

# Perceptive Motion Planning with Friction Constraints for Quadruped Robot Locomotion

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**Abstract**—The legged robots, such as quadruped robots, have the advantage of being able to traverse the types of complex, cluttered environments we are interested in. This advantage brings quadruped robots one step closer to being used in challenging fields from the research perspective, such as industrial and disaster sites. For this to be possible, the robots must be aware of the complex area to traverse and plan the desired footsteps according to the desired walking direction considering the environment. In this paper, we described an end-to-end pipeline of a novel framework from environment perception to footstep planning in order to let the quadruped robots traverse complex environments with friction constraints. The environment perception extracts planar regions that the legged robots can step on from the 2.5-dimension height map with the collected point cloud data. To overcome locomotion over rough terrain, footstep planning optimizes the desired footstep by considering the friction coefficients, and body pose planning determines the pose using the movement strategy for the stair position. The entire computation process of the environment perception completes to detect the planar regions within 80 Hz, and the motion planner generates the optimal foot trajectory and body trajectory to get a specified location. We experimentally validate our approach with the quadruped robot Canine by autonomously navigating stairs, stepping stones over virtual world environments.

## I. INTRODUCTION

Compared to wheeled robots, the primary advantage of legged robots is their exceptional mobility on rough terrain, which has led to the development of robust locomotion for various challenging environments. To make a stable motion in a complex environment, perception ability is important because the robots need to be aware of the environment and select safe foot positions and body poses based on the current state of the robot. However, for stable walking on terrain, the robots need to consider not only the geometry information of the environments, but also the friction. For example, walking safely on slippery surfaces, such as dusty and sandy fields or wet floors from rain, is not easy with general perceptive planning and control algorithms. Among the various unstructured environments, stairs are basic and important because legged robots need to navigate to help and

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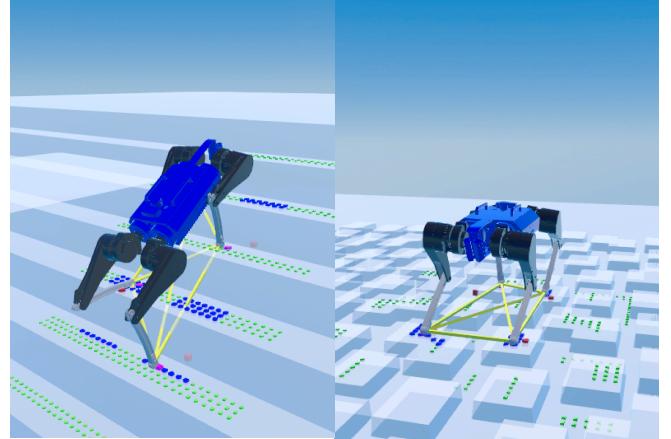


Fig. 1. The quadruped robot Canine is walking on stairs and stepping stones with varying frictions in simulation environments. For stable locomotion, the robot uses the perceptive information of the terrain and the friction coefficients.

replace humans, so we need to be prepared for challenging staircase environments.

In this work, we focus on the footstep and body pose planning framework using perceptive information and friction coefficients information of the floor for the stable traverse of the quadruped robot in staircase environments with various frictions. We show an efficient 2.5-dimensional map processing for the quadruped robot for optimal motion generation and robust locomotion in challenging terrain. The map data processed in real-time has surface normal vectors and filtered segmented planar regions, and is used for scoring in footstep planning with friction coefficients. We also propose a framework to efficiently use the generated map data for stabilizing the body pose when walking on stairs. Based on the map information, the robot obtains the geometric information of the stairs and finds the optimal safe footstep and body pose to prevent slipping and falling considering the four different walking strategies. Fig. 1 shows that the quadruped robot Canine [1] walks without falling on stairs and stepping stones with a friction coefficient of 0.2 in simulation using the proposed motion planning framework.

## II. RELATED WORK

Locomotion over rough terrain for the legged robots is constantly being researched in various ways. Recently, deep reinforcement learning has been used to study walking in limited situations, such as without terrain information from vision sensors [2], [3], but terrain awareness is necessary

for dynamic and agile movement through more complex and unstructured terrain.

A well-known method for environmental perception in quadruped robots is to use a 2.5-dimensional map [4], [5] with height information about the X-Y plane of the ground. A large amount of environment information obtained through 3D cameras such as depth, stereo camera, and LiDAR is mapped for efficient handling and use with an occupancy grid structure. Among previous work in this mapping research, the latest work [6] uses neural networks to implement the elevation map that is more noise-resistant and reconstructs occluded areas. In the same way that bipedal robots such as humanoids detect planar regions of the terrain to consider the sole shape of their feet for walking [7], [8], plane detection is important for quadruped robots to determine the Ground Reaction Force (GRF) relative to the Center of Mass (CoM) accurately. Our terrain mapping framework uses simple methods to efficiently generate 2.5-dimension height maps in real-time, consisting of surface normal vectors and filtered segmented planar regions.

In the approaches presented by [4], [9]-[11], foot positions and body poses are planned using a perceptive map for locomotion. Based on the given terrain map and the robot's state, the robot optimizes by searching and scoring the foot positions and body motions that satisfy the conditions such as stepability and traversability. In the case of motion planners for humanoids [8], [12], [13], the rotation of the foot is considered in the optimization of the footstep and body pose due to the feature that the foot has the support area. The navigation associated with safe traveling planned path using algorithms such as A\*, RRT\* and BIT\* to get the robot to the target point [14], [15]. The path planner uses terrain information to find collision-free paths to traverse and optimizations such as path smoothing to improve traversability. TOWR [16] is a widely used trajectory optimization that optimizes motion using parameterized goals and the states of the robot to generate dynamic locomotion, but it has the disadvantage of taking quite a time to compute.

Model Predictive Control (MPC) is a widely researched method for optimal control under real-time constraints [17]-[20]. It uses a single rigid-body dynamics (SRBD) model to represent the robot dynamics problem as a convex optimization problem with constraints, which can be solved in real-time by reducing the computational complexity. If the dynamics model cannot be defined or the terrain information for locomotion is not available, deep reinforcement learning can be used to achieve control. This approach can be used to learn motion for challenging conditions such as slippery ground, complex rough terrain, and obstacles to perform dynamic locomotion and navigation [21]-[25].

Our approach plans foot positions and body poses for model-based locomotion in challenging environments with friction constraints. The motion planner uses terrain information, the robot's state, and friction coefficients to optimize a safe footstep and pose based on scoring.

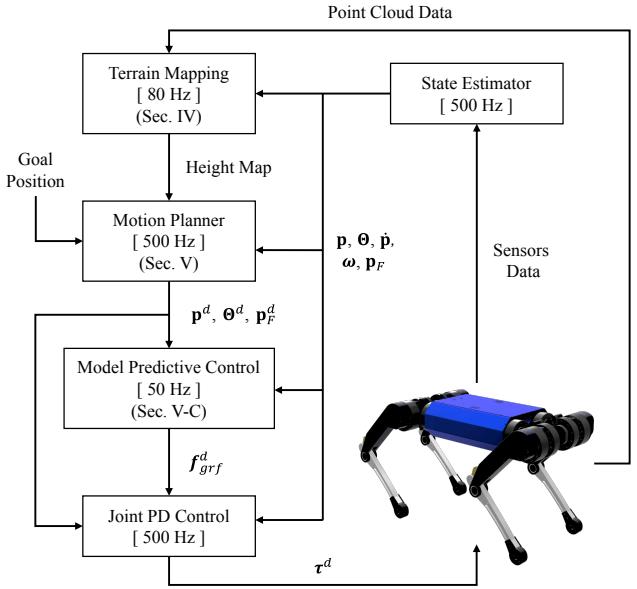


Fig. 2. Overview of the proposed perceptive data processing and motion planning framework for the quadruped robot.

### III. SYSTEM OVERVIEW

We used the quadruped robot Canine which weighs 18 kg and has 0.45 m legs with a fully stretched length, walking at a height of 0.35 m for this work. It walks using the crawl gait, which is a simple and stable gait, and swing legs in the sequence LF, RB, RF, and LB. Our proposed pipeline for locomotion on challenging terrain with friction constraints is shown in Fig. 2. The state estimator fuses the data from sensors such as the IMU and encoder to determine the state of the robot, such as CoM position, position, velocity, orientation, and contact with the ground, and this data is used for mapping, planning and control.

The Terrain mapping module generates the 2.5-dimension height map using the perception data and the robot's pose from the state estimator. The height map has surface normal vectors, planar region detection, and filtering process, and is processed at over 80 Hz. The motion planner, which consists of the footstep planner and the body pose planner, uses the terrain map data, the state of the robot, and the goal position from the user command. It finds footsteps, poses, and generates desired trajectories for safe locomotion. The footstep planner gets candidates from the terrain map, scores them based on kinematic, friction, distance, and edge cost, and selects the highest scoring position as the next desired foot position for robust locomotion. The body pose planner divides the robot's state on the stairs into four states using the terrain map, and uses different strategies for each state to find a stable pose. the position of the CoM and the orientation of the body are calculated by the foot positions, the slope of the stairs, and the friction coefficients.

We utilize the MPC and Joint PD Controller as controllers for optimal locomotion. The MPC control method tracks the predefined desired trajectory generated by the motion

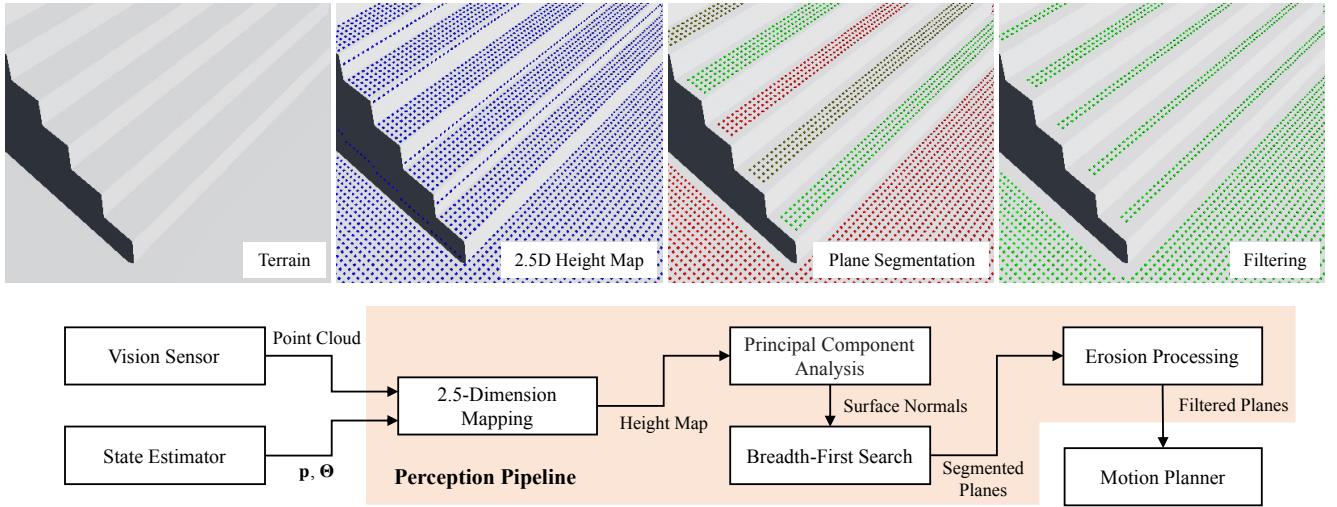


Fig. 3. Overview of the perception pipe. From *top-left* to *top-right*: Staircase terrain created in the simulation environment. Point cloud data for staircase terrain compressed into a 2.5-dimension height map with 3 cm resolution using pose data from the state estimator. The height map is segmented using PCA to get surface normal vectors and BFS to segment the plane. The segmented planes are filtered for noise and margins for edges by erosion process.

planner. Specifically, we employ Linearized Model Predictive Control (LMPC) to govern the robot's movements. LMPC simplifies the robot's dynamics to enhance computational efficiency, allowing for real-time adjustments of the robot's actions and continuous computation of optimal control inputs. This enables accurate tracking of the desired trajectory throughout the robot's operation. In addition, to increase control stability and follow the joint desired by the motion planner, the Joint PD Controller is added to provide feedback on the joint state values.

#### IV. TERRAIN AWARENESS

The planning and control for locomotion use a 2.5-dimensional height map with terrain information. The map used is the local map with the planar region and edge information at 3 cm x 3 cm resolution in a 1.0 m x 1.0 m range from the quadruped robot and is generated by the CPU over 80 Hz. Fig. 3 shows the pipeline of terrain mapping about the staircase.

##### A. Height Map

One of the representative vision processing used for legged robots walking is the height map, which has height information about the ground. Also known as a 2.5 dimension map, it compresses a large amount of 3D data of the entire physical world so that it only has height information for two dimensions, X-Y plane of the ground, to reduce the amount of computation and make it easier to use. The point cloud data, which has three-dimensional information about the terrain, processes the data collected in each cell using a grid of X-Y plane in a certain range based on the robot, and selects a representative height Z value to generate a map.

The map is generated with a resolution of 3 cm x 3 cm and a range of 1.0 m x 1.0 m to be aware of the terrain. In this work, the height map was generated using the value of the map information used in the simulation, and in the

physical world, the height map can be generated using 3D vision sensors such as depth, stereo camera and LiDAR.

##### B. Planar Region Segmentation

Planar region segmentation is performed using a height map. To get planar regions, find the surface normal vector of each grid cell in the map and compare the values to form a plane. Use principal component analysis (PCA) to estimate the surface normal vector by finding the eigenvectors of the cell's neighbors.

To construct the planes using the estimated surface normal vectors, use the Breadth-First Search (BFS) algorithm, which is a simple and effective method. It determines whether planes are constructed by comparing the surface normal vectors of all cells in the height map to check if they are over a certain threshold. Also, if the number of cells that compose a constructed plane is under a certain value, it is not considered to be a plane. This process not only detects

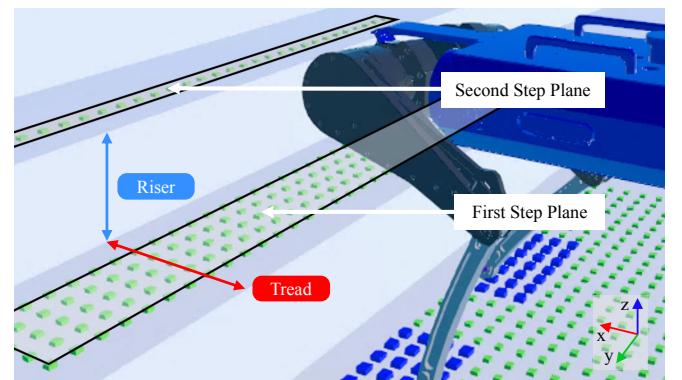


Fig. 4. Determine the size of the stairs using the segmented plane information from the height map. The riser is derived as the distance between the Z-axis of the two planes, and the tread is calculated using the resolution of the map and the number of data that compose the plane.

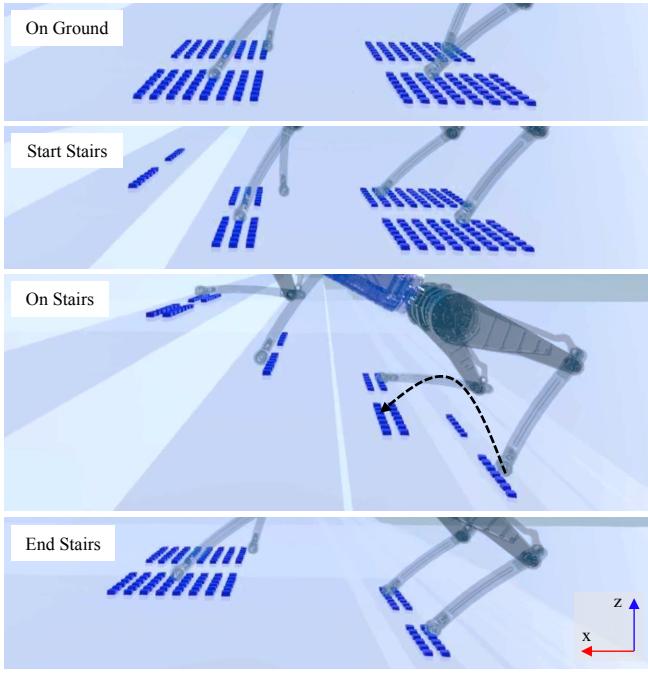


Fig. 5. Use map data to determine the states of the robot for the staircase. The difference in the number of search candidates for footstep planning in the filtered map is used to classify four states; On Ground, Start Stairs, On Stairs, and End Stairs (From top to bottom). When the number of candidates changes, the robot switches to the next state, and body pose planning is applied to each strategy.

planes but also removes edges of environments.

### C. Filtering

The filtering process is performed to remove noise such as outliers in the generated height map and adds margins to the edges. The filter uses the erosion operation, which is one of the morphology processing [18], and is performed on the occupancy information of the height map. The erosion process is calculated as a convolution of a  $3 \times 3$  cross-shaped kernel with the entire height map data, and the kernel is shown below:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad (1)$$

The filtered height map is used for safe motion planning, and especially, the edges of the segmented planes are eroded to give the margin to prevent stepping on edges.

### D. Stairs Awareness

The more information the robot has about the stairs, the better for stable navigation. The geometric information such as tread, riser, and angle of the stairs from the map data and robot state estimates for the stairs are used for locomotion planning.

Since we know the resolution value of the height map, we can use the size of the segmented plane to calculate the stairs' tread value. In order to use the complete planar region for the size of the first step stair, the calculation must be done

at the time when the second step plane is partially generated. Therefore, if the complete first step plane is generated, use the resolution to find the tread and compare its height with the second stair to get the riser. (Fig. 4)

The stair climbing strategies are divided into four states based on the robot's position on the stairs as shown in Fig. 5. The blue cubes represent candidate positions for the quadruped robot to step, which are generated from the height map information of a specific range at the  $x$ ,  $y$  position of each shoulder of the robot. We constructed a state machine that switches states and strategies by comparing the difference between the number of candidates in the front with the back. On the ground with the same number of candidates in front and back, the first time the number of candidates starts to differ, the robot identifies that a stair is starting, the next time the number of candidates is the same, it identifies that the body is on the stairs, and the next time the number of candidates differs, it identifies that the body has reached the end of the stairs. It determines that the state change has occurred when the number of candidates differs by more than a specific value. For each state, the planning strategies are divided to optimize the motion for stable traversal of challenging terrain.

## V. MOTION PLANNING AND CONTROL

To traverse complex terrains with friction constraints, it is important to optimize footstep and body pose. The robot plans the next desired foot position by scoring the footstep safety using the height map information introduced in the previous section, and plan the desired body pose according to the stair information obtained from the map, state, and foot positions. The optimal desired feet and body poses obtained by the motion planner are created optimal trajectories by the trajectory generator for the controller.

### A. Footstep Planning

Footstep planning is achieved by scoring steppable candidates using the terrain map, kinematic constraints of the



Fig. 6. The desired foot position found in the footstep planning process is determined by scoring on the search candidate, which is the value of the search region created from the robot's shoulder  $x$ ,  $y$  position in the height map data.

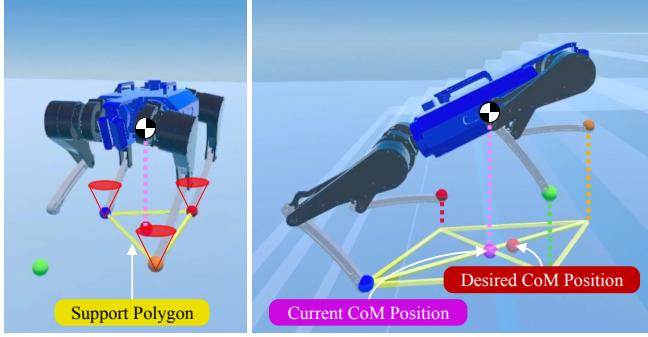


Fig. 7. The desired CoM position and the body orientation are determined by considering the friction cone, the size of the stairs and the support polygon, which is determined by the foot position.

leg depending on the robot's state, and friction constraints. In Fig. 6, the green cube is the generated height map, blue is the candidate, red is the nominal foot position, and pink is the next desired foot position derived by planning. The nominal foot position for a stable stance is the position on the terrain corresponding to the  $x$ ,  $y$  position of each shoulder of the robot. Candidates are height map data that is contained in a certain range from the shoulders of the body. Footstep planner selects the position with the highest cost score from the candidates as the next footstep with the following equations:

$$\mathbf{p}_{F,i}^d = \arg \max_{p \in \mathbf{P}_{c,i}} f_{score}(p) \quad (2)$$

$$f_{score}(p) = w_1 c_{kin} + w_2 c_{frict} + w_3 c_{dist} + w_4 c_{edge} \quad (3)$$

In Eq. (2),  $\mathbf{p}_{F,i}^d$  is the next desired foot position to be selected, and  $p$  is the navigation candidate of  $\mathbf{P}_{c,i}$ .  $i$  denotes the index of the foot and represents LF, RF, LB and RB in order of index. The kinematic cost  $c_{kin}$  is calculated by considering the kinematic constraints as the reachability of the leg, and the friction cost  $c_{frict}$  is calculated using the friction coefficients. These two values relate to the search region that determines the candidates, and the lower the friction coefficient, the smaller the region. We added the distance cost  $c_{dist}$ , which gives a higher cost the farther away from the current foot position the robot is heading, and the edge cost  $c_{edge}$ , which gives a higher cost the farther away from the edge of the terrain map. Our approach finds the appropriate footstep to efficiently and moderately move the body in the next gait sequence while considering the friction coefficients. The weight factors  $w_1$ ,  $w_2$ ,  $w_3$  and  $w_4$  experimentally tuned to the better behavior.

### B. Body Pose Planning

Finding the optimal body positions and orientations is very important for walking on unstructured terrain, such as climbing stairs. To navigate terrain safely, the robots need to be careful not only when stepping foot, but also when moving the body to consider the friction cone. When climbing a staircase, the pose planning for the body uses the currently located positions of the foot, geometric information of stairs,

stairs states based on the robot position (Section IV-B), and the friction constraints of environments.

The desired pitch angle of the body  $\theta^d$  is found by the following equation, depending on the states of the robot with regard to the stairs:

$$\theta^d = \begin{cases} 0 & \text{on ground} \\ \theta_{stairs} & \text{on stairs} \\ f_\theta(\mathbf{p}_F) & \text{otherwise} \end{cases} \quad (4)$$

where  $\theta_{stairs}$  is the angle that gets from the stairs awareness process with the tread and riser information of the stairs, the function  $f_\theta$  computes a pitch angle using current foot positions  $\mathbf{p}_F = \{\mathbf{p}_{F,0}, \mathbf{p}_{F,1}, \mathbf{p}_{F,2}, \mathbf{p}_{F,3}\}$ . The desired roll angle and desired yaw angle of the body are calculated regardless of the states of the stairs, with the roll kept at 0 and the yaw calculated using the goal direction.

The desired position of body  $\mathbf{p}^d$  is the position of the center of mass (CoM), which is calculated as follows depending on the states:

$$\mathbf{p}^d = f_{SP}(\mathbf{p}_F) + \begin{cases} 0 & \text{on ground} \\ \lambda_{start}\theta^d & \text{start stairs} \\ \lambda_{on}\theta^d + \lambda_{frict}\mu & \text{on stairs} \\ \lambda_{end}\theta^d & \text{end stairs} \end{cases} \quad (5)$$

To determine the desired position of the CoM of a stable body, the robot uses the support polygon that is constructed from the positions of the foot in the stance phase of the gait table and places the CoM inside the support polygon. In Eq. (5), the function  $f_{SP}$  calculates the basic CoM position by generating a support polygon using positions of foot  $\mathbf{p}_F$ . For more stable stepping at low frictions, additional calculations are made for offset polygons. Using the desired body pitch angle  $\theta_B^d$  found from Eq. (4) and friction coefficient  $\mu$ , calculate the stable CoM position at the location of each stage. The parameters  $\lambda_{start}$ ,  $\lambda_{on}$ ,  $\lambda_{end}$  and  $\lambda_{frict}$  used in the calculation are tuned heuristically to the preferred robot behavior.

Fig. 7 shows the support polygons and CoM positions considered in the body pose planning process in the On Ground state and On Stairs state. Our planning method finds the appropriate orientation of the body for the four stairs states and determines the CoM position by considering the friction.

### C. Model Predictive Control

The robot is controlled by LMPC to follow the estimated footstep and body position trajectory. The dynamics of LMPC represented in Eq. (6) is simplified to SRBD for computational efficiency [1], [26].

$$\begin{aligned} m\ddot{\mathbf{p}} &= \sum_{i=1}^c \mathbf{f}_i - mg\mathbf{g} \\ \frac{d}{dt}(\mathbf{I}\omega) &= \sum_{i=1}^c (\mathbf{r}_i \times \mathbf{f}^i) \end{aligned} \quad (6)$$

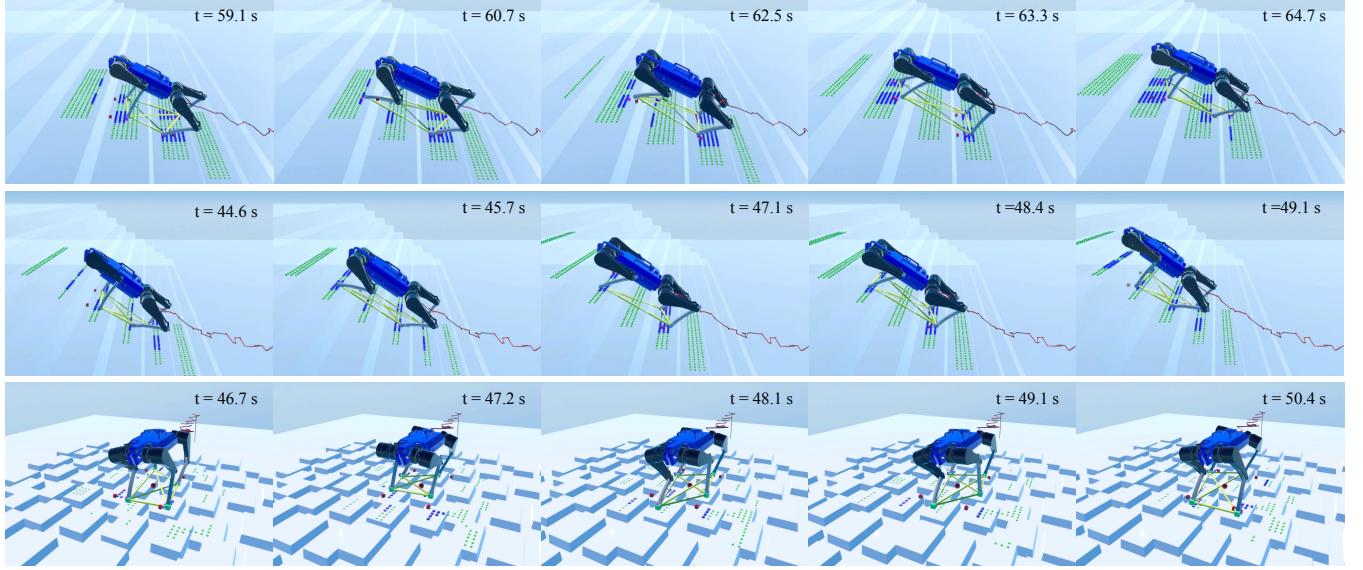


Fig. 8. Results of the Canine walking over three different simulation cases using our motion planning framework in the Raisim. The friction coefficient  $\mu$  is the same in these three results, 0.2. From *top* to *bottom*: Staircase with 30 cm tread and 13.5 cm riser. Staircase with 20 cm tread and 15 cm riser. Stepping stones at a 5 m distance.

where  $m$  is the robot math simplified,  $\mathbf{p}$  is the global robot position,  $c$  is the number of leg contacted with ground, which means stand leg,  $f_i$  is the GRF on the  $i$ -th leg,  $g$  is gravity,  $I$  is inertia moment of the simplified robot,  $w$  is the angular velocity, and  $r_i$  is the foot position of  $i$ -th stand leg on the robot frame.

The robot state,  $\mathbf{X} = [\Theta^\top \quad \mathbf{p}^\top \quad \omega^\top \quad \dot{\mathbf{p}}^\top]^\top$ , consists of orientation, global position, angular velocity, and linear velocity. We solve the LMPC problem given below in Eq. (7) to estimate force vector  $\mathbf{U}_k = [f_1^\top \quad \dots \quad f_i^\top]^\top$  with the number of contact leg,  $i$ .

$$\min_{\mathbf{X}, \mathbf{U}} \sum_{k=1}^{n-1} \|\mathbf{X}_{k+1} - \mathbf{X}_{k+1}^d\|_{Q_k} + \|\mathbf{U}_k\|_{R_k} \quad (7)$$

$$\text{subject to } \begin{aligned} \mathbf{X}_{k+1} &= \mathbf{A}_k \mathbf{X}_k + \mathbf{B}_k \mathbf{U}_k + g \\ \underline{\mathbf{c}}_k &\leq \mathbf{C}_k \mathbf{U}_k \leq \bar{\mathbf{c}}_k \end{aligned} \quad (8)$$

where Eq. (8) represents the dynamics equality constraint and friction cone inequality constraints. Notably, the friction constraint takes into account the ground friction coefficient specific to each experiment. By considering this coefficient, the controller ensures that the foot forces and body motion are optimized to satisfy the necessary constraints, thus facilitating stable locomotion.

## VI. RESULTS

We set up simulation environments and tested them with different friction coefficients to validate our proposed motion planning framework. Compare the baseline and our framework with respect to success rate and slip distance over the terrains.

TABLE I  
RESULTS OF STAIRS CASES AND STEPPING STONES

Cases	Tread	Riser	Avg. Speed [cm/s]		Success Rate	
	[cm]	[cm]	Baseline	Ours	Baseline	Ours
Stairs I	30	13.5	3.7	3.9	13/20	20/20
Stairs II	20	15	3.4	3.8	5/20	18/20
Step. Stones	.	.	4.3	4.1	17/20	20/20

### A. Experimental Setup

We used the physics simulation environment [27] to create various environments with different friction coefficients and experimented with the quadruped robot Canine. Four friction coefficients are used in the test, and two types of stairs are constructed, one with a gentle incline and the other with a steep incline. In addition, a stepping stone environment is also configured to show the applicability not only to the staircase environment but also to various terrains. In the experiments described in the results, the robot traversed using the given goal position.

### B. Validation Results

To validate our proposed motion planner, the baseline against which it is compared is planning with only goal position and basic terrain map information. As shown in Fig. 8, we set experiments on two types of stairs and the case of a stepping stone, and the friction coefficient used was 0.2. The height map generated through our terrain mapping framework for the perceptive motion planning has a resolution of 3 cm, and can be found in Fig. 8, where segmented planes and filtered for edges' margins. Margins are placed at 6 cm offset from the edge of the plane, twice the resolution of the map. The robot in the snapshots also

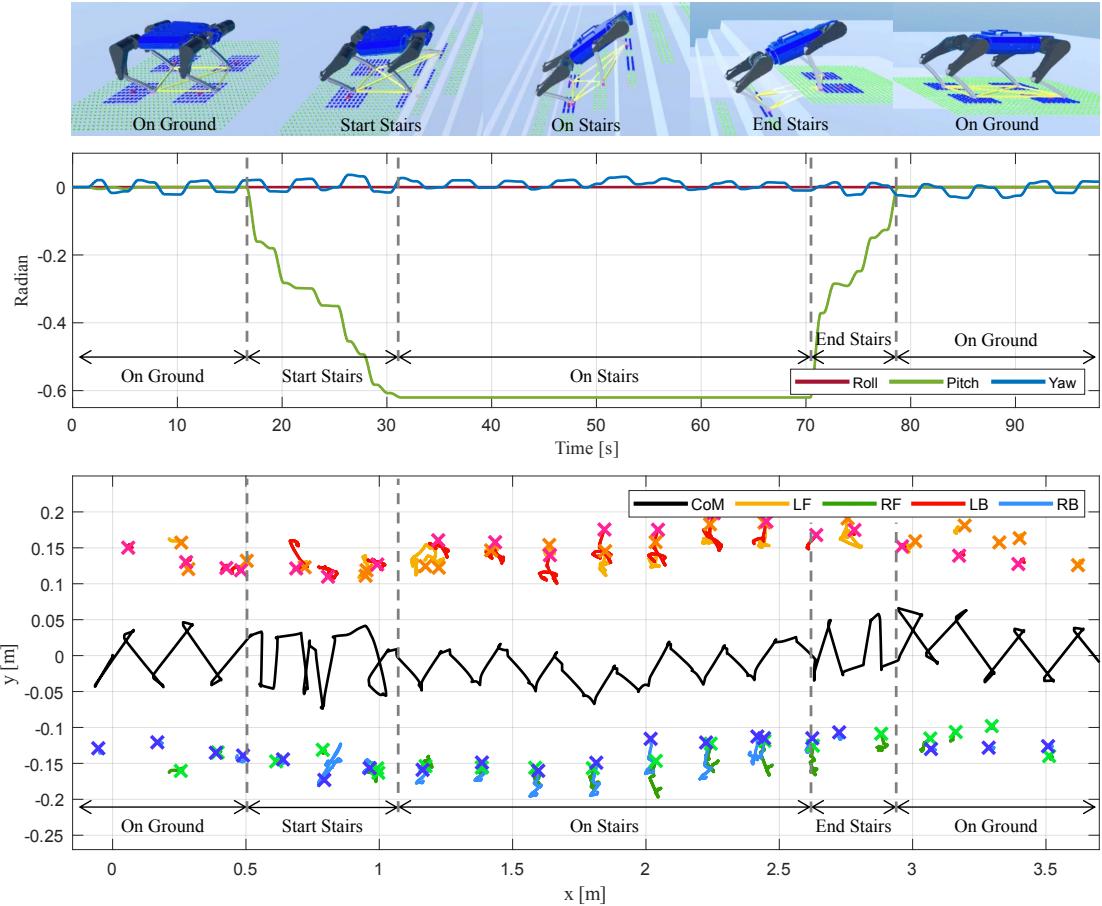


Fig. 9. The snapshots of climbing the stairs (*top*), the orientation of body (*middle*), global positions of body and foot (*bottom*) while climbing Stairs case II with a friction coefficient of 0.2. These results show that the pose of the body and footstep are determined by the four states on the stairs. In the results for foot positions, the cross represents the position where the first step was taken, and the line shows that the slip occurred during locomotion.

TABLE II  
MAXIMUM SLIPPED DISTANCE OF FEET ABOUT  
DIFFERENT FRICTION COEFFICIENTS

	Friction Coefficient			
	0.8	0.4	0.2	0.1
Baseline	1.684 cm	6.172 cm	8.095 cm	17.655 cm
Ours	2.002 cm	2.216 cm	4.639 cm	8.344 cm

recognizes that it is in the On Stairs state through the number of candidates and plans motion using the On Stairs strategy.

The results are summarized in Table I. The stairs used in the experiment are designed as Stairs I with a gentle incline of 24.288 degrees and Stairs II with a steep incline of 36.870 degrees with 10 steps in both cases. The average speeds are calculated by taking the horizontal distance traveled and the time when the robot successfully traverses without falling over. Since the case of the stepping stones has a small incline, the baseline also has a high success rate compared to the other cases. However, the experimental results in the stairs cases show that walking with our method is more stable. The GRF required to climb terrain with an incline is more significant than walking on a flat surface. For this reason,

climbing stairs with low friction coefficients are easy to slip because the GRF can easily move out of the friction cone. The results for the success rate show that our proposed motion planning avoids falls in challenging environments, but the bigger the incline angle, the more instability, even for the same coefficient of friction.

Fig. 9 shows the snapshots (*top*) of the traverse using our proposed motion planning framework in the experiment on staircase II and the results about the body orientation (*middle*), CoM position, and foot position (*bottom*) obtained at that traverse. These show that according to the proposed strategy, the pitch angle determination maintains the pitch angle using the foot positions in the Start Stairs state and End Stairs state, and the incline of the stairs obtained through the map in the On Stairs state. The CoM position is also shown to be planned based on each strategy. In the On Stairs state, the CoM movement is minimized so that the GRF does not exceed the friction cone. The cross indicates the first stepping position of each foot position, and if slipping occurs during the climb, the path of the slip is represented by the line. When walking on the ground, there is not much slip, but in other states, the GRF is out of the friction cone and slip occurs. In this result, the slips are up to about 5 cm from

the first stepping position.

Lastly, we experimented with different friction coefficients for the Stairs II case to see how much slip happens. The friction coefficients used in the experiment are 0.8, 0.4, 0.2, and 0.1. The slipped distance, shown in Table II is calculated from the first step position to the slipped position and is the maximum value among the values before falling. As the friction coefficients become smaller, the distance at which both the baseline and our framework slip increases, but the results show that our method slips more than twice as less as the baseline.

## VII. CONCLUSION

We present a motion planning framework for perceptive locomotion with friction coefficients for quadruped robots. Based on information about the terrain and the friction coefficients, the framework generates footsteps and body poses that make the robot more stable. The framework efficiently processes a 2.5-dimensional height map and generates it in real-time. Using the terrain map, the robot plans its footsteps through safe scoring, finds a stable pose depending on the strategies for the states of stairs, and generates a foot trajectory and CoM trajectory for control. For evaluation, we built an environment of stairs and stepping stones with different friction coefficients in the simulation, and successfully navigated to the specified goal position at a moderate speed and without falling. However, the user must give the friction coefficient information to be used in the proposed method. For future work, we will study how to determine the friction coefficient of the ground during locomotion in various environments and test it in a real-world environment. We also aim to make walking more stable, using different gaits in different environments and situations.

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