Assignment 1

(1)

(1.1) Negative Binomial Distribution

In a series of Bernoulli trials (independent with constant probability *p* of a success), let the random variable *X* denote the number of trials required to obtain *r* successes. Then *X* is a **negative binomial random variable** with parameter and and the probability mass function of *X* is

With mean and variance,

Example:

Bob is a high school basketball player. He is a 70% free throw shooter. That means his probability of making a free throw is 0.70 and each throw is independent.

Let *X* denotes the number of throws until Bob makes his third free throw, what is the

① mass function

② cumulative probability the probability that Bob makes his third free throw with fewer than six shots?

③ mean and variance of random variable *X* ?

Each throw of Bob can be treated as a Bernoulli trial with a probability of success Hence *X* has a Negative Binomial distribution with parameter *p*= and *r=* 3. It has

① mass function

② cumulative probability

③ mean and variance

(1.2) Poisson Distribution

Consider a random experiment where an interval *T* of real numbers partitioned into subintervals of small length and as ,

(1) the probability of more than one event in a subinterval tends to zero,

(2) the probability of one event in a subinterval tends to ,

(3) the event in each subinterval is independent of other subintervals.

A process with these properties is called a **Poisson process**. The random variable *X* that equals the number of events in this process is a **Poisson random variable** with parameter , and the probability mass function of *X* is

With mean and variance,

Example:

The number of flaws in bolts of cloth in textile manufacturing is assumed to be Poisson distributed with a mean of 0.1 flaws per square meter.

Let *X* denote the number of flaws in 10 square meter of cloth. What is the

① mass function

② cumulative probability and the probability that fewer than 3 flaws in 10 square meter of cloth

③ mean and variance of random variable *X* ?

*X* has a Poisson distribution with parameter =0.1 flaws /=1 flaws. It has

① mass function

② cumulative probability

③ mean and variance

(1.3) Normal Distribution and its standardization

A random variable *X* with probability density function

is a **normal random variable** with parameters , where and, and

Also with mean and variance,

and the notation can be used to denote this distribution.

A normal random variable with and is called a **standard normal random variable** and is denoted as *Z*. Any normal distribution with random variable *X* with mean and variance can be **standardized** through equation

where and *Z* is a standard normal random variable**.**

Example:

The weight of a sophisticated running shoe is normally distributed with a mean of 12 ounces and a standard deviation of 0.5 ounce.

Let *X* denote the weight of a shoe in ounces. What is the

① mass function

② cumulative probability and the probability that a shoe weighs more than 13 ounces

③ mean and variance of random variable *X* ?

*X* has a Normal distribution with mean and variance. It has

① mass function

② cumulative probability

③ mean and variance

(1.4) Exponential Distribution

The random variable *X* that equals the distance between successive events where the events follow a **Poisson distribution** with mean number of per unit interval is an **exponential** **random variable** with parameter .The probability density function of *X* is

With mean and variance,

Example:

Assume that the flaws along a magnetic tape follow a Poisson distribution with a mean of 0.2 flaws per meter.

Let *X* denote the distance between two successive flaws. What is the

① mass function

② cumulative probability and the probability that there are no flaws in 10 consecutive meters of tape

③ mean and variance of random variable *X* ?

*X* denotes the distance between two successive flaws where the flaws follow a Poisson distribution with a mean, hence *X* is an exponential random variablewith parameter . It has

① mass function

② cumulative probability

③ mean and variance

(1.5) Gamma Distribution

The random variable *X* that equals the length until *r* counts occur in a Passion process with mean number of per unit interval is a **gamma random variable** with parameters and. The probability density function of *X* is

With mean and variance,

Example:

In a data communication system, several messages that arrive at a node are bundled into a packet before they are transmitted over the network. Assume the messages arrive at the node according to a Poisson process with 30 messages per minute. Five messages are used to form a packet.

Let *X* denote the time a packet is formed. What is the

① mass function

② cumulative probability and the probability that a packet is formed in less than 10 seconds

③ mean and variance of random variable *X* ?

*X* denotes the time when 5 messages arrive (and a packet is formed) and the messages arrive follow a Poisson distribution with mean=30 messages/minute hence *X* has a Gamma distribution with parameter=30 messages/minute and *r=* 5. It has

① mass function

② cumulative probability

③ mean and variance

(2)Please show for an exponential random variable *X*,

Proof:

For an exponential random variable X with parameter ,

QED