Applied Statistics for Business & Economics [STA103, SS2 2025]
Wookyeong Song, UC Davis

Assignment #**1** (Due by noon, Aug 12nd, Tue)

Instructions

- Show detailed calculations and clearly state conclusions in context.
- Round results to 4 decimal places unless otherwise noted.
- The in-person midterm may include similar problems, and you will need to solve them on your own from scratch. Use LLMs at your own risk. Overreliance on LLMs may hinder your understanding of the underlying concepts.

1 Summation (Lecture 1)

Let $x_1, \ldots, x_n$ be observations and the sample mean be

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

(a) (10pt) Show that

$$\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right) - \bar{x}^2.$$

Hint 1: Start by expanding the square inside the sum in the left hand side: formula $(a-b)^2 = a^2 - 2ab + b^2$.

Hint 2: Note that $\bar{x}$ *does not depend on the index* i, *meaning that* $\bar{x}$ *can be considered as a fixed constant. We know that* $\sum_{i=1}^{n} c = n \cdot c$, *where* c *is a fixed constant.*

(b) (5pt) Verify the identity for the data $(x_1, x_2, x_3) = (1, 3, 5)$ by computing both sides numerically.

2 Expected Profit and Risk of a Stock Investment (Lecture 2)

A certain stock currently trades at \$3 per share. Suppose your friend tells you that tomorrow a certain stock will with go up to \$10 with probability 0.8 or will go down to \$1 with probability 0.2.

If you purchase 1 share today at the current price, let random variable X denote your profit tomorrow.

(a) (10pt) Provide a probability mass function (PMF) of X in **either** table form or function form.

(b) (10pt) Compute the expected profit $E[X]$ and the standard deviation $\mathrm{sd}(X)$ of the profit.

If you purchase 100 shares today at the current price, let random variable Y denote your profit tomorrow.

(c) (10pt) Compute the expected profit $E[Y]$ and the standard deviation $\mathrm{sd}(Y)$ of the profit.

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3 Monopoly Game (Lecture 3)

In the board game Monopoly, a player in jail may get out by rolling doubles with two fair dice. If the player does not roll doubles, they remain in jail for another turn. Let X be the first roll on which the player rolls doubles.

(a) (10pt) Find a PMF of X in function form.

Hint: You may need to search "Geometric distribution" on Wikipedia.

Suppose that if the player has not rolled doubles by the end of their third try, they must pay $50 to get out.

(b) (10pt) What is the probability that a player who is in jail will get out of jail before having to pay the $50 fine?

4 Variance and Independence in Ticket Draws (Lecture 4)

Suppose you randomly choose two tickets (with replacement) from a box containing the following 4 tickets:

$$\boxed{\begin{smallmatrix}0\\0\end{smallmatrix}} \quad \boxed{\begin{smallmatrix}0\\1\end{smallmatrix}} \quad \boxed{\begin{smallmatrix}1\\0\end{smallmatrix}} \quad \boxed{\begin{smallmatrix}1\\1\end{smallmatrix}}$$

Let $\boxed{\begin{smallmatrix}X_1\\Y_1\end{smallmatrix}}$ and $\boxed{\begin{smallmatrix}X_2\\Y_2\end{smallmatrix}}$ denote these two random tickets.

(a) (10pt) Compute $\mathrm{Var}(X_1 - Y_1)$ and $\mathrm{Var}(X_2 - Y_2)$.

(b) (5pt) Compute $\mathrm{Var}(X_1 - Y_2)$?

(c) (5pt) Determine whether X_1 and Y_1 are independent. Justify your answers.

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5 Lecture Note Problems

(a) (5pt) Let X have PMF given by:

$$p(x) = \begin{cases} \binom{2}{x}(0.4)^x(0.6)^{2-x} & \text{for } x \in \{0, 1, 2\}, \\ 0 & \text{otherwise.} \end{cases}$$

Note: The binomial coefficient, $\binom{n}{k}$, is defined by the expression:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \qquad k! = k \cdot (k-1) \cdot (k-2) \cdots 2 \cdot 1.$$

Find $E(X)$.

(b) (5pt) Suppose random variables W have the following PMF table:

w	$p(w)$
0	35/47
2	2/47
4	10/47

Find $E(W^2)$, $\mathrm{Var}(W)$, $E(5 - W)$, and $E(\exp(W))$?

(c) (5pt) Let X represent the number of units of a product sold per day at a local street market. The probability mass function (PMF) is given below:

x (Units Sold)	0	1	2	3
$p(x)$	0.1	0.3	0.4	0.2

Let $Y = 2X - 3$. Find the variance $\mathrm{Var}(Y)$ and standard deviation $\mathrm{sd}(Y)$?