

Tuning and Temperament

Class 5: Meantone Temperament

Today's Class

- Where just intonation falls short
- Temperament
 - What is temperament?
- 1/4 comma meantone temperament
 - Building a 12-note scale
 - How it is different than a just scale?
- **Listening:** Mozart's Fantasie KV397 in Three Different Temperaments
- **Listening:** Yale's Divinity School Meantone Organ

Where does just intonation fall short?

Let's look at a passage of music and assume that our tuning system is just intonation (5-limit). Let's also try to keep it *totally* in tune such that every interval between every note is just (i.e. no commas or wolf intervals).

A musical score consisting of four staves, each with a treble clef and a key signature of one flat (B-flat). The staves are grouped by a large left brace. The music is written in a style that emphasizes intervals and ratios, with notes and their corresponding ratios labeled below the staves. The ratios are as follows:

Staff	Measure 1	Measure 2	Measure 3	Measure 4	Measure 5	Measure 6
Staff 1	6/5	4/3 (3/2)	5/4	5/4	4/3 (3/2)	6/5
Staff 2	4/3 (3/2)	6/5	4/3 (3/2)	6/5	5/4	4/3 (3/2)
Staff 3	3/2	5/4	3/2	1/1	6/5	3/2
Staff 4						

The notes are positioned on the staves as follows: Staff 1 has notes on the 4th and 5th lines; Staff 2 has notes on the 3rd and 4th lines; Staff 3 has notes on the 2nd and 3rd lines; Staff 4 has notes on the 1st line. The ratios are placed below the notes, indicating the interval between the notes in each measure.

A musical score for a four-part setting, likely a canon or a similar contrapuntal piece. The score is written for four staves, each with a different clef: Treble (top), Treble (second), Treble (third), and Bass (bottom). The music is organized into six measures, with each measure containing a pair of notes (one on the staff, one below it) connected by a bracket. Above each pair of notes is a ratio, and below each pair is an interval. The ratios are: 6/5, 6/5, 9/8, 81/80, 27/20, and 243/200. The intervals are: 1/1, 9/5, 81/50, 81/80, 81/80, and 81/80. The notes are: C4, D4, E4, F4, G4, A4, B4, C5, D5, E5, F5, G5, A5, B5, C6, D6, E6, F6, G6, A6, B6, C7, D7, E7, F7, G7, A7, B7, C8, D8, E8, F8, G8, A8, B8, C9, D9, E9, F9, G9, A9, B9, C10, D10, E10, F10, G10, A10, B10, C11, D11, E11, F11, G11, A11, B11, C12, D12, E12, F12, G12, A12, B12, C13, D13, E13, F13, G13, A13, B13, C14, D14, E14, F14, G14, A14, B14, C15, D15, E15, F15, G15, A15, B15, C16, D16, E16, F16, G16, A16, B16, C17, D17, E17, F17, G17, A17, B17, C18, D18, E18, F18, G18, A18, B18, C19, D19, E19, F19, G19, A19, B19, C20, D20, E20, F20, G20, A20, B20, C21, D21, E21, F21, G21, A21, B21, C22, D22, E22, F22, G22, A22, B22, C23, D23, E23, F23, G23, A23, B23, C24, D24, E24, F24, G24, A24, B24, C25, D25, E25, F25, G25, A25, B25, C26, D26, E26, F26, G26, A26, B26, C27, D27, E27, F27, G27, A27, B27, C28, D28, E28, F28, G28, A28, B28, C29, D29, E29, F29, G29, A29, B29, C30, D30, E30, F30, G30, A30, B30, C31, D31, E31, F31, G31, A31, B31, C32, D32, E32, F32, G32, A32, B32, C33, D33, 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Temperaments

What is a temperament anyway? What is a tuning?

Tuning:

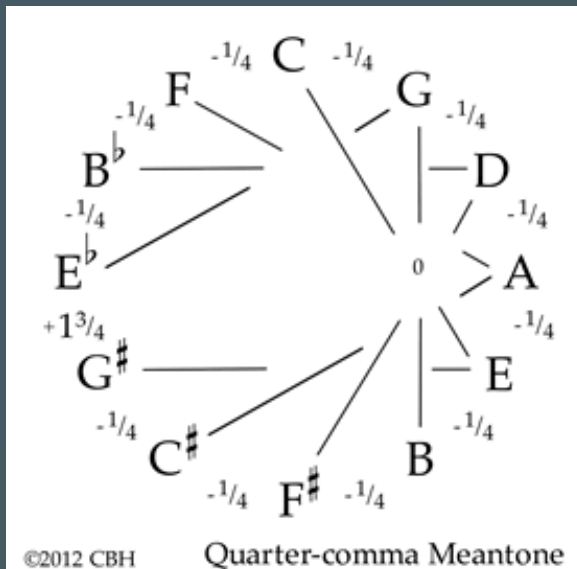
1. A system whose intervals all can be expressed by rational numbers.

Temperament:

1. A system whose intervals cannot all be expressed by rational numbers.
2. A system which "tempers" just intervals.
3. Partch: "... a system which deliberately robs its intervals of their purity in order to implement the idea of every-tone-in-several senses."

1/4 Comma Meantone Temperament

- Pure thirds ($\frac{5}{4}$, $\frac{6}{5}$)
- Flat fifths, by $1/4$ of a syntonic comma ($\frac{81}{80}$)
- Wolf fifth between G# and Eb (if tuned on C)



Building a 12-note $1/4$ comma meantone scale

We want pure major thirds, $\frac{5}{4}$, so we want to slightly flatten the fifth, $\frac{3}{2}$. If starting on C, the major third is E. We can get to E either by 1) stacking four $\frac{3}{2}$'s or two octaves and a $\frac{5}{4}$.

In a Pythagorean style of tuning using fifths, the E would be $\frac{81}{64}$ whereas a pure third is $\frac{5}{4}$ (the difference is $\frac{81}{80}$). So, we need to lower each $\frac{3}{2}$ by one quarter of the difference, $\frac{81}{80}$, such that getting to an E by four $\frac{3}{2}$'s or two octaves and a $\frac{5}{4}$ are the same.

We can rewrite $\frac{5}{4}$ as $\frac{5}{1}$ by moving it up two octaves. Therefore, letting r be the ratio of our meantone fifth:

$$r^4 = \frac{5}{1} = 5$$

$$r = \sqrt[4]{5}$$

So in "musical" terms:

$$r \approx 1.49535 \approx \frac{643}{430} \approx 696.587 \text{ cents}$$

What does this mean for the rest of the scale?

We construct it exactly as a Pythagorean scale, substituting $\sqrt[4]{5}$ for $\frac{3}{2}$. For example, a whole step is:

$$\sqrt[4]{5} \times \sqrt[4]{5} \times \frac{1}{2} = \frac{\sqrt[4]{5} \times \sqrt[4]{5}}{2} = \frac{5^{1/4} \times 5^{1/4}}{2} = \frac{5^{1/2}}{2} = \frac{\sqrt{5}}{2}$$

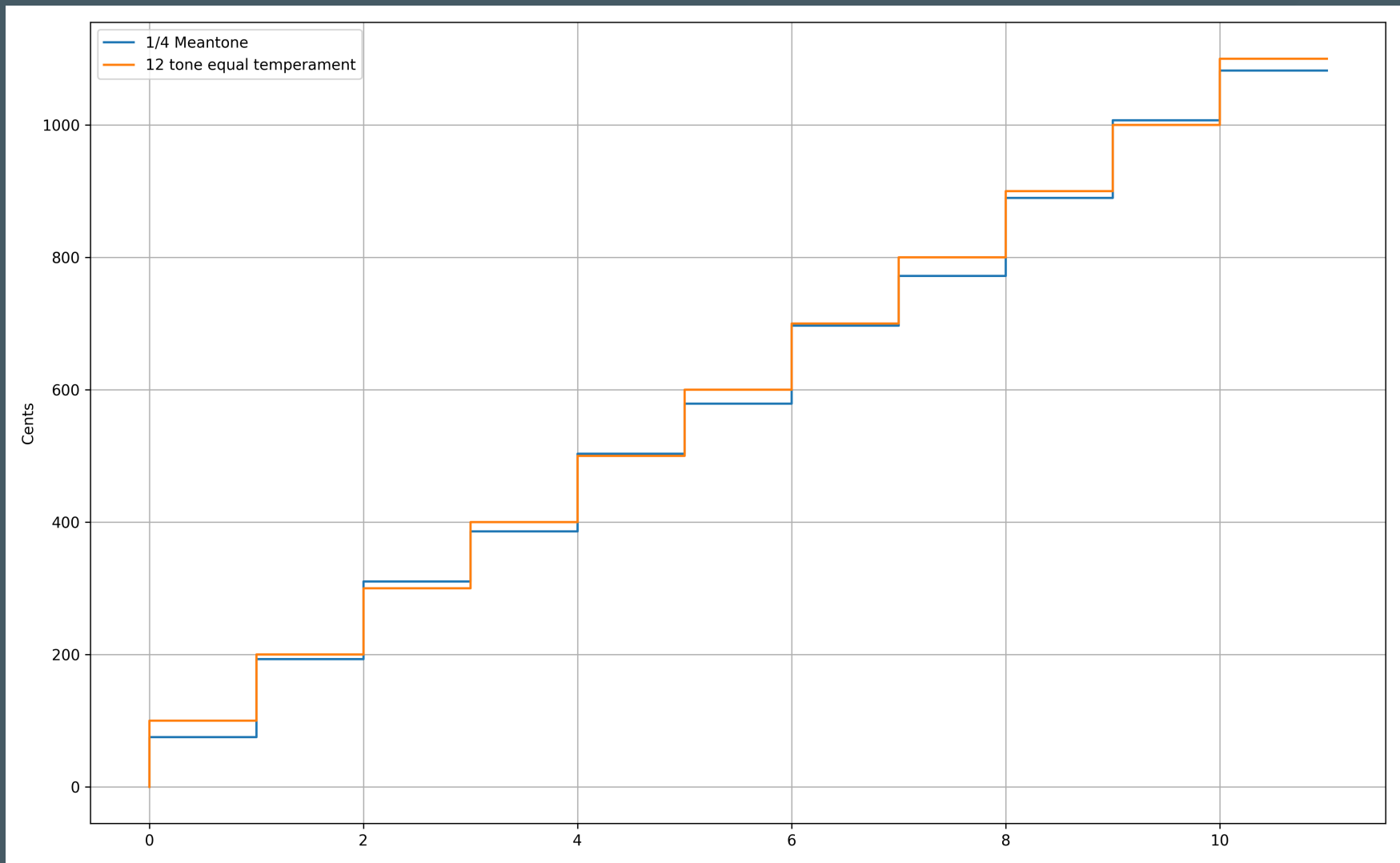
Confirm by taking two whole steps to make a major third:

$$\frac{\sqrt{5}}{2} \times \frac{\sqrt{5}}{2} = \frac{5}{4}$$

Formula for pitches of a major scale where r is the fifth:

Note	Formula	Cents	Note	Formula	Cents
C	$r^0 \times 2^0 = 1$	0	G	$r^1 \times 2^0 = r$	696.6
D	$r^2 \times 2^{-1} = \frac{\sqrt{5}}{2}$	193.2	A	$r^3 \times 2^{-1} = \frac{r\sqrt{5}}{2}$	889.7
E	$r^4 \times 2^{-2} = \frac{5}{4}$	386.3	B	$r^5 \times 2^{-2} = \frac{5r}{4}$	1082.9
F	$r^{-1} \times 2^2 = \frac{2r\sqrt{5}}{5}$	503.4	C	$r^0 \times 2^1 = 2$	1200

Wolf fifth between G# and Eb so it's symmetrical (common).



Listening

[Mozart's Fantasie KV397 in Three Different Temperaments](#)

Listen to about 2 min in Equal Temperament (0:00), 1/4 comma meantone (11:40), and just for kicks, Prelleur temperament (5:40).

[Yale's Divinity School Meantone Organ](#)

Split sharps?

Other flavors of meantone:

- $1/5$ comma meantone (Pythagorean comma)
- $1/6$ comma meantone
- Extended meantone (building extra notes so all/most keys can have pure thirds)
 - [Ascanio Mayone - Examples for the Cimbalo Cromatico](#)
 - [Nicola Vicentino: "Musica prisca caput"](#)

