

Tuning and Temperament

Class 5: Meantone Temperament

Today's Class

- Where just intonation falls short
- Temperament
 - What is temperament?
- Meantone temperaments
 - $1/4$ comma
 - $1/5$ comma



Where does just intonation fall short?

Let's look at a passage of music and assume that our tuning system is just intonation (5-limit). Let's also try to keep it *totally* in tune such that every interval between every note is just (i.e. no commas or wolf intervals).

A musical score for a four-part setting, likely a canon or a similar contrapuntal exercise. The score is written on four staves, each with a different clef: Treble, Treble 8va, Treble 8va, and Bass. The music is organized into six measures, with a double bar line at the end of the sixth measure. Each measure contains a ratio, which is a fraction representing the interval between the notes of the four parts. The ratios are as follows:

Measure	Ratio
1	6/5
2	4/3 (3/2)
3	5/4
4	5/4
5	4/3 (3/2)
6	6/5

The ratios are written below the notes of the four parts in each measure. The notes are half notes, and the intervals are indicated by the ratios. The ratios are: 6/5, 4/3 (3/2), 5/4, 5/4, 4/3 (3/2), 6/5. The ratios are written in a way that suggests a sequence of intervals, with the first ratio being 6/5, the second 4/3 (3/2), the third 5/4, the fourth 5/4, the fifth 4/3 (3/2), and the sixth 6/5.

A musical score for a four-part setting, likely a canon or a similar contrapuntal piece. The score is written for four staves, each with a different clef: Treble (top), Treble (second), Treble (third), and Bass (bottom). The music is organized into six measures, with ratios and intervals indicated above and below the notes.

Measure	Staff 1 (Treble)	Staff 2 (Treble)	Staff 3 (Treble)	Staff 4 (Bass)
1	6/5	1/1	4/3 (3/2)	3/2
2	6/5	9/5	6/5	5/4
3	9/8	9/5	4/3 (3/2)	3/2
4	81/80	81/50	6/5	1/1
5	27/20	81/80	5/4	6/5
6	243/200	81/80	4/3 (3/2)	3/2

The ratios and intervals are as follows:

- Measure 1: 6/5, 1/1, 4/3 (3/2), 3/2
- Measure 2: 6/5, 9/5, 6/5, 5/4
- Measure 3: 9/8, 9/5, 4/3 (3/2), 3/2
- Measure 4: 81/80, 81/50, 6/5, 1/1
- Measure 5: 27/20, 81/80, 5/4, 6/5
- Measure 6: 243/200, 81/80, 4/3 (3/2), 3/2

Temperaments

What is a temperament anyway? What is a tuning?

Tuning:

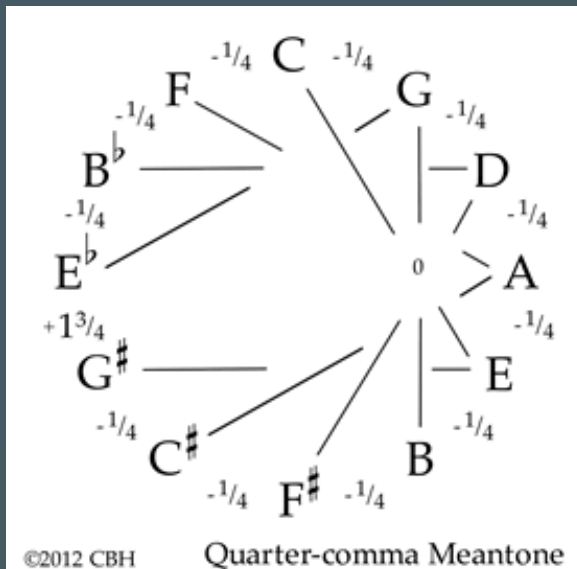
1. A system whose intervals all can be expressed by rational numbers.

Temperament:

1. A system whose intervals cannot all be expressed by rational numbers.
2. A system which "tempers" just intervals.
3. Partch: "... a system which deliberately robs its intervals of their purity in order to implement the idea of every-tone-in-several senses"

1/4 Comma Meantone Temperament

- Pure thirds ($\frac{5}{4}$, $\frac{6}{5}$)
- Flat fifths, by 1/4 of a syntonic comma ($\frac{81}{80}$)
- Wolf fifth between G# and Eb (if tuned on C)



Building a 12-note $1/4$ comma meantone scale

We want pure major thirds, $\frac{5}{4}$, so we want to slightly flatten the fifth, $\frac{3}{2}$. If starting on C, the major third is E. We can get to E either by 1) stacking four $\frac{3}{2}$'s or two octaves and a $\frac{5}{4}$.

In a Pythagorean style of tuning using fifths, the E would be $\frac{81}{64}$ whereas a pure third is $\frac{5}{4}$ (the difference is $\frac{81}{80}$). So, we need to lower each $\frac{3}{2}$ by one quarter of the difference, $\frac{81}{80}$, such that getting to an E by four $\frac{3}{2}$'s or two octaves and a $\frac{5}{4}$ are the same.

We can rewrite $\frac{5}{4}$ as $\frac{5}{1}$ by moving it up two octaves. Therefore, letting r be the ratio of our meantone fifth:

$$r^4 = \frac{5}{1} = 5$$

$$r = \sqrt[4]{5}$$

So in "musical" terms:

$$r \approx 1.49535 \approx \frac{643}{430} \approx 696.587 \text{ cents}$$

What does this mean for the rest of the scale?

We construct it exactly as a Pythagorean scale, substituting $\sqrt[4]{5}$ for $\frac{3}{2}$. For example, a whole step is:

$$\sqrt[4]{5} \times \sqrt[4]{5} \times \frac{1}{2} = \frac{\sqrt[4]{5} \times \sqrt[4]{5}}{2} = \frac{5^{1/4} \times 5^{1/4}}{2} = \frac{5^{1/2}}{2} = \frac{\sqrt{5}}{2}$$

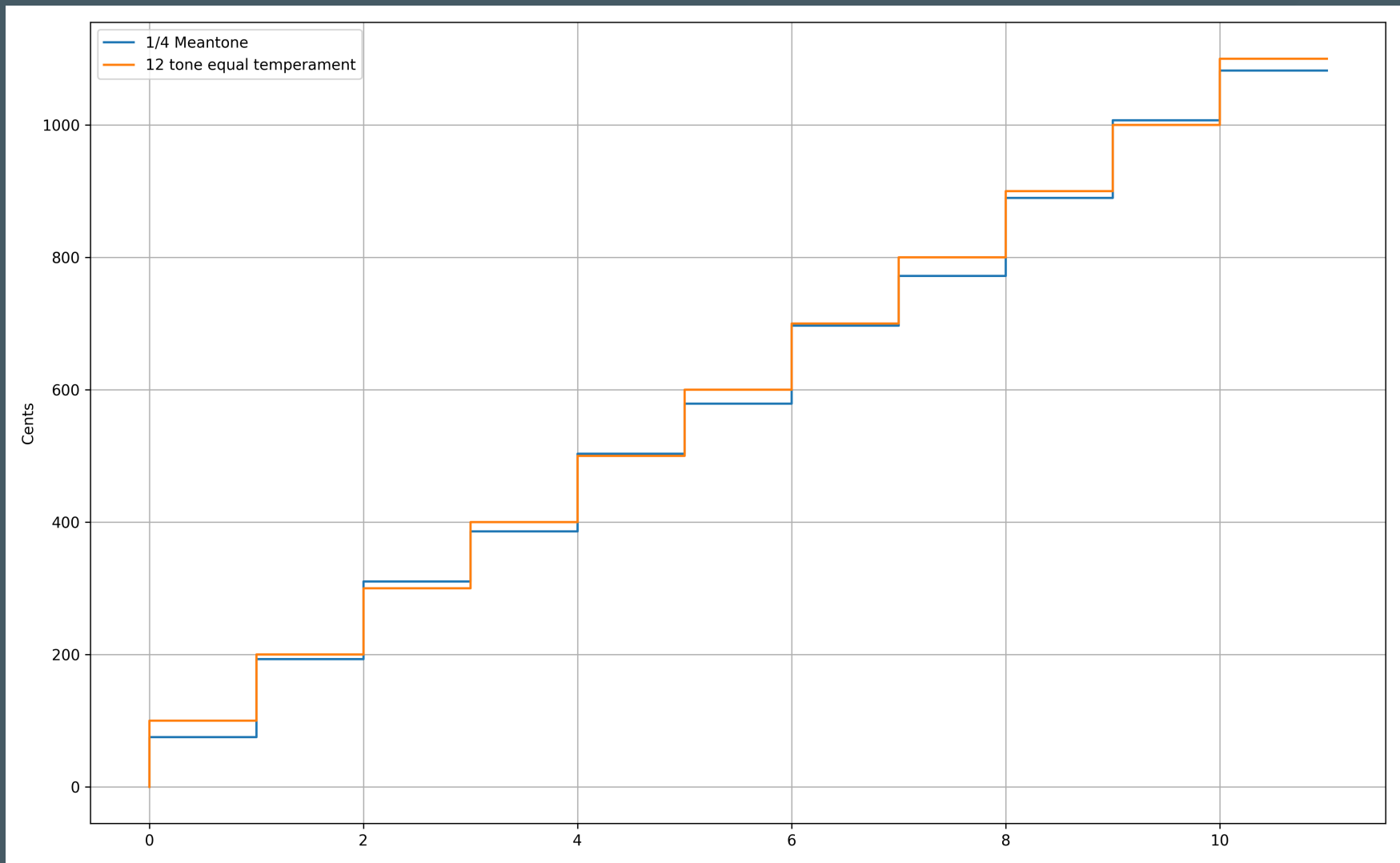
Confirm by taking two whole steps to make a major third:

$$\frac{\sqrt{5}}{2} \times \frac{\sqrt{5}}{2} = \frac{5}{4}$$

Formula for pitches of a major scale where r is the fifth:

Note	Formula	Cents	Note	Formula	Cents
C	$r^0 \times 2^0 = 1$	0	G	$r^1 \times 2^0 = r$	696.6
D	$r^2 \times 2^{-1} = \frac{\sqrt{5}}{2}$	193.2	A	$r^3 \times 2^{-1} = \frac{r\sqrt{5}}{2}$	889.7
E	$r^4 \times 2^{-2} = \frac{5}{4}$	386.3	B	$r^5 \times 2^{-2} = \frac{5r}{4}$	1082.9
F	$r^{-1} \times 2^2 = \frac{2r\sqrt{5}}{5}$	503.4	C	$r^0 \times 2^1 = 2$	1200

Wolf fifth between G# and Eb so it's symmetrical (common).



Listening

[Mozart's Fantasie KV397 in Three Different Temperaments](#)

Listen to about 2 min in Equal Temperament (0:00), 1/4 comma meantone (11:40), and just for kicks, Prelleur temperament (5:40).

[Yale's Divinity School Meantone Organ](#)

Split sharps?

Other flavors of meantone:

- 1/5 comma meantone (Pythagorean comma)
- 1/6 comma meantone
- Extended meantone (building extra notes so all/most keys can have pure thirds)
 - [Ascanio Mayone - Examples for the Cimbalo Cromatico](#)
 - [Nicola Vicentino: "Musica prisca caput"](#)

