Applied Statistics for Business & Economics [STA103, SS2 2025]

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Assignment #**2** (Due by noon, Aug 25th, Mon)

Instructions

- Show detailed calculations and clearly state conclusions in context.
- Round results to 4 decimal places unless otherwise noted.
- The in-person midterm may include similar problems, and you will need to solve them on your own from scratch. Use LLMs at your own risk. Overreliance on LLMs may hinder your understanding of the underlying concepts.

1 Investment Portfolio Risk Hedging (Lecture 5)

Imagine you are investing in two assets, X_1 and X_2, and your total profit is:

$$\text{Total Profit } = X_1 + X_2.$$

We will consider two different scenarios for how these two assets behave. In each scenario, there are two possible outcomes, and each outcome (each "ticket") has equal probability $\frac{1}{2}$. A ticket shows the returns of both assets together in the form $\boxed{\begin{matrix}X_1\\X_2\end{matrix}}$.

- Case 1:

$$\boxed{\begin{matrix}100\\100\end{matrix}} \quad \boxed{\begin{matrix}-50\\-50\end{matrix}}$$

- Case 2:

$$\boxed{\begin{matrix}100\\-50\end{matrix}} \quad \boxed{\begin{matrix}-50\\100\end{matrix}}$$

(a) (5pt) For **Case 1**, show that $\mathrm{Cov}(X_1, X_2) > 0$.

(b) (5pt) For **Case 2**, show that $\mathrm{Cov}(X_1, X_2) < 0$.

(c) (5pt) Compare the values of $\mathrm{Var}(X_1 + X_2)$ in the two cases. Comparison here means that identify $>$ or $<$ relationship between the variances for **Case 1** and **Case 2**.

(d) (5pt) Explain which case produces a riskier portfolio and which case produces a more stable (hedged) portfolio.

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2 Air-Quality Index Forecast (AQI) (Lecture 6)

A company runs two sensors, **Sensor A** and **Sensor B**, to measure daily air pollution in AQI units.

- Let random variable X represent the measurement AQI from **Sensor A**, with $X \sim N(70, 9)$.
- Let random variable Y represent the measurement AQI from **Sensor B**, with $Y \sim N(65, 16)$.

The two sensors are not independent. Their measurement errors are partly correlated, with $\mathrm{Cov}(X, Y) = 5$.

(a) (5pt) Compute the probability that Sensor A reports a value between 68 and 74. Show the Z-score steps clearly.

(b) (10pt) The city combines the two sensor readings into a single index:

$$W = 0.4X + 0.6Y.$$

Find the distribution of W by computing its mean and variance. Clearly specify **distribution name** and **parameters**.

The city issues an **alert** if the combined index W is greater than **74**.

(c) (5pt) Compute the probability that the city issues an alert.

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3 Daily Commute Times (Lecture 7)

In a large city, the daily commute time (in minutes) for workers is approximately normally distributed:

$$C \sim N(50, 100).$$

(a) (5pt) Find the 90th percentile of commute times.

(b) (10pt) Find the symmetric interval around the mean that contains 95% of all commute times. (Hint: use the fact that $z_{0.975} = 1.96$).

One commuter reports a daily commute of 75 minutes.

(c) (10pt) Compute this commuter's Z-score and decide whether this commute time is "unusual" under the 95% rule from part (b). Explain your reasoning.

(d) (10pt) Find the symmetric interval around the mean that contains 99% of all commute times. (Hint: use the fact that $z_{0.995} = 2.58$). Decide whether this 75 minutes commute time is "unusual" under the 99% rule.

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4 Loan Approval Rate (Lecture 8)

A bank approves 75% of credit applications. In one day the bank reviews $n = 80$ applications. Let X be the number approved in the sample. We know that X follows Binomial distribution with $n = 80$ and $p = 0.75$. Unless noted, show your Z-score steps.

(a) (5pt) Explain why the Central Limit Theorem (CLT) can be applied to the **count** X (the number of approved applications). Provide your reasoning clearly.

(b) (5pt) Based on the result from part (a), approximate the probability that fewer than 55 applications are approved.

Now define the sample proportion of approved applications

$$\hat{p} = \frac{X}{n}.$$

(c) (10pt) What are the mean and standard deviation of $\hat{p}$? Make sure to substitute the given values of n and p, so that your final answers should be numeric values.

(d) (5pt) Using the CLT for the **proportion** $\hat{p}$, approximate the probability that $\hat{p} < 55/80$. Make sure to substitute the given values of n and p, so that your final answers should be numeric value.