# Tuning and Temperament

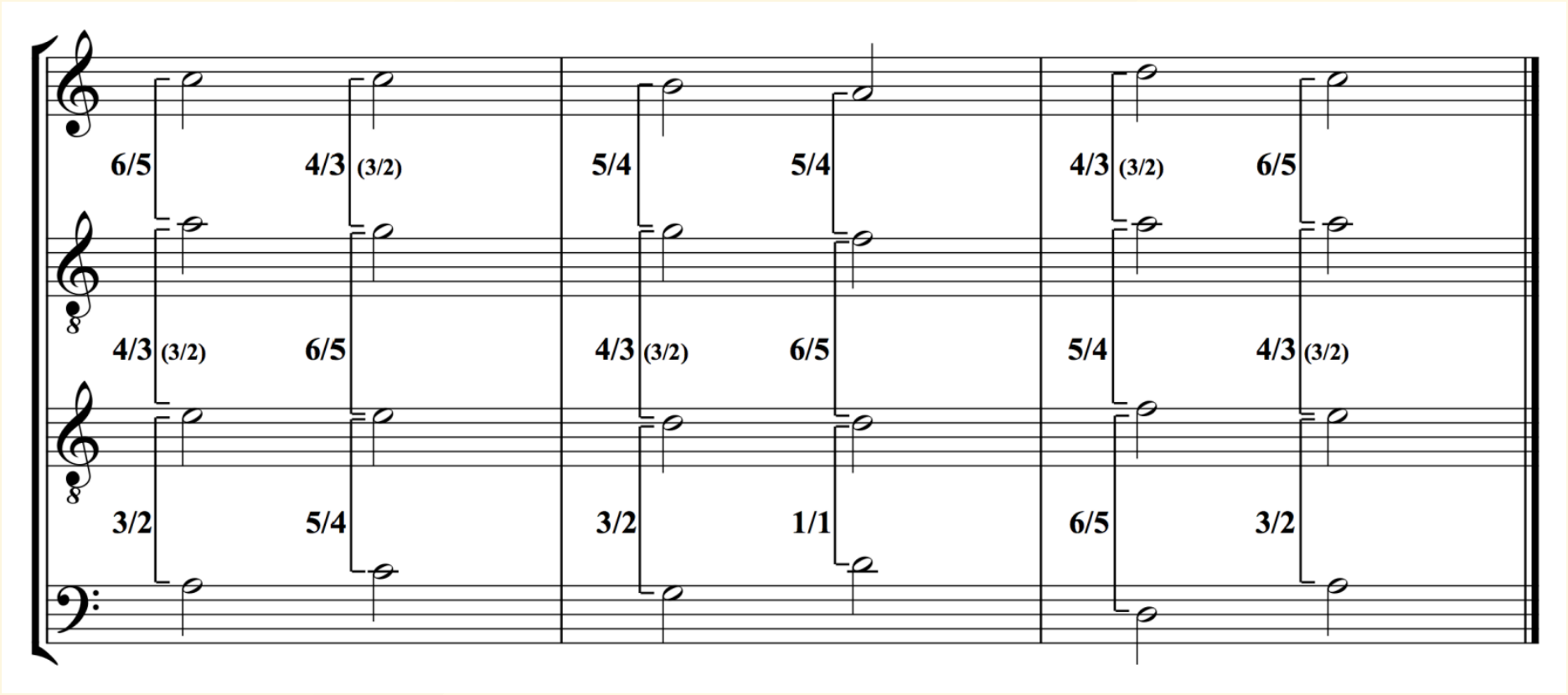
Class 5: Meantone Temperament

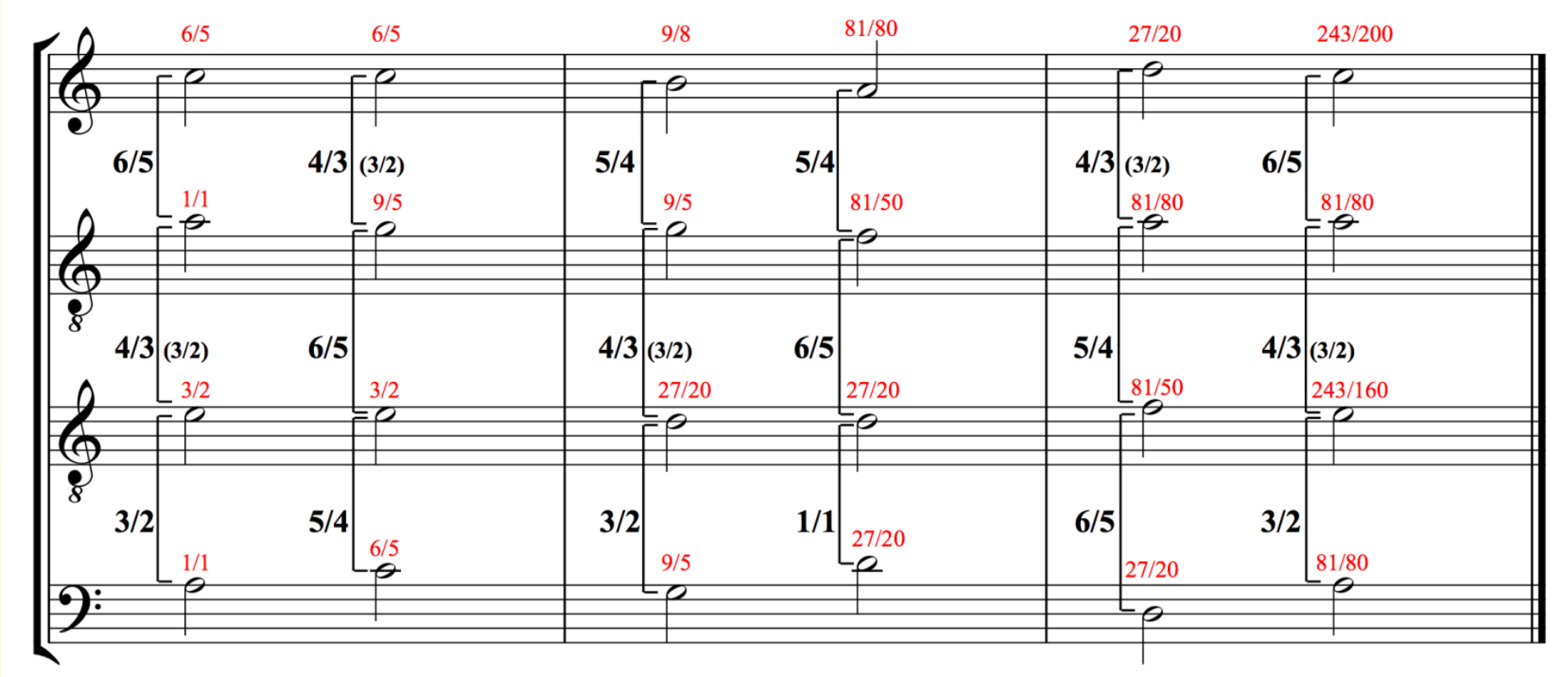
# Today's Class

- Where just intonation falls short
- Temperament
  - What is temperament?
- Meantone temperaments
  - 1/4 comma
  - 1/5 comma

# Where does just intonation fall short?

Let's look at a passage of music and assume that our tuning system is just intonation (5-limit). Let's also try to keep it *totally* in tune such that every interval between every note is just (i.e. no commas or wolf intervals).


6/5
4/3 (3/2)
5/4
5/4
4/3 (3/2)
6/5
4/3 (3/2)
6/5
4/3 (3/2)
5/4
4/3 (3/2)
3/2
5/4
3/2
1/1
6/5
3/2


6/5
6/5
9/8
81/80
27/20
243/200
6/5
4/3 (3/2)
5/4
5/4
4/3 (3/2)
6/5
1/1
9/5
9/5
81/50
81/80
81/80
4/3 (3/2)
6/5
4/3 (3/2)
6/5
5/4
4/3 (3/2)
3/2
3/2
27/20
27/20
81/50
243/160
3/2
5/4
3/2
1/1
6/5
3/2
1/1
6/5
9/5
27/20
27/20
81/80

# Temperaments

What is a temperament anyway? What is a tuning?

**Tuning:**

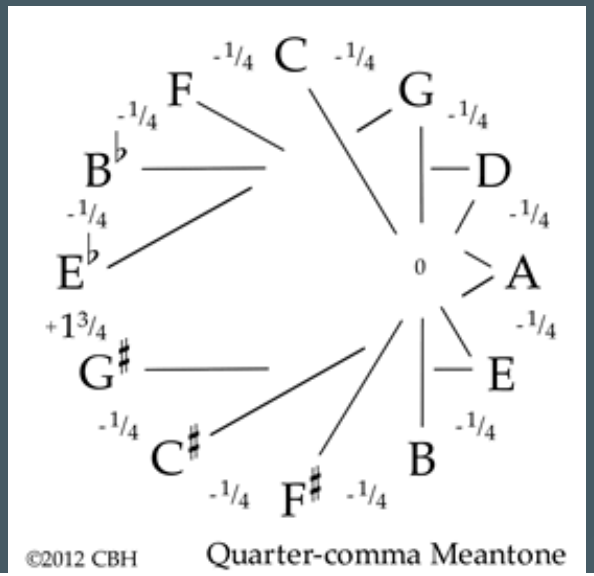
1. A system whose intervals all can be expressed by rational numbers.

**Temperament:**

1. A system whose intervals cannot all be expressed by rational numbers.
2. A system which "tempers" just intervals.
3. Partch: "... a system which deliberately robs its intervals of their purity in order to implement the idea of every-tone-in-several senses."

# 1/4 Comma Meantone Temperament

- Pure thirds ($\frac{5}{4}$, $\frac{6}{5}$)
- Flat fifths, by 1/4 of a syntonic comma ($\frac{81}{80}$)
- Wolf fifth between G# and Eb (if tuned on C)



# Building a 12-note 1/4 comma meantone scale

We want pure major thirds, $\frac{5}{4}$, so we want to slightly flatten the fifth, $\frac{3}{2}$. If starting on C, the major third is E. We can get to E either by 1) stacking four $\frac{3}{2}$'s or two octaves and a $\frac{5}{4}$.

In a Pythagorean sytle of tuning using fifths, the E would be $\frac{81}{64}$ whereas a pure third is $\frac{5}{4}$ (the difference is $\frac{81}{80}$). So, we need to lower each $\frac{3}{2}$ by one quarter of the difference, $\frac{81}{80}$, such that getting to an E by four $\frac{3}{2}$'s or two octaves and a $\frac{5}{4}$ are the same.

We can rewrite $\frac{5}{4}$ as $\frac{5}{1}$ by moving it up two octaves. Therefore, letting $r$ be the ratio of our meantone fifth:

$$r^4 = \frac{5}{1} = 5$$

$$r = \sqrt[4]{5}$$

So in "musical" terms:

$$r \approx 1.49535 \approx \frac{643}{430} \approx 696.587 \text{ cents}$$

## What does this mean for the rest of the scale?

We construct it exactly as a Pythagorean scale, substituting $\sqrt[4]{5}$ for $\frac{3}{2}$. For example, a whole step is:

$$\sqrt[4]{5} \times \sqrt[4]{5} \times \frac{1}{2} = \frac{\sqrt[4]{5} \times \sqrt[4]{5}}{2} = \frac{5^{1/4} \times 5^{1/4}}{2} = \frac{5^{1/2}}{2} = \frac{\sqrt{5}}{2}$$

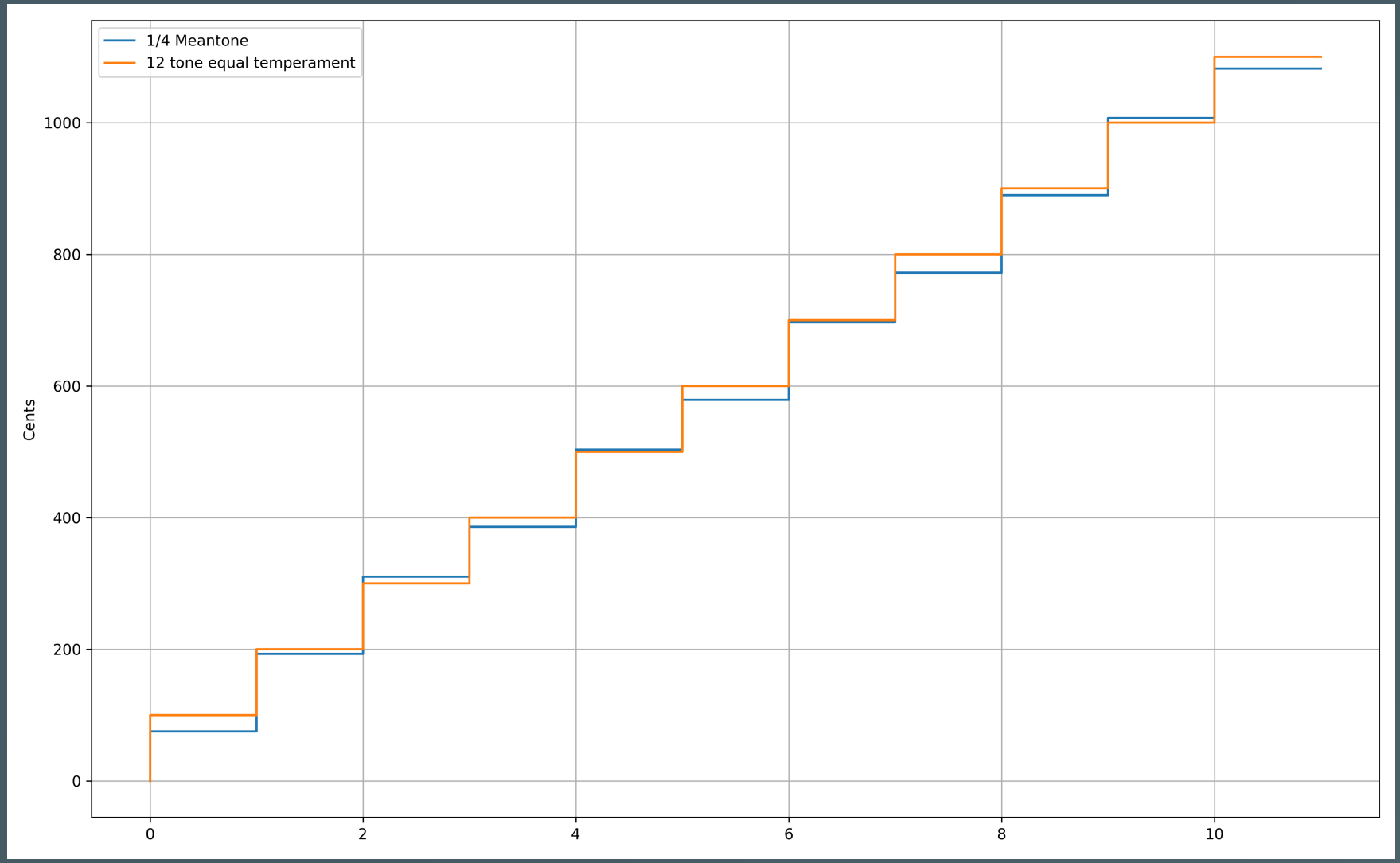
Confirm by taking two whole steps to make a major third:

$$\frac{\sqrt{5}}{2} \times \frac{\sqrt{5}}{2} = \frac{5}{4}$$

**Formula for pitches of a major scale where $r$ is the fifth:**

| Note | Formula | Cents | Note | Formula | Cents |
|---|---|---|---|---|---|
| C | $r^0 \times 2^0 = 1$ | 0 | G | $r^1 \times 2^0 = r$ | 696.6 |
| D | $r^2 \times 2^{-1} = \frac{\sqrt{5}}{2}$ | 193.2 | A | $r^3 \times 2^{-1} = \frac{r\sqrt{5}}{2}$ | 889.7 |
| E | $r^4 \times 2^{-2} = \frac{5}{4}$ | 386.3 | B | $r^5 \times 2^{-2} = \frac{5r}{4}$ | 1082.9 |
| F | $r^{-1} \times 2^2 = \frac{2r\sqrt{5}}{5}$ | 503.4 | C | $r^0 \times 2^1 = 2$ | 1200 |

Wolf fifth between G# and Eb so it's symmetrical (common).


1/4 Meantone
12 tone equal temperament
Cents
0
200
400
600
800
1000
0
2
4
6
8
10

## Listening

Mozart's Fantasie KV397 in Three Different Temperaments

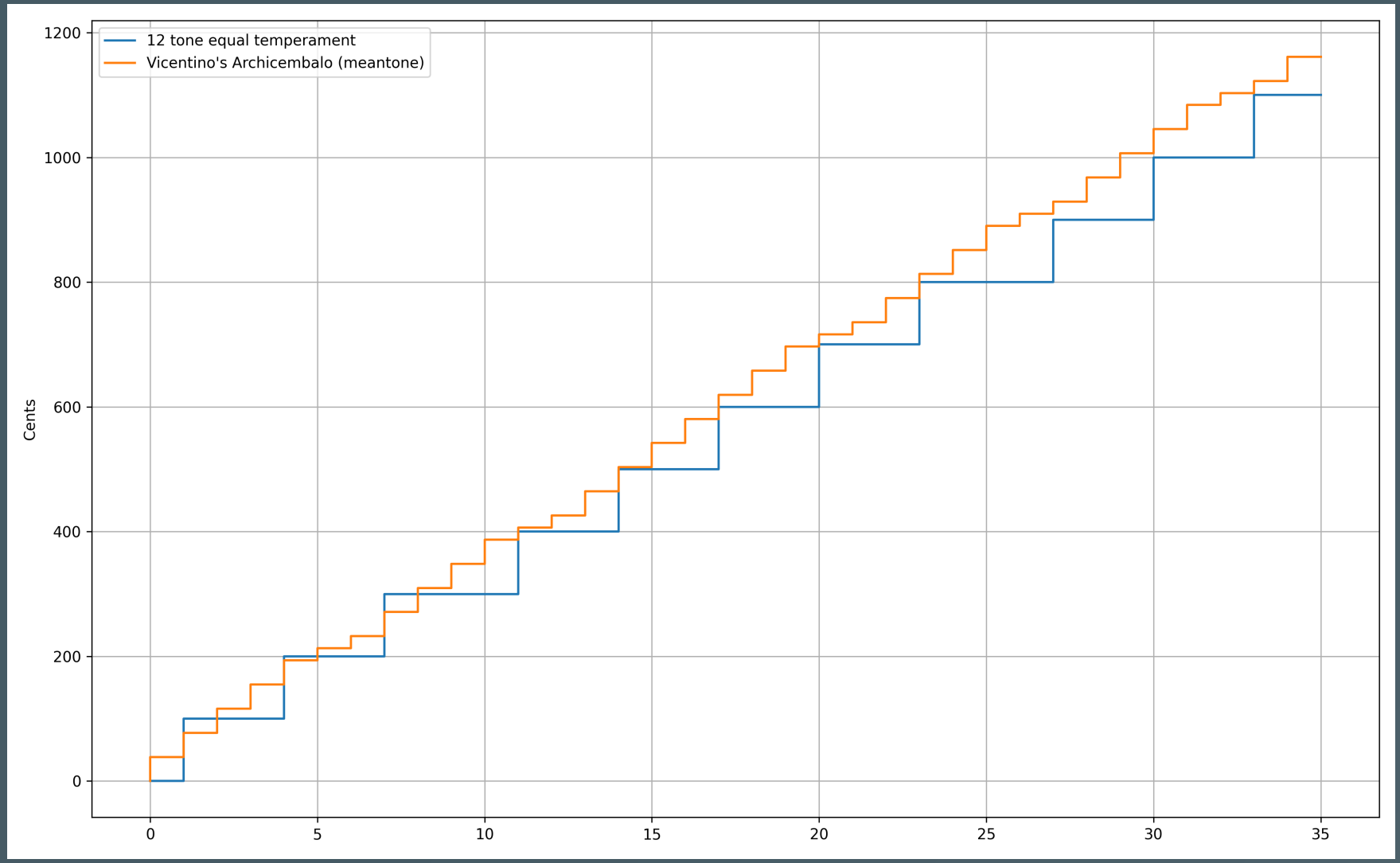
Listen to about 2 min in Equal Temperament (0:00), 1/4 comma meantone (11:40), and just for kicks, Prelleur temperament (5:40).

Yale's Divinity School Meantone Organ

Split sharps?

**Other flavors of meantone:**

- 1/5 comma meantone (Pythagorean comma)
- 1/6 comma meantone
- Extended meantone (building extra notes so all/most keys can have pure thirds)
  - Ascanio Mayone - Examples for the Cimbalo Cromatico
  - Nicola Vicentino: "Musica prisca caput"


12 tone equal temperament
Vicentino's Archicembalo (meantone)
Cents
1200
1000
800
600
400
200
0
0
5
10
15
20
25
30
35