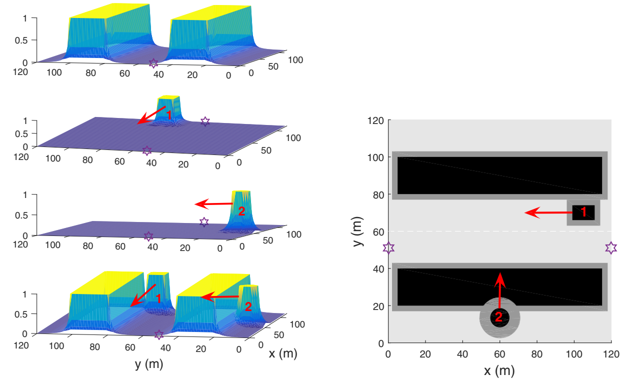


Fig. 2: Repulsive potential field in which the vehicle moves, with all matrix levels

When first created (from the car sensors data extraction), the PF is mapped in a three-dimensional matrix U_{mat}^{rep}: two spatial dimensions, and one dimension for the different obstacles. Each obstacle presenting a different motion is set on a different "level" of the matrix. The last layer of the matrix contains U_{tot}^{rep}. On Figure 2, two obstacles have a speed, marked with an arrow. Then, the danger value of the trajectory is calculated as in equation (14):

$$J_{danger} = \sum_{l,k} U_{mat}^{tot}(x_{l,k}^{mob}, y_{l,k}^{mob}, k) \tag{14}$$

where t is the time of the considered point, and $x_{l,k}^{mob}$ and $y_{l,k}^{mob}$ are the coordinates of the vehicle in the potential matrix, considering the motion of obstacles, such as:

$$\begin{cases} x_{l,k}^{mob} &= x_l^{int} - v_k^x t \\ y_{l,k}^{mob} &= y_l^{int} - v_k^y t, \end{cases} \tag{15}$$

where $k \in [0, K]$ index stands for the considered obstacle, *i.e.* for each level of the potential field matrix. v_k^x and v_k^y are respectively the x and y speed of the k-th obstacle. t stands for the time, knowing that the initial time is always considered to be zero.

The PF method does not require that the trajectories of the obstacles are known *a priori*. Instead, the motion planning only needs the online measurements of the obstacles information. Another advantage of this method is its reactivity when an unexpected event occurs, like an obstacle crossing the trajectory. The main drawback of this method is that it only reacts to the environment: forces are calculated from the negative gradient of the danger, and then acceleration from Newton's second law. Finally acceleration is integrated to get speed and again to get position, with no way to control the whole trajectory appart from the potential shape.

In the next part, the GA optimization method and how it completes the PF method is explained.

B. Genetic Algorithm

GAs solve optimization problems, based on the natural selection principle. They modifie, in an iterative way, a population of individuals which are potential solutions. For a complex optimization space, with two or more variables, classic deterministic algorithms can easily fall into local minima or get computationally too complicated. For instance, if the cost function is not continuous, not differentiable, or if some variables are integers, deterministic methods might be powerless. GAs use some specific terminology defined in [11].

As was said in section III-A, the EGV is attracted towards the target, under the influence of the attractive force. The EGV cannot move but within the potential field, which means that, to generate a trajectory close to an optimal one, it is the potential field which needs to be optimal. The potential field cannot be optimal by itself when moving obstacles are present. A way is to use an optimal moving target, *i.e.* an optimal attractive point moving so as to permanently attract the EGV where it should be. The GA tests intermediate points (one set of intermediate points constitutes an individual, given in Table I) and generates a spline trajectory between these points, represented by (x^{int}, y^{int}). The final result is an optimal trajectory represented by the variables (x^{ref}, y^{ref}).

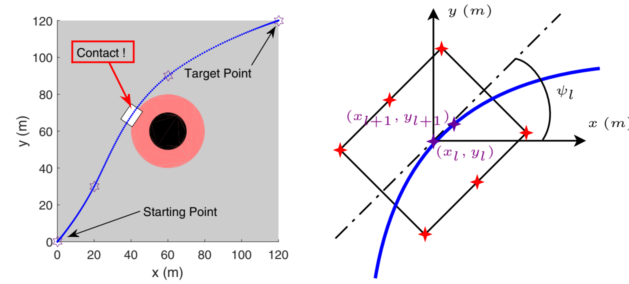
x_1	y_1	t_1	...	$x_{n_{opt}}$	$y_{n_{opt}}$	$t_{n_{opt}}$	t_f

TABLE I: Chromosome structure for the trajectory point search. n_{opt} is the number of intermediate points and t_f is the time needed to reach the final point.

Without consideration for the fitness function itself, the complexity of a GA is known to be $O(n_{pop} n_{gen} n_{var})$ where n_{pop}, n_{gen}, and $n_{var} = 3n_{opt} + 1$ are respectively the population size, the number of generation and the size of an individual. The fitness function itself has a complexity of n_{interp}, without recalculation of the PF, but would be of $n_{interp} T_x T_y$ if the whole PF had to be recalculated for every point of the trajectory, which represents a major issue.

In a nutshell, in this method, the GA brings *optimality* and *anticipation*, and its output trajectory is inserted in the PF for *reactivity* and *flexibility*. The combination of the two is efficient and promising.

1) EGV volume: The danger criterion was previously calculated using the trajectory points only, without consideration for the EGV dimensions and orientation. This can lead to bad results as shown on Figure 3 (a). In this study, the dimensions



(a) (b)

Fig. 3: (a) Illustration of the possible bad result without taking into consideration the EGV volume (b) EGV volume and ψ_l definitions

of the EGV are considered with 6 additional "dimension" points, marked with red crosses on Figure 3 (b), so as to avoid contact between the EGV and obstacles. The yaw angle of the car is $\psi_l \simeq atan\left(\frac{y_{l+1}-y_l}{x_{l+1}-x_l}\right)$.

IV. Simulations and Results

As was said in section III-B, the GA is not meant to replace the PF method and therefore its advantages (particularly its reactivity). GAs are an offline method, and not a real time control method. Here, GAs rather come as an improvement of the existing PF method by adding an optimal virtual target in the potential field. In this section, two types of trajectories are presented: off-line GA-generated trajectories are called "optimal" trajectories, and PF trajectories are called "real time" trajectories. Two ideas are being demonstrated: the anticipation of the motion of obstacles while doing the trajectory optimization can bring major improvement to the trajectory, and the creation of a dynamic target in the PF following this optimal trajectory brings major improvements to the potential field used alone. Two scenarios are presented to illustrate these points, a simple mobile obstacle case and a characteristic road environment with mobile obstacles.

As the optimization is periodical, obstacles motion might change between two computations, but for each computation the motion is supposed to stay identical: acceleration and speed are supposed to stay equal to their value when the optimization starts.

First, only offline optimal trajectories are presented. Then, real time trajectories, in the potential field, with and without an optimal target, will be compared between them and also with the optimal one. In the figures, the x-values of the trajectory function of time will be called "position".

A. Optimal trajectories with mobile obstacle

Let us first consider the optimal trajectory obtained with the GA. No PF is involved here, and the effect of mobile obstacle is already visible. On Figure 3 (a) the volume of the EGV was not taken into account. On Figure 4 (a), not only is it taken into account but the obstacle is mobile too. As can be seen, the obstacle is considered too fast for the EGV to go before it, so it goes behind. On Figure 4 (b), the trajectory is optimized for a realistic urban crossroads scenario with other moving cars, but without consideration for those moving cars. It results in a possible collision between the EGV and another vehicle. On Figure 4 (c) the trajectory is optimized too, but mobiles vehicle are taken into account, and the EGV accelerates more slowly to let the first car pass on the crossroads. The EGV passes just behind, *i.e.* waits the optimal time needed for the other car to pass. It shows the importance of considering mobile obstacles, as long as they keep the same motion.

B. Consequences for real-time potential field trajectories

In previous studies using PF, the trajectory reacts to obstacles, avoiding them at a given distance. There is no anticipation of the best trajectory, the vehicle reacts considering forces from the closest obstacles. Considering that most obstacles will keep the same motion they have during a computation cycle, anticipation can be added in the planification, by following an optimal target, acting as an optimal moving minimum of the potential.

Once the optimal trajectory is calculated, an optimal virtual target is created to be followed in the PF, according to the law given in section III-A.2. What is most important to notice on Figure 5 is the consequences of the optimization compared to a non-optimized trajectory: the optimal trajectory enables a smooth passage in real time behind the obstacle (Figure 5(b)) whereas without optimization, the EGV reaches the obstacles and struggles trying to avoid it (Figure 5(a)). The danger criterion value calculated on this last trajectory is close to 10^5 whereas it is null when the EGV follows the optimized trajectory.

The optimal and real-time trajectory for a crossroads scenario are given on Figure 6(a), along with the optimal and real-time speed curves on 6(b). The values for the time criteria are, for the optimal trajectory $J_{time}^{GA} = 15.29s$ and for the real-time one $J_{time}^{GAPF} = 15.34s$. The difference between them is close to 0.3%, which is within our objectives (less than 1% difference between the optimal and the real-time trajectory). $1.5s$ is the time needed to let the other vehicle pass.

V. CONCLUSION

GAs, a well spread method in the optimization field, enable us to improve the PF method [2] by adding anticipation, and particularly mobile obstacle anticipation. The GA brings optimality and anticipation, and its output trajectory is inserted in the PF for reactivity and flexibility. The combination of the two is efficient and promising. Effectively, from a given position of the vehicle, the environment is always partially known, including current obstacle motion, and motion can be planned in advance so as to avoid pure reaction as it is the case in the potential field method used alone. For this purpose, optimization criteria are defined, and aggregated with weight factors. In addition, test scenarios are created with mobile obstacles. Thus, with the GA, a dynamic optimal target is added in the PF, generating a smooth and adaptable path between the starting point and the final point. Moreover, the GA enables us to optimize jointly the trajectory, the speed and the acceleration curves by adding time variables in the optimization, and therefore, the choice of a strategy function of the obstacle motion is possible. The volume of the vehicle is also considered.

For further works, the potential field method needs to be improved to enable the ego-vehicle to follow more smoothly the virtual target lessening acceleration gaps due to the discontinuities in the potential. Moreover, in a real situation, the vehicle is moving, and therefore discovering new environment all the time, such as new obstacles, making the trajectory locally sub-optimal. All this suggests that the optimal trajectory needs to be periodically recalculated and the calculation time must be very inferior to the recalculation period.

This study could also be extended by adding the real dynamics of the vehicle instead of those of a mass-point.

VI. APPENDIX: PARAMETER VALUES

Parameter values are given in Table II, along with the parameter name and section where it is used.

Section	Parameter	Parameter value
II	M	900 kg
II-B	n_{interp}	300
II-B	b_1, b_2	1000
III-A.1	n_k (orders of repulsive potential)	between 1 and 3 excluded
III-A.2	α_p, α_v	168750, 22500
II-A	T_x,T_y	120 m
IV-A	V_3	3 m/s
IV-A	V_4, V_5, V_6	-2.2, 4.1, 1.5 m/s
II-B	α_{danger}	0.35
II-B	α_{length}	0.2
II-B	α_{time}	0.2
II-B	α_{energy}	0.15
II-B	$\alpha_{dcomfort}$	0.1

TABLE II: Parameter values for simulations

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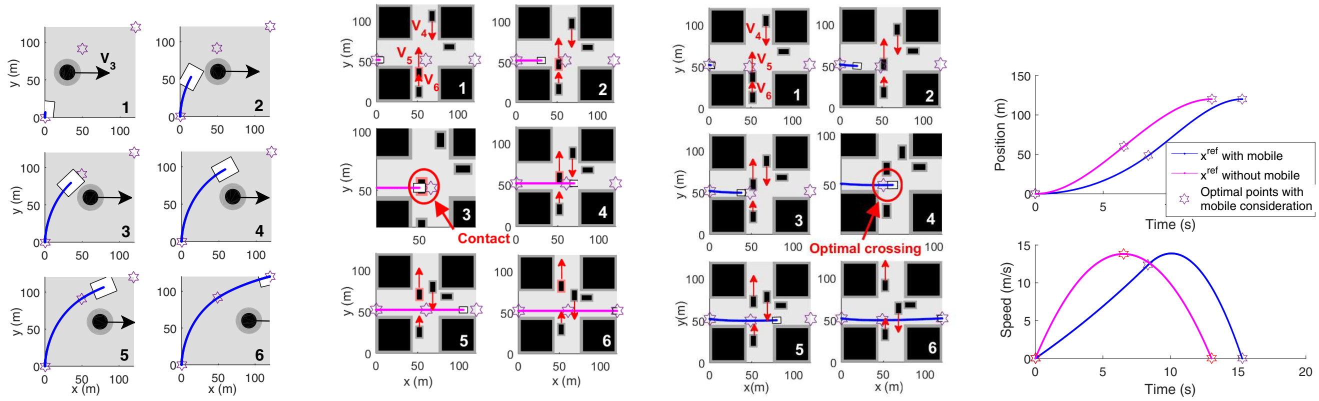
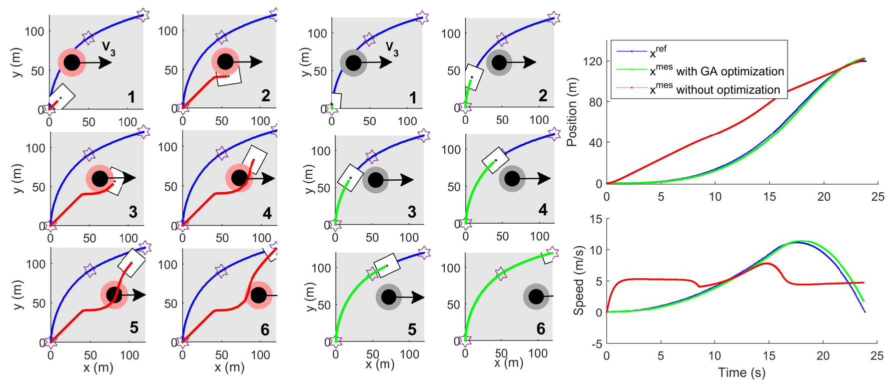


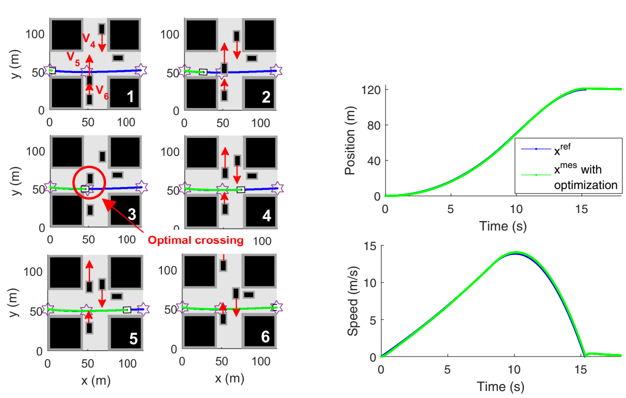
(a) (b) (c) (d)

Fig. 4: (a) Optimal trajectory (x^{ref}, y^{ref}) for a simple scenario (b) Crossroads scenario, comparison between an optimal trajectory without consideration for mobile obstacles and (c) an optimal trajectory with consideration for the mobile obstacles. On (d) are their respective position and speed curves.



(a) (b) (c)

Fig. 5: Real-time trajectory (–) without optimization (a) and with optimization (–) (b) compared to the optimal one (–) for a simple scenario. On (c) are the position and speed curves function of time.



(a) (b)

Fig. 6: (a) Real-time trajectory compared to the optimal one for a crossroads scenario, (b) corresponding position and speed curves

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