

Tuning and Temperament

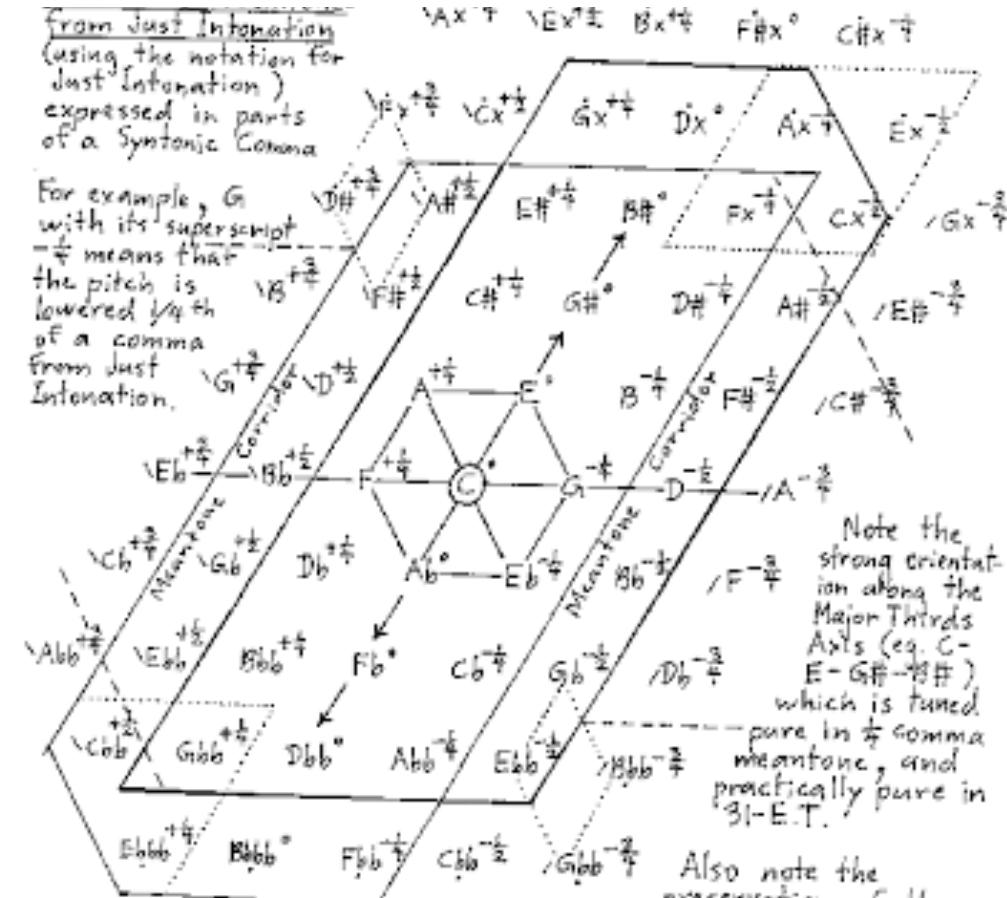
Class 3: (Extended) Just Intonation

Today's Class

- Introduction to just intonation
 - 3-limit, 5-limit, etc
- Tone lattices: a useful tool
- **Analysis:** Ben Johnston's Crossings: String Quartet No. 4, "Amazing Grace"
- **Analysis:** Partch: Delusion of the Fury (excerpt)
 - More of a discussion of the work since even a cursory analysis would require more than we've discussed.
- **Analysis:** La Monte Young's Well-Tuned Piano (1985) (excerpt)

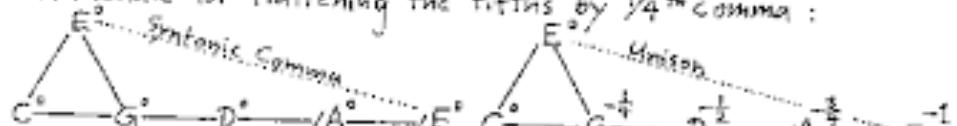
Tone Lattices (or squares)

- Useful to visualize interval relationships and to build out harmonies
- Can exist in multiple dimensions (!)
- Bread and butter for extended just intonation tunings in the 20th and 21st centuries

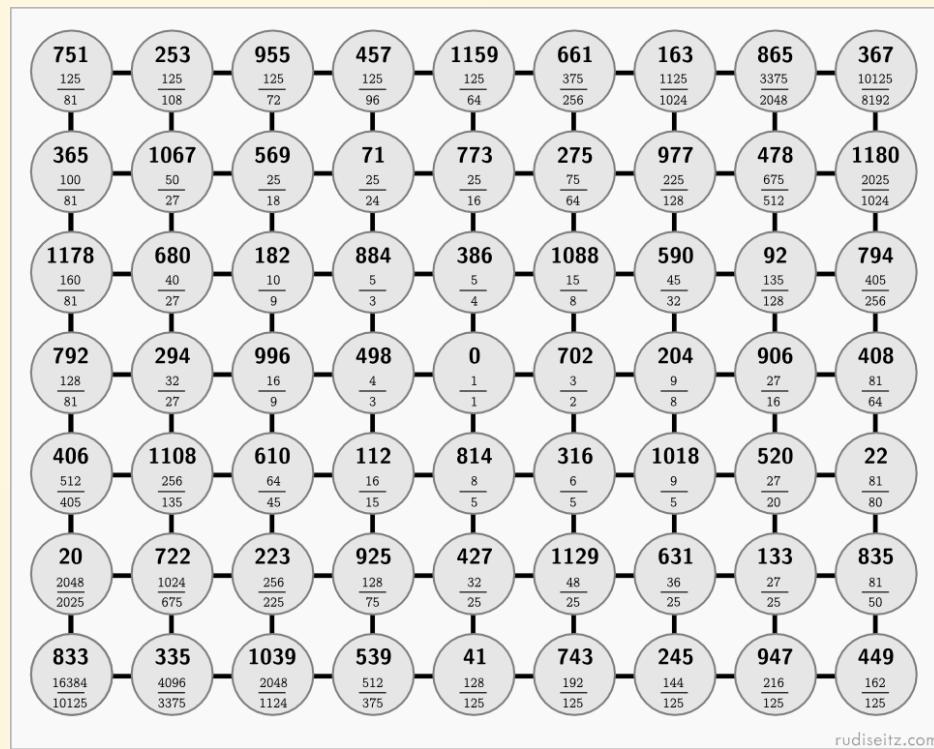


Also note the preservation of the bottom of the Field. In the closed cycle of 31-E.T., these two ends meet in a "strange loop" at the opposite side of the circle of Fifths from the Generator tone "C".

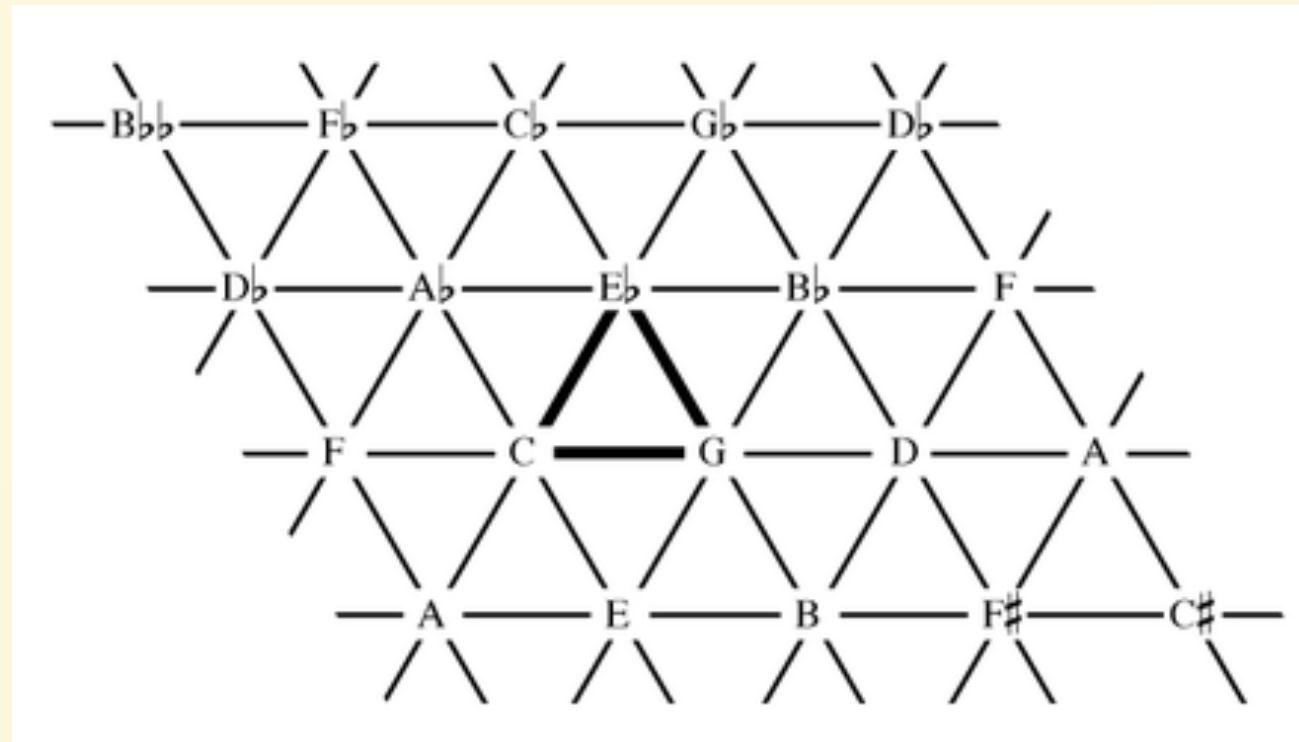
Rationale for flattening the Fifths by $\frac{1}{4}$ th comma:



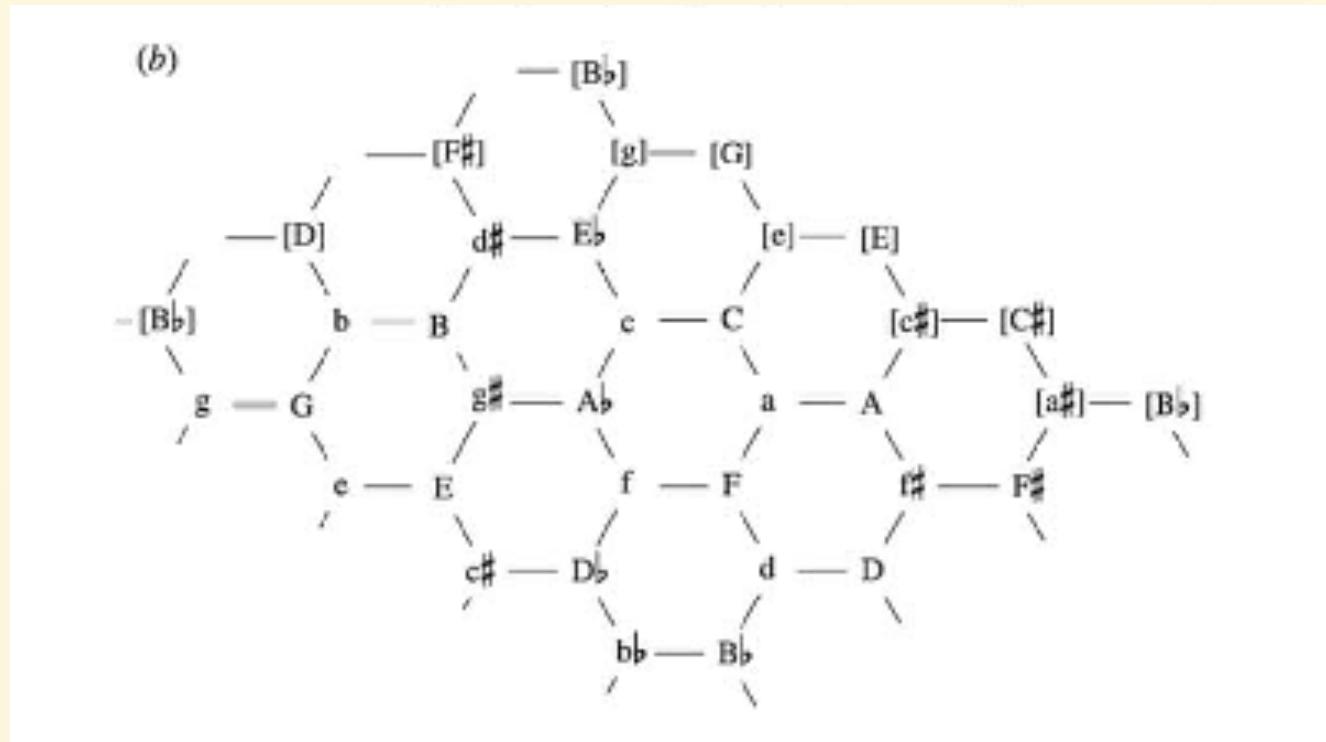
2D with square connections (5-limit in this instance)



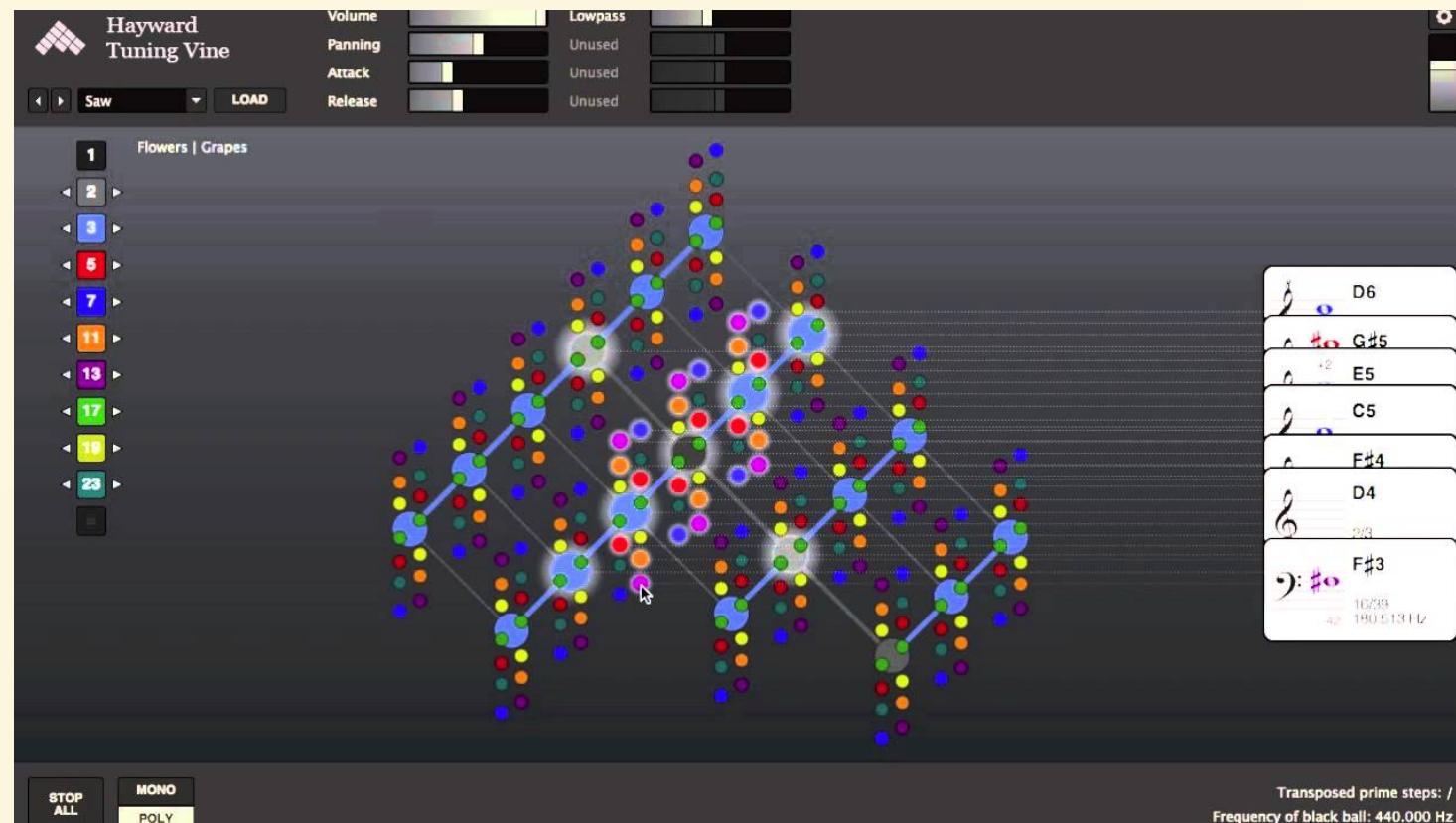
2D with triangular connections



2D with hexagonal connections



3D



Analysis

Ben Johnston's Crossings:
String Quartet No. 4,
"Amazing Grace"

III Variation

solemnity $\lambda = 80$

32

Progressively more complex tuning for each variation

I and II: Pythagorean pentatonic

$$\frac{1}{1} - \frac{9}{8} - \frac{81}{64} - \frac{3}{2} - \frac{27}{16} - \frac{2}{1}$$

A musical staff in G clef and common time. It shows six notes corresponding to the ratios: $\frac{1}{1}$, $\frac{9}{8}$, $\frac{81}{64}$, $\frac{3}{2}$, $\frac{27}{16}$, and $\frac{2}{1}$. The notes are placed on the first, second, third, fifth, and sixth spaces from left to right. Below the staff, the word "Pythagorean" is written.

III: 5-limit Just Intonation

$$\frac{1}{1} - \frac{9}{8} - \frac{5}{4} - \frac{4}{3} - \frac{3}{2} - \frac{5}{3} - \frac{15}{8} - \frac{2}{1}$$

$\frac{1}{1}$ $\frac{9}{8}$ $\frac{5}{4}$ $\frac{4}{3}$ $\frac{3}{2}$ $\frac{5}{3}$ $\frac{15}{8}$ $\frac{2}{1}$

The scale ratios multiplied by 1/2 =

$$\frac{1}{2} \quad \frac{9}{16} \quad \frac{5}{8} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{5}{6} \quad \frac{15}{16} \quad \frac{1}{1}$$

The above fractions subtracted from 1/1 =

$$\frac{1}{2} \quad \frac{7}{16} \quad \frac{3}{8} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{16} \quad \frac{0}{0}$$

The above series of fractions from 0/0 to 1/1 times 48 =

$$0 \ 3 \ 8 \ 12 \ 16 \ 18 \ 21 \ 24 \ 27 \ 30 \ 32 \ 36 \ 40 \ 45 \ 48
(3+5+4+4+2+3+3+3+3+2+4+4+5+3)$$

The meters used are then:

$$\begin{matrix} 3 & 5 & 4 & 4 & 2 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \end{matrix}$$

IV and V: 7-limit justly intoned "blues"

The musical staff shows 13 notes corresponding to 7-limit just intonation ratios. The notes are: $\frac{1}{1}$, $\frac{28}{27}$, $\frac{10}{9}$, $\frac{7}{6}$, $\frac{5}{4}$, $\frac{21}{16}$, $\frac{45}{32}$, $\frac{3}{2}$, $\frac{14}{9}$, $\frac{5}{3}$, $\frac{7}{4}$, $\frac{15}{8}$, and $\frac{2}{1}$. The staff has a treble clef and a common time signature. Below the staff, the ratios are repeated below each note.

Notice the different "flavors" of intervals. There are even more when accounting for those not against $\frac{1}{1}$.

Did you catch the Partch quote from *Greek Studies* in Variation V?



Listening

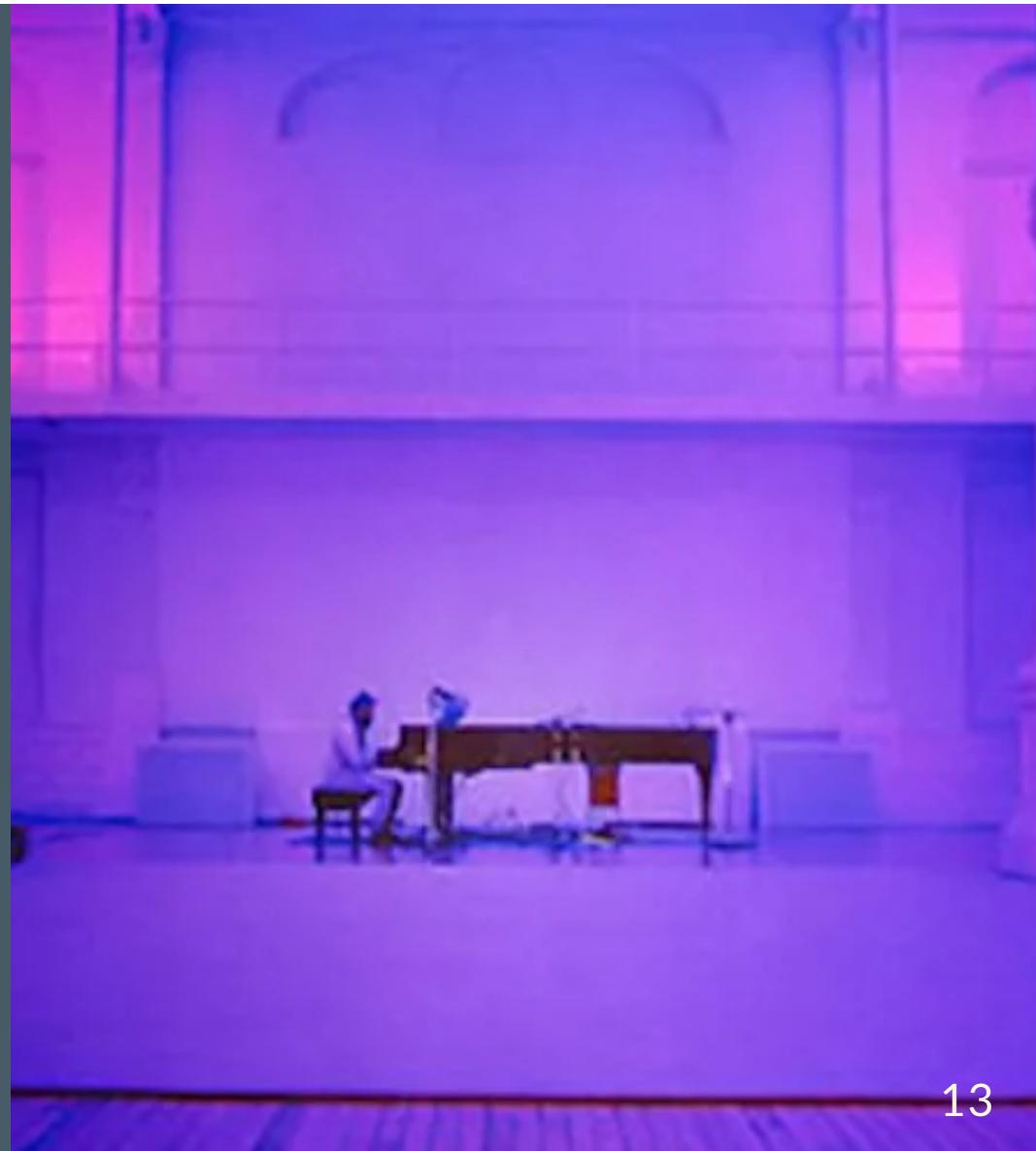
[Pauline Oliveros: Accordion & Voice, Horse Sings from Cloud \(1982\)](#)

"[My instrument] is also tuned in just intonation: a five limit system in the left hand, a seven limit system in the right hand.""

Analysis

La Monte Young: the Well-Tuned Piano

(Yes, it's 5 hours long)



Since it's on a piano, Young fixed the pitches:

Young



ratio: $\frac{1}{1}$ $\frac{567}{512}$ $\frac{9}{8}$ $\frac{147}{128}$ $\frac{21}{16}$ $\frac{1323}{1024}$ $\frac{189}{128}$ $\frac{3}{2}$ $\frac{49}{32}$ $\frac{7}{4}$ $\frac{441}{256}$ $\frac{63}{32}$

cents: 0 177 204 240 471 444 675 702 738 969 942 1173

$\frac{1}{1}$ is an Eb. What do you notice about these pitches? (Hint: think about their prime factors). How did he get this tuning?

				$\times \ 3/2$
	$\frac{49}{32}$	$\frac{147}{128}$	$\frac{441}{256}$	$\frac{1323}{1024}$
	B	F ♯	C ♯	G ♯
$\times \ 7/4$	$\frac{7}{4}$	$\frac{21}{16}$	$\frac{63}{32}$	$\frac{189}{128}$
	C	G	D	A
	$\frac{1}{1}$	$\frac{3}{2}$	$\frac{9}{8}$	
	E♭	B♭	F	

The opening chord of *Well-Tuned Piano*:

$$\frac{1}{1} - \frac{3}{2} - \frac{7}{4} - \frac{9}{8}$$

The Opening Chord The Magic Chord

The image shows two staves of musical notation. The left staff, labeled 'The Opening Chord', consists of two measures. The first measure has a treble clef, a key signature of one flat, and a common time signature. It contains notes on the G, B, and D lines. The second measure has a bass clef, a key signature of one flat, and a common time signature. It contains notes on the B, D, and F lines. Below the staff, the ratios 4 : 6 : 7 : 8 : 9 : 12 and 2 : 3 are listed. The right staff, labeled 'The Magic Chord', also consists of two measures. The first measure has a treble clef, a key signature of one sharp, and a common time signature. It contains notes on the A, C, and E lines. The second measure has a bass clef, a key signature of one sharp, and a common time signature. It contains notes on the C, E, and G lines. Below the staff, the ratios 81 : 84 : 108 : 112 : 144 : 162 : 192 : 216 and 27 : 28 : 27 : 28 : 8 : 9 : 8 : 9 are listed. The ratios 7 : 9 and 27 : 32 are also present below the staff.

4 : 6 : 7 : 8 : 9 : 12 81 : 84 : 108 : 112 : 144 : 162 : 192 : 216

2 : 3 27 : 28 : 27 : 28 : 8 : 9 : 8 : 9

7 : 9 27 : 32