

Tuning and Temperament

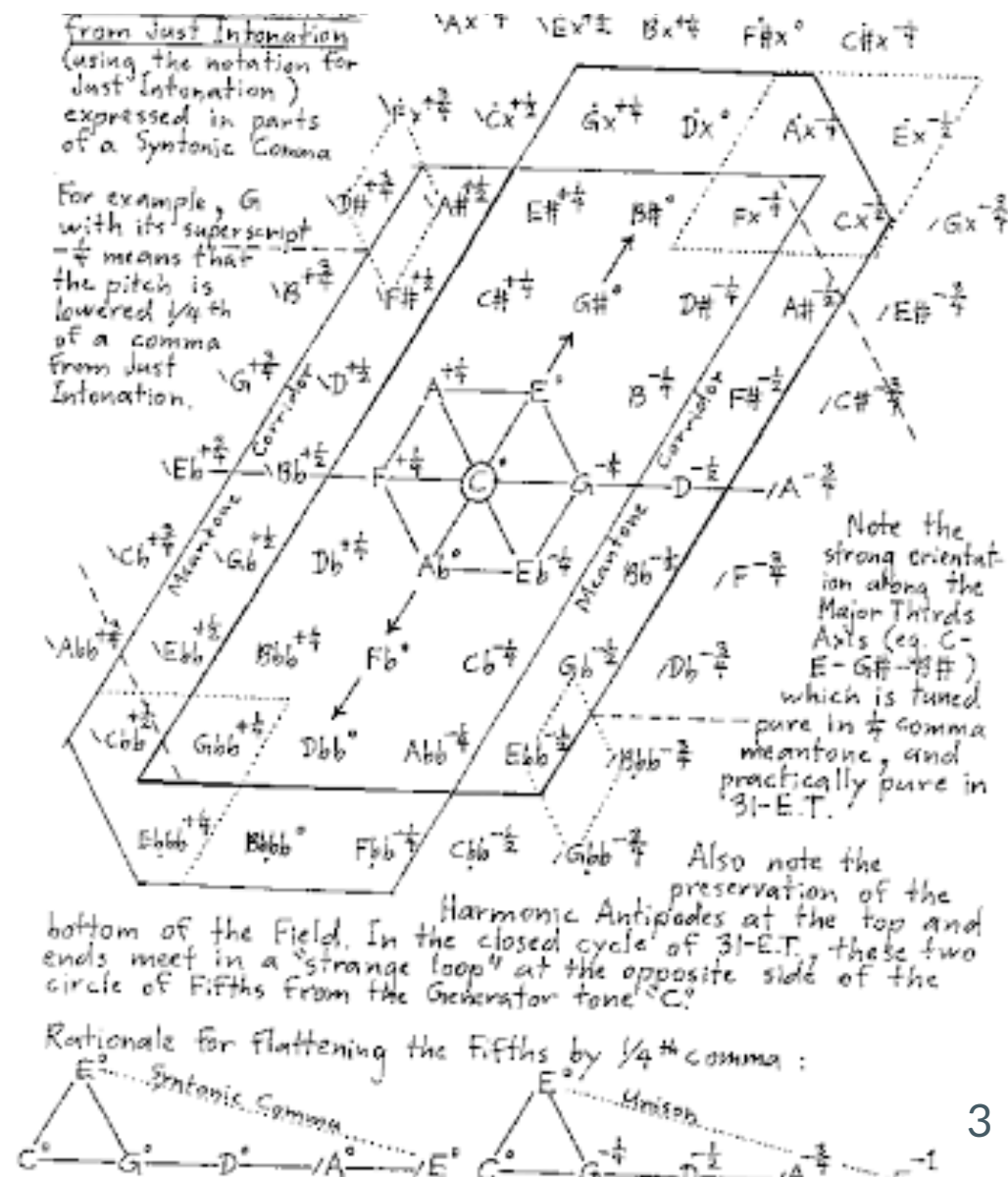
Class 3: (Extended) Just Intonation

Today's Class

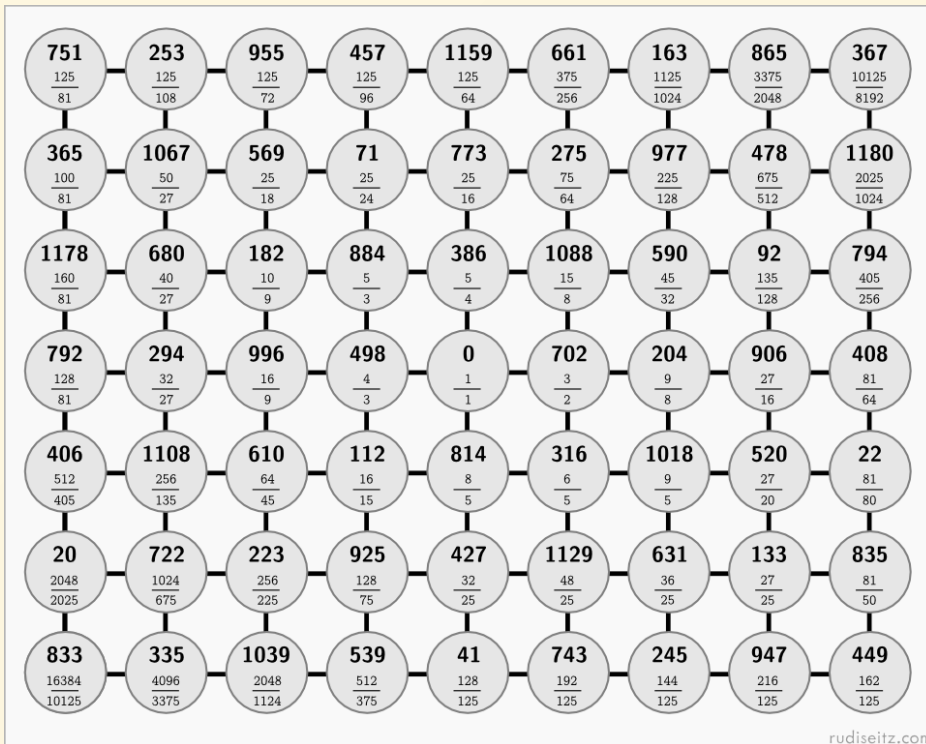
- Introduction to just intonation
 - 3-limit, 5-limit, etc
- Tone lattices: a useful tool
- **Analysis:** Ben Johnston's Crossings: String Quartet No. 4, "Amazing Grace"
- **Analysis:** Partch: Delusion of the Fury (excerpt)
 - More of a discussion of the work since even a cursory analysis would require more than we've discussed.
- **Analysis:** La Monte Young's Well-Tuned Piano (1985) (excerpt)

Tone Lattices (or squares)

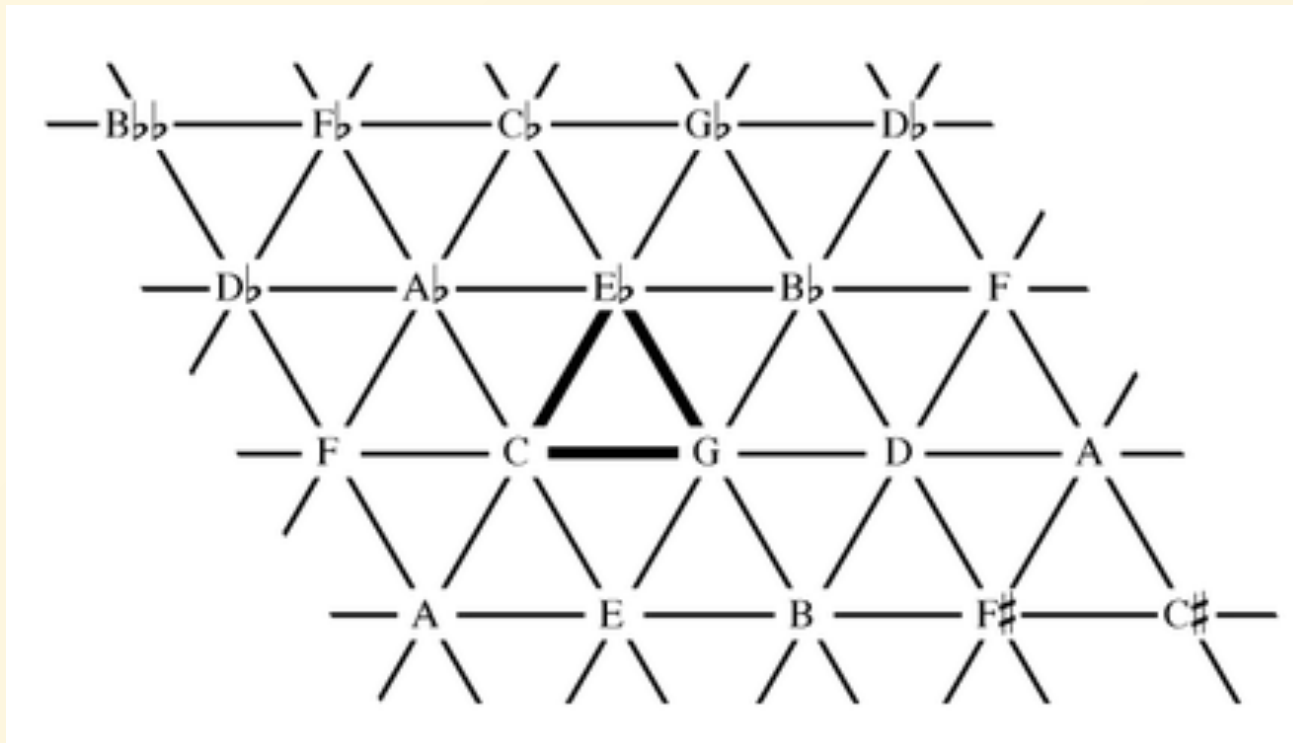
- Useful to visualize interval relationships and to build out harmonies
- Can exist in multiple dimensions (!)
- Bread and butter for extended just intonation tunings in the 20th and 21st centuries



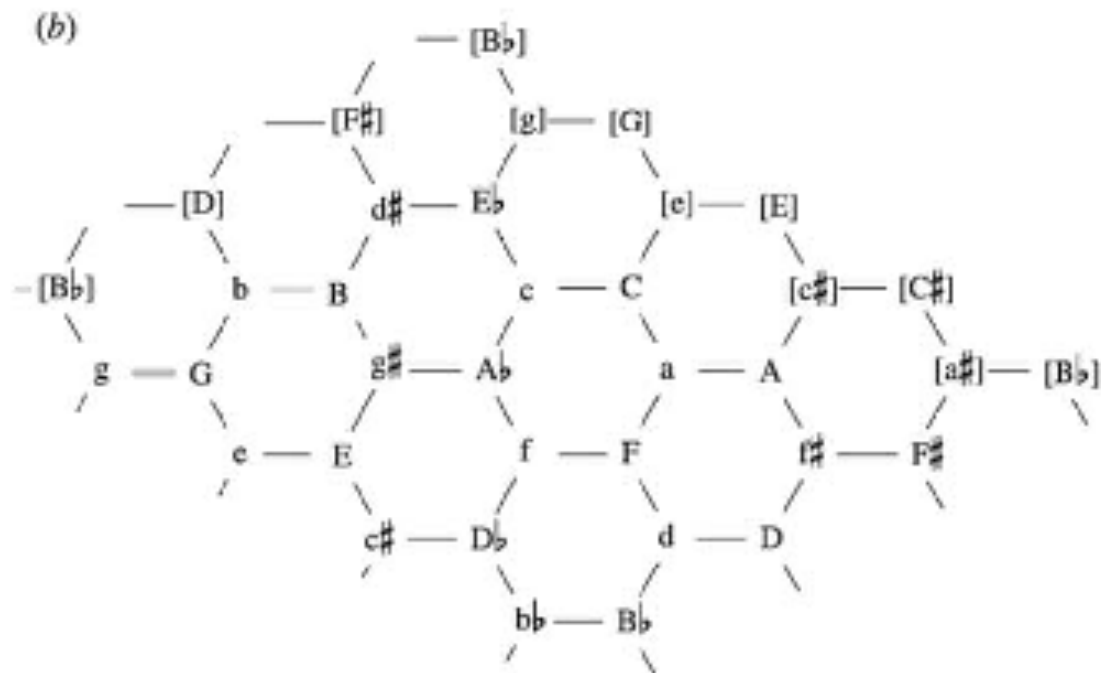
2D with square connections (5-limit in this instance)



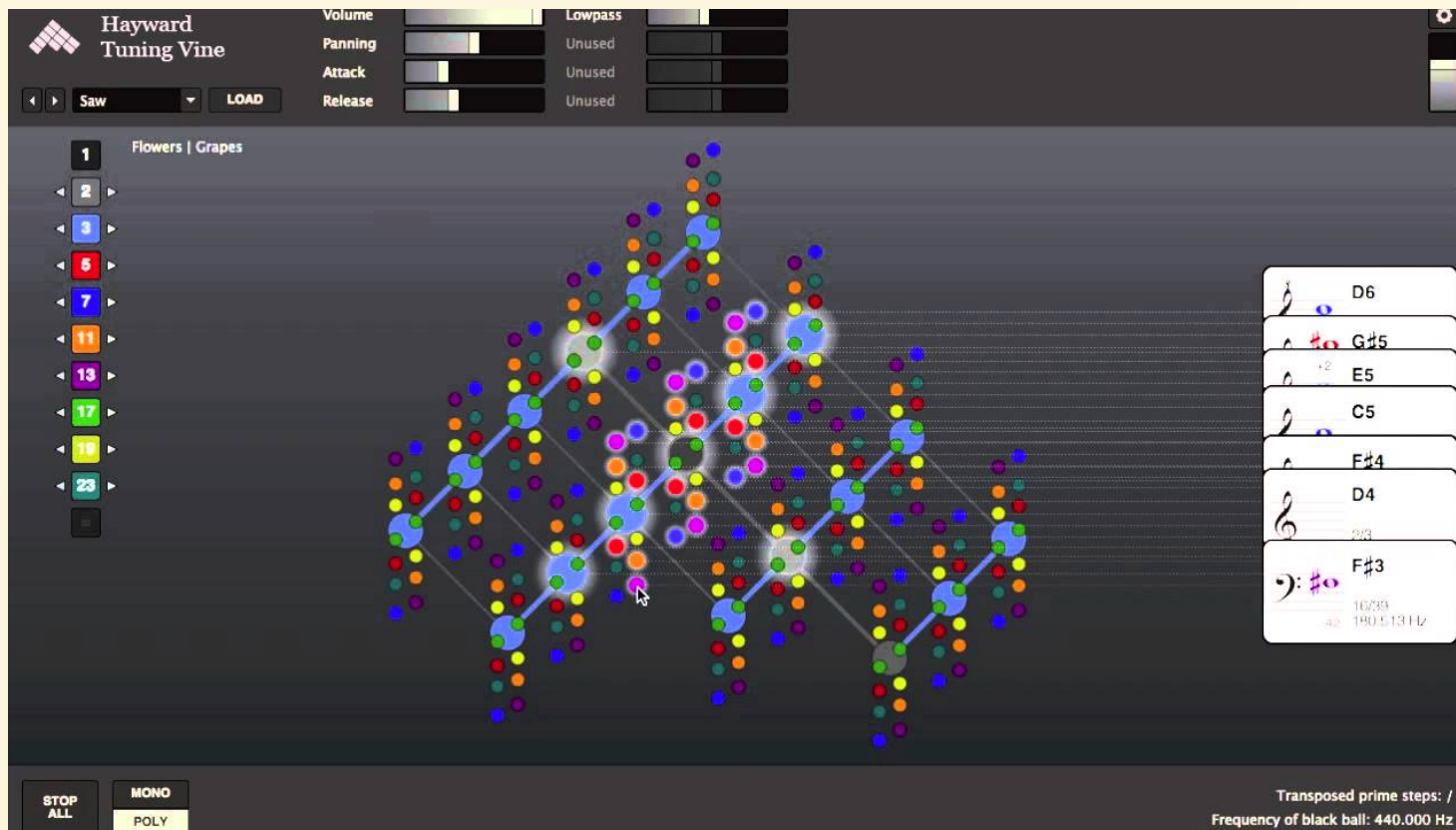
2D with triangular connections



2D with hexagonal connections



3D



Analysis

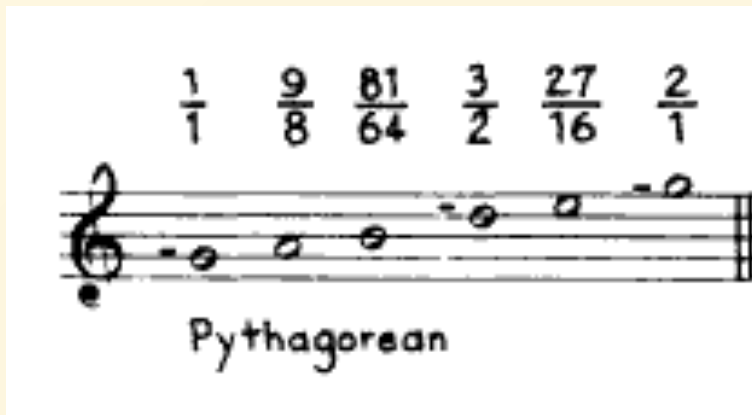
[Ben Johnston's Crossings:](#)
[String Quartet No. 4,](#)
["Amazing Grace"](#)



Progressively more complex tuning for each variation

I and II: Pythagorean pentatonic

$$\frac{1}{1} - \frac{9}{8} - \frac{81}{64} - \frac{3}{2} - \frac{27}{16} - \frac{2}{1}$$



III: 5-limit Just Intonation

$$\frac{1}{1} - \frac{9}{8} - \frac{5}{4} - \frac{4}{3} - \frac{3}{2} - \frac{5}{3} - \frac{15}{8} - \frac{2}{1}$$



The scale ratios multiplied by 1/2 =

$$\frac{1}{2} \quad \frac{9}{16} \quad \frac{5}{8} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{5}{6} \quad \frac{15}{16} \quad \frac{1}{1}$$

The above fractions subtracted from 1/1 =

$$\frac{1}{2} \quad \frac{7}{16} \quad \frac{3}{8} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{16} \quad \frac{0}{0}$$

The above series of fractions from 0/0 to 1/1 times 48 =

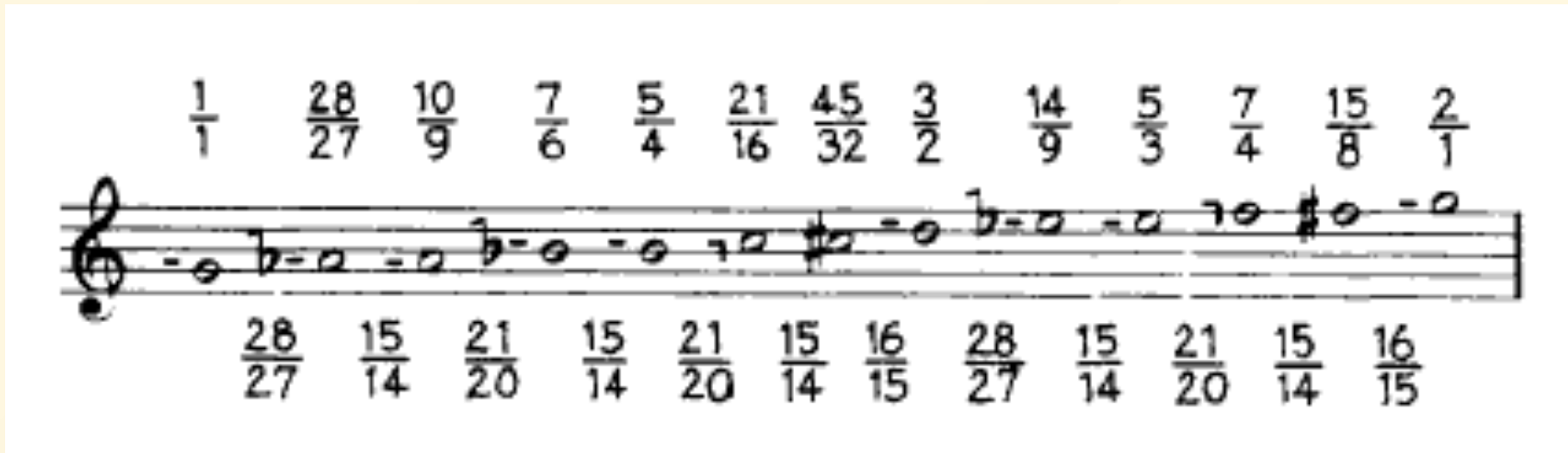
$$0 \quad 3 \quad 8 \quad 12 \quad 16 \quad 18 \quad 21 \quad 24 \quad 27 \quad 30 \quad 32 \quad 36 \quad 40 \quad 45 \quad 48$$

$$(3+5+4+4+2+3+3+3+3+2+4+4+5+3)$$

The meters used are then:

$$\begin{array}{cccccccccccccc} 3 & 5 & 4 & 4 & 2 & 3 & 3 & 3 & 3 & 2 & 4 & 4 & 5 & 3 \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \end{array}$$

IV and V: 7-limit justly intoned "blues"



Notice the different "flavors" of intervals. There are even more when accounting for those not against $\frac{1}{1}$.

Did you catch the Partch quote from *Greek Studies* in Variation V?



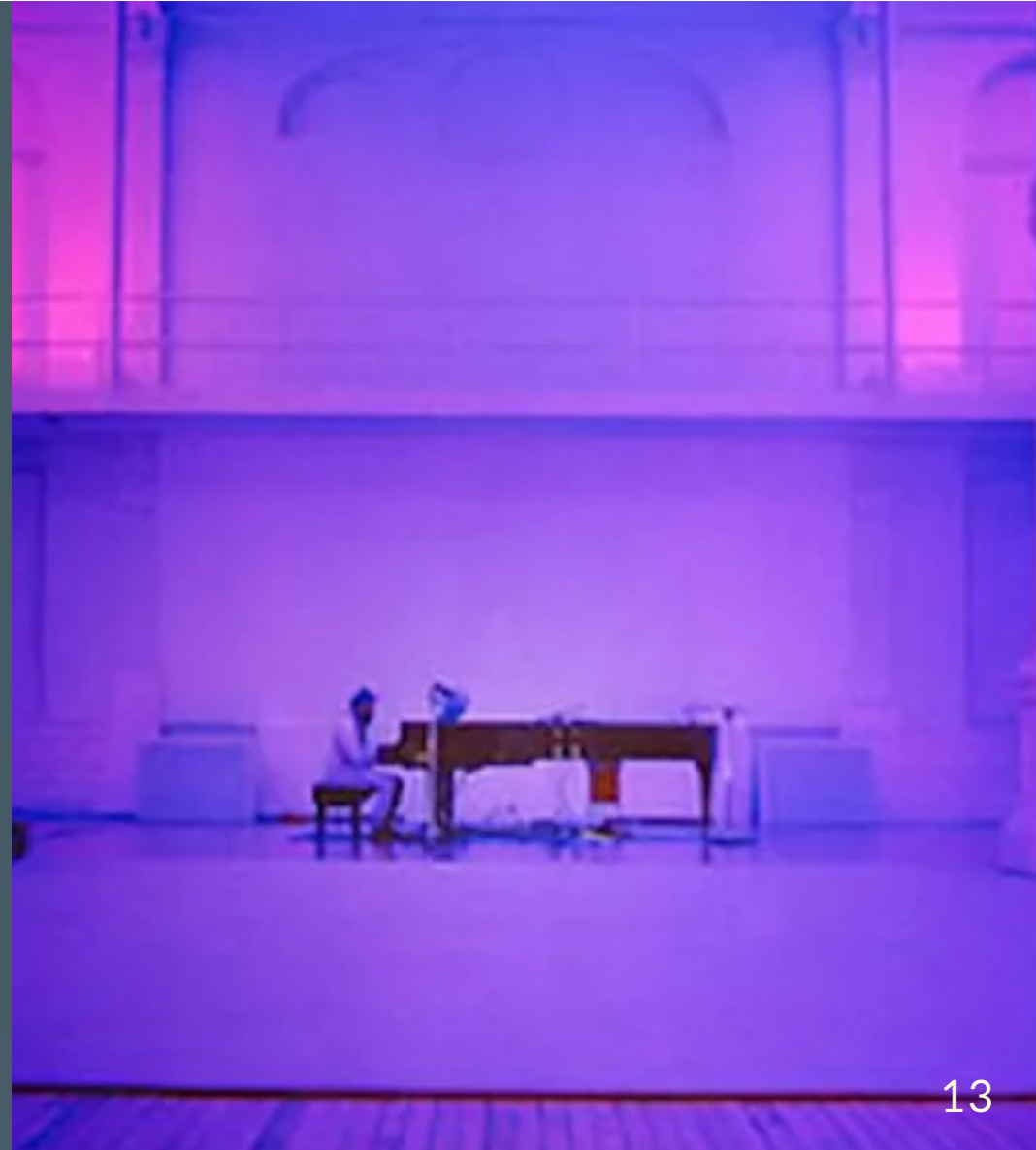
Analysis

[Harry Partch: Delusion of the Fury \(1969\)](#)

Analysis

La Monte Young: the Well-Tuned Piano

(Yes, it's 5 hours long)



Since it's on a piano, Young fixed the pitches:

Young


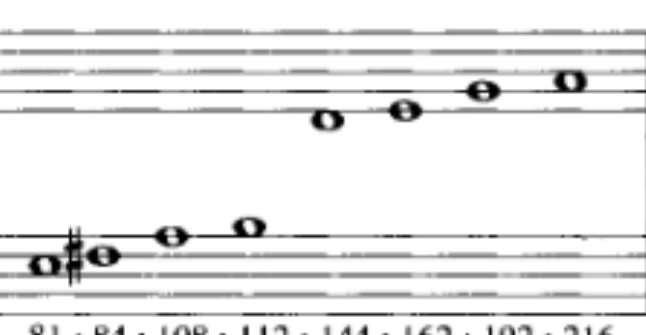
ratio:	$\frac{1}{1}$	$\frac{567}{512}$	$\frac{9}{8}$	$\frac{147}{128}$	$\frac{21}{16}$	$\frac{1323}{1024}$	$\frac{189}{128}$	$\frac{3}{2}$	$\frac{49}{32}$	$\frac{7}{4}$	$\frac{441}{256}$	$\frac{63}{32}$
cents:	0	177	204	240	471	444	675	702	738	969	942	1173

$\frac{1}{1}$ is an Eb. What do you notice about these pitches? (Hint: think about their prime factors). How did he get this tuning?

		$\times \frac{3}{2}$			
	$\frac{49}{32}$	$\frac{147}{128}$	$\frac{441}{256}$	$\frac{1323}{1024}$	
	B	F \sharp	C \sharp	G \sharp	
$\times \frac{7}{4}$	$\frac{7}{4}$	$\frac{21}{16}$	$\frac{63}{32}$	$\frac{189}{128}$	$\frac{567}{512}$
	C	G	D	A	E
	$\frac{1}{1}$	$\frac{3}{2}$	$\frac{9}{8}$		
	E \flat	B \flat	F		

The opening chord of *Well-Tuned Piano*:

$$\frac{1}{1} - \frac{3}{2} - \frac{7}{4} - \frac{9}{8}$$

The Opening Chord		The Magic Chord	
			
4 : 6 : 7 : 8 : 9 : 12		81 : 84 : 108 : 112 : 144 : 162 : 192 : 216	
2 : 3		27 : 28 27 : 28 8 : 9 8 : 9	
2 : 3		7 : 9 7 : 9 27 : 32	