(1) Do the problem from Sun 1.2-3

Construct a know-show table for each of the following statements and then write a formal proof for one of the statements.

(a) If m is an even integer and n is an integer, then $m \cdot n$ is an even integer.
(b) If n is an even integer, then n^2 is an even integer.
(c) If n is an odd integer, then n^2 is an odd integer.

Know-show table **(a)**

Step	**Know**	**Reason**
P	m is an even integer, n is an integer	Hypothesis
P1	$m = 2 \cdot k$, for $k \in \mathbb{Z}$	Definition of an even integer
P2	$m \cdot n = (2 \cdot k) \cdot n$	Substitution
P3	$m \cdot n = 2 \cdot (k \cdot n)$	Algebra
Q	$m \cdot n$ is an integer multiple of two and hence, even	Definition of an even integer
Step	**Show**	**Reason**

Know-show table **(b)**

Step	**Know**	**Reason**
P	n is an even integer	Hypothesis
P1	$n = 2 \cdot m$, for $m \in \mathbb{Z}$	Definition of an even integer
P2	$n^2 = (2 \cdot m) \cdot n$	Substitution
P3	$n^2 = 2 \cdot (m \cdot n)$	Algebra
Q	n^2 is an integer multiple of two and hence, even	Definition of an even integer
Step	**Show**	**Reason**

Know-show table **(c)**

Step	**Know**	**Reason**
P	n is an odd integer	Hypothesis
P1	$n = 2 \cdot m + 1$, for $m \in \mathbb{Z}$	Definition of an odd integer
P2	$n^2 = (2 \cdot m + 1) \cdot n$	Substitution
P3	$n^2 = 2 \cdot m \cdot n + n$	Algebra
P4	$2 \cdot m \cdot n$ is even, as $m, n \in \mathbb{Z}$	Definition of an even integer
P5	n^2 is n greater than an even integer.	Algebra
Q	Since n is odd and n^2 is n greater than an even integer, n^2 is odd.	Definition of an odd integer
Step	**Show**	**Reason**

Theorem. *If m is an even integer and n is an integer, then $m \cdot n$ is an even integer.*
Proof. We assume that m is an even integer and n is an integer. Since m is an even integer, there exists

$k \in \mathbb{Z}$ such that

$$m = 2 \cdot k.$$

Using algebra, we obtain

$$\begin{aligned} m \cdot n &= (2 \cdot k) \cdot n \\ &= 2 \cdot (k \cdot n). \end{aligned}$$

Since $k, n \in \mathbb{Z}$, we can conclude $k \cdot n \in \mathbb{Z}$ and hence, $2 \cdot (k \cdot n)$ is a multiple of two. By definition, all multiples of two are even integers. Therefore, $m \cdot n$ is even. $\blacksquare$