

Tuning and Temperament

Class 7: Into the weeds

Today's Class

- Non-octave scales??
 - The tritave
- The Bohlen-Pierce Scale
- The Bohlen-Pierce Clarinet



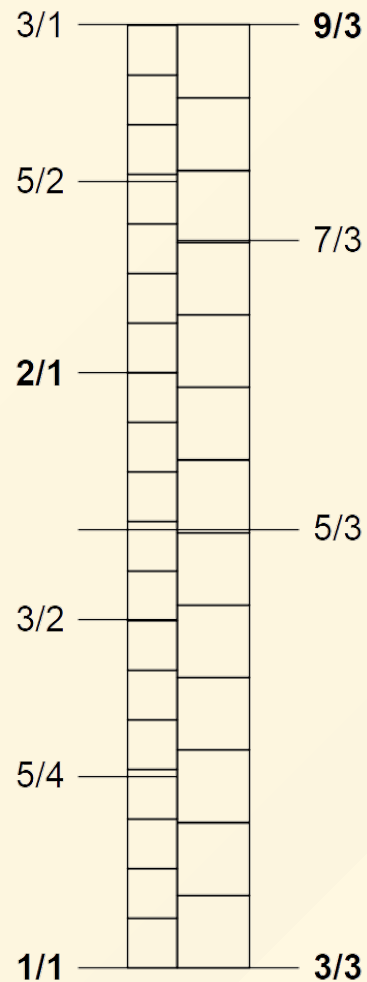
A fundamental question

Why should we only use octave-based scales?

Recall the harmonic series:

The image displays the harmonic series for the note C, spanning from the second line of the bass clef (C2) to the second line of the treble clef (C8). The notes are numbered 1 through 32. The first line (bass clef) contains notes 1 through 16, and the second line (treble clef) contains notes 17 through 32. A dashed line labeled '8va' indicates the octave jump between note 16 and note 17. The notes are: 1 (C2), 2 (C3), 3 (G2), 4 (F2), 5 (E2), 6 (D2), 7 (C3), 8 (C4), 9 (G3), 10 (F3), 11 (E3), 12 (D3), 13 (C4), 14 (B3), 15 (A3), 16 (G3), 17 (C5), 18 (B4), 19 (A4), 20 (G4), 21 (F4), 22 (E4), 23 (D4), 24 (C5), 25 (B5), 26 (A5), 27 (G5), 28 (F5), 29 (E5), 30 (D5), 31 (C6), 32 (C6). The notes are written as whole notes on a five-line staff. The first line (bass clef) has a C2 note on the second line, a C3 note on the first space, a G2 note on the first line, a F2 note on the first space, an E2 note on the first line, a D2 note on the first space, a C3 note on the first line, a C4 note on the first space, a G3 note on the first line, a F3 note on the first space, an E3 note on the first line, a D3 note on the first space, a C4 note on the first line, a B3 note on the first space, an A3 note on the first line, and a G3 note on the first space. The second line (treble clef) has a C5 note on the first space, a B4 note on the first line, an A4 note on the first space, a G4 note on the first line, an F4 note on the first space, an E4 note on the first line, a D4 note on the first space, a C5 note on the first space, a B5 note on the first line, an A5 note on the first space, a G5 note on the first line, an F5 note on the first space, an E5 note on the first line, a D5 note on the first space, a C6 note on the first space, and a C6 note on the first space.

Apart from $\frac{2}{1}$, what is the next most simple interval?



The third harmonic, $\frac{3}{1}$, is an octave plus a fifth $\left(\frac{2}{1} \times \frac{3}{2} \right)$. This interval is also known as a *tritave*.

What happens when we build a scale based on the tritave as opposed to the octave?

The Bohlen-Pierce Scale

- Described independently by Heinz Bohlen (left), John Pierce (middle), and Kees van Prooijen (right). Both Bohlen and Pierce were microwave engineers.
- Replaces $\frac{2}{1}$ with $\frac{3}{1}$ as the scale's fundamental interval.



Let's Listen

[Kjell Hansen: Bohlen-Pierce Cannon](#)

Principle of Equidistance

- Essentially, it is a mathematical expression of how the exponential frequency sensitivity of our ears relates to the "aesthetic pleasure"* of a monophonic musical scale.

$$\frac{f_n}{f_0} = K^{n/N}$$

where f_n is the pitch of step n of the scale, f_0 is the fundamental tone (2 for octave, 3 for tritave, etc), K the frame interval and N the total number of steps. Both f 's are measured in Hz.

*Whatever that means.

Principle of Equidistance (cont.)

The closer a (tempered) musical scale's steps follow the equation, the more "aesthetically pleasing" it is supposed to be.

Example:

Let's take 12edo based on A = 440Hz.

$$f_0 = 440$$

$$K = 2$$

$$N = 12$$

Principle of Equidistance (cont.)

Substituting those values:

$$\frac{f_n}{f_0} = K^{n/N}$$
$$\frac{f_n}{f_0} = 2^{n/12}$$

Note that this is the definition of 12 equal divisions of the octave where each half-step is $\sqrt[12]{2}$.

Principle of Equidistance (cont.)

Example: check for the 7th scale degree:

$$\frac{f_n}{f_0} = 2^{n/12}$$

$$\frac{659.25511382467}{440} = 2^{7/12}$$

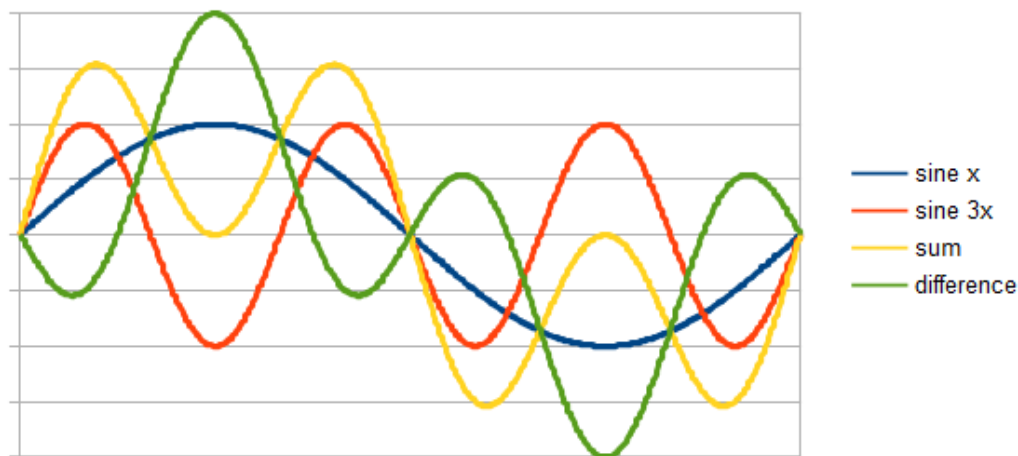
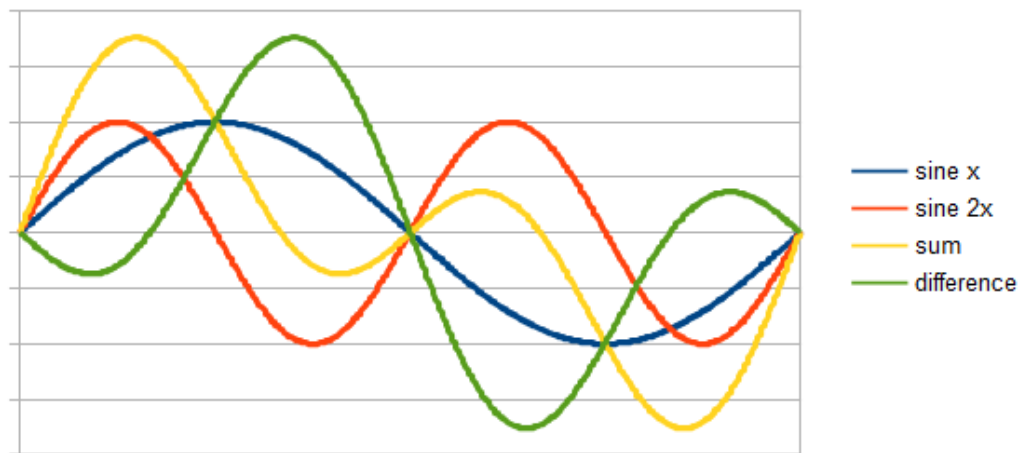
$$1.49830707 = 1.49830707$$

Principle of consonance

- The principle of equidistance applies only to monophonic music. With the "principle of consonance", theorists are attempting to take account of the Gestalt compatibility between intervals.

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- A Gestalt impression "views the formation of a general impression as the sum of several interrelated impressions." To take this to tone-space, we need to account for how different tones produced by physical instruments (with their individual spectrum) combine to create a new sensation by their combination tones and by altering each others harmonics.



Aside: Combination Tones?

Combination tones are psychoacoustical phenomenon by which artificial tones are perceived as the **sum** and/or **difference** between two physically present tones.

Related: Diana Deutsch's *Musical Illusions*

Essentially, we want consonance between combination tones:

$pf_0 - qf_x = f_x$ or $pf_x - qf_0 = f_0$ where $p \geq 1$ and $0 < q \leq p$ and are both integers. Simplified:

$$\frac{f_x}{f_0} = \frac{p}{(q+1)}$$

and

$$\frac{f_x}{f_0} = \frac{(q+1)}{p}$$

where $p, q \geq 1$.

Using both the principles of both consonance and equidistance, where does this lead us? Let's assume that:

$$\begin{aligned} &\{p, q \geq 1 | p, q \in \mathbb{Z}\}, \\ &p \text{ are odd numbers,} \\ &p + q \text{ are odd numbers,} \\ &q < p \end{aligned}$$

What occurs?

Using the above criteria, we get a 13-note scale in the frame of a twelfth (tritave):

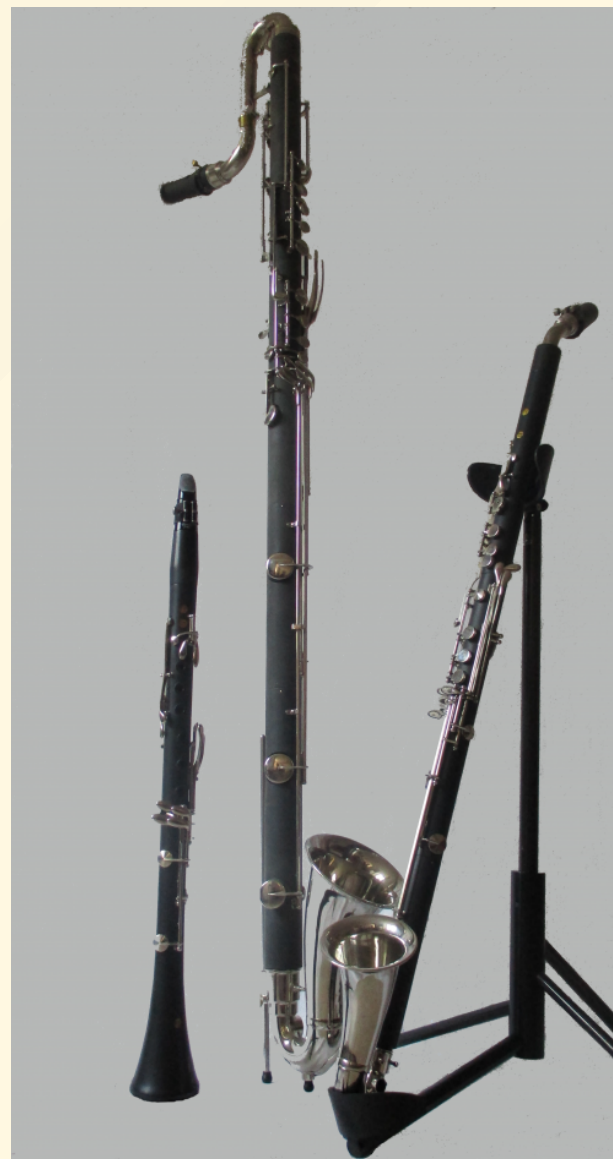
$$\frac{1}{1} - \frac{27}{25} - \frac{25}{21} - \frac{9}{7} - \frac{7}{5} - \frac{75}{49} - \frac{5}{3} - \frac{9}{5} - \frac{49}{25} - \frac{15}{7} - \frac{7}{3} - \frac{63}{25} - \frac{25}{9} - \frac{3}{1}$$

Note that since the frame interval is no longer $\frac{2}{1}$ but rather $\frac{3}{1}$, we want to ratios to fall between 1 and 3. The subset below is the actual Bohlen-Pierce:

$$\frac{1}{1} - \frac{27}{25} - \frac{9}{7} - \frac{7}{5} - \frac{5}{3} - \frac{9}{5} - \frac{15}{7} - \frac{7}{3} - \frac{25}{9} - \frac{3}{1}$$

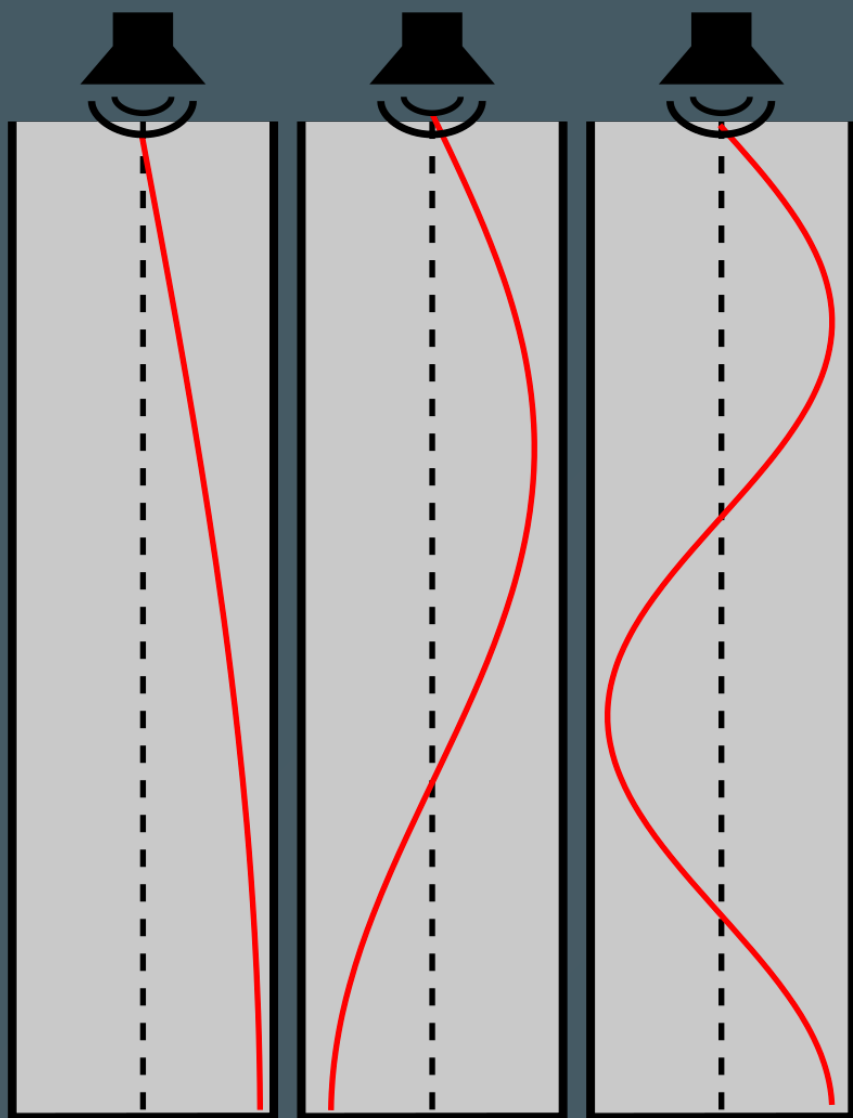
Of course, this is then approximated with equal temperament \Rightarrow :

$$\frac{f_n}{f_0} = K^{n/N}$$
$$\frac{f_n}{f_0} = 3^{n/13}$$



Enough, let's hear some music.

Improvisation by Nora-Louise Müller
Concert 3: Bohlen-Pierce Symposium 2010



What's with the clarinet?

Lucky for us, the clarinet works on the principle of the twelfth. The acoustics of a closed-tube favor odd-harmonics and emphasize 1-3-5-etc.

On a clarinet, the thumb key (on the back of the instrument, played with the left hand) makes the pitch jump a twelfth so the clarinet is particularly suited to the Bohlen-Pierce scale.



Homework Four

Propose a final project.