STA 103 (SS2 2025): Formula Sheet 1
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- Summation notation: $\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n$
- Linear properties of summation:
  1. $\sum_{i=1}^{n}(ax_i) = a(\sum_{i=1}^{n} x_i)$
  2. $\sum_{i=1}^{n}(x_i + b) = (\sum_{i=1}^{n} x_i) + nb$
  3. $\sum_{i=1}^{n}(x_i + y_i) = (\sum_{i=1}^{n} x_i) + (\sum_{k=1}^{n} y_i)$
- Suppose $y_i = ax_i + b$ and $\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = v$, then:
  1. $\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = (\frac{1}{n}\sum_{i=1}^{n} x_i^2) - (\overline{x})^2$
  2. $\sum_{i=1}^{n} y_i = a(\sum_{i=1}^{n} x_i) + nb$
  3. $\overline{y} = a\overline{x} + b$
  4. $\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2 = a^2 v$
- PMFs (Probability Mass Functions)
  1. Valid PMF Requirements
  2. Table format
  3. Function format
  4. Modifying PMFs for functions of a random variable.
  5. Modifying PMFs for conditional distributions.
- Expected value
  1. $E(X) = \sum_{i=1}^{n} x_i p(x_i)$
  2. $E(f(X)) = \sum_{i=1}^{n} f(x_i)p(x_i)$
- Variance and Standard Deviation
  1. $\text{Var}(X) = E[(X - E(X))^2]$
  2. $\text{Var}(X) = E(X^2) - (E(X))^2$
  3. $\text{sd}(X) = \sqrt{\text{Var}(X)}$
- Bernoulli Distribution
  1. $X \sim \text{Bernoulli}(p)$, Parameters: p.
  2. PMF:

$$p(x) = \begin{cases} p^x(1-p)^{1-x} & \text{if } x = 0 \text{ or } x = 1, \\ 0 & \text{otherwise.} \end{cases}$$

  3. Mean $E(X) = p$, Variance $\text{Var}(X) = p(1-p)$.
- Binomial Distribution
  1. $X \sim \text{Binomial}(n, p)$, Parameters: n, p.
  2. PMF:

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}.$$

  3. $\binom{n}{x} = \frac{n!}{(n-x)!x!}$
  4. Mean $E(X) = np$,
     Variance $\text{Var}(X) = np(1-p)$.
- Poisson Distribution
  1. $X \sim \text{Poisson}(\lambda)$, Parameters: λ.
  2. PMF:

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, \text{ for } x = 0, 1, 2, \ldots$$

  3. $\binom{n}{x} = \frac{n!}{(n-x)!x!}$
  4. Mean $E(X) = \lambda$, Variance $\text{Var}(X) = \lambda$.
- JPMFs (Joint Probability Mass Functions)
  1. Valid JPMF Requirements
  2. Table format
  3. Formula format
  4. Computing marginal distributions from JPMFs.
  5. Modifying JPMFs for functions of random variables.
  6. Modifying JPMFs for conditional distributions.
- Expected value of a function of X and Y:
  1. $E(f(X, Y)) = \sum_{x,y} f(x, y)p(x, y)$.
  2. $E(X \mid Y = y) = \sum_{x} x p_X(x \mid Y = y)$.
  3. $E(Y \mid X = x) = \sum_{y} y p_Y(y \mid X = x)$.
- Independence: two random variables X and Y are independent if and only if:
  1. $p(x, y) = p_X(x) \cdot p_Y(y)$ for all possible x, y **or**
  2. $p_X(x|Y = y) = p_X(x)$ for all possible x, y **or**
  3. $p_Y(y|X = x) = p_Y(y)$ for all possible x, y
- Covariance of X and Y:

$$\begin{aligned} \text{cov}(X, Y) &= E[\,(X - E(X))(Y - E(Y))\,] \\ &= E(XY) - E(X)E(Y) \end{aligned}$$

- Covariance Properties
  1. $\text{cov}(aX, bY) = a \cdot b \cdot \text{cov}(X, Y)$
  2. $\text{cov}(X, c) = \text{cov}(c, X) = 0$
  3. $\text{cov}(X + b, Y + c) = \text{cov}(X, Y)$
  4. $\text{cov}(X, Y) = \text{cov}(Y, X)$
  5. $\text{cov}(X + Y, Z) = \text{cov}(X, Z) + \text{cov}(Y, Z)$
  6. If X & Y are independent then $\text{cov}(X, Y) = 0$