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Decision-Making of Closed-Loop Supply Chain Strategic Alliance Considering the Quality Level of Remanufactured Products and Patent Protection

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In this paper, a closed-loop supply chain consisting of a manufacturer, a retailer and a third-party remanufacturer responsible for collecting used products and remanufacturing is constructed. Considering the quality level of remanufactured products, four kinds of closed-loop supply chain alliance structure models are constructed. The optimal equilibrium decisions of these four models are compared and analyzed. The optimal decisions of the models are verified by numerical analysis. Furthermore, the impacts of the quality of remanufactured products and the decision influence of the third-party remanufacturer

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in the alliance on the remanufacturer's decision are further analyzed. The results show that remanufactured products are competitive with new products, and the improvement of the remanufactured product quality will reduce the market demand of new products. The equilibrium decision of the closed-loop supply chain is affected by the alliance behavior of members in the closed-loop supply chain and the quality level of remanufactured products. The higher the decision concentration of the closed-loop supply chain is, the more favorable the supply chain is; the higher the remanufacturing quality level is, the more favorable the supply chain is, and the alliance decision of the third-party remanufacturer is affected by the quality level of remanufactured products and the decision-making influence of the third-party remanufacturer in the alliance structure. We find that the improvement of the concentration degree of closed-loop supply chain decision can benefit the supply chain by improving the remanufacturing quality level, which has direct effect on the alliance decision of the third-party remanufacturer. In most cases, the choice of the alliance is the dominant decision of the third-party remanufacturer.

Keywords: Closed-loop supply chain; strategic alliance structure; quality level of remanufactured products; patent protection; Stackelberg game.

1. Introduction

In the recent years, global warming and shortage of resources have become very serious. Environmental protection has attracted increased attention. A growing number of people are willing to sell used products and buy green environmental friendly products, which makes the remanufacturing and marketing of products increasingly important. Closed-loop supply chain management mode which combines product forward sales with reverse recycling has been widely practiced. Many well-known companies at home and abroad (such as HP, Lenovo, and IBM) have actively implemented management and achieved great success.

According to Potter (1980), the essence of competitive strategy lies in how to make the right choice. Because all kinds of resources of enterprises are limited, it is necessary to ensure that these resources are used reasonably, and focus on establishing their core competitiveness. If we diversify blindly, we will diversify the resources, lose the focus of development and weaken the competitive advantage. So, many patent owners are reluctant to participate in remanufacturing business because of many unfavorable factors such as low marginal profit of remanufactured products, occupying company resources and influencing the company's main business (Majumder *et al.*, 2001). However, when a third-party remanufacturer makes remanufacturing privately, it will encroach on the intellectual property rights of the patent owner, so the third-party remanufacturer often has to pay the licensing fee to the patent owner. Otherwise, the remanufacturing action would constitute a patent infringement action on the patent owner, and there are many patent infringement cases of remanufacturing such as "recycled ink cartridge case". In 2007,in the "recycled ink cartridge case", after three trials, the Supreme Court of Japan finally ruled that recycle assist's recycling of disposable cartridges infringed Canon's patent right, and the failure of the recycling enterprise sounded an alarm in the manufacturing industry (Zhang, 2008). Thus, the patentee's patent right should be taken into account in recycling and reusing patented products.

From the perspective of supply chain, centralized decision-making often has better economic and social benefits than decentralized decision-making. However, with the increase of supply chain members and the complexity of supply chain structure, alliances among supply chain members are inevitable. For example, the strategic alliance between Procter & Gamble Company (P&G) and Wal-Mart Inc. (Wal-Mart)reduced the transaction cost of P&G and Wal-Mart, making it easier to create and implement the enterprise marketing plan with the help of the Man Machine Interface (MMI) system, as well as the inventory cost through the automatic ordering system. Indirect costs have been saved through paperless trade, what's more, they also improved the production efficiency of P&G, excluded the intermediate relationship and the circulation was lower at the same time. Conversely, in 2004, Gome Co. Ltd. (Gome) and Gree Electric Appliances Inc. (Gree) were the largest electric appliance retailer and air conditioner manufacturer in China. However, Gome launched air conditioning promotion campaign without Gree's consent, greatly damaging the brand image of Gree. After several rounds of negotiations between Gome and Gree, there was still no consensus, so Gree stopped supplying Gome. As a result, the conflict escalated, the two sides did not cooperate for a long time, and the competitive advantage of Gome gradually began to decline. "P&G - Wal-Mart" and "Gree-Gome" cases show that supply chain alliance cooperation can improve the decision-making efficiency of supply chain, reduce costs, and make the members of supply chain win-win. The noncooperation often damages the interests of one or both sides, and will inevitably lead to the break-up of the alliance, which will eventually result in the loss of the interests of both sides. Therefore, this paper will discuss the optimal alliance decision-making of the third-party remanufacturer in closed-loop supply chain. Considering consumers' decision-making behavior, patent protection and strategic alliance, this paper studies the impact of remanufactured product quality level and patent protection on closed-loop supply chain decision-making. More specifically, we will address the following research questions:

(i) How do consumer decision-making behavior, remanufactured product quality level, patent protection and strategic alliance affect the decision-making of the closed-loop supply chain?
(ii) How to choose the optimal strategic alliance for the third-party remanufacturer in a closed-loop supply chain?

The remainder of this paper is organized as follows. In Sec. 2, we briefly review the related literature. In Sec. 3, we describe the model and set the assumptions. In Sec. 4, we investigate the equilibrium decisions of the closed-loop supply chain in four types of alliance models. In Sec. 5, we further study the strategic alliance selection decision of the third-party remanufacturer. In Sec. 6, we conduct the numerical analysis to verify the theoretical results. In Sec. 7, we conclude with a brief discussion and suggestions for future work.

2. Literature Review

At present, many scholars have conducted lots of researches on the pricing decisions, consumer preference, member cooperation, and intellectual property rights, and have achieved many results. In the research of supply chain pricing, Dekker *et al.* (2004) use qualitative analysis to study the main factors of closed-loop supply chain coordination, and point out that pricing decision plays an important role in reverse channel coordination. Savaskan *et al.* (2004) construct a pricing decision model for a closed-loop supply chain consisting of an independent manufacturer and multiple retailers with competitive relationships, and give a contract to coordinate the closed-loop supply chain. Arya *et al.* (2007) analyze the impact of dual channel operations on remanufacturing decisions. Ferguson *et al.* (2010) analyze the competition between new and remanufactured products produced by the monopoly manufacturer, as well as the external remanufacturing competition. José *et al.* (2015) study the impact of interactive supply chains between freight forwarders on public policy initiatives based on pricing and incentives. Jadidi *et al.* (2016) study the dual price strategy of newsvendor product supply chain considering the influence of time and price sensitivity on demand. Sarkar *et al.* (2017) build a supply chain of retailers and two manufacturers that produce complementary products, and study pricing decisions in the supply chain. Genc *et al.* (2018) constructed two different discount modes and studied their effects on pricing decisions. The above literature has made a comprehensive study of supply chain pricing decisions from different aspects. Unlike the above literature, this paper comprehensively analyzes the impact of alliance decision-making, consumer preferences and patent protection on supply chain pricing strategy.

Many scholars have studied consumer preferences in supply chains. Conrad (2005) establishes a duopoly decision model based on consumers' willingness to pay for product environmental attributes, and analyzes the impact of consumer environmental awareness on member decision-making in supply chain. Ferrer (2010) hypothesizes that consumers have heterogeneous preferences for new and remanufactured products, and study the optimal yield and pricing decisions for multiple remanufacturing systems. Atasu *et al.* (2008) show that green consumers like corporate remanufacturing and they are willing to buy remanufactured products to increase corporate profits. Gan *et al.* (2016) consider consumer price preferences for new and remanufactured products, and study dual-channel closed-loop supply chain pricing and coordination decisions. Abbey *et al.* (2017) analyze the consumer's perception of the risk perception of remanufactured product quality on product pricing and find that consumer quality perception plays an important role in product pricing. When considering consumer preferences, the existing literature uses the theory of consumer utility, which provides a theoretical basis for this study. Unlike the previous literature, the model we built contains both new products and remanufactured products, and consumers have different preferences for different types of products. In the following chapters, we further analyze the impact of the quality level of remanufactured products on supply chain decision-making.

Many achievements have been made in the research of member cooperation in supply chain. Jeuland *et al.* (1983) study the problem of cooperation in different channels and find that the use of joint pricing and coordination can achieve optimal supply chain performance. Nagarajan *et al.* (2008) and Granot *et al.* (2008) conduct a series of discussions on the stability of the farsighted supplier alliance in the assembly supply chain system. Bhattacharya *et al.* (2010) construct four closed-loop supply chain structures with different cooperation modes, and study the optimal order quantity of retailers ordering new and remanufactured products. Vanclay *et al.* (2011) show that consumer's low-carbon preference behavior can have an impact on consumer behavior, which in turn affects product demand. Nagarajan (2008) constructs a variety of supply chain models and studies the relationship between profit distribution and alliance stability in supply chain alliance. In the case of upstream manufacturers providing quantitative discounts, Chen *et al.* (2011) study the profitability of two downstream competitors after co-ordering. Benjaafar *et al.* (2013) study the impact of corporate alliances on carbon emissions reduction and product abatement costs. Sarat *et al.* (2014) find that corporate cooperation increases the performance of supply chains by comparing pricing strategies for different products in three cases: non-cooperation, channel cooperation, and cooperation. In the above literature, the supply chain member alliance model has been studied in detail, but the problems faced by the vulnerable third-party remanufacturer in choosing alliance have not been studied. In this paper, we will examine the profit of the third-party remanufacturer under the influence of several representative decision-making, and use Nash equilibrium to provide the basis for the alliance decision-making of the third-party remanufacturer.

With regard to the conflict of intellectual property rights between the original manufacturer and the remanufacturer in the closed-loop supply chain, Wang (2002) shows that imposing royalties is more favorable to patent holders than levying fixed fees. Xiong *et al.* (2011) show that the original manufacturer can influence the decision-making of the third-party remanufacturer by charging the unit patent licensing fee. Cao *et al.* (2014) construct a patent-protected closed-loop supply chain to study the optimal decision of the supply chain under centralized decision-making and decentralized decision-making, and propose a cost-sharing contract to coordinate the supply chain system. Chen *et al.* (2017) construct two models of patent licensing fee collection, such as product-based strategy and component-based strategy, and study the different decision-making efficiencies of two manufacturers under these two models. The results show that component-based strategy is the occupancy decision-making of component suppliers. The above literature studies a variety of patent licensing models, and also shows that charging per unit product is a more efficient way of operation, which is also adopted in this paper. In this paper, the patent licensing fee is taken into account in the construction of the model, but the supply chain alliance is still the main line, which is different from the direction of the previous literature.

The above literatures provide important ideas and references for the related research of the closed-loop supply chain, but there are still some shortcomings, which are mainly manifested in the following aspects: First, the existing research does not take into account the impact of the cumulative effects of consumer preferences, remanufactured product quality levels, and the alliance behaviors of supply chain members on closed-loop supply chain decisions, this paper has a certain innovation in the above directions. Second, although the alliance decision-making of the manufacturer in closed-loop supply chain is concerned in the literature, the decision-making of the third-party remanufacturer in the closed-loop supply chain has not been considered. Finally, the literatures above do not take into account the patent protection issue in analyzing the alliance decisions of the closed-loop supply chain members. Based on game theory, this paper draws on the theoretical research results of consumer preference, member cooperation in the supply chain and patent protection, and constructs a closed-loop supply chain model considering the quality level of remanufactured products and patent protection, and discusses the alliance decisions of members in the closed-loop supply chain. Furthermore, we investigate the impact of the quality level of remanufactured products on pricing decisions and profits of the closed-loop supply chain and the decision-making of the optimal strategic alliance for the third-party remanufacturer.

3. Model Description and Assumptions

This paper considers a closed-loop supply chain consisting of a manufacturer, a retailer and a third-party remanufacturer. The manufacturer is responsible for producing new products, the third-party remanufacturer is responsible for recycling used products for remanufacturing, and the retailer is responsible for wholesaling new products from the manufacturer and wholesaling remanufactured products from the third-party remanufacturer.

The retail prices of new and remanufactured products are denoted by p_n and p_r, respectively, and the wholesale prices of new and remanufactured products are denoted by ω_n and ω_r, respectively. The cost of new products mainly includes the depreciation of fixed assets required for the production of new products, as well as the cost of new product materials and production assembly, etc. For the purpose of simplifying the analysis, c_n will denote the unit cost of a new product. The quality of used products recovered in real life is mixed, and as a result, recycling and remanufacturing become a highly complex process. It is difficult to express the costs involved in remanufacturing with a single parameter. The cost of remanufactured products mainly includes the return cost to consumers, the recycling, disassembly, inspection cost, and reproduction cost of used products. c_r will denote the unit cost of remanufactured products in this paper. The manufacturer owns the patent of the product and does not participate in the recycling and remanufacturing of used

Table 1. The description of the symbols.

Symbol	Description
Model parameters	
α	The quality level of remanufactured products
θ	The preference coefficient of a consumer for product quality
s	The quality of new products
u_n	The utility of customers to buy new products
u_r	The utility of customers to buy remanufactured products
α	The quality level of remanufactured products
c_n	The unit cost of a new product
c_r	The unit cost of remanufactured products
q_n	The demand of new products
q_r	The demand of remanufactured products
φ	Decision-making influence of members in the supply chain
Decision variables	
p_n	Retail price of new product
p_r	Retail price of remanufactured product
ω_n	Wholesale price of new products
ω_r	Wholesale price of remanufactured products
t	Patent licensing feefor per unit of remanufactured product
Other notations	
Π_m	The profit of the manufacturer
Π_r	The profit of the retailer
Π_p	The profit of the third-party remanufacturer
Superscript j	$j \in \{D, MP, RP, C\}$ refers to Model D, Model MP, Model RP and Model C, respectively
Subscript i	$i \in \{m, r, p\}$ refers to the manufacturer, the retailer and the third-party remanufacturer, respectively

products. Instead, it allows the third-party remanufacturer to recycle and remanufacture through patent licensing. Generally, there are two ways to charge for patent authorization: unit patent licensing fee and fixed patent licensing fee. This paper uses the unit patent licensing fee, that is, the third-party remanufacturer must pay a unit patent licensing fee to the manufacturer for each unit of remanufactured product. Here, the unit patent licensing fee is denoted by t. The model symbols are described in Table 1.

Assumption 1. The quality of new and remanufactured products is different and consumers can identify them. We use the following notation throughout the paper: s will denote the quality of new products, and αs denotes the quality of remanufactured products, where $\alpha \in (0, 1)$ represents the quality level of remanufactured products. The quality level of remanufactured products α is an exogenous variable, which is affected by many factors such as the quality of idle used products, the degree of modularization and the technical level of the third-party remanufacturer.

Assumption 2. Each consumer buys at most one item, for example, if a consumer purchases a new product, he will not purchase the remanufactured product.

Assumption 3. The market demand of the product is composed of a group of heterogeneous consumers, the preference coefficient of a consumer for the product quality is θ, which is uniformly distributed in [0, 1], and the probability density of θ is $f(\theta) = 1$. When the consumer's product quality preference coefficient is θ, the willingness to pay for the new product is θs, and the willingness to pay for remanufactured products is $\theta\alpha s$. By analogy, the utility of the consumers to buy new and remanufactured products is $u_n = \theta s - p_n$, $u_r = \theta\alpha s - p_r$, respectively.

Assumption 4. The members of the closed-loop supply chain have sufficient manufacturing capacity and can accurately predict the market demand, and the products can enter the market. That is to say, the production volume of new and remanufactured products is equal to their respective market demand, expressed by q_n and q_r, respectively.

Assumption 5. In order to satisfy the positive demands of new and remanufactured products, we assume the following constraints on the parameter α hold:

(i) In D model, $\frac{c_r}{c_n} < \alpha < \frac{2(s-c_n)+c_r}{2s-c_n}$.
(ii) In MP, RP and C models, $\frac{c_r}{c_n} < \alpha < \frac{s-c_n+c_r}{s}$.

Assumption 5 shows that the above models can be established only if α is within a certain range. When the quality of remanufactured products is lower than the threshold $\frac{c_r}{c_n}$, the quality of remanufactured products is quite different from that of new products. Poor quality level of remanufactured products cannot attract consumers' desire to buy, so at this time there will be no market demand for remanufactured products. From Lemma 1, Propositions 1 and 2, it can be seen that the improvement of the quality of remanufactured products will reduce the demand of new products, when the quality of remanufactured products is higher than the threshold $\frac{2(s-c_n)+c_r}{2s-c_n}$ or $\frac{s-c_n+c_r}{s}$, the difference between the quality of new and remanufactured products is not obvious, and remanufactured products have certain advantages in price. At this time, the demand of new products will drop to zero. Therefore, α needs to meet certain conditions, and the model can be established.

According to Assumptions 1–3, purchase decisions of consumers with different quality preferences can be obtained, as shown in Lemma 1.

Lemma 1. *Consumers with a quality preference of* $\left[\frac{p_r}{\alpha s}, \frac{p_n-p_r}{s(1-\alpha)}\right]$ *will buy remanufactured products, and consumers with a quality preference of* $\left[\frac{p_n-p_r}{s(1-\alpha)}, 1\right]$ *will choose to buy new products.*

According to Lemma 1, the demand functions of new and remanufactured products can be expressed by

$$q_n = \int_{\frac{p_n-p_r}{s(1-\alpha)}}^{1} f(\theta)d\theta = 1 - \frac{p_n - p_r}{s(1-\alpha)}$$

and

$$q_r = \int_{\frac{p_r}{\alpha s}}^{\frac{p_n - p_r}{s(1-\alpha)}} f(\theta) d\theta = \frac{p_n - p_r}{s(1-\alpha)} - \frac{p_r}{\alpha s},$$

respectively, and total sales volume is $q_n + q_r = 1 - \frac{p_r}{\alpha s}$.

According to the above formulas, the sales volumes of new and remanufactured products are affected by the retail prices of new and remanufactured products and the quality level of remanufactured products. Total sales volume is positively correlated with the remanufactured product quality α, and negatively correlated with the remanufactured products price p_r.

4. Decision Models of the Closed-Loop Supply Chain

The following four decision models are considered: (i) Non-alliance (D model), in which each member of the closed-loop supply chain makes decisions with their respective optimal profits. (ii) The manufacturer and the third-party remanufacturer alliance (MP model), the manufacturer and the third-party remanufacturer form a decision-making body and make decisions based on the alliance's optimal profit. (iii) The retailer and the third-party remanufacturer alliance (RP model). (iv) The alliance of the manufacturer, the retailer and the third-party remanufacturer is a centralized decision body (C model), which makes a unified decision to obtain the overall optimal profit of the supply chain. The above four supply chain structures are shown in Fig. 1.

In this paper, the optimal solution is recorded as $()^*$, and the correlation results of different models are recorded as $()^A$, $A \in \{D, MP, RP, C\}$.

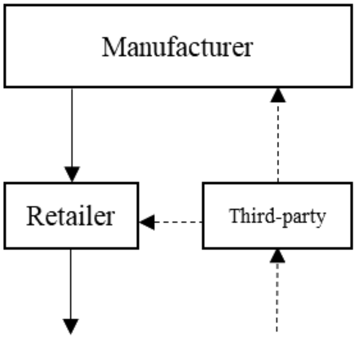
4.1. *Situation without alliance (Model D)*

Without the alliance, each member of the closed-loop supply chain makes decisions for maximizing their respective profits. The manufacturer's profit comes from two parts: one is to produce new products, and the other is to collect the third-party patent licensing fees. Therefore, the manufacturer's optimization problem is $\max \Pi_m^D(\omega_n, t) = (\omega_n - c_n)q_n + q_r t$. The retailer's profit comes from the sales of new and remanufactured products, then the retailer's optimization problem is: $\max \Pi_r^D(p_n, p_r) = (p_n - \omega_n)q_n + (p_r - \omega_r)q_r$. The profit path of the third-party remanufacturer is to remanufacture the used products, so the optimization problem of the third-party remanufacturer is $\max \Pi_p^D(\omega_r) = (\omega_r - c_r - t)q_r$. Therefore, the optimization problem in the D model is

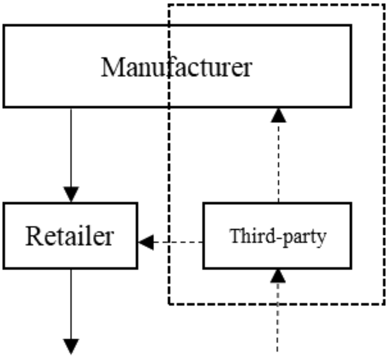
$$\max \;\; \Pi_m^D(\omega_n t) = (\omega_n - c_n)q_n + q_r t,$$

$$\text{s.t.} \;\; \begin{cases} \max \Pi_p^D(\omega_r) = (\omega_r - c_r - t)q_r \\ \max \Pi_r^D(p_n, p_r) = (p_n - \omega_n)q_n + (p_r - \omega_r)q_r. \end{cases}$$

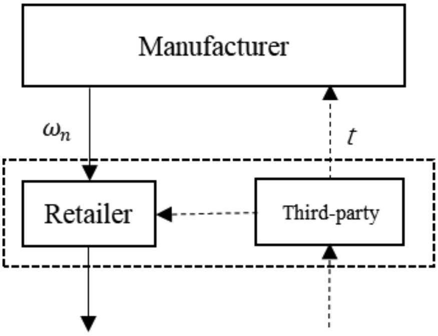
The closed-loop supply chain can be divided into a forward supply chain and a reverse supply chain. In the forward supply chain, the manufacturer as the leader



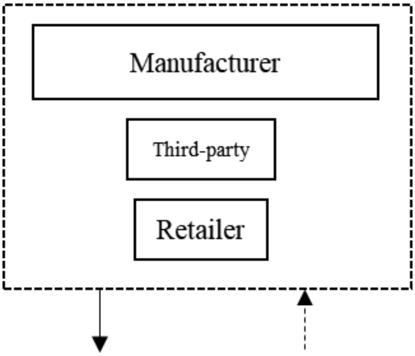
(a) Model D



(b) Model MP



(c) Model RP



(d) Model C

Fig. 1. Closed-loop supply chain model of different alliance structures.

determines the wholesale price of the new product ω_n, after that, the retailer determines the retail price of the new product p_n. In the reverse supply chain, the manufacturer still determines the patent licensing fee for per unit of remanufactured product t, and then the third-party remanufacturer determines the wholesale price of remanufactured products ω_r, and finally the retailer determines the retail price of remanufactured products p_r.

Proposition 1 gives the optimal decision results for Model D.

Proposition 1. *In Model D, the optimal decisions of the closed-loop supply chain are as follows:*

(i) *The optimal retail prices of new and remanufactured products are*

$$p_n^{D*} = \frac{1}{4}(3s + c_n) \quad \textit{and} \quad p_r^{D*} = \frac{1}{8}(6s\alpha + \alpha c_n + c_r), \quad \text{respectively.}$$

(ii) *The optimal wholesale prices of new and remanufactured products are* $\omega_n^{D*} = \frac{s+c_n}{2}$ *and* $\omega_r^{D*} = \frac{1}{4}(2s\alpha + \alpha c_n + c_r)$, *respectively.*

(iii) *The optimal patent licensing fee per unit of remanufactured product is* $t^{D*} = \frac{s\alpha - c_r}{2}$;

(iv) *The optimal sales volumes of new and remanufactured products are* $q_n^{D*} = \frac{2s-2s\alpha-2c_n+\alpha c_n+c_r}{8s-8s\alpha}$ *and* $q_r^{D*} = \frac{\alpha c_n - c_r}{8s\alpha - 8s\alpha^2}$, *respectively.*

Since $q_n^{D*} q_r^{D*} > 0$ is required to be satisfied, the following discussion is performed within the scope of $\frac{c_r}{c_n} < \alpha < \frac{2(s-c_n)+c_r}{2s-c_n}$. Proposition 1 shows that

(i) In the D model, the optimal wholesale price and retail price of the new product are not affected by the quality level of the remanufactured product, and the quality level of the remanufactured product only affects the sales volume of the new product, and the sales volume of new products will decrease due to the increase of quality level of the remanufactured product. This is because the wholesale and retail prices of new products are mainly determined by the cost of the new product, the expected profits of the manufacturer and retailer, and not related to the profits from remanufactured products. In the case of a fixed retail price, higher quality level of remanufactured products will increase the utility of the purchase of remanufactured products. Due to the exclusivity of the product, higher remanufactured quality will reduce the sales volume of new products. The retail price, wholesale price, sales volume, the manufacturer profit, the retailer profit, and the total profit of the closed-loop supply chain all increase with the quality of remanufactured products.

(ii) The optimal patent licensing fee per unit of remanufactured product is positively correlated with the quality level of the remanufactured product. This is because the improvement of the quality of remanufactured products will lead to a decrease in the sales volume of new products, while the wholesale price and the manufacturing cost of new products are not affected by the quality level of remanufactured products. Therefore, if the revenue of new products is reduced, the manufacturer will inevitably increase the patent licensing fee to obtain the optimal profit.

(iii) The profits of each member of the closed-loop supply chain has a quantitative relationship of $\Pi_m^{D*} = 2\Pi_r^{D*} + \Pi_p^{D*}$, which indicates that the manufacturer's profit accounts for most of the profit in the supply chain and exceeds half of the profit of the closed-loop supply chain.

4.2. *The manufacturer and the third-party remanufacturer alliance (MP model)*

In this model, the manufacturer and the third-party remanufacturer form an alliance for decision making. First, the alliance of the manufacturer and the third-party remanufacturer determines the wholesale prices of new and remanufactured products, and then the retailer chooses the retail prices of its new and remanufactured products for the alliance's decision. The optimization problem is similar to the above and is given directly as follows:

$$\max\ \Pi_{mp}^{MP}(\omega_n, \omega_r) = (\omega_n - c_n)q_n + (\omega_r - c_r)q_r;$$

$$\text{s.t.}\ \max \Pi_r^{MP}(p_n, p_r) = (p_n - \omega_n)q_n + (p_r - \omega_r)q_r.$$

The optimal decisions of the closed-loop supply chain are given in Proposition 2.

Proposition 2. *In MP model, the optimal decisions of the closed-loop supply chain are as follows*:

(i) *The optimal retail prices of new and remanufactured products are* $p_n^{MP*} = \frac{3s+c_n}{4}$ *and* $p_r^{MP*} = \frac{3s\alpha+c_r}{4}$, *respectively.*
(ii) *The optimal wholesale prices of new and remanufactured products are* $\omega_n^{MP*} = \frac{s+c_n}{2}$ *and* $\omega_r^{MP*} = \frac{s\alpha+c_r}{2}$, *respectively.*
(iii) *The optimal sales volumes of new and remanufactured products are* $q_n^{MP*} = \frac{s-s\alpha-c_n+c_r}{4s-4s\alpha}$ *and* $q_r^{MP*} = \frac{\alpha c_n-c_r}{4s\alpha-4s\alpha^2}$, *respectively.*

Since $q_n^{MP*}q_r^{MP*} > 0$ are required to be satisfied, the following discussion is performed in the range of $\frac{c_r}{c_n} < \alpha < \frac{s-c_n+c_r}{s}$. According to Proposition 2, in the MP model, the sales volume of new products will decrease with α, while sales volume of remanufactured products, the wholesale and retail prices of remanufactured products, the profits of the manufacturer and retailer, and the total profit of the supply chain all increase with α. The profit relationship of each member in the closed-loop supply chainis $\Pi_m^{MP*} + \Pi_p^{MP*} = 2\Pi_r^{MP*}$. Compared with the D model, it can be seen that the alliance of the manufacturer and the third-party remanufacturer will expand the retailer's profit share in the closed-loop supply chain.

4.3. *The retailer and the third-party remanufacturer alliance* (*RP model*)

In this model, the retailer and the third-party remanufacturer form an alliance to make decisions based on the alliance's optimal profit. Since the retailer is a follower in the forward supply chain and the third-party remanufacturer is also a follower in the reverse supply chain, the alliance of the retailer and the third-party remanufacturer remains a follower in the closed-loop supply chain. Then, the decision-making sequence of the closed-loop supply chain is: The manufacturer chooses the wholesale price of new products and the patent licensing fee for each unit of remanufactured products, and then the RP alliance determines the retail prices of new and remanufactured products. Similar to the above, the optimization problem is given directly as follows

$$\max \Pi_m^{RP}(\omega_n, t) = (\omega_n - c_n)q_n + tq_r;$$

$$\max \Pi_{rp}^{RP}(p_n, p_r) = (p_n - \omega_n)q_n + (p_r - c_r - t)q_r.$$

The optimal decisions of the closed-loop supply chain are given in Proposition 3.

Proposition 3. *In RP model, the optimal decisions of the closed-loop supply chain are as follows*:

(i) *The optimal retail prices of new and remanufactured products are* $p_n^{RP*} = \frac{3s+c_n}{4}$ *and* $p_r^{RP*} = \frac{3s\alpha+c_r}{4}$, *respectively.*

(ii) *The optimal wholesale price for new products is* $\omega_n^{RP*} = \frac{s+c_n}{2}$;

(iii) *The optimal patent licensing fee per unit of remanufactured product is* $t^{RP*} = \frac{s\alpha - c_r}{2}$;

(iv) *The optimal sales volumes of new and remanufactured products are* $q_n^{RP*} = \frac{s - s\alpha - c_n + c_r}{4s - 4s\alpha}$ *and* $q_r^{RP*} = \frac{\alpha c_n - c_r}{4s\alpha - 4s\alpha^2}$, *respectively.*

Substituting the above results into the corresponding profit functions, we can obtain the corresponding optimal profits.

Similar to Proposition 2, the optimal sales volumes of new and remanufactured products are subject to constraints $q_n^{RP*} > 0, q_r^{RP*} > 0$, which are still discussed in the range of $\frac{c_r}{c_n} < \alpha < \frac{s - c_n + c_r}{s}$. It can be seen from Proposition 3 that the optimal retail and wholesale prices of new products in RP model, and the optimal retail price decision of remanufactured products are completely consistent with the optimal decisions in MP model. There is a difference in the relationship between the optimal profits of each member in the closed-loop supply chain. In RP model, there is a relationship $\Pi_m^{RP*} = 2(\Pi_p^{RP*} + \Pi_r^{RP*})$, and the manufacturer's profit accounts for 2/3 of the total profit of the closed-loop supply chain, and is greater than the ratio in D model and MP model. The alliance between the retailer and the third-party remanufacturer will expand the profit ratio of the manufacturer in this closed-loop supply chain.

4.4. *Centralized decision alliance of the manufacturer, the retailer and the third-party remanufacturer (C model)*

In the centralized decision model, the manufacturer, the retailer, and the third-party remanufacturer work closely together, share information between the three, and aim to achieve the maximum revenue of the supply chain, so the optimization problem of the closed-loop supply chain is as follows:

$$\max \Pi_{mrp}^{C}(p_n, p_r) = (p_n - c_n)q_n + (p_r - c_r)q_r.$$

The optimal decisions of the closed-loop supply chain are given in Proposition 4.

Proposition 4. *In C model, the optimal decisions of the closed-loop supply chain are as follows:*

(i) *The optimal retail prices of new and remanufactured products are* $p_n^{C*} = \frac{s+c_n}{2}$ *and* $p_r^{C*} = \frac{s\alpha + c_r}{2}$, *respectively.*

(ii) *The optimal sales volumes of new and remanufactured products are* $q_n^{C*} = \frac{s - s\alpha - c_n + c_r}{2s - 2s\alpha}$ *and* $q_r^{C*} = \frac{\alpha c_n - c_r}{2s\alpha - 2s\alpha^2}$, *respectively.*

Corollary 1. *In D model and RP model, the optimal patent licensing fee per unit of remanufactured products satisfies* $t^{D*} = t^{RP*}$, *and the patent licensing fees in both models are positively correlated with the quality level of remanufactured products.*

According to $t^{D*} = t^{RP*} = \frac{s\alpha - c_r}{2}$ in Proposition 1 and Corollary 1, the optimal patent licensing fee per unit of remanufactured products in the D model and

RP model must be satisfied by the range of $\frac{sc_r - c_r c_n}{2c_n} < t^{D*} < \frac{2s^2 - 2sc_n - sc_r + c_r c_n}{2s - 2c_n}$, $\frac{sc_r - c_r c_n}{2c_n} < t^{RP*} < \frac{s - c_n}{2}$.

Corollary 2. *In the above cooperation modes, the retail price of new products satisfies* $p_n^{C*} < p_n^{MP*} = p_n^{RP*} = p_n^{D*}$; *and the retail price of remanufactured products satisfies* $p_r^{C*} < p_r^{MP*} = p_r^{RP*} < p_r^{D*}$.

Corollary 2 indicates that the retail price of new and remanufactured products is related to the alliance strategy adopted by the retailer, the insights behind which are as follows:

(i) The retail price of the new products in the C model is the lowest, and keeps equal in D model, MP model, and RP model. This is because the retail price of the new products is mainly influenced by the decisions of the retailer and manufacturer. The retailer forms an alliance with the manufacturer in the C model, eliminating the double marginal effect and making the retail price lower than that in other models. In D model, MP model and RP model, the actual participants of decision-making in the forward supply chain are the manufacturer and retailer, and the alliance decision made by the third-party remanufacturer does not affect the structure of the forward supply chain, so the retail prices of new products in the D model, MP model and RP model remain equal.

(ii) The retail price of remanufactured products is determined by the participants in the reverse supply chain. In the D model, the manufacturer, the retailer and the third-party remanufacturer have not reached a coalition relationship with the highest degree of dispersion, so the retail price is the highest; there are alliance actions with two participants in both MP and RP models, and the degree of dispersion is second, so the retail price is second. In the C model, the three members of the closed-loop supply chain form a coalition with the highest concentration, which completely eliminates the double marginal effect. Therefore, the retail price of remanufactured products is the lowest.

Corollary 3. *The following relationships hold*:

(i) *The wholesale prices of new products in D model, MP model and RP model keep the same, and are equal to the retail price of new products in C model, that is* $\omega_n^{D*} = \omega_n^{MP*} = \omega_n^{RP*} = p_n^{C*}$.

(ii) *The wholesale price of remanufactured products in D model is higher than that in MP model, that is* $\omega_r^{D*} > \omega_r^{MP*}$, *and the wholesale price of remanufactured products is lower than that of new products, that is* $\omega_r^{D*} < \omega_n^{D*}$.

Corollary 3 indicates that the wholesale price of new products is not affected by the alliance behavior and the quality level of remanufactured products in the supply chain. This is because the alliance of the third-party remanufacturer with other members of the closed-loop supply chain will only influence the decision-making of the reverse supply chain, and will not affect the decision-making of the forward supply chain. The wholesale price of new products is determined by the manufacturer in the forward supply chain. The manufacturer does not reach an alliance relationship with the retailer in D model, MP model and RP model, so the manufacturer will make the same decision, which also confirms the correctness of the discussion on the retail price of new products in Corollary 2. Similarly, the wholesale price of remanufactured products is determined by members of the reverse supply chain. The degree of dispersion in the MP model is lower than that of the D model, which eliminates the double marginal effect between the manufacturer and the third-party remanufacturer, so we have $\omega_r^{MP*} < \omega_r^{D*}$. The retail price of remanufactured products in MP model is lower than that in D model due to the lower wholesale price of remanufactured products in MP model, which confirms the discussion in Corollary 2.

Corollary 4. *In the above closed-loop supply chain models, the demand of remanufactured products satisfies* $q_r^{C*} > q_r^{MP*} = q_r^{RP*} > q_r^{D*}$. *The demand of new products has the following relationships*:

(i) *if* $\frac{c_r}{c_n} < \alpha < \frac{2(s-c_n)+3c_r}{2s+c_n}$, $q_n^{C*} > q_n^{D*} > q_n^{MP*} = q_n^{RP*}$;
(ii) *if* $\frac{2(s-c_n)+3c_r}{2s+c_n} < \alpha < \frac{2(s-c_n)+c_r}{2s-c_n}$, $q_n^{D*} > q_n^{C*} > q_n^{MP*} = q_n^{RP*}$;
(iii) *if* $\frac{2(s-c_n)+c_r}{2s-c_n} < \alpha < \frac{s-c_n+c_r}{s}$, *then only* $q_n^{D*} > 0$ *is satisfied, and* $q_n^{C*} = q_n^{MP*} = q_n^{RP*} = 0$.

According to Lemma 1, the demand of new and remanufactured products is related to the retail prices of new and remanufactured products, and the quality level of remanufactured products. Combined with Corollary 2, the retail prices of new and remanufactured products in MP model are equal to those in RP model, respectively. So the demands of new and remanufactured products in MP model are equal to those in RP model, respectively. It can be seen from Corollary 4 that the demand of new products in C model is most affected by the quality level of remanufactured products α, while that in D model is least affected by α, and the demand of new products can be maintained when α is increased to a certain extent. Therefore, compared to decentralized decision-making, centralized decision-making of the closed-loop supply chain tends to reduce the retail price to promote consumer demand for products.

Corollary 5. *In the same decision model, the profits of each member of the closed-loop supply chain satisfy the following relationships*:

(i) *In D model*, $\Pi_m^{D*} = 2\Pi_r^{D*} + \Pi_p^{D*}$;
(ii) *In MP model*, $\Pi_m^{MP*} = 2\Pi_r^{MP*} - \Pi_p^{MP*}$;
(iii) *In RP model*, $\Pi_m^{RP*} = \Pi_r^{RP*} + \Pi_p^{RP*}$.

Corollary 5 shows that in different closed-loop supply chain structures, the profit share of each member is not the same. (i) In all alliance structures, the manufacturer has the highest profit share, and is no less than 1/3 (the ratio in D model exceeds 1/2, the ratio in MP model exceeds 1/3, and the ratio in RP model is 1/2). This is because the manufacturer plays a leading role in both the forward and reverse supply chains, and the manufacturer often makes decisions in favor of itself. (ii) The alliance between the manufacturer and the third-party remanufacturer increases the retailer's profit share; the alliance between the retailer and the third-party remanufacturer will also increase the manufacturer's profit share.

Corollary 6. *In the four models*, (i) *the total profit of the closed-loop supply chain increases with the quality level of remanufactured products*, *and the following relationship exists*: $\Pi_T^{C*} > \Pi_T^{MP*} = \Pi_T^{RP*} > \Pi_T^{D*}$; (ii) *the profits of members of the closed-loop supply chain in different decision models satisfy the following relationships*: $\Pi_m^{RP*} > \Pi_m^{D*}$ *always holds for the manufacturer*; $\Pi_r^{MP*} \geq \Pi_r^{D*}$ *always holds for the retailer.*

Corollary 6 shows that improving the quality of remanufactured products is beneficial to increase the profit of the closed-loop supply chain without threatening the market demand of new products. Different alliance strategies and remanufactured product quality strategies adopted by the third-party remanufacturer will affect the overall profit of the supply chain. The total profit of the closed-loop supply chain is the highest in C model, second in MP and RP models, and lowest in D model. This is because the concentration degree of the C model is the highest in the closed-loop supply chain, while the concentration degree of the D model is the lowest. The supply chain with the high concentration degree can make the decision based on the overall optimum of the supply chain. Higher concentration level can often make better decisions, which shows that the closed-loop supply chain with high concentration level can obtain higher overall profit, and lower retail price is also beneficial to consumers and has higher social benefits.

5. The Strategic Alliance Selection of the Third-Party Remanufacturer

The above discussions find the optimal equilibrium solution for the third-party remanufacturer to choose different alliance decisions, but do not explicitly determine the optimal profit of the third-party remanufacturer in MP model, RP model, and C model, and cannot help the third-party remanufacturer to choose the optimal alliance. This section discusses the allocation of incremental profits in different alliances to determine the alliance selection decision for the third-party remanufacturer.

As shown in Zheng *et al.* (2018), defining the incremental profit of the alliance: on the basis of the case of no alliance, the incremental profit of the alliance compared to the case of no alliance will be denoted by $\Delta\Pi$. Taking the MP alliance as an example, the incremental profit of the MP alliance is $\Delta\Pi^{MP} = \Pi_{mp}^{MP*} - \Pi_m^{D*} - \Pi_p^{D*}$. Then the incremental profits of RP alliance and C alliance are: $\Delta\Pi^{RP} = \Pi_{rp}^{RP*} - \Pi_r^{D*} - \Pi_p^{D*}$ and $\Delta\Pi^C = \Pi_{mrp}^{C*} - \Pi_m^{D*} - \Pi_r^{D*} - \Pi_p^{D*}$, respectively. On the issue of profit distribution inside the alliance, it is assumed that the profit of D model is regarded as the retained profit, and the incremental profit is allocated according to the decision influence of each member in the alliance. The decision-making influence of members in the supply chain is expressed as φ_b^X, $X \in \{MP, RP, C\}$, $b \in \{m, r, p\}$.

We have the following results.

Proposition 5. *For the third-party remanufacturer,* (i) *if* $\frac{\varphi_p^{MP}}{\varphi_p^{RP}} > \frac{1}{2}$ *and* $\frac{\varphi_p^{MP}}{\varphi_p^{C}} > B$, *choosing MP alliance is a dominant decision*; (ii) *if* $\frac{\varphi_p^{RP}}{\varphi_p^{MP}} > 2$ *and* $\frac{\varphi_p^{RP}}{\varphi_p^{C}} > \frac{B}{2}$, *choosing RP alliance is the dominant decision*; (iii) *the conditions for choosing C alliance are* $\frac{\varphi_p^{C}}{\varphi_p^{MP}} > \frac{1}{B}$ *and* $\frac{\varphi_p^{C}}{\varphi_p^{RP}} > \frac{2}{B}$, *where* $B = \frac{9}{2} + \frac{2(1-\alpha)\alpha(s-c_n)^2}{(\alpha c_n - c_r)^2}$.

It can be seen from Proposition 5 that strategic alliance selection of the third-party remanufacturer is determined by both the remanufactured product quality coefficient and the third-party remanufacturer's decision-making influence in the alliance. In order to facilitate the analysis, it is assumed that the third-party remanufacturer has the same decision-making influence in different alliance structures, that is $\varphi_p^{MP} = \varphi_p^{RP} = \varphi_p^{C}$. According to $\Delta\Pi^C > \Delta\Pi^{MP} > \Delta\Pi^{RP}$, it can be seen that the selection of C alliance is always the dominant decision of the third-party remanufacturer. In reality, the influence of the third-party remanufacturer in different alliance structures is often asymmetrical. For example, in MP alliance, the influence of decision-making of the third-party remanufacturer is often weaker than that of the manufacturer. In RP alliance, the third-party remanufacturer has stronger decision-making influence. The analysis of the asymmetry case of decision-making influence of the third-party remanufacturer in different alliances is given in the next section.

6. Numerical Analysis

In order to explain the above analysis more intuitively and effectively and verify the correctness of the above analysis, this section combines numerical examples to analyze and verify the above results, and further supplements the analysis of the strategic alliance selection of the third-party remanufacturer.

Suppose $s = 30$, $c_n = 10$ and $c_r = 4$. α is an exogenous variable. According to the Corollary 1, $\frac{c_r}{c_n} < \alpha < \frac{2(s-c_n)+c_r}{2s-c_n}$ needs to be satisfied in the D model; α needs to be satisfied $\frac{c_r}{c_n} < \alpha < \frac{s-c_n+c_r}{s}$ in MP, RP and C models, so the domain of α in D model is $(0.40, 0.88)$, and the domain of α in MP, RP, and C models is $(0.40, 0.80)$.

It can be seen from Fig. 2 that the sales volumes of new products in the D model, MP model, RP model and C model decrease with the quality level of the remanufactured product, where C model, MP model and RP model all fall to zero when $\alpha = 0.8$, while D model is greater than zero when $\alpha = 0.85$. In C model, the decline rate of new product sales is the fastest, followed by MP and RP model, and the decline rate of new product sales in D model is the slowest. According to the results of Lemma 1, the demand of new products is affected by the retail price of new and remanufactured products, and the quality level of remanufactured products.

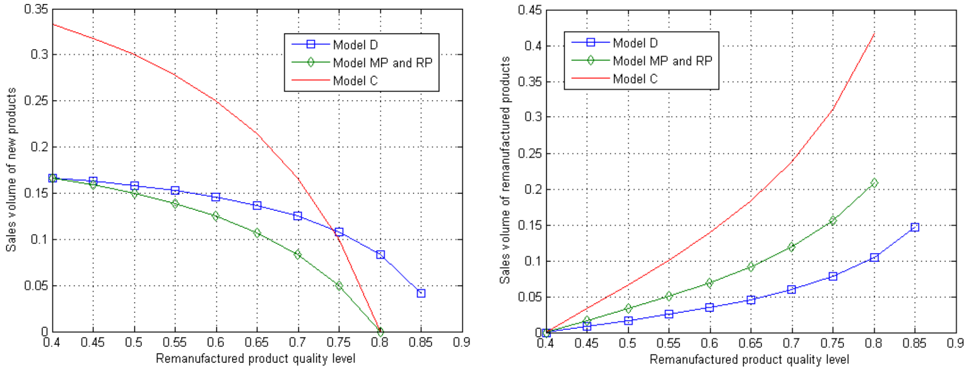


Fig. 2. Sales volumes of new and remanufactured products.

As the price of new products remains the same, the sales volume of new products is affected by the retail price of remanufactured products and the quality level of remanufactured products, which indicates that consumers of C model have higher sensitivity to the dual impacts of the retail price and quality level of remanufactured products, while consumer sensitivity in D model is relatively low. The range of α in D model is larger than that of C model, MP model and RP model, which further shows that D model has lower sensitivity. The sales volume of new products of C model is always larger than that of MP and RP models, and is bigger than that of D model when α is between 0.4 and 0.75, but is overtaken by that of D model when α is in the range of 0.75 to 0.8, which is consistent with Corollary 5.

The sales volume of remanufactured products is affected by the alliance strategy of the closed-loop supply chain and the quality level of remanufactured products. The sales volume of remanufactured products starts from $\alpha = 0.4$ and increases with α in a certain range. In the range of $0.4 < \alpha < 0.8$, C model has the highest sales volume of remanufactured products, MP and RP models take the second place, and model D has the lowest sales volume. It can be seen that the rate at which the sales volume of remanufactured products increases with α is less than the rate at which the sales volume of new products decreases with α, which indicates that the improvement of the quality of remanufactured products leads to the decrease of new product sales volume because the remanufactured product encroaches on the new product market.

The improvement of the level of remanufactured products can reduce the sales of new products and increase the sales of remanufactured products. From the aspect of social benefits, the total sales will increase. From the aspect of environmental protection, it can improve the utilization rate of social waste products. Therefore, remanufacturers need to improve their manufacturing level in order to improve environmental protection and social benefits.

As can be seen from Fig. 3, the retail and wholesale prices of remanufactured products increase with α. The retail price of remanufactured products in D model

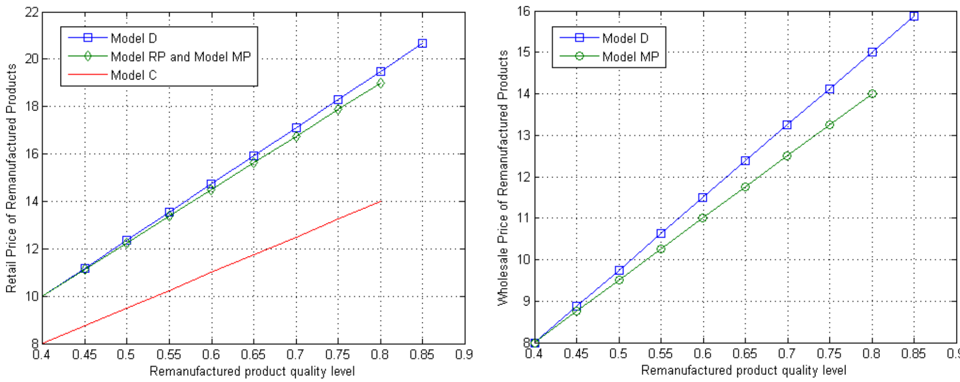


Fig. 3. The retail price and wholesale price of remanufactured products.

is the highest; the retail prices of remanufactured products of MP and RP models are slightly lower than those of D model, and are tied for second place. Model C has the lowest retail price of remanufactured goods, which shows that the decision made by the closed-loop supply chain with high concentration is beneficial to consumers. The wholesale price of remanufactured products in D model is also higher than that of MP model, which is consistent with Corollary 4.

As a third-party remanufacturer, improving the level of remanufacturing can directly increase the retail and wholesale prices of remanufactured goods. In the C model, the retail price rises least and has the highest social effect. Therefore, improving the level of remanufacturing and promoting a higher level of cooperation is the direction of third-party remanufacturer efforts.

It can be seen from Fig. 4 that the total profit of the supply chain is affected by the quality level of remanufactured products and the alliance strategy in the closed-loop supply chain. With the improvement of the quality level of the remanufactured products, the total profit of the supply chain of the above four closed-loop supply

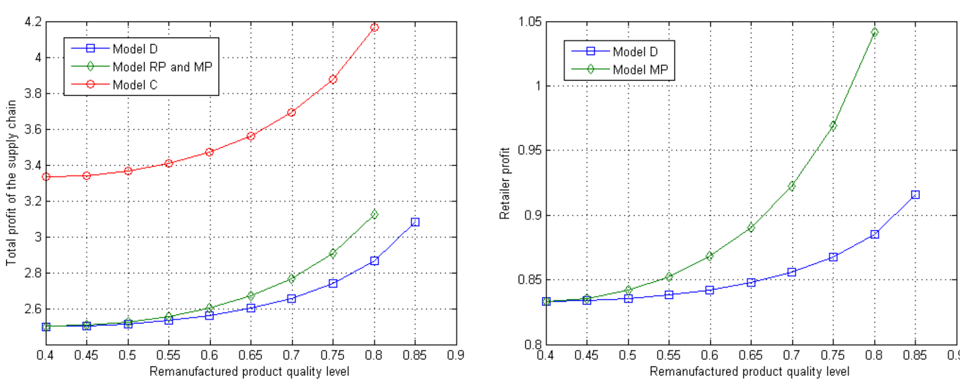


Fig. 4. Total profit of the supply chain and the retailer's profit.

chain structures will rise accordingly. This is because the improvement of the quality level of remanufactured products can promote the sales of remanufactured products. Although it will erode a part of the new product market, the total sales volume can still maintain a certain degree of stability, and the retail price of new products remains unchanged, while the retail price of remanufactured products will continue to rise with the improvement of the quality level of remanufactured products. So the total profit of the supply chain will continue to rise with the improvement of the quality level of remanufactured products.

Obviously, an increase in the quality level of the remanufactured products will increase the profits of the retailer and overall supply chain, but it is not linear. We can see that this curve is first slow and then fast, so when a third-party remanufacturer develops or invests in the early stage, there may not be a rapid increase in profits. This requires a third-party remanufacturer to guard against arrogance. Resilience will increase the level of remanufacturing, but when the manufacturing level rises to a certain level, it will usher in the blue ocean with rapid growth in profits.

The retailer's profit increases with the improvement of the quality level of remanufactured products. When $0.4 < \alpha < 0.8$, the profit of the retailer in MP model is higher than that of D model, and the growth rate of profit with α is higher than that of D model. This is mainly because the wholesale price of remanufactured products in MP model is lower than that of D model. Correspondingly, the retail price of the remanufactured products in MP model is lower than that of D model, which will promote the sales of remanufactured products, and greatly increase the profit of the retailer.

It can be seen from Fig. 5 that the manufacturer's profit increases with the improvement of the quality level of the remanufactured product, and the rate of increase in RP model is higher than that of D model. The patent licensing fee charged by the manufacturer increases linearly with the quality level of remanufactured products, mainly because the improvement in the quality of

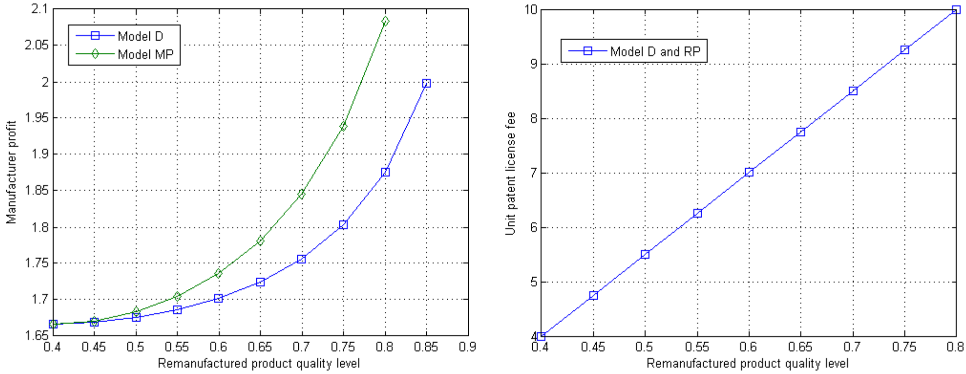


Fig. 5. The manufacturer's profit and unit patent licensing fee.

Table 2. Impact of α on thethird-party remanufacturer alliance selection.

	Model D			Model MP				Model RP				Model C			
α	Π_m	Π_r	Π_p	Π_m	Π_r	Π_p	$\Delta\Pi$	Π_m	Π_r	Π_p	$\Delta\Pi$	Π_m	Π_r	Π_p	$\Delta\Pi$
0.4	1.67	0.83	0.00	1.67	0.83	0.00	0.00	1.67	0.83	0.00	0.00	2.08	1.00	0.25	0.83
0.5	1.68	0.84	0.00	1.68	0.84	0.01	0.00	1.68	0.84	0.00	0.00	2.10	1.01	0.26	0.85
0.6	1.70	0.84	0.02	1.71	0.87	0.02	0.02	1.74	0.85	0.02	0.01	2.16	1.02	0.29	0.91
0.7	1.76	0.86	0.04	1.79	0.92	0.06	0.04	1.85	0.87	0.05	0.02	2.27	1.06	0.35	1.03
0.8	1.88	0.89	0.10	1.95	1.04	0.14	0.10	2.08	0.92	0.12	0.05	2.53	1.15	0.49	1.30

remanufactured products reduces the profit of the manufacturer in producing new products. Increasing the patent licensing fee is an effective way for the manufacturer to ensure its own profit. When the quality of remanufactured products is improved, although the sales volume of new products has declined, the manufacturer can still guarantee the growth of the overall profits by raising the patent licensing fee. From the manufacturer's point of view, promoting alliance with third-party remanufacturer will make more profits for itself. From the aspect of remanufacturer, higher quality level of remanufactured products will lead to higher licensing fees, but the remanufacturer should not worry about this because it will gain more profits from the premium of quality improvement.

The following study analyzes the impact of α on the choice of strategic alliance by the third-party remanufacturer, using the control variable method, assuming that the third-party remanufacturer has the same decision-making influence in the alliance. Suppose that $\varphi_p^{MP} = \varphi_p^{RP} = \varphi_p^C = 0.3$, $\varphi_m^C = 0.5, \varphi_r^C = 0.2$. Table 2 gives the profit of various supply chain structures under different values of α.

It can be seen from Table 2 that for any kind of supply chain alliance structure, the improvement of the quality level of remanufactured products can increase the profits of each member of the closed-loop supply chain. In each of the above cases, the third-party remanufacturer can obtain the maximum profit in C model, so forming alliance with the manufacturer and the retailer is the dominant decision of the third-party remanufacturer.

In reality, the third-party remanufacturer often has different decision-making influences in different closed-loop supply chain structures. The following will analyze the situation of unbalanced decision-making influence in different closed-loop supply chain structures. Considering the following three cases:

$(\varphi_p^{MP}, \varphi_p^{RP}, \varphi_p^C) = (0.9, 0.1, 0.1)$, $(0.1, 0.9, 0.1)$ or $(0.1, 0.1, 0.9)$, the profits of each member of the supply chain under the dual role of the quality of the remanufactured product are given in Table 3.

It can be seen from Table 3 that, in most of the above cases, the choice of model C is the dominant decision of the third-party remanufacturer; when $\alpha = 0.88$ and $(\varphi_p^{MP}, \varphi_p^{RP}, \varphi_p^C) = (0.9, 0.1, 0.1)$, choosing MP alliance is the dominant decision of the third-party remanufacturer. In summary, in most cases, the formation of a tripartite alliance (model C) is a dominant decision of the third-party remanufacturer;

Table 3. Impact of α and φ on the third-party remanufacturer alliance selection.

		Model D			Model MP			Model RP			Model C		
α	$(\varphi_p^{MP}\varphi_p^{RP}\varphi_p^{C})$	Π_m	Π_r	Π_p	Π_m	Π_r	Π_p	Π_m	Π_r	Π_p	Π_m	Π_r	Π_p
0.6	(0.9,0.1,0.1)	1.70	0.84	0.02	1.70	0.87	0.03	1.74	0.85	0.02	2.11	1.25	0.11
	(0.1,0.9,0.1)	1.70	0.84	0.02	1.72	0.87	0.02	1.74	0.84	0.03	2.11	1.25	0.11
	(0.1,0.1,0.9)	1.70	0.84	0.02	1.72	0.87	0.02	1.74	0.85	0.02	1.75	0.89	0.84
0.7	(0.9,0.1,0.1)	1.76	0.86	0.04	1.76	0.92	0.08	1.85	0.88	0.05	2.22	1.32	0.15
	(0.1,0.9,0.1)	1.76	0.86	0.04	1.80	0.92	0.05	1.85	0.86	0.06	2.22	1.32	0.15
	(0.1,0.1,0.9)	1.76	0.86	0.04	1.80	0.92	0.05	1.85	0.88	0.05	1.81	0.91	0.98
0.8	(0.9,0.1,0.1)	1.88	0.89	0.10	1.89	1.04	0.20	2.08	0.93	0.11	2.46	1.47	0.23
	(0.1,0.9,0.1)	1.88	0.89	0.10	1.97	1.04	0.11	2.08	0.89	0.15	2.46	1.47	0.23
	(0.1,0.1,0.9)	1.88	0.89	0.10	1.97	1.04	0.11	2.08	0.93	0.11	1.94	0.95	1.28
0.88	(0.9,0.1,0.1)	2.12	0.95	0.23	2.14	1.29	0.43	2.58	1.05	0.24	2.96	1.78	0.41
	(0.1,0.9,0.1)	2.12	0.95	0.23	2.33	1.29	0.25	2.58	0.96	0.33	2.96	1.78	0.41
	(0.1,0.1,0.9)	2.12	0.95	0.23	2.33	1.29	0.25	2.58	1.05	0.24	2.21	1.04	1.90

when the quality of remanufactured products is close to that of new products, and the decision-making influence of the third-party remanufacturer in MP model is much stronger than that of C model, the MP alliance is the dominant decision of the third-party remanufacturer.

7. Concluding Remarks

This paper builds a closed-loop supply chain system consisting of a single manufacturer with product intellectual property, a single retailer, and a single third-party remanufacturer. In the case of considering the quality of remanufactured products, using the Stackelberg game method, four closed-loop supply chain structure models are constructed. The impact of the quality of remanufactured products and the decision-making influence of the third-party remanufacturer in the alliance on the decision-making of the third-party remanufacturer is further analyzed. Main conclusions are as follows:

(i) There is a competitive relationship between new and remanufactured products. Therefore, the quality level of remanufactured products needs to meet a certain range. Too low a quality level will result in no market for remanufactured products, and the high quality level will completely eliminate the need for new products.

(ii) The equilibrium decision of the closed-loop supply chain is influenced by the alliance behavior of members in the closed-loop supply chain and the quality level of remanufactured products, but the wholesale and retail prices of new products are not affected. The wholesale and retail prices and the unit patent licensing fee for remanufactured products will increase with the quality of remanufactured products. In different alliance situations, the profits of supply chain members and supply chain as a whole increase with the improvement of the quality of remanufactured products. For the supply chain as a whole, the higher the degree of centralization of decision-making, the higher the overall profit of the supply chain. An alliance of two members in the supply chain will also benefit another member who does not participate in the alliance.

(iii) The alliance decision of the third-party remanufacturer is influenced by the quality level of remanufactured products and the decision-making influence of the third-party remanufacturer in the alliance structure. In most cases, the choice of C alliance is the dominant decision of the third-party remanufacturer. Only in some extreme cases (where the remanufacturer has absolute influence in MP alliance and the quality level of remanufactured products is close to that of new products), the choice of MP alliance is the best decision.

However, there are still several limitations in this paper. First, the quality level of remanufactured products is not only affected by the technical level of the third-party remanufacturer, but also affected by various factors such as the quality level of new products. Accordingly, it would be interesting to study the case considering the quality level of new products. Second, this paper only considers the situation that the third-party remanufacturer participates in the recycling of used products. In practice, the manufacturer or the retailer often participates in the recycling and remanufacturing. Therefore, we believe that cases of multiple members participating in recycling are worthy of further exploration. Third, there are usually two patent licensing methods: fixed patent licensing fee and unit patent licensing fee. However, we only consider the unit patent licensing fee method in this paper, and different patent licensing methods will produce different results, so which method is the optimal choice from the viewpoints of the manufacturer and different alliances? These points are very interesting and worth noting for further research.

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Appendix

Proof of Lemma 1.

The conditions for consumers to choose remanufactured products are: $u_r > u_n$ and $u_r > 0$. That is $\theta\alpha s - p_r > \theta s - p_n, \theta\alpha s - p_r > 0$, and the range of θ is obtained as $\frac{p_r}{\alpha s}<\theta < \frac{p_n - p_r}{s(1-\alpha)}$. If and only if $\frac{p_r}{\alpha s} < \frac{p_n - p_r}{s(1-\alpha)}$, the inequality has a solution, and $p_n\alpha > p_r$ is obtained. Therefore, under the premise of $p_n\alpha > p_r$, customers whose quality preference satisfies $\frac{p_r}{\alpha s}<\theta < \frac{p_n - p_r}{s(1-\alpha)}$ will purchase remanufactured products.

Purchasing new products should satisfy the conditions $u_n > u_r$ and $u_n > 0$, that is $\theta s - p_n > \theta\alpha s - p_r, \theta s - p_n > 0$. $\theta > \max\left\{\frac{p_n - p_r}{s\left(1-\alpha\right)}, \frac{p_n}{s}\right\}$ is obtained. This paper discusses the situation that both new products and remanufactured products have markets. The premise for consumers to buy remanufactured products is that $\frac{p_r}{p_n} < \alpha < 1$. It is easy to know that $\max\left\{\frac{p_n - p_r}{s(1-\alpha)}, \frac{p_n}{s}\right\} = \frac{p_n - p_r}{s(1-\alpha)}$, so $\theta > \frac{p_n - p_r}{s(1-\alpha)}$, and as θ distributes in $[0, 1]$, so $\frac{p_n - p_r}{s(1-\alpha)} < \theta \leq 1$.

Proof of Proposition 1.

The above closed-loop supply chain conforms to the Stackelberg game. According to the inverse induction method, the retailer's Hessian matrix with respect to p_n and p_r is first obtained: $H_1 = \begin{bmatrix} -\frac{2}{s-s\alpha} & \frac{2}{s-s\alpha} \\ \frac{2}{s-s\alpha} & -\frac{2}{s\alpha - s\alpha^2} \end{bmatrix}$. The Hessian matrix is negative definite, then Π_r is a strictly concave function with respect to p_n and p_r. According to $\frac{\partial \Pi_r^D}{\partial p_n} = 0$, $\frac{\partial \Pi_r^D}{\partial p_r} = 0$, the retail prices of new and remanufactured products are given by $p_n = \frac{s+\omega_n}{2}$ and $p_r = \frac{s\alpha + \omega_r}{2}$, respectively.

The third-party remanufacturer makes the second priority decision, and $p_n = \frac{s+\omega_n}{2}$ and $p_r = \frac{s\alpha+\omega_r}{2}$ are substituted into Π_p^D. Because of $\frac{\partial^2 \Pi_p^D}{\partial \omega_r^2} = -\frac{1}{s(1-\alpha)\alpha} < 0$, then Π_p^Dhas a maximum value, and $\omega_r = \frac{t + c_r + \alpha\omega_n}{2}$ is obtained from $\frac{\partial \Pi_p^D}{\partial \omega_r} = 0$.

Manufacturer's Hessian matrix $H_2 = \begin{bmatrix} -\frac{2-\alpha}{2s(1-\alpha)} & \frac{1}{2s(1-\alpha)} \\ \frac{1}{2s(1-\alpha)} & -\frac{1}{2s\alpha(1-\alpha)} \end{bmatrix}$, and it is easy to know that this Hessian matrix is negative definite, then Π_m is a strictlyconcave function with respect to ω_n and t, so $\omega_n = \frac{s+c_n}{2} t = \frac{s\alpha - c_r}{2}$ are obtained from $\frac{\partial \Pi_r^D}{\partial p_n} = 0$, $\frac{\partial \Pi_r^D}{\partial p_r} = 0$.

Proof of Proposition 2.

In the closed-loop supply chain, the retailer acts as a follower, and the Hessian matrix of its profit function with respect to p_n and p_r is $\begin{bmatrix} -\frac{2}{s-s\alpha} & \frac{2}{s-s\alpha} \\ \frac{2}{s-s\alpha} & -\frac{2}{s\alpha - s\alpha^2} \end{bmatrix}$. It is easy to know that the Matrix is negative definite, then there is a maximum value. According to $\frac{\partial \Pi_r^{MP}}{\partial p_n} = 0$, $\frac{\partial \Pi_r^{MP}}{\partial p_r} = 0$, we can obtain the result $p_n = \frac{s+\omega_n}{2} p_r = \frac{s\alpha+\omega_r}{2}$.

Substituting the above formula into Π_{mp}^{MP}, its Hessian matrix with respect to ω_n and ω_r is $\begin{bmatrix} -\frac{1}{s(1-\alpha)} & \frac{1}{s(1-\alpha)} \\ \frac{1}{s(1-\alpha)} & -\frac{1}{s(1-\alpha)\alpha} \end{bmatrix}$, and the Matrix is negative definite, so $\omega_n = \frac{s+c_n}{2}$ and $\omega_r = \frac{s\alpha + c_r}{2}$ are obtained from $\frac{\partial \Pi_{mp}^{MP}}{\partial \omega_n} = 0$, $\frac{\partial \Pi_{mp}^{MP}}{\partial \omega_r} = 0$.

Proof of Proposition 3.

The decision of the closed-loop supply chain is in line with the Stackelberg game.

The alliance of the retailer and the third-party remanufacturer is a follower in this game. The Hessian matrix of Π_{rp}^{RP} with respect to p_n and p_r is $\begin{bmatrix} -\frac{2}{s-s\alpha} & \frac{2}{s-s\alpha} \\ \frac{2}{s-s\alpha} & -\frac{2}{s\alpha - s\alpha^2} \end{bmatrix}$, and it is easy to know that the matrix is negative definite. According to $\frac{\partial \Pi_m^{RP}}{\partial \omega_n} = 0$, $\frac{\partial \Pi_m^{RP}}{\partial t} = 0$, we have $\omega_n = \frac{s+c_n}{2} t = \frac{s\alpha - c_r}{2}$.

Proof of Proposition 4.

The Hessian matrix of $\Pi_{mrp}$with respect top_n and p_r is $\begin{bmatrix} -\frac{2}{s-s\alpha} & \frac{2}{s-s\alpha} \\ \frac{2}{s-s\alpha} & -\frac{2}{s\alpha - s\alpha^2} \end{bmatrix}$, and it is easy to know that the Matrix is negative definite. According to $\frac{\partial \Pi_{mrp}^C}{\partial p_n} = 0$, $\frac{\partial \Pi_{mrp}^C}{\partial p_r} = 0$, we have $p_n = \frac{s+c_n}{2} p_r = \frac{s\alpha + c_r}{2}$.

Proof of Proposition 5.

The followings give direct incremental profits for different alliances:

$$\Delta\Pi^{MP} = \Pi_{mp}^{MP*} - \Pi_m^{D*} - \Pi_p^{D*} = \frac{A}{32},$$

$$\Delta\Pi^{RP} = \Pi_{rp}^{RP*} - \Pi_r^{D*} - \Pi_p^{D*} = \frac{A}{64},$$

$$\Delta\Pi^{C} = \Pi_{mrp}^{C*} - \Pi_m^{D*} - \Pi_r^{D*} - \Pi_p^{D*} = \Pi_1 + \frac{5A}{64},$$

where $\Pi_1 = \frac{s^2(1-\alpha)\alpha + \alpha c_n^2 + c_r^2 - 2\alpha c_n(s - s\alpha + c_r)}{16s(1-\alpha)\alpha}$; $A = \frac{(\alpha c_n - c_r)^2}{s(1-\alpha)\alpha}$.

It can be seen from the above formula that $\Delta\Pi^{MP}$, $\Delta\Pi^{RP}$ and $\Delta\Pi^{C}$ are all greater than zero, so the third-party remanufacturer can choose any kind of alliance to obtain incremental profits, and the third-party remanufacturer is profit-driven in choosing alliances. Then the mechanism for the third party to select the alliance is: while $\frac{\varphi_p^{MP}}{\varphi_p^{RP}} > \frac{\Delta\Pi^{RP}}{\Delta\Pi^{MP}}$ and $\frac{\varphi_p^{MP}}{\varphi_p^{C}} > \frac{\Delta\Pi^{C}}{\Delta\Pi^{MP}}$, the third-party remanufacturer chooses the MP alliance; when $\frac{\varphi_p^{RP}}{\varphi_p^{MP}} > \frac{\Delta\Pi^{MP}}{\Delta\Pi^{RP}}$ and $\frac{\varphi_p^{RP}}{\varphi_p^{C}} > \frac{\Delta\Pi^{C}}{\Delta\Pi^{RP}}$, it chooses the RP alliance. otherwise the third-party remanufacturer chooses C alliance.

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