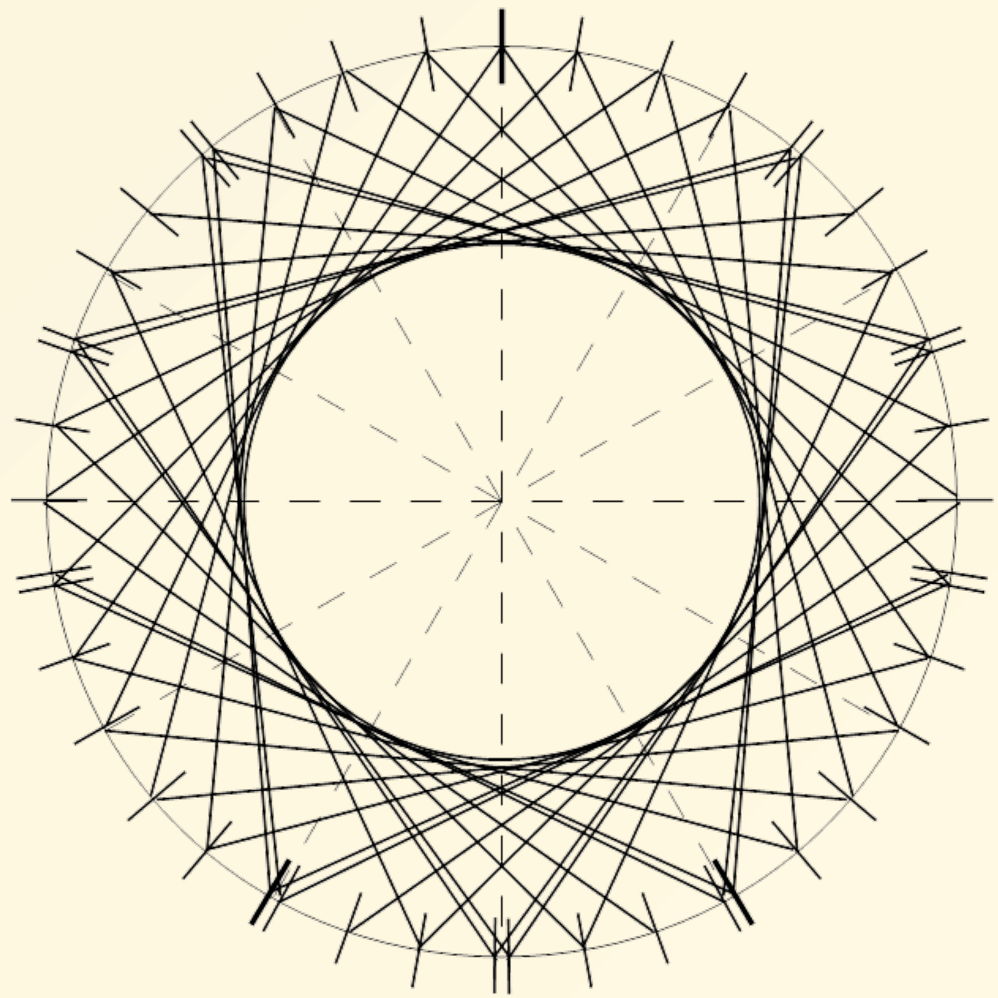
Tuning and Temperament

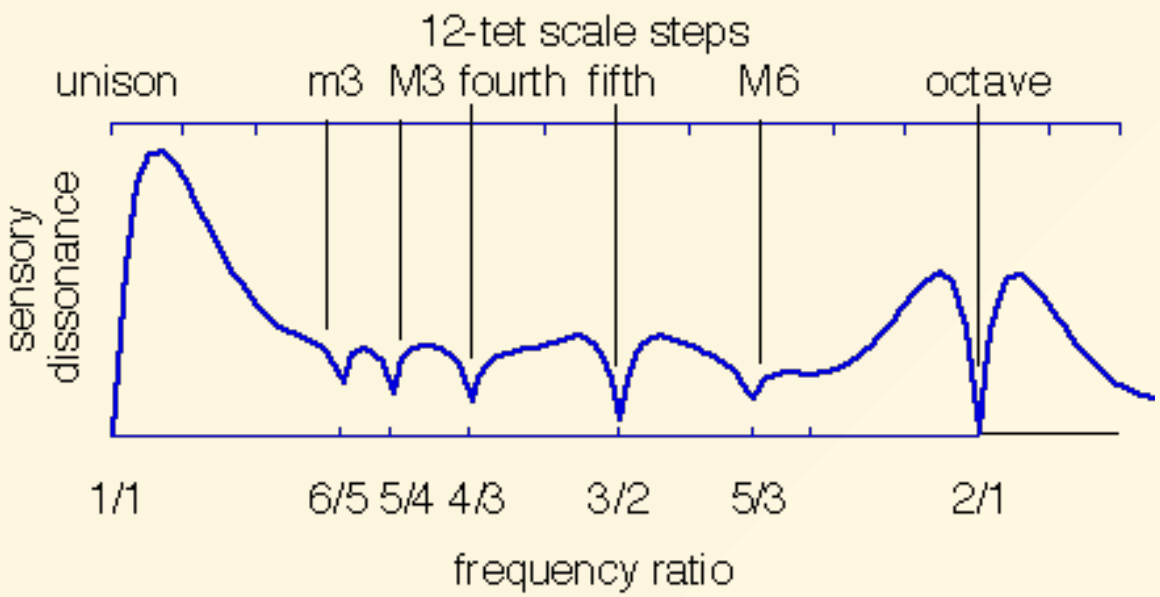
Class 9: Into the weeds: Part II

Today's Class

- William Sethares
  - Spectra and scale
- Bohlen
  - 833 cents scale, A12 scale
- Sacrificing the Octave: **Wendy Carlos**
  - Alpha, beta, gamma, delta



Sethares: Spectra, consonance, and scales



- What is the effect of timbre on consonance?
  - In other words, how does consonance work when given more complex sounds?

Sethares (cont.)

Let's listen to these two recordings: what do you think the difference is?

[Ten Fingers (1)]
[Ten Fingers (2)]

Sethares (cont.)

Let's listen to these two recordings: what do you think the difference is?

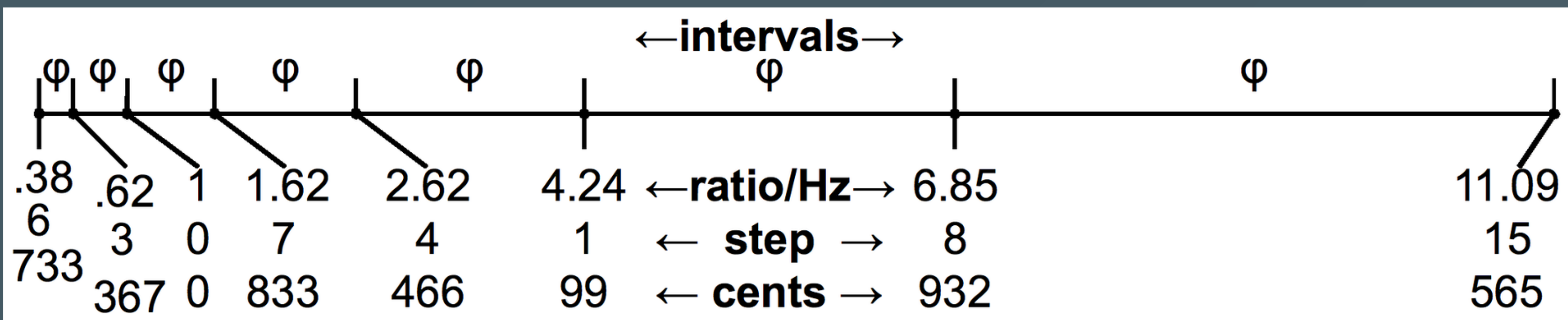
Ten Fingers (1)
Ten Fingers (2)

Apart from the first recoding being longer, the only difference is the way the *spectra* of the notes conform (or don't) with the underlying scale. This is known as *spectral mapping*.

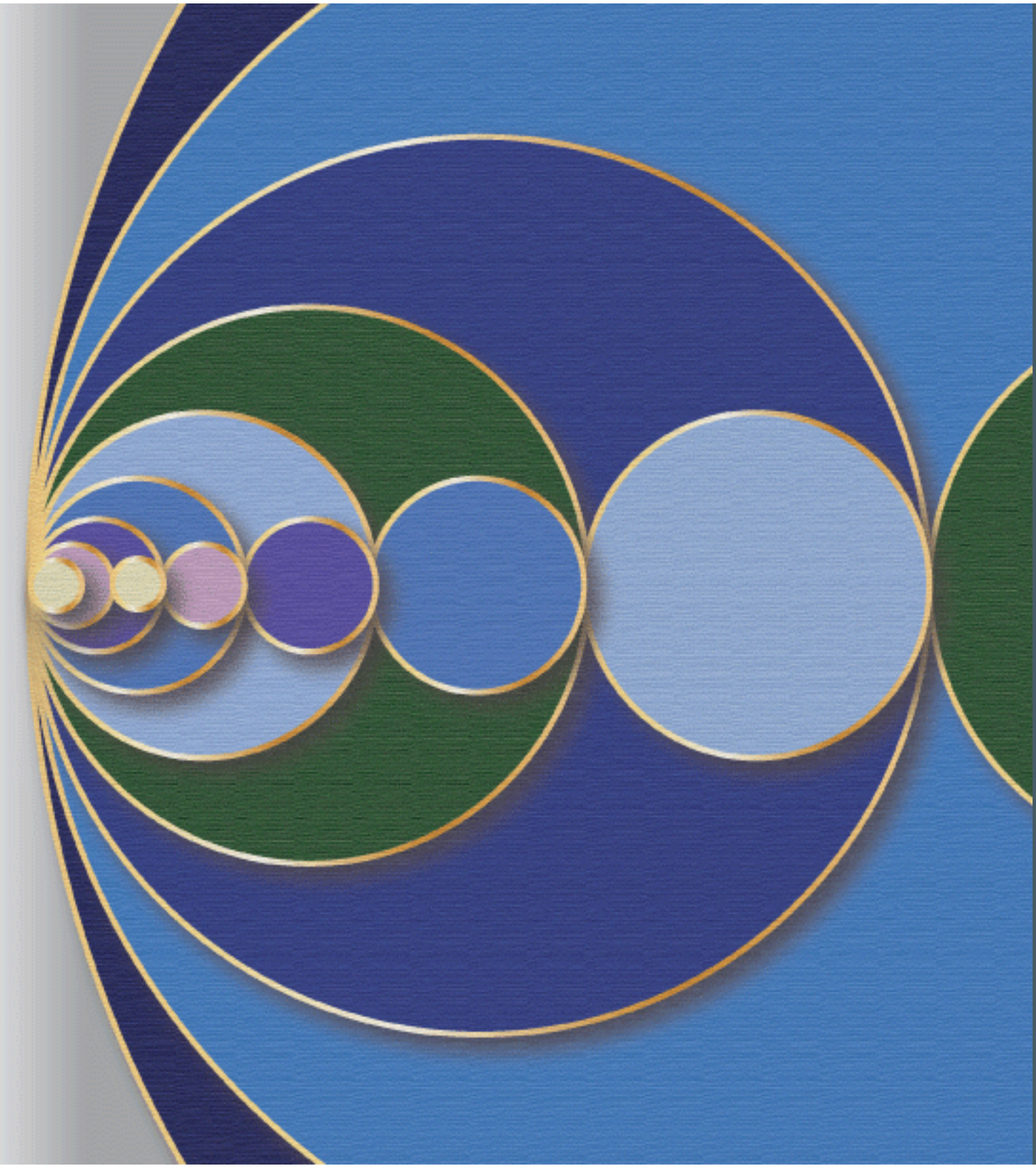
Welcome back, Heinz Bohlen

833 Cents Scale

Based on combination tones but actually works out to the Fibonacci series, too.



Essentially, stack and "reduce" combination tones until convergence.



833 Cents Scale (cont.)

Example: take any interval, find the combination tone produced, add that new note to the system and repeat. This is most easily illustrated using Hz:

$$1.)\ \frac{2}{1} = \frac{440}{220} \to 660 \to \frac{660}{440} = \frac{2}{3} = 701.96 \text{ cents}$$

$$2.)\ \frac{3}{2} = \frac{660}{440} \to 1100 \to \frac{1100}{660} = \frac{5}{3} = 884.36 \text{ cents}$$

$$3.)\ \frac{5}{3} = \frac{1100}{660} \to 1760 \to \frac{1760}{1100} = \frac{8}{5} = 813.69 \text{ cents}$$

Etc...

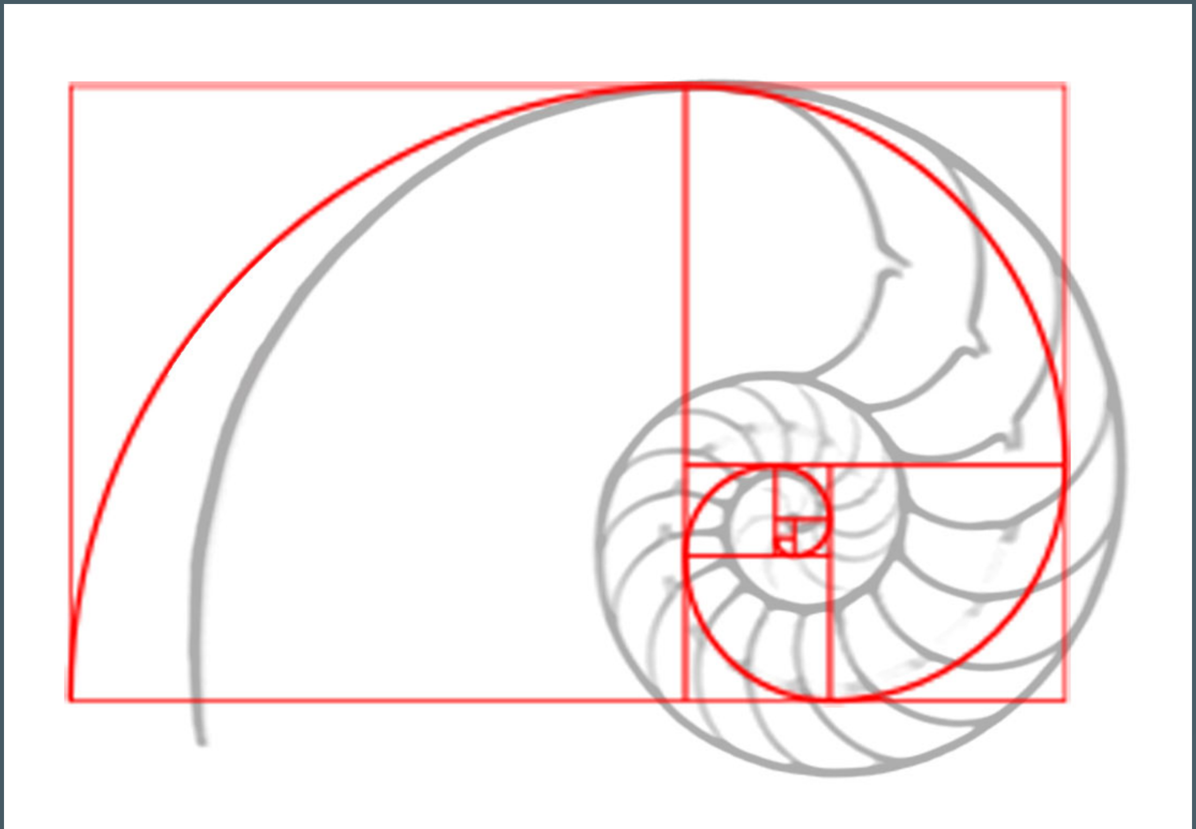
Base Interval	New Interval [Ratio]	New Interval [Cents]
2 : 1	3 : 2	701.96
3 : 2	5 : 3	884.36
5 : 3	8 : 5	813.69
8 : 5	13 : 8	840.53
13 : 8	21 : 13	830.25
21 : 13	34 : 21	834.17
34 : 21	55 : 34	832.68
55 : 34	89 : 55	833.25
89 : 55	144 : 89	833.03
144 : 89	233 :144	833.11

We can actually calculate the exact convergence. Assume $A : B : C$ is the desired ratio where $x = C/B = B/A$, and $C = A + B$. By substituting and simplifying, we get:

$$x = 1/x + 1$$

With solutions:

$$x_1 = 1/2 + (5/4)1/2 = 1.618034$$
$$x_2 = 1/2 - (5/4)1/2 = -0.618034$$



John Chowning: Stria (1977)

A12 Scale

- Also "discovered" by Bohlen; named by Enrique Moreno in 1995.
- Based on the 4th, 7th, and 10th harmonics (as opposed to 4:5:6 of 12edo).
- I could not find any freely available existing music (that was any good) written using the tuning.

Wendy Carlos

- An important figure in electronic music, especially in the 1980's.
- Searched for asymmetric divisions of the octave that create the most "consonance". Found three:
    - Alpha
    - Beta
    - Gamma

Carlos (cont.)

Essentially asked, "Since we can easily create octaves, why build them into the tuning?" In other words, she argued that octaves create "redundancy" and can therefore be sacrificed.

So, "[s]everal years ago I wrote a computer program to perform a precise deep-search investigation into this kind of Asymmetric Division, based on the target ratios of: 3/2, 5/4, 6/5, 7/4, and 11/8."

What did it find?

Carlos (cont.)

Three divisions that are more consonant that all the others:

Name	Step Size (cents)	Steps per Octave
Alpha	78.0	15.385
Beta	63.8	18.809
Gamma	35.1	34.188

An interesting property of these three are that they are all almost exactly equal divisions of the fifth ($\frac{3}{2}$).

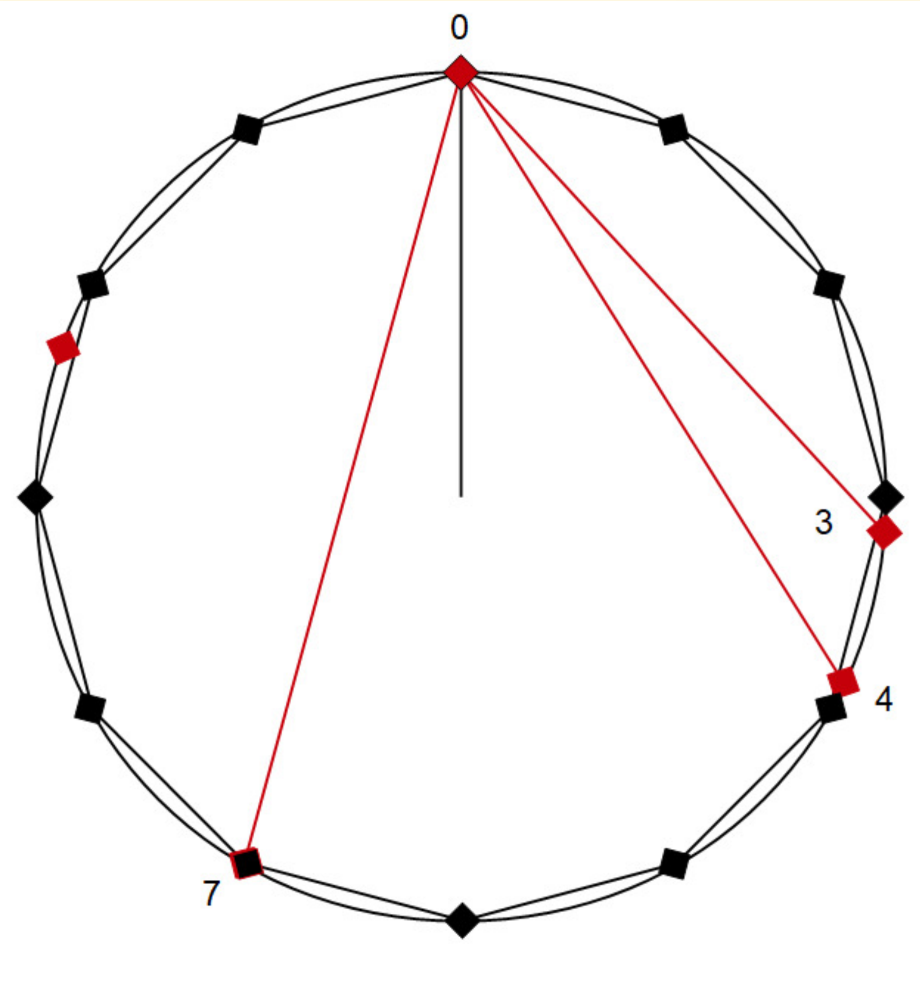
Carlos (cont.)

Each of Carlos' scales can be approximated by minimizing the mean square deviation for given intervals (I_n) and steps desired (s_n):

$$\frac{s_1 \log_2(I_1) + s_2 \log_2(I_2) + s_3 \log_2(I_3)}{(s_1)^2 + (s_2)^2 + (s_3)^2}$$

Example: alpha scale

$$\frac{9 \log_2(3/2) + 5 \log_2(5/4) + 4 \log_2(6/5)}{9^2 + 5^2 + 4^2} \approx 0.064970824 \rightarrow 77.965 \text{ cents}$$



Alpha (α)

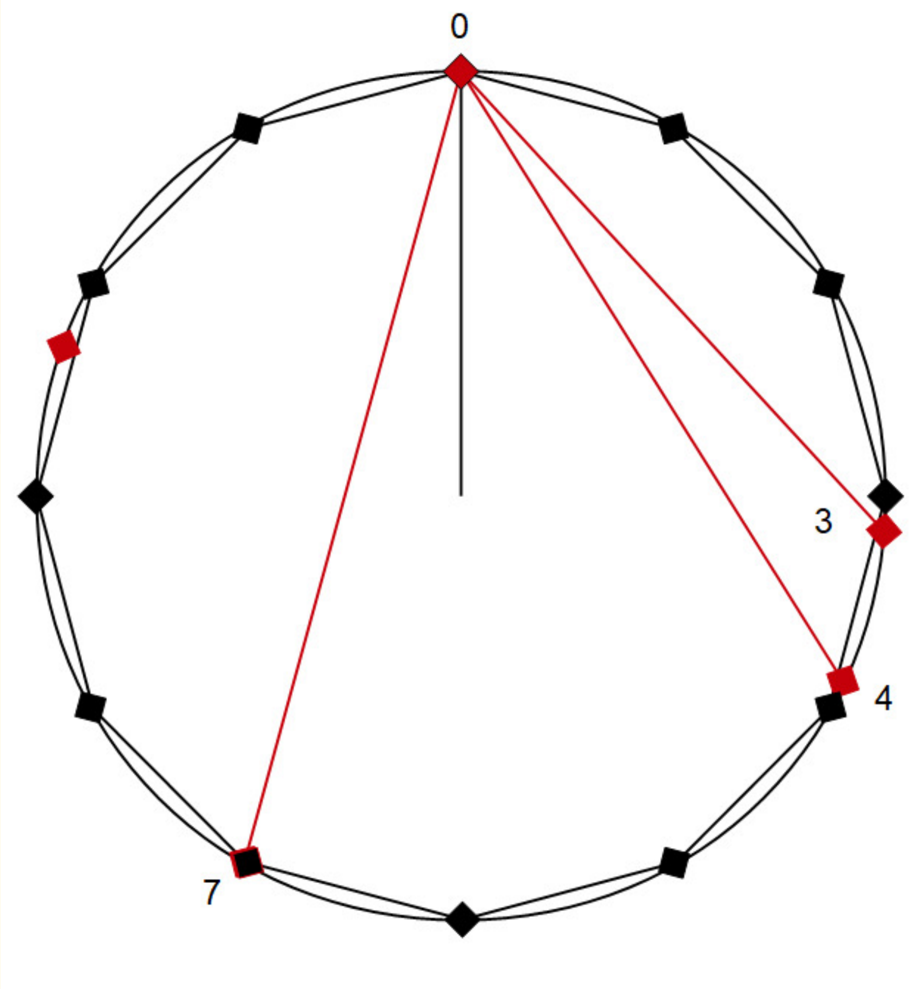
- About 9 equal divisions of the fifth (9edf)
- Equally divides the minor third into quarters.

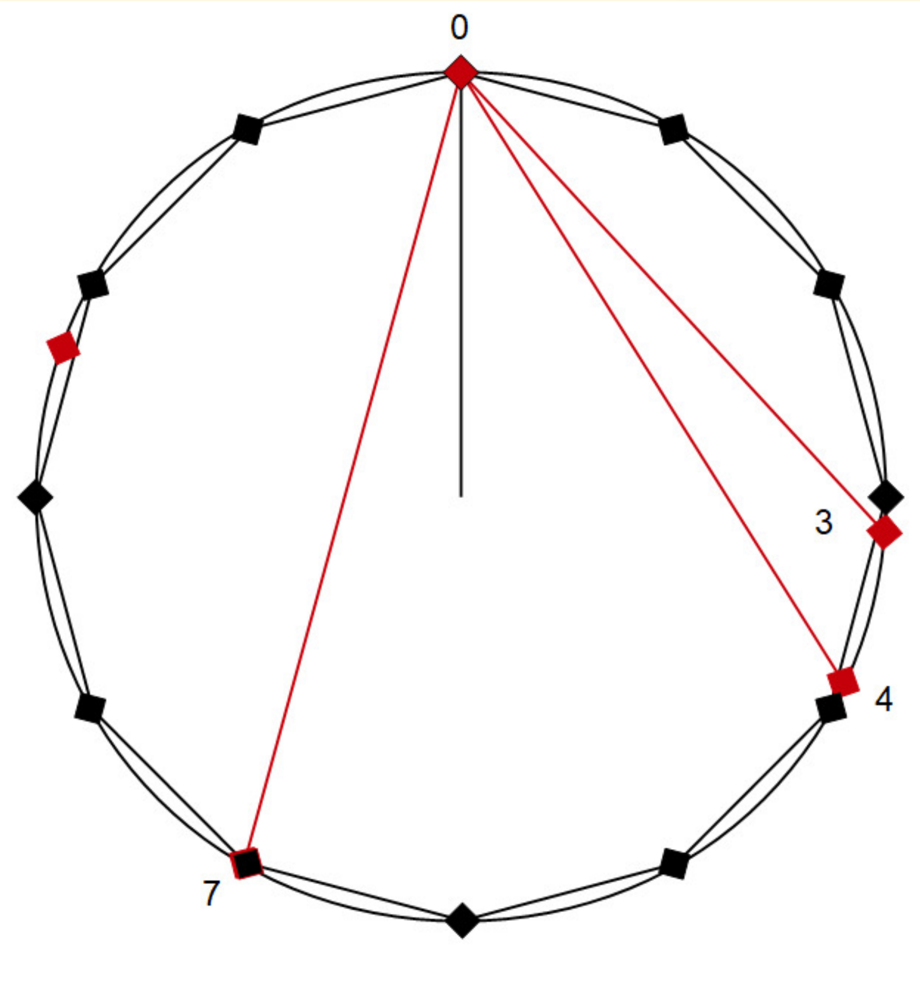
interval name	size (steps)	size (cents)	just ratio	just (cents)	error
septimal major second	3	233.90	8:7	231.17	+2.72
major third	5	389.83	5:4	386.31	+3.51
perfect fifth	9	701.69	3:2	701.96	−0.27
harmonic seventh	12	935.58	7:4	968.83	−33.25
octave	15	1169.48	2:1	1200.00	−30.52
octave	16	1247.44	2:1	1200.00	+47.44

Beta (β)

- About 11 equal divisions of the fifth (11edf)
- Very close to 19edo
- Divides the fourth, $\frac{4}{3}$, into quarters.

interval name	size (steps)	size (cents)	just ratio	just (cents)	error
minor third	5	319.00	6:5	315.64	+3.35
major third	6	382.80	5:4	386.31	−3.52
perfect fifth	11	701.79	3:2	701.96	−0.16
harmonic seventh	15	956.99	7:4	968.83	−11.84





Gamma (γ)

- Hard to tell the difference between gamma and just harmonies.
- About 20 equal divisions of the fifth (20edf)

interval name	size (steps)	size (cents)	just ratio	just (cents)	error
minor third	9	315.89	6:5	315.64	+0.25
major third	11	386.09	5:4	386.31	−0.22
perfect fifth	20	701.98	3:2	701.96	+0.02

Carlos (cont.)

Each of these scales were used in the title track of Carlos' 1986 album, *Beauty in the Beast*.

[Beauty in the Beast]