Cost and Math Primer

Plan

- Slowly build up some micro theory and math
- Lots of jumping up and down to work on the board

Single Product Cost Functions

$$Cost = C(q)$$

- Single parameter q, which is output measured with some logical unit.
- Cost functions must be monotonically increasing, $\forall q' > q,\ C(q') > C(q)$
- Commonly divided into fixed and variable cost $C(q) = F + VC(q)$
    - F is fixed, uncorrelated with output.
    - $VC(q)$ is variable, correlated. Just like $C(q)$, is monotonic in q.

Examples

We will work with these two as concrete examples

1. $C_1(q) = F + \alpha q^2$
2. $C_2(q) = F + \alpha q$

Grab pens and plot one out. If you have not had calculus, pick C_2

We have lots of derived costs

- Average Cost: $AC(q) = \frac{C(q)}{q}$
- Average Variable Cost : $AVC(q) = \frac{VC(q)}{q}$
- Marginal Cost: $MC(q) = \frac{\partial}{\partial q}C(q) = \frac{\partial}{\partial q}VC(q)$

Find AC, AVC and MC for your cost functions. Then plot them out.

Sub and Super Additivity

Subadditive: $f(x+y) \leq f(x) + f(y)$

Example: Pour 1 C water into 1 C salt and get 1 C salty water – not 2 C.

Some functions are globally subadditive, some over ranges.

Relation to Costs

$$C(x+y) \leq C(x) + C(Y)$$

Subadditive costs mean that it is cheaper for one firm to produce than two.

Are your functions subadditive?

- One is globally subadditive.
- One is subadditive over a range.
  - Try assuming subadditivity and find the range where it is true.
  - You should find subadditive when F is high and superadditive when F is low.

Subadditivity By AC

- Subadditivity implies Economies of Scale.
- Economies of scale (One Definition):
  $AC(q') \leq AC(q) \text{ when } q' > q$
- Helps decide if we should, for low production costs, have one big firm or many small firms.
- Also, if $MC < AC$ then subadditive.

General Pattern in Economics

- Each agent has a way of evaluating how well off they are from choices, utility, profit.
- They have constraints on achieving those objectives, income, legal limits.
- This allows you to predict behavior.

If you know what someone wants and how they are constrained you can predict their behavior.

An Objective For Firms – Profit

$$\Pi(q) = R(q) - C(q)$$

- The idea is that firms are trying to make as much money in total as possible.
- Not, most per unit, or most sales.
- R is "Revenue", Pq.

Different Revenue Function for Each Example

We need to do this because there is not always a solution for every revenue function.

- q use $P = P_0 - \gamma q$, i.e., a firm with market power.
- q^2 use $P = P_0$, a firm with no market power.

What to Do

- Set up your objective function
- Find profit maximizing output, q.
- Find Revenue, Costs, and Profit and the optimum.

Now Graphically

- Put your P function in the graph.
- Create marginal revenue, $MR(q) = \frac{\partial}{\partial q} R(q)$.
- Profit maximizing is generally, $MC = MR$.
- Cost, Revenue and Profits are areas.