

Ace Hardware has the following slogan: *If we don't have it, you don't need it.* Does this mean the same thing as: *If you don't need it, we don't have it? If you need it, we have it?*

We can make the proof using symbolic logic.

**Proof**

Let  $H$  = “we do have it” and  $N$  = “you need it.” Then, the original statement  $P$  = “if we don't have it, you don't need it” can be written as  $\neg H \rightarrow \neg N$ . Using logical equivalences,

$$\begin{aligned} P &= \neg H \rightarrow \neg N \\ &= \neg \neg H \vee \neg N \\ &= H \vee \neg N \\ &= \neg N \vee H \\ &= N \rightarrow H \end{aligned} \tag{1}$$

Let the second statement  $P' =$  “if you don't need it, we don't have it”  $= \neg N \rightarrow \neg H$ . The form of  $P'$  is called the inverse of  $P$ , and since inversions are not logically equivalent to the original statement,  $P'$  is not logically equivalent to  $P$ .

Let the third statement  $P'' =$  “if you need it, we have it”  $= N \rightarrow H$ . Since this is logically equivalent to (1), which is logically equivalent to  $P$ ,  $P''$  is logically equivalent to  $P$ .