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Automated Vehicle Attitude and Lateral Velocity Estimation Using A 6-D IMU Aided by Vehicle Dynamics

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Abstract— **In this paper, an estimation method for attitude and lateral velocity for automated vehicle has been proposed using a six degree of freedom inertial measurement unit(IMU) aided by vehicle dynamics. This estimation method makes full use of the advantage of the IMU and vehicle dynamics and could run autonomously without aid from other outside information such as GNSS or camera. Based on Kalman filter theory, three observers have been developed: a kinematic model based attitude observer for pitch angle and roll angle using IMU, a kinematic model based lateral velocity observer for lateral velocity using IMU and a dynamic model based observer using vehicle dynamics. In small excitation condition, the estimated lateral velocity from dynamic model based observer is more reliable and it is forwarded to the two kinematic model based observers to prevent the accumulated estimation error. In larger excitation condition, the two kinematic model based observers run in open-loop mode. Slalom maneuver and double lane change(DLC) maneuvers have been conducted to validate the estimation method. The experiment results have proved the effectiveness of this estimation method.**

I. INTRODUCTION

Automated driving technology has aroused much attention recently. Implementing high level automated driving technology to on-road vehicle needs to address many cutting-edge issues. Among those, accurate attitude and lateral velocity are with high significance [1]. For example, image processing or feature recognition could be aided by external pitch and roll angle of vehicle body for vision technology. Also attitude and velocity are the prerequisite for determining vehicle location [2][3]. From vehicle dynamics control perspective, lateral velocity whose estimation methods researched more than 20 years is the basis for vehicle steering behavior control [4].

Unfortunately, the devices like RT3000 from OxTS or S-Motion from Kistler which can directly measure vehicle attitude and lateral velocity is too expensive to be used for commercial vehicle. The more feasible ways to get them indirectly are estimation methods by fusing the information from different sensors in intelligent vehicle. Usually, the sensors include lidar, radar, camera, IMU and so on. Naturally, integrating the angular rate and acceleration sensor to attitude and velocity is an approach. However, angular rate and acceleration sensors are contaminated by unstable bias and long-term accumulation error when automotive grade MEMS (Micro-Electro-Mechanical System) IMU is used [5]. Thus integrating MEMS IMU to estimate attitude and lateral velocity should be conducted with help from other sensors such as GNSS or camera [6][7]. However, there are also some drawbacks when aiding IMU with sensors like GNSS or camera. GNSS signal is weak and easy to be blocked and suffers from multi-path problem. Camera performance relies on the light condition. In dynamic situation such as emergency steering maneuver, it is difficult to track the features all the time. In addition, delay and low sample rate issues come out when GNSS and camera are used. Much care should be paid when implementing GNSS and camera to estimate attitude and lateral velocity [8]. Besides GNSS and camera, there are some other standard sensors like steering wheel angle and its speed sensor and wheel speed sensor in vehicle. Those sensors could be input to an observer based on vehicle dynamics for lateral velocity estimation [9][10]. Using this estimated velocity, the acceleration part due to translation movement could be removed and then the residual part due to gravity could be used to calculate attitude [11]. However, in critical maneuver, the performance of dynamic model based observers would degrade due to model discrepancy.

In this paper, we made full use of 6-D IMU and vehicle dynamics to estimate vehicle attitude and lateral velocity simultaneously. Based on the tri-axial gyroscope and tri-axial accelerometers, we developed a kinematic model based observer to estimate attitude including pitch and roll angle and lateral velocity simultaneously. In the meantime, we also developed a dynamic model based observer to estimate lateral velocity. In small lateral excitation maneuver, this dynamic model based observer could give accurate lateral velocity. Since both the kinematic model based observer and dynamic model based observer could output lateral velocity, on the one hand, the difference between them could be forwarded to kinematic model based velocity observer to compensate the accumulation error in small lateral excitation maneuver. On the other hand, the lateral velocity from dynamic model based observer was used to remove translation acceleration from the lateral accelerometer, and the residual gravity part was used to calculate roll angle. Then this calculated roll angle was forwarded to the kinematic model based attitude method. In critical maneuver, this feedback was cut off. The attitude and lateral velocity were estimated by open loop integration. Thanks to the rapid development of MEMS technology, full-dimension short term integration under critical maneuver would not generate large accumulated error which has been validated by experiments under Slalom and DLC maneuvers.

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The remainder of this paper is organized as follows: Section II introduces the state of the art. Section III explains estimation method design procedure. Section IV shows the experiment results. Finally this paper is concluded in Section V.

II. Related Work

Extensive work has been done to estimate vehicle attitude and lateral velocity. Below we provide a brief review of state of the art. The attitude estimation methods could be divided into IMU-based methods and integration methods. The lateral velocity estimation methods could be divided into kinematic model based methods and dynamic model based methods.

A. Attitude estimation

The tri-axial gyroscope was used to estimate attitude in direct cosine matrix and quaternion representation forms [12]. As discussed before, only using high performance and cost gyroscope like ring-laser or fiber optic gyroscope could generate accurate attitude estimation result in a relatively short term. Long term integration without aids from other sensor inevitably leads to huge accumulation error. Some other research used the gravity part due to attitude from tri-axial accelerometer to calculate the pitch and roll angle to aid gyroscope [5][13]. The main challenge to abstract the attitude information from accelerometer is variable acceleration due to translation movement. Threshold methods in [14], fuzzy logic method and adaptive method in [15] were adopted to handle the translation movement problem.

Some other sensor could be used to give accessible measurement of attitude. Magnetometer could measure the direction of the sensor but the magnetometer is easy to be disturbed by surrounding magnetic materials in [16]. Also the pitch angle and roll angle is usually small and the signal to noise ratio is very low for magnetometer. Integrating GNSS to attitude estimation has been researched widely [6][17], the main challenge is that the GNSS signal could be blocked by trees or high buildings which limit the performance of GNSS [18]. In recent years camera has been used to estimate vehicle attitude through computer vision in [19]. Another limitation of GNSS and camera is that the measurement from them is usually delayed and sampled at a low rate which would cause extra measurement error if the delay and low sample problems are not well addressed [8].

B. Lateral velocity estimation

Kinematic model based methods use the kinematic relationship between the sensors such as the inertial measurement unit (IMU), GNSS, or camera to estimate the lateral velocity. In [20], a direct integrator was used to estimate the lateral velocity based on the basic relationship of sideslip angle and lateral acceleration. In [11], a 6 degree of freedom of IMU was used to address the coupling problem between the velocity and the angular rate and then a Luenberger like observer was proposed to estimate the lateral velocity. In [21], an IMU combined with GNSS was chosen to estimate the sensor bias and the lateral velocity. In [22], camera was innovatively introduced to estimate the optical flow and then the lateral velocity could be estimated. However, the accuracy of those kinds of observers depends on the output accuracy of the sensors. However, sensors like IMU suffer from the zero-bias error, temperature drift and random noise when measure the longitudinal, lateral acceleration and yaw rate. Besides that, under critical situation, the roll angle and pitch angle of the vehicle is large enough to create a gravity part to the accelerometer [5]. More care should be taken to remove bias, noise and gravity part when integrating. Otherwise huge estimation drift may arise after long term integration.

Besides those kinds of methods, the basic principle for dynamic model based methods is using the measurable input signal such as the steering wheel angle, driving torque or braking torque exerted on the actuators to drive a virtual vehicle dynamic model to generate the lateral velocity and then use the measurable output of the actual vehicle such as yaw rate and lateral acceleration as feedback term to correct the virtual lateral velocity, ie the estimated lateral velocity. Since different vehicle models, tire models or estimation methods could be adopted to address different problems when estimating sideslip angle, a lot of methods have arisen[9][10]. Such as in [23], a three freedom vehicle model and magic formula tire model was used to describe the vehicle and tire dynamics. Then a unscented Kalman filter(UKF) was applied to estimate the sideslip angle. In [9], based on the two track vehicle model with 3 degree and magic formula tire model, a variable structure EKF method was designed to estimate the sideslip angle. Considering the road friction, a high gain observer was first introduced to estimate the sideslip angle with small calculation load based on a single track vehicle model [24]. However it is hard to estimate due to the nonlinear and uncertain vehicle and tire dynamics [25].The accuracy of those kinds of observers rely on the accuracy of vehicle dynamic model. Model discrepancy will result in estimation error relatively. Some researchers have combined the kinematic model based method and dynamic model based method to make full use of the merits of each kind of method [4][7][26].

III. Attitude and Lateral velocity estimation

A. Process model

a) Sensor model

We analyzed the gyroscope and acceleration sensor through Allan variance method to determine the error composition in IMU sensor [27]. The gyroscope or accelerometer measurement was composed of real value, constant bias term b_0, random walk bias term b_1 and wide band noise term w. A first order Markov model could be used to show the random walk bias. τ is the time constant and w_b is the wide band noise. The gyroscope model is given by (1)~(2).

$$\begin{aligned} \dot{\phi}_s &= \dot{\phi}_r + b_{\varphi 0} + b_{\varphi 1} + w_{\phi} \\ \dot{\theta}_s &= \dot{\theta}_r + b_{\theta 0} + b_{\theta 1} + w_{\theta} \\ \dot{\varphi}_s &= \dot{\varphi}_r + b_{\varphi 0} + b_{\varphi 1} + w_{\varphi} \end{aligned} \tag{1}$$

$$\begin{cases} \dot{b}_{\phi 1} = -\dfrac{1}{\tau_{\phi 1}} b_{\phi 1} + w_{b_{\phi 1}} \\ \dot{b}_{\theta 1} = -\dfrac{1}{\tau_{\theta 1}} b_{\theta 1} + w_{b_{\theta 1}} \\ \dot{b}_{\varphi 1} = -\dfrac{1}{\tau_{\varphi 1}} b_{\varphi 1} + w_{b_{\varphi 1}} \end{cases} \tag{2}$$

Accelerometer model is given by (3)~(4).

$$\begin{aligned} a_{x_s} &= a_{x_r} + b_{x0} + b_{x1} + w_{a_x} \\ a_{y_s} &= a_{y_r} + b_{y0} + b_{y1} + w_{a_y} \\ a_{z_s} &= a_{z_r} + b_{z0} + b_{z1} + w_{a_z} \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{b}_{x1} &= -\frac{1}{\tau_{x1}} b_{x1} + w_{b_{x1}} \\ \dot{b}_{y1} &= -\frac{1}{\tau_{y1}} b_{y1} + w_{b_{y1}} \\ \dot{b}_{z1} &= -\frac{1}{\tau_{z1}} b_{z1} + w_{b_{z1}} \end{aligned} \tag{4}$$

Where the subscript s means the measurement of sensor, the superscript • means derivative of the variable, ϕ, θ and φ are roll , pitch, and yaw angle, a_x, a_y and a_z are longitudinal, lateral and vertical acceleration respectively.

b) Euler angle and velocity dynamics [11]

In this paper, we choose Euler angle to represent the attitude of the vehicle body. The rotation sequence is Z-Y-X. Rotating about each angle, we have yaw, pitch and roll angle respectively. Then the dynamics of Euler angle is given by (5).

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi}_r \\ \dot{\theta}_r \\ \dot{\varphi}_r \end{bmatrix} \tag{5}$$

The dynamics of the velocity in vehicle body frame is given by (6). v_x, v_y and v_z are longitudinal, lateral and vertical velocity respectively. g is gravity acceleration.

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} a_{x_r} \\ a_{y_r} \\ a_{z_r} \end{bmatrix} - \begin{bmatrix} 0 & -\dot{\varphi}_r & \dot{\theta}_r \\ \dot{\varphi}_r & 0 & -\dot{\phi}_r \\ -\dot{\theta}_r & \dot{\phi}_r & 0 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} - \begin{bmatrix} -g\sin\theta \\ g\sin\phi\cos\theta \\ g\cos\phi\cos\theta \end{bmatrix} \tag{6}$$

c) Tire model

Estimation algorithm's performance depends on the precision of dynamic model, especially the tire model. In order to obtain the nonlinear lateral force, the brush tire model is used in this paper. It is assumed that there is no longitudinal force. Then the formula of brush tire model is shown by (7).

$$F_y = \begin{cases} -\mathrm{sgn}(\alpha) \bullet 3\mu F_z \dfrac{c_1|\alpha|}{3\mu F_z}\left[1 - \dfrac{c_1|\alpha|}{3\mu F_z} + \dfrac{1}{3}\left(\dfrac{c_1|\alpha|}{3\mu F_z}\right)^2\right] & ;|\alpha| < \dfrac{3\mu F_z}{c_1} \\ -\mathrm{sgn}(\alpha) \bullet \mu F_z & |\alpha| \geq \dfrac{3\mu F_z}{c_1} \end{cases} \tag{7}$$

Where μ is the maximum road friction coefficient; α is the tire slip angle; F_z is the tire normal force; F_y is the lateral force; d is the length of tire contact; c_1 is the cornering stiffness of lateral force.

d) Vehicle dynamics

$$\begin{cases} \dot{v}_y = f_1(x,u) = \dfrac{(F_{yfl} + F_{yfr}) \cdot \cos\delta + (F_{yrl} + F_{yrr})}{m} - v_x \cdot \gamma \\ \dot{\gamma} = f_2(x,u) \\ \quad = \dfrac{l_{f0} \cdot (F_{yfl} + F_{yfr}) \cdot \cos\delta - l_{ro}(F_{yrl} + F_{yrr})}{I_z} + \dfrac{N_z}{I_z} \end{cases} \tag{8}$$

Equation.(8) gives the vehicle dynamics equation of traditional single track vehicle model where γ is the yaw rate, F_{yf} and F_{yr} are the lateral force on front and rear axle, N_z is the yaw moment caused by longitudinal tire force , m is the total mass, l_f and l_r is the distance between the acting point of lateral force of COG(center of gravity) for front and rear axle, I_z is moment inertia and $x=[v_y,\gamma]^T$ is the state variable.

In order to calculate tire force, the slip angle could be calculated by (9).

$$\alpha = \arctan(\frac{v_y}{v_x}) \tag{9}$$

The normal load could be estimated according to [28].

B. Measurement model

As statement before, three observers have been developed in this paper. For the two kinematic model based observers, in small lateral excitation maneuver, the translation acceleration part in a_{y_s} was removed using the lateral velocity from the dynamic model based observer. Since in this paper we focus on the attitude and lateral velocity estimation, as for the longitudinal velocity, we assume under small longitudinal acceleration, the longitudinal velocity could be obtained from wheel speed sensor. Then we propose the flow charts as Fig.1 and Fig.2 to detect the exact time when the lateral velocity from dynamic model based observer could be used for roll angle feedback and the kinematic model based lateral velocity estimation, and when the longitudinal velocity from wheel speed sensor could be used for pitch angle and longitudinal velocity feedback for the kinematic model based observer according (6).

Fig.1 gives the roll angle and pitch angle feedback mechanism and Fig.2 gives longitudinal and lateral velocity feedback mechanism.

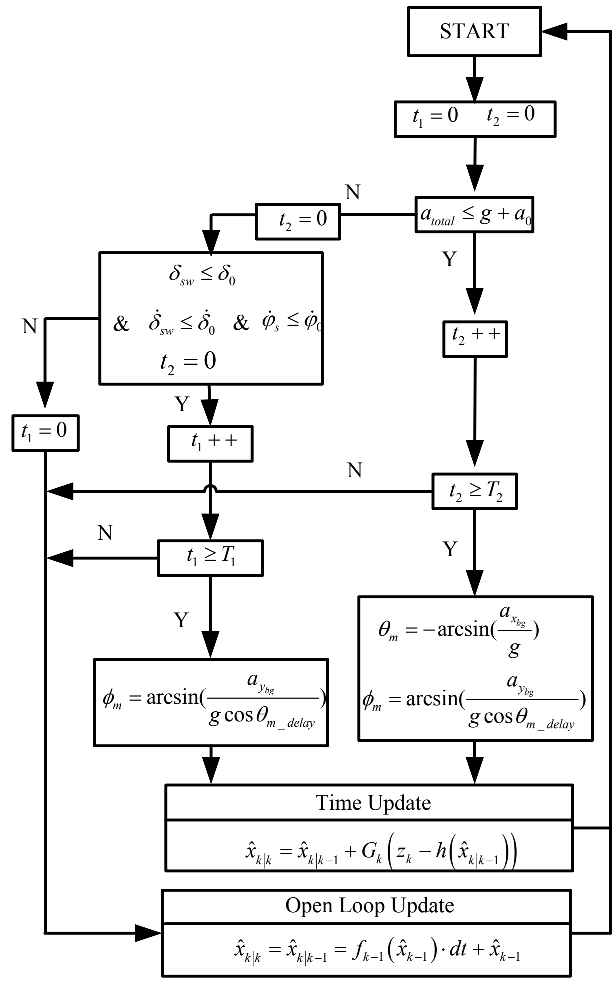


Fig.1 Feedback mechanism of attitude estimation

Where t_1 and t_2 are the accumulated time variable, T_1 and T_2 are the threshold time for judge if the vehicle is under small excitation maneuver, a_0 is the threshold for total acceleration, δ_{sw} is steering wheel angle, δ_0 is the threshold for steering wheel angle, $\dot{\varphi}_0$ is yaw rate threshold, $a_{x_{bg}}$ and $a_{y_{bg}}$ is the acceleration caused by gravity in longitudinal and lateral direction which could be obtained according to (6).

Since later we will use extended Kalman filter and Kalman filter for attitude and lateral velocity estimation respectively, we can see from Fig.1 and Fig.2, when the vehicle is under small excitation maneuver, the innovation term $z_k - h\left(\hat{x}_{k|k-1}\right)$ could be used for time update process and when the vehicle is under large excitation maneuver, the feedback is cut off.

As for dynamic model based observer, it works all the time. The measurement variable of this observer is yaw rate which is the feedback term.

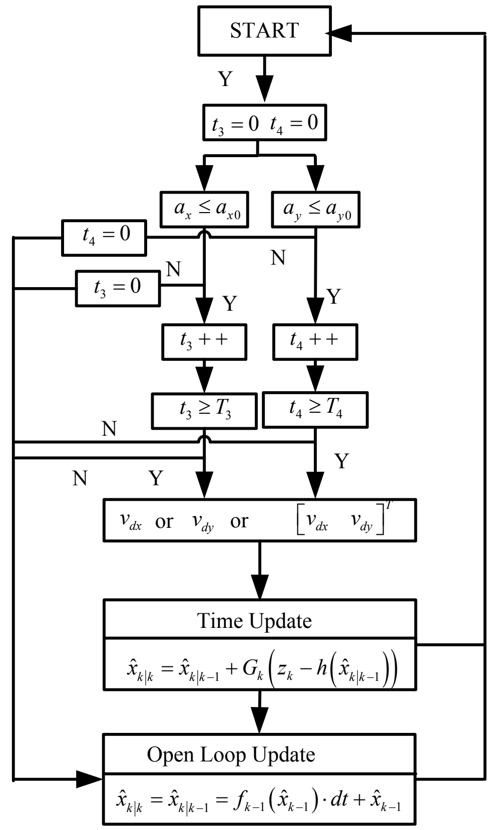


Fig.2 Feedback mechanism of velocity estimation

Where t_3 and t_4 are the accumulated time variable, T_3 and T_4 are the threshold time, a_{x0} and a_{y0} are threshold for longitudinal and lateral acceleration.

C. Estimation method design

In this paper, we applied extended Kalman filter and Kalman filter in three parts. Since the dynamics of Euler angle given by (5) is nonlinear, we used the extended Kalman filter for attitude estimation. The dynamics for velocity of the kinematic model based observer is linear, given by (6), then the Kalman filter was used for velocity estimation. As for the vehicle dynamic model based observer for lateral velocity, we used a extended Kalman filter again to dealing with the nonlinear characteristics of the tire model. Details about the filter design could be referred to [29].

The structure of the three parts is shown by Fig.4. The three blue modules represent the three observers. The green modules represent the sensor modules. The two yellow modules represent the feedback mechanism of Fig.1 and Fig.2.

For the attitude estimation, based on (1), (2) and (5), we applied extended Kalman filter for the nonlinear problem in equation (5) and noise problem in (1)(2). The state variable here we used contained $\left[\phi \quad \theta \quad \varphi \quad b_{\phi 1} \quad b_{\theta 1} \quad b_{\varphi 1}\right]$ and the measurement variable is ϕ_m or ϕ_m and θ_m together according Fig.1.

For the kinematic model based lateral velocity estimation, based on (3), (4) and (6), we applied Kalman filter since (3) is linear system. The state variable here we used contained $\begin{bmatrix} v_x & v_y & v_z & b_{x1} & b_{y1} & b_{z1} \end{bmatrix}$ and the measurement variable is v_{dy}, v_{dx} or v_{dx} and v_{dy} together from the dynamic model based observer according Fig.2.

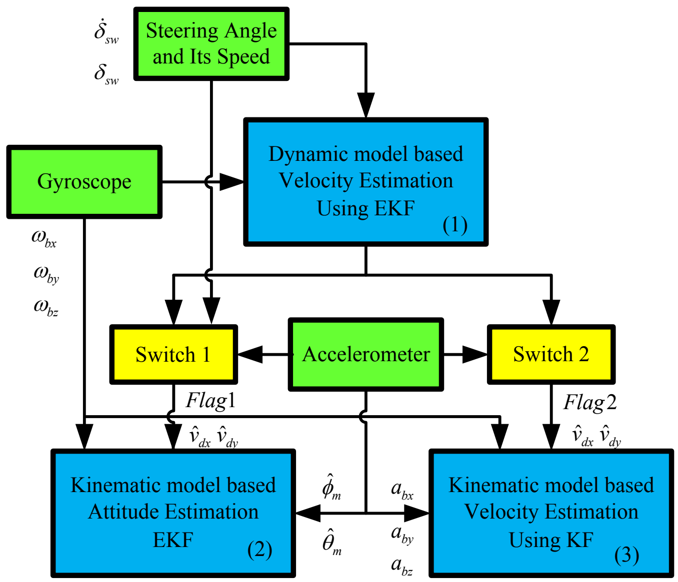


Fig.3 The structure of the estimation method in this paper

For the dynamic model based lateral velocity estimation, based on (7), (8) and (9), we applied extended Kalman filter since the tire model is nonlinear system. The state variable here we used containing $\begin{bmatrix} v_y & \gamma \end{bmatrix}$ and the measurement variable was γ.

IV. VEHICLE IMPLEMENTATION

A. Hardware configuration

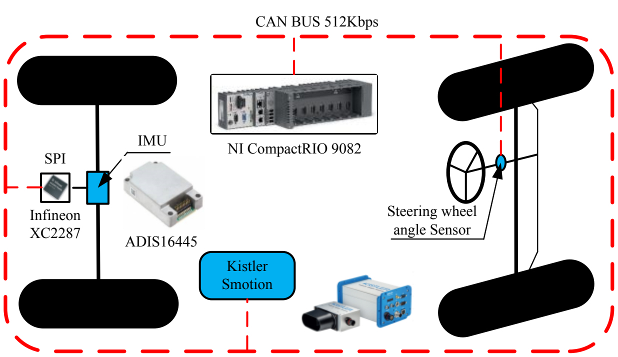


Fig.4 Hardware configuration

The hardware configuration is shown by Fig.4. The ADIS16445 is a compact IMU with tri-axial gyroscope and tri-axial accelerometer from Analog Devices used by the estimation method proposed in this paper. The XC2287 micro-control unit from Infineon is used to read data from ADIS16445 through SPI communication protocol and then transmit it to CAN with 500Kbps. The S-Motion from Kistler is used to provide the reference signal of roll angle, pitch angle, and lateral velocity. The CompactRIO from NI is used for data acquisition. The real hardware implementation is shown by Fig.5.

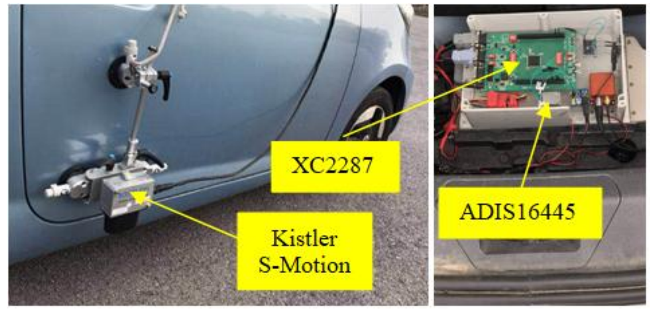
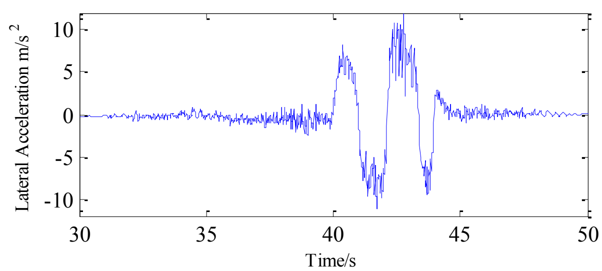


Fig.5 Hardware Implementation

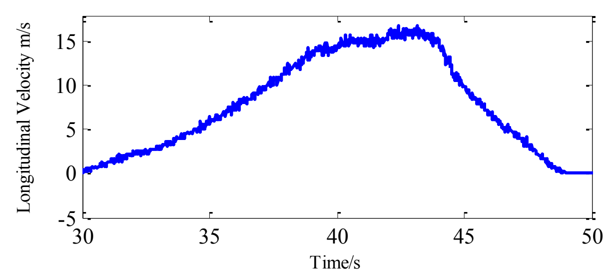
B. Experiment results

In order to validate the estimation method, we have conducted experiments under slalom and double lane change maneuvers at nearly 50~60km/h. The estimation results are given and discussed below.

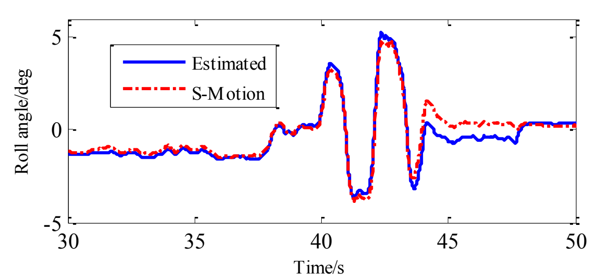
a) Slalom maneuver



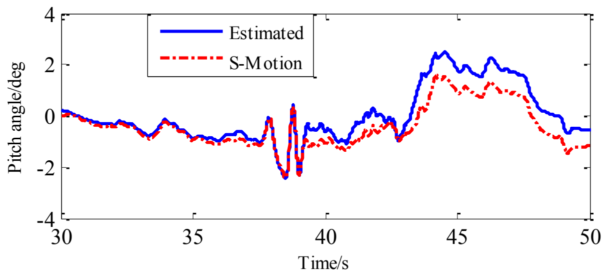
(a) Lateral Acceleration



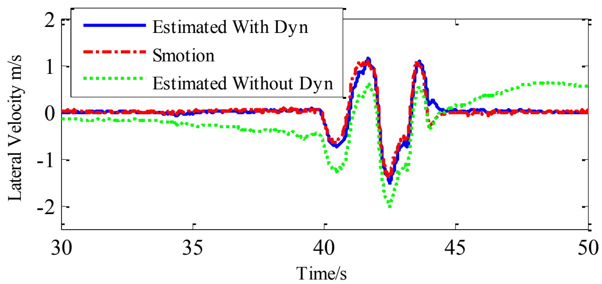
(b) Longitudinal Velocity



(c) Roll angle



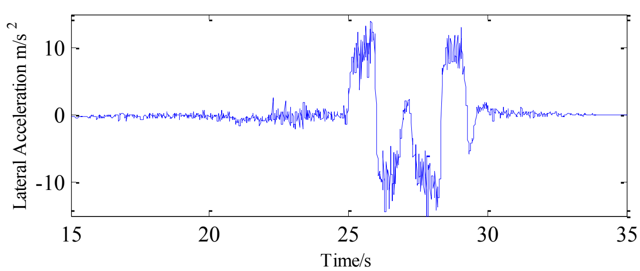
(d) Pitch angle



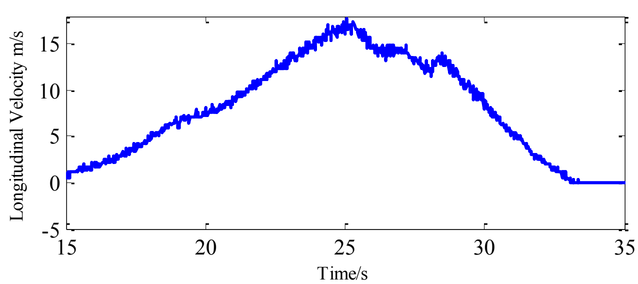
(e) Lateral Velocity

Fig.6 Slalom Experiment Results

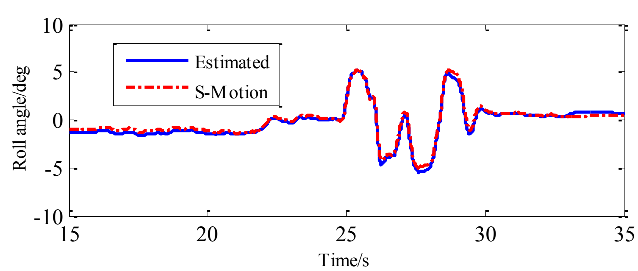
b) DLC maneuver



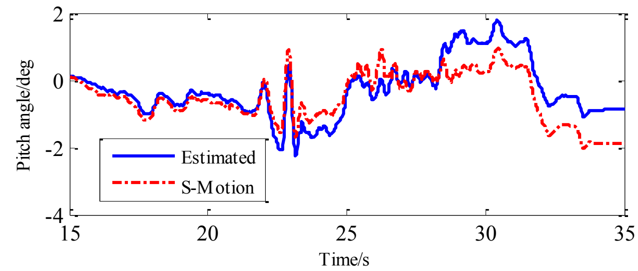
(a) Lateral Acceleration



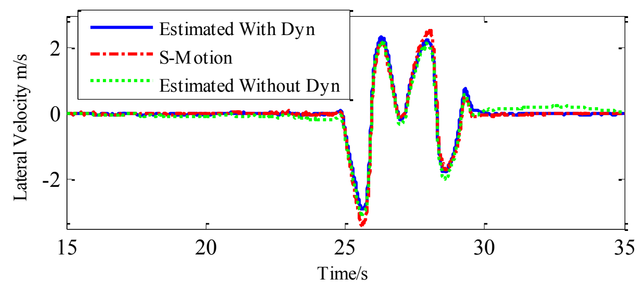
(b) Longitudinal Velocity



(c) Roll angle



(d) Pitch angle



(e) Lateral Velocity

Fig.7 DLC Experiment Results

C. Discussion

The slalom and DLC experiment results are shown by Fig.6 and Fig.7. The longitudinal velocity in both maneuvers is maintained at 50~60km/h by the driver. In both maneuvers, the maximum lateral acceleration reaches about 10m/s^2 which indicates that the vehicle is under critical and highly dynamic maneuver. The estimation method in this paper could still estimate the attitude and lateral velocity well. For example, from Fig.6(c), we can see that the estimated roll angle can track the roll angle from S-Motion well before t=43s. After that instant, the vehicle starts to run in line. However, due to the accumulated error, there is little estimation error smaller than 0.6deg. Thanks for the roll angle feedback mechanism, after t=48s, the estimated roll angle has been pull back to the roll angle from S-Motion. As for the pitch angle, since the absolute value is small and little longitudinal velocity error from wheel speed sensors would cause relatively large pitch angle estimation error, we set the a_0 in a small value. Therefore the feedback to pitch angle lasts shorter than the roll angle feedback and this is why the pitch angle estimation error is larger than roll angle estimation result. On the other hand, since the pitch angle is usually small, from the dynamics we can see that pitch angle appears as the cosine function, the pitch angle estimation error would not contribute a lot to the roll angle estimation result. In addition, from Fig.6.e we know that the blue line tracks the red line better than the green line which indicates the effectiveness for the lateral velocity feedback from dynamic based observer for the kinematic model based observer. The maximum lateral velocity estimation error is smaller than 0.3m/s which showing good performance. The same estimation results could be concluded from the DLC experiment result.

V. CONCLUSION AND FUTURE WORK

In this paper, an estimation method for attitude and lateral velocity has been proposed for automated driving vehicle. This method fuses the information from the 6 DOF integrated IMU and vehicle dynamics and it can run autonomously without aid form extra information. With consideration of the complement characteristics of the IMU and vehicle dynamics, we developed three observers: a kinematic model based observer for attitude estimation, a kinematic model based observer for lateral velocity estimation, and an observer for lateral velocity based on vehicle dynamics. In small excitation, the velocity from the vehicle dynamics based observer is valuable for the two kinematic model based observers and was used as feedback term to correct the kinematic model based attitude and lateral velocity observer. In larger excitation, the kinematic model based observers run autonomously without aids from the vehicle dynamics based observer. This estimation method has been validated through slalom and DLC maneuvers under critical condition and shown a promising effect.

However, since under large condition, the two kinematic model based observer run in open loop without aids from other sensors, the estimation performance depends on the accuracy of IMU and inevitably accumulated error will appear. In future work, we will focus on fusing more information for the short period of the open loop estimation to improve the estimation accuracy further.

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