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Probabilistic Prediction from Planning Perspective: Problem Formulation, Representation Simplification and Evaluation Metric

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***Abstract*— Accurate probabilistic prediction for intention and motion of road users is a key prerequisite to achieve safe and high-quality decision-making and motion planning for autonomous driving. Typically, the performance of probabilistic predictions was only evaluated by learning metrics for approximation to the motion distribution in the dataset. However, as a module supporting decision and planning, probabilistic prediction should also be evaluated from decision and planning perspective. Moreover, the evaluation of probabilistic prediction highly relies on the problem formulation variation and motion representation simplification, which lacks a formal foundation in a comprehensive framework. To address such concerns, we provide a systematic and unified framework for the analysis of three under-explored aspects of probabilistic prediction: problem formulation, representation simplification and evaluation metric. More importantly, we address the omitted but crucial problems in the three aspects from decision and planning perspective. In addition to a review of learning metrics, metrics to be considered from planning perspective are highlighted, such as planning consequence of inaccurate and erroneous prediction, as well as violations of predicted motions to planning constraints. We address practical formulation variations of prediction problems, such as decision-maker view and blind view for viewpoint, as well as reactive prediction for interaction, so that decision and planning can be facilitated.**

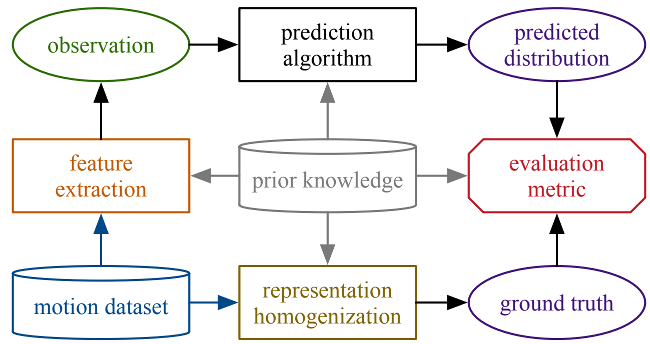

Fig. 1: Typical structure and data flow for probabilistic prediction. Observations are generated via feature extraction of historical motions in the dataset combined with maps from prior knowledge. Prediction algorithm provides predicted distribution according to the observation input by incorporating prior knowledge such as traffic rules, vehicle models and maps. Since the motion representation in the predicted distribution is often simplified, the representation of the future motions in the dataset should be homogenized so that the ground truth can be used to evaluate the performance of the algorithm by appropriate metrics.

I. Introduction

The behavior of traffic participants is full of uncertainties in the real world. Autonomous vehicles need to well estimate such uncertainties to increase driving quality (time-efficiency, comfort, etc.) and safety level for the decision-making and motion planning. To drive safely, autonomous vehicles should predict possible intentions and motions of other road participants, and avoid collisions accordingly. To enhance driving quality, autonomous vehicles should take threats of high probability seriously, yet not overreact to threats of low probability. Therefore, probabilistic intention and motion predictions are inevitable for safe and high-quality decision-making and motion planning for autonomous vehicles.

A typical structure of probabilistic prediction is shown in Figure 1. Most of the research efforts on probabilistic prediction [1] were focused on prediction algorithm design for specific scenarios. Some recent works addressed incorporating prior knowledge to construct prediction frameworks which can deal with a variety of scenarios [2] [3]. Solutions for other parts in Figure 1 were often arbitrarily adopted, which lacks sufficient investigation and solid foundation.

A. Evaluation metric

Evaluation metrics are required to measure the performance of predictions. Distance-based trajectory similarity metrics were investigated and employed in [4] [5] [6] [7]. Comprehensive reviews on distance-based metrics were provided and novel measures were proposed in [8] [9]. Distance-based metrics are well applicable to evaluate deterministic predictions. However, the evaluation of probabilistic prediction cannot be provided by using distance-based metrics directly. A variety of learning metrics were used to evaluate the performance of probabilistic prediction models on whether the distribution in the dataset is well approximated, such as area under the curve (AUC) [10], root mean square error (RMSE) [11], likelihood [12], and Kullback-Leibler (KL) divergence [13].

The executed motion of a vehicle typically satisfies planning constraints on feasibility (vehicle model) and safety (collision avoidance). However, small perturbations making a trajectory infeasible or unsafe, can hardly make a large difference for learning and distance-based metrics. The afore-

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mentioned problem has not been sufficiently addressed for evaluation. Moreover, the ultimate goal of the probabilistic prediction model is to support decision-making and planning. Several practical aspects should also be considered from such perspective. The potential consequence of inaccurate predictions to safety and driving quality should be taken into account. It is also a concern for the decision and planning module whether accurate prediction is provided timely with a sufficient preview horizon. Neither the learning nor distance-based metrics can take into account these aspects.

The performance of probabilistic predictions should be evaluated by appropriate metrics so that it can best approximate the distribution in datasets, while avoid infeasible and unsafe motions in the prediction, and desirably provide indications to the subsequent decision and planning modules. Thus our *primary motivation* is to provide a summary of learning metrics, and address the crucial but omitted aspects on prediction evaluation from the perspective of decision-making and planning, so that novel metrics can be inspired.

B. Representation simplification

Indicators are commonly used to simplify the motion representation to enable the probabilistic description of the predictions since the original space of trajectories can be extremely high with continuous variables. As shown in Figure 1, the representation of future motions should be homogenized so that same indicators are used for comparing the predicted distribution and the ground truth to make the evaluation metric meaningful. Motion patterns such as route [14] and pass-yield [15] patterns, as well as spatiotemporal representations such as prototype trajectory [16] and reachable set [17], were used as indicators, which need to be summarized.

Intention is one of the most common indicators for simplification. However, the meaning of "intention" can be twofold, namely, original desire and executed motion pattern. In highly interactive scenarios, an entity may not be able to achieve its original desire due to the motion of others. In fact, the decision and planning module checks the potential collision with others according to their possible motions to execute, but not the original desire. Moreover, the original desire cannot be labeled as the ground truth in datasets. Instead, only the executed motion patterns can be labeled. Therefore, the difference between pattern and desire should be clarified. Thus our *second motivation* is to summarize methods for motion representation simplification, and to clarify the distinction between motion pattern and desire.

C. Problem formulation

The variation of the assumptions and settings for the input and output of the prediction algorithm can significantly change the forms of the observation and predicted distribution in Figure 1. It is hard to compare algorithms with different problem formulations due to the viewpoint and interaction-involvement of the input, as well as the number of predicted entities and motion representation of the output. Such variation can also completely change the problem complexity, as well as model practicability for decision-making and planning of the host vehicle.

For instance, NGSIM dataset [18] provides bird's-eye view of the dynamic scenes so that the surroundings of a predicted vehicle are fully observable. However, autonomous vehicles in the real traffic can only obtain local view, which means that the surroundings of a predicted vehicle are partially observable with uncertainties. Thus our *third motivation* is to summarize the variations of input and output for the problem formulation of probabilistic prediction from decision and planning perspective so that the difficult but practically significant problems can be addressed.

The main contributions of this paper can be summarized as follows. First, we address the under-explored but crucial aspects to formulate, simplify and evaluate probabilistic prediction from decision-making and planning perspective. Second, we provide a systematic framework to summarize the assumptions and settings for problem formulation, indicators for motion representation simplification, and metrics for performance evaluation. Third, existing probabilistic prediction works are reviewed from the three highlighted aspects, instead of methodologies and scenarios, in order to inspire novel evaluation metrics.

II. Problem Formulation

Suppose x_i is an input observation in Figure 1, which contains the extracted features from historical motions in the dataset with map context. $\hat{y}_i$ denotes a representation of the future motion of the predicted entity in the prediction algorithm corresponding to x_i. y_i is the ground truth of the future motion from dataset homogenized to the same form of $\hat{y}_i$. X and Y are the corresponding random variables. Then the problem is to design prediction algorithms so that the predicted distribution $p(Y|X)$ can best approximate the distribution in the dataset $q(Y|X)$. Note that typically $q(Y|X)$ can only be obtained or represented by ground truth data points (x_i, y_i). There are several variations of the original problem by changing the input and output of the model.

A. Input variation 1: viewpoint

An important variation of the observation input is the viewpoint. The main distinction for different viewpoints lies in occlusions, such as whether the surroundings of the predicted entity is occluded to the prediction module, and whether the predicted entity itself is occluded.

1) Bird's-eye view (host prediction): The viewpoint is a *bird's-eye view* when the surroundings of a predicted entity are fully observable. An algorithm designed with such viewpoint is actually predicting the motion of a host vehicle (first-person viewpoint), since all the adjacent entities can be assumed to be fully observable from the viewpoint of the host vehicle. The most representative bird's-eye view vehicle motion dataset is the NGSIM dataset, which was utilized by many recent works on probabilistic prediction [11] [12] [15] [19].

2) Local view (surrounding prediction): The viewpoint is a *local view* when the surroundings of a predicted object are partially observable due to occlusions. An algorithm designed with such viewpoint can be used to predict the motion of a surrounding vehicle (third-person viewpoint). Although the data used in many existing works were collected in local view, probabilistic prediction with partial occlusion has not been sufficiently investigated by researchers yet. It is possible to take into account the uncertainty caused by occlusion via using the raw sensor data as the input [20] or constructing noisy measurement input [21].

3) Blind view (occluded entity prediction): The predicted entity can also be partially or fully occluded by vehicles or buildings, which leads to a *blind view*. Algorithms designed with such viewpoint can provide prediction for blind corner decision-making [22] [23] [24] or occluded object tracking [25].

B. Input variation 2: interaction

Another input variation is interaction. The main distinction lies in whether the behavior of the predicted vehicle is influenced by the surrounding entities. Another distinction is whether there is a decision-maker asking "what if my motion is like this in the future".

1) Independent prediction: The "maneuver-based" prediction model was defined in [1] as the behavior of the predicted vehicle is independent from others. Since the word "maneuver" was used in some literatures for behaviors influenced by others, we use *independent prediction* to describe the same problem.

2) Interdependent prediction: The "interaction-aware" prediction model was defined in [1] when the behavior of the predicted entity is influenced by others. In this paper we emphasize whether the behavior is impacted by historical or future motions of the surroundings. When only historical surrounding motions are used as the input, the problem can be defined as *interdependent prediction*, namely, the motions of the entities are only historically interactive.

3) Reactive prediction: From the host vehicle decision-making and planning perspective, the most desirable prediction for a highly interactive scenario is an algorithm that can answer the question "what if my motion is like this in the future". Therefore, we define *reactive prediction* as the problem to obtain the algorithm whose input can take into account the future motion of a host vehicle or a pattern (simplification) of the motion, and provide the distribution of the predicted entity accordingly.

C. Output variation 1: number of predicted entities

The number of predicted entities as the output can change the formulation of the prediction problem.

1) Single-entity prediction: Most of the recent works are focused on *single-entity prediction*, where the future behavior of only one entity is provided.

2) Situation prediction: The combinatorial decision and planning [26] of the host vehicle may strongly depend on the possible behavior of several entities as a group in complicated scenarios. However, it is hard to directly combine the predictions of single entities since the future motions may be exclusive. Instead, situation prediction [27] [28] can provide the joint distribution of the motions of surroundings.

D. Output variation 2: motion representation

The original and intuitive representation of predicted motion is to use continuous trajectories. However, due to the high dimension and difficulty in describing the distribution for continuous random variables, the motion representation is often simplified in order to describe the probability distribution for complicated scenarios. Detailed discussion on representation simplification is provided in Section III.

III. Representation Simplification

As discussed in Section II, the representation of long-term motions is usually simplified by indicators since it is intractable to directly use continuous trajectories to describe the distribution of predicted motions in complicated scenarios. In this section, the most commonly used indicators are categorized as continuous motions, motion patterns or spatiotemporal indicators. We also categorize motion patterns according to the hierarchy from decision and planning perspective. Moreover, the spatiotemporal indicators are discussed based on whether motion patterns are considered.

A. Continuous motions

A sequence of positions and yaw angles [29], as well as velocities [15] and accelerations [19] [11] are typically used as the continuous motions. Such representation is relatively more applicable in car following [19] [30] [31] and ramp merging [11] [15] [32] scenarios, where only longitudinal motions need to be considered. When the preview horizon is relatively long, it is intractable for a model to directly output the distribution of long-term motions. Instead, the models can output the distribution for one step look ahead, then structures such as Bayesian filtering [21] [29] [33] and long short-term memory (LSTM) [19] [34] can be employed to propagate the motion to the long-term future.

B. Motion patterns with hierarchical categorization

For decision-making and planning of the host vehicle, the destination to reach is the first to be considered. A corresponding route can be planned offline, which is typically independent from the situation encountered in real time. Then the local decision-making and planning module can deal with the specific situation, such as whether to pass a conflict region before or after another entity. Such a hierarchical architecture can also be applied for prediction, namely, the route pattern and pass-yield pattern are hierarchical. One example for such hierarchy is predicting potential right turn with pedestrian yielding at an intersection [35], in which route patterns were turn right and go straight, and the pass-yield patterns were go and stop for straight, and turn and yield for turn right. Another example is predicting routes and right-of-way at a four-way-stop intersection [36]. The route patterns were turn left/right and go straight, and the pass-yield pattern was the right-of-way at intersections.

Moreover, there are also subtle motion patterns which cannot be defined as route or pass-yield patterns, such as slow down, go as expected, accelerate, etc. A comprehensive example for the hierarchy of route, pass-yield and subtle patterns is predicting potential left turn with proceeding and oncoming vehicles at an intersection [10]. The route patterns were go straight and turn left. The pass-yield patterns were go in front and yield for the oncoming vehicle when turning left. The subtle patterns were free drive and influenced for the proceeding vehicle, and full stop and slow down for yielding oncoming vehicle when turning left.

1) Route pattern: Route pattern (intention) denotes the discrete pattern shaped by the spatial road structure, such as road branches, lanes and parking lots. It depicts which parking lot the entity wants to occupy, or which branch of road it wants to take at nodes such as intersections, roundabouts, exit ramp, etc. Dedicated lanes (left/right turn only) at these nodes are decisive in estimating the intention. Route pattern were estimated for vehicle [12] and cyclists [37] with probabilistic models. Also, probabilistic models for route patterns of vehicles at intersections were also proposed along with other motion patterns [35] [10] [14] [36].

2) Pass-yield pattern: Pass-yield pattern depicts which entity occupies the conflict region first when the potential routes of two entities have an overlap (conflict). Similar concept was also used in cooperative driving as homotopy class [38]. Such pattern was predicted for ramp merging [15], pedestrian yielding [35], oncoming vehicle yielding for left turn [10], four-way-stop right-of-way [36], gap selection for lane change [39] etc. The patterns such as pass or stop during yellow light, red-light violation or stop [40], and stop or violate the stop sign [41] [42] can also be categorized as pass-yield pattern.

3) Subtle pattern: Subtle pattern denotes the motion patterns which cannot be explicitly categorized via spatial road structure or conflict region for two entities. For example, there can be several motion patterns for vehicles near stop signs such as conservative/normal stop and rolling/moderate/severe violation [41], which can be clustered via unsupervised learning. The longitudinal motions of vehicles can be simplified as acceleration, deceleration and expected behavior pattern [27]. In [10], the motion of the predicted vehicle impacted by the proceeding one was simplified as free drive and influenced, and the left-turn motion yielding the oncoming vehicle was simplified as full stop and slow down.

C. Distinctions between pattern and desire

In literatures, terminologies such as "intention", "maneuver" and "behavior" are often used for motion clusters. However, the meaning of motion clusters can be twofold. One is "motion pattern", which clusters the motions executed by the entities, and it can be observed or labeled in the motion data to serve as the ground truth. The other is "motion desire", which is the motivation inside the mind of humans, and it cannot be fully observed or labeled in the motion data. In fact, the desire in the human minds can rapidly change from time to time with the dynamic behavior of surroundings.

In previous works, using the terminologies such as "intention" is ambiguous to express the distinctions between pattern and desire. It is acceptable to mix pattern and desire for route since the final desire of the entity related to route is typically not influenced by surroundings so that the desire in the data can be time-invariant. However, for pass-yield and subtle patterns, the corresponding desires may jump from one to another rapidly, and the surroundings may restrict the predicted entity to achieve its desire, especially in highly interactive scenarios. Therefore, the ground truth cannot be labeled for such desires in the data, and only motion patterns can be labeled.

D. Spatiotemporal indicators

Motion patterns can be incorporated in the spatiotemporal domain by two kinds of indicators, namely, prototype trajectory and reachable set. The spatiotemporal domain can also be represented without considering any motion patterns and semantic meaning by using occupancy grid.

1) Prototype trajectory: The terminology "prototype trajectory" was defined in [1] for one or a set of trajectories which can represent a motion pattern. It was employed in [36] [43] [16] to represent possible motions.

2) Reachable set: Reachable set is a widely used motion representation describing the area which may be occupied by the body of an entity. Stochastic reachable set [17] can represent the probabilistic prediction and take into account information from motion patterns. It is also more direct to indicate drivable area for the planning module.

3) Occupancy grid: Occupancy grid divides the spatial domain [44] [45], or other domains of motion variables such as velocity [45] [14], into discrete grids evenly. It can provide a uniform representation of the environment regardless of the number of entities in the scene, which is favorable for learning models such as deep neural networks [44] [45], and for complex scenarios with a number of entities interacting with each other.

IV. Evaluation metrics

In many recent works, probabilistic prediction was regarded as a pure machine learning problem, that is, to find a learning model which can best represent the distribution in the dataset in the sense of some training metrics. However, distribution learned from data alone for prediction is not sufficient to support high-quality and safe decision-making and planning. In this section, the most commonly used learning metrics are summarized. Also, we discuss potential metrics based on prior knowledge on planning constraints, and aspects of metric construction which are crucial for decision-making and planning.

A. Learning metrics

Typical classification and regression metrics are widely used to evaluate the performance of learning models for probabilistic prediction. Despite the most intuitive metric, accuracy, the following metrics are also widely used for evaluation of probabilistic predictions.

1) Receiver operating characteristic (ROC) curve: ROC curve is typically used to illustrate the performance of a binary classifier with different thresholds of discrimination. The area under the curve (AUC) is often used as a quantitive metric. ROC and AUC were employed to evaluate the classification performance of intention estimation, such lane change [46] and intersection maneuvers [10] [35], as well as situation prediction [27].

2) Root mean square error (RMSE): RMSE is one of the most widely used metric for regression evaluation. Some variations in different literatures evaluate the performance from similar perspective, such as root-weighted square error (RWSE) [19] [11] [13], mean absolute error (MAE) [46] [44], Mean Error (ME) [37]. RMSE is mostly used to represent the errors of continuous motions between the (sampled) prediction and ground truth, such as acceleration [30] [47], velocity [19] [11] [13] [14], as well as position and distance [46] [13] [14]. A special implementation of RMSE is for evaluating the error of discretized probability weighted by the corresponding occupancy grid distance [44].

3) Likelihood: Likelihood is commonly used metric for training probabilistic models [19] [11] [44] [15] [48] [33], although it is not used as widely to evaluate the performance of the trained models since the likelihood values are not well interpretable. Likelihood is used as an evaluation metric in [37] [13] [12].

4) Kullback-Leibler (KL) divergence: KL divergence is a widely-used metric in machine learning field to measure discrepancies between two probability distributions. It is used as an evaluation metric for probabilistic prediction in [19] [13]. KL divergence is also employed in [49] to assess the similarity of data observation and provide the value of additional motion data.

B. Prior metrics

The decision-making and planning module of the host vehicle need to take into account feasibility constraints according to vehicle kinematics and dynamics, as well as safety constraints for collision avoidance and hard traffic rules. The predicted motions have similar requirements as planned motions, although the requirements are not as strict. A small perturbation of a trajectory may change it from a safe and feasible one to an unsafe or infeasible one, especially when the trajectory is near the constraint boundary. However, such perturbation may not make a large difference on learning metrics or other distance-based metrics. Therefore, in order to achieve a comprehensive evaluation of predicted motion distribution, prior metrics based on planning constraints should also be considered, which is rarely mentioned in existing works.

1) Feasibility violation: Some prediction methods or frameworks can inherently guarantee feasibility. For instance, [50] incorporated Rapidly-exploring Random Tree (RRT) into the prediction frameworks so that the generated motions can be dynamically feasible. It is an ideal case if the generated motions are sampled from obtained distributions via approaches with feasibility guarantee. Otherwise, violation verification and evaluation are necessary, except for rare cases when one can recognize a driver completely losing control of the vehicle.

2) Safety violation: The requirements on safety are different for prediction and planning. The planned motions for the host vehicle need to consider safety as the top priority, and try its best to guarantee safety if possible. On the contrary, the most important aspect for prediction is to be human-like. Violations to hard traffic rule (red light, stop sign, etc.) and collisions between road users are not common in realistic human driving, but is not extremely rare. Therefore, it is not appropriate to forbid all unsafe motions in predictions, but some obviously unrealistic collisions or violations should not be generated. Values of negative distance headway and negative speed were employed in [19] to evaluate unrealistic motions generated. Collision rate and average distances between the host and the closet merging cars are used as the metric for safety violation and margin of the predicted motions in [15].

C. Decision-related metrics

The learning and prior metrics can help us evaluate how human-like the generated prediction is. There are still several aspects to be considered to evaluate predictions according to the consequences when the prediction is adopted by decision-making and planning modules.

1) Fatality: The consequential fatalities of different inaccurate or incorrect predictions can be completely different when they are adopted by the decision and planning module. The inaccuracy and incorrectness should not be treated equally in the metric. Fatality and criticality differences should be revealed.

2) Defensiveness and conservatism: It is crucial for the safety and driving quality of the host autonomous vehicle to survive in the worst case (defensive driving strategy), yet not overreact to threats of low or zero probability (non-conservative strategy) [42]. In order to achieve such a non-conservatively defensive driving strategy, the decision and planning module requires the prediction module to acquire the following two capabilities. One is to enhance *defensiveness*, namely, to provide all possible future motions (completeness), including possible violations to the traffic rules, and possible careless or dangerous behaviors, so that the host vehicle can drive defensively to potential threats. On the other hand, *conservatism* should be reduced, that is, to provide zero probability (or a safe to pass indicator) when the corresponding motion is impossible, so that the host vehicle can proceed without hesitation. Similar evaluations were also [28] as *miss detection rate* corresponding to *defensiveness* and *false alarm rate* corresponding to *conservatism*.

3) Timeliness: Decision and planning module expect correct and accurate predictions to be provided timely, especially for those causing potential fatal accidents. Correct and accurate predictions are meaningless if such motion has already been completed or an accident is inevitable. Therefore, time-to-event (TTE) [37] and time-to-intersections (TTI) [10] variables were used as the horizontal axis with learning metrics as the vertical axis, so that the timeliness of accurate predictions can be revealed. Distance-to-event variables were also employed in [36], in which specific quantile, mean and standard deviation of distance until correct classification (DCC) were used as metrics. [28] also addressed timeliness from threat estimation perspective as time-to-collision when dangerous situation is correctly classified.

4) Preview horizon: The length of the preview horizon is crucial for a motion planning module. Long-term horizon is preferred for safety and driving quality. Accordingly, the predicted motion should have the same horizon as required by the planning module. Therefore, the ideal case is to provide the distribution of the predicted motion within a long-term horizon, and compare the accuracy over the horizon length, as was illustrated in [3] [46].

5) Computational cost: When a probabilistic method serves as an online prediction module, the host vehicle requires it to provide results in real time. The computational cost should be considered as a metric, especially when the size of the model is relatively large. Computational time was provided in a few recent works [51] for comparison.

V. Conclusion

In this paper three under-explored aspects of probabilistic prediction were highlighted for on-road autonomous driving from decision and planning perspective. A systematic and unified framework for the analysis of prediction problems and methods was provided. Practical input and output variations for decision and planning, such as local and blind view, reactive prediction, and situation prediction were addressed for problem formulation. Indicators for motion representation simplification were summarized as continuous motions, motion patterns and spatiotemporal indicators. The distinctions between pattern and desire were clarified based on the existence of ground truth. In addition to reviewing learning metrics which were commonly used for model evaluation, we also recommended omitted but crucial metrics to be considered according to the prior knowledge and requirements from the decision and planning module. It was emphasized that the predicted motion should avoid feasibility and safety violations, and decision-related factors should be taken into account, such as fatality, defensiveness and conservatism, timeliness and horizon, as well as computational cost.

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