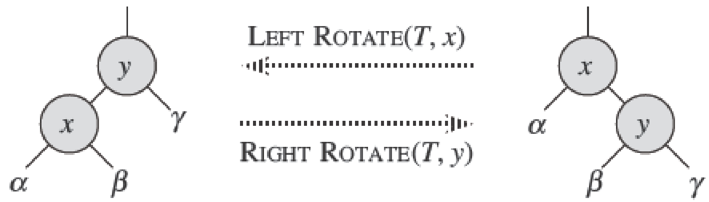


Figure 13.2

13.2-3

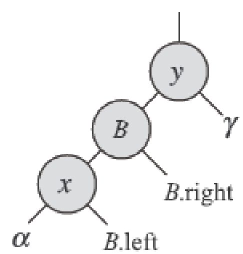
Let a, b, and c be arbitrary nodes in subtrees α, β, and γ, respectively, in the left tree of Figure 13.2. How do the depths of a, b, and c change when a left rotation is performed on node x in the picture?

We assume that β is the subtree starting at x.right. Then, we let $B = x.\text{right} = \beta.\text{root}$. Since b is an arbitrary node of β, which is rooted at B, we know that $B \neq \text{NIL}$. That is to say, β is not an empty binary tree. We can make similar arguments to show that α and γ are both not empty trees.

After a left rotation, since $x = x.\text{p.left}$ $(= y.\text{left})$, we perform the following operations by LEFT-ROTATE:

$$x.\text{right} = B.\text{left},$$
$$y.\text{left} = B,$$
$$B.\text{left} = x,$$

and likewise set the parents of the changed nodes appropriately. Then, our resulting tree looks roughly as so, where B.left denotes the left subtree which was originally rooted at B, and B.right denotes the right subtree which was originally rooted at B.



Hence, it is easy to see that c does not change whatsoever in regards to depth as the depth of γ remains unchanged. We can also easily see that the depth of a is increased by one as the depth of α is increased by one. However, the depth of b depends on its location. If b lies in B.right, whose depth is decremented by one, then b has a depth decremented by one. Likewise, if b lies in B.left, whose depth is unchanged, then b has an unchanged depth.