STA 103 (SS2 2025): Problem Set 1
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TOPICS COVERED

- Summation notation: $\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n$
- Linear properties of summation:
  1. $\sum_{i=1}^{n}(ax_i) = a(\sum_{i=1}^{n} x_i)$
  2. $\sum_{i=1}^{n}(x_i + b) = (\sum_{i=1}^{n} x_i) + nb$
  3. $\sum_{i=1}^{n}(x_i + y_i) = (\sum_{i=1}^{n} x_i) + (\sum_{i=1}^{n} y_i)$
- Suppose $y_i = ax_i + b$ and $\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = v$, then:
  1. $\sum_{i=1}^{n} y_i = a(\sum_{i=1}^{n} x_i) + nb$
  2. $\overline{y} = a\overline{x} + b$
  3. $\frac{1}{n}\sum_{i=1}^{n}(y_i - \overline{y})^2 = a^2 v$
- Probability Mass Functions (PMFs); Box Models; PMF tables; PMF formulas; and converting between various formats
- Expected value $E(X)$, variance $\mathrm{Var}(X)$, and standard deviation $\mathrm{sd}(X)$.
- Variance computing formula $\mathrm{Var}(X) = E(X^2) - (E(X))^2$ versus variance conceptual formula $\mathrm{Var}(X) = E[(X - E(X))^2]$.
- $E(X)$ for prediction; $\mathrm{sd}(X)$ for typical prediction error.
- Expected value of a function of a random variable.
- Conditional PMF: modifying a PMF given partial information on the outcome of the random variable.
- Dependent random variables
- Joint probability mass functions
- Conditional expected value
- Independent random variables

PRACTICE PROBLEMS

Lecture 1

1. Suppose $\sum\limits_{i=1}^{n} x_i = 118$, $\sum\limits_{i=1}^{n} x_i^2 = 750$ and $n = 25$. Find $\sum\limits_{i=1}^{n}(x_i - \bar{x})^2$.

2. Suppose $\sum\limits_{i=1}^{n} x_i = 81, \sum\limits_{i=1}^{n} x_i^2 = 509, n = 75$ and $y_i = (x_i - 2)^2$. Find $\sum\limits_{i=1}^{n} y_i$.

3. Let $x_1 = 2, x_2 = 2, x_3 = 5, x_4 = 7$. Compute $\sum\limits_{n=1}^{4}(n + x_n)$.

4. Let $x_1, x_2, \ldots, x_n$ be a list of n numbers such that
$$\frac{1}{n}\sum_{i=1}^{n} x_i^2 = 10 \quad \text{and} \quad \frac{1}{n}\sum_{i=1}^{n} x_i = 2.$$
Find $\frac{1}{n}\sum\limits_{i=1}^{n}(x_i - 1)(x_i + 1)$.

5. Suppose $\frac{1}{n}\sum\limits_{i=1}^{n}(x_i - \bar{x})^2 = 1202$ and define $y_i = 2x_i - 5$. Find $\frac{1}{n}\sum\limits_{i=1}^{n}(y_i - \bar{y})^2$.

Lecture 2

1. Consider a box full of tickets, where 79% of the tickets have a "1" written on it, the rest have a "0". I randomly choose one ticket and let X be the ticket number I get. What is $\mathrm{Var}(X)$?

2. Let X be a random variable with PMF given by the following table:

x	$p(x)$
-1	0.2
0	0.1
1	0.7

Find $E\big(X/n\big)$, where n is a fixed constant.

3. Let W be a random random variable with PMF given by:
$$p(w) = \begin{cases} 0.7 & \text{if } w = -2 \\ 0.15 & \text{if } w = 0 \\ 0.05 & \text{if } w = 1 \\ 0.1 & \text{if } w = 5 \end{cases}$$
Find $E(W)$.

4. Let $\boxed{\begin{smallmatrix} X \\ Y \end{smallmatrix}}$ be a random draw from the following box of tickets:

$$\begin{array}{ccccc} 3\times & 3\times & 2\times & 4\times & 3\times \\ \boxed{\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}} & \boxed{\begin{smallmatrix} 0 \\ 2 \end{smallmatrix}} & \boxed{\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}} & \boxed{\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}} & \boxed{\begin{smallmatrix} 2 \\ 0 \end{smallmatrix}} \end{array}$$

Box with 15 tickets

Note: the number on the top of each ticket tells you how many copies of that ticket are in the box.

Let $W = X^2 + Y^2$. Find $E(W)$.

5. Let X be a random variable from the following list

$$0, 0, 0, 0, 1, 2, 2, 7.$$

Find $E(X)$.

6. Suppose Z is a random variable with the following probability mass function

$$p(z) = \begin{cases} 0.09 & \text{if } z = 10 \\ 0.32 & \text{if } z = 4 \\ 0.17 & \text{if } z = -14 \\ 0.24 & \text{if } z = 7 \\ c & \text{if } z = 1 \end{cases}$$

What is the value of c?

7. Suppose X is a random variable such that $E(X^2) = 14$, $\text{Var}(X) = 10$, and $E(X)$ is known to be negative. What is $E(X)$?

8. Let $X = Y^2$ and PMF of Y is given by

$$p(y) = \begin{cases} .7 & \text{if } y = -1 \\ .25 & \text{if } y = 0 \\ .05 & \text{if } y = 1 \end{cases}$$

Find the PMF of X in a table form of the random variable.

9. Let X be a random variable with probability mass function given in the following table

x	$p(x)$
0	0.091
1	0.218
2	0.261
3	0.209
4	0.125
$\vdots$	$\vdots$

Find $P(X \geq 3 \text{ or } X = 0)$.

10. Suppose X is a random variable such that $E(X) = -5$, $\text{Var}(X) = 100$. What is $E(X^2)$?

11. Let X be a random variable following probability mass function:

$$p(x) = \begin{cases} \frac{2^x e^{-2}}{x!}, & \text{if } x = 0, 1, 2, 3, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

Calculate $P(X = 0 \text{ or } X = 2)$.

12. Suppose X is a random variable with probability mass function

$$p(x) = \begin{cases} 2\left(\frac{1}{3}\right)^x & \text{if } x = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X \leq 2)$.

13. Let W be a random random variable with PMF given by:

$$p(w) = \begin{cases} .7 & \text{if } w = -2 \\ .15 & \text{if } w = 2 \\ .05 & \text{if } w = 1 \\ .1 & \text{if } w = 5 \end{cases}$$

Find $E(1/W)$.

14. Suppose your friend has an investment for you: If you invest \$20,000 there is a 30% chance the investment will mature to \$100,000 but there is a 70% chance the investment will become worthless (i.e. you loose it all). Let X be the profit of this investment. Compute the expected profit $E(X)$?

15. Find $P(-1.5 \leq X \leq 1)$ where X is a random variable with PMF given by

$$p(x) = \begin{cases} \frac{1}{|x|^2} & \text{if } x = 2 \text{ or } x = -2 \\ \frac{1}{4|x|} & \text{if } x = 1 \text{ or } x = -1 \\ 0 & \text{otherwise} \end{cases}$$

Lecture 3

1. Consider a box which has 80% blue marbles and 20% red marbles. Randomly pick a marble and let X be the random variable which is 1 if the marble is blue and 0 if the marble is red. Find the variance of X.

2. The probability that a customer responds to a survey is $p = 0.2$. Suppose we have 40 customers contacted. What are the expected number of responses and the standard deviation?

3. Let X be a random variable following Binomial distribution with total trial $n = 400$, and proportion of success $p = 0.3$. Calculate $E(X)$, and $E(X^2)$?

4. Let X be a random variable following Poisson distribution with rate $\lambda = 2$, given by:

$$p(x) = \begin{cases} \frac{2^x e^{-2}}{x!}, & \text{if } x = 0, 1, 2, 3, \dots \\ 0, & \text{otherwise.} \end{cases}$$

   Calculate $P(X = 0 \text{ or } X = 2)$.

5. The Prussian Cavalry is highly concerned with fatal horse kicks. Let X be the number of soldiers killed by horse kicks in a particular year. We assume that X follows Poisson distribution with parameter $\lambda = 0.61$, where PMF is

$$p(x) = \frac{0.61^x e^{-0.61}}{x!}$$

   What is the probability that at least one soldier will be killed by a horse kick in the next year?

6. The number of times a certain web server is accessed per minute (X) can be modeled by the Poisson distribution with parameter $\lambda = 2.5$,

$$p(x) = \frac{2.5^x e^{-2.5}}{x!}$$

   where $x = 0, 1, 2, 3, \dots$. Calculate the probability that the web server is accessed at most once during any given minute.

7. Let X denote the number of mutations in a given stretch of DNA after exposed to a certain amount of radiation. It turns out that the randomness in X is well modeled by a PMF of the following form

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

   where $x = 0, 1, 2, 3, \dots$. Then number λ depends on the amount of radiation and the number of base pairs in the DNA strain. Use this PMF to compute $P(X = 0)$ (i.e. that there are no mutations) when $\lambda = 0.12$.

8. Suppose X is a random variable with PMF given by the following table

x	$p(x)$
1	0.4
2	0.1
3	0.2
4	0.3

   Find $P(X = 1 | X \leq 2)$.

Lecture 4

1. Suppose X and Y are the random variables with joint PMF given by:

$X \backslash Y$	22.1	-5.75
-3	0	0.07
-2	0.2	0.08
-1	0.1	0.1
0	0.15	0.1
1	0	0.2

   Compute $E(X^2 | Y = 22.1)$.

2. Suppose X and Y are random variables with joint PMF:

$X \backslash Y$	0	1
0	0.1	0.4
1	0.3	0.2

   Are X and Y independent?

3. Suppose X and Y are the random variables with joint PMF given by:

$X \backslash Y$	-2	0	3
-1	0.1	0.3	0.1
0	0.2	0	0.1
1	0	0.1	0.1

   Compute $E(Y)$, and $P\big(Y > 0 \,\big|\, X + Y \geq 0\big)$.

4. Let X, Y be two random variables with the following joint probability mass function.

$X \backslash Y$	0	1
-1	0	1/3
0	1/3	0
1	0	1/3

   Find $P(X + Y = 1)$.

5. Let $\boxed{\begin{matrix} X \\ Y \end{matrix}}$ be a random draw from the following box of tickets:

$3\times$	$3\times$	$2\times$	$4\times$	$3\times$
$\boxed{\begin{matrix} 0 \\ 1 \end{matrix}}$	$\boxed{\begin{matrix} 0 \\ 2 \end{matrix}}$	$\boxed{\begin{matrix} 1 \\ 0 \end{matrix}}$	$\boxed{\begin{matrix} 1 \\ 1 \end{matrix}}$	$\boxed{\begin{matrix} 2 \\ 0 \end{matrix}}$

Box with 15 tickets

Note: the number on the top of each ticket tells you how many copies of that ticket are in the box.

Find $E(X^2|Y \geq 1)$.

6. Let X, Y be two random variables with the following joint probability mass function

$X \backslash Y$	-1	0	1
-1	0.2	0	0.05
0	0.25	0.05	0
1	0.10	0.05	0.3

Find $P(X \geq 0|Y \geq 0)$.

7. Suppose X and Y are random variables with joint PMF given by:

$X \backslash Y$	-1	0	1
-1	0.1	0.3	0.1
0	0.1	0	0.1
1	0.1	0.1	0.1

Find $E(X|X = Y)$

8. Let $\boxed{\begin{matrix} X \\ Y \end{matrix}}$ be a random draw from the following box of tickets:

$\boxed{\begin{matrix} 1 \\ 0 \end{matrix}}$ $\boxed{\begin{matrix} 1 \\ 0 \end{matrix}}$ $\boxed{\begin{matrix} 1 \\ 0 \end{matrix}}$ $\boxed{\begin{matrix} 1 \\ 5 \end{matrix}}$ $\boxed{\begin{matrix} -1 \\ 5 \end{matrix}}$ $\boxed{\begin{matrix} -1 \\ 5 \end{matrix}}$ $\boxed{\begin{matrix} -1 \\ 0 \end{matrix}}$ $\boxed{\begin{matrix} -1 \\ 0 \end{matrix}}$

8 tickets

Find the covariance between X and Y.