Chapter II. GREEK TUNINGS

Greek music theory is highly complex and difficult, with its alphabetical notation, the dependence of musical rhythm upon poetic meter, and all the rest of it. Our confusion is not lessened by the fact that scholars quarrel about the exact interpretation of the modal scales and that a pitifully scant remnant of the music itself is available for study today. Fortunately it is possible to understand the essentials of Greek tuning theories without entering into the other and more controversial aspects of Greek musical science. Moreover, it is advisable that the Greek tuning lore be presented in some detail in order that the attitude of many sixteenth and seventeenth century theorists may be clarified.

The foundation of the Greek scale was the tetrachord, a descending series of four notes in the compass of the modern perfect fourth. Most typical was the Dorian tetrachord, with two tones and then a semitone, as A G F E or E D C B. Two or more tetrachords could be combined by conjunction, as the above tetrachords would be with E a common note. Or they might be combined by disjunction, as the above tetrachords would be in reverse order, with a whole tone between B and A. Tetrachords combined alternately by conjunction and by disjunction correspond to our natural heptatonic scale.

The Greeks had three genera—diatonic, chromatic, and enharmonic. A diatonic tetrachord contained two tones and a semitone, variously arranged, the Dorian tetrachord having the order shown above, as A G F E. In the chromatic tetrachord the second string (as G) was lowered until the two lower intervals in the tetrachord were equal. Thus A G^{b} F E represents the process of formation better than the more commonly shown A $F^{\#}$ F E. In the enharmonic tetrachord the second string was lowered still further until it was in unison with the third string; the third string was then tuned half way between the second and fourth strings. In notes the enharmonic tetrachord would be A G^{bb} $\overset{\flat}{F}$ E or A F $\overset{\flat}{F}$ E. Thus in the chromatic tetrachord there were the consecutive semitones that we associate with the modern chromatic genus; but the enharmonic tetrachord contained real quarter tones, whereas our enharmonically equivalent notes, as F^{b} and E, differ by a comma, 1/9 tone, or at most by a diesis, 1/5 tone.

Claudius Ptolemy has presented the most complete list of tunings advocated by various theorists, including himself.[1] These (with one exception to be discussed later) were shown by the ratios of the three consecutive intervals that constituted the tetrachord, and also by string-lengths for the octave lying between 120 and 60, using sexagesimal fractions where necessary. The octave is the Dorian octave, as from E to E, with B–A the disjunctive tone, always with 9:8 ratio. Ptolemy's tables are given here (Tables 1–21) with comments following. The fractions have been changed into decimal notation.

Greek Enharmonic Tunings

Table 1. Archytas' Enharmonic

Lengths	60.00		75.00		77.14		80.00		90.00		112.50		115.71		120.00
Names	E		C		C♭		B		A		F		F♭		E
Ratios		5/4		36/35		28/27		9/8		5/4		36/35		28/27	
Cents	1200		814		765		702		498		112		63		0

Table 2. Aristoxenus' Enharmonic

Lengths	60.00		76.00		78.00		80.00		90.00		114.00		117.00		120.00
Names	E		C		C♭		B		A		F		F♭		E
Parts		16		2		2		10		24		3		3	
Cents	1200		791		746		702		498		89		44		0

Table 3. Eratosthenes' Enharmonic

Lengths	60.00		75.00		77.50		80.00		90.00		112.50		116.25		120.00
Names	E		C		C♭		B		A		F		F♭		E
Ratios		5/4		24/23		46/45		9/8		5/4		24/23		46/45	
Cents	1200		814		740		702		498		112		38		0

[1] Claudii Ptolemaei _Harmonicorum libri tres_. Latin translation by John Wallis (London, 1699).

Greek Chromatic Tunings

Table 4. Archytas' Chromatic

Lengths	60.00	71.11	77.14	80.00	90.00	106.67	115.71	120.00
Names	E	D^b	C	B	A	G^b	F	E
Ratios	32/27	243/224	28/27	9/8	32/27	243/224	28/27	
Cents	1200	906	765	702	498	204	63	0

Table 5. Aristoxenus' Chromatic Malakon

Lengths	60.00	74.67	77.33	80.00	90.00	112.00	116.00	120.00
Names	E	D^b	C	B	A	G^b	F	E
Parts	$14\frac{2}{3}$	$2\frac{2}{3}$	$2\frac{2}{3}$	10	22	4	4	
Cents	1200	821	761	702	498	119	59	0

Table 6. Aristoxenus' Chromatic Hemiolion

Lengths	60.00	74.00	77.00	80.00	90.00	111.00	115.50	120.00
Names	E	D^b	C	B	A	G^b	F	E
Parts	14	3	3	10	21	$4\frac{1}{2}$	$4\frac{1}{2}$	
Cents	1200	837	768	702	498	135	66	0

Table 7. Aristoxenus' Chromatic Tonikon

Lengths	60.00	72.00	76.00	80.00	90.00	108.00	114.00	120.00
Names	E	D^b	C	B	A	G^b	F	E
Parts	12	4	4	10	18	6	6	
Cents	1200	884	791	702	498	182	89	0

Table 8. Eratosthenes' Chromatic

Lengths	60.00		72.00		76.00		80.00		90.00		108.00		114.00		120.00
Names	E		D^b		C		B		A		G^b		F		E
Ratios		6/5		19/18		20/19		9/8		6/5		19/18		20/19	
Cents	1200		884		791		702		498		182		89		0

Table 9. Didymus' Chromatic

Lengths	60.00		72.00		75.00		80.00		90.00		108.00		112.50		120.00
Names	E		D^b		C		B		A		G^b		F		E
Ratios		6/5		25/24		16/15		9/8		6/5		25/24		16/15	
Cents	1200		884		814		702		498		182		112		0

Table 10. Ptolemy's Chromatic Malakon

Lengths	60.00		72.00		77.14		80.00		90.00		108.00		115.71		120.00
Names	E		D^b		C		B		A		G^b		F		E
Ratios		6/5		15/14		28/27		9/8		6/5		15/14		28/27	
Cents	1200		884		765		702		498		182		63		0

Table 11. Ptolemy's Chromatic Syntonon

Lengths	60.00		70.00		76.36		80.00		90.00		105.00		114.55		120.00
Names	E		D^b		C		B		A		G^b		F		E
Ratios		7/6		12/11		22/21		9/8		7/6		12/11		22/21	
Cents	1200		933		783		702		498		231		81		0

Greek Diatonic Tunings

Table 12. Archytas' Diatonic

Lengths	60.00	67.50	77.14	80.00	90.00	101.25	115.71	120.00
Names	E	D	C	B	A	G	F	E
Ratios	9/8	8/7	28/27	9/8	9/8	8/7	28/27	
Cents	1200	996	765	702	498	294	63	0

Table 13. Aristoxenus' Diatonic Malakon

Lengths	60.00	70.00	76.00	80.00	90.00	105.00	114.00	120.00
Names	E	D	C	B	A	G	F	E
Parts	10	6	4	10	15	9	6	
Cents	1200	933	791	702	498	231	89	0

Table 14. Aristoxenus' Diatonic Syntonon

Lengths	60.00	68.00	76.00	80.00	90.00	102.00	114.00	120.00
Names	E	D	C	B	A	G	F	E
Parts	8	8	4	10	12	12	6	
Cents	1200	983	791	702	498	281	89	0

Table 15. Eratosthenes' Diatonic

Lengths	60.00	67.50	75.94	80.00	90.00	101.25	113.91	120.00
Names	E	D	C	B	A	G	F	E
Ratios	9/8	9/8	256/243	9/8	9/8	9/8	256/243	
Cents	1200	996	792	702	498	294	90	0

Table 16. Didymus' Diatonic

Lengths	60.00	67.50	75.00	80.00	90.00	101.25	112.50	120.00
Names	E	D	C	B	A	G	F	E
Ratios	9/8	10/9	16/15	9/8	9/8	10/9	16/15	
Cents	1200	996	814	702	498	294	112	0

Table 17. Ptolemy's Diatonic Malakon

Lengths	60.00	68.57	76.19	80.00	90.00	102.86	114.27	120.00
Names	E	D	C	B	A	G	F	E
Ratios	8/7	10/9	21/20	9/8	8/7	10/9	21/20	
Cents	1200	969	787	702	498	265	85	0

Table 18. Ptolemy's Diatonic Toniaion

Lengths	60.00	67.30	77.14	80.00	90.00	101.25	115.71	120.00
Names	E	D	C	B	A	G	F	E
Ratios	9/8	8/7	28/27	9/8	9/8	8/7	28/27	
Cents	1200	996	765	702	498	294	63	0

Table 19. Ptolemy's Diatonic Ditoniaion

Lengths	60.00	67.50	75.94	80.00	90.00	101.25	113.91	120.00
Names	E	D	C	B	A	G	F	E
Ratios	9/8	9/8	256/243	9/8	9/8	9/8	256/243	
Cents	1200	996	792	702	498	294	90	0

Table 20. Ptolemy's Diatonic Syntonon

Lengths	60.00	66.67	75.00	80.00	90.00	100.00	112.50	120.00
Names	E	D	C	B	A	G	F	E
Ratios	10/9	9/8	16/15	9/8	10/9	9/8	16/15	
Cents	1200	1018	814	702	498	316	112	0

Table 21. Ptolemy's Diatonic Hemiolon

Lengths	60.00	66.67	73.33	80.00	90.00	100.00	110.00	120.00
Names	E	D	C	B	A	G	F	E
Ratios	10/9	11/10	12/11	9/8	10/9	11/10	12/11	
Cents	1200	1018	853	702	498	316	151	0

Only two of these seventeen or eighteen independent tunings have had any great influence upon modern music theory—the third and fourth of Ptolemy's diatonic scales, commonly called the "ditonic" and the "syntonic." The former is the same as Eratosthenes' diatonic, and is the old Pythagorean tuning. It gains its name from the fact that its major third (ditone) consists of a pair of equal tones. The latter, the "tightly stretched" in contrast to the "soft" (malakon), is what we know as just intonation. Didymus' diatonic contains the same intervals as Ptolemy's syntonic diatonic, but with the minor tone (10:9) below the major tone (9:8) instead of the reverse. Didymus' arrangement is the more logical for constructing a monochord; Ptolemy's in terms of the harmonic series.

The theorists of the sixteenth and seventeenth centuries, eager to bolster their ideas with classical prototypes, pointed out that the just tuning was that of Didymus and Ptolemy. But they ignored the other diatonic tunings of Ptolemy. They liked to point out further that in three of the enharmonic tunings the pure major third (5:4) appears, and in four of the chromatic tunings the pure minor third (6:5). But only Didymus used enharmonic and chromatic tunings that really resembled just intonation. His chromatic is tuned precisely as E, C#, C, etc., would be in just intonation, using the chromatic semitone, 25:24, which appears in no other tuning. In his enharmonic, not only does the major third have the ratio 5:4, but the small intervals are "equal" quarter tones, resulting from an arithmetical division of the 16:15 semitone.* The other nine enharmonic and chromatic tunings depart more or less from Didymus' standard.

*Didymus' enharmonic is not included in the above tables.

Let us examine more of the peculiarities of these Greek tunings. Archytas has used the same ratio (28:27) for the lowest interval in each genus, thus having an interval (63 cents) that is much smaller than most of the semitones and larger than the quarter tones. The ditonic semitone, 256:243, is about the same size as Ptolemy's "soft" semitone, 21:20, being a comma smaller than the syntonic semitone, 16:15. The tones range from minimum, 11:10, through minor, 10:9, and major, 9:8, to maximum, 8:7. Archytas' minor third, 32:27, is a comma larger than the syntonic third, 6:5, and more than a comma smaller than Ptolemy's minor third, 7:6. Eratosthenes' major third, 19:15, is about the same size as the Pythagorean ditone, 81:64, and is about a ditonic comma larger than the syntonic third, 5:4.

Ever since his own age a great controversy has raged about the teachings of Aristoxenus. Instead of using ratios, he divided the tetrachord into 30 parts, of which, in his diatonic syntonon, each tone has 12 parts, each semitone 6. Thus, if we are to take him at his word, Aristoxenus was here describing equal temperament. The sixteenth and seventeenth century theorists were of the opinion that such was his intention, the advocates of equal temperament opposing the name of Aristoxenus to that of Ptolemy.

Ptolemy himself did not so understand Aristoxenus' doctrines. With a fundamental of 120 units, the perfect fourth above has 90 units. Thus, as shown in the tables, Ptolemy subtracted Aristoxenus' "parts" from 120. His enharmonic then agrees with that of Eratosthenes, and his chromatic tonikon with the latter's chromatic. But Aristoxenus' diatonic syntonon does not then quite agree with the Pythagorean (ditonic) diatonic, although the latter is the only Greek tuning that contains two equal tones. His diatonic malakon, as Ptolemy has shown it, is unlike any of the other tunings; whereas in its succession of intervals—large, medium, small — it resembles Ptolemy's diatonic malakon or chromatic syntonon.

So it seems quite likely that Aristoxenus did not intend to express any new tunings by his adding together of parts of a tone, but simply to indicate in a general way the impression that current tunings made upon the ear. But his vagueness has made possible all sorts of wild speculations. It is even possible, by an improper manipulation of the figures, to argue that Aristoxenus was a proponent of just intonation. Take his enharmonic: 24 + 3 + 3. Add these numbers to 90 in reverse order as before, getting 90 93 96 120. Then consider these numbers to be frequencies rather than string-lengths. The result is practically the same as Didymus': 5/4 x 32/31 x 31/30. Or take Aristoxenus' diatonic syntonon: 12 + 12 + 6. Treat it as we have just treated his enharmonic, getting 90 96 108 120. If these are then taken as frequencies, we have Ptolemy's syntonic, 10/9 x 9/8 x 16/15.

The paramount principle in Ptolemy's tunings was the use of superparticular proportion, a ratio in which the antecedent exceeds the consequent by unity. (The Latin prefix "sesqui" is conveniently used to describe these ratios, e.g., "sesquiquarta," meaning 5/4.) Ptolemy used 5/4, 6/5, 7/6, 8/7, etc. Seven of the eight tunings that bear his own name are constructed entirely of superparticular proportions, the eighth being the ditonic, or Pythagorean. Seven tunings that he has ascribed to other writers also use these ratios exclusively, including all of Didymus' tunings, Archytas' enharmonic and diatonic, and Eratosthenes' chromatic (Aristoxenus' chromatic tonikon). In just intonation the ratios are, of course, superparticular, and this feature only would have appealed to Ptolemy and his contemporaries. For, despite the many apparently just intervals used in the given tunings, Ptolemy recognized no consonances other than those of the Pythagorean tuning—fourth, fifth, octave, eleventh, twelfth, and fifteenth.

It is easy to obtain, by algebra, all the possible divisions of the tetrachord built up entirely by superparticular proportions. (A theory for the superparticular division of tones is shown in connection with Colonna, in Chapter VII.) Eliminating those in which one interval is considerably smaller than the smallest enharmonic quarter tone (46:45), we find that, collectively, the Greeks had not omitted many possibilities. Other enharmonic tunings similar to Ptolemy's would be 5/4 x 22/21 x 56/55 and 5/4 x 26/25 x 40/39. Chromatic tunings would include 6/5 x 13/12 x 40/39; 7/6 x 9/8 x 64/63; 7/6 x 10/9 x 36/35; and 7/6 x 15/14 x 16/15. Two others are difficult to classify: 8/7 x 13/12 x 14/13 might best be considered a chromatic tuning, something like 14 + 8 + 8 in Aristoxenus' parts. And 8/7 x 8/7 x 49/48 is undoubtedly a variant of the ditonic tuning, but with a quarter tone instead of a semitone at the bottom, perhaps 14 + 14 + 2.

In later chapters we shall see many echoes of Greek tuning methods, not only in such well-known systems as the Pythagorean and the just, but also in the modified systems, such as Ganassi's, and in irregular systems, such as Dowland's. Unusual superparticular intervals are used by Colonna in the poorest tuning system shown in this book, and also by Awraamoff, whose system is even worse.