

2. Prove the following

$$T(n) = 4T(n/2) + n = \Theta(n^2).$$

Conjecture. $T(n) = 4T(n/2) + n = \Theta(n^2)$.

Proof. We will prove the conjecture by the Master Theorem. We assume that $a = 4, b = 2, f(n) = n$, and $\log_b a = \lg 4 = 2$. Note that $f(n) = O(n)$ and that letting $\epsilon = 2$, $O(n^{\log_b a - \epsilon}) = O(n^{\lg 2}) = O(n)$. Since $f(n) = O(n^{\log_b a - \epsilon})$, then $T(n) = \Theta(n^{\log_b a}) = \Theta(n^2)$. Hence, we have proven the conjecture by the Master Theorem. ■