Applied Statistics for Business & Economics [STA103, SS2 2025]
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Assignment #**3** (Due by noon, Sep 8th, Mon)

Instructions

- Show detailed calculations and clearly state conclusions in context.
- Round results to 4 decimal places unless otherwise noted.
- The in-person midterm may include similar problems, and you will need to solve them on your own from scratch. Use LLMs at your own risk. Overreliance on LLMs may hinder your understanding of the underlying concepts.

1 Inverse variance weighted averaging (Lecture 9)

We want to estimate an unknown mean parameter μ using three independent random samples:

$$X_1 \sim N(\mu, 4), \quad X_2 \sim N(\mu, 9), \quad X_3 \sim N(\mu, 16).$$

A natural first estimator is the simple average:

$$\hat{\mu} = \frac{X_1 + X_2 + X_3}{3}.$$

(a) (10pt) Compute the mean squared error (MSE) of $\hat{\mu}$, i.e., MSE($\hat{\mu}$).
Hint: First prove that $\hat{\mu}$ has no bias. For an unbiased estimator, the MSE($\hat{\mu}$) equals its variance $\mathrm{Var}(\hat{\mu})$.

A more general class of estimators is given by weighted averages:

$$\hat{\mu}^w = w_1 X_1 + w_2 X_2 + w_3 X_3,$$

where the weights w_1, w_2, w_3 are non-negative and $w_1 + w_2 + w_3 = 1$.

The **inverse-variance weights** are given by

$$w_i = \frac{\frac{1}{\mathrm{Var}(X_i)}}{\frac{1}{\mathrm{Var}(X_1)} + \frac{1}{\mathrm{Var}(X_2)} + \frac{1}{\mathrm{Var}(X_3)}}, \text{ for } i = 1, 2, 3.$$

(b) (5pt) Compute the explicit values of w_1, w_2 and w_3 for this problem.

(c) (5pt) Compute the mean squared error (MSE) of $\hat{\mu}^w$, i.e., $\mathrm{MSE}(\hat{\mu}^w)$. *Hint: As in part (a), first prove that $\hat{\mu}^w$ has no bias, then MSE($\hat{\mu}^w$) equals its variance* $\mathrm{Var}(\hat{\mu}^w)$.

(d) (5pt) Compare the MSE of the simple average $\hat{\mu}$ in (a) and the inverse-variance weighted average $\hat{\mu}^w$ in (c). Which estimator do you think is better, and why?

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2 Quality Control for Light Bulbs (Lecture 10 - 11)

You are in charge of quality control for a warehouse full of light bulbs. You want to estimate the average lifetime (in hours) of the bulbs in the warehouse.

Let the unknown mean lifetime of the old bulbs be μ_A with variance σ_A^2. You take a sample of 36 bulbs, $X_1, X_2, \ldots, X_{36}$, and measure their lifetimes. The results are summarized as:

$$\hat{\mu}_A = \frac{X_1 + X_2 + \cdots X_{36}}{36} = 831, \quad \hat{\sigma}_A = \sqrt{\frac{1}{35}\sum_{i=1}^{36}(X_i - \hat{\mu}_A)^2} = 42.$$

(a) (5pt) Find the 95% confidence interval for μ_A based on the CLT and the z-score approximation.

(b) (5pt) Suppose past records claim that the true mean lifetime is $\mu_A = 850$. Based on your confidence interval from (a), does this claim seem reliable? Explain why or why not.

(c) (10pt) Perform a hypothesis test at significance level $\alpha = 0.05$ that the true average lifetime of bulbs is 850 hours,

$$H_0 : \mu_A = 850, \quad H_a : \mu_A \neq 850.$$

State the test statistic, p-value, and conclusion.

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The company has developed a new technology for producing bulbs. Let the mean and variance of the new bulb lifetimes be μ_B and σ_B^2.

You take a sample of 49 bulbs, with results:

$$\hat{\mu}_B = \frac{Y_1 + Y_2 + \cdots Y_{49}}{49} = 868, \quad \hat{\sigma}_B = \sqrt{\frac{1}{48}\sum_{i=1}^{48}(Y_i - \hat{\mu}_B)^2} = 58.$$

(d) (10pt) Perform a hypothesis testing at significance level $\alpha = 0.01$, whether the new bulbs have a longer lifetime on average than the old bulbs:

$$H_0 : \mu_A \geq \mu_B, \quad H_a : \mu_A < \mu_B.$$

State the test statistic, p-value, and conclusion.

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3 Express Shipping Usage (Lecture 10-11)

Historically, 70% of customers have chosen the express shipping option. To check if this rate has changed, you take a recent sample of $n = 150$ orders. In the sample, 96 customers chose express shipping.

(a) (5pt) Compute the sample proportion of express shipping orders. Then construct a 95% confidence interval for the true proportion. (Use the plug-in estimator for the standard error of the sample proportion.)

(b) (5pt) Based on your confidence interval from (a), does this historic data that 70% of customers chose the express shipping seem reliable? Explain why or why not.

(c) (5pt) Our ultimate goal is to test at the 5% significance level, whether the true proportion of customers choosing express shipping has changed compared to the historical value of 0.70. As a first step, clearly state the null and alternative hypotheses.

(d) (10pt) Carry out the hypothesis test from (c). Report the test statistic, p-value, and conclusion at $\alpha = 0.05$.

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4 Advertising and Monthly Revenue (Lecture 12)

A retail company collects 12 months of data on ad spending (x, in \$1000s) and revenue (y, in \$1000s). The fitted model is:

$$\hat{y} = 120 + 8.5x \quad \text{with} \quad \mathrm{SE}(\hat{\beta}_1) = 1.2.$$

(a) (5pt) Interpret the slope and intercept in context.

(b) (10pt) Perform a hypothesis test at significance level $\alpha = 0.01$ to assess if advertising spending significantly affects revenue. (Use the CLT and the z-score approximation).

(c) (5pt) Construct a 95% confidence interval for the slope.