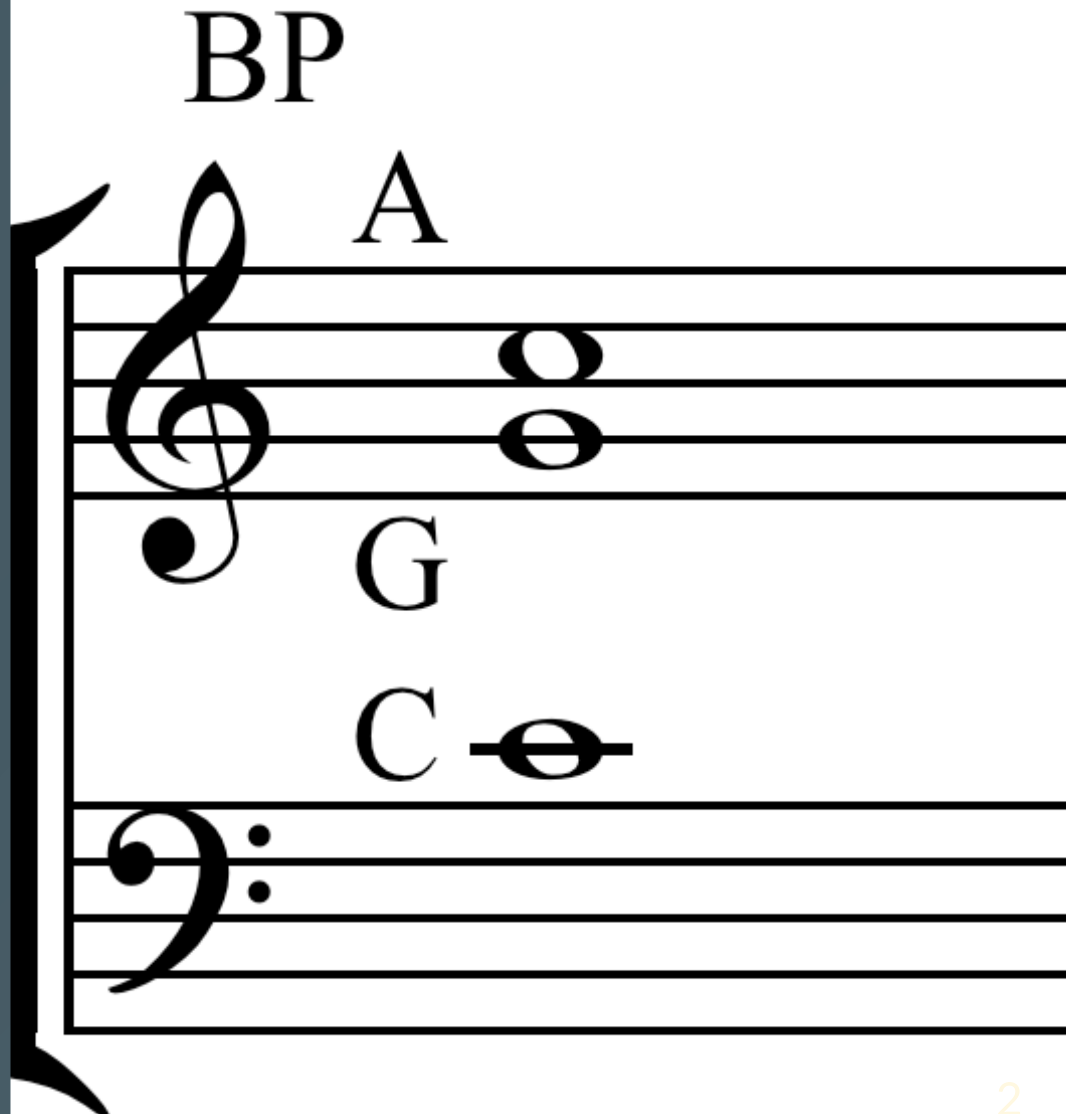
Tuning and Temperament

Class 7: Into the weeds

Today's Class

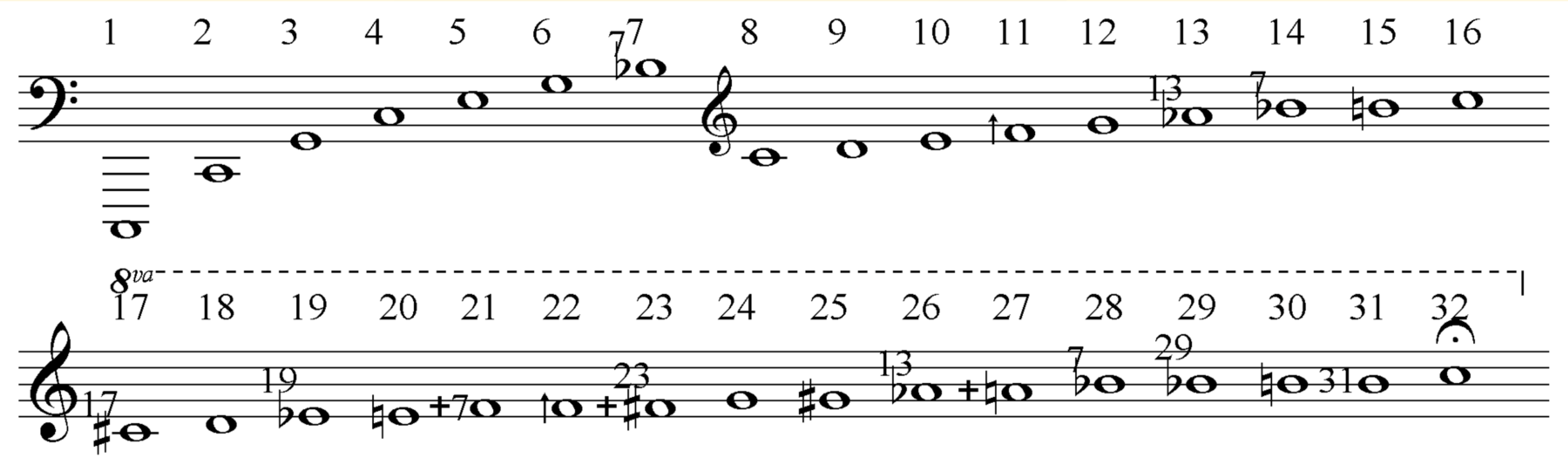
- Non-octave scales??
  - The tritave
- The Bohlen-Pierce Scale
- The Bohlen-Pierce Clarinet



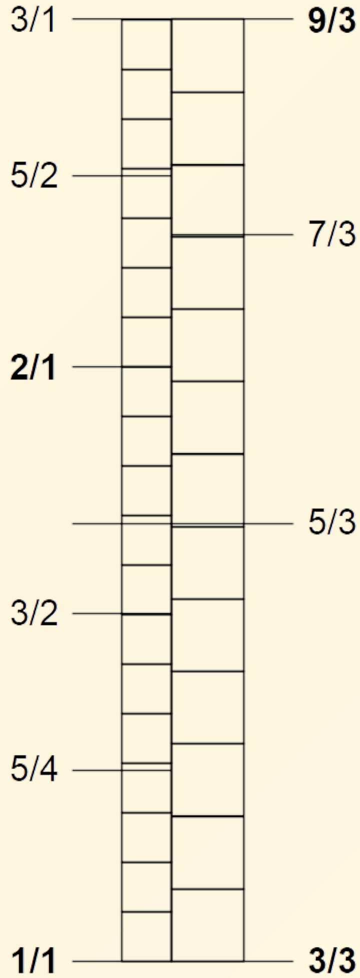
A fundamental question

Why should we only use octave-based scales?

Recall the harmonic series:



Apart from $\frac{2}{1}$, what is the next most simple interval?



The third harmonic, $\frac{3}{1}$, is an octave plus a fifth $\left(\frac{2}{1} \times \frac{3}{2}\right)$. This interval is also known as a *tritave*.

What happens when we build a scale based on the tritave as opposed to the octave?

The Bohlen-Pierce Scale

- Described independently by Heinz Bohlen (left), John Pierce (middle), and Kees van Prooijen (right). Both Bohlen and Pierce were microwave engineers.
- Replaces $\frac{2}{1}$ with $\frac{3}{1}$ as the scale's fundamental interval.

Let's Listen

Kjell Hansen: Bohlen-Pierce Cannon

Principle of Equidistance

- Essentially, it is a mathematical expression of how the exponential frequency sensitivity of our ears relates to the "aesthetic pleasure"* of a monophonic musical scale.

$$\frac{f_n}{f_0} = K^{n/N}$$

where f_n is the pitch of step n of the scale, f_0 is the fundamental tone (2 for octave, 3 for tritave, etc), K the frame interval and N the total number of steps. Both f's are measured in Hz.

*Whatever that means.

Principle of Equidistance (cont.)

The closer a (tempered) musical scale's steps follow the equation, the more "aesthetically pleasing" it is supposed to be.

Example:

Let's take 12edo based on A = 440Hz.

$$f_0 = 440$$
$$K = 2$$
$$N = 12$$

Principle of Equidistance (cont.)

Substituting those values:

$$\frac{f_n}{f_0} = K^{n/N}$$
$$\frac{f_n}{f_0} = 2^{n/12}$$

Note that this is the definition of 12 equal divisions of the octave where each half-step is $\sqrt[12]{2}$.

Principle of Equidistance (cont.)

Example: check for the 7th scale degree:

$$\frac{f_n}{f_0} = 2^{n/12}$$

$$\frac{659.25511382467}{440} = 2^{7/12}$$

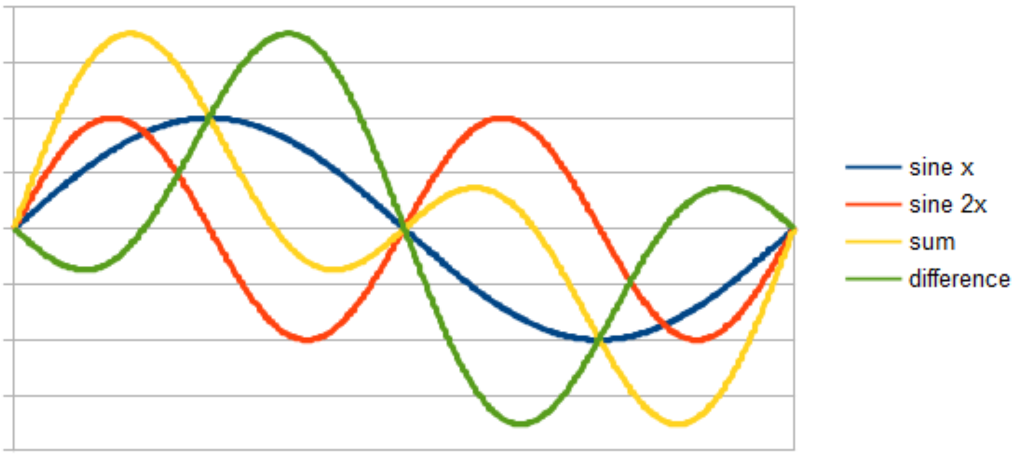
$$1.49830707 = 1.49830707$$

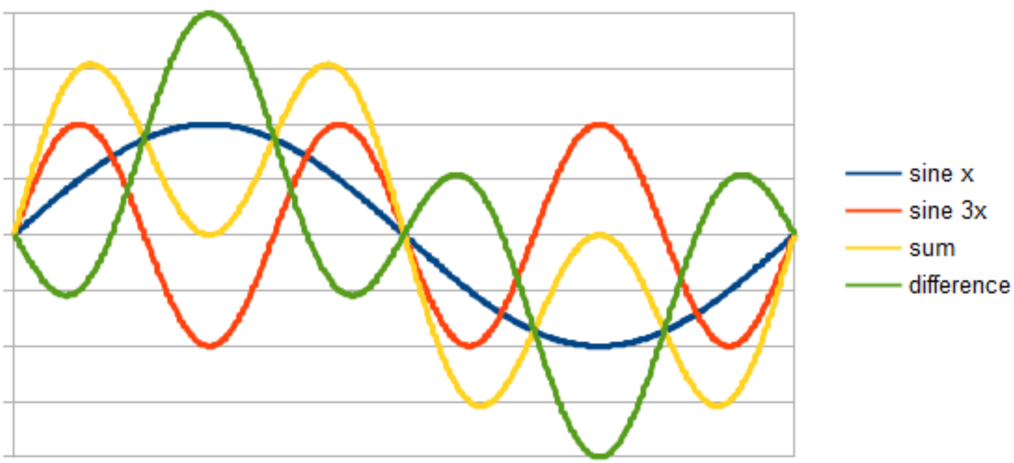
Principle of consonance

- The principle of equidistance applies only to monophonic music. With the "principle of consonance", theorists are attempting to take account of the Gestalt compatibility between intervals.

Principle of consonance

- The principle of equidistance applies only to monophonic music. With the "principle of consonance", theorists are attempting to take account of the Gestalt compatibility between intervals.
- A Gestalt impression "views the formation of a general impression as the sum of several interrelated impressions." To take this to tone-space, we need to account for how different tones produced by physical instruments (with their individual spectrum) combine to create a new sensation by their combination tones and by altering each others harmonics.





Aside: Combination Tones?

Combination tones are psychoacoustical phenomon by which artificial tones are percieved as the **sum** and/or **difference** between two physically present tones.

Related: Diana Deutsch's *Musical Illusions*

Essentially, we want consonance between combination tones: $pf_0 - qf_x = f_x$ or $pf_x - qf_0 = f_0$ where $p \geq 1$ and $0 < q \leq p$ and are both integers. Simplified:

$$\frac{f_x}{f_0} = \frac{p}{(q+1)}$$
$$\text{and}$$
$$\frac{f_x}{f_0} = \frac{(q+1)}{p}$$

where $p, q \geq 1$.

Using both the principles of both consonance and equidistance, where does this lead us? Let's assume that:

$$
\begin{aligned}
&\{p, q \geq 1 | p, q \in \mathbb{Z}\},\\
&p \text{ are odd numbers},\\
&p + q \text{ are odd numbers},\\
&q < p
\end{aligned}
$$

What occurs?

Using the above criteria, we get a 13-note scale in the frame of a twelfth (tritave):

$$\frac{1}{1} - \frac{27}{25} - \frac{25}{21} - \frac{9}{7} - \frac{7}{5} - \frac{75}{49} - \frac{5}{3} - \frac{9}{5} - \frac{49}{25} - \frac{15}{7} - \frac{7}{3} - \frac{63}{25} - \frac{25}{9} - \frac{3}{1}$$

Note that since the frame interval is no longer $\frac{2}{1}$ but rather $\frac{3}{1}$, we want to ratios to fall between 1 and 3. The subset below is the actual Bohlen-Pierce:

$$\frac{1}{1} - \frac{27}{25} - \frac{9}{7} - \frac{7}{5} - \frac{5}{3} - \frac{9}{5} - \frac{15}{7} - \frac{7}{3} - \frac{25}{9} - \frac{3}{1}$$

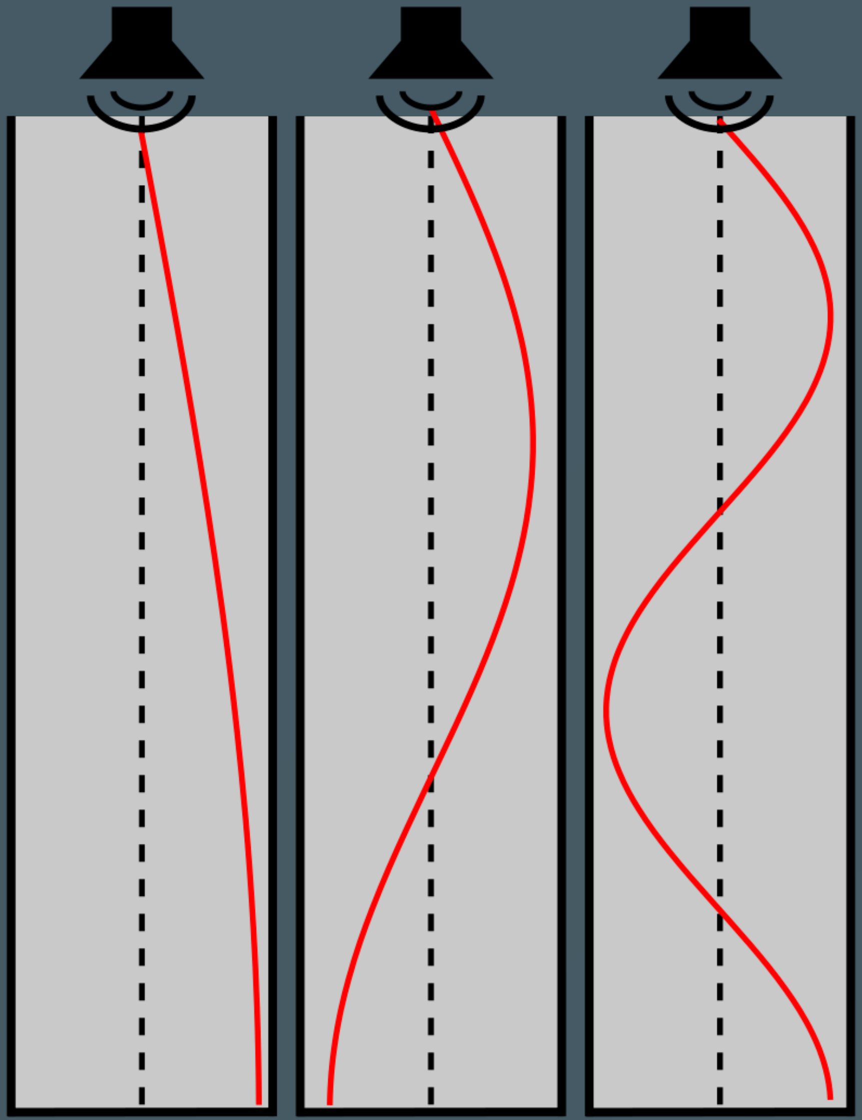
Of course, this is then approximated with equal temperament 😑:

$$\frac{f_n}{f_0} = K^{n/N}$$

$$\frac{f_n}{f_0} = 3^{n/13}$$

Enough, let's hear some music.

Improvisation by Nora-Louise Müller
Concert 3: Bohlen-Pierce Symposium 2010



What's with the clarinet?

Lucky for us, the clarinet works on the principle of the twelfth. The acoustics of a closed-tube favor odd-harmonics and emphasize 1-3-5-etc.

On a clarinet, the thumb key (on the back of the instrument, played with the left hand) makes the pitch jump a twelfth so the clarinet is particularly suited to the Bohlen-Pierce scale.

Homework Four

Propose a final project.