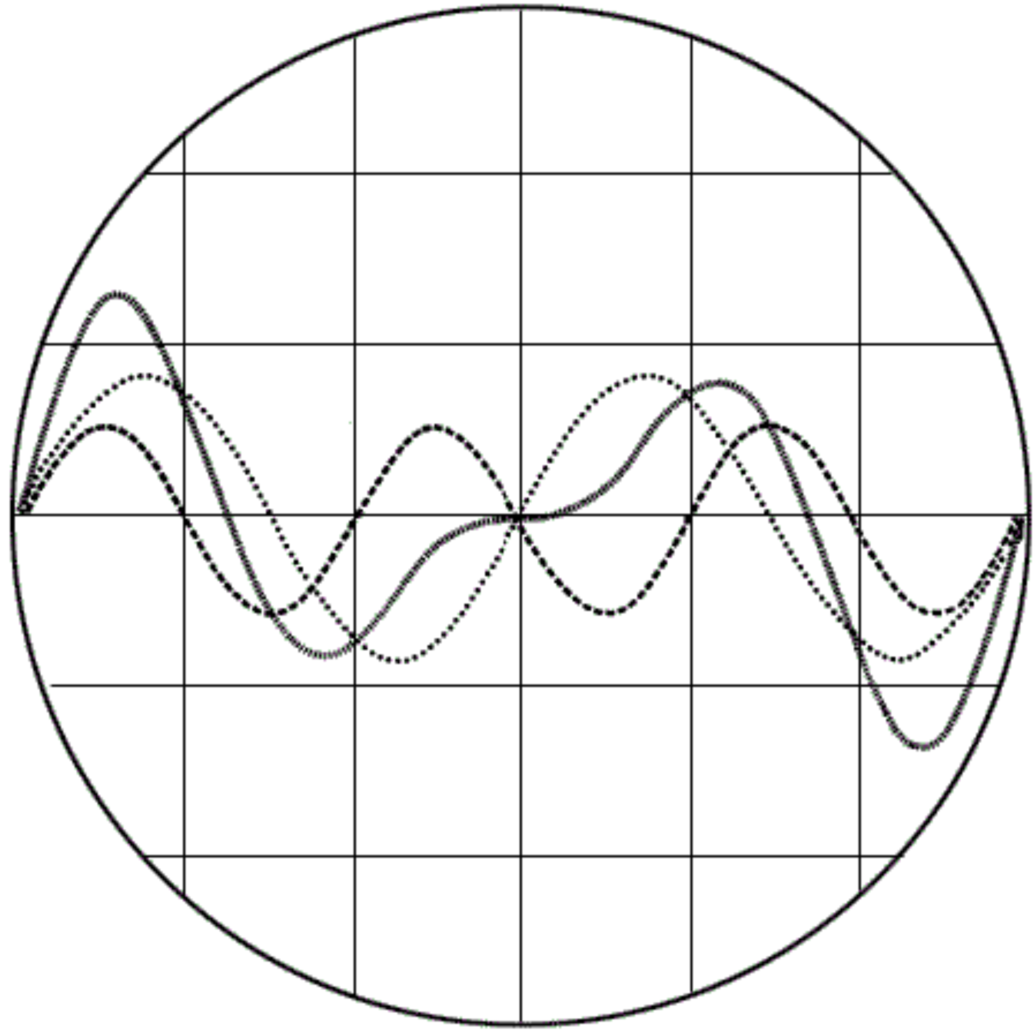
Tuning and Temperament

Music 80

Syllabus

Course website is [here]

- Listening and Reading
- Assignments & Final
  - Listening Responses
- Grading Information



The Way we Hear

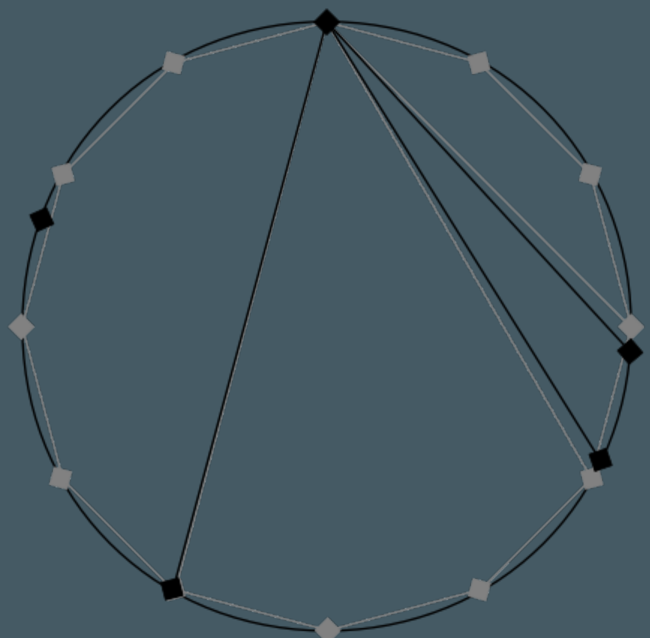
We hear frequency *logarithmically* and not linearly.

In other words, if we have a tone of 200Hz, the next octave is 400Hz, the octave after is 800Hz, etc. This means that a increase of, say, 50Hz, is a different “distance” depending on where we start.

EXAMPLE:
100Hz + 50Hz = 150Hz (i.e. a tritone). 1000Hz + 50Hz = 1050Hz (i.e. something a little less than a halfstep).

What does this mean for tuning?

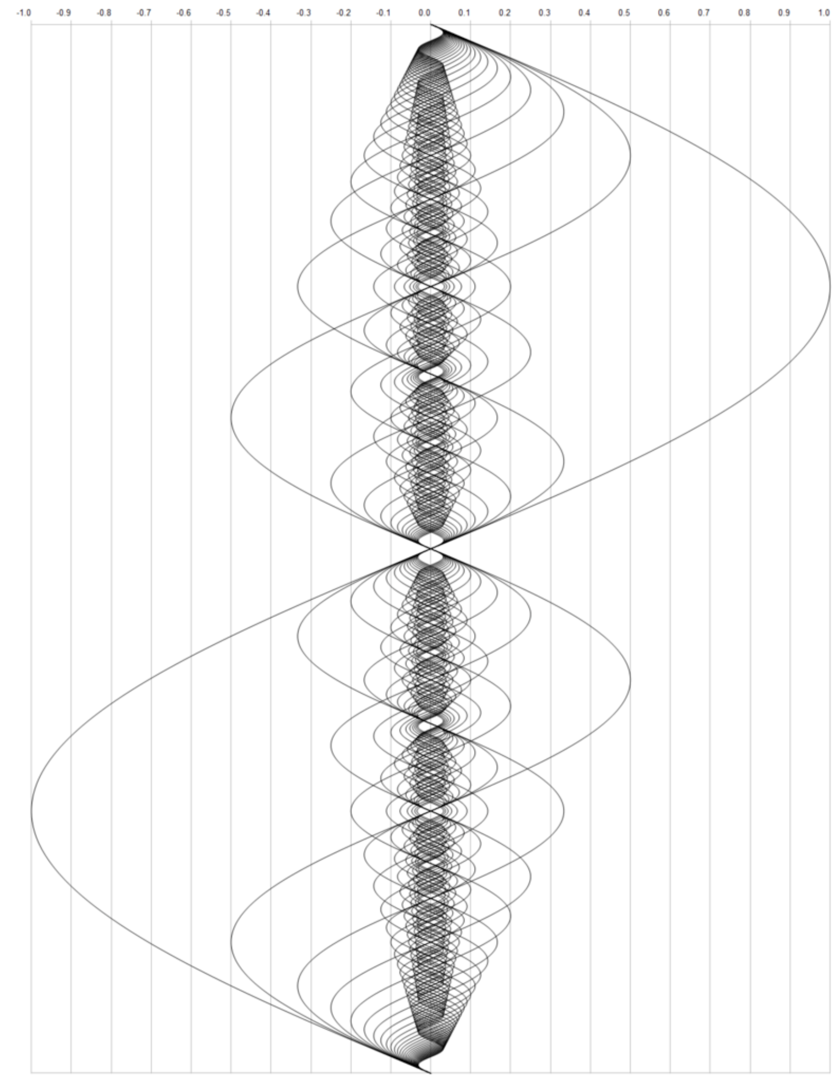
It means that the best way to describe musical intervals, from a mathematical perspective, is with ratios.

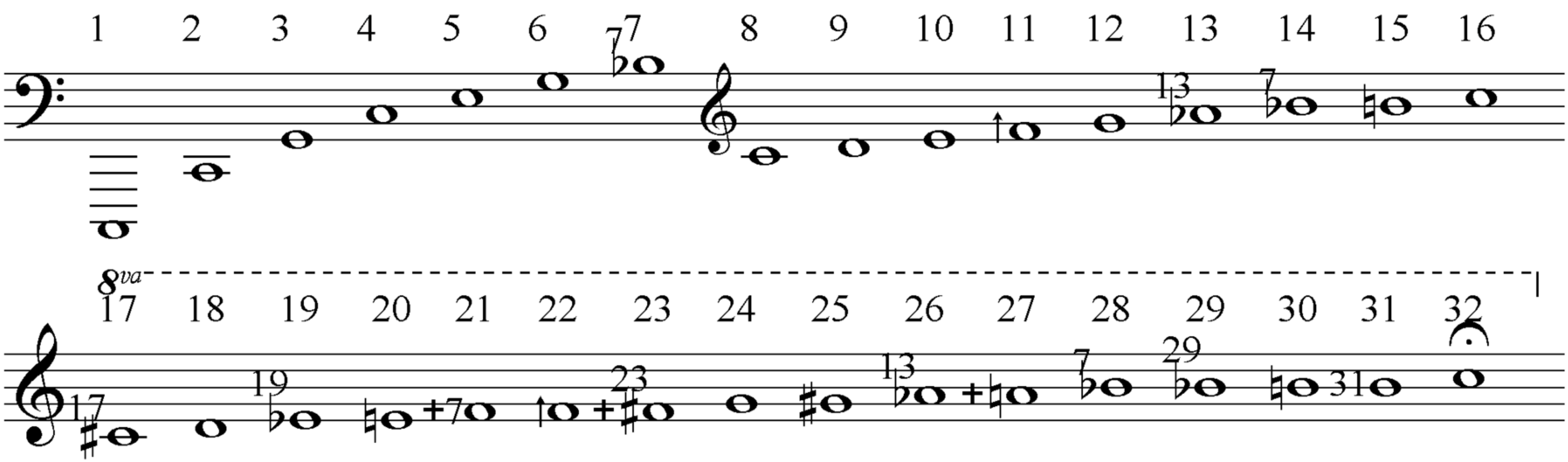


The Harmonic Series

The harmonic series is the most fundamental way of examining musical ratios.

1. 100Hz: fundamental
2. 200Hz: octave
3. 300Hz: fifth
4. 400Hz: next octave
5. 500Hz: major third




1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
8va
17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

Common Intervals Contained within the Harmonic Series

Interval	Harmonics	Common Name
2:1	1st & 2nd	octave
3:2	2nd & 3rd	fifth
4:3	3rd & 4th	fourth
5:4	4th & 5th	major third
6:5	5th & 6th	minor third
7:6	6th & 7th	???
7:4	1st & 7th	minor seventh

Ratios are almost always reduced to the interval between 1 and 2; i.e. 3:2, 5:4, 8:9, etc.

Bringing a Ratio into Spec

The equation for getting the frequency of a harmonic is:

$$\text{Hz for a harmonic} = \text{fundamental} \times \text{harmonic } \#$$

Let's look at an arbitrary interval: that between the second harmonic and the ninth. Assume our fundamental is 100Hz (doesn't actually matter).

$$\begin{aligned} \text{9nd harmonic} &= 100\text{Hz} \times 9 = 900\text{Hz} \\ \text{2nd harmonic} &= 100\text{Hz} \times 2 = 200\text{Hz} \end{aligned}$$

The ratio is 9:2 but note that $\frac{9}{2} > 2$. This means we need to reduce.

To bring a ratio into the interval 2:1, we need to do one or both of the following to the numerator and denominator, perhaps multiple times:

1. Divide by two (bring it *down* an octave)
2. Multiply by two (take it *up* an octave)

NOTE: to make things readable, we only want integers in our ratio.

$$\frac{9}{2} = \frac{9}{4} = \frac{9}{8}$$

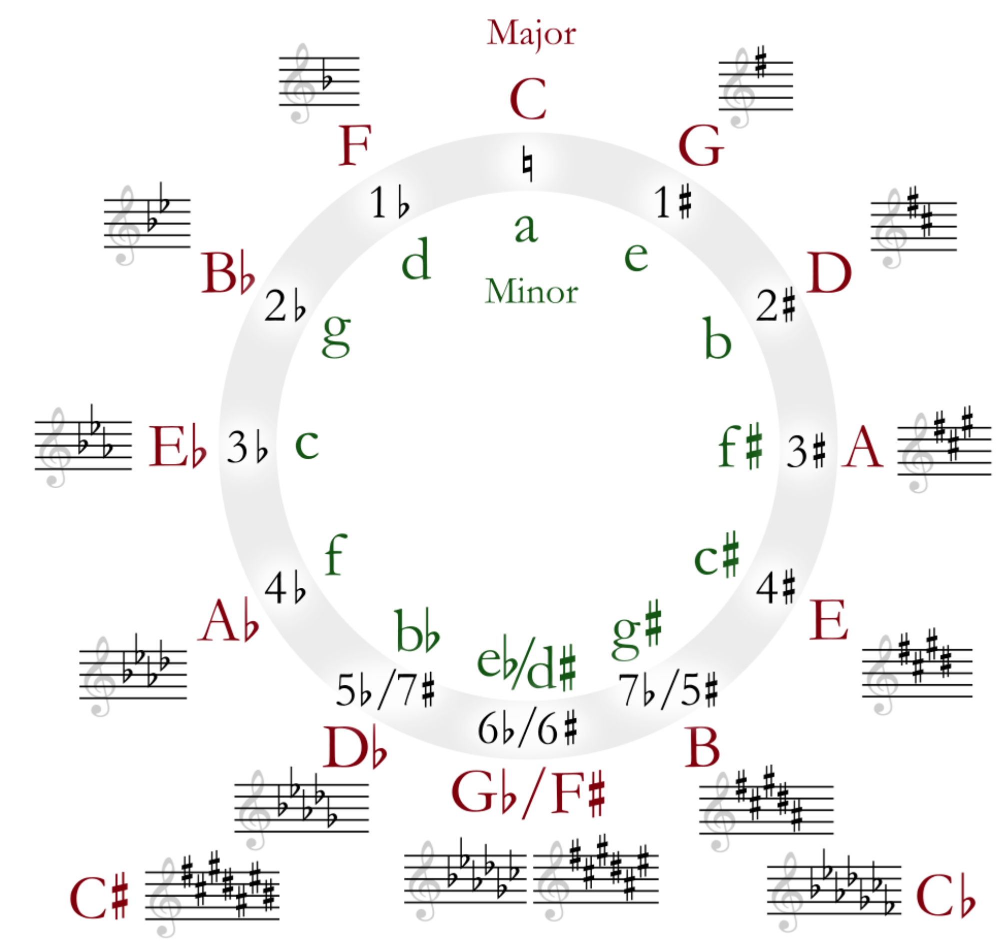
(9:8 happens to be just about a major second)

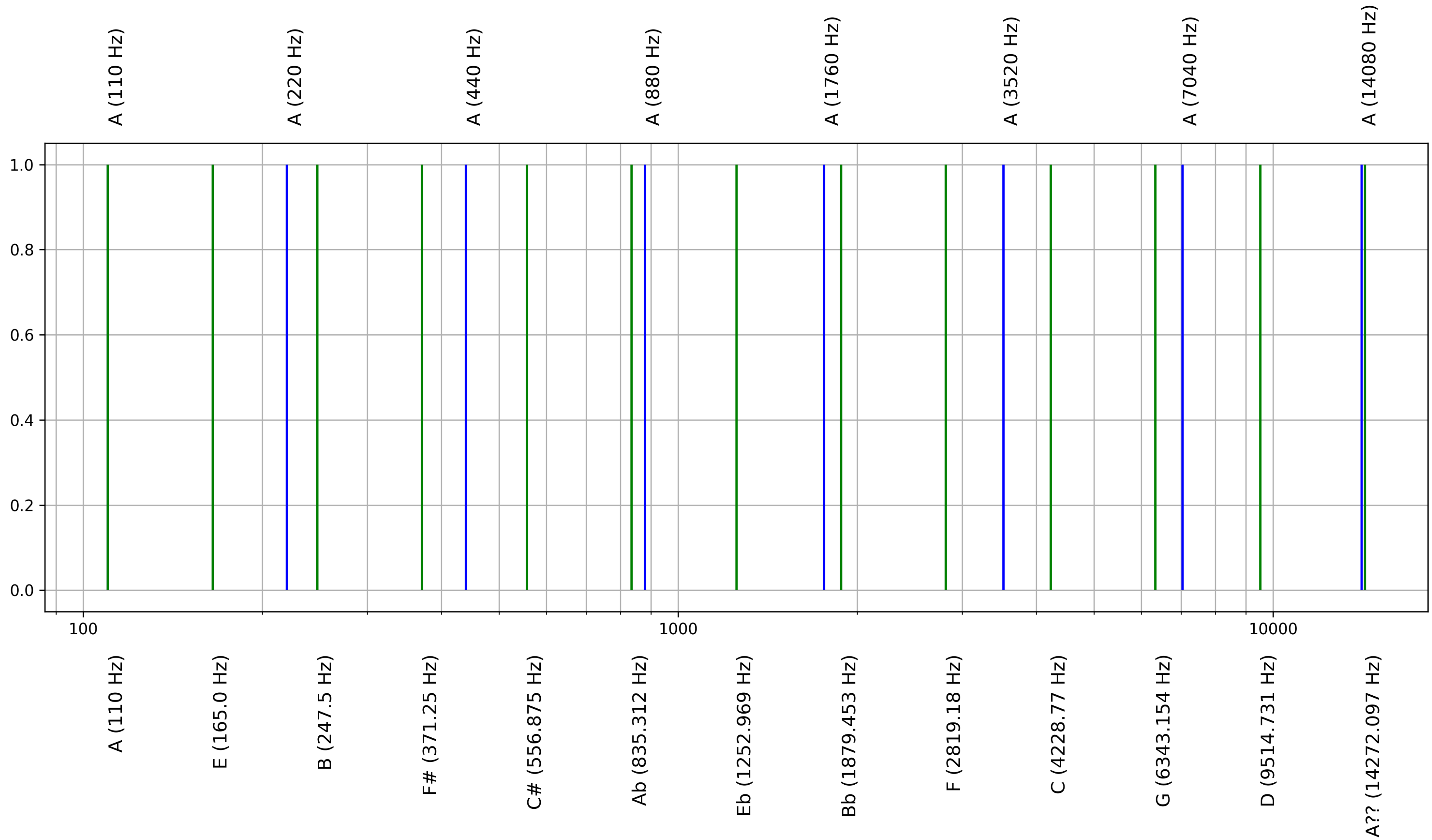
QUESTION: if we know what frequencies to put together for things to be "in tune", what is the issue?

Things don't actually work out... Let's look at the two most simple intervals in the harmonic series, 2:1 (octave) and 3:2 (fifth).

Recall that by the circle of fifths, we should "wrap around" to the same pitch by going up by fifths (3:2).

This should mean that if we take a pitch and go up by octaves on one hand, and on the other go up by fifths, we'll eventually run into each other.




A (110 Hz)
A (220 Hz)
A (440 Hz)
A (880 Hz)
A (1760 Hz)
A (3520 Hz)
A (7040 Hz)
A (14080 Hz)
1.0
0.8
0.6
0.4
0.2
0.0
100
1000
10000
A (110 Hz)
E (165.0 Hz)
B (247.5 Hz)
F# (371.25 Hz)
C# (556.875 Hz)
Ab (835.312 Hz)
Eb (1252.969 Hz)
Bb (1879.453 Hz)
F (2819.18 Hz)
C (4228.77 Hz)
G (6343.154 Hz)
D (9514.731 Hz)
A?? (14272.097 Hz)

Well it doesn't actually work out...

2:1: 110 Hz, 220 Hz, 440 Hz, 880 Hz, 1760 Hz, 3520 Hz, 7040 Hz, 14080 Hz, etc...

3:2: 110 Hz, 165 Hz, 247.5 Hz, 371.25 Hz, 556.88 Hz, 835.31 Hz, 1252.97 Hz, 1879.45 Hz, 2819.18 Hz, 4228.77 Hz, 6343.15 Hz, 9514.73 Hz, 14272.1 Hz, etc...

Notice the last values: 14080 ≠ 14272.1!!! When we reduce this ratio, we get exactly 531441:524288 or approximately 74:73. This is 23.46 cents (about 1/4th of a half step).

The Comma

This phenomenon is called a "comma". The comma produced by creating new pitches with the octave and a fifth is called the **Pythagorean Comma**, named after the same person for whom the trigonometric theorem is named.

This is a concept to which we will return.

Early Tuning Systems

(An Introduction)

Pythagorean Tuning

Why Pythagoras? It comes from a tuning he (probably a pupil) developed that is based on the ratios 2:1 (octave) and 3:2 (fifth). From these two ratios we can create entire diatonic scales.

To create a Pythagorean scale, choose a fundamental. Then, moving by 3:2, go *up* six pitches and *down* six pitches from the same fundamental.

We'll do that next time. :)