2018 IEEE Intelligent Vehicles Symposium (IV)
Changshu, Suzhou, China, June 26-30, 2018

# Vehicle Platoon Control with Communication Scheduling

L. Y. Wang, G. Guo, *Member, IEEE*, and W. Yue

*Abstract*—**This paper investigates vehicle platoon control problem in vehicular ad hoc networks with seriously limited communication capacity. By introducing binary sequences as the basis of network access scheduling, the vehicle platoon system is modeled as a discrete-time switching system. Then, based on the new system model, a framework for stability analysis, channel scheduling and controller design is developed. The resulting platoon control and scheduling algorithm can resolve network access conflicts in vehicular ad hoc networks and guarantee exponential stability with $H_\infty$ performance. The effectiveness of the results is demonstrated by numerical simulations.**

## I. Introduction

Inspired by the overwhelming popularity of nowadays wireless local area networks (WLAN, or Wi-Fi), researchers and developers have found their wide application perspectives in a vehicular environment, which spawns a new type of vehicular ad hoc networks (VANETs). The initiative of VANETs is to establish a physical platform for vehicles gathering data sets, including distances, speeds, accelerations, etc., and sharing data sets with other vehicles in the network. Combining the wireless communication provided by a VANET with sensing devices installed in vehicles, vehicular sensor networks serves as an enabling technique for accurate traffic monitoring and efficient vehicle cooperative control that guarantees satisfactorily enhanced mobility, greatly increased traffic capacity, largely reduced adverse environmental effect, and a safer and more comfortable driving experience [1-3].

However, due to the bandwidth constraint of wireless network, only a limited number of vehicles can gain access the channel for transmitting data sets to their following vehicles. If not effectively schedule the channel accessing sequence of vehicles, channel competition may cause some vehicles losing connect with their preceding vehicles for a long time and get off the vehicle platoon in the end.

To address the bandwidth constraint problem of platoon control system, the design of the vehicle controllers should be accompanied by a channel scheduling policy for the vehicles to gain access to the network. This is difficult because the control and scheduling aspects are strongly coupled with each other. Some existing works studied this problem for general networked control systems. Specifically, [4] studied a problem of scheduling a set of plants for remote stabilization via a shared communication channel. For the same problem, [5] introduced binary communication sequences as the basis for channel scheduling policy, [6] investigated how to design the controller and the scheduling scheme jointly; and [7] extended the results in [6] to linear quadratic Gaussian control problems. From a different viewpoint, Liu et al. [8, 9] addressed a problem of scheduling a set of sensors for stability analysis via some existing communication scheduling protocols, such as round robin protocol, try-once-discard protocol and stochastic protocol. Most of the results are very limited in that: the results are only applicable to general linear time-invariant systems, the extension to connected vehicle platoon system is still open and challenging. In vehicle platoon system, the data sets are acquired by two different ways: measured by the sensor directly or transmitted by the network indirectly. Then the control signal cannot be treated as zero simply if the vehicle losing the channel accessing right as [4-9]. Instead, it can still use information measured by the sensor onboard to produce the control signal. Recently, Guo et al [10] investigated a different co-design problem for vehicle platoon control systems whose access to multiple channels is controlled by a random event modeled by a Markov chain. For the same problem, [11] introduced a so-called most regular binary sequence binary communication sequences as the basis for channel scheduling, but the robust performance of vehicle platoon control system has never been investigated.

In this paper, we investigate the channel scheduling and control co-design problem for vehicular platoon in a VANET. We propose a uniform co-design framework for channel scheduling and platoon control that can resolve channel competition, guarantee exponential stability and achieve $H_\infty$ performance at the same time. The channel access status of vehicles is described binary-valued function. When a vehicle is assigned access to a channel, it is authorized to communicate with its preceding vehicle, then it can make use of the information of preceding vehicle to compute its desired control input; otherwise, the vehicle is not accessing any channel and can only use the measurements from on-board sensors. The parameters of the scheduling policy are determined jointly with the vehicle controller gains based on a common set of conditions on schedulability and stability.

The remainder of the paper is organized as follows. In Section 2, we introduce the problem formulation and system models. In Section 3, we provide the sufficient conditions for the exponential stability for a single vehicle system. A scheduling and control co-design procedure is developed to stabilize the vehicle platoon system with a guaranteed $H_\infty$

This work is supported by the National Natural Science Foundation of China under 61703072 and 61673084, and the Fundamental Research Fund for the Central Universities under Grant 3132017128.

L. Y. Wang is with College of Mechanical and Electronic Engineering, Dalian Minzu University, Dalian 116600, China, and also with the Key Laboratory of Intelligent Perception and Advanced Control, State Ethnic Affairs Commission, Dalian 116600, China (e-mail: wangliyuandmu@163.com; zhangrubo@dlnu.edu.cn)

G. Guo and W. Yue are with School of Information Science and Technology, Dalian Maritime University, Dalian 116026, China (e-mail: geguo@yeah.net).

performance for each of them. Section 4 contains the simulation study to demonstrate the effectiveness of the proposed method. Finally, the conclusions and future work are given in Section 5.

*Notation*: Throughout this paper, $R^n$ and $R^{n\times m}$ represent the $n$-dimensional Euclidean space and the set of $n\times m$ real matrices, respectively. Superscript "$T$" represents the transpose. For Hermitian matrices $X = X^T \in R^{n\times n}$ and $Y = Y^T \in R^{n\times n}$, $X > Y$ means that matrix $X - Y$ is positive definite. $\|\cdot\|$ represents the Euclidean norm for a vector.

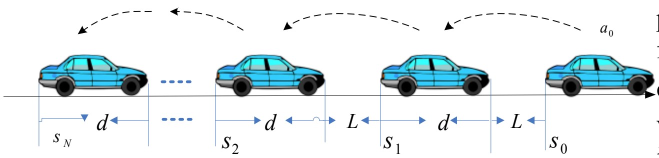

Figure 1. *Platoon of vehicles*

## II. PROBLEM FORMULATION

Consider a platoon of $n+1$ vehicles running on a horizontal road as in Fig. 1. Let $s_i$, $v_i$ and $a_i$, respectively, denote the position, velocity and acceleration of the $i$-th following vehicle ($i = 1,\dots,n$, with $i = 0$ denoting the leading vehicle). Then the dynamics of vehicle $i$ can be written as [12]

$$\begin{aligned} \dot{s}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= a_i(t) \\ \dot{a}_i(t) &= -\frac{1}{\tau}a_i(t) + \frac{1}{\tau}u_i(t) \end{aligned} , \tag{1}$$

Let $e_i = s_{i-1} - s_i - d - L$ and $\dot{e}_i = v_{i-1} - v_i$ denote the spacing error and velocity error between two adjacent vehicles respectively, where $L$ is the desired vehicle spacing and $d$ is the vehicle length. Then the dynamics of vehicle $i$ can be rewritten as

$$\dot{x}_i(t) = \overline{A}x_i(t) + \overline{B}u_i(t) + \overline{C}w_i(t),$$

where $x_i(t) = \begin{bmatrix} e_i(t) \\ \dot{e}_i(t) \\ \ddot{e}_i(t) \end{bmatrix}$, $\overline{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/\tau \end{bmatrix}$, $\overline{B} = \begin{bmatrix} 0 \\ 0 \\ -1/\tau \end{bmatrix}$, $\overline{C} = \begin{bmatrix} 0 \\ 0 \\ 1/\tau \end{bmatrix}$, $w_i(t) = u_{i-1}(t)$, $\tau$ is the time constant of the lag in tracking any desired acceleration command.

After discretization with sampling time $T$ and zero-order hold, the platoon error model can be written as

$$x_i(k+1) = Ax_i(k) + Bu_i(k) + Cw_i(k), \tag{2}$$

where $A = e^{\overline{A}T}$, $B = \int_0^T e^{\overline{A}t}dt\overline{B}$, $C = \int_0^T e^{\overline{A}t}dt\overline{C}$.

At each sample time, the upper layer controller computes the desired acceleration as ($i = 1,\dots,N$)

$$u_i(k) = k_e e_i(k) + k_v\dot{e}_i(k) + k_a\ddot{e}_i(k),$$

where $k_e$, $k_v$, $k_a$ are the controller gains.

Each follower makes use of the spacing error, velocity error and acceleration error to ensure its stability. The spacing error $e_i(k)$ and vehicle error $\dot{e}_i(k)$ are computed by on-board sensors directly but the acceleration error $\ddot{e}_i(k)$ must be calculated using the proceeding vehicle acceleration $a_{i-1}(k)$ (which is received via the wireless network).

It is assumed that the $n$ following vehicles share $m(<n)$ wireless channels to receive the information of their preceding. Without loss of generality, let the binary-valued function $\theta_i(k): R \to \{0,\ 1\}$, $i = 1,2,\dots,N$, denote the channel-access status of vehicle $i$ at time $k$. When $\theta_i(k) = 1$, vehicle $i$ is accessing the wireless channel to receive the information from its preceding vehicle; otherwise, $\theta_i(k) = 0$, vehicle $i$ is not accessing the wireless network, then the controller of vehicle $i$ implements the modified control action that uses only measurements from on-board sensors: $u_i(k) = k_e e_i(k) + k_v\dot{e}_i(k)$. To summarize, each follower uses the following switching logic:

$$u_i(k) = \begin{cases} k_{eo}e_i(k) + k_{vo}\dot{e}_i(k), & \text{if } \theta_i(k) = 0 \\ k_{ec}e_i(k) + k_{vc}\dot{e}_i(k) + k_{ac}\ddot{e}_i(k), & \text{if } \theta_i(k) = 1 \end{cases} \tag{3}$$

Due to bandwidth constraint, the parameter $\theta_i(k)$ should satisfy $\theta_1(k) + \theta_2(k) + \cdots + \theta_N(k) \le m$ at any time $k$. As $\theta_i(k)$ is a binary function, describing network access status, each vehicle system is actually a switching system comprising the following two modes

Mode 1: $x_i(k+1) = (A + BK_oM_o)x_i(k) + Cw_i(k)$, if $\theta_i(k) = 0$, where $K_o = [k_{eo}\quad k_{vo}\quad 0]$, $M_o = diag\{1,1,0\}$.

Mode 2: $x_i(k+1) = (A + BK_c)x_i(k) + Cw_i(k)$, if $\theta_i(k) = 1$, where $K_c = [k_{ec}\quad k_{vc}\quad k_{ac}]$.

Let $\Omega$ be a periodic schedule which assigns values to $\theta_i(k)$, i.e., $\Omega = \{\theta_i(0),\cdots,\theta_i(T-1)\}$. For a given periodic scheduling policy $\Omega$, let $\alpha_o(k)$ and $\alpha_c(k)$ denote the number of sampling periods in which vehicle $i$ operates in mode 1 and mode 2 over time interval [0, $k$), respectively. Then it is easy to know that, $\alpha_c(k) = \Sigma_{l=1}^k \theta_i(l)$ and $\alpha_o(k) = k - \Sigma_{l=1}^k \theta_i(l)$. Denote $\beta(k) = \alpha_c(k)/k$ as the instantaneous attention rate of vehicle $i$ at time $k$. Then due to bandwidth constraint, $\beta(k) \le r/n$ should always be met.

The times of switching of vehicle $i$ between mode 1 and mode 2 over time interval $[0,k)$ is called the *chatter frequency*, which is denoted by $N(k)$ in this paper. In switching system studies (e.g., [13]), *chatter frequency* and *average dwell time* are two important parameters of essential importance for a scheduling rule. For some $N_0 \ge 0$ and $\sigma > 0$, if chatter frequency $N(k) \le N_0 + k/\sigma$, then $\sigma$ is called the average dwell time and $N_0$ is the chatter bound

[13]. Details regarding how these parameters are selected will appear later in the paper.

To give our objective, we need the following definition.

*Definition 1:* System $x(k+1) = f(x(k))$ with $f(0) = 0$ is said to be exponentially stable if there exist two scalars $c > 0$ and $\eta > 1$ such that, for any $x(0) \in R^n$,

$$\| x(k) \|^2 \le c\eta^{-k} \| x(0) \|^2,$$

where $\eta$ is called the decay rate.

*Definition 2:* For given scalars $\gamma > 0$ and $0 < \varsigma < 1$, $x(k+1) = f(x(k))$ is said to be exponential stable with $H_\infty$ performance, if the following conditions are satisfied:

i) The vehicle platoon system (2) with $w_i(k) = 0$ is exponentially stable.

ii) Under zero-initial condition, the controlled vehicle state $x_i(k)$ satisfies

$$\sum_{k=0}^{\infty} \varsigma^k \left\| x_i(k) \right\|^2 < \gamma^2 \sum_{k=0}^{\infty} \left\| w_i(k) \right\|^2,$$

for all nonzero $w_i(k)$, where $\gamma > 0$ is a given scalar.

*The objective* of this paper is to find a co-design framework for a communication scheduling policy and platoon feedback controller such that each following vehicle can keep a desired constant distance with its preceding vehicle. Specifically, our intention is to determine the period scheduling policy $\Omega$ and the controller gains $K_o$, $K_c$ such that the vehicle platoon control system is stabilized and the $H_\infty$ performance constraint is satisfied.

## III. Main Results

In this section, sufficient conditions for exponential stability of the single vehicle system are given first, which are followed by simultaneous stability conditions for the vehicle platoon system. We then give a co-design procedure for the communication scheduling policy and feedback controller.

### A. Single vehicle stability

The following result is concerned with the exponential stability of a single vehicle *i*.

*Theorem 1:* Vehicle *i* is exponentially stable with weighted $H_\infty$ performance, if the following conditions hold:

1) For three given positive scalars $\eta_o > \eta_c$ and $\gamma$, there exist two positive matrices $0 < P_o, P_c \in R^{n\times n}$ such that

$$\begin{bmatrix} R_o + I & (A + BK_o M_o)^T P_o C \\ C^T P_o (A + BK_o M_o) & C^T P_o C - \gamma^2 I \end{bmatrix} < 0, \quad (4)$$

$$\begin{bmatrix} R_c + I & (A + BK_c)^T P_c C \\ C^T P_c (A + BK_c) & C^T P_c C - \gamma^2 I \end{bmatrix} < 0, \quad (5)$$

where $R_o = (A + BK_o M_o)^T P_o (A + BK_o M_o) - \eta_o P_o$, $R_c = (A + BK_c)^T P_c (A + BK_c) - \eta_c P_c$;

2) There exists a constant scalar $\mu > 1$ such that

$$P_o \le \mu P_c, \; P_c \le \mu P_o; \quad (6)$$

3) The attention rate satisfies

$$\beta(k) \ge \frac{2\ln\eta + \ln\eta_o}{\ln\eta_o - \ln\eta_c}; \quad (7)$$

4) The chatter frequency and average dwell time satisfy that

$$N(k) \le N_0 + k/\sigma \text{ and } \sigma \ge \sigma' = \ln\mu / \ln\eta, \quad (8)$$

where $N_0 > 0$ is the chatter bound.

*Proof:* Assume $w_i(k) = 0$, without loss of generality, we assume that vehicle *i* operates in mode 1 during $[k_{2j}, k_{2j+1})$, and in mode 2 during $[k_{2j+1}, k_{2j+2})$, where $j = 0,1,\ldots$, $k_0 = 0$. Namely,

$$\theta_i(k) = \begin{cases} 0, & k \in [k_{2j}, k_{2j+1}) \\ 1, & k \in [k_{2j+1}, k_{2j+2}) \end{cases}. \quad (27)$$

Choose a piecewise quadratic Lyapunov-like function candidate as follows

$$V(k) = \begin{cases} V_o(k) = x_i^T(k) P_o x_i(k), \; \theta_i(k) = 0 \\ V_c(k) = x_i^T(k) P_c x_i(k), \; \theta_i(k) = 1 \end{cases}. \quad (9)$$

Then it holds from (6) that

$$V_o(k) \le \mu V_c(k), \; V_c(k) \le \mu V_o(k). \quad (10)$$

It is obvious that (4) and (5) imply the following condition

$$R_o < 0, \; R_c < 0. \quad (11)$$

Then from (11), it is easy to know that

$$\begin{aligned} & V_o(k+1) - \eta_0 V_o(k) \\ &= x_i^T(k+1) P_o x_i(k+1) - \eta_o x_i^T(k) P_o x_i(k) \\ &= x_i^T(k) R_o x_i(k) < 0. \end{aligned} \quad (12)$$

$$\begin{aligned} & V_c(k+1) - \eta_c V_c(k) \\ &= x_i^T(k+1) P_c x_i(k+1) - \eta_c x_i^T(k) P_c x_i(k) \\ &= x_i^T(k) R_c x_i(k) < 0. \end{aligned} \quad (13)$$

Combine (12) and (13) we have

$$V(k) < \begin{cases} \eta_o^{k-k_{2j}} V(k_{2j}), & \text{if } k \in [k_{2j}, k_{2j+1}) \\ \eta_c^{k-k_{2j+1}} V(k_{2j+1}), & \text{if } k \in [k_{2j+1}, k_{2j+2}) \end{cases}. \quad (14)$$

Therefore, if $k \in [k_{2j+1}, \; k_{2j+2})$, if follows from (10) and (14) that

$$\begin{aligned} V(k) &< \eta_c^{k-k_{2j+1}} V_c(k_{2j+1}) \le \mu\eta_c^{k-k_{2j+1}} V_c(k_{2j+1}^-) \\ &< \mu\eta_c^{k-k_{2j+1}} \eta_o^{k_{2j+1}-k_{2j}} V_o(k_{2j}) \end{aligned}$$

$$\leq \cdots \leq \mu^{N(k)}\eta_c^{\alpha(k)}\eta_o^{k-\alpha(k)}V(0),$$

where $k_{2j+1}^-$ denotes the time instant that is immediately before $k_{2j+1}$.

Similarly, for $k \in [k_{2j}, k_{2j+1})$, we have that

$$V(k) < \mu^{N(k)}\eta_c^{\alpha(k)}\eta_o^{k-\alpha(k)}V(0). \qquad (15)$$

where $\eta$ is derived based on the fact that $\eta = \mu^{\ln\eta/\ln\mu}$. Note that this derivation comprises the following two facts. First, from (7) and $\beta(k) = \alpha(k)/k$, we can have that $(\ln\eta_o - \ln\eta_c)\alpha(k) \geq (2\ln\eta + \ln\eta_o)k$, which is equivalent to

$$\eta_c^{\alpha(k)}\eta_o^{k-\alpha_i(k)} \leq \eta^{-2k}. \qquad (16)$$

Second, from (8), we have

$$\mu^{N(k)} \leq \mu^{N_0+k/\sigma} \leq \mu^{N_0}\mu^{k/\sigma} \leq \mu^{N_0}\mu^{k\ln\eta/\ln\mu} = \mu^{N_0}\eta^k. \qquad (17)$$

In addition, from the quadratic Lyapunov-like function in (9), we can have

$$V(k) \geq \| x_i(k)\|^2 / \max\{\| P_o^{-1}\|, \| P_c^{-1}\|\}, \qquad (18)$$

and

$$V_i(0) \leq \max\{\| P_o\|, \| P_c\|\}\, \| x_i(0)\|^2. \qquad (19)$$

Finally, from (15)-(19), we can obtain that

$$\| x_i(k)\|^2 < c\eta^{-k}\| x_i(0)\|^2,$$

where $c = \mu^{N_0}\max\{\| P_o^{-1}\|, \| P_c^{-1}\|\} \times \max\{\| P_o\|, \| P_c\|\}$. Then, according to Definition 1, we know that vehicle $i$ is exponentially stable with decay rate equal to $\eta$. This completes the proof of Theorem 1.

Next, the performance of the platoon system will be established.

From (4), it is easy to know that

$$V_o(k+1) - \eta_o V_o(k) + x^T(k)x(k) - \gamma w^T(k)w(k)$$

$$= z^T(k)\begin{bmatrix} R_o + I & * \\ C^T P_o(A + BK_oM_o) & C^T P_o C - \gamma^2 I\end{bmatrix} z(k) < 0, \qquad (20)$$

From (5), it is easy to know that

$$V_c(k+1) - \eta_c V_c(k) + x^T(k)x(k) - \gamma w^T(k)w(k)$$

$$= z^T(k)\begin{bmatrix} R_c + I & * \\ C^T P_c(A + BK_c) & C^T P_c C - \gamma^2 I\end{bmatrix} z(k) < 0, \qquad (21)$$

Let $\Gamma(k) = x^T(k)x(k) - \gamma^2 w^T(k)w(k)$, then from (20) and (21), we have

$$V(k+1) < \begin{cases} \eta_o V_o(k) - \Gamma(k), & if\ k \in [k_{2j}, k_{2j+1}) \\ \eta_c V_c(k) - \Gamma(k), & if\ k \in [k_{2j+1}, k_{2j+2}) \end{cases} \qquad (22)$$

If $k \in [k_{2j+1}, \ k_{2j+2})$, if follows from (22) that

$$V(k) < \eta_c^{k-k_{2j+1}}V_c(k_{2j+1}) - \sum_{s=k_{2j+1}}^{k-1}\eta_c^{k-s-1-\alpha_o(s+1,k)}\Gamma(s)$$

$$\leq \mu\eta_c^{k-k_{2j+1}}V_o(k_{2j+1}^-) - \sum_{s=k_{2j+1}}^{k-1}\eta_c^{k-s-1-\alpha_o(s+1,k)}\Gamma(s)$$

$$< \mu\eta_c^{k-k_{2j+1}}\eta_o^{k_{2j+1}-k_{2j}}V_o(k_{2j}) - \sum_{s=k_{2j}}^{k-1}\eta_o^{\alpha_o(s+1,k)}\eta_c^{k-s-1-\alpha_o(s+1,k)}\Gamma(s)$$

$$\leq \cdots \leq \mu^{N(k)}\eta_c^{k-\alpha_o(k)}\eta_o^{\alpha_o(k)}V(0)$$

$$-\sum_{s=0}^{k-1}\mu^{N(s,k)}\eta_o^{\alpha_o(s+1,k)}\eta_c^{k-s-1-\alpha_o(s+1,k)}\Gamma(s) \qquad (23)$$

Under zero initial condition, $V(k) \geq 0$ and (23) imply

$$\sum_{s=0}^{k-1}\mu^{N(s,k)}\eta_o^{\alpha_o(s+1,k)}\eta_c^{k-s-1-\alpha_o(s+1,k)}\Gamma(s) < 0. \qquad (24)$$

Multiplying both sides of (24) by $\mu^{-N(k)}$ we have

$$\sum_{s=0}^{k-1}\mu^{-N(s)}\kappa^{\alpha_o(s+1,k)}\eta_c^{k-s-1}x^T(s)x(s)$$

$$\leq \gamma^2\sum_{s=0}^{k-1}\kappa^{\alpha_o(s+1,k)}\eta_c^{k-s-1}w^T(s)w(s). \qquad (25)$$

where $\kappa = \eta_o/\eta_c > 1$.

Multiplying $\kappa^{-\alpha_o(k)}$ on both sides of (25) yields

$$\sum_{s=0}^{k-1}\mu^{-N(s)}\kappa^{-\alpha_o(s+1)}\eta_c^{k-s-1}x^T(s)x(s) \leq \gamma^2\sum_{s=0}^{k-1}\kappa^{-\alpha_o(s+1)}\eta_c^{k-s-1}w^T(s)w(s)$$

Then it follows from (8) and $\kappa > 1$ that

$$\sum_{s=0}^{k-1}\mu^{-s/\sigma}\kappa^{-s-1}\eta_c^{k-s-1}x^T(s)x(s) \leq \gamma^2\sum_{s=0}^{k-1}\eta_c^{k-s-1}w^T(s)w(s)$$

$$\Leftrightarrow \kappa^{-1}\sum_{s=0}^{k-1}\xi^s\eta_c^{k-s-1}x^T(s)x(s) \leq \gamma^2\sum_{s=0}^{k-1}\eta_c^{k-s-1}w^T(s)w(s)$$

$$\Leftrightarrow \kappa^{-1}\sum_{k=0}^{\infty}\sum_{s=0}^{k-1}\xi^s\eta_c^{k-s-1}x^T(s)x(s) \leq \gamma^2\sum_{k=0}^{\infty}\sum_{s=0}^{k-1}\eta_c^{k-s-1}w^T(s)w(s)$$

$$\Leftrightarrow \kappa^{-1}\sum_{k=s+1}^{\infty}\eta_c^{k-s-1}\sum_{s=0}^{\infty}\xi^s x^T(s)x(s) \leq \gamma^2\sum_{k=s+1}^{\infty}\eta_c^{k-s-1}\sum_{s=0}^{\infty}w^T(s)w(s)$$

$$\Leftrightarrow \sum_{s=0}^{\infty}\xi^s x^T(s)x(s) \leq \kappa\gamma^2\sum_{s=0}^{\infty}w^T(s)w(s) = \gamma_1^2\sum_{s=0}^{\infty}w^T(s)w(s)$$

where $\xi = \mu^{-1/\sigma}\kappa^{-1}$, $\gamma_1 = \gamma\sqrt{\kappa}$.

According to Definition 2, vehicle $i$ has weighted $H_\infty$ performance $\gamma_1$. This ends the proof.

*B. Simultaneous Stability*

A sufficient condition for the exponential stability of the vehicle platoon system is given in the following theorem.

*Theorem 2:* Suppose that conditions 1) and 2) in Theorem 1 hold for each vehicle and the following condition holds

$$\frac{2\ln\eta + \ln\eta_o}{\ln\eta_o - \ln\eta_c} \le \frac{m}{n}, \tag{26}$$

then there exists a period scheduling policy $\Omega$ that guarantees the exponential stabilization of the vehicle platoon with decay rates $\eta$ and $H_\infty$ performance $\gamma_1$.

*Proof:* The scheduling policy is given as below.

*Scheduling policy A:*

1) Choose the scheduling period $T = dh$, where $d = 2\ln\mu / \ln\eta$, $\mu = \lambda_{\max}\{P_o / P_c, P_c / P_o\}$, $h$ is the sampling time.
2) During each scheduling period *T*, let vehicle 1,…, *N*, gains access to the shared wireless channel in order and each works for a time interval of length *fh*, where $f = (2\ln\eta - \ln\eta_o)/(\ln\eta_d - \ln\eta_o)$.

The proof of exponential stability of the vehicle platoon system is similar to Lemma 2 in [13], so we omit the details here.

### C. Controller design

Note that inequality (4) is nonlinear since gain matrix $K_o$ and $P_o$ are all unknown variables. In order to reformulate matrix inequality (4) into a linear matrix inequality that can be easily calculated with engineering software, we introduce a new variable $Q_o$ to replace $P_o^{-1}$ in inequality (4). Then by using Schur complement, we have that

$$\begin{bmatrix} R_o + I & (A + BK_oM_o)^T P_oC \\ C^T P_o(A + BK_oM_o) & C^T P_oC - \gamma^2 I \end{bmatrix}$$

$$= \begin{bmatrix} -\eta_o P_o + I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} (A + BK_oM_o)^T \\ C^T \end{bmatrix} P_o \begin{bmatrix} A + BK_oM_o & C \end{bmatrix}$$

$$= \begin{bmatrix} -\eta_o P_o + I & 0 & (A + BK_oM_o)^T \\ 0 & -\gamma^2 I & C^T \\ A + BK_oM_o & C & -Q_o \end{bmatrix} < 0. \tag{27}$$

Now by using the cone complementarity method [15], the feedback controller gain $K_o$ can be obtained by solving the following minimization problem

*Problem 1:* minimize $Tr(P_oQ_o)$

$$\text{subject to (27) and } \begin{bmatrix} P_o & I \\ I & Q_o \end{bmatrix} \ge 0.$$

Similarly, by introducing a new variable $Q_c$ to replace $P_c^{-1}$ in inequality (5), we have that

$$\begin{bmatrix} R_c + I & (A + BK_c)^T P_cC \\ C^T P_c(A + BK_c) & C^T P_cC - \gamma^2 I \end{bmatrix}$$

$$= \begin{bmatrix} -\eta_c P_c + I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} (A + BK_c)^T \\ C^T \end{bmatrix} P_c \begin{bmatrix} A + BK_c & C \end{bmatrix} < 0$$

$$= \begin{bmatrix} -\eta_c P_c + I & 0 & (A + BK_c)^T \\ 0 & -\gamma^2 I & C^T \\ A + BK_c & C & -Q_c \end{bmatrix} < 0. \tag{28}$$

Now by using the cone complementarity method [14], the feedback controller gains $K_c$ can be obtained by solving the following minimization problem

*Problem 2:* minimize $Tr(P_cQ_c)$

$$\text{subject to (28) and } \begin{bmatrix} P_c & I \\ I & Q_c \end{bmatrix} \ge 0.$$

### D. Scheduling and Control Co-design

By using Theorem 1 and Theorem 2, a scheduling and control co-design algorithm for the platoon vehicles can be obtained, which is given as below.

*Algorithm 1:*

Step 1. Choose initial values for $\eta$, $\gamma$, $\eta_o$ and $\eta_c$.

Step 2. Test the feasibility of (26). If (26) is feasible, go to Step 3. Otherwise, decrease $\eta$, $\eta_o$ or $\eta_c$, and return to Step 2.

Step 3. Find $P_o$, $P_c$, $K_o$, $K_c$ by solving Problem 1 and Problem 2.

Step 4. Choose the scheduling period $T = dh$, where $d = 2\ln\mu / \ln\eta$, $\mu = \lambda_{\max}\{P_o / P_c, P_c / P_o\}$, $h$ is the sampling time.

Step 5. During each scheduling period *T*, let vehicle 1,…, *N*, gains access to the shared wireless channel in order and each works for a time interval of length *fh*, where $f = (2\ln\eta - \ln\eta_o)/(\ln\eta_d - \ln\eta_o)$.

## IV. Simulation

In the simulations, we consider a platoon of 4 vehicles. The three following vehicles share two communication channels. The sampling time and time constant of the lag are given as $T = 0.2s$, $\tau = 0.2s$. The length of vehicle and the desired vehicle spacing are given as $L = 4m$, $d = 1m$. For prescribed $r = 4$, $\eta = 1.02$, $\eta_c = 0.85$ and $\eta_o = 1.25$, it is easy to check by using the Matlab LMI Toolbox that the controller gains $K_o$, $K_c$ and matrices $P_o$, $P_c$ are obtained as follows:

$$K_o = [2.4214 \quad 3.7187 \quad 0.8806],$$

$$K_c = [0.2238 \quad 1.1332 \quad 0].$$

$$P_o = \begin{bmatrix} 3.8585 & 1.7644 & 0.2565 \\ 1.7644 & 4.4596 & 0.3470 \\ 0.2565 & 0.3470 & 1.0256 \end{bmatrix},$$

$$P_c = \begin{bmatrix} 24.7441 & 14.8168 & 1.1232 \\ 14.8166 & 20.6175 & 1.5565 \\ 1.1232 & 1.5565 & 1.3046 \end{bmatrix}.$$

According to Algorithm 1, the channel scheduling policy is given as below

$$\begin{cases} \underbrace{V1 \cdots V1}_{8} & \underbrace{V2 \cdots V2}_{4} \\ \underbrace{V2 \cdots V2}_{4} & \underbrace{V3 \cdots V3}_{8} \end{cases},$$

where $\underbrace{V_i \cdots V_i}_{b}$ denotes vehicle $i$ gains access the channel for a time interval of length $bh$.

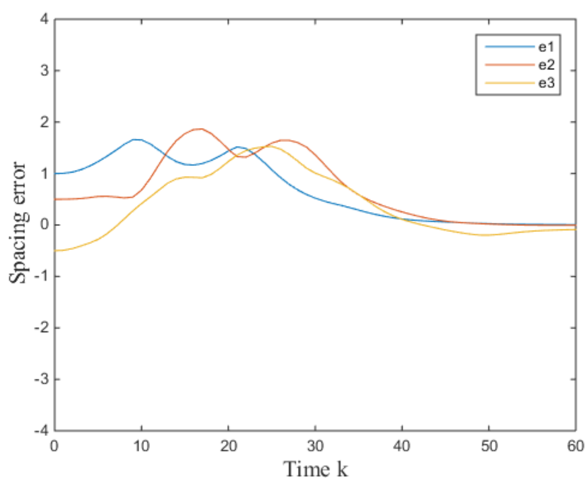

Figure 2. *Spacing error*

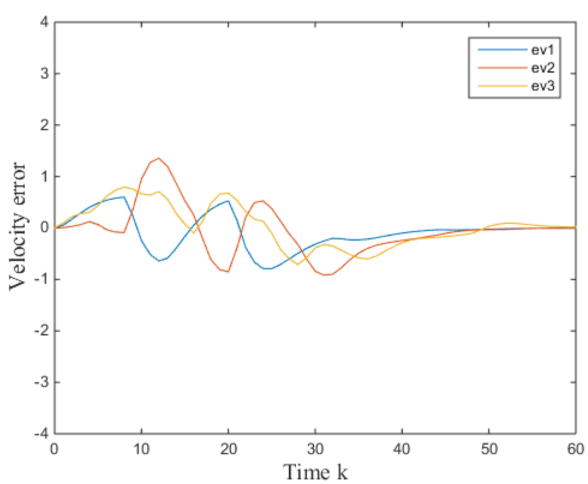

Figure 3. *Velocity error*

Assume that the input of the leading vehicle over time interval $60h$ is given by $[\underbrace{1 \cdots 1}_{20} \quad \underbrace{0 \cdots 0}_{40}]$. The spacing error and velocity error are shown in Figure 2 and Figure 3. From Figure 2 and Figure 3 we can see that the spacing errors increase due to the impact of the input of the leading vehicle during the first $20h$. Then the spacing errors converge to zero with the effect of channel scheduling policy and controller. Therefore, we can conclude that the scheduling policy and controller can guarantee the stability of the three vehicles and eliminate the effect of disturbance.

## V. Conclusion

This paper addressed a joint problem of control, and communication scheduling for stabilization of a vehicle platoon systems sharing a limited bandwidth communication network. A co-design methodology of communication scheduling and control was presented. The methodology can guarantee the stabilization of vehicle platoon system with $H_\infty$ performance. The periodic scheduling policy suggested is easy for implementation.

## References


[1] B. Placzek, "Selective data collection in vehicular networks for traffic control applications," *Transp. Res. C, Emerging Technol.*, vol. 23, pp. 14–28, Aug. 2012.

[2] Y. Toor, P. Muhlethaler, and A. Laouiti, "Vehicle ad hoc networks: Applications and related technical issues," *IEEE Commun. Surveys Tuts.*, vol. 10, no. 3, pp. 74–88, 2008.

[3] F. W. Fu and V. D. S. M, "Structure-aware stochastic control for transmission scheduling," *IEEE Trans. Veh. Technol.*, vol. 61, no. 9, pp. 3931–3945, Nov. 2012.

[4] L. Y. Wang, G. Guo, Y. Zhuang. "Networked control of battery-powered systems with communication scheduling and power allocation," *Int. J. Robust. Nonlinear Control*, vol. 27, pp. 3488-3507, 2017.

[5] H. Rehbinder, M. Sanfridson, "Scheduling of a limited communication channel for optimal control," *Automatica* vol. 40, no. 3, pp. 491–500. 2004.

[6] L. Zhang, D. Hristu-Varsakelis, "Communication and control co-design for networked control systems," *Automatica,* vol. 42, no. 6, pp. 953–958, 2006.

[7] D. Hristu-Varsakelis, L. Zhang L, "LQG control of networked control systems with access constraints and delays," *Int. J. Control*, vol. 81, no. 8, pp. 1266–1280, 2008.

[8] K. Liu, E. Fridman, "Discrete-time network-based control under scheduling and actuator constraints," *Int. J. Robust. Nonlinear Control*, vol. 25, pp. 1816–1830, 2015.

[9] K. Liu, E. Fridman, "Johansson KH. Networked control with stochastic scheduling," *IEEE Trans. Automat. Control*, vol. 60, no. 11, pp. 3071–3076, 2015.

[10] G. Guo, L. Y. Wang, "Control over medium-constrained vehicular networks with fading channels and random access protocol: a networked systems approach," *IEEE Trans. Veh. Technol.*, vol. 64, no. 8, pp. 3347-3358, 2015.

[11] G. Guo, S. X. Wen, "Communication scheduling and control of a platoon of vehicles in VANETS," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 6, pp. 1551-1563, 2016.

[12] W. Yue, L. Y. Wang, G. Guo, "Event-triggered platoon control of vehicles with time-varying delay and probabilistic faults," *Mech. Syst. Signal Pro.*, vol. 87, pp. 96-117, 2017.

[13] S. L. Dai, H. Lin, S. S. Ge, "Scheduling-and-control co-design for a collection of networked control systems with uncertain delays," *IEEE Trans. Contr. Syst. Technol.*, vol. 18, no. 1, pp. 66–78, 2010.

[14] L. E. Ghaoui, L. Oustry, and M. AitRami, "A cone complementarity linearization algorithm for static output feedback and related problems," *IEEE Trans. Autom. Control*, vol. 42, no. 8, pp. 1171–1176, Aug. 1997.