Verification and Application of the Unified Coordinate System in Preliminary Design

Nathan Woods Ryan Starkey

Busemann Advanced Concepts Laboratory
Department of Aerospace Engineering Sciences
University of Colorado at Boulder

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Outline

The Project Cycle

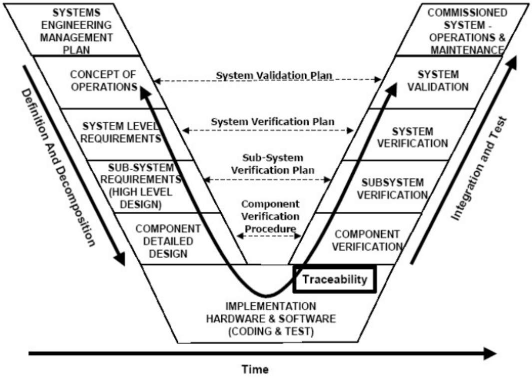


Figure: The life cycle of a project. (Wikimedia Commons)

Wind Tunnels and CFD

Wouldn't it be great if CFD actually worked like a wind tunnel?

- Quick tests for design shapes
- No grid generation

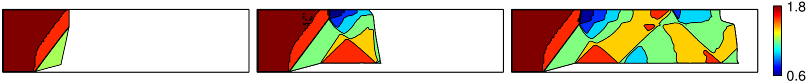


Figure: Mach contours for a transonic duct flow. See also http://www.youtube.com/watch?v=g-a9oGpKiEw

The Virtual Wind Tunnel

- Lagrangian Fluid dynamics
  - Natural analogue to wind tunnel testing
  - Fails due to severe grid distortion
- Unified Coordinates
  - Largely preserves benefits of Lagrangian systems
  - Distortion can be controlled

Original Development of UCS

Developed by W. H. Hui[1] from from Lagrangian coordinates

- The time-dependent Euler equations[2][3]
- Extension to viscous flows[4]
- External flows and oscillating airfoils[5],[6]

Extensions of UCS

Applications of UCS to other systems

- Reactive flows[7]
- Multimaterial flows[8]
- Plasma dynamics[9]
- Gas-kinetic (BGK) aerodynamics[10]

Outline

The Unified Coordinate Transformation

$$\begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ U & A & L & P \\ V & B & M & Q \\ W & C & N & R \end{pmatrix} \begin{pmatrix} d\lambda \\ d\xi \\ d\eta \\ d\zeta \end{pmatrix} \tag{1}$$

Conservation Equations

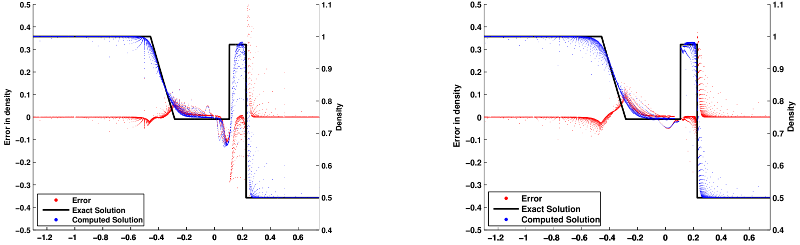
$$\frac{\partial \mathbf{E}}{\partial \lambda} + \frac{\partial \mathbf{F}}{\partial \xi} + \frac{\partial \mathbf{G}}{\partial \eta} = 0 \qquad (2)$$

$$\mathbf{E} = \begin{pmatrix} \rho\Delta \\ \rho\Delta u \\ \rho\Delta v \\ \rho\Delta e \\ A \\ B \\ L \\ M \end{pmatrix}, \mathbf{F} = \begin{pmatrix} \rho\left(1-h\right)I \\ \rho\left(1-h\right)Iu + pM \\ \rho\left(1-h\right)Iv - pL \\ \rho\left(1-h\right)Ie + pI \\ -hu \\ -hv \\ 0 \\ 0 \end{pmatrix}, \mathbf{G} = \begin{pmatrix} \rho\left(1-h\right)J \\ \rho\left(1-h\right)Ju - pB \\ \rho\left(1-h\right)Jv + pA \\ \rho\left(1-h\right)Je + pJ \\ 0 \\ 0 \\ -hu \\ -hv \end{pmatrix} \qquad (3)$$

$$\Delta = AM - BL,\ I = uM - vL,\ J = Av - Bu \qquad (4)$$

Outline

The Riemann Problem



(a) $h = 0$

(b) $h = 0.999$

Figure: The similarity solution of the Riemann problem and the corresponding error in the numerical solution, computed throughout the simulation region.

The Riemann Problem - Convergence

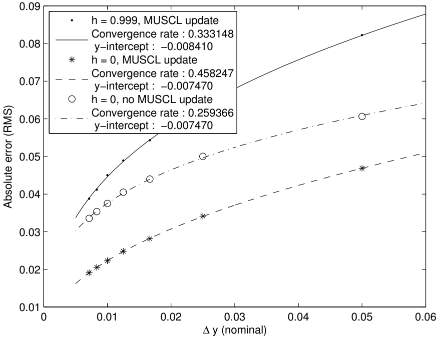
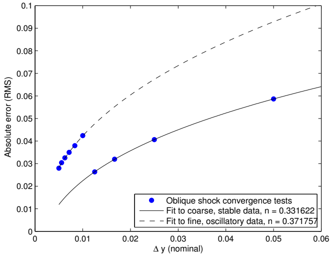
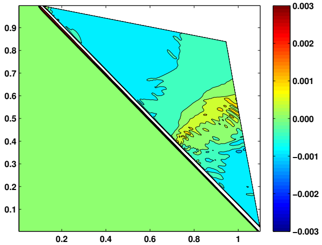


Figure: Root-mean-squared error for the riemann problem, with order of convergence *n*.

The Oblique Shock



(a) Root-mean-squared error for the oblique shock problem. The appearance of grid instabilities leads to two distinct error curves with different rates of convergence *n*.



(b) A plot of normalized error in pressure, highlighting the oscillations which propogate downstream from the oblique shock.

Figure: The oblique shock wave

The Expansion Corner

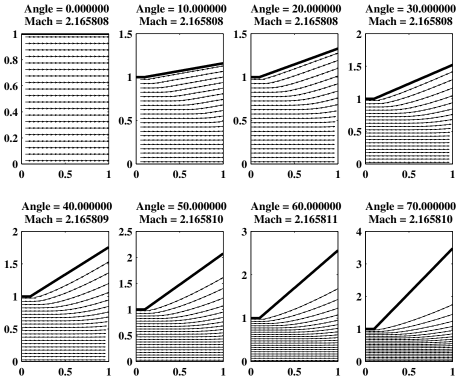


Figure: Computed streamlines for Prandtl-Meyer expansion at increasing expansion angles. Angles are given in degrees.

The Diamond Shock Train

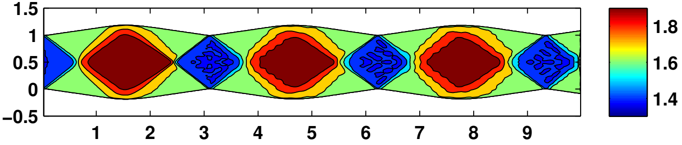
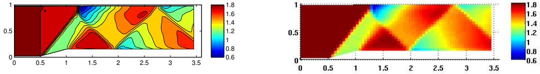
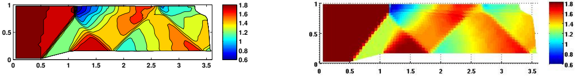


Figure: Computed Mach number for an under-expanded nozzle flow, showing the diamond-shock train. Note the presence of oscillations which grow as the flow progresses downstream.

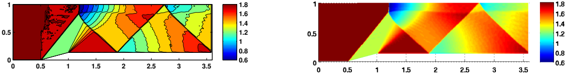
The Transonic Duct



(a) $h = 0$, nominal grid size: 72×20



(b) $h = 0.25$, nominal grid size: 72×20



(c) $h = 0$, nominal grid size: 360×100

Figure: Qualitative accuracy comparison between UCS and Eulerian simulations for a transonic duct flow. Notice the improved resolution of the slip line and the walls for the UCS solution.

Boundary-layer flow

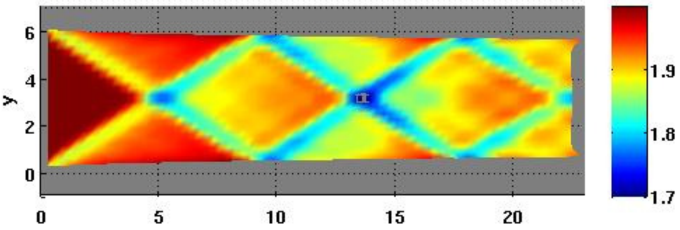


Figure: Oblique shock train produced by a turbulent boundary layer in an otherwise uniform channel

A more interesting application

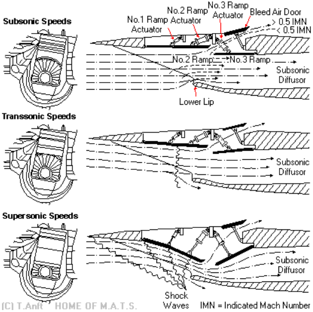


Figure: Diagram of the variable inlet geometry of the USAF F-14 Tomcat. Courtesy Home of M.A.T.S., http://www.anft.net/f-14/f14-detail-airintake.htm

A more interesting application

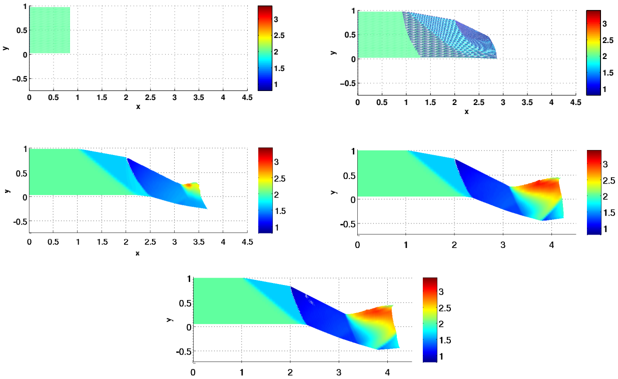


Figure: Time-lapse images of Mach number in a geometry modeled after Fig. 9.

Outline

Summary

- The major advantage of UCS is the automatic mesh
- UCS must be implemented in a way that improves stability and accuracy

- Future Work
  - Optimize for speed
  - Boundary layer solver
  - Shock-aligned grid

References and Further Reading I

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