

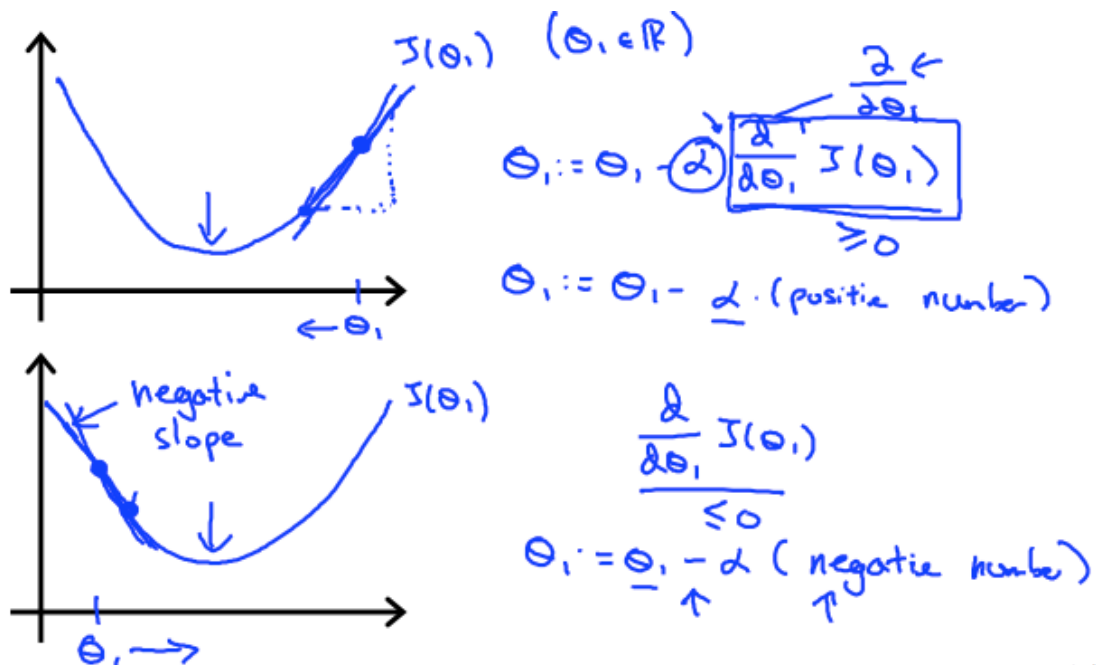
# Gradient Descent Intuition

In this video we explored the scenario where we used one parameter  $\theta_1$  and plotted its cost function to implement a gradient descent. Our formula for a single parameter was :

Repeat until convergence:

$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

Regardless of the slope's sign for  $\frac{d}{d\theta_1} J(\theta_1)$ ,  $\theta_1$  eventually converges to its minimum value. The following graph shows that when the slope is negative, the value of  $\theta_1$  increases and when it is positive, the value of  $\theta_1$  decreases.

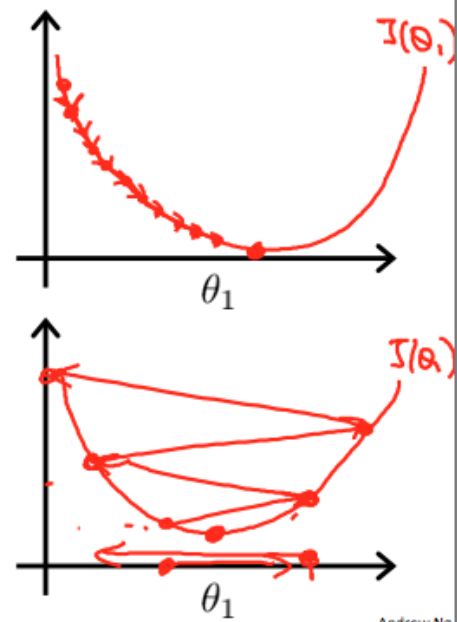


On a side note, we should adjust our parameter  $\alpha$  to ensure that the gradient descent algorithm converges in a reasonable time. Failure to converge or too much time to obtain the minimum value imply that our step size is wrong.

$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If  $\alpha$  is too small, gradient descent can be slow.

If  $\alpha$  is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.



How does gradient descent converge with a fixed step size  $\alpha$ ?

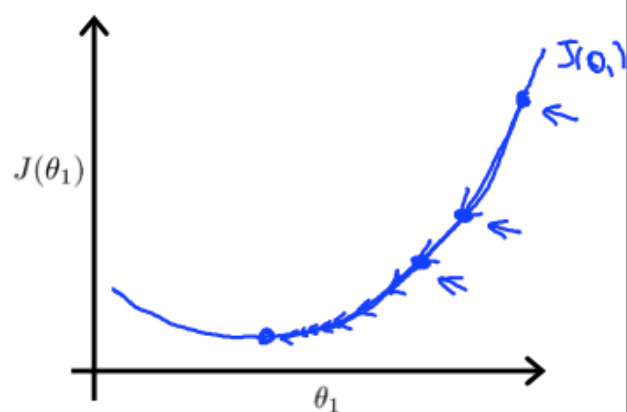
The intuition behind the convergence is that  $\frac{d}{d\theta_1} J(\theta_1)$  approaches 0 as we approach the bottom of our convex function. At the minimum, the derivative will always be 0 and thus we get:

$$\theta_1 := \theta_1 - \alpha * 0$$

Gradient descent can converge to a local minimum, even with the learning rate  $\alpha$  fixed.

$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease  $\alpha$  over time.



✓ Complete