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Motorcycle inertial parameters identification via algorithmic computation of state and design sensitivities

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Abstract— **Recent advanced riding assistance safety systems, such as electronic yaw stability control, adaptive cruise control, and lane-keeping systems, require good approximations of motorcycle inertial properties, such as yaw, roll and pitch moment of inertia, this parameter can vary significantly with the rider's weight and heavy baggage placed on the luggage rack. This paper presents further research on parametric identification of two wheeler vehicles, carried out using a recursive Levenberg-Marquardt parameter identification formulation. This approach needs the use of sensitivity functions to identify acceleration responses in time domain by updating coupled inertial parameters value. The identification algorithm is implemented in *MATLAB/Simulink* software. Data and prior value are taken from the professional motorcycle simulation software *BikeSim* (based on high fidelity virtual motorcycle models).**

I. INTRODUCTION

Over the last few decades, the challenge of creating more accurate models for Active safety systems, is increased. Furthermore, much attention has been paid to powered two wheelers vehicles (PTWv) as the most vulnerable transportation mean, in order to enhance road safety systems including traction control systems, such as electronic yaw stability control, rollover prevention, and lane-keeping systems [1] [2]. Advanced riding assistance safety systems use various vehicle parameters, e.g; the tire-road friction coefficient, moment of inertia, mass and position of the center of gravity, to produce control signals. However, most of vehicle control systems, assume that vehicle parameters are known and fixed.

It is well recognized in the automotive research community that knowledge of the real-time pertinent vehicles parameters can be extremely valuable for active safety applications. If vehicle parameters were identified accurately, then the performance of model based vehicle control systems could be improved [3] [4] .

In order to reduce fatalities, out-of-plane motion should be taken into consideration in two wheelers control systems.

Previous research results in motorcycle literature have focused on state estimation based observer [5], [6], [7], [8], [9], [10]. To the knowledge of the author, works on the parametric identification of the inertial parameters for PTWv have been very few, main research were achieved without considering the physical model of the two-wheeled vehicle. In [11], the author considers an autoregressive motorcycle model to estimate the state space model of lateral dynamics without identifying parameters. There are other research axes which are concerned with the identification of the controller parameters to stabilize the two-wheeled vehicle, in ([12],[13]) the authors consider the identification of rider control as a linear PID controller. In [14], the author uses identification method to determine the physical parameters, using static tests and an algebraic method based on a one body motorcycle model, in [15], the author identifies motorcycle parameters in cascade from different kind of riding scenario with a steepest descent algorithm.

This paper demonstrates the implementation of Levenberg-Marquardt (LM) parameter estimation algorithms [16] [17]. As the combination of the steepest descent algorithm and the Gauss-Newton algorithm, the LM algorithm switches between the two algorithms during the training process. One explores the development of Levenberg-Marquardt (LM) algorithms, which are capable of working with a four degree of freedom, nonlinear motorcycle model. The mostly used dynamic model to describe the lateral motorcycle motion is the well- known Sharp model [18]. The LM algorithm allows reliable identification of motorcycle moment of inertia at the front and rear frame, based on minimizing the difference between measured generic responses and the calculated ones from a mathematical model. This approach needs the use of sensitivity functions to identify the acceleration responses by updating coupled inertial parameters value, which may be more complex in multibody systems.

The effect of measurement noise has been considered and autocorrelation of error is studied. Although the identification approach is tested on software simulation *BikeSim* implemented in conjunction with *MATLAB*.

II. PROBLEM STATEMENT

Contemporary intelligent active safety systems have great potential in improving passenger comfort and safety. With an increase of two wheelers on the road, motorcycle safety is becoming more important each day. There is a rising demand in research on powered two wheeled vehicles. With a rising of emphasis on motorcycle safety, in the vehicles economic market different controls systems have been developed and implemented

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in today's cars to assist vehicle safety development, however there are less typically safety systems for motorcycles and those developed for four-wheeled vehicles are not necessarily transferable for powered two wheelers vehicles; Also, many of these control systems do not take into account changes of vehicle parameters. With knowledge of important vehicle parameters, these control systems could be more effective in decreasing the number of crashes.
To answer safety issue, it is important to develop identification technique with the aim of improving advanced rider assistance systems (ARAS), taking into account motorcycle dynamic behavior, which depends heavily on motorcycle and rider's inertia parameters. The most important moments of inertia are the roll, pitch and yaw moments of the main frame, the moment of inertia of the front frame with respect to the steering axis, the moments of the wheels and the inertia moment of the engine. The yaw moment of inertia influences the maneuverability of the motorcycle. In particular, high values of the yaw moment (obtained, for example by heavy baggage placed on the luggage rack) reduce handling. The roll moment of inertia influences the speed of the motorcycle in roll motion. High values of the roll inertia, maintaining the same height of the center of gravity, slow down the roll motion in both entry and exit of a curve [19].

III. Lateral dynamics of PTWv

A. Motorcycle Description

The motorcycle inertial parameters are identified based on a two bodies motorcycle model with out of plane motion inspired from those presented in [18] [19]. This study uses nonlinear four degree-of-freedom (4 Dof) model in which: the main frame is subject to lateral motion, roll motion about the x-axis and yaw motion about the z-axis, and the front frame is subject to steering motion.
Figure 1 presents the motorcycle model describing the lateral motion by considering the following assumptions:

- The main frame G_r, including rider and rear bodies
- The front frame G_f enclosing the steering actions
- Longitudinal dynamics is omitted but the forward velocity v_x will be considered varying.

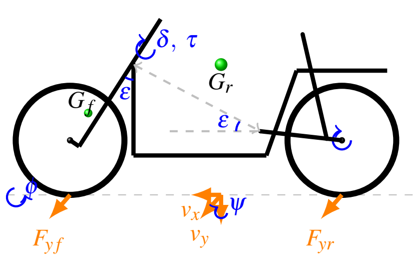

Fig. 1. Geometrical representation of the motorcycle model

The study of such a model aims to identify inertia moment of motorcycle in cornering situation.

1) Motorcycle motion: The motions of the motorcycle can be modeled by the following equations:

$$\begin{cases} e_{33}\dot{v}_y + e_{34}\ddot{\psi} + e_{35}\ddot{\phi} + e_{36}\ddot{\delta} = a_{34}\dot{\psi} + \sum F_y \\ e_{34}\dot{v}_y + e_{44}\ddot{\psi} + e_{45}\ddot{\phi} + e_{46}\ddot{\delta} = a_{44}\dot{\psi} + a_{45}\dot{\phi} + a_{46}\dot{\delta} + \sum M_z \\ e_{35}\dot{v}_y + e_{45}\ddot{\psi} + e_{55}\ddot{\phi} + e_{56}\ddot{\delta} = a_{54}\dot{\psi} + a_{56}\dot{\delta} + \sum M_x \\ e_{36}\dot{v}_y + e_{46}\ddot{\psi} + e_{56}\ddot{\phi} + e_{66}\ddot{\delta} = a_{64}\dot{\psi} + a_{65}\dot{\phi} + a_{66}\dot{\delta} + \sum M_s \\ \dot{F}_{yf} = a_{71}\phi + a_{72}\delta + a_{73}v_y + a_{74}\dot{\psi} + a_{76}\dot{\delta} + a_{77}F_{yf} \\ \dot{F}_{yr} = a_{81}\phi + a_{83}v_y + a_{84}\dot{\psi} + a_{88}F_{yr} \end{cases} \tag{1}$$

where:

$$\begin{cases} \sum F_y = F_{yf} + F_{yr} = Ma_y \\ \sum M_z = a_{47}F_{yf} + a_{48}F_{yr} \\ \sum M_x = a_{51}\sin(\phi) + a_{52}\sin(\delta) \\ \sum M_s = a_{61}\sin(\phi) + a_{62}\sin(\delta) + a_{67}F_{yf} + \tau \end{cases}$$

In these equations not all parameters are known, $\theta = [e_{44}, e_{45}, e_{46}, e_{55}, e_{56}, a_{54}, a_{56}, a_{64}, a_{65}, a_{46}]$ is the unknown vector of interest which represent the combined inertial parameters of motorcycle of the front and rear wheels, depending on : $I_{f_{x,y,z}} = \{I_{f_x}, I_{f_y}, I_{f_z}\}$, $I_{r_{x,y,z}} = \{I_{r_x}, I_{r_y}, I_{r_z}\}$, C_{rxz} and $i_{y_{f,x}} = \{i_{r_y}, i_{f_y}\}$.

For further details on the motorcycle parameters (e_{ij}, a_{ij}) and expressions refer to table II.

TABLE I

Motorcycle dynamic parameters signification	
ϕ, δ, ψ	roll, steering, yaw angles.
$\dot{\phi}$, $\dot{\delta}$, $\dot{\psi}$	respectively rate angles.
v_y, v_x	denotes lateral and forward velocity.
F_{yf}, F_{yr}	cornering front and rear forces .
τ	the torque is the input applied to the handle bar.

The dynamics of the state variables are derived to construct the continuous state-space representation of the motorcycle model,

$$\dot{x} = F(x, v_x, \theta, \tau)$$

whereas $x = [\phi, \delta, v_y, \dot{\psi}, \dot{\phi}, \dot{\delta}, F_{yf}, F_{yr}]^T$ denotes the state vector.

IV. Levenberg-Marquardt

The identification of parameter values in complex multibody systems constitutes a challenging field. Not only are the dynamics of such systems algorithmically complex, which makes the implementation of parameter identification methods harder, but also industrial motorcycle models and test data are fairly costly to obtain.

In these approaches, the objective function is usually defined as the difference between the model response and the experimental response, expressed through some kind of metric such as a definite integral of the variable under consideration. The design parameters, in turn, are the unknown model properties.

For the parametric identification, we suggest the Levenberg-Marquardt (LM) algorithm. In the following, the optimization methods improve objective function by varying the value of certain design parameters.

Let us assume that $f(y,\dot{y}(\theta),\ddot{y}(\theta))$, is a generic response function depends on a set of unidentified inertia parameters (θ), measurement output or observable state : $y,\dot{y},\ddot{y}$ (positions, velocities and accelerations), time t.

$$f=\left\{\begin{array}{c} f_1\\ f_2\\ f_3\end{array}\right\}=\left\{\begin{array}{c}\ddot{\psi}(y,\dot{y}(\theta),\ddot{y}(\theta))\\ \ddot{\phi}(y,\dot{y}(\theta),\ddot{y}(\theta))\\ \ddot{\delta}(y,\dot{y}(\theta),\ddot{y}(\theta))\end{array}\right\}=\left\{\begin{array}{c} f_1(y,\dot{y}(\theta),\ddot{y}(\theta))\\ f_2(y,\dot{y}(\theta),\ddot{y}(\theta))\\ f_3(y,\dot{y}(\theta),\ddot{y}(\theta))\end{array}\right\}. \tag{2}$$

The sensitivities of the generic functions are as follow :

$$f_\theta=\frac{df}{d\theta}=\frac{\partial f}{\partial y}\frac{dy}{d\theta}+\frac{\partial f}{\partial \dot{y}}\frac{d\dot{y}}{d\theta}+\frac{\partial f}{\partial \ddot{y}}\frac{d\ddot{y}}{d\theta}$$

Where, operator $\frac{d(.)}{d(.)}$ denotes total derivatives, and operator $\frac{\partial(.)}{\partial(.)}$ denotes partial derivatives. In this expression, an additional type of derivative is required: state sensitivities y_θ, $\dot{y}_\theta$ and $\ddot{y}_\theta$.

$$f_\theta=\left\{\begin{array}{c} f_{1_\theta}\\ f_{2_\theta}\\ f_{3_\theta}\end{array}\right\}=\frac{\partial f}{\partial y}y_\theta+\frac{\partial f}{\partial \dot{y}}\dot{y}_\theta+\frac{\partial f}{\partial \ddot{y}}\ddot{y}_\theta \tag{3}$$

A. Principle of method:

The basic idea of least squares methods is fitting a mathematical model to a sequence of observed data, minimizing the sum of the squares of the difference between observed and computed data. By doing so, any noise or inaccuracies in the observed data are expected to have less effect on the accuracy of the mathematical model.

The Leverndberg-Marquardt algorithm is an improvement of the algorithm of gradient and Gauss-Newton, hence it's way of reducing the number of iterations of an optimization algorithm through the use of second derivatives of criterion.
Identification methods are generally based on minimizing the difference between the measured outputs and the estimated outputs. The criterion J to minimize, represents the quadratic deviation between the generic responses function defined in equation (2) and true measurement from sensors:

$$\min J(t)=\frac{1}{2}\varepsilon^2(t) \tag{4}$$

The prediction error is defined as follow:

$$\varepsilon(t)=f(t)-\hat{f}(t)$$

The LM algorithm requires that you supply it with the Jacobian of the vector f. The Jacobian is merely a matrix representation of all the first derivatives of the components of the vector.

Normally, gradient J_θ is defined as the first-order derivative of total error function ((3)), the elements of gradient vector can be calculated as

$$J_\theta=\frac{\partial J}{\partial\theta}=\frac{\partial J}{\partial\varepsilon}\frac{\partial\varepsilon}{\partial\theta}=[\frac{\partial J}{\partial\theta_1}...\frac{\partial J}{\partial\theta_j}]^T$$

$$J_\theta=\varepsilon\frac{\partial\varepsilon}{\partial\theta}=-\varepsilon\frac{\partial f}{\partial\theta}=-\varepsilon f_\theta$$

By combining the definition of gradient vector J_θ, it could be determined that

$$\frac{\partial J_{\theta_i}}{\partial\theta_j}=\frac{\partial^2 J}{\partial\theta_i\partial\theta_j}$$

$$J_{\theta\theta}=\frac{\partial J'_{\theta_i}}{\partial\theta_j}=\frac{\partial(\varepsilon f_{\theta_i})}{\partial\theta_j}=f^T_{\theta_j}f_{\theta_i}$$

In order to make sure that the approximated Hessian matrix $J_{\theta\theta}$ is invertible, Levenberg-Marquardt algorithm introduces an approximation to Hessian matrix H :

$$H\simeq(J_{\theta\theta}+\lambda I)$$

where

- λ is always positive, called combination coefficient;
- I is the identity matrix.

From the above Hessian equation, one may notice that the elements on the main diagonal of the approximated Hessian matrix will be larger than zero. Therefore, with this approximation, it can be sure that matrix H is always invertible.
The correction term H should allow us to minimize in each step the criterion.

The update rule of Levenberg-Marquardt algorithm can be presented as

$$\theta^{(i+1)}=\theta^{(i)}-\{[J_{\theta\theta}+\lambda I]^{-1}J_\theta\}_{(\hat{\theta}=\theta^{(i)})}$$

This algorithm is based on the calculation of the gradient (J_θ) and the Hessian $(J_{\theta\theta})$ which uses the sensitivities functions.

One can defined a generic matrix Q, the calculation of the LM is given by the following algorithm :

$$Q_i=\left\{\begin{array}{cccc}\varepsilon^T\varepsilon & \varepsilon^T\frac{\partial f_i}{\partial\theta_1} & ... & \varepsilon^T\frac{\partial f_i}{\partial\theta_n}\\ (\frac{\partial f_i}{\partial\theta_1})^T\varepsilon & (\frac{\partial f_i}{\partial\theta_1})^T\frac{\partial f_i}{\partial\theta_1} & ... & (\frac{\partial f_i}{\partial\theta_1})^T\frac{\partial f_i}{\partial\theta_n}\\ ... & ... & ... & ...\\ (\frac{\partial f_i}{\partial\theta_n})^T\varepsilon & (\frac{\partial f_i}{\partial\theta_n})^T\frac{\partial f_i}{\partial\theta_1} & ... & (\frac{\partial f_i}{\partial\theta_n})^T\frac{\partial f_i}{\partial\theta_n}\end{array}\right\} \tag{5}$$

The practical implementation of the Algorithm is given by the following diagram:

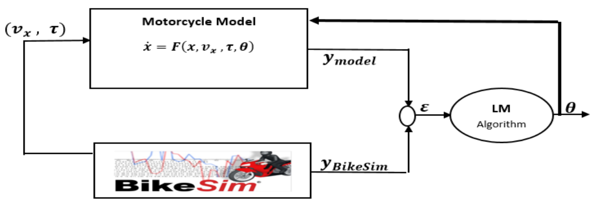

Fig. 2. Block diagram of the LM method.

The standard LM process can be illustrated in the following algorithm:

Algorithm 1: `Levenberg-Marquardt (LM):`(θ)

1 $\text{Initialisations}\begin{cases} \theta_0(t_0),\ \lambda = 1,\ fini = 0, \\ epsCrit = 10^{-5} \end{cases}$

. **if** *stop*$\neq$ *1* **then**

2 **for**$(t_{k-1} = t_0) \Rightarrow$ Calculate $J_0(\theta_0)$, $G_0(\theta_0)$, $H_0(\theta_0)$, then : f_{i_θ}= the sensitivies function

3 $S_i = [\varepsilon_i \ f_{i_{\theta_j}}]$, i=(1,2,3), j=(1,..,10)

4 $Q_i = S_i^T \times S_i$,

5 $J_i = Q_i(1,1)$, $G_i = Q_i(2:end,1)$, $H_i = Q_i(2:end,2:end)$

6 $J(\theta_0) = \sum_1^3 J_i$, $G(\theta_0) = \sum_1^3 G_i$, $H(\theta_0) = \sum_1^3 H_i$

7 then

$$\hat{\theta}_1 = \theta_0(t) + (H(\theta_0) + \lambda I)^{-1} G(\theta_0)$$

for$(\hat{\theta}_1) \Rightarrow$ Calculate $J(\theta_1)$, $G(\theta_1)$, $H(\theta_1)$, then : **if** $J(\theta_1) < J(\theta_0)$ **then**

8 $\begin{cases} \theta_1 = \theta_0 \\ \lambda = \lambda/10 \end{cases}$

9 **else if then**

10 $\{ \ \lambda = \lambda \times 10$

return to step 7

11 **if** $(|J(\theta_1) - J(\theta_0)| < epsCrit)$ **then**

12 **return** stop = 1;

V. Test Simulation

The proposed estimation algorithm is evaluated in simulations by using data from BikeSim, an industry-standard motorcycle dynamics simulation software developed to simulate and analyze motorcycle dynamic responses. The motorcycle model from BikeSim [20] chosen for this simulation is a Sport Touring, Wishbone; Baseline "Touring Bikes" with 8 bodies, and default parameters.
It should be noted that while the lateral model estimation algorithm was developed using two bodies motorcycle model (4 Dof), the data in simulations are being collected using a high-fidelity motorcycle dynamics model. The proposed approach is evaluated under different driving scenarios using BikeSim in Matlab/Simulink. Double lane change (DLC) on a (152.4 m) Circle flat road and an oncoming traffic with variable Speed on handling road course with a high road friction coefficient $\mu = 0.85$ are selected to be the test scenarios.
The motorbike in the BikeSim simulation undergoes a double lane change maneuver at a speed of 100 km/h to generate the lateral motorcycle dynamics, The simulation scenario involved both accelerating and braking in a turn on flat circle road surfaces (152.4 m), for a high-μ surface with $\mu = 0.85$. The double lane change maneuver can generate complicated dynamics in the vehicle, such as lateral rotation motion about the lateral y-axis.
Then, the second driving conditions is set as follows: the motorcycle is assumed to drive at oncoming traffic variable speed with a high adhesion coefficient of 0.85, and a handling road course maneuver is performed.
In Fig. 3 and Fig 7, the figures indicate steering torque τ, forward speed v_x, and the trajectory from the BikeSim model for the riding maneuver. The properties and performances of the proposed LM identification algorithm were investigated by simulations. As mentioned in the previous section, the objective function of acceleration converges to true state when the unknown parameters are updating estimated values of the combined inertial parameters, as shown in Fig. 4 and Fig. 9 for initial parameters (for the two maneuvers). Then in Fig. 5 and Fig. 10 after updating the inertial parameters convergence.
The identification of the inertial parameters are given in Table. II and for the parameters which depend on the forward speed are given in Fig. 6 and Fig. 13.

Interpretation

The results show that in the beginning when applying the initial value of parameters the model didn't match the Bikesim responses output, however, by updating the values of unknown parameters by LM method, the responses estimation from the identified value closely match the BikeSim simulated output.
Gaussian noise was added to output state for the LM identifier to realistically recreate real application scenarios. The variances for Gaussian noise is $(0.05[rad/s^2])$ and it was generated from *randn* MATLAB function.
The final step in validating the method is to analyze the residuals generated by it. For a good identification, these residuals should be white, which statistically means that we have insignificant correlations for non-zero lags, in Fig 7 and Fig 12, one shows plots of the autocorrelation of residuals error compared to that of the white noise, the residuals are mostly uncorrelated at nonzero lags, and The prediction error is approximately a white Gaussian noise. The model obtained has the same characteristics as a white noise, hence the validity of the model that explains the BikeSim data.

A. Test 1

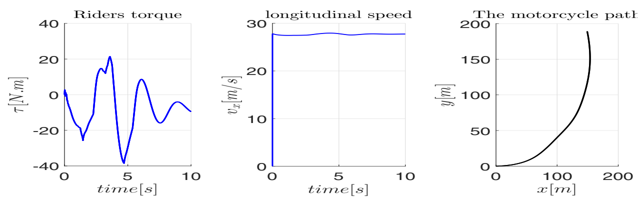

Fig. 3. Test maneuver : double line change, $v_x = 100km/h$ in circle road, on high-friction surface $\mu = 0.85$: Rider torque τ - Longitudinal velocity - Path.

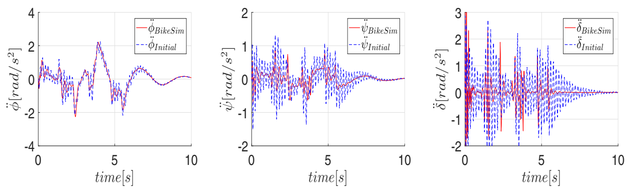

Fig. 4. Generic responses estimation (dashed blue) in initial parameters value θ_0 compared to actual BikeSim responses (red).

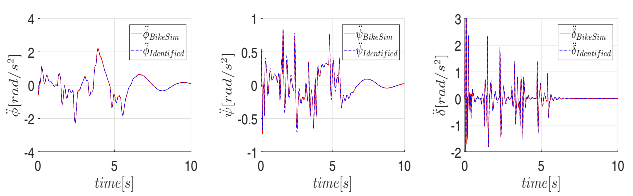

Fig. 5. Generic responses estimation (dashed blue) after updating parameters estimates value θ_i, compared to actual BikeSim responses (red)

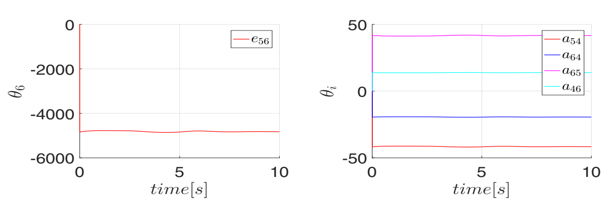

Fig. 6. Combined inertial parameters estimates $\hat{\theta}_i$, $\hat{\theta}_6$.

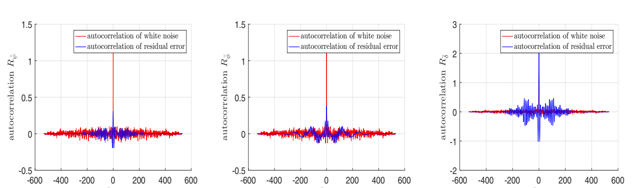

Fig. 7. Correlation for white noise (red) and residual error (blue).

B. Test 2

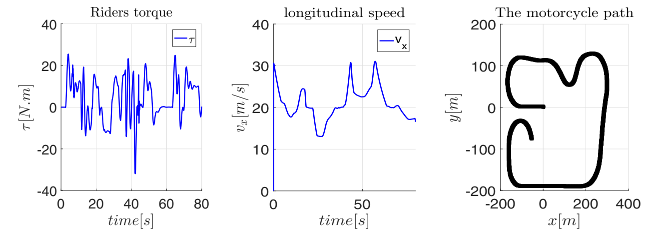

Fig. 8. Test maneuver : oncoming variable speed, in road course with $\mu = 0.85$: Rider torque τ - Longitudinal velocity - Path..

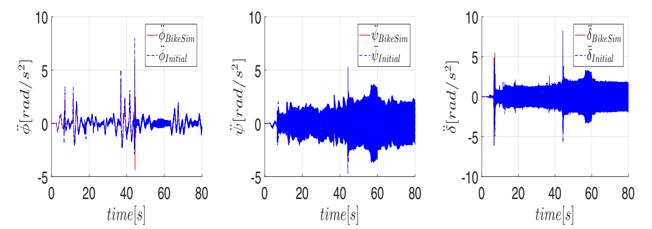

Fig. 9. Generic responses estimation (dashed blue) in initial parameters value θ_0 compared to actual BikeSim responses (red).

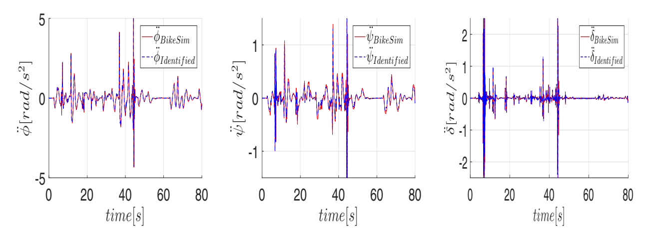

Fig. 10. Estimation (dashed blue) after updating parameters estimates value θ_i, compared to actual BikeSim responses (red).

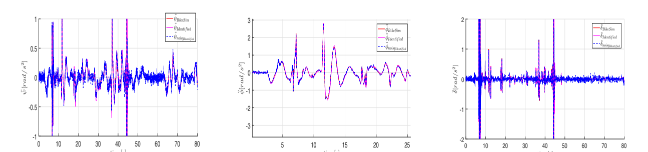

Fig. 11. Estimation (magenta) after updating parameters estimates value θ_i, compared to actual BikeSim responses (red) and noisy responses (dashed blue).

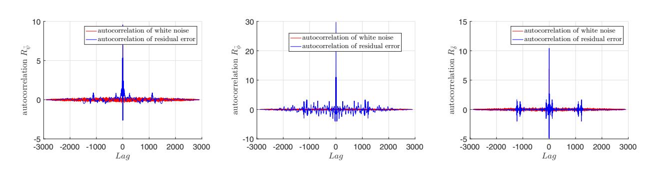

Fig. 12. Correlation graph for white noise (red) and residual error (blue).

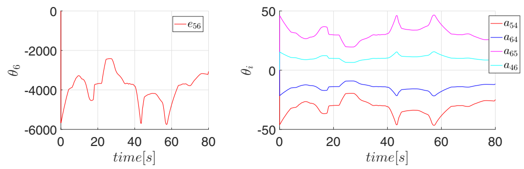

Fig. 13. Combined inertial parameters estimates.

Table II presents parameters values estimates .

TABLE II

PARAMETERS θ

	Initial	*True values*	*Scenario*1	*Scenario*2
parameters	(θ_0)	(θ_r)	(θ_i)	(θ_i)
e_{44}	1	34.7228	31.7913638	28.2429
e_{45}	2	1.9632	0.968	-4.0424
e_{46}	0.1	0.6584	0.5	0.5
e_{55}	113	118.0202	119.78	119.9828
a_{54}	$-170 \times v_x$	$-175.0479 \times v_x$	$*$	$*$
e_{56}	$-1 \times v_x$	$-1.4622 \times v_x$	$*$	$*$
a_{56}	0.1	0.3827	0.5	0.5
a_{64}	$-1 \times v_x$	$-0.8685 \times v_x$	$*$	$*$
a_{65}	$1 \times v_x$	$1.4622 \times v_x$	$*$	$*$
a_{46}	$0.2 \times v_x$	$0.6818 \times v_x$	$*$	$*$

For $(*)$ parameters, the estimated convergence is shown in figures 6 13.

VI. Conclusion

In this paper we have described the design process of Levenberg-Marquardt (LM) identifier to estimate motorcycle inertial parameters, this approach uses sensitivity functions of generic responses developed using the Sharp motorcycle model to optimal estimation. The LM estimation improved convergence characteristics for state by updating inertial parameters, which are most apparent in the estimation of the generic responses: yaw, roll and the steering acceleration. The designed LM method, identified combined expression of inertial parameters and predicted the objective functions. The simulation results were very promising, the LM formulation is a good method to predict the response with very high accuracy.

Acknowledgments

This work has been done as part of *ANR project* : *VIROLO*$^{++}$ and it has been supported by *Evry Val d'Essonne University* (UEVE) and *Informatique, Biologie Intégrative et Systèmes Complexes Laboratory* (IBISC).

TABLE III

Motorcycle parameters expressions

$e_{33} = M, e_{35} = M_f j + M_r h,\ e_{34} = M_f k\ ,\ a_{44} = -M_f k v_x,\ e_{36} = M_f e\ a_{34} = -M v_x\ a_{45} = \frac{i_{fy}}{R_f} + \frac{i_{ry}}{R_r},$ $a_{52} = M_f eg - \eta Z_f,\ a_{51} = (M_f j + M_r h)g,\ a_{61} = M_f eg - \eta Z_f, a_{62} = (M_f eg - \eta Z_f)\sin(\varepsilon), a_{74} = -\frac{C_{f1}}{\sigma_f} l_f,$ $e_{66} = M_f e^2\ , a_{66} = -K, a_{67} = -\eta,\ a_{71} = \frac{C_{f2}}{\sigma_f} v_x,\ a_{73} = -\frac{C_{f1}}{\sigma_f},\ a_{72} = \frac{(C_{f1}\cos(\varepsilon) + C_{f2}\sin(\varepsilon))}{\sigma_f} v_x,$ $a_{76} = \frac{C_{f1}}{\sigma_f}\eta;\ a_{47} = l_f,\ a_{48} = -l_r,\ a_{77} = a_{88},\ a_{81} = \frac{C_{r2}}{\sigma_r} v_x,\ a_{83} = -\frac{C_{r10}}{\sigma_r},\ a_{84} = \frac{C_{r1}}{\sigma_r} l_r.$
$\theta = \{I_{fx}, I_{fy}, I_{fz}, I_{rx}, I_{ry}, I_{rz}, C_{rxz}, i_{ry}, i_{fy}\}$
$e_{44} = M_f k^2 + I_{rz} + I_{fx}\sin^2\varepsilon + I_{fz}\cos^2\varepsilon\ ,\ e_{45} = M_f jk - C_{rxz} + (I_{fz} - I_{fx})\sin\varepsilon\cos\varepsilon\ ,\ a_{65} = \frac{i_{fy}}{R_f}\cos(\varepsilon) v_x,$ $e_{46} = M_f ek + I_{fz}\cos\varepsilon\ ,\ e_{55} = M_f j^2 + M_r h^2 + I_{rx} + I_{fx}\cos^2\varepsilon + I_{fz}\sin^2\varepsilon\ ,\ a_{56} = M_f ej + I_{fz}\sin\varepsilon$ $a_{46} = \frac{i_{fy}}{R_f}\sin\varepsilon v_x\ a_{54} = -(M_f j + M_r h + \frac{i_{fy}}{R_f} + \frac{i_{ry}}{R_r})v_x\ ,\ e_{56} = -\frac{i_{fy}}{R_f}\cos\varepsilon v_x\ ,\ a_{64} = -(M_f e + \frac{i_{fy}}{R_f}\sin\varepsilon)v_x$

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