

Ace Hardware has the following slogan: *If we don't have it, you don't need it.* Does this mean the same thing as: *If you don't need it, we don't have it? If you need it, we have it?*

We can make the proof using symbolic logic.

Proof

Let H = “we do have it” and N = “you need it.” Then, the original statement P = “if we don't have it, you don't need it” can be written as $\neg H \rightarrow \neg N$. Using logical equivalences,

$$\begin{aligned} P &= \neg H \rightarrow \neg N \\ &= \neg \neg H \vee \neg N \\ &= H \vee \neg N \\ &= \neg N \vee H \\ &= N \rightarrow H \end{aligned} \tag{1}$$

Let the second statement $P' =$ “if you don't need it, we don't have it” $= \neg N \rightarrow \neg H$. The form of P' is called the inverse of P , and since inversions are not logically equivalent to the original statement, P' is not logically equivalent to P .

Let the third statement $P'' =$ “if you need it, we have it” $= N \rightarrow H$. Since this is logically equivalent to (1), which is logically equivalent to P , P'' is logically equivalent to P . ■