Microeconomics II

Pricing with Market Power

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Introduction

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Firms with monopoly power must worry about the characteristics of demand

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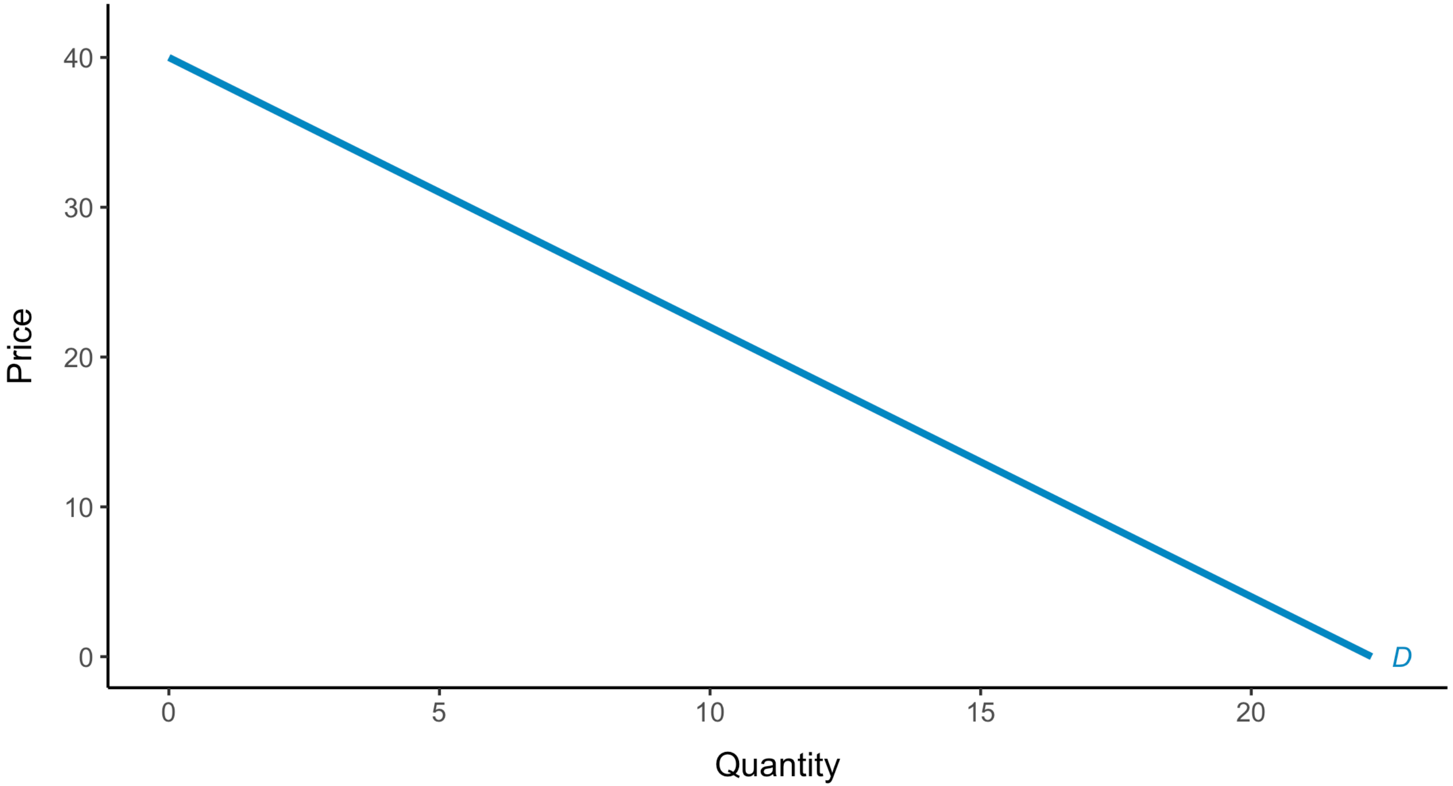
Capturing Consumer Surplus

Capturing Consumer Surplus

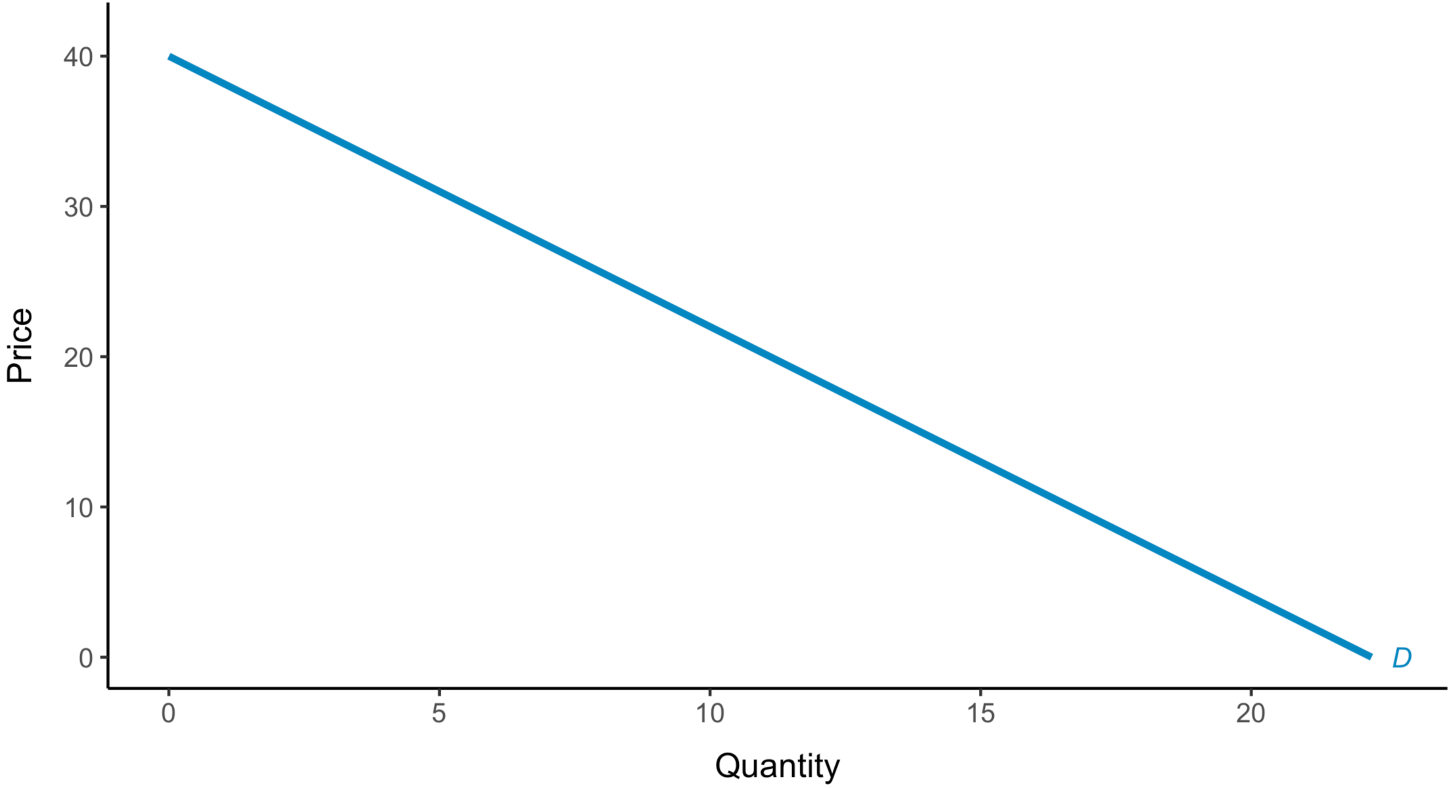
A firm faces the following (hypothetical) demand curve: $P(Q) = 40 - 1.8Q$.



What is the MR? $MR = 40 - 3.6Q$

Capturing Consumer Surplus

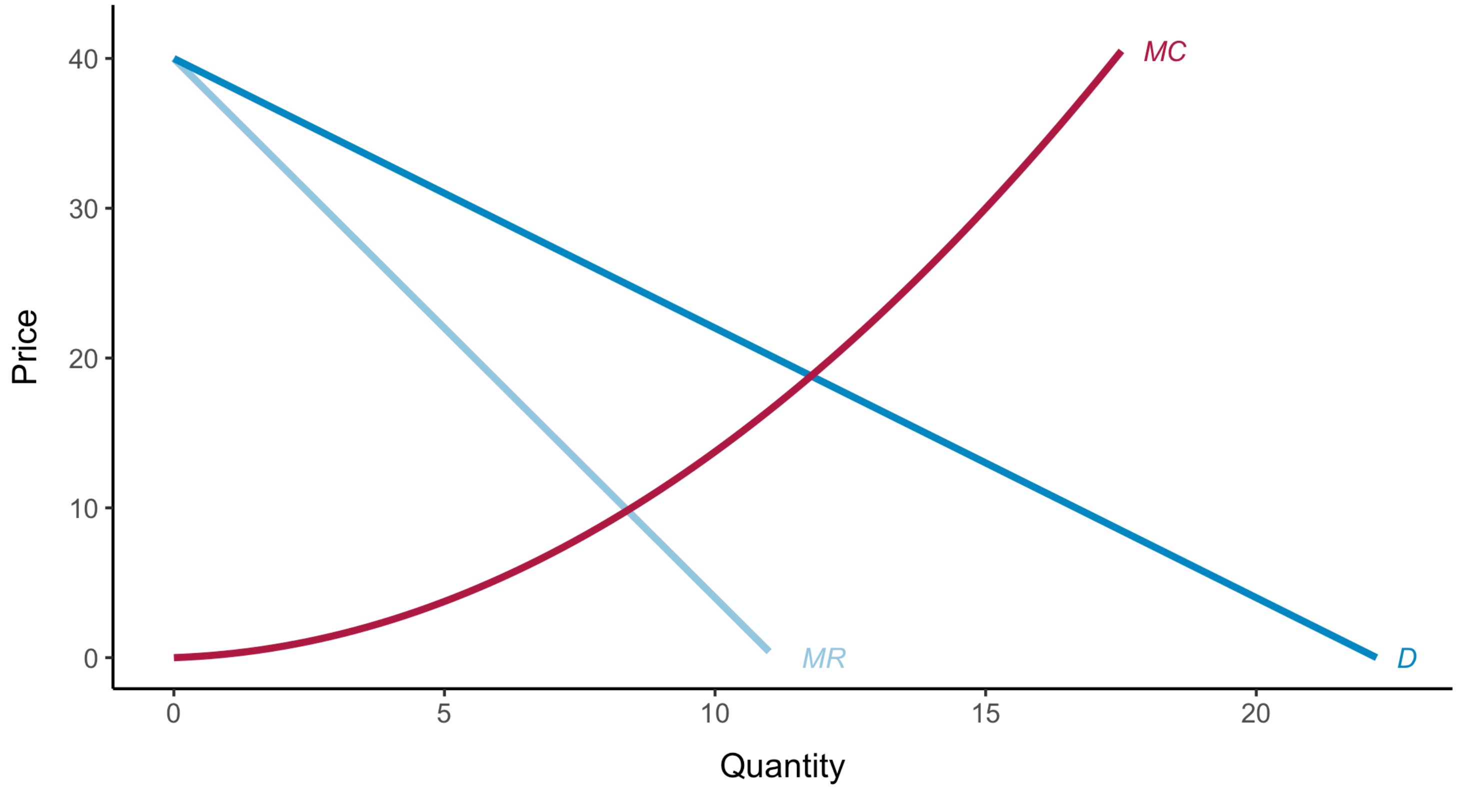
Suppose $C(Q) = \frac{1}{16}Q^2 + \frac{1}{24}Q^3$.



What is the MC? $MC = \frac{1}{8}Q + \frac{1}{8}Q^2$.

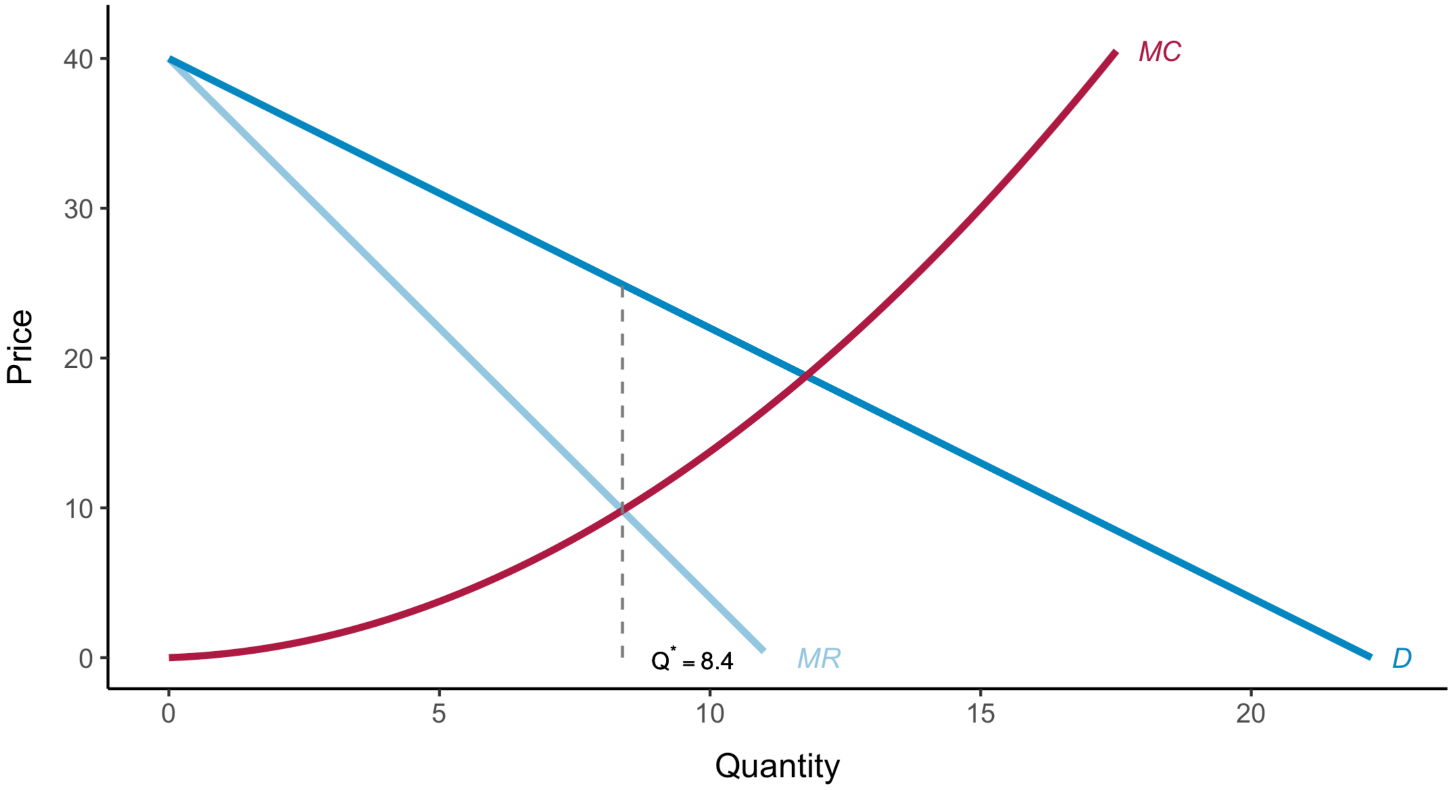
Capturing Consumer Surplus

With market power, the firm would choose Q^* such that $MC = MR$.



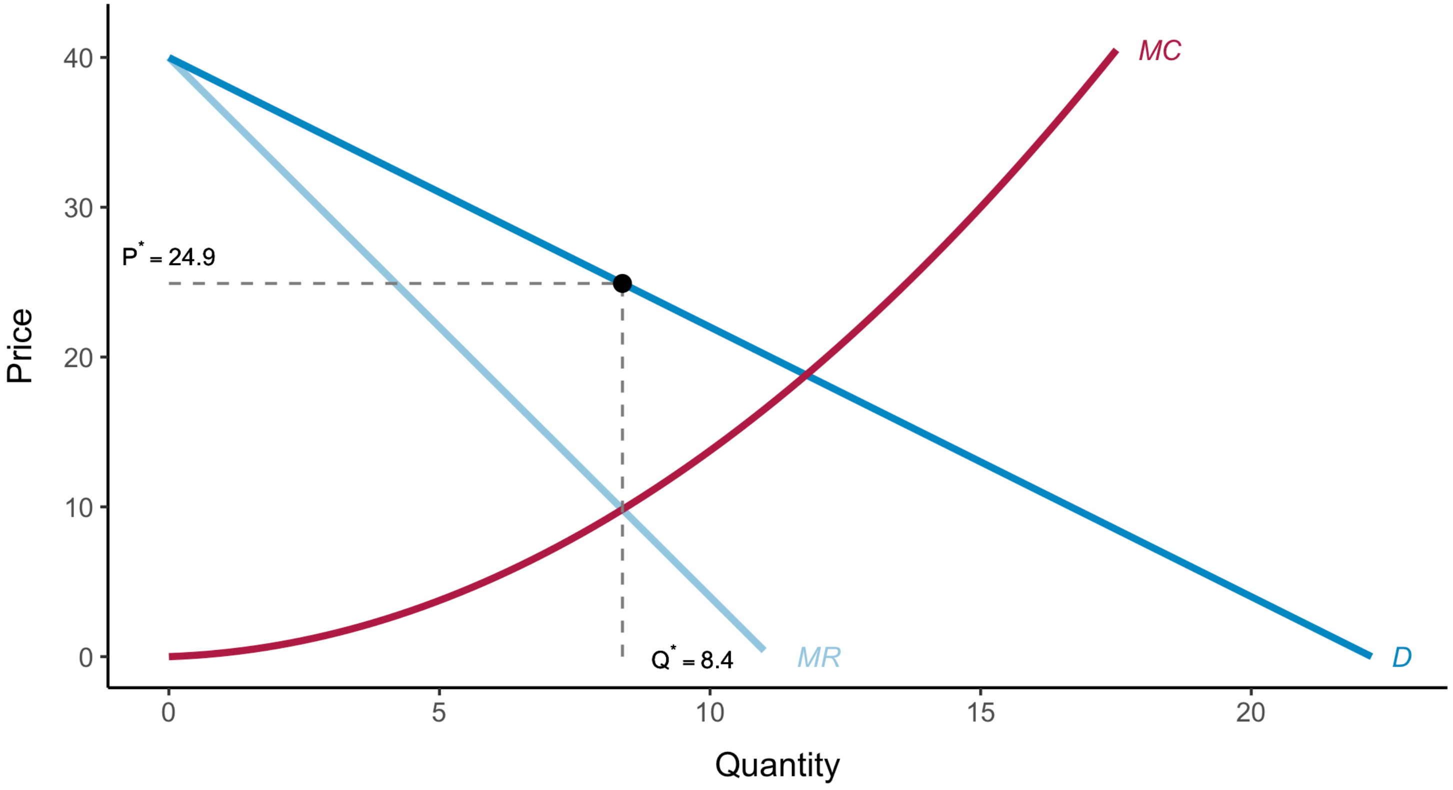
Capturing Consumer Surplus

With market power, the firm would choose Q^* such that $MC = MR$. Thus, $Q^* = 8.4$.



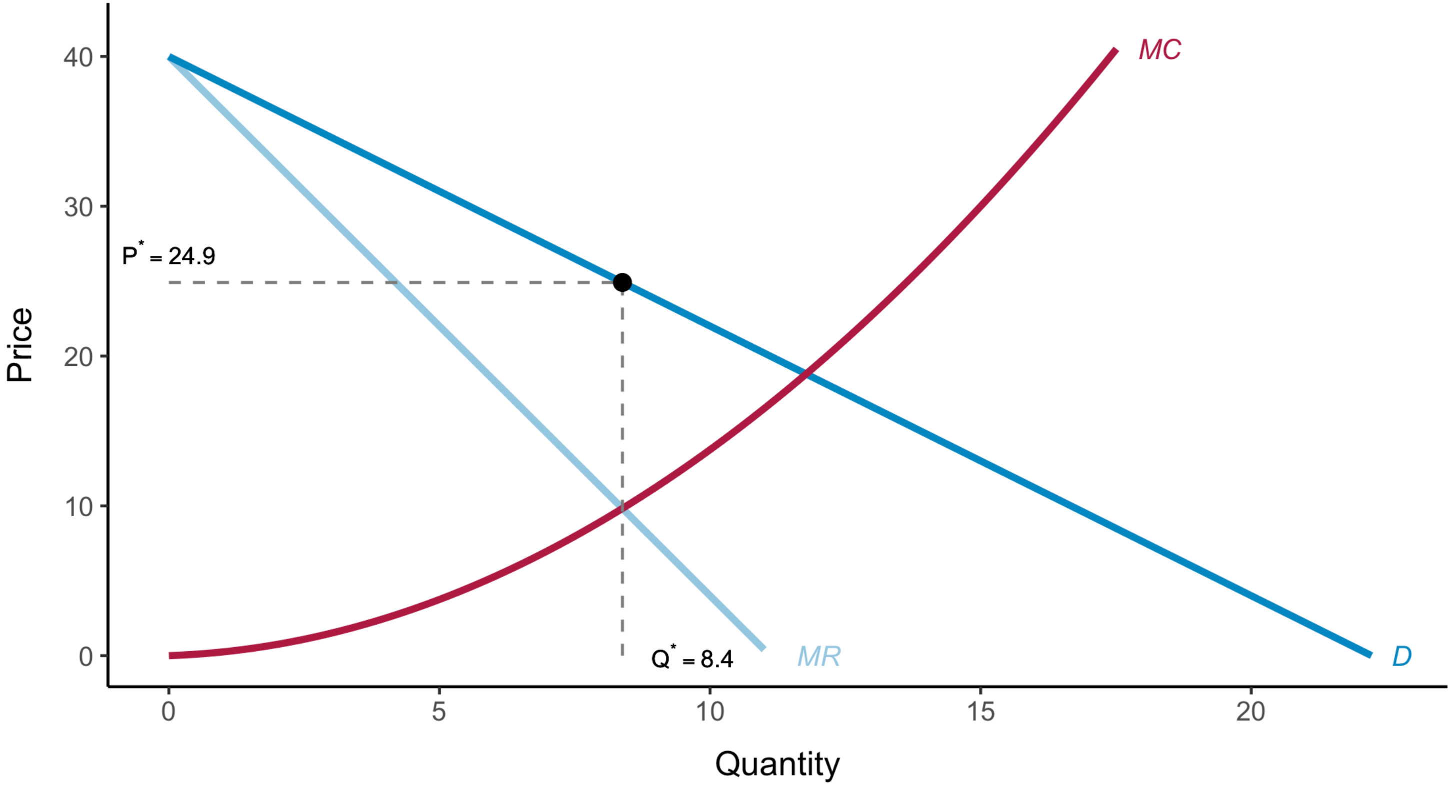
Capturing Consumer Surplus

With market power, the firm would choose Q^* such that $MC = MR$. Thus, $Q^* = 8.4$. At $P(Q^*) \approx 25$.



Capturing Consumer Surplus

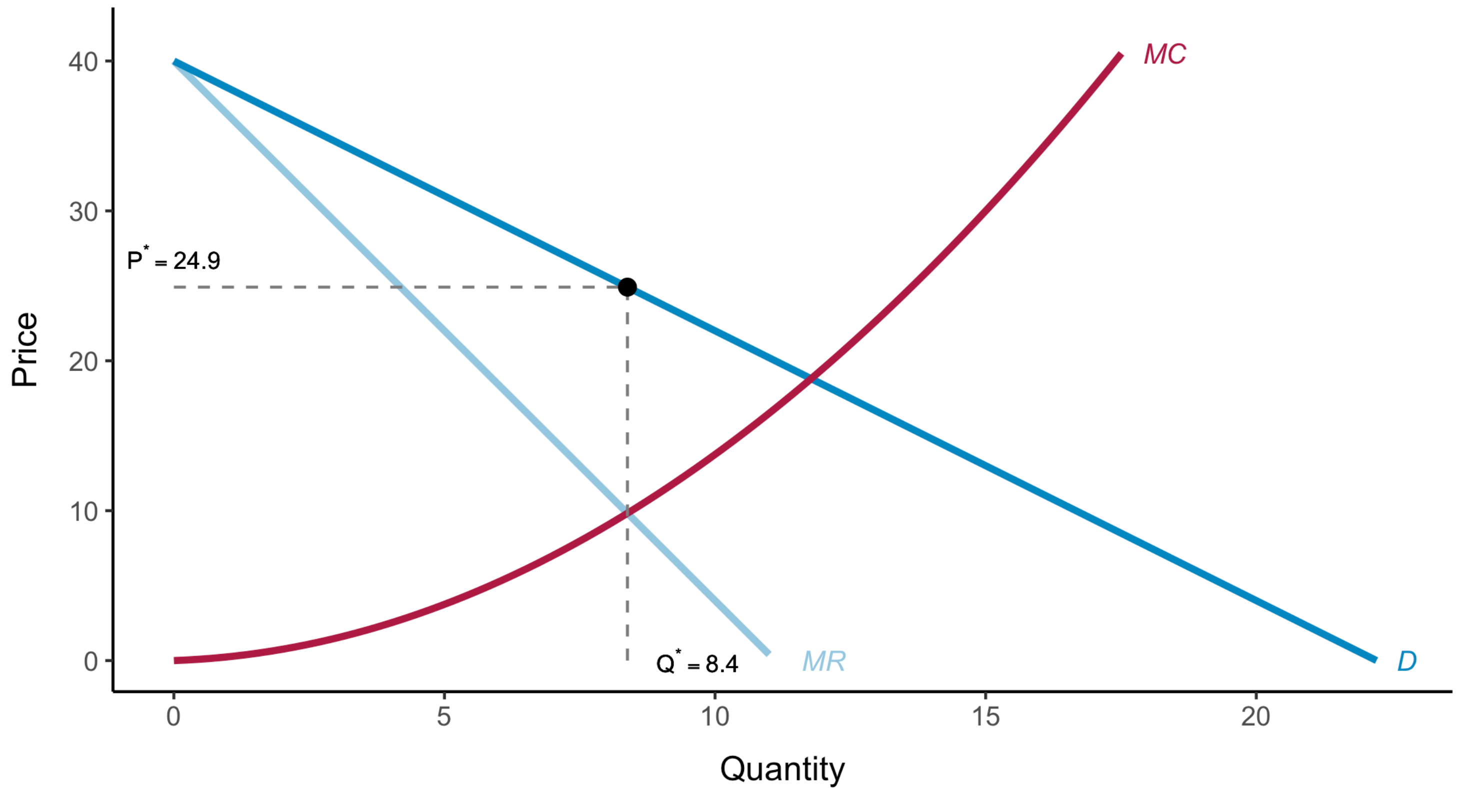
Some consumers are willing to pay more than P^*.



Raising prices would reduce sales. This would lead to fewer customers and lower profits.

Capturing Consumer Surplus

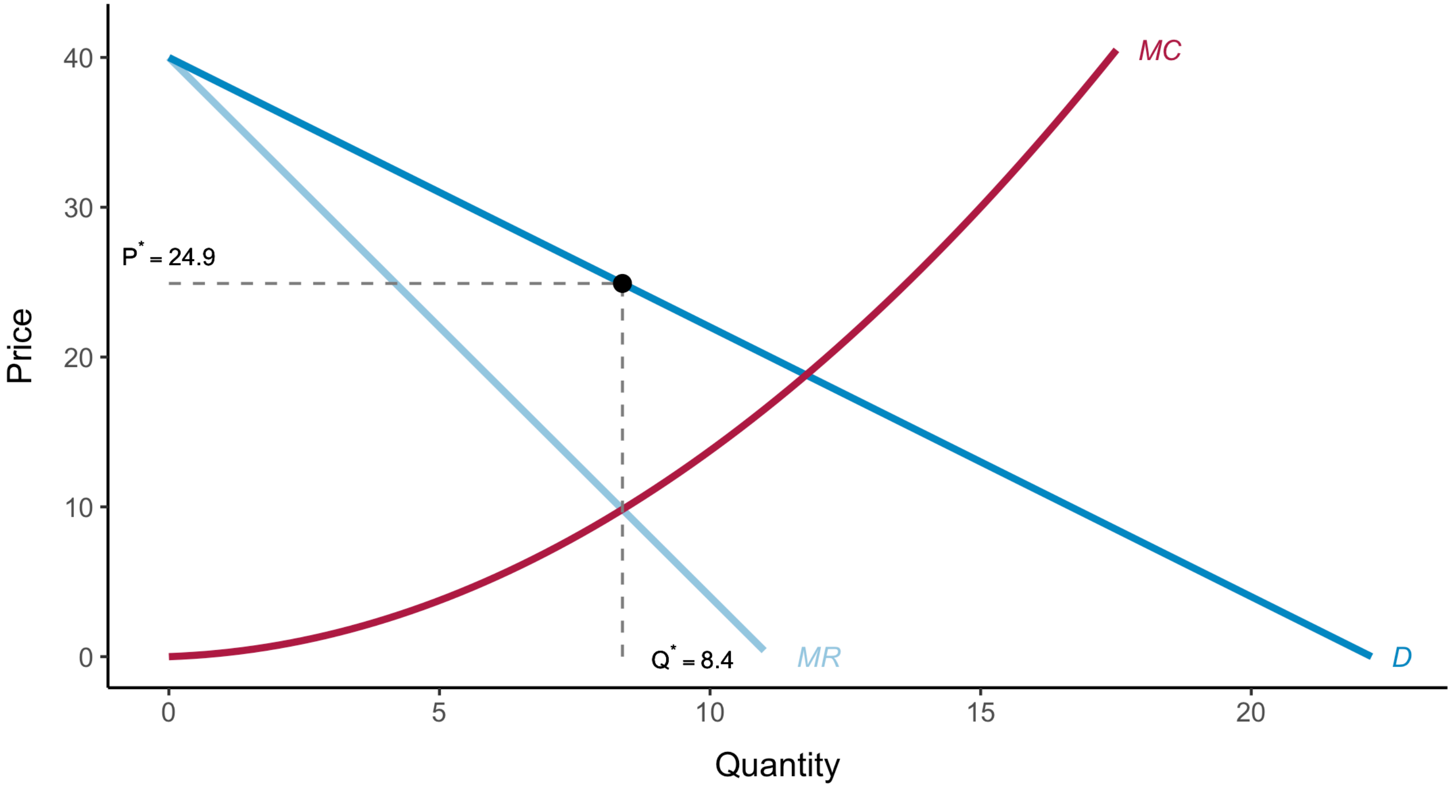
Some potential customers won't buy at P^*, but many would pay more than the firm's marginal cost.



This suggests missed sales opportunities at lower prices.

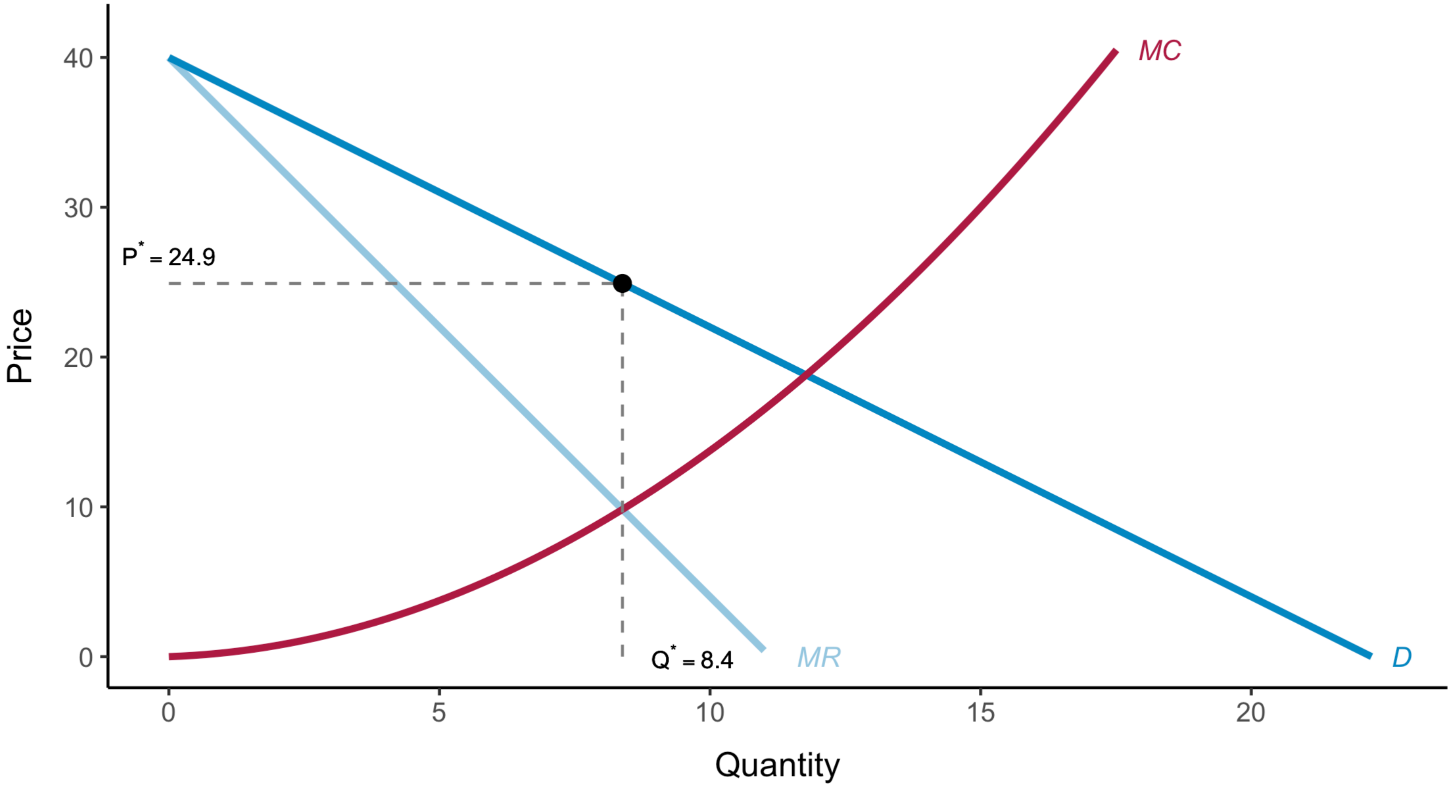
Capturing Consumer Surplus

The firm might charge different prices to different customers, according to where the customers are along the demand curve.



Capturing Consumer Surplus

Pricing strategies have one thing in common: means of capturing consumer surplus and transferring it to the producer.



Price Discrimination

Price Discrimination

Practice of charging different prices to different consumers for similar goods.

Price discrimination can take three broad forms:

First-degree

Second-degree

Third-degree

First-degree Price Discrimination

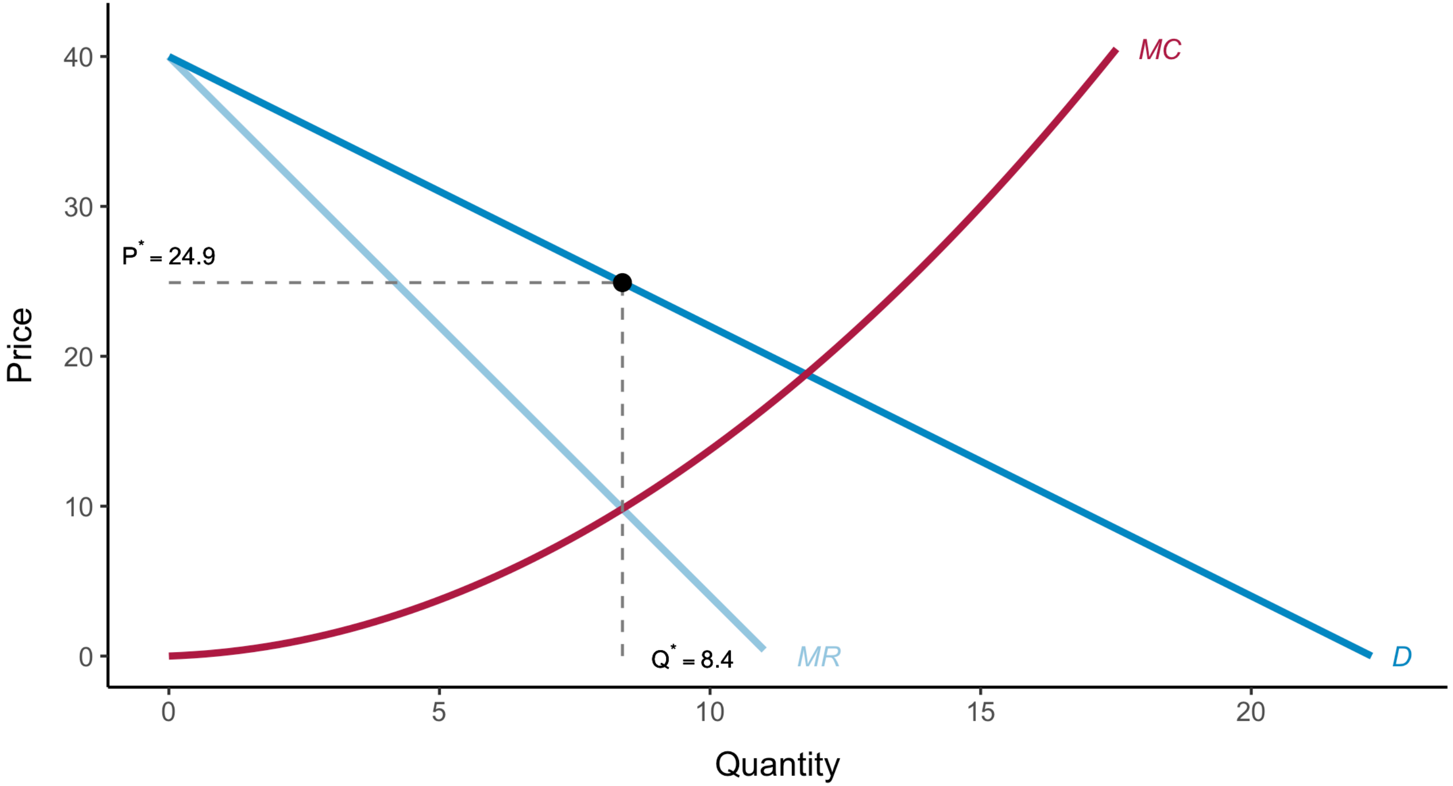
A firm ideally wants to charge each customer their maximum willingness to pay, known as their **reservation price**.

This strategy, called perfect first-degree price discrimination, maximizes revenue by tailoring prices to individual customers.

Is it profitable? We need to know the profit that the firm earns when it charges only the single price P^*.

First-degree Price Discrimination

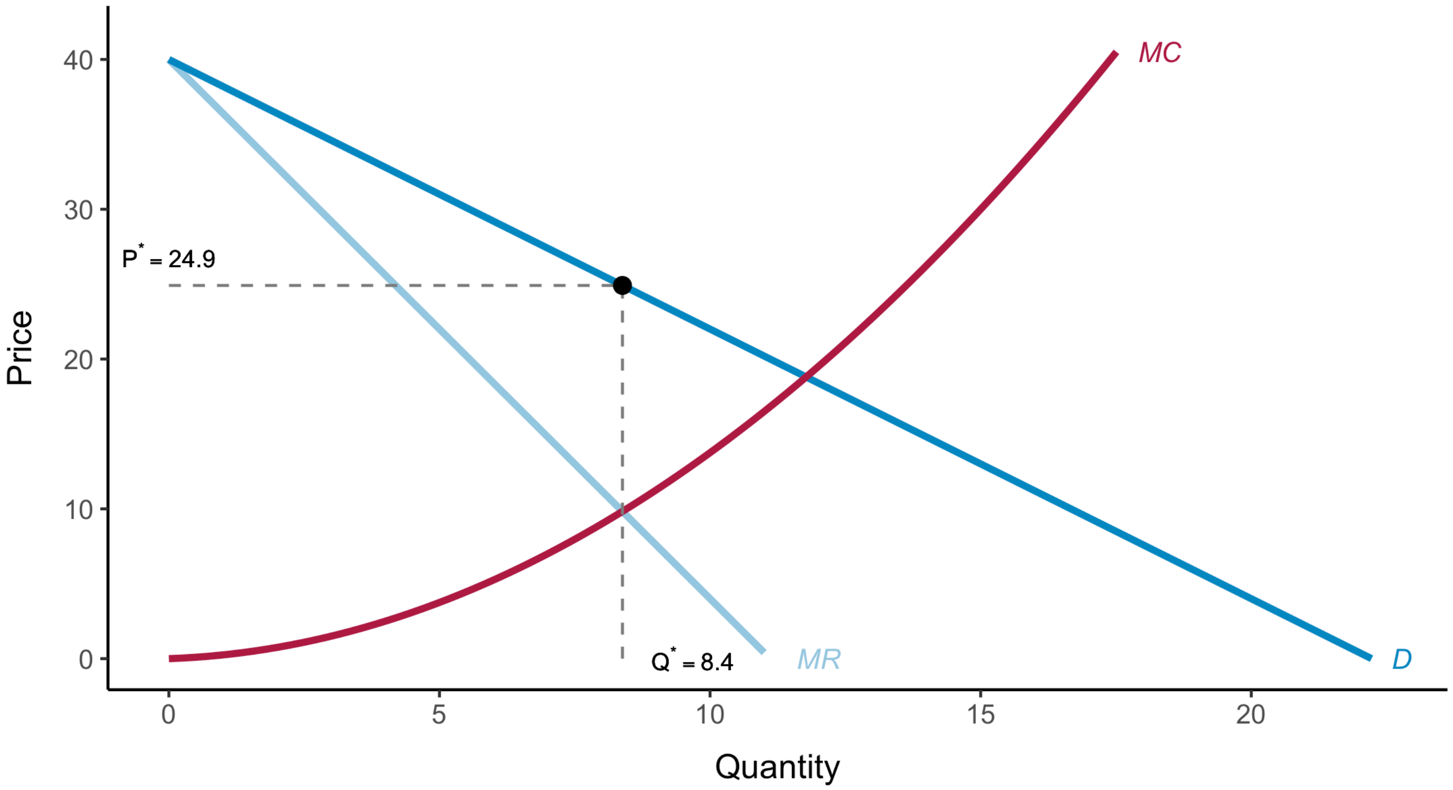
We need to know the profit that the firm earns when it charges only the single price P^*.



First-degree Price Discrimination

Pricing P^*.

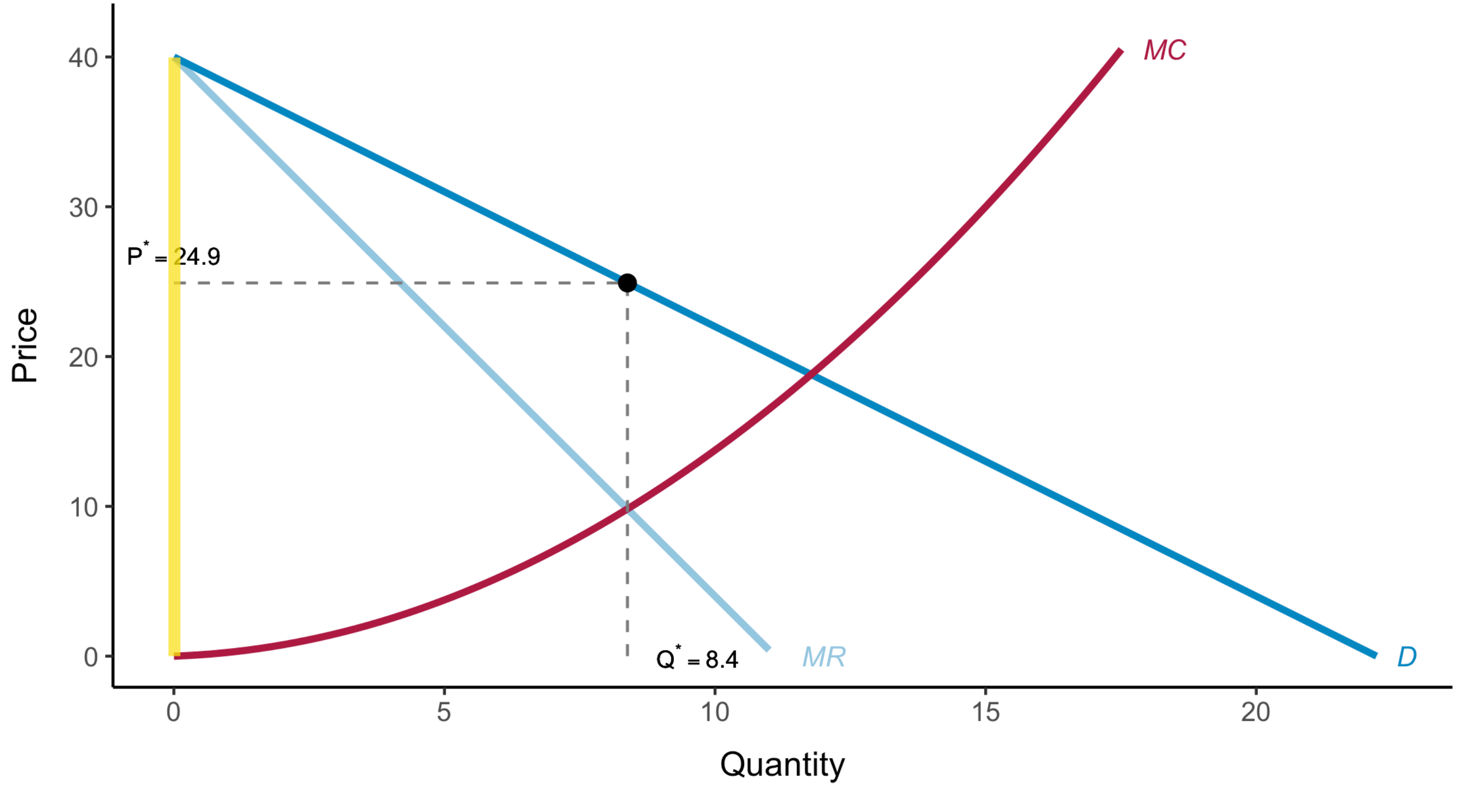
Profit: $\Pi = R - C \;\Rightarrow\; \Delta\Pi = \Delta R - \Delta C = MR - MC.$



First-degree Price Discrimination

Pricing P^*.

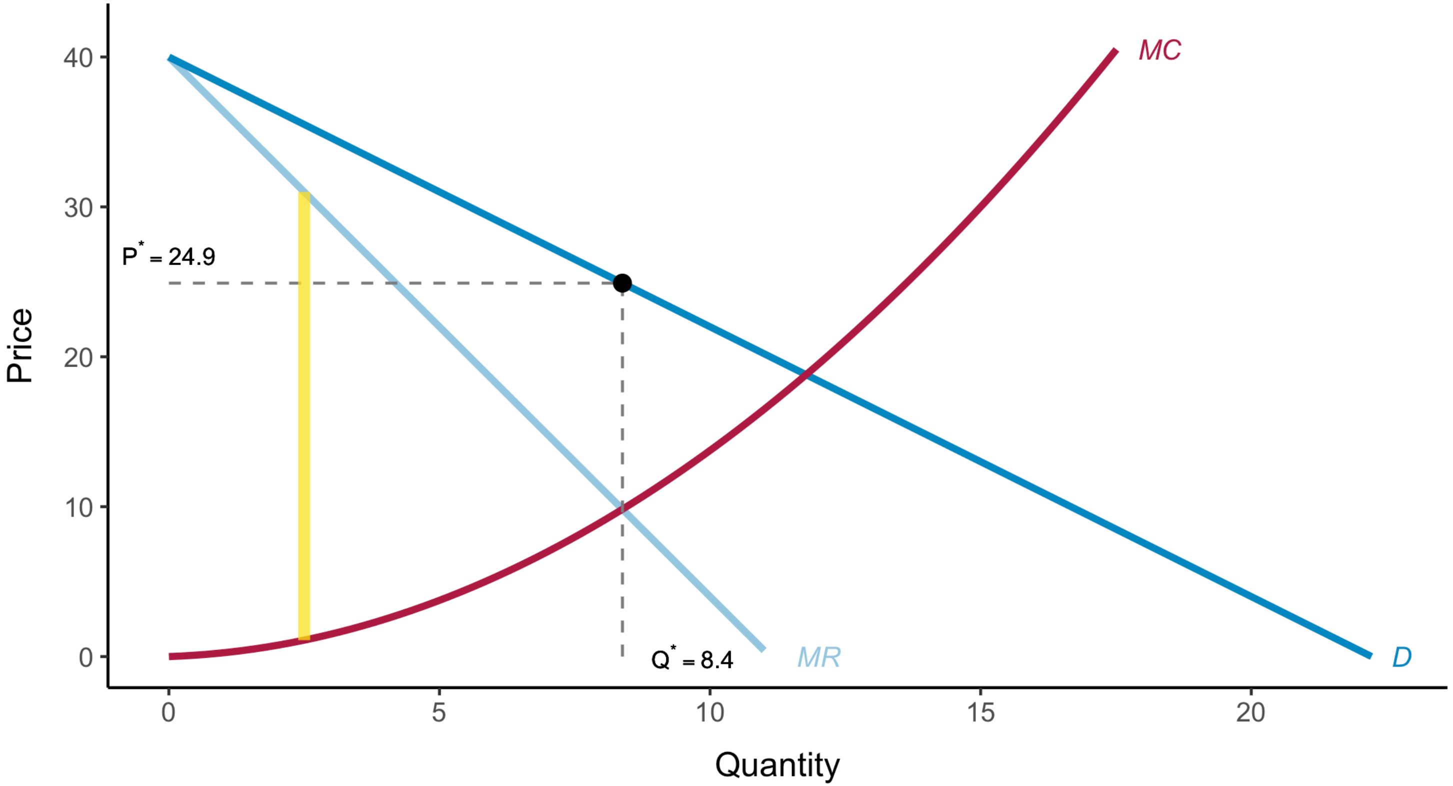
Close to $Q = 0 \quad \Rightarrow \Delta\Pi = MR - MC = 40 - 0 = 40.$



First-degree Price Discrimination

Pricing P^*.

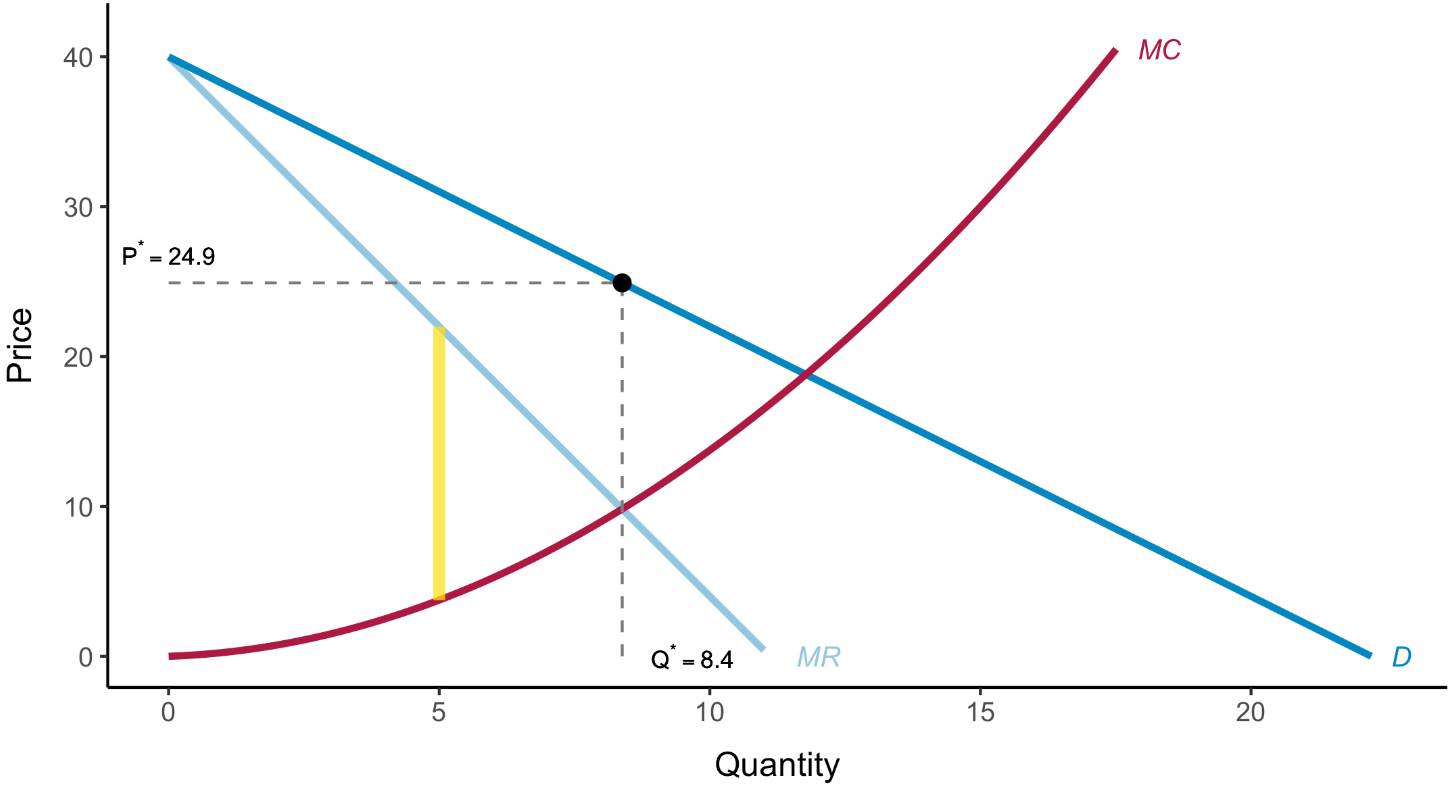
When $Q = 2.5 \quad \Rightarrow \Delta\Pi = MR - MC \approx 31 - 1 = 30.$



First-degree Price Discrimination

Pricing P^*.

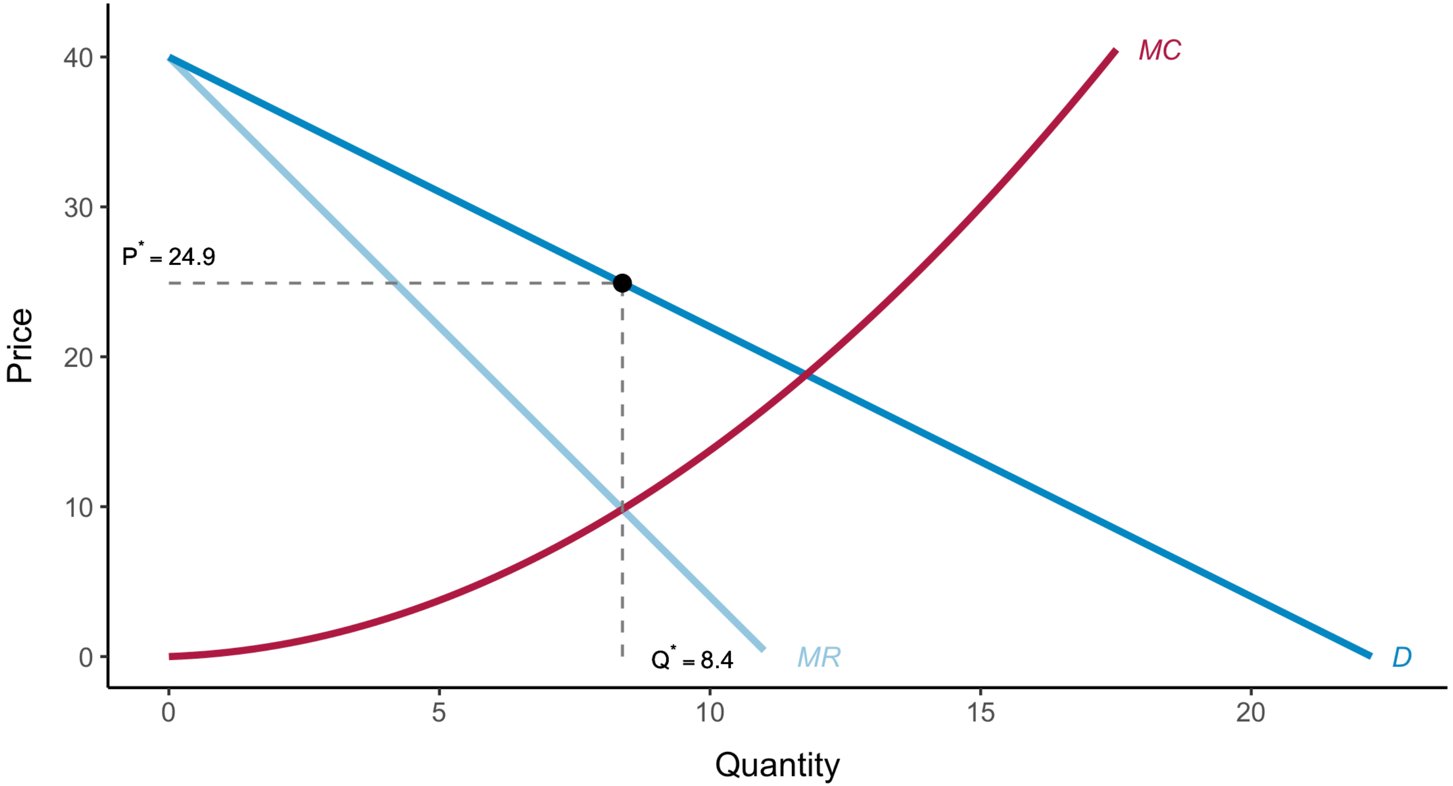
When $Q = 5 \quad \Rightarrow \Delta\Pi = MR - MC \approx 22 - 3.75 = 18.25.$



First-degree Price Discrimination

Pricing P^*.

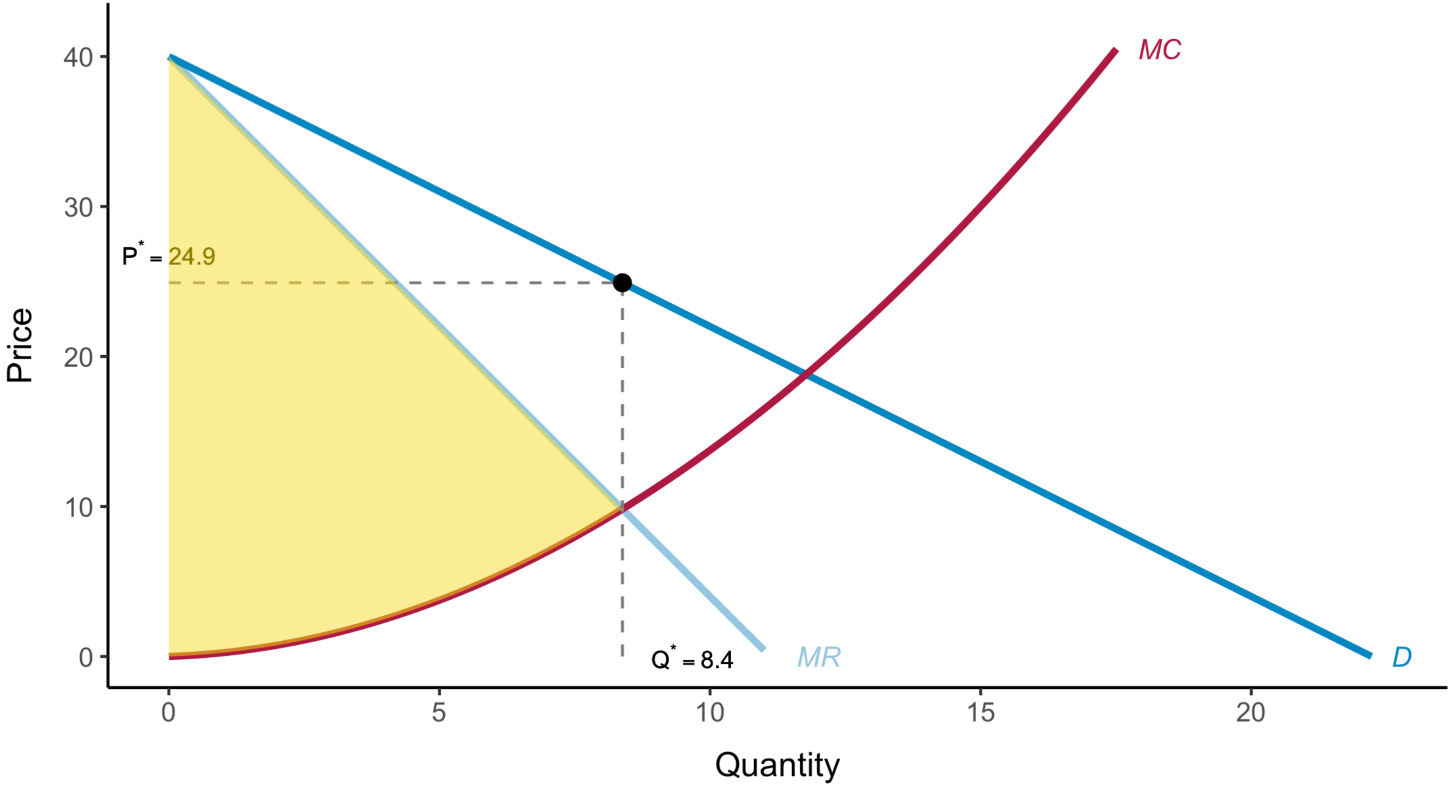
When $Q = Q^* \approx 8.4 \quad \Rightarrow \Delta\Pi = MR - MC \approx 9.83 - 9.83 = 0.$



First-degree Price Discrimination

Pricing P^*.

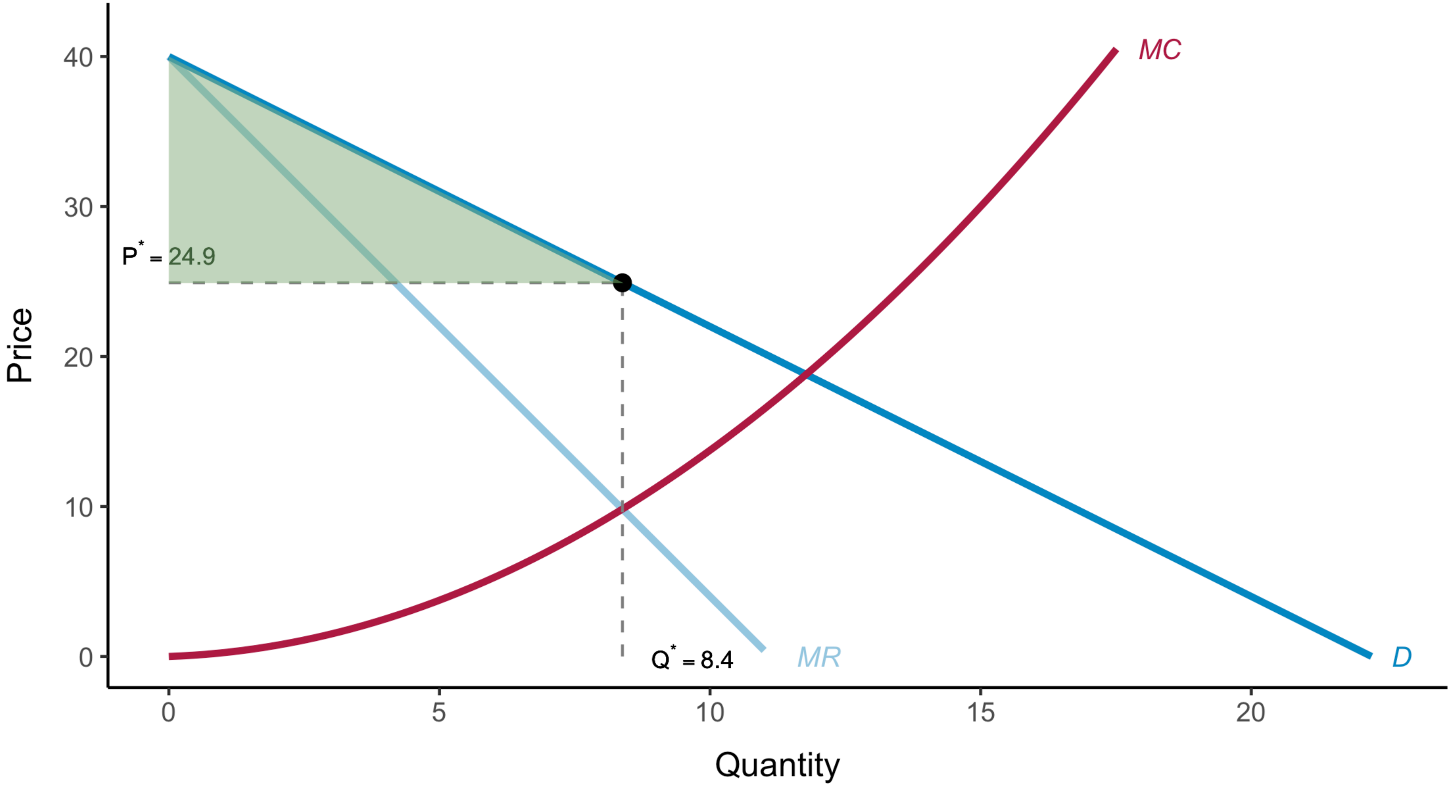
Variable profit: $\sum_{Q=0}^{Q^*} MR(Q) - MC(Q)$



First-degree Price Discrimination

Pricing P^*.

Consumer surplus: $\sum_{Q=0}^{Q^*} P(Q) - P^*$



First-degree Price Discrimination

Perfect Price Discrimination: Each consumer is charged exactly what she is willing to pay.

Is the MR informative for the firm under such pricing situation?

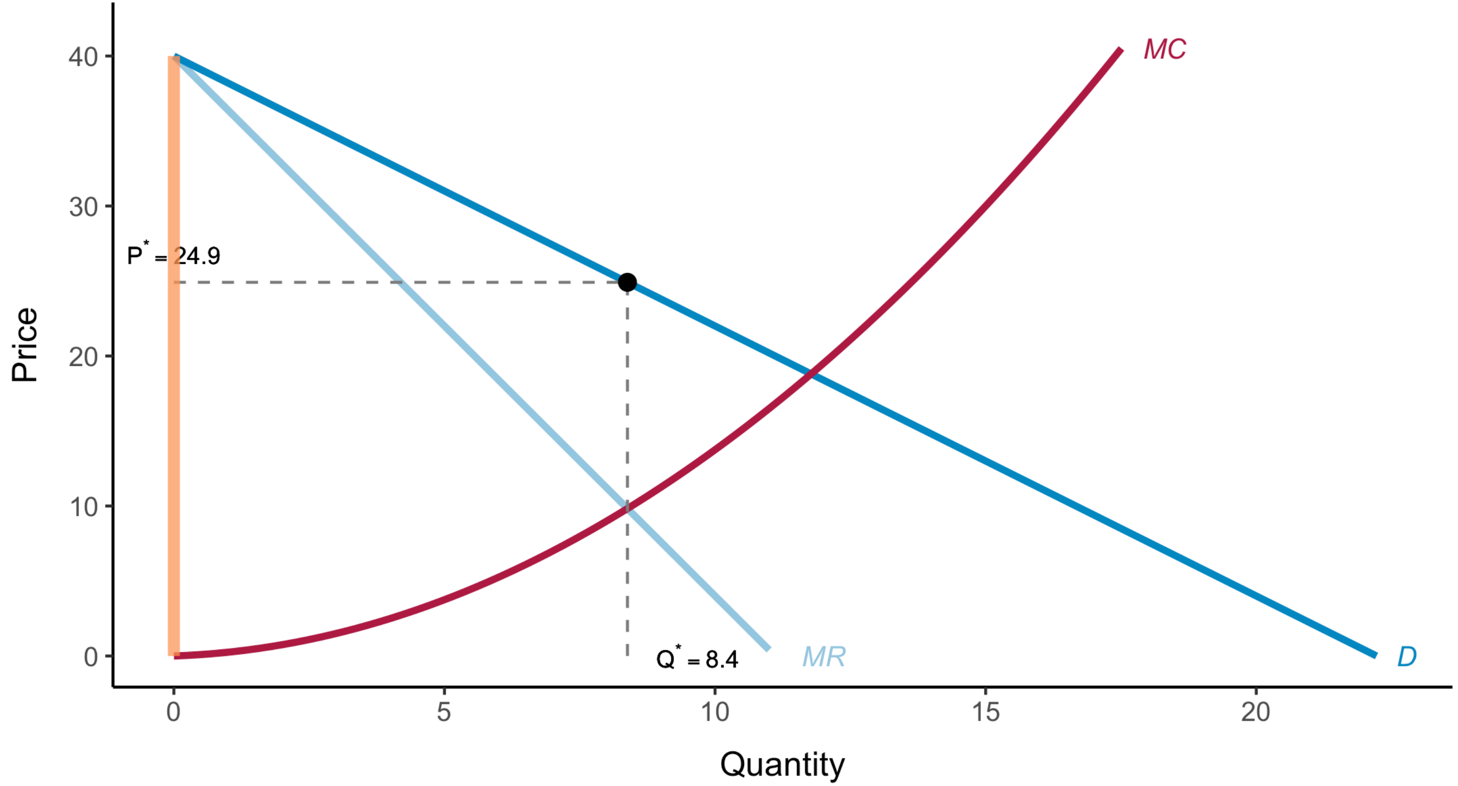
The revenue from each additional unit sold equals its price, as shown by the demand curve.

The additional profit, $\Delta\Pi$, from producing and selling an incremental unit is now the difference between demand, $P(Q)$, and marginal cost, $MC(Q)$.

First-degree Price Discrimination

Perfect Price Discrimination: Each consumer is charged exactly what she is willing to pay.

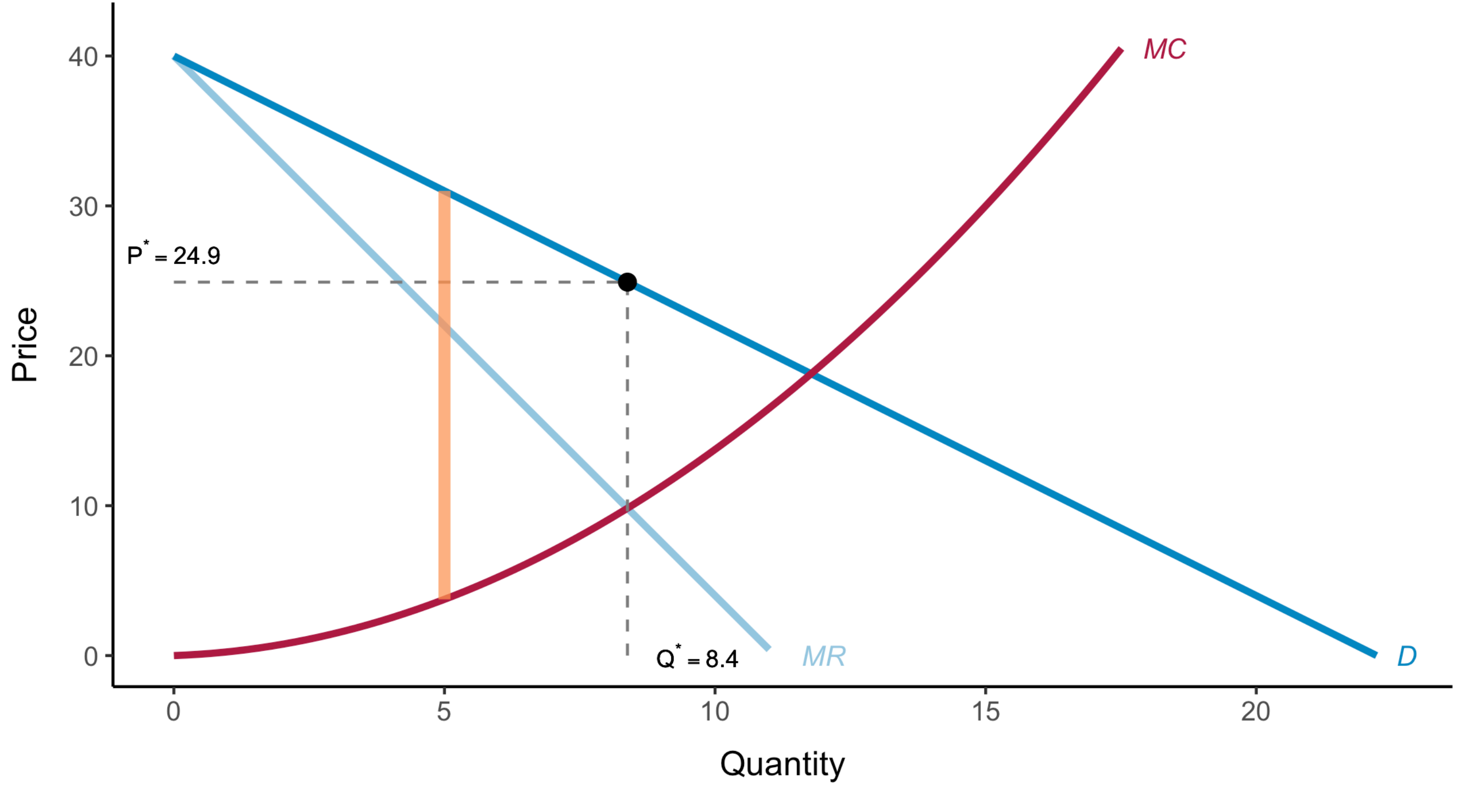
Close to $Q = 0 \quad \Rightarrow \Delta\Pi = AR(Q) - MC(Q) = 40 - 0 = 40.$



First-degree Price Discrimination

Perfect Price Discrimination: Each consumer is charged exactly what she is willing to pay.

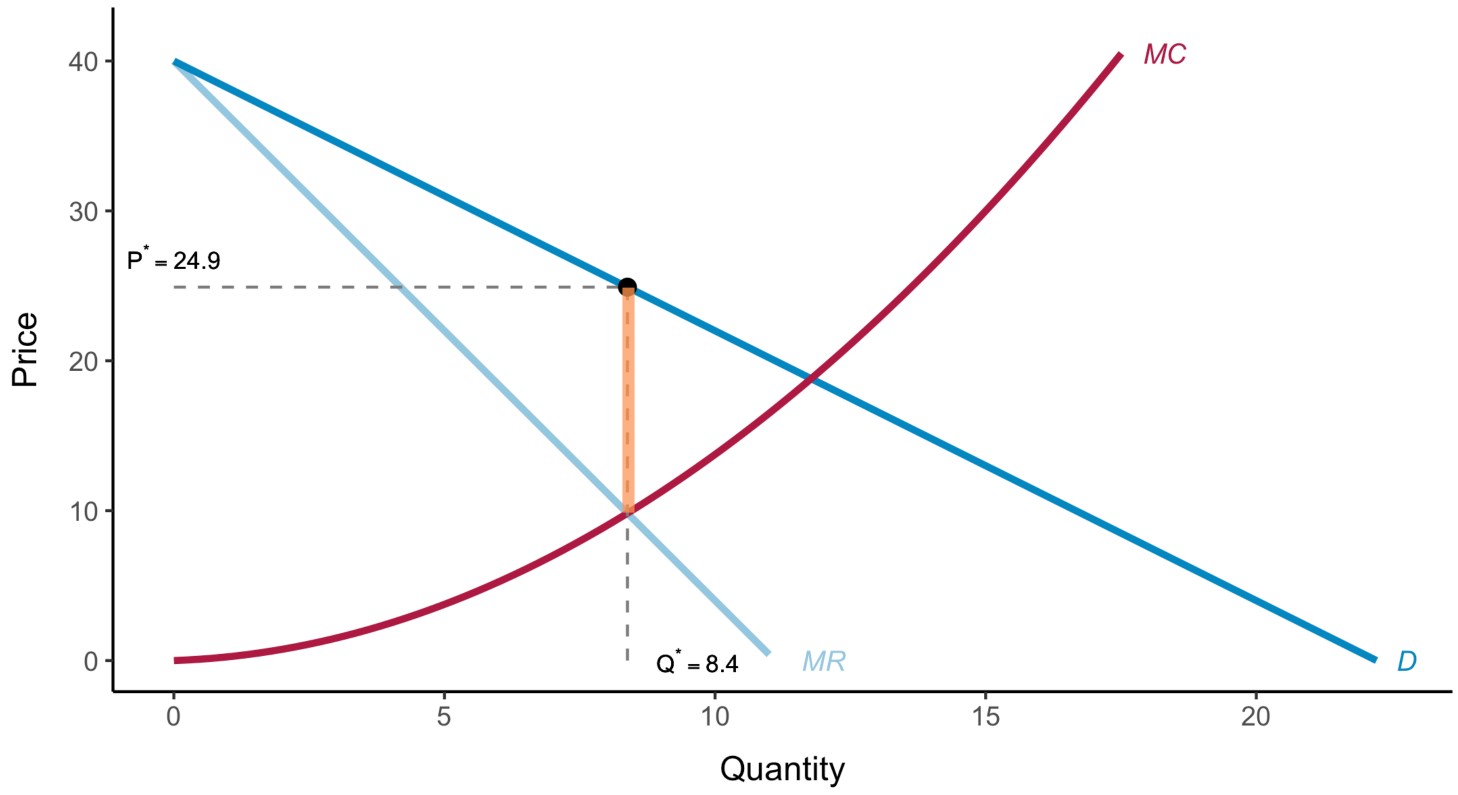
When $Q = 5 \quad \Rightarrow \Delta\Pi = AR(Q) - MC(Q) = 31 - 3.75 = 27.25.$



First-degree Price Discrimination

Perfect Price Discrimination: Each consumer is charged exactly what she is willing to pay.

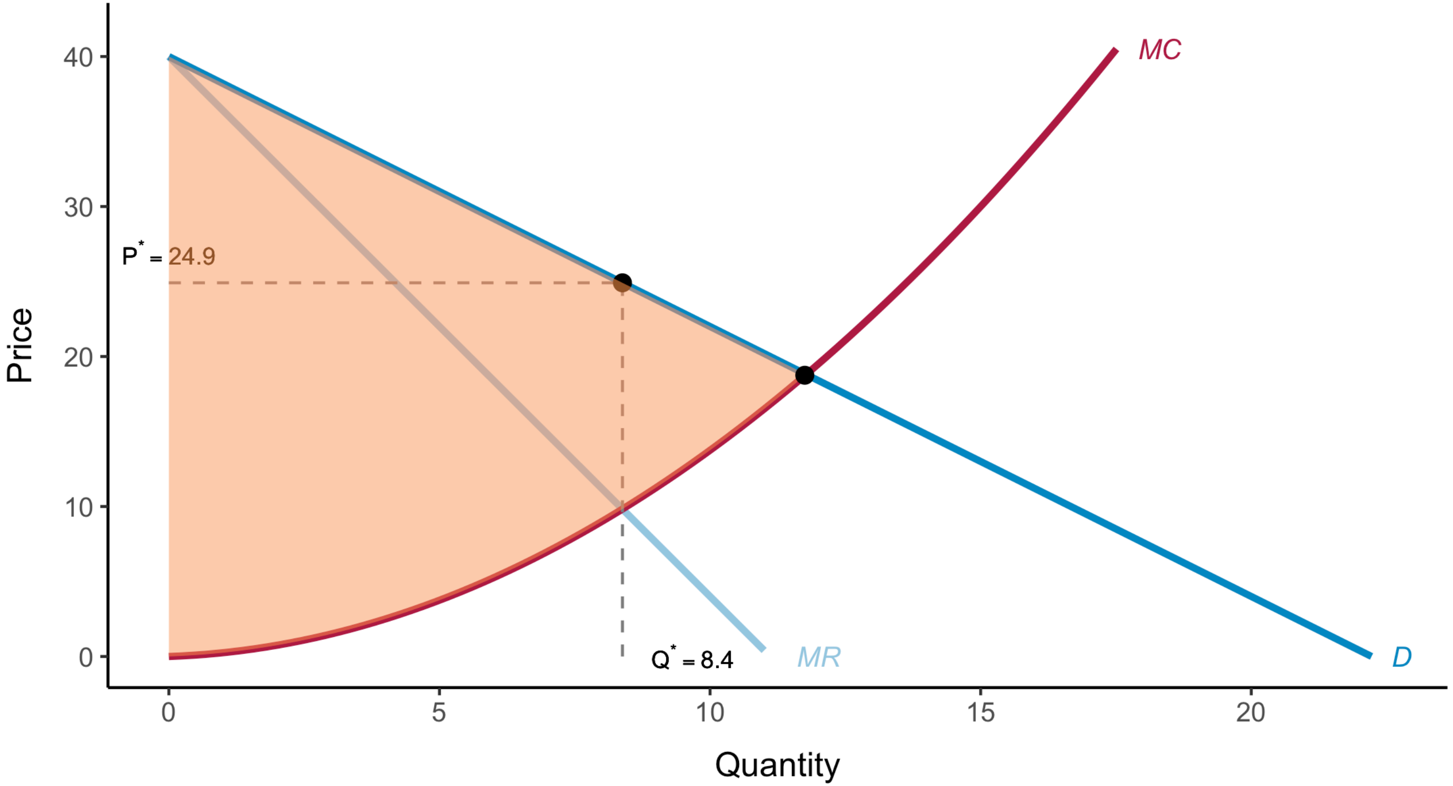
When $Q = 8.4 \quad \Rightarrow \Delta\Pi = AR(Q) - MC(Q) \approx 24.9 - 9.9 = 15.$



First-degree Price Discrimination

Perfect Price Discrimination: Each consumer is charged exactly what she is willing to pay.

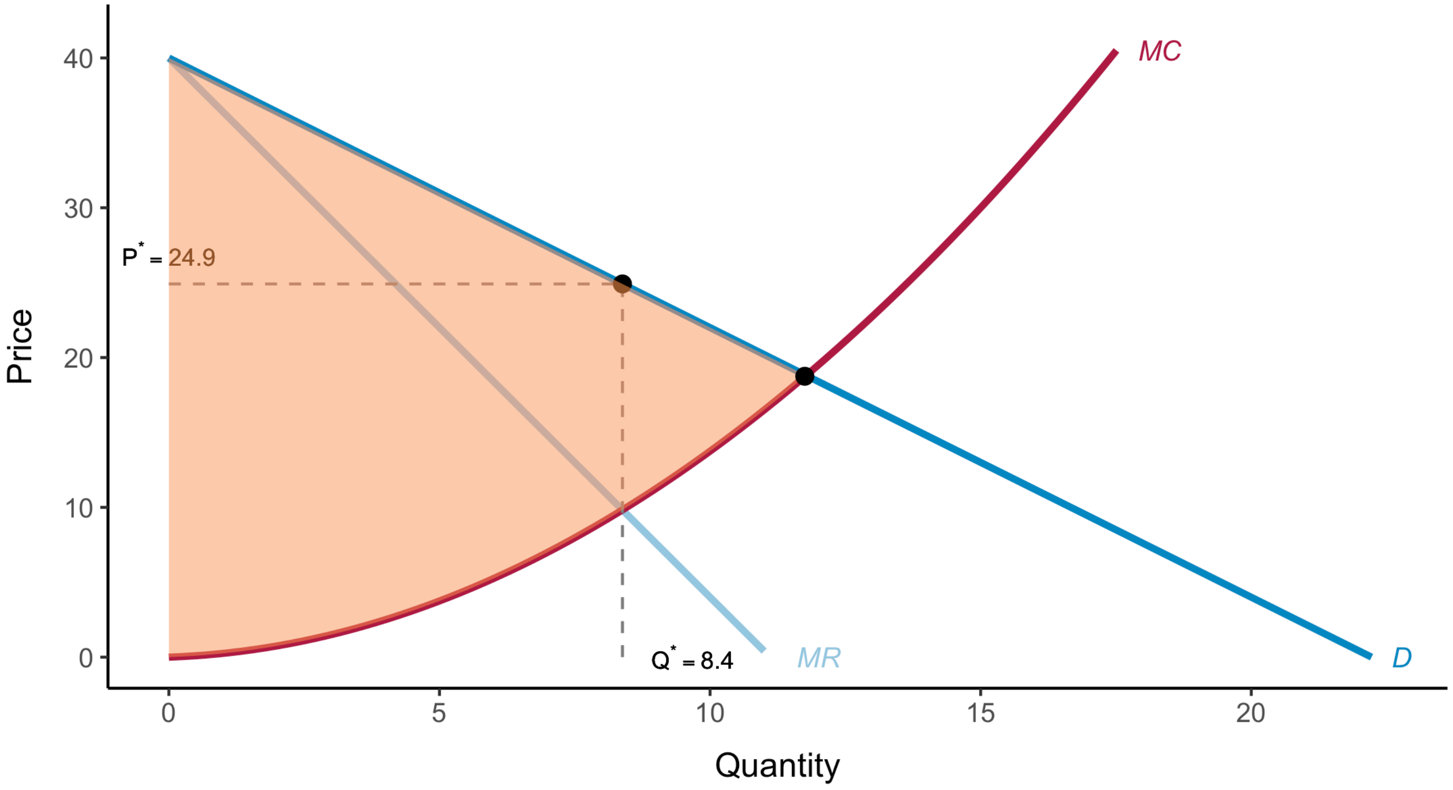
Variable profit: $\sum_{Q=0}^{Q^*} AR(Q) - MC(Q)$



First-degree Price Discrimination

Perfect Price Discrimination: Each consumer is charged exactly what she is willing to pay.

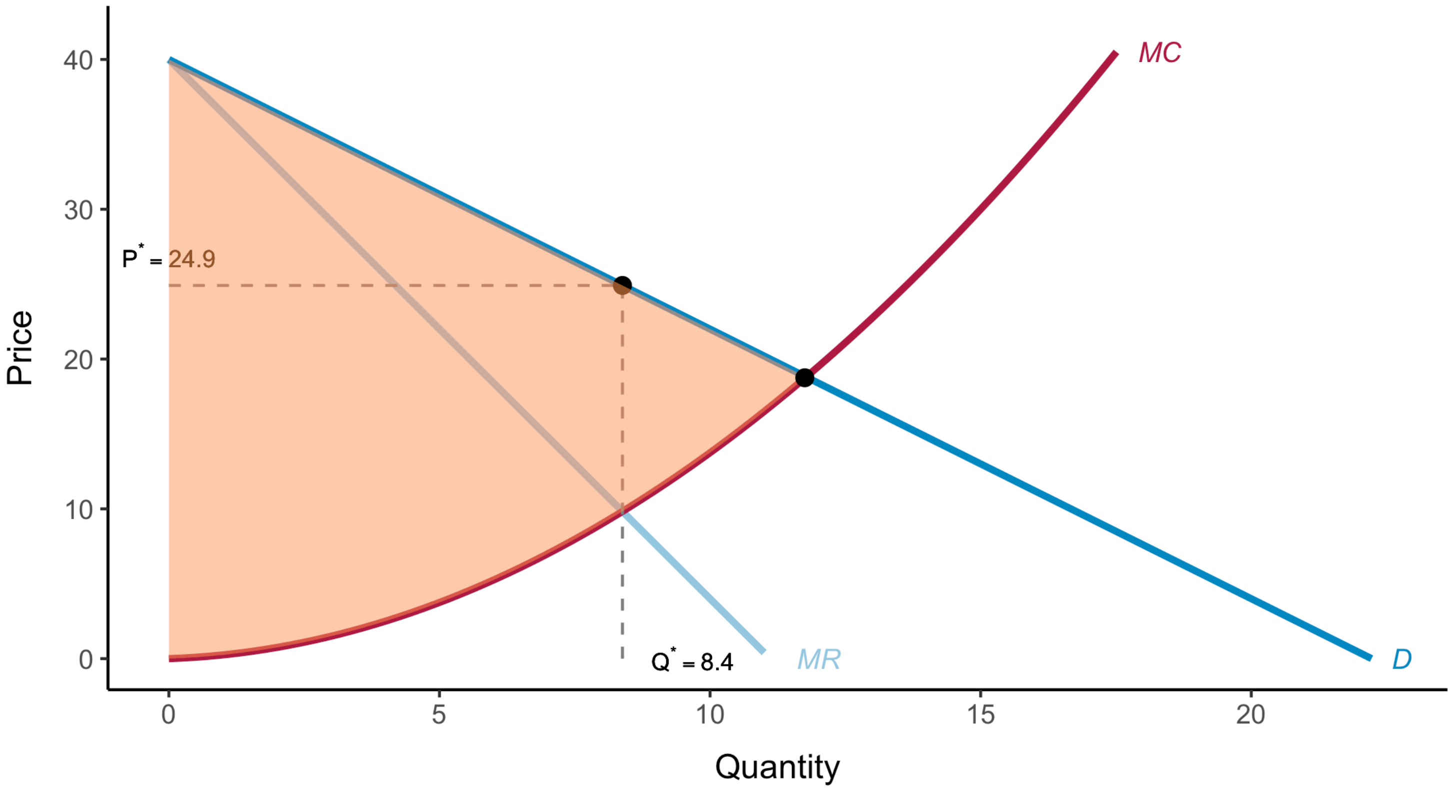
The firm increases production as long as demand exceeds marginal cost, stopping at output $Q^{**} \approx 11.7$ to maximize profit.



First-degree Price Discrimination

Perfect Price Discrimination: Each consumer is charged exactly what she is willing to pay.

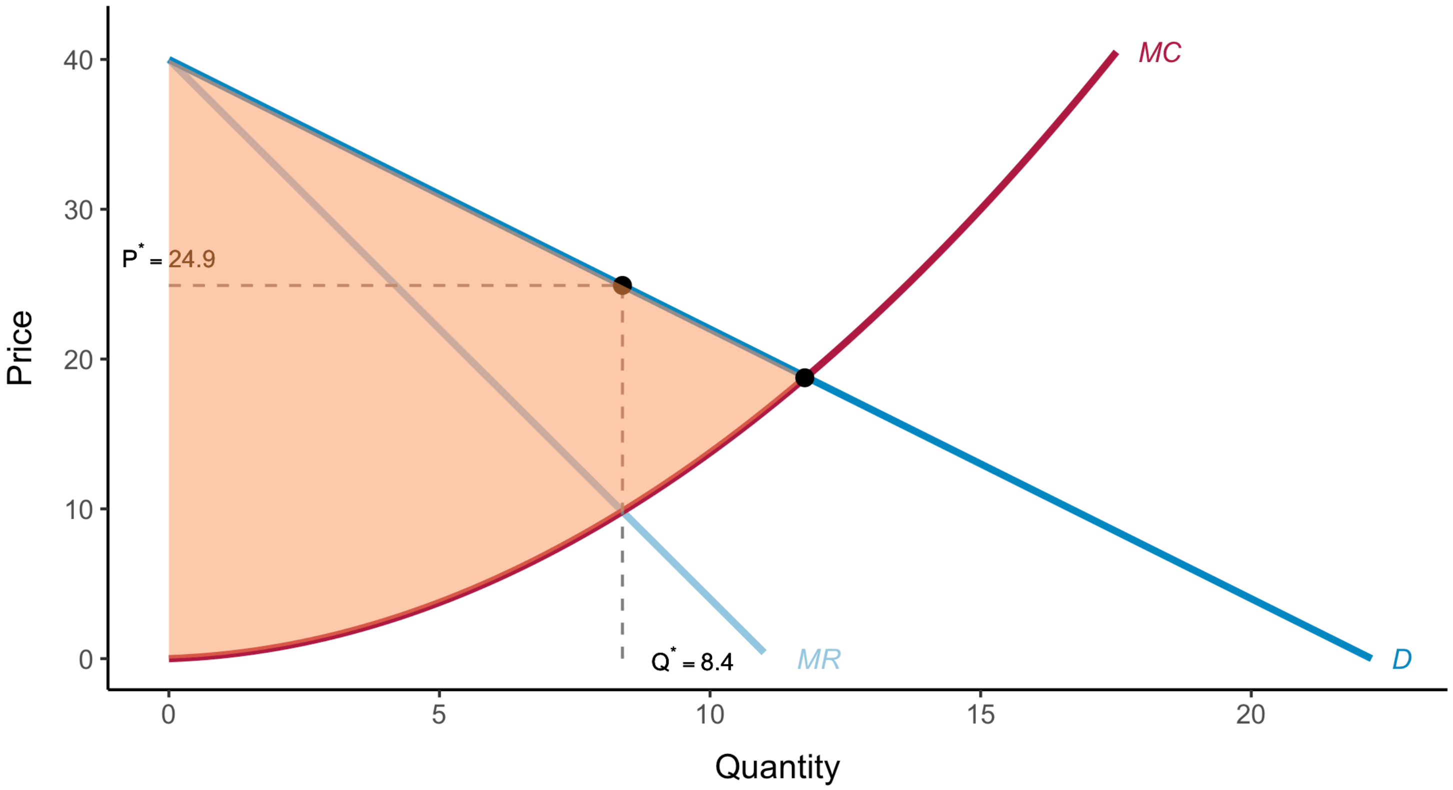
1) The firm's profit has increased.



First-degree Price Discrimination

Perfect Price Discrimination: Each consumer is charged exactly what she is willing to pay.

2) Since each customer pays their maximum willingness to pay, the firm captures all consumer surplus.



First-degree Price Discrimination

Perfect first-degree price discrimination is almost never possible.

1) Impractical to charge each and every customer a different price

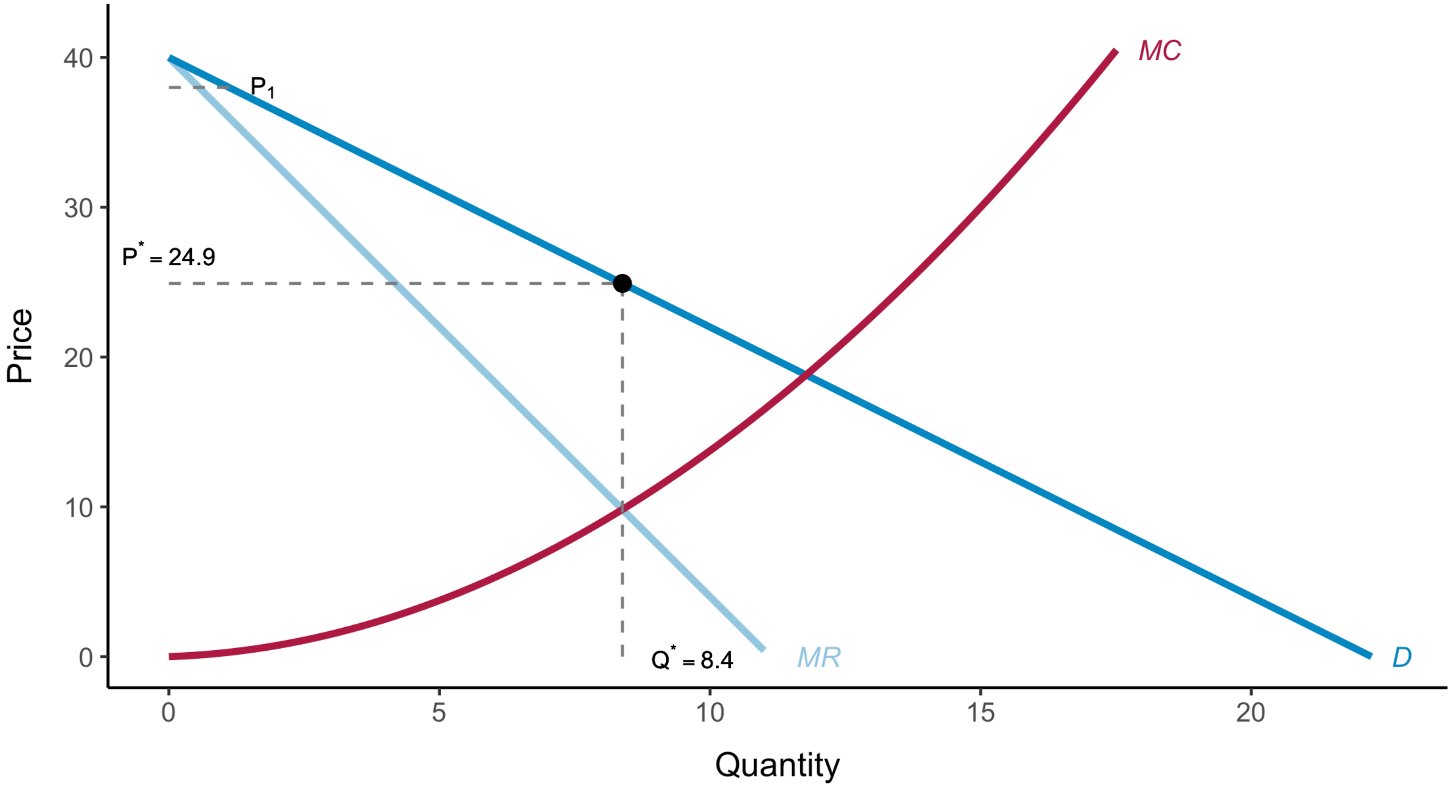
2) Firms usually do not know the reservation price of each customer

Imperfect Price Discrimination: charging a few different prices based on estimates of customers' reservation prices

- Professionals (i.e. doctors, lawyers, architects)
- Salesperson
- Universities

First-degree Price Discrimination

Imperfect Price Discrimination: charging a few different prices based on estimates of customers' reservation prices



First-degree Price Discrimination

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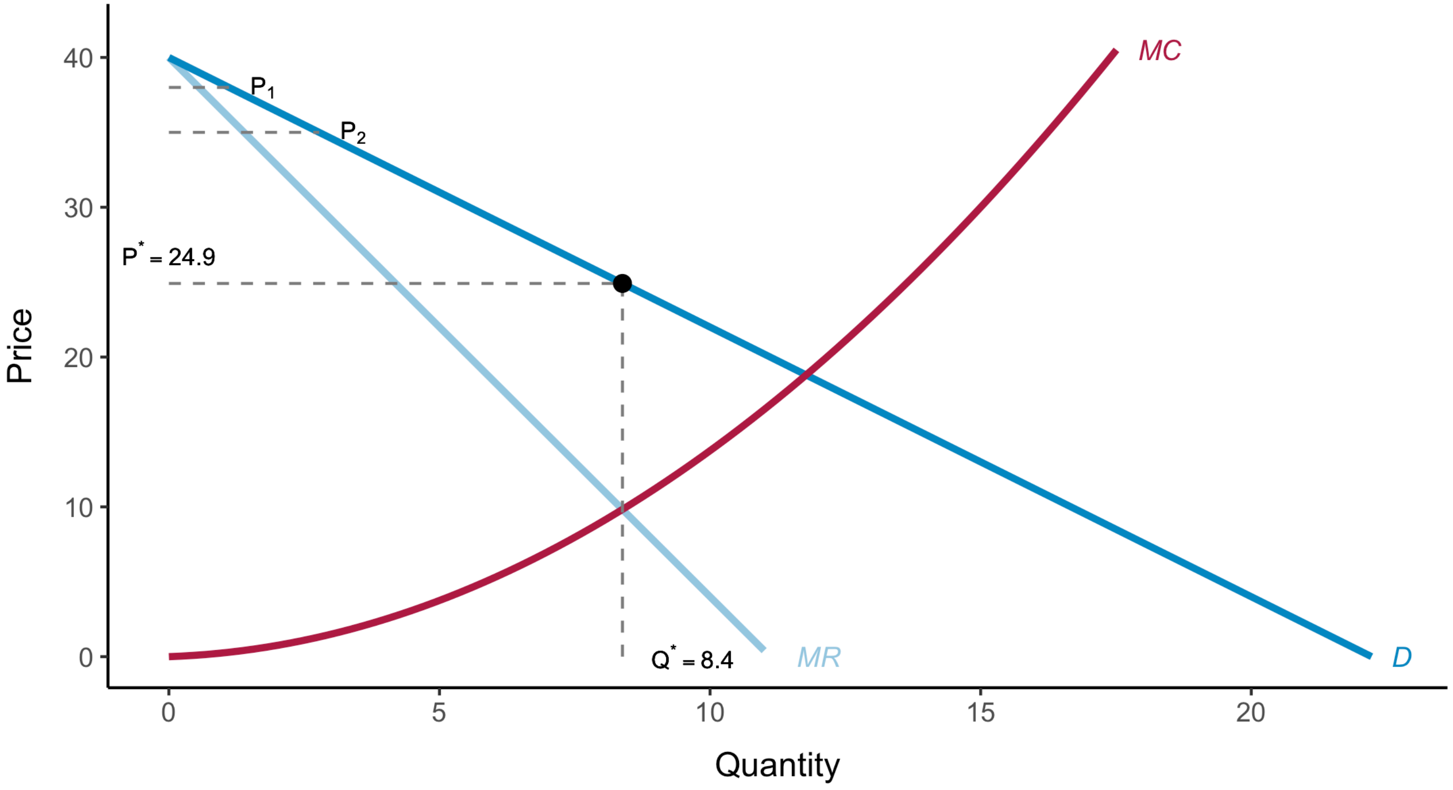

First-degree Price Discrimination

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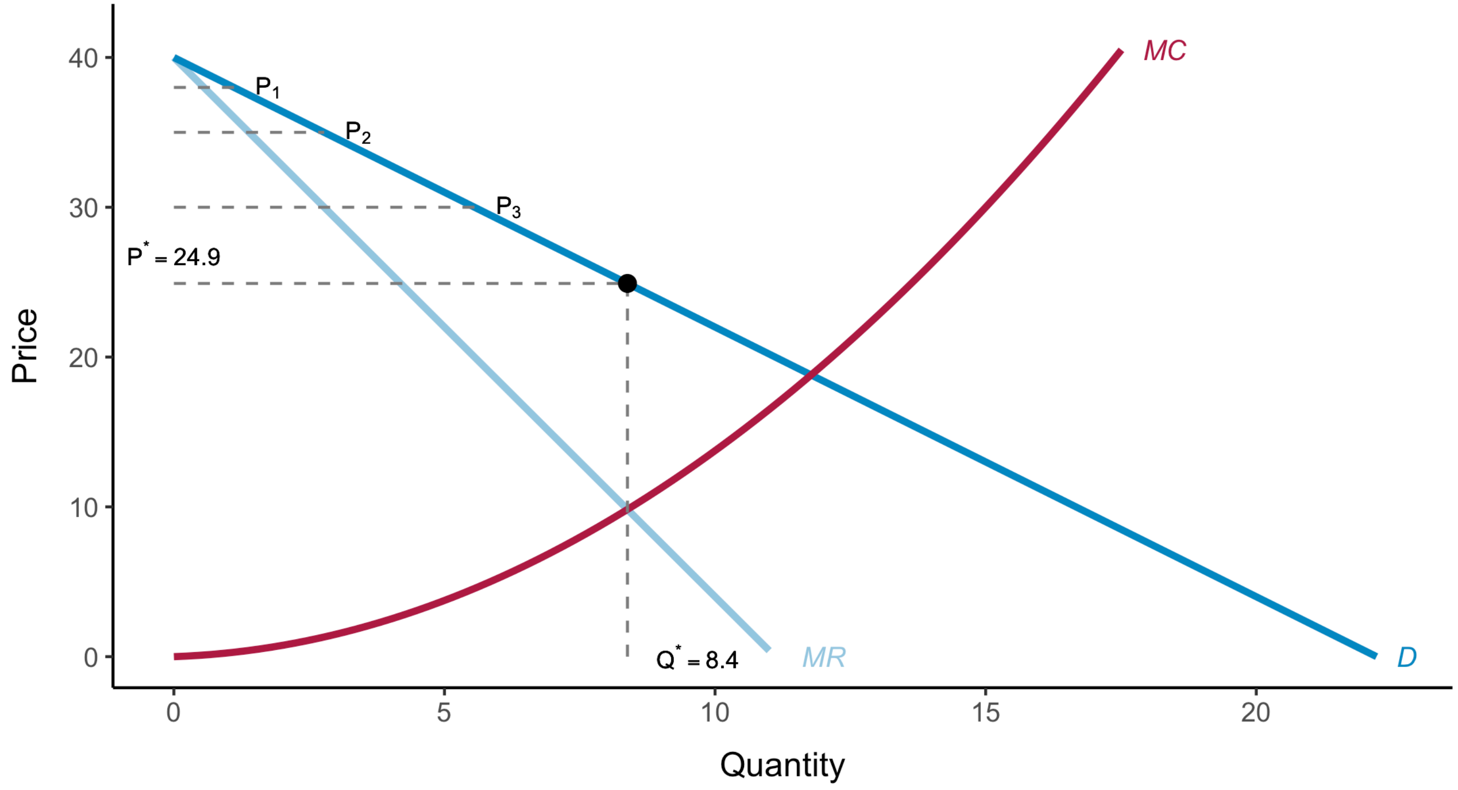

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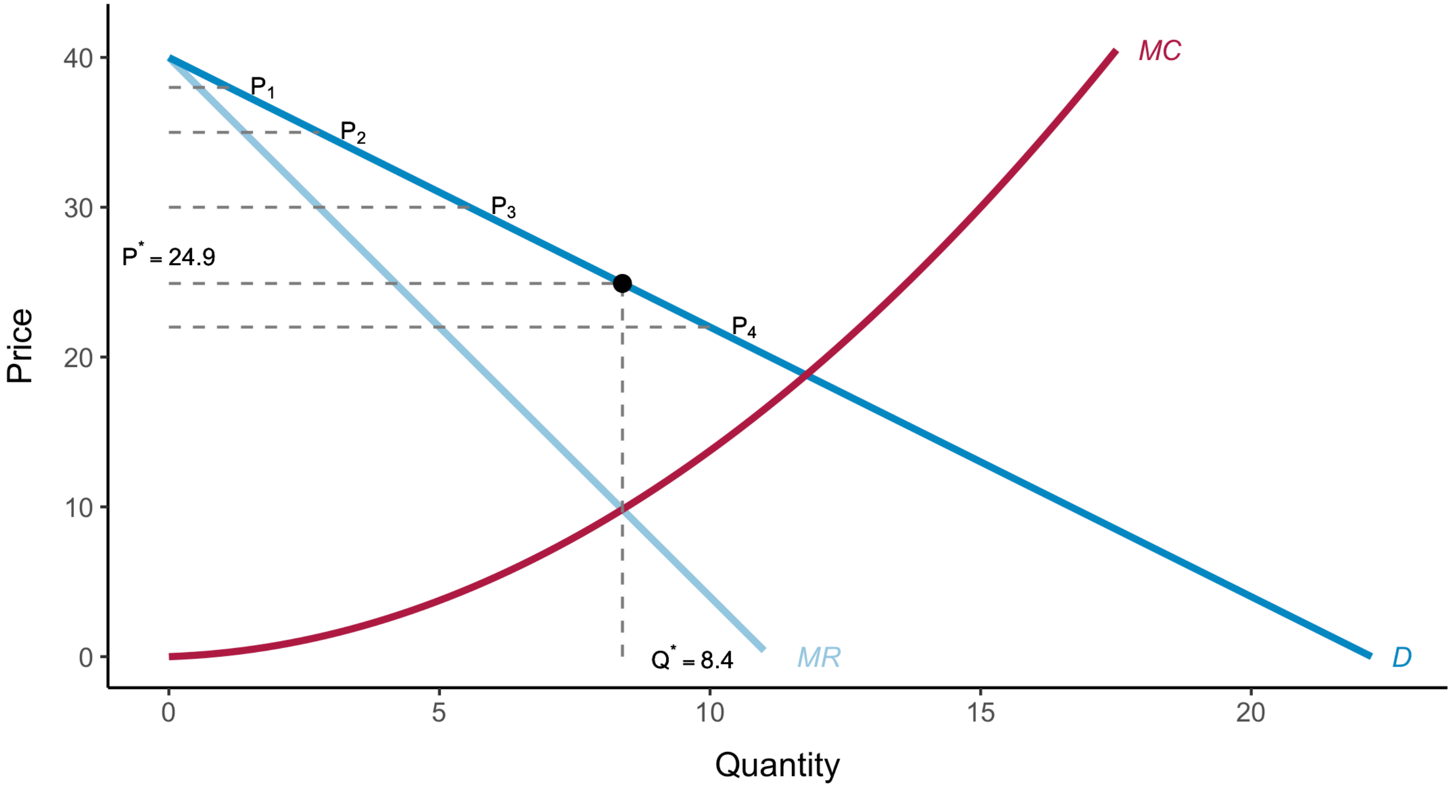

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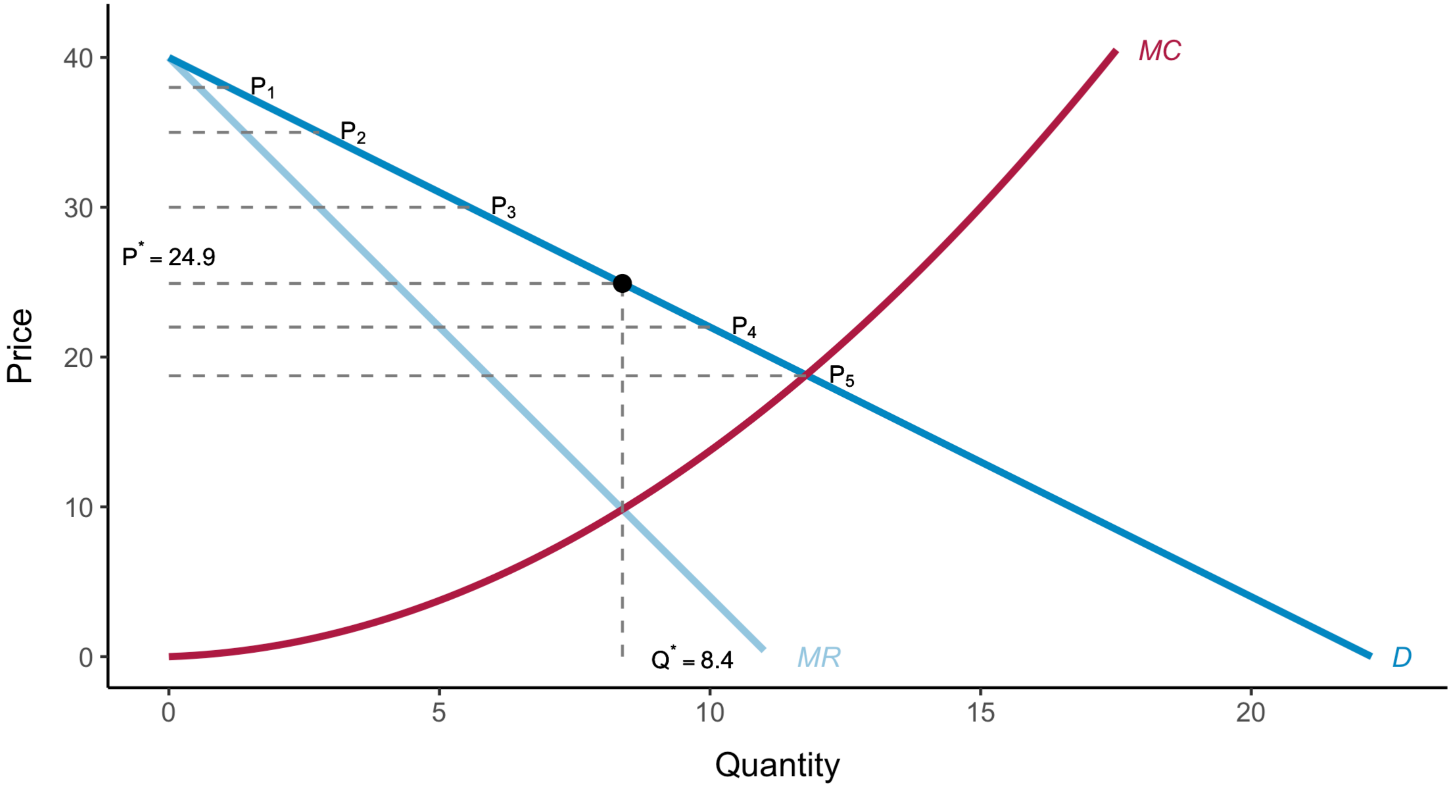
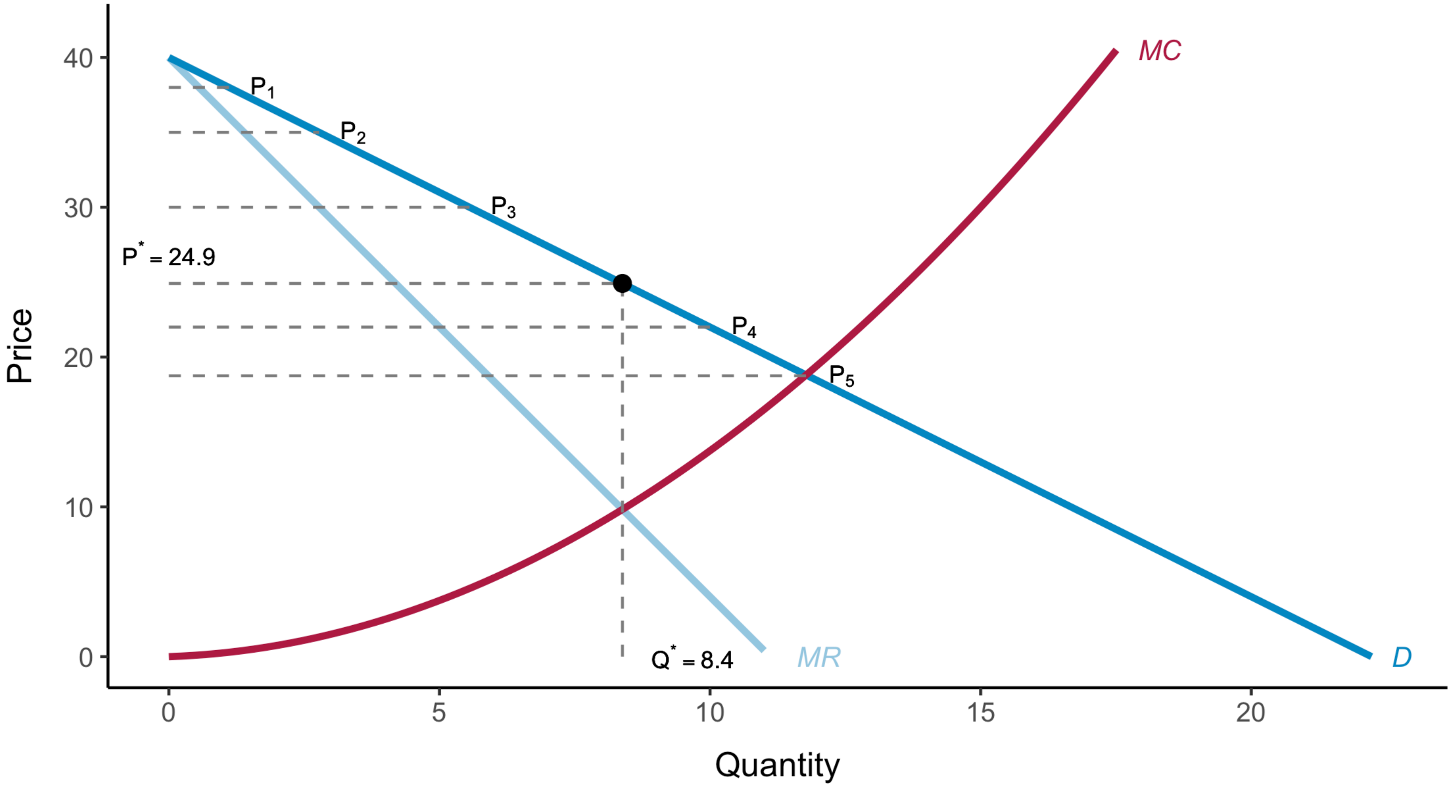


First-degree Price Discrimination

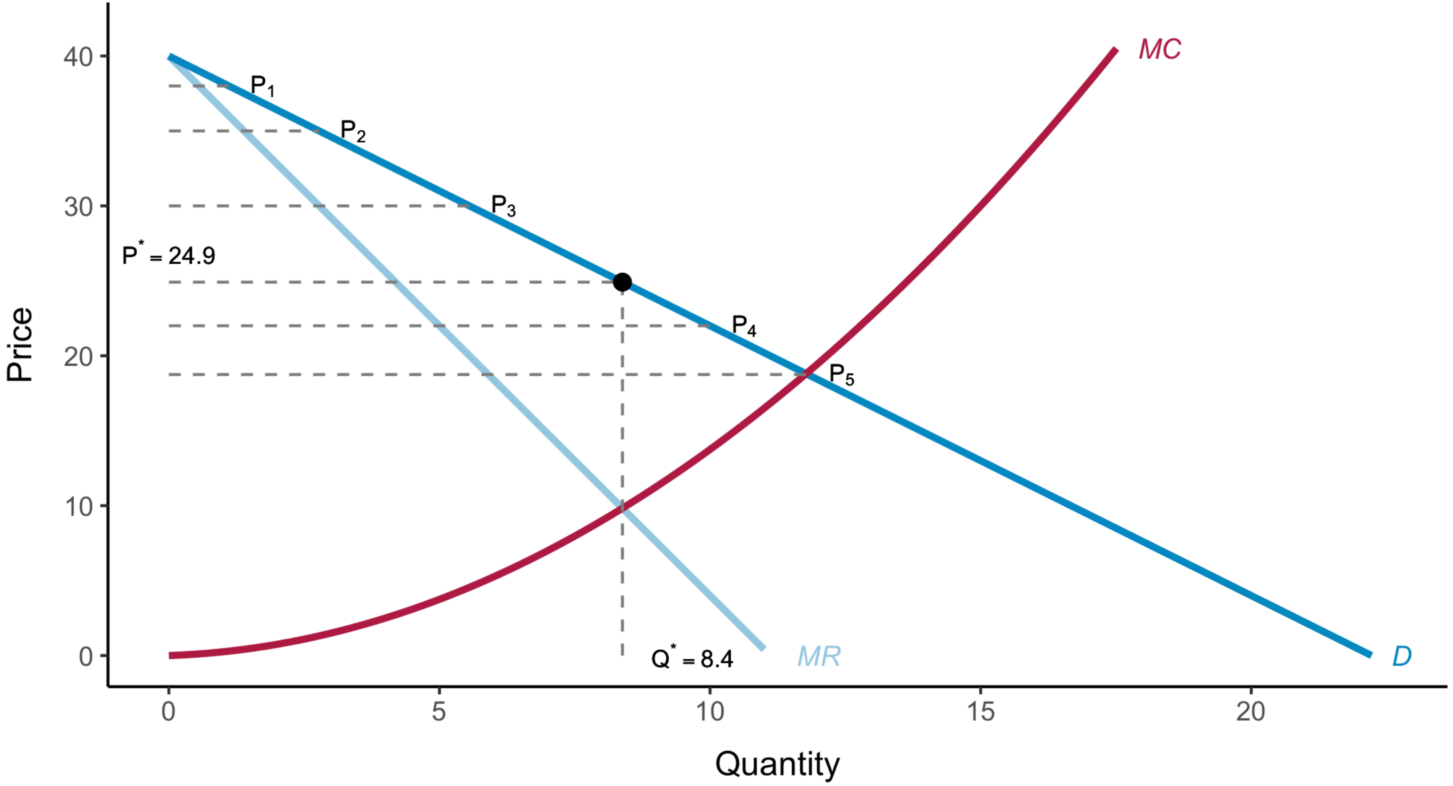
Imperfect Price Discrimination: charging a few different prices based on estimates of customers' reservation prices



First-degree Price Discrimination

Imperfect Price Discrimination: charging a few different prices based on estimates of customers' reservation prices

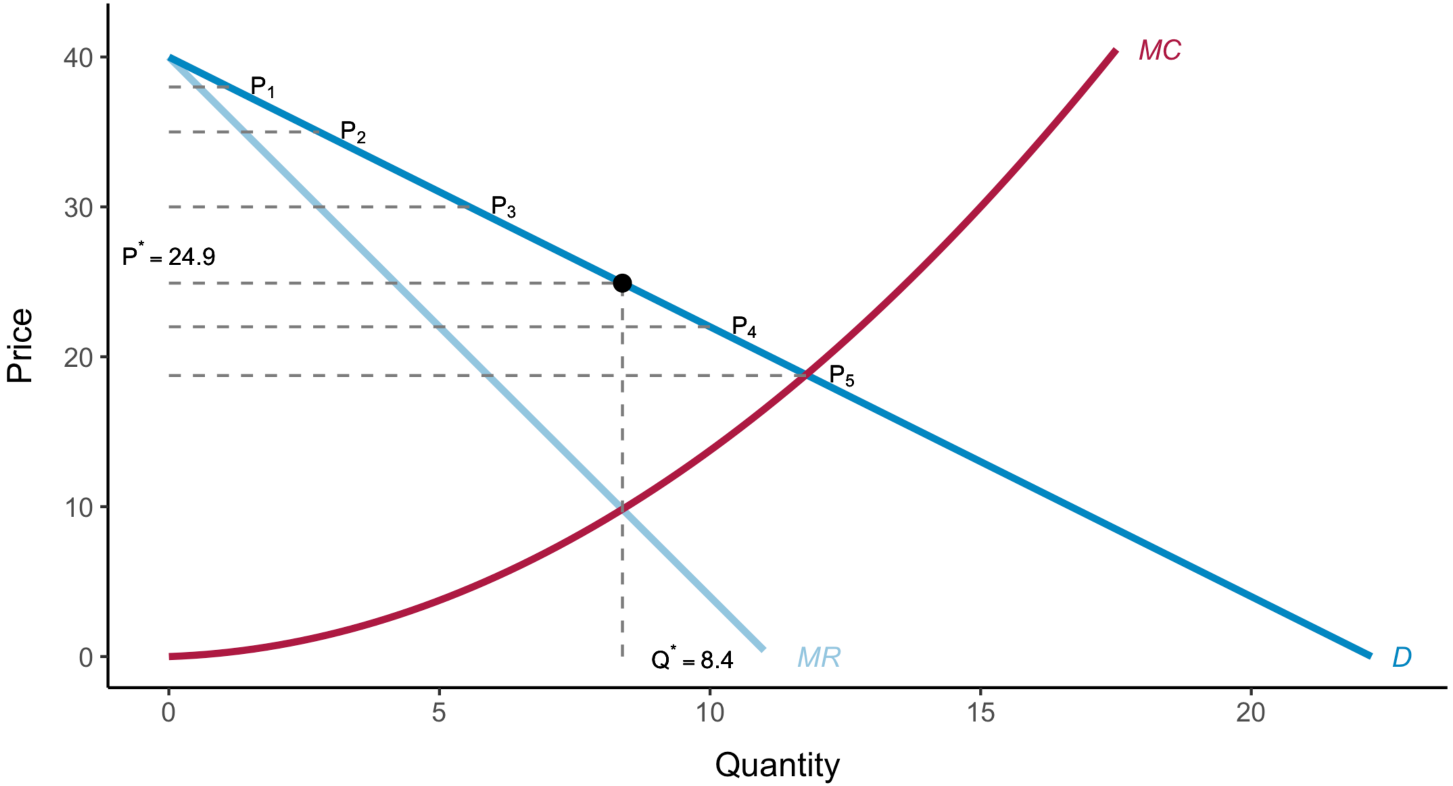
Customers unwilling to pay P^* or more benefit from entering the market, gaining some consumer surplus.



First-degree Price Discrimination

Imperfect Price Discrimination: charging a few different prices based on estimates of customers' reservation prices

If price discrimination attracts enough new buyers, overall welfare can improve for both producers and consumers.



Second-Degree Price Discrimination

In some markets, a consumer’s reservation price decreases as they buy more units over time.

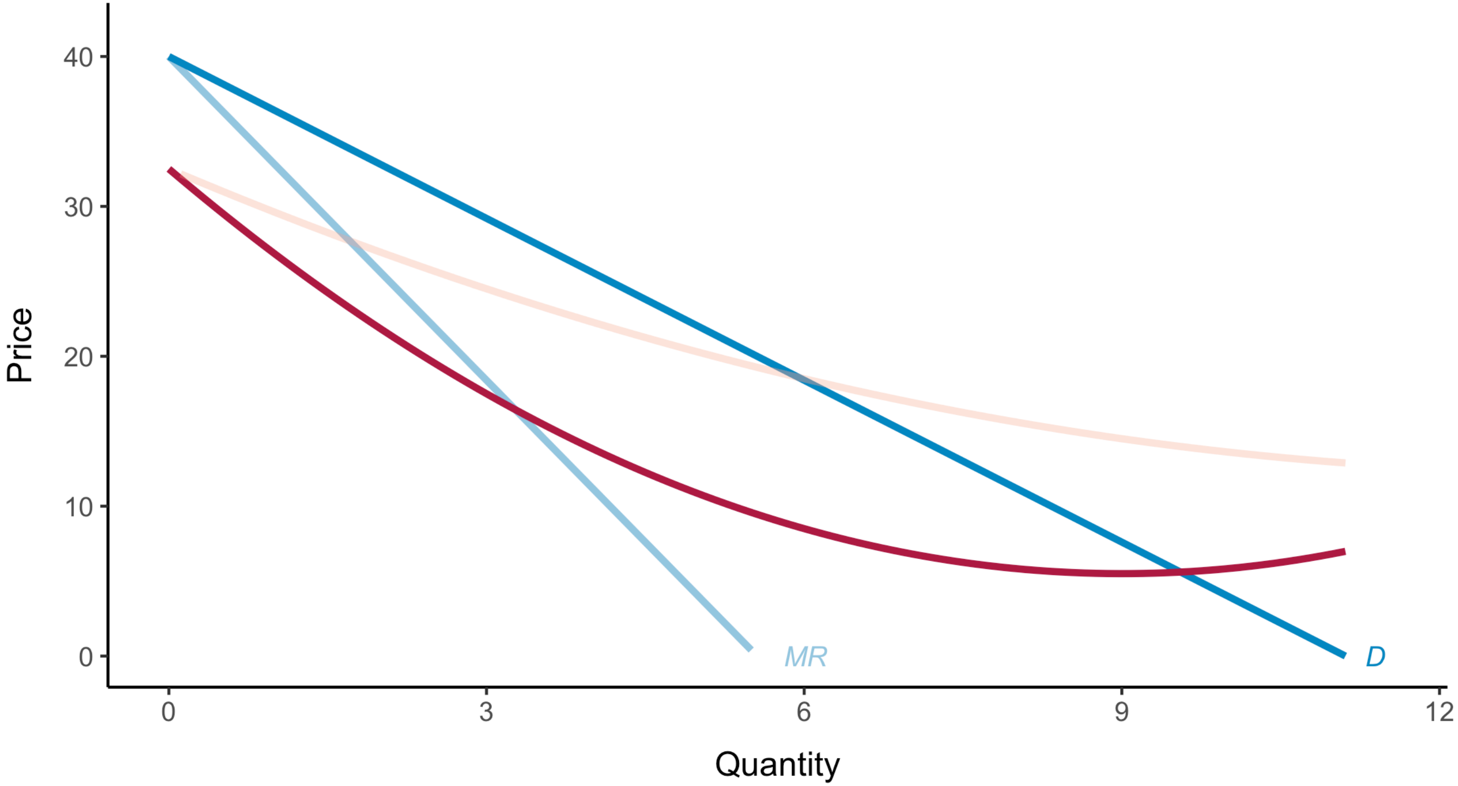
Second-degree price discrimination: Charging different prices per unit based on the quantity consumed.

Block pricing Practice of charging different prices for different quantities or “blocks” of a good.

- Electric power companies
- Natural gas utilities
- Municipal water companies

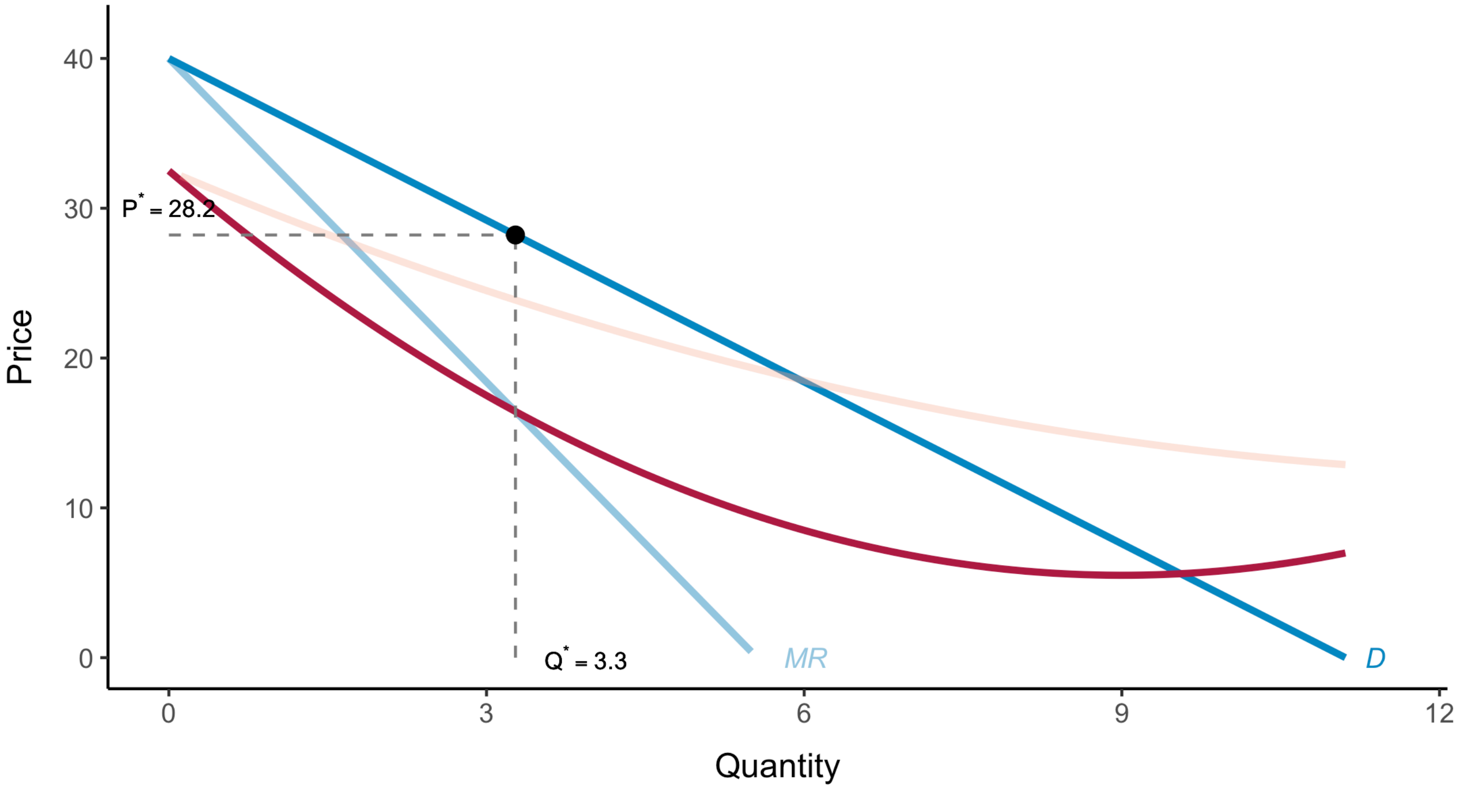
Second-Degree Price Discrimination

Suppose a firm with declining average and marginal costs.



Second-Degree Price Discrimination

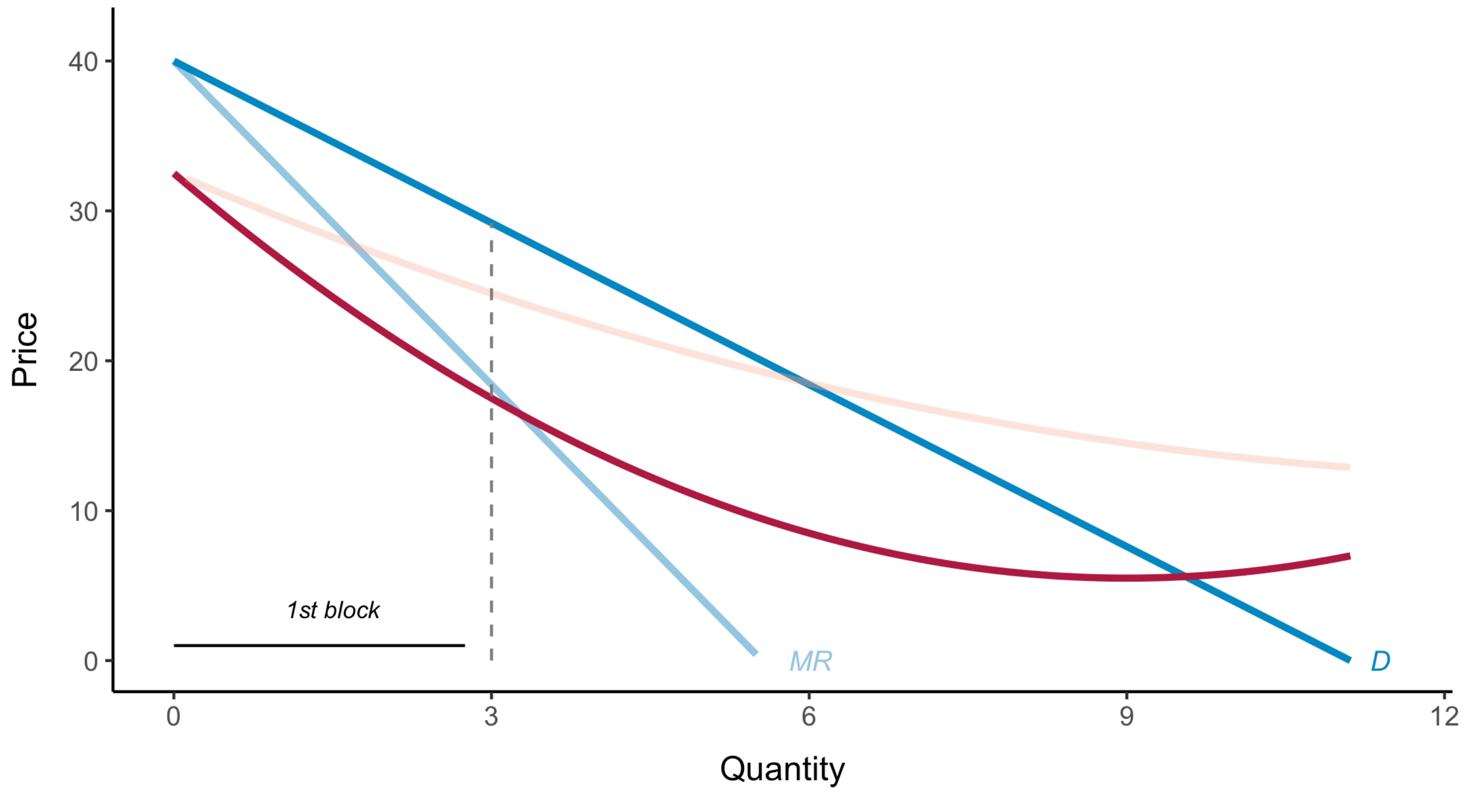
Under single pricing, P^* and Q^*.



Second-Degree Price Discrimination

Under block pricing.

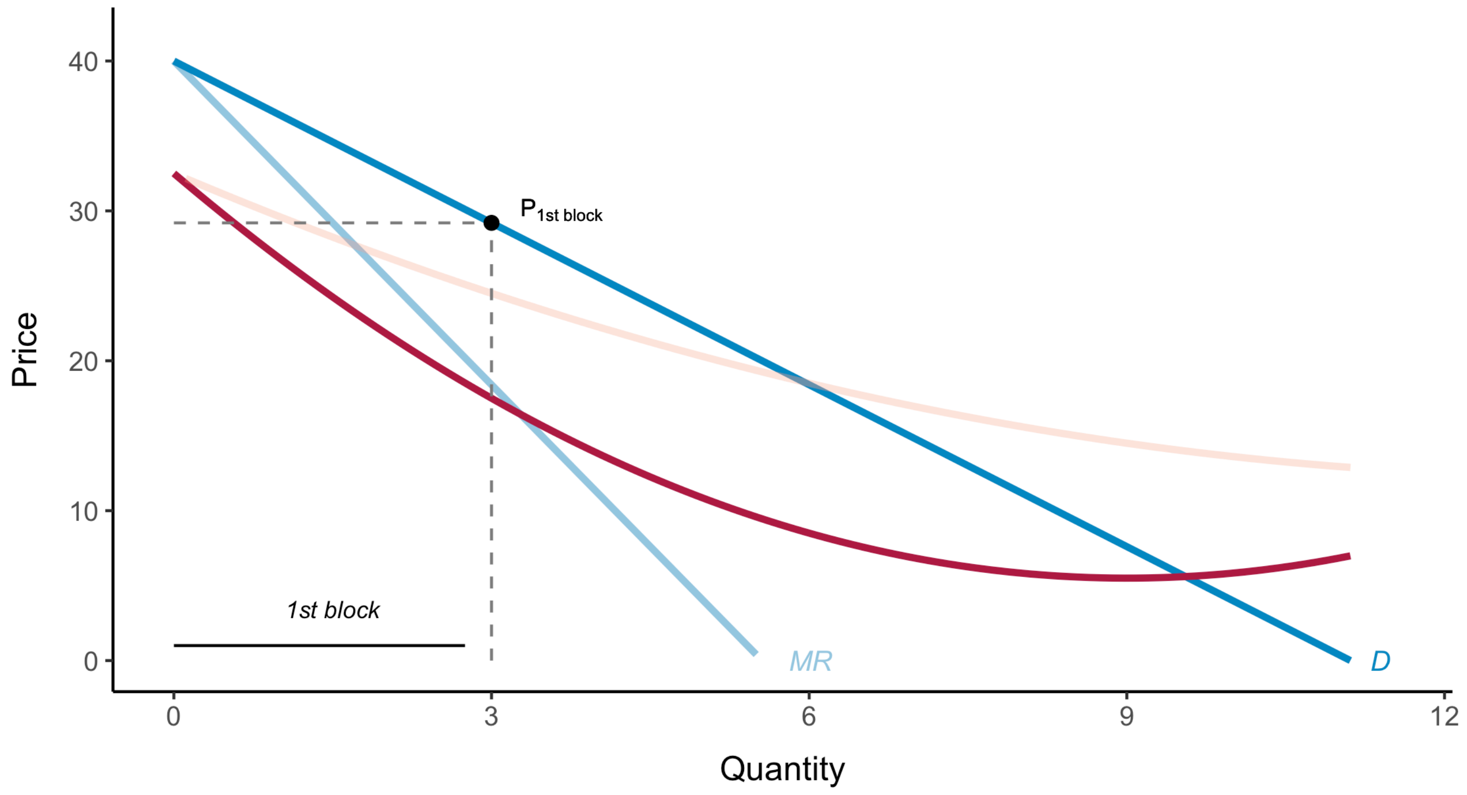
Define blocks: 1st block $Q \in [0, 3]$.



Second-Degree Price Discrimination

Under block pricing.

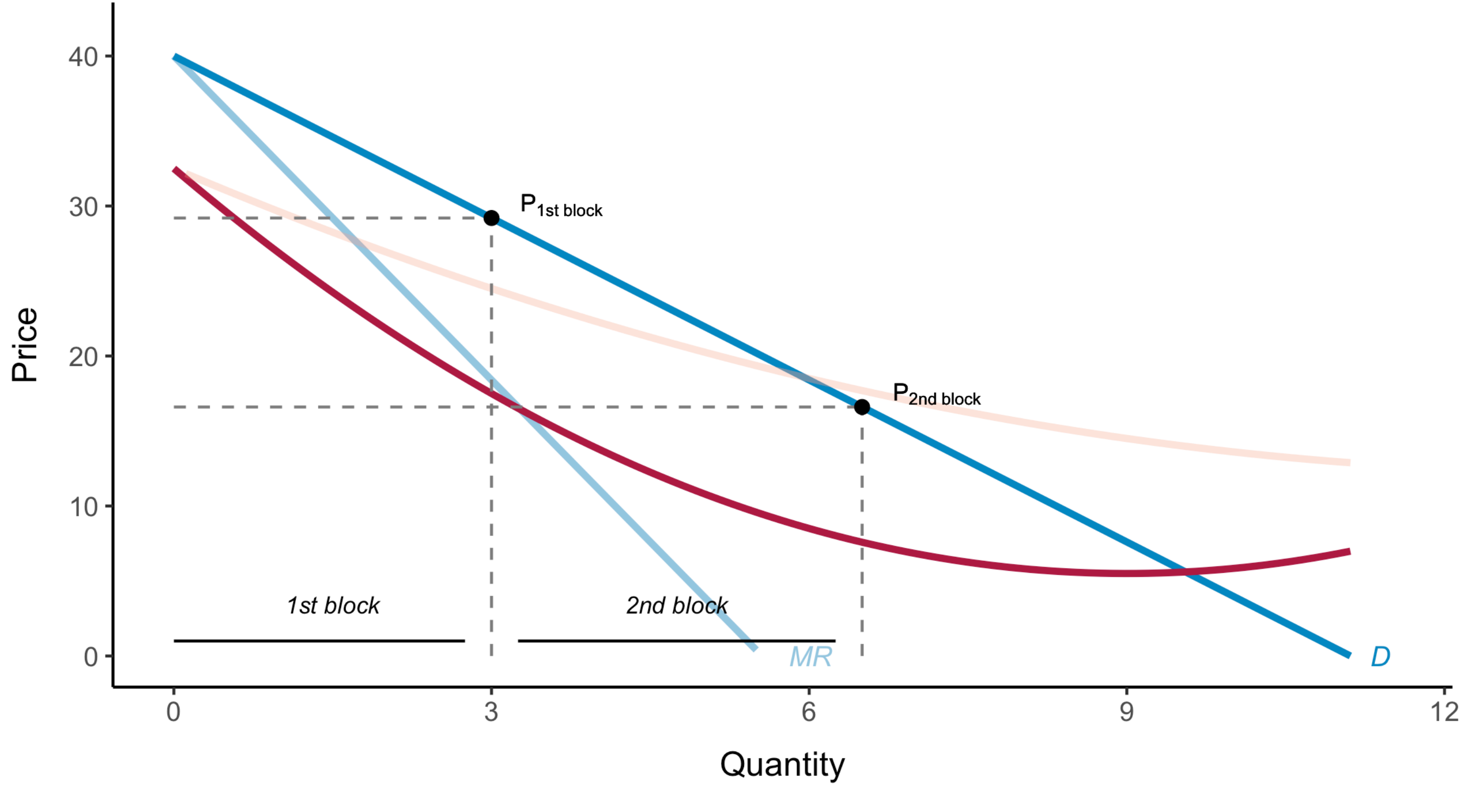
For all quantities in 1st block $Q \in [0, 3]$, firm charges $P_{\mathit{1st\ block}} = P(Q_{\mathit{max\ 1st\ block}})$.



Second-Degree Price Discrimination

Under block pricing.

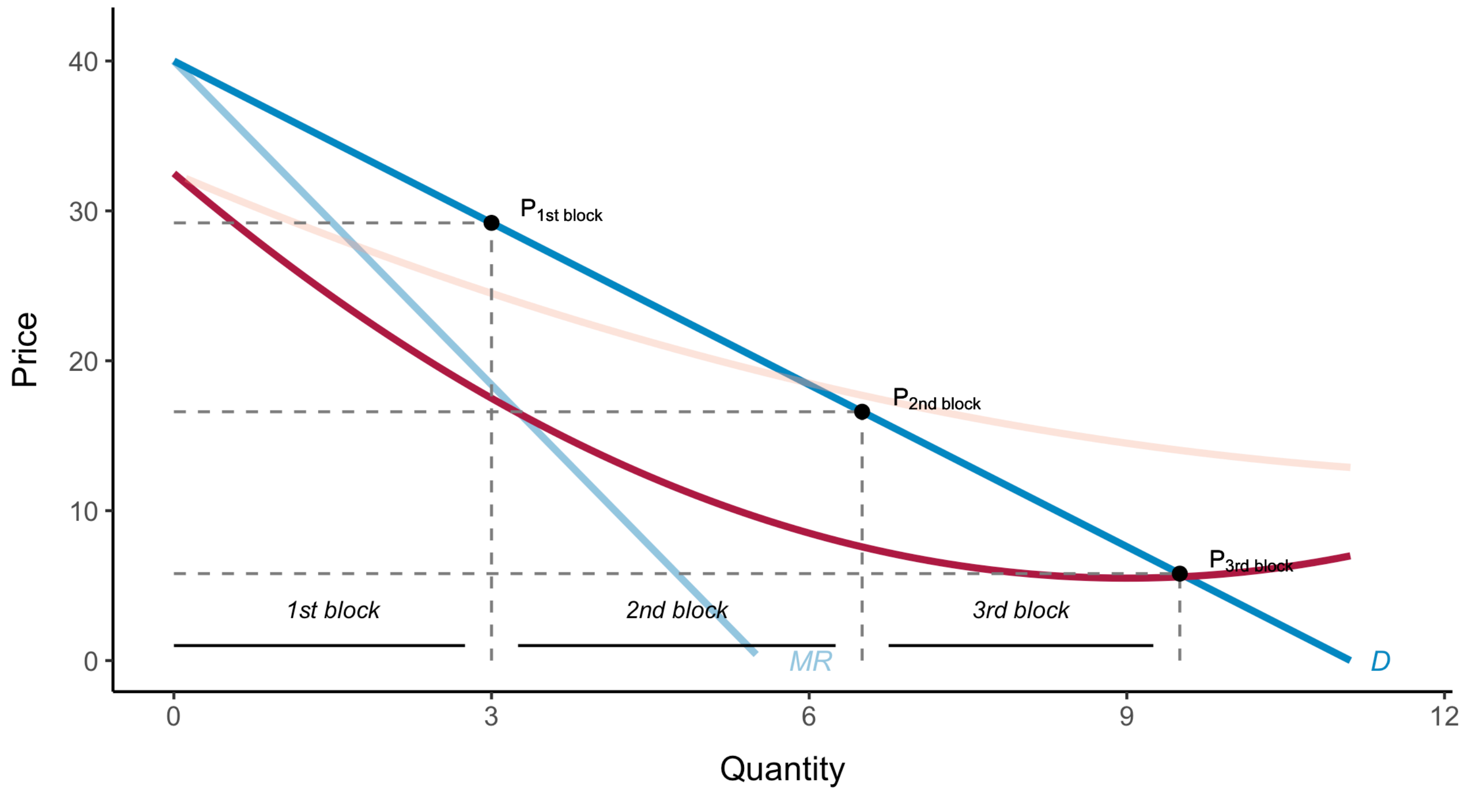
For all quantities in 2nd block $Q \in (3, 6.5]$, firm charges $P_{2nd\ block} = P(Q_{max\ 2nd\ block})$.



Second-Degree Price Discrimination

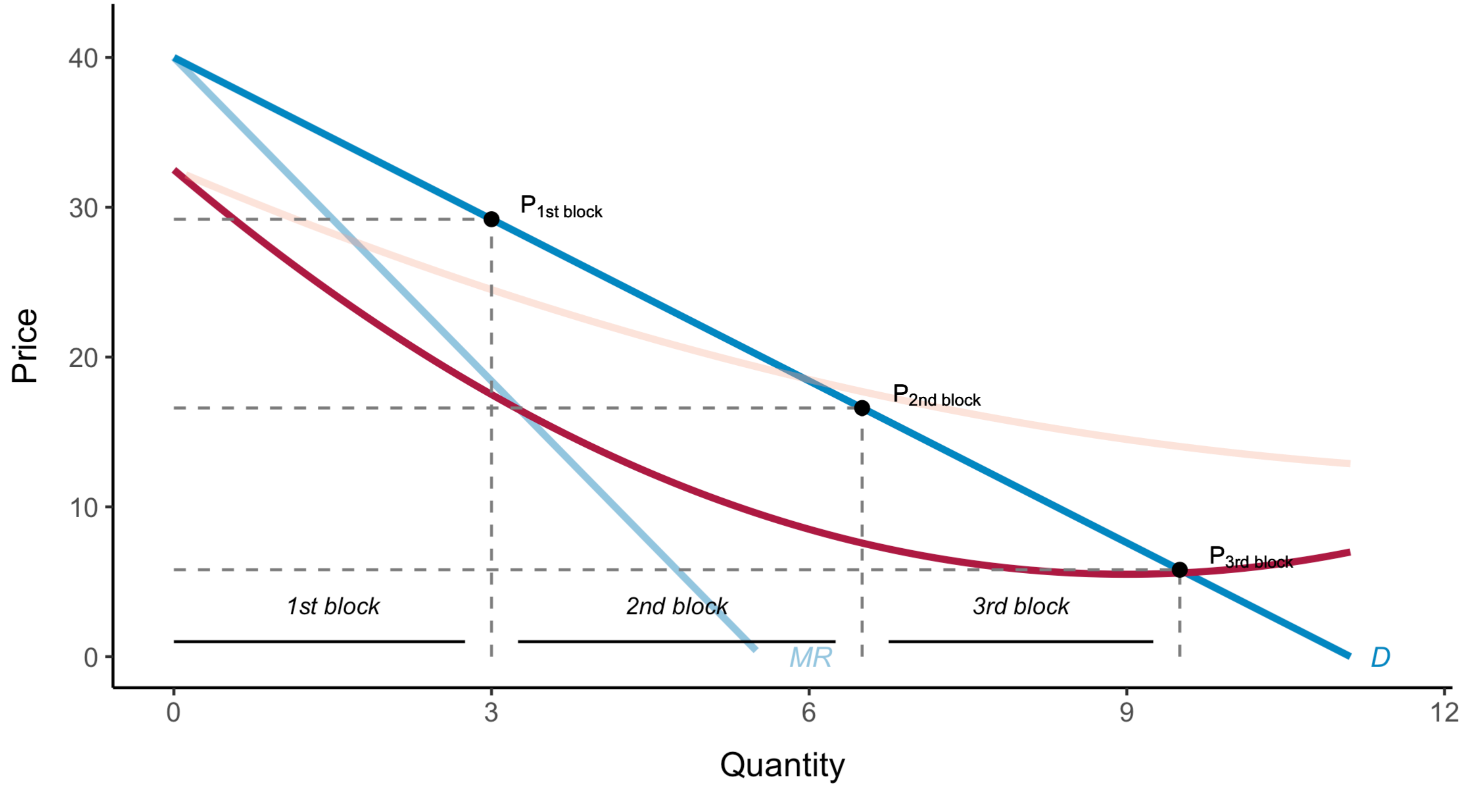
Under block pricing.

For all quantities in 3rd block $Q \in (6.5, 9.5]$, firm charges $P_{3rd\ block} = P(Q_{max\ 3rd\ block})$.



Second-Degree Price Discrimination

Second-degree price discrimination can then make consumers better off by expanding output and lowering cost.



Third-Degree Price Discrimination

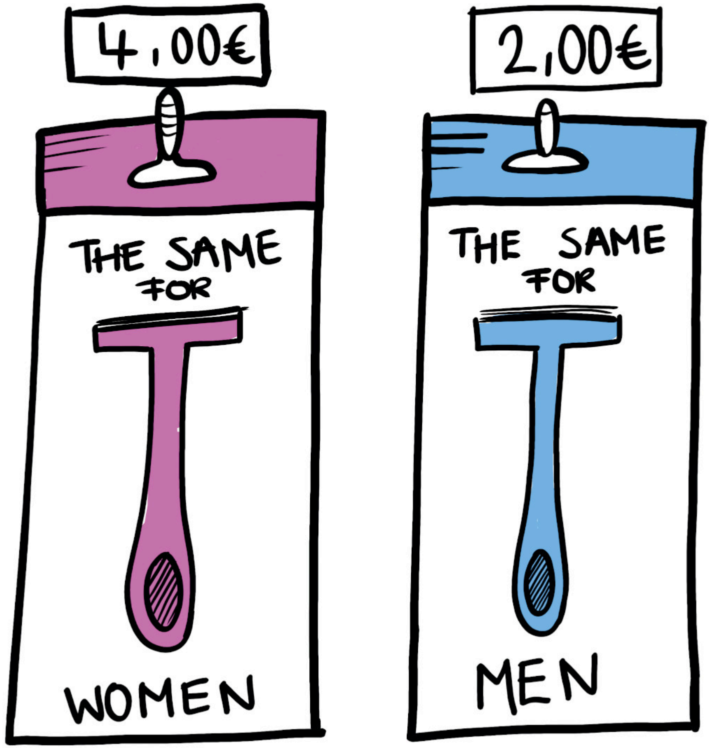
What is the most prevalent form of price discrimination?

Third-degree price discrimination: Practice of dividing consumers into two or more groups with separate demand curves and charging different prices to each group.

- regular versus “special” airline fares
- premium versus nonpremium brands of liquor,
- discounts to students and senior citizens

Third-Degree Price Discrimination

Creating consumer groups. Based on observable characteristics and identities.



Third-Degree Price Discrimination

Creating consumer groups. Based on observable characteristics and identities.

Third-Degree Price Discrimination Strategy:

1) Equalizing Marginal Revenues Across Groups

- Total output should be divided between the groups of customers so that marginal revenues (MR) for each group are equal.
- If MR differs, the firm can increase profit by shifting output between groups, adjusting prices accordingly

2) Setting Output Where MR = MC

- Total output should be set so that MR for each group equals marginal cost (MC).
- If MR exceeds MC, the firm can increase profit by expanding output; if MR is below MC, it should reduce output.

Third-Degree Price Discrimination

Suppose there are two groups: $G \in \{1, 2\}$.

Let P_1 be the price charged for the first group, and P_2 to the second.

Total output is: $Q_T = Q_1 + Q_2$.

Let $C(Q_T)$ be the total cost of producing output. Note that costs vary with total output, not with the output of each individual group.

Then, total profit is: $\Pi = P_1 \cdot Q_1 + P_2 \cdot Q_2 - C(Q_T)$

The firm should increase its sales to each group of consumers, Q_1 and Q_2, until the incremental profit from the last unit sold is zero.

$$\frac{\Delta\Pi}{\Delta Q_1} = \frac{\Delta(P_1 \cdot Q_1)}{\Delta Q_1} - \frac{\Delta C(Q_T)}{\Delta Q_1} = 0$$

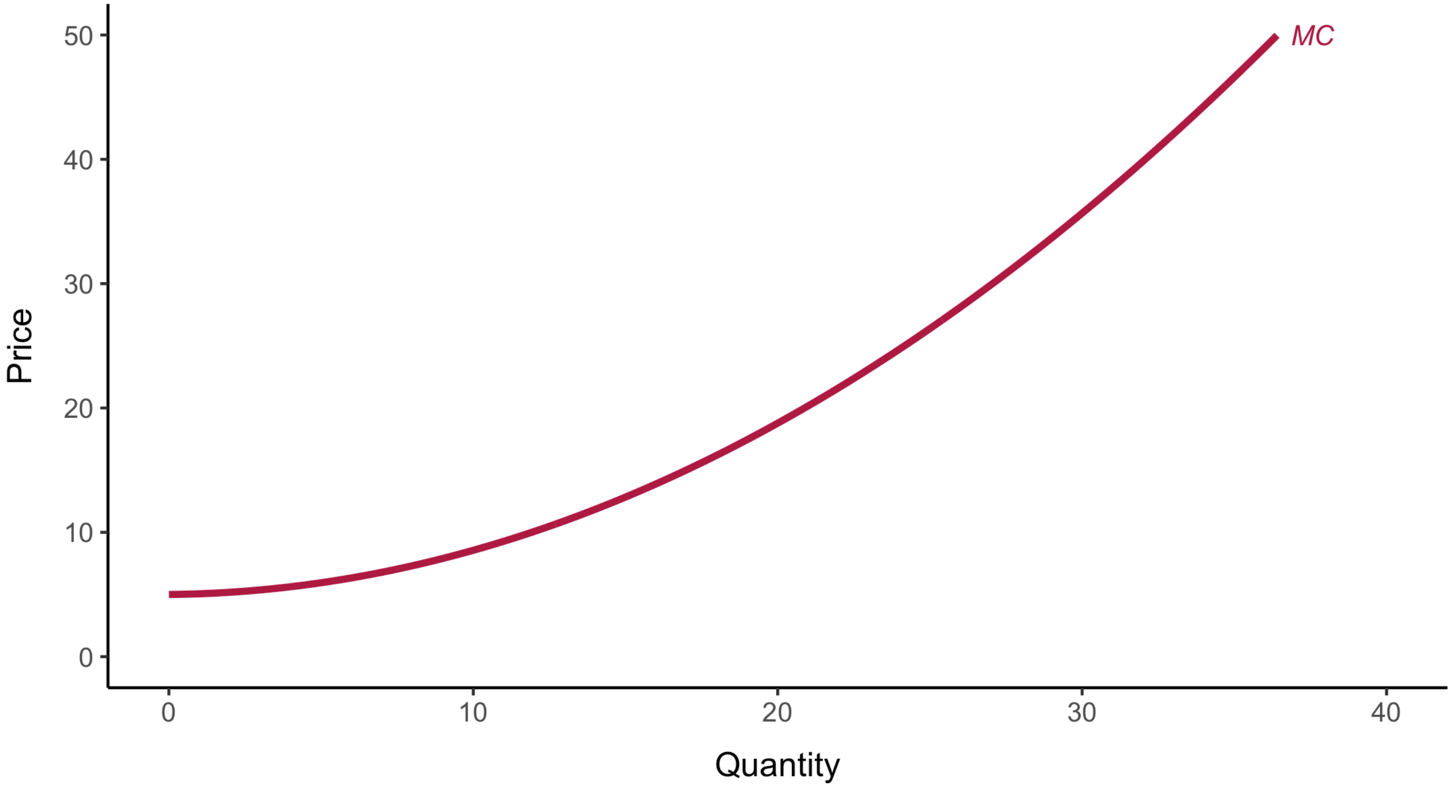
$$\frac{\Delta\Pi}{\Delta Q_2} = \frac{\Delta(P_2 \cdot Q_2)}{\Delta Q_2} - \frac{\Delta C(Q_T)}{\Delta Q_2} = 0$$

Optimality condition:

$$MR_1 = MR_2 = MC$$

Third-Degree Price Discrimination

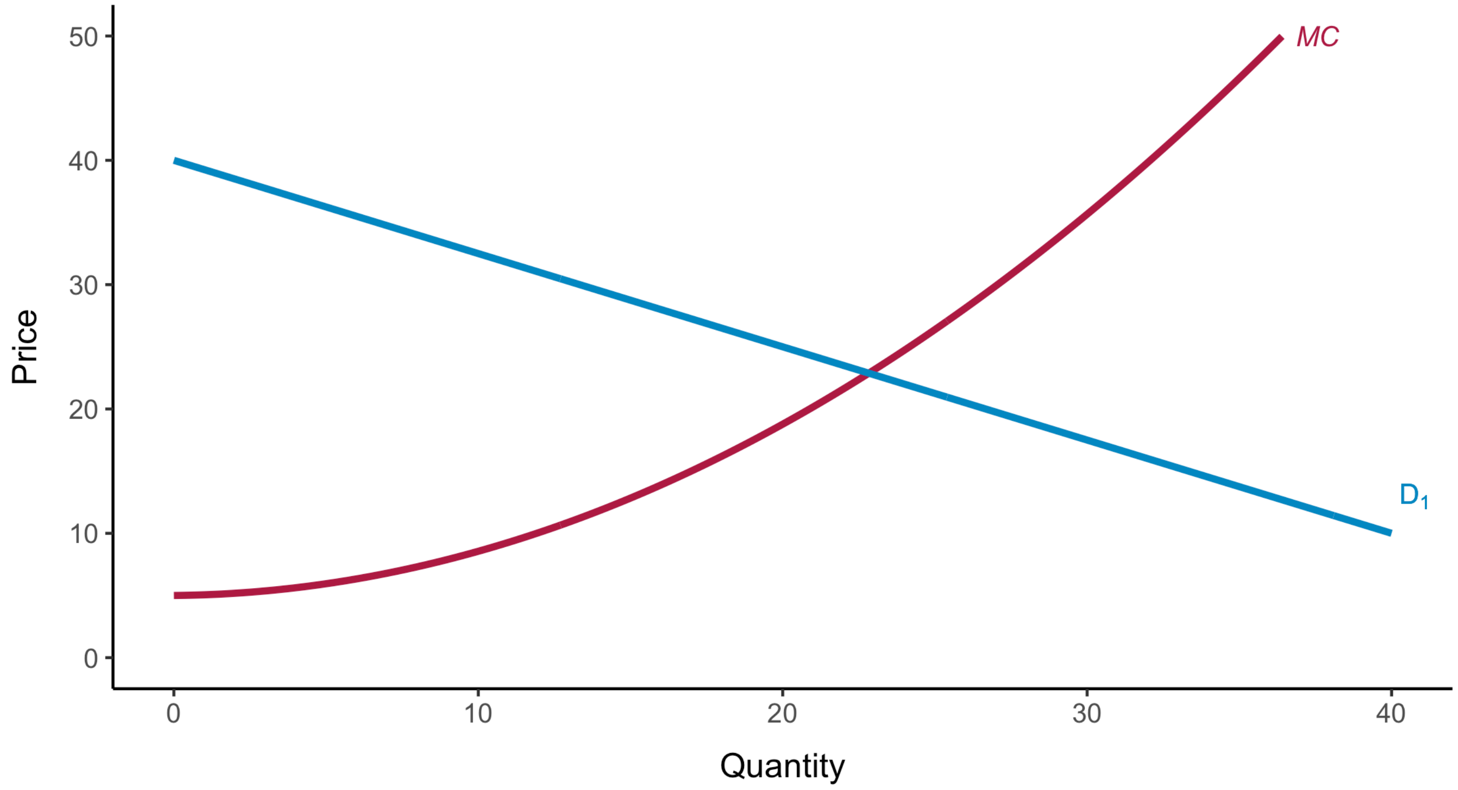
Suposse a firm with the following total cost function: $C(Q_T) = 50 \cdot Q_T + \frac{1}{90} \cdot Q_T^2 + \frac{1}{90} \cdot Q_T^3$. Thus, $MC(Q_T) = 50 + \frac{1}{45} \cdot Q_T + \frac{1}{30} \cdot Q_T^2$.



Third-Degree Price Discrimination

Suppose the firm can identify two groups and their respective demands.

$$P(Q_1) = 40 - 0.75 \cdot Q_1$$

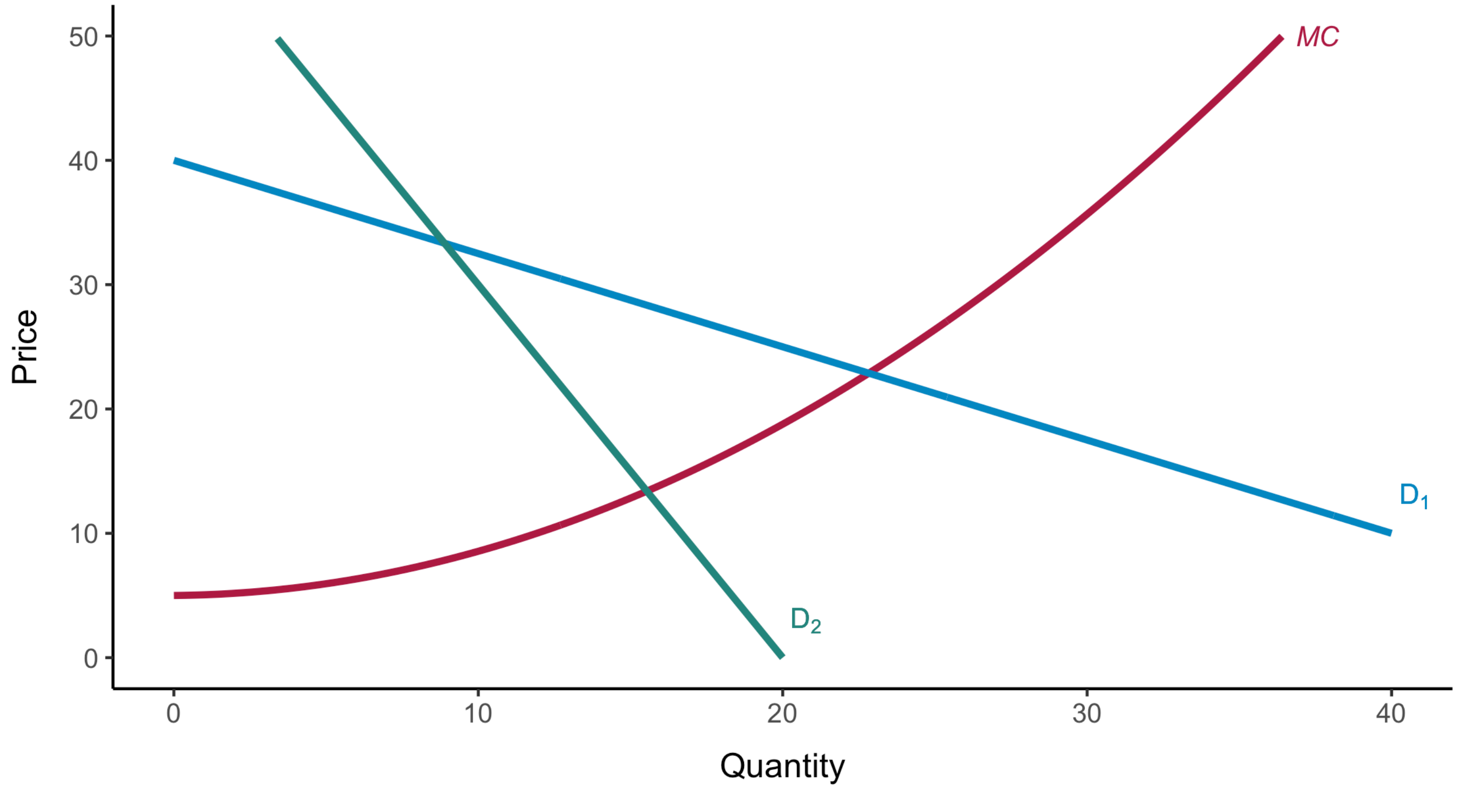


Third-Degree Price Discrimination

Suppose the firm can identify two groups and their respective demands.

$P(Q_1) = 40 - 0.75 \cdot Q_1$

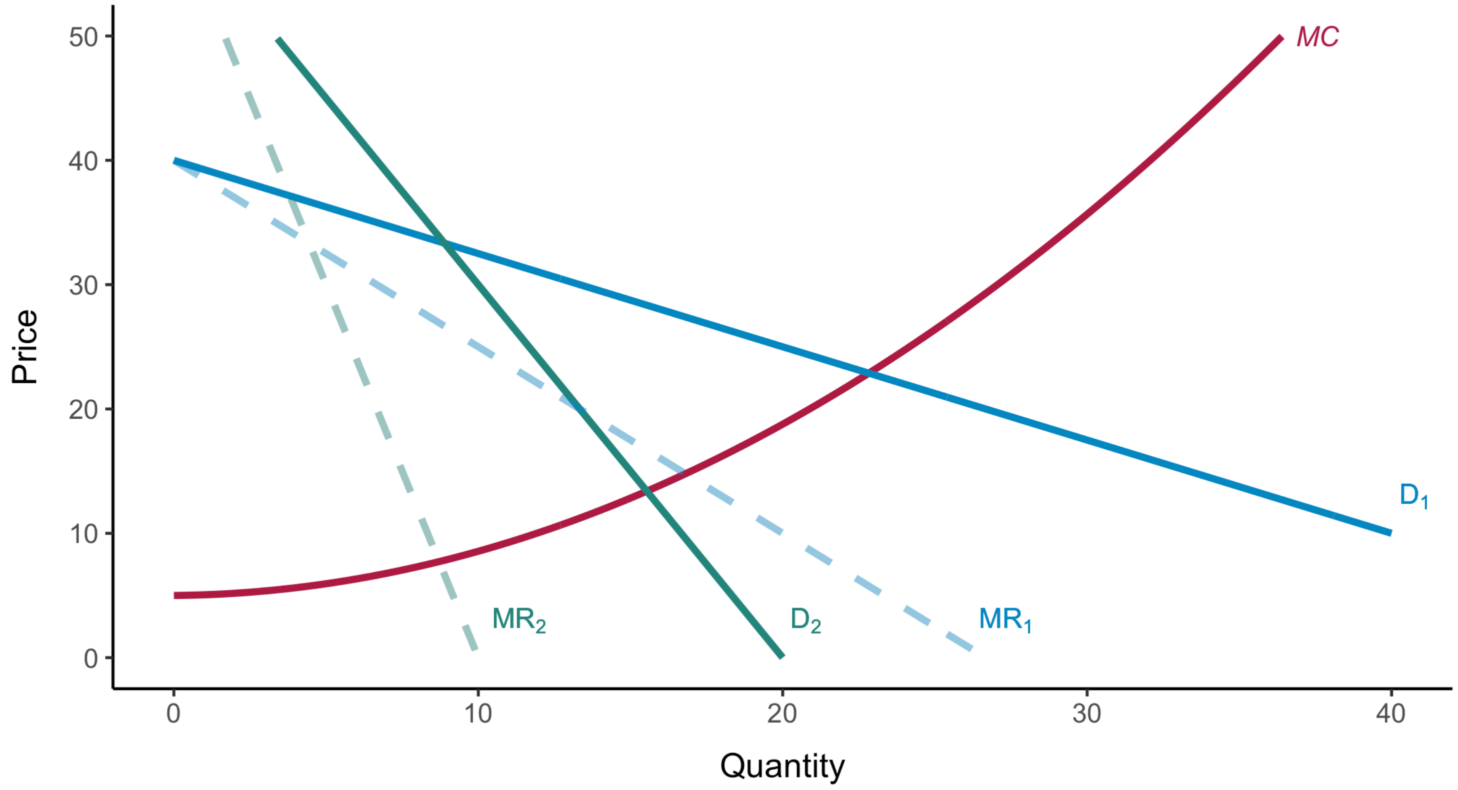
$P(Q_2) = 60 - 3 \cdot Q_2$



Third-Degree Price Discrimination

Suppose the firm can identify two groups and their respective demands.

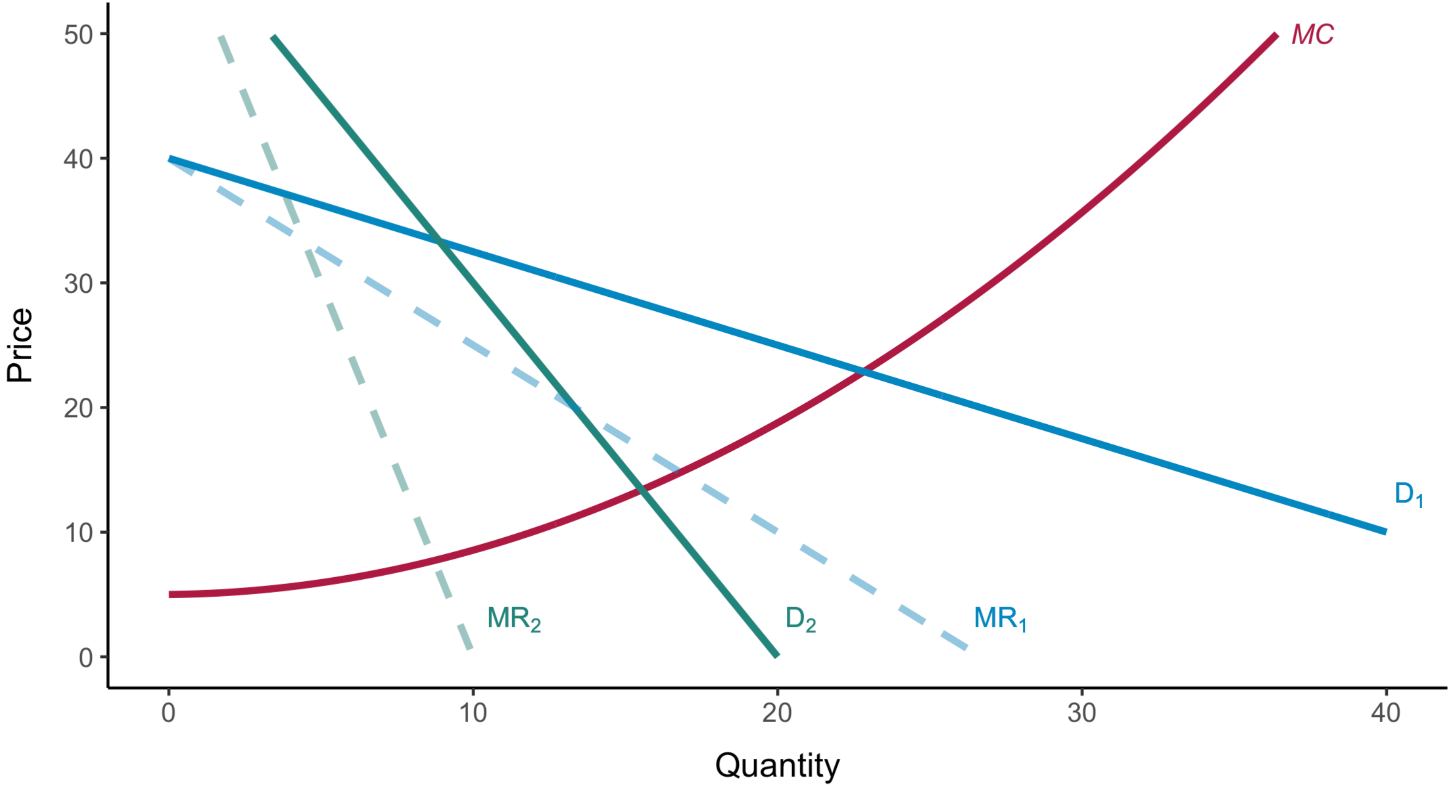
$$P(Q_1) = 40 - 0.75 \cdot Q_1 \;\Rightarrow\; MR(Q_1) = 40 - 1.5 \cdot Q_1 \;\; P(Q_2) = 60 - 3 \cdot Q_2 \;\Rightarrow\; MR(Q_2) = 60 - 6 \cdot Q_1$$



Third-Degree Price Discrimination

How to define the total quantity produced?

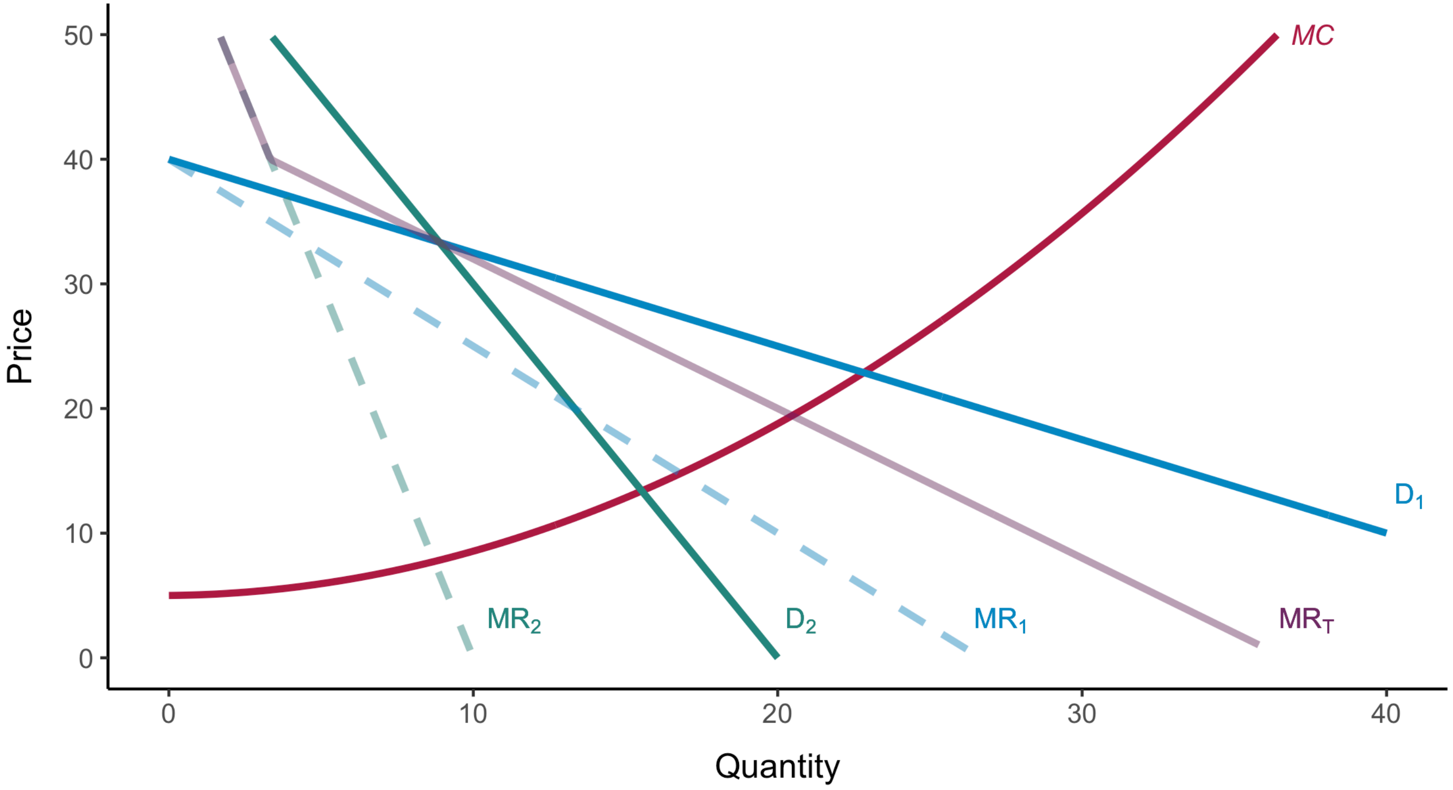
At the optimal point, we need a reference line in the price axis where the following conditions hold: $MR_1 = MR_2 = MC$.



Third-Degree Price Discrimination

Horizontally summing MR curves into MR_T: adding the quantities that different consumer groups are willing to buy at the same price.

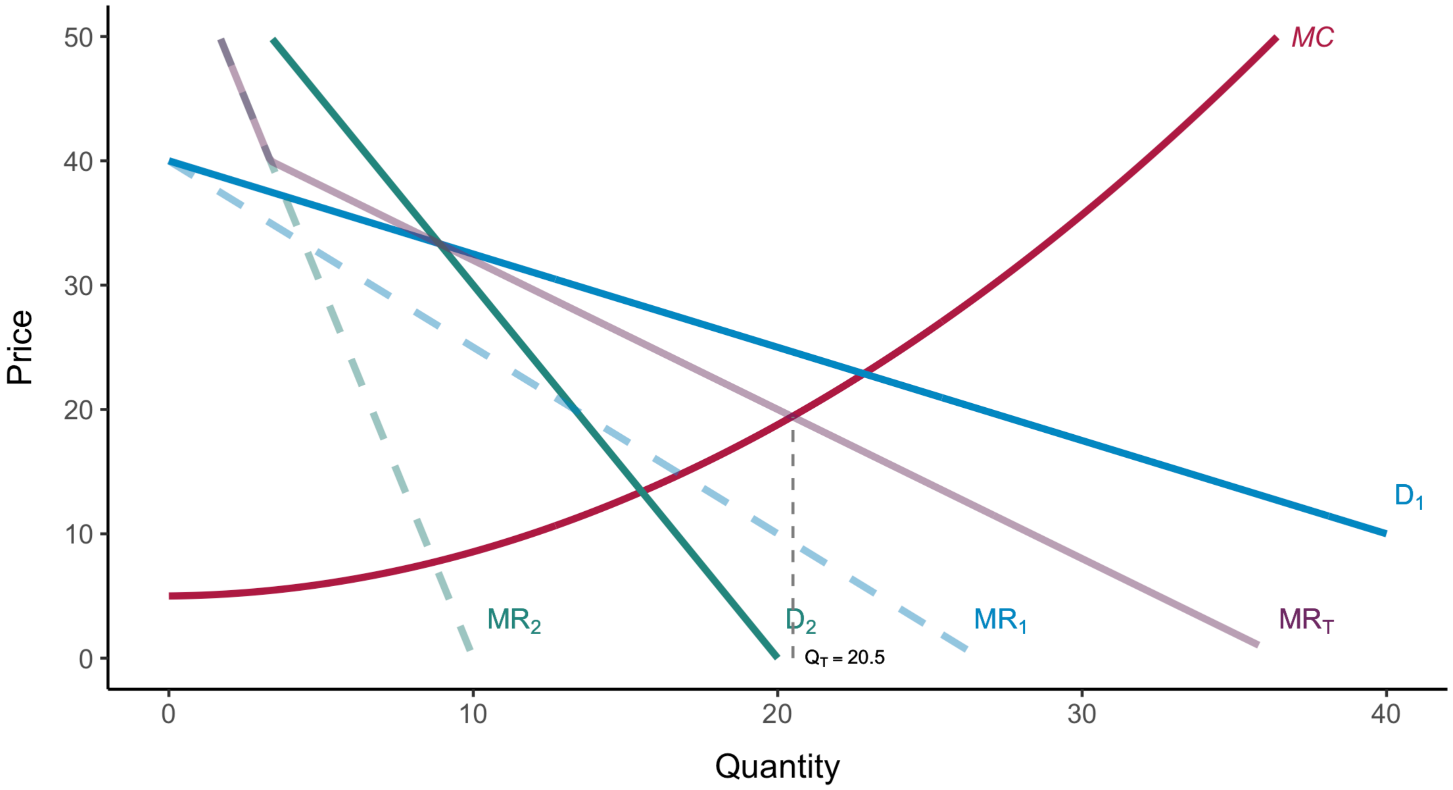
Instead of viewing MR as a function of quantity, we determine the total quantity demanded by both groups at each price level.



Third-Degree Price Discrimination

First, find Q_T such that $MR_T = MC$.

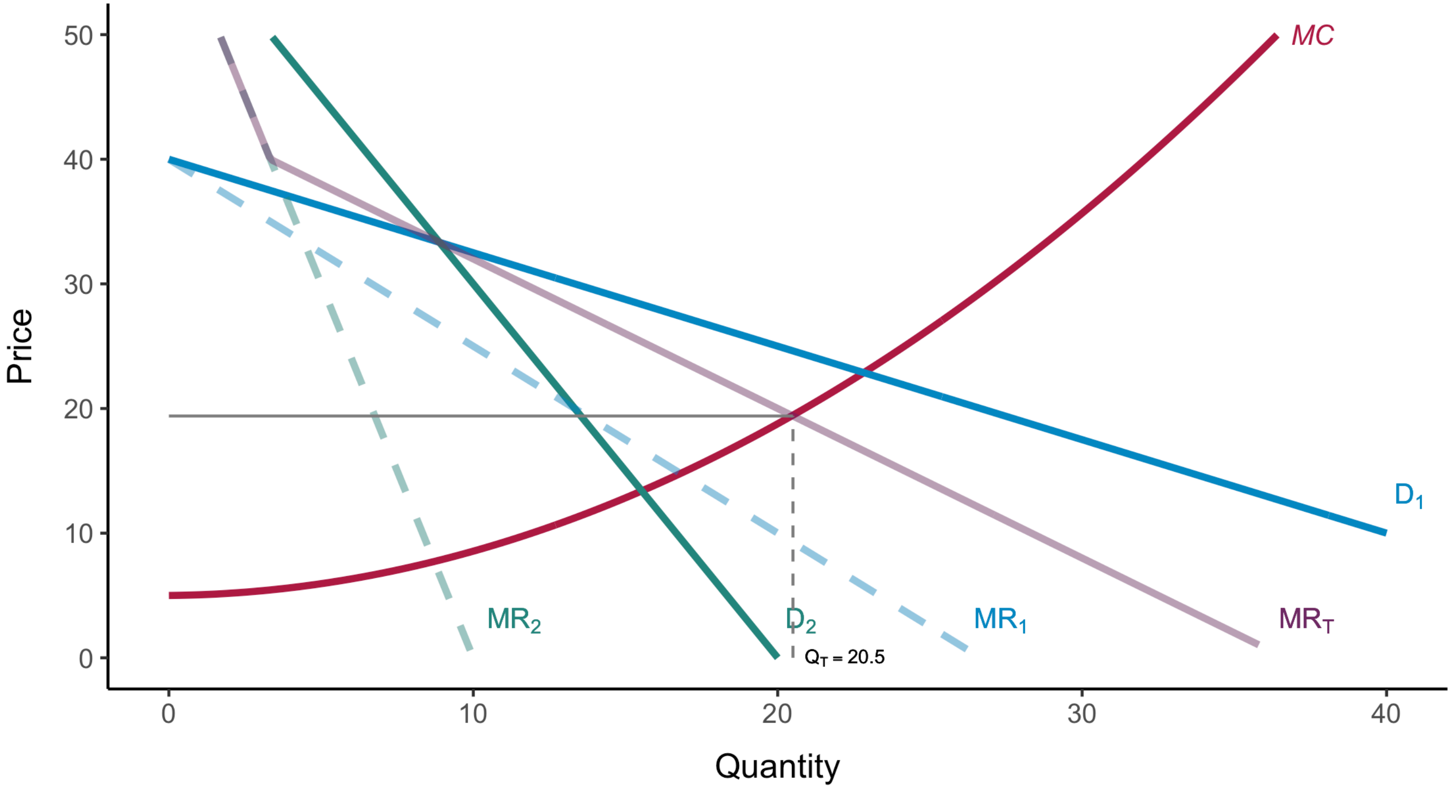
In this case, $Q_T = 20.5$.



Third-Degree Price Discrimination

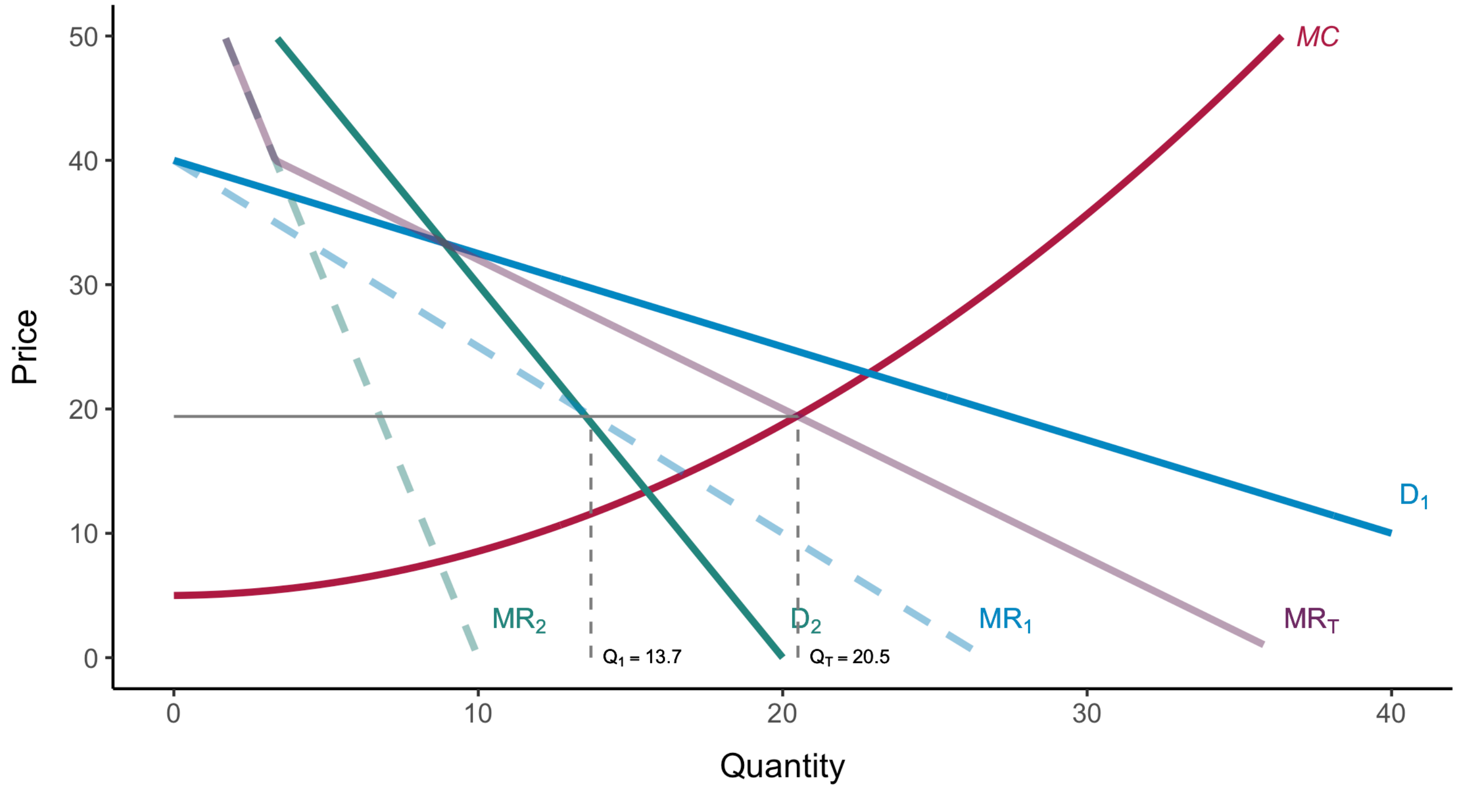
Then, find a reference number in price-axis such that $MR_1 = MR_2 = MC$.

For $Q_T = 20.5, \Rightarrow 19.4 = MR_1 = MR_2 = MC$



Third-Degree Price Discrimination

Find the quantities Q_1 and Q_2.



Third-Degree Price Discrimination

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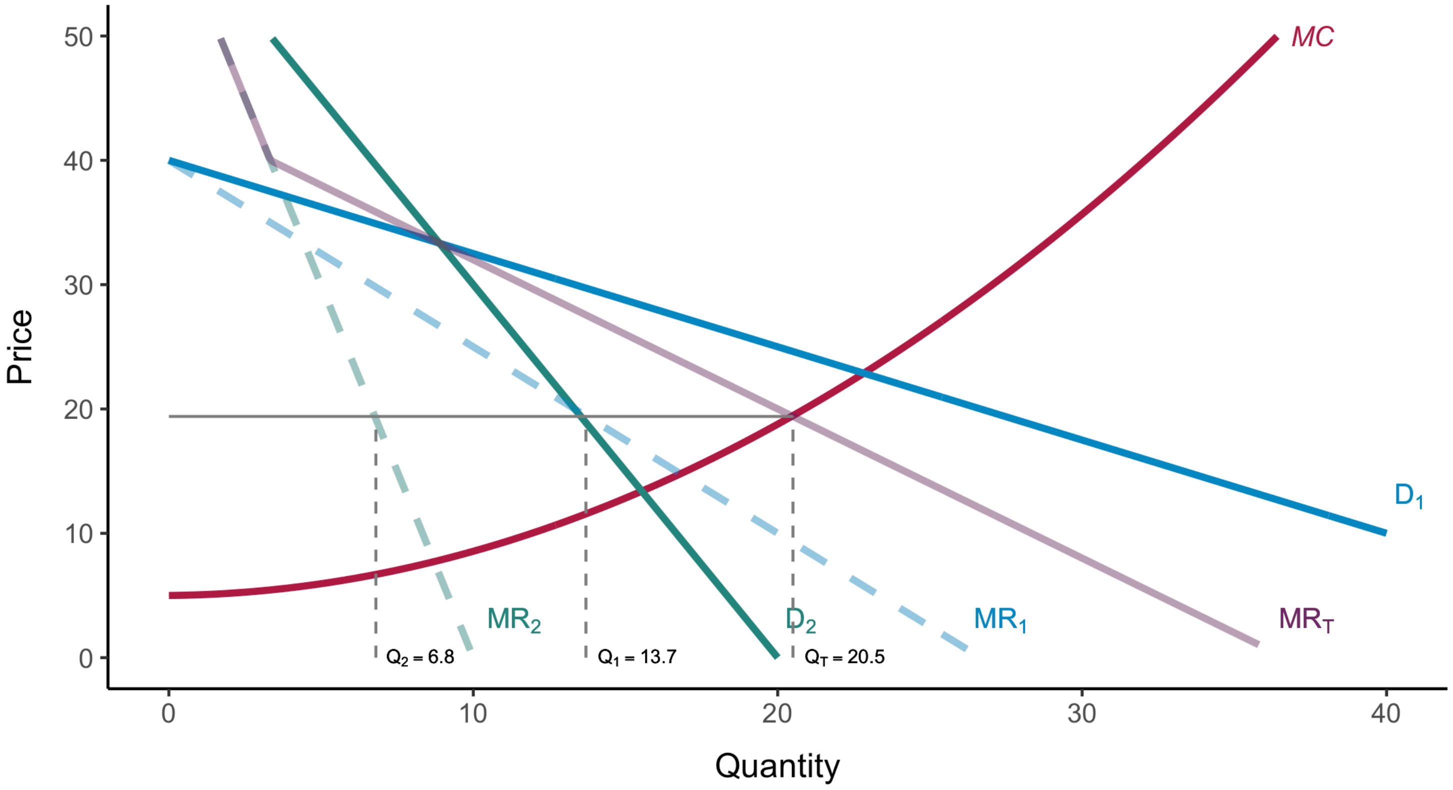
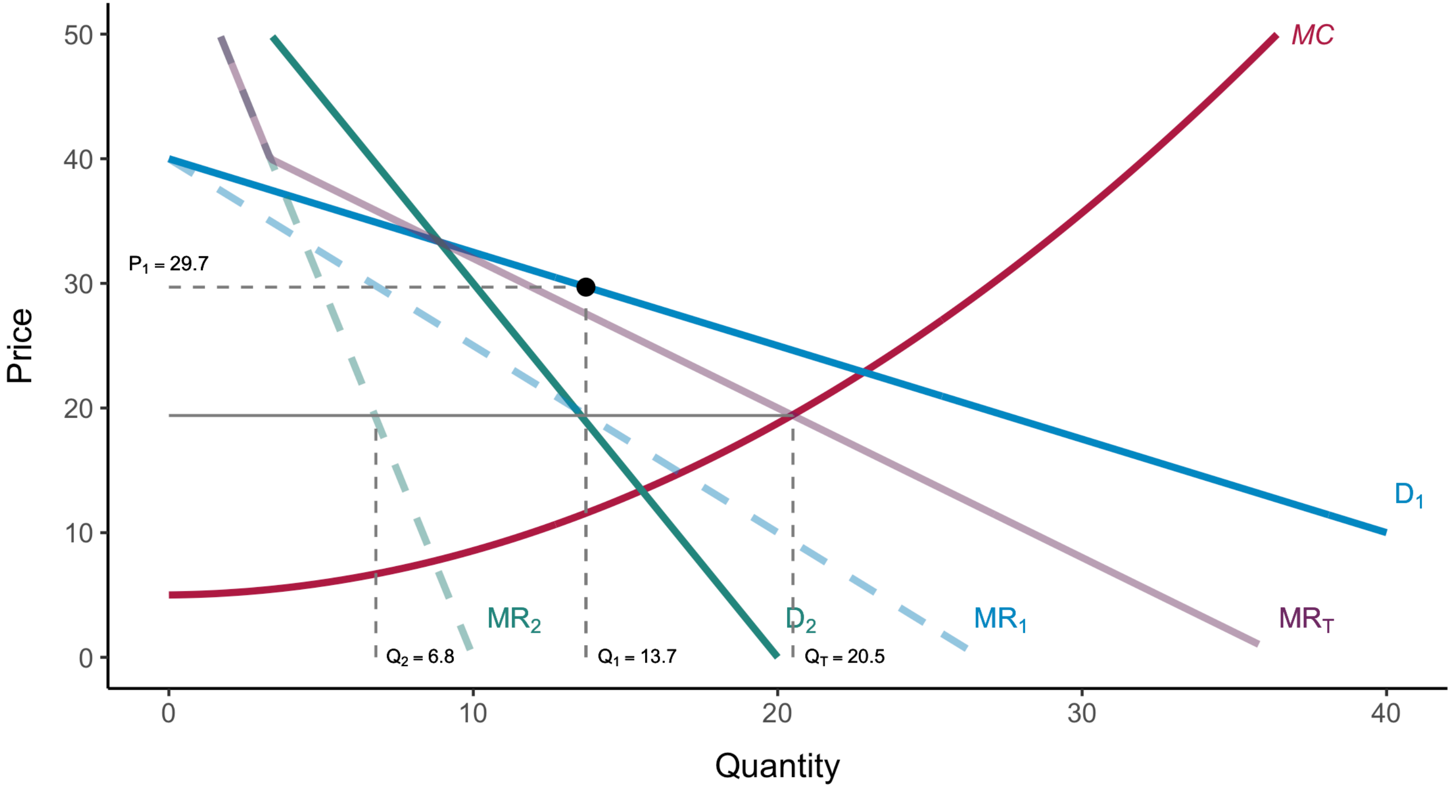


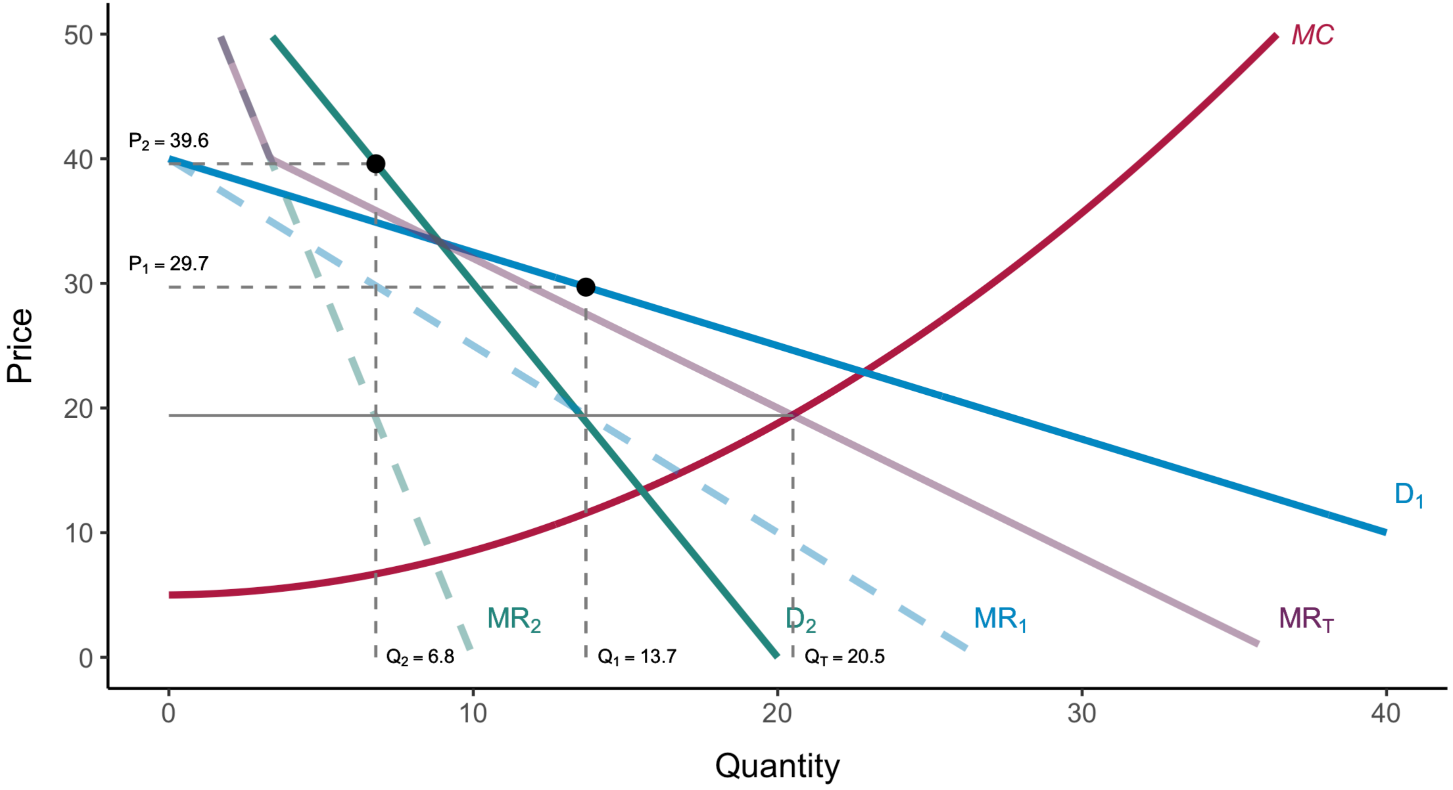
Third-Degree Price Discrimination

Find the respective prices P_1 and P_2.



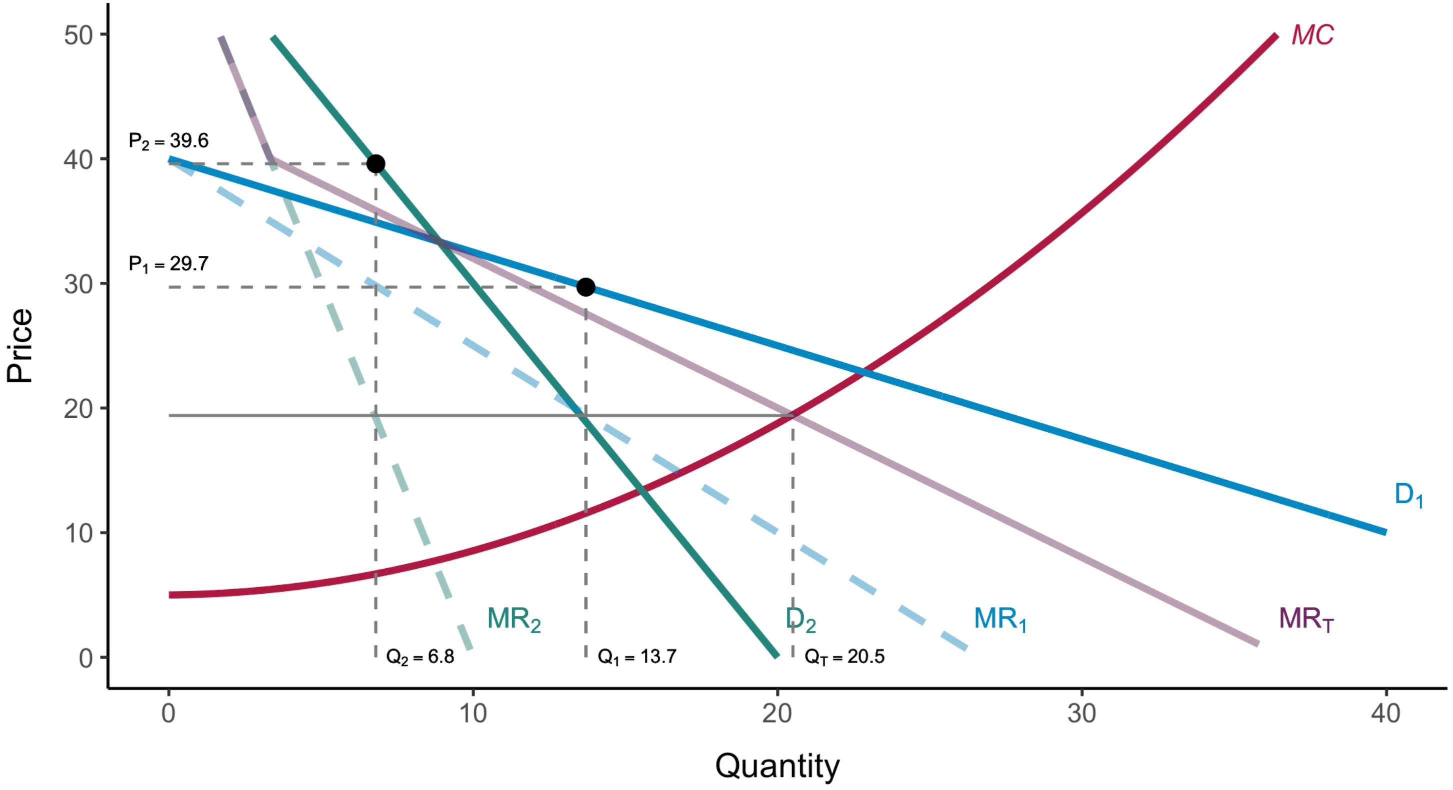
Third-Degree Price Discrimination

Find the respective prices P_1 and P_2.



Third-Degree Price Discrimination

Note $P_2 > P_1$. Why?



Third-Degree Price Discrimination

Note $P_2 > P_1$. Why? Elasticities.

Recall: $MR = P(1 + \frac{1}{\varepsilon_d})$

$$MR_1 = P_1(1 + \frac{1}{\varepsilon_{d1}}) \qquad MR_2 = P_2(1 + \frac{1}{\varepsilon_{d2}})$$

If the optimality condition is $MR_1 = MR_2$:

$$P_1(1 + \frac{1}{\varepsilon_{d1}}) = P_2(1 + \frac{1}{\varepsilon_{d2}})$$

$$\frac{P_1}{P_2} = \frac{1 + \frac{1}{\varepsilon_{d2}}}{1 + \frac{1}{\varepsilon_{d1}}}$$

Third-Degree Price Discrimination

Recall $\varepsilon_d = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$. Thus, in our example, with $P_1 = 29.7$ and $P_2 = 39.6$:

$$P_1 = 40 - 0.75 \cdot Q_1 \qquad\qquad P_2 = 60 - 3 \cdot Q_2$$

$$\Rightarrow \ \Delta P_1 = -0.75 \cdot \Delta Q_1 \qquad\qquad \Rightarrow \ \Delta P_2 = -3 \cdot \Rightarrow \ \Delta Q_2$$

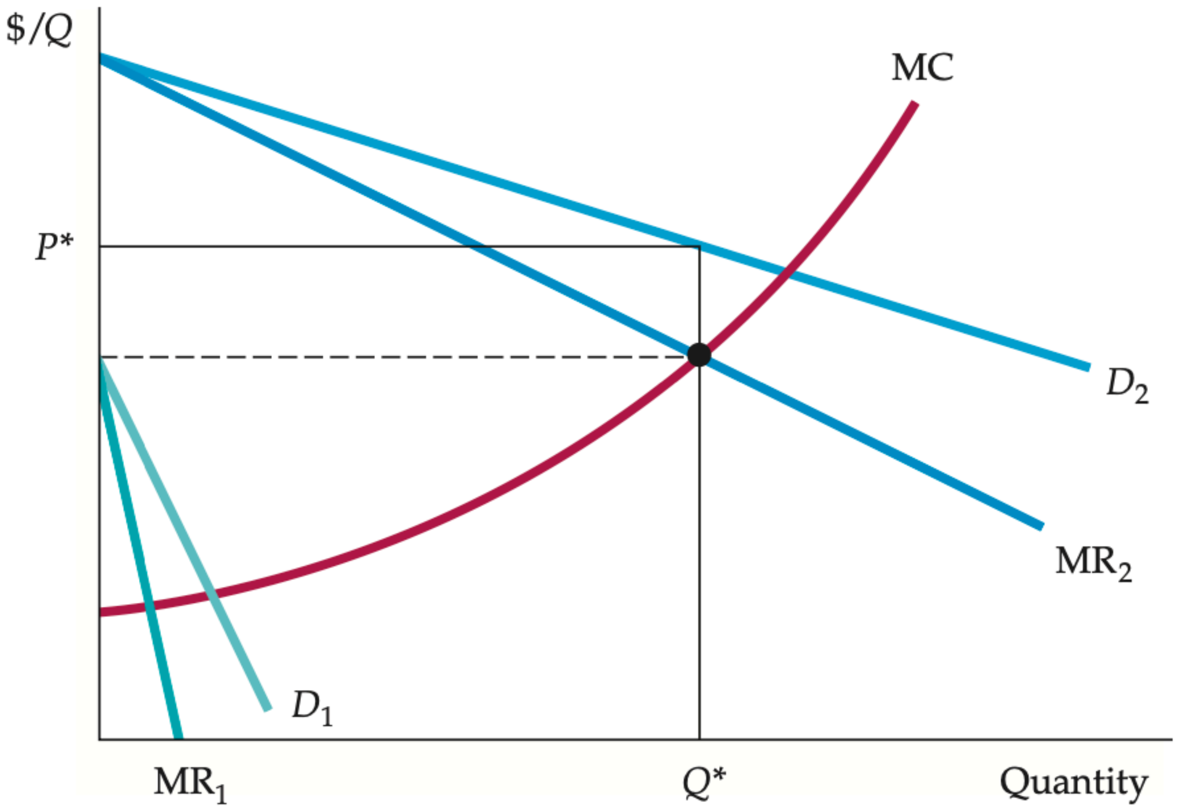
$$\iff \ \frac{\Delta P_1}{\Delta Q_1} = -0.75 \qquad\qquad \iff \ \frac{\Delta P_2}{\Delta Q_2} = -3$$

$$\Rightarrow \ \frac{1}{\varepsilon_{d1}} = -0.75 \cdot \frac{29.7}{13.7} \approx -0.35 \qquad\qquad \Rightarrow \ \frac{1}{\varepsilon_{d2}} = -3 \cdot \frac{39.6}{6.8} \approx -0.515$$

$$\frac{P_1}{P_2} = \frac{1 + \frac{1}{\varepsilon_{d2}}}{1 + \frac{1}{\varepsilon_{d1}}} \ \Rightarrow \ \frac{1 - 0.515}{1 - 0.35} \approx 0.75$$

Third-Degree Price Discrimination

It may not pay to sell to both groups of consumers if marginal cost is rising and one group is not willing to pay much for the product.



Intertemporal Price Discrimination and Peak-Load Pricing

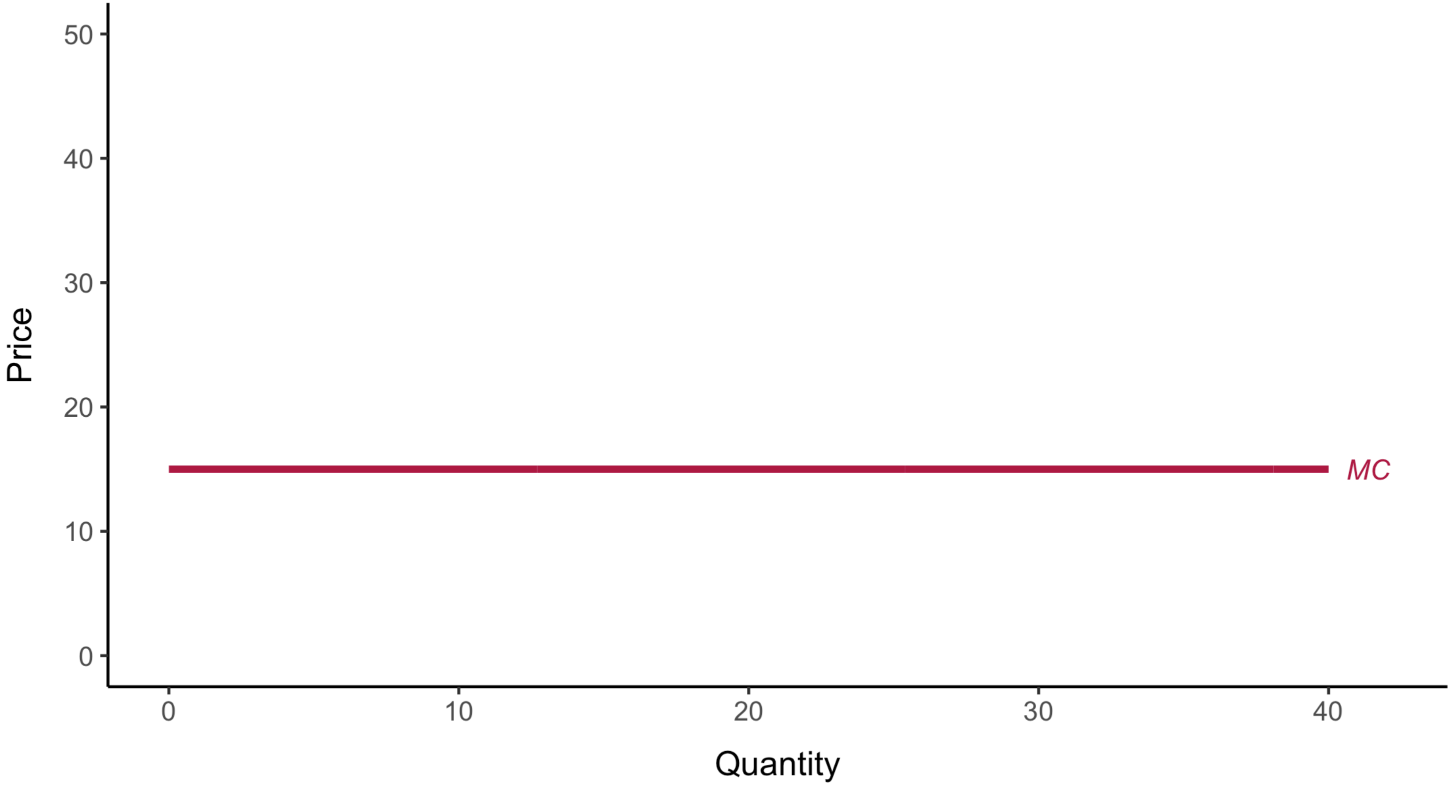
Intertemporal Price Discrimination

Divide consumers into high-demand and low-demand groups by charging a price that is high at first but falls later.

- New technological equipments (i.e. cellphones, computers)
- Movies and books
- Apparel

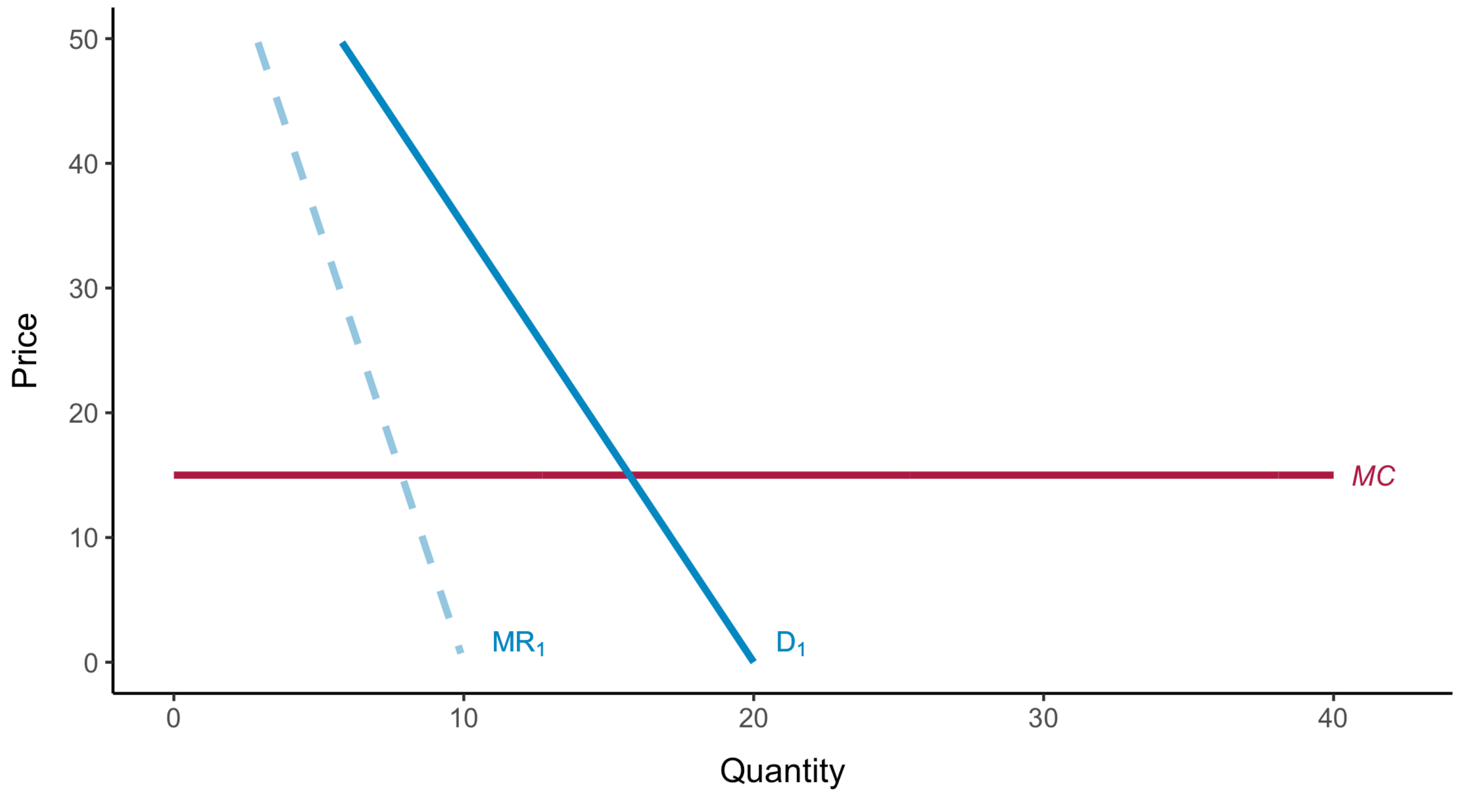
Intertemporal Price Discrimination

Suppose a firm with constant or decreasing MC. (What is the intuition?)



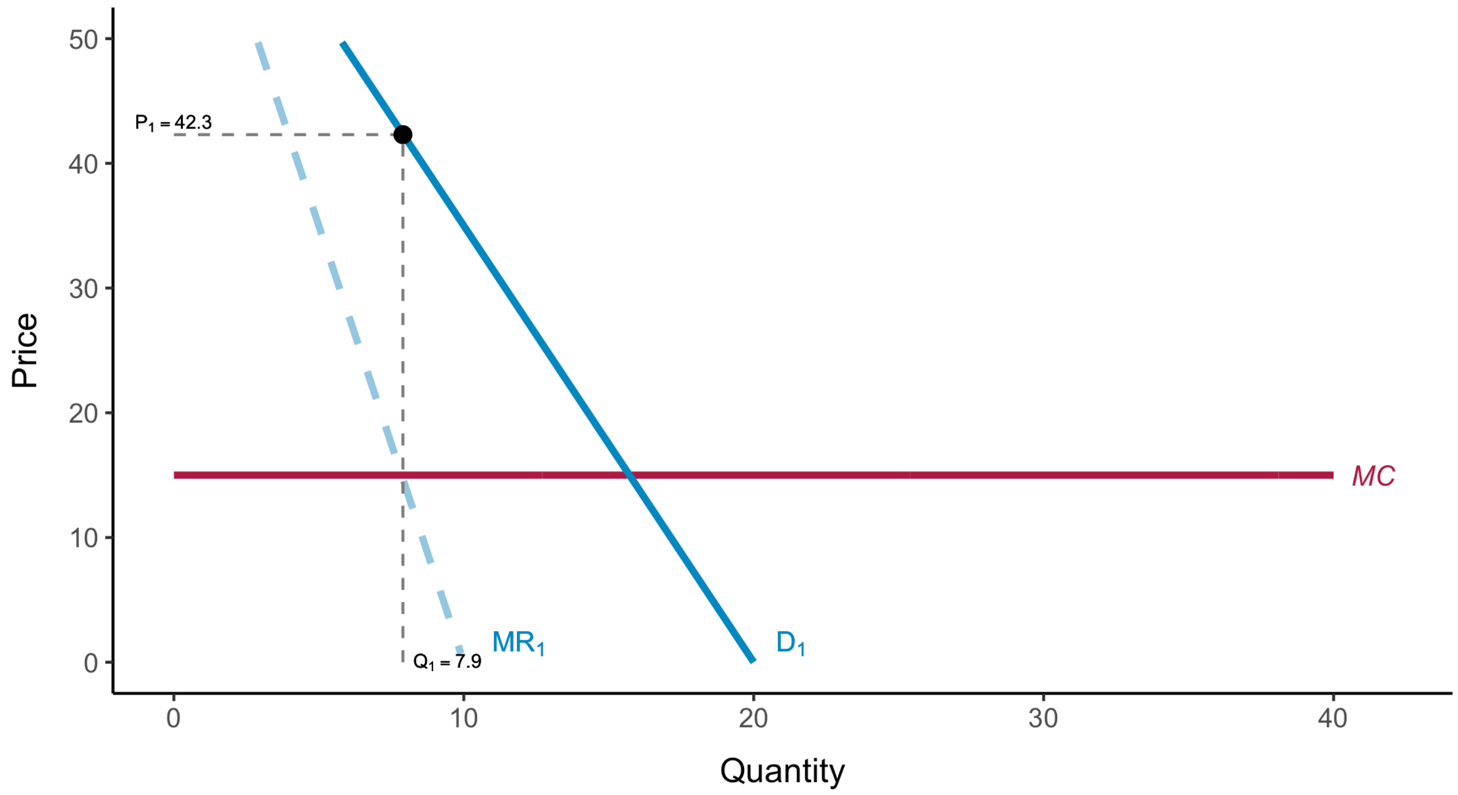
Intertemporal Price Discrimination

D_1: group of consumers who value the product highly and do not want to wait to buy it.



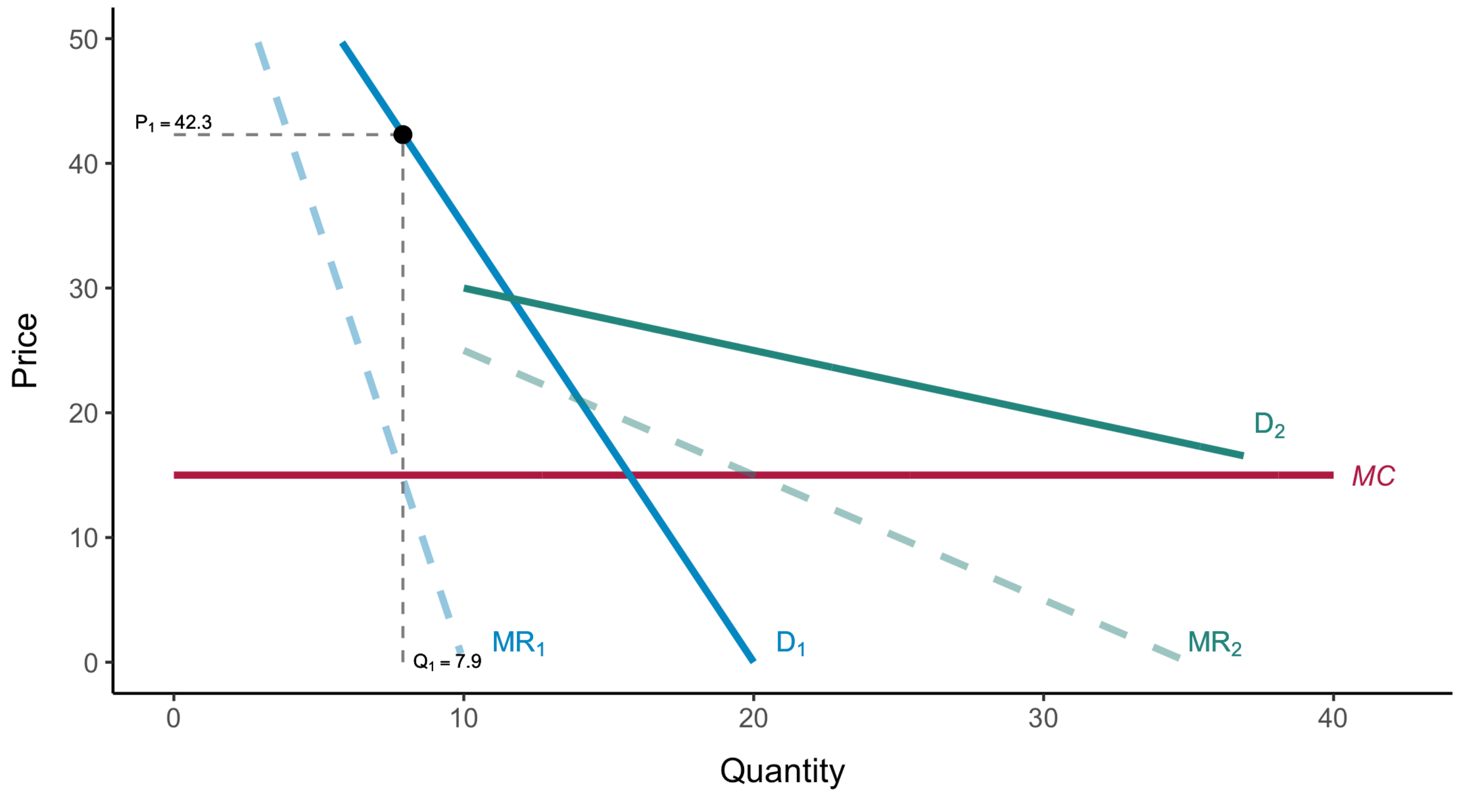
Intertemporal Price Discrimination

Offer the product initially at the high price P_1, selling mostly to consumers on demand curve D_1.



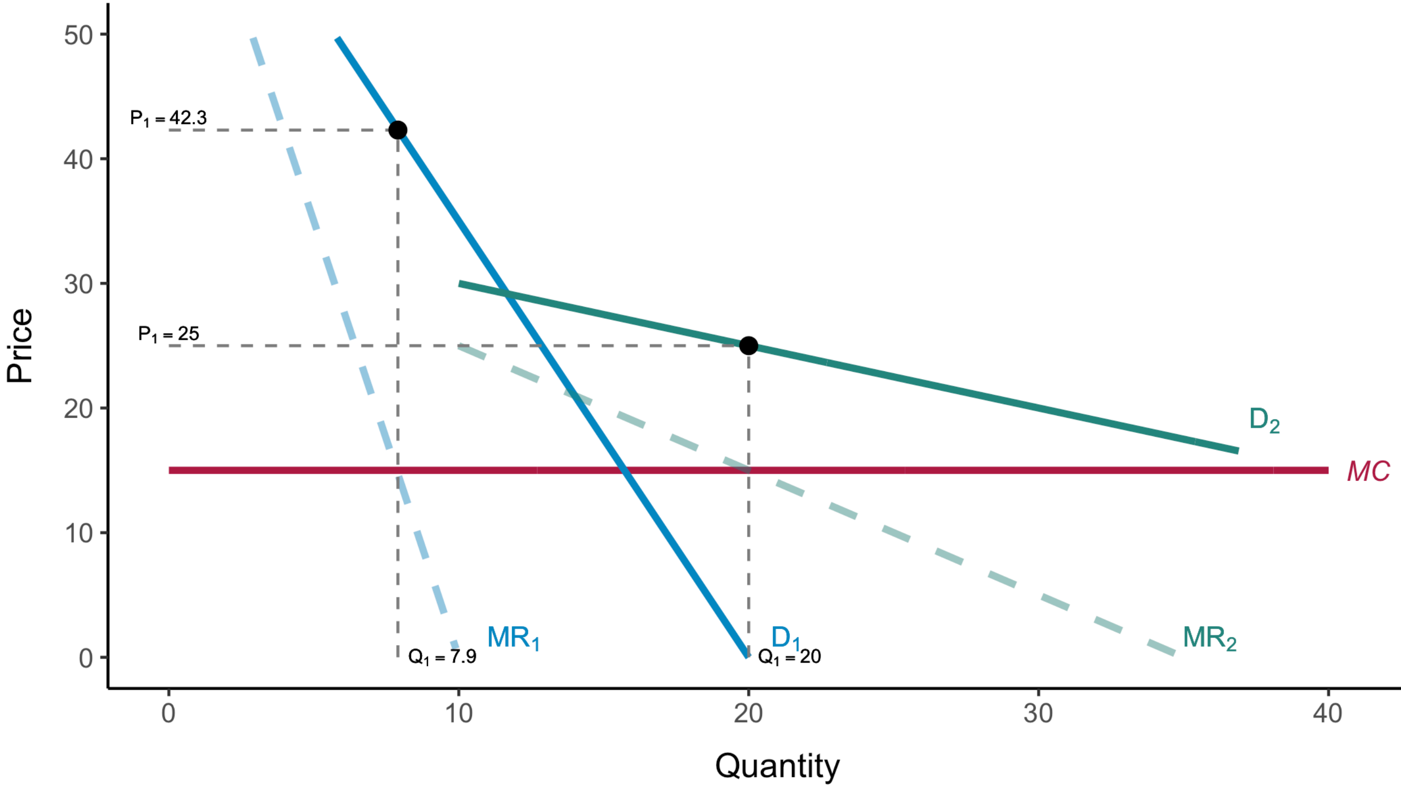
Intertemporal Price Discrimination

D_2: consumers who are more price-sensitive and likely to forgo the product if the price is high.



Intertemporal Price Discrimination

After D_1 has bought the product, the price is lowered to P_2, and sales are made to the larger group D_2.



Peak-load Pricing

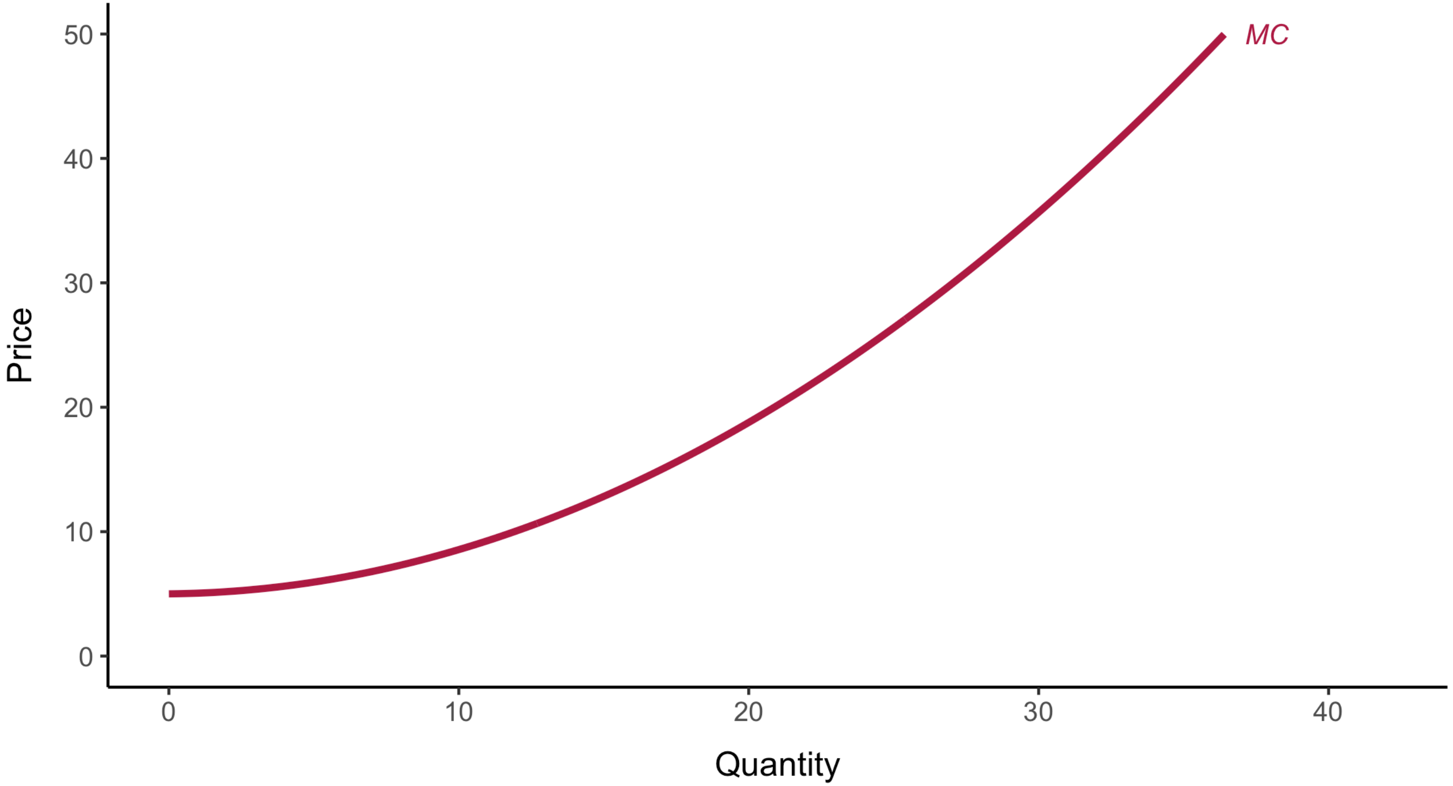
Charging different prices at different points in time.

The objective is to increase economic efficiency by charging consumers prices that are close to marginal cost.

- Uber, Bolt, etc.
- Hotels, Ski resorts, amusement parks
- Movie tickets

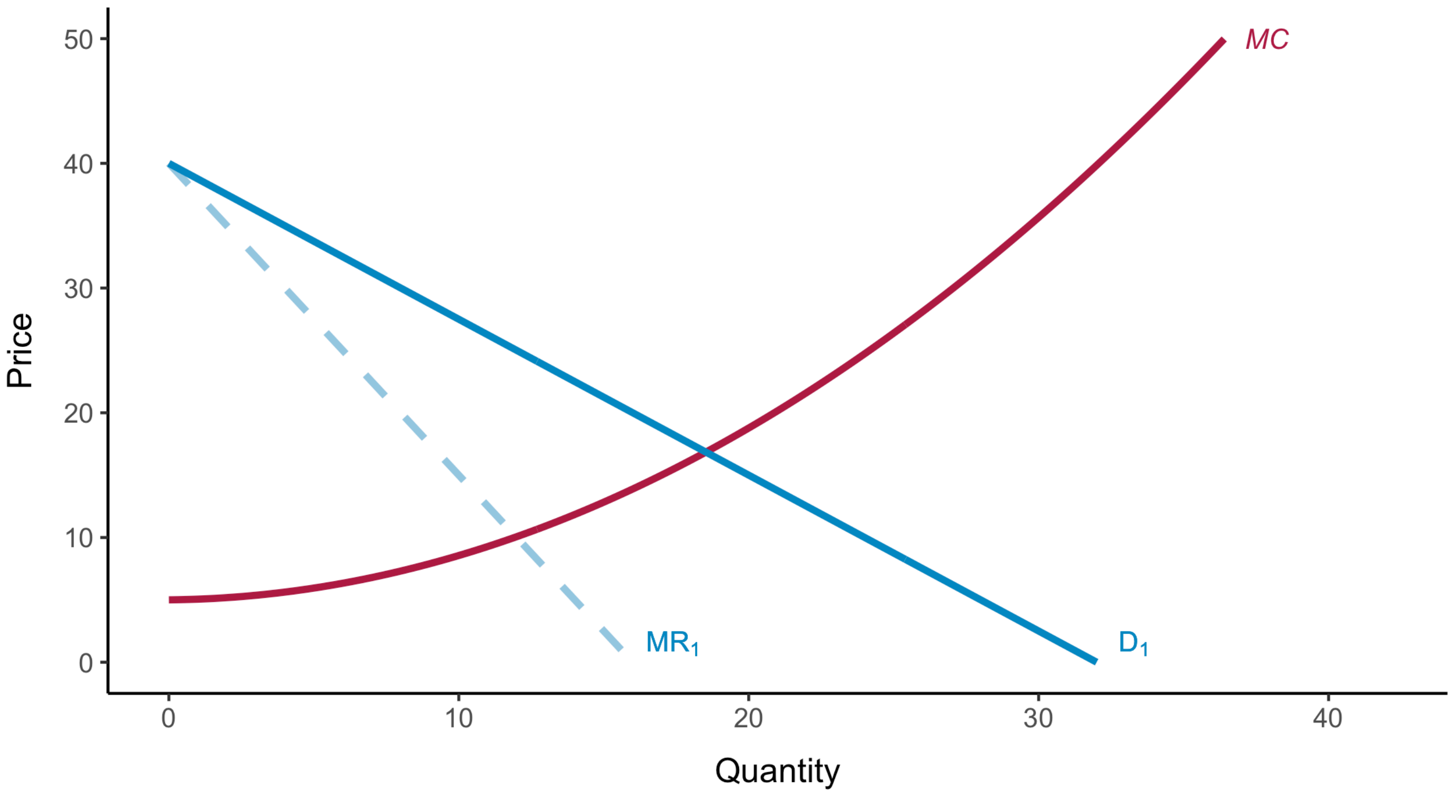
Peak-load Pricing

When demanded quantity peaks, marginal cost is also high because of capacity constraints.



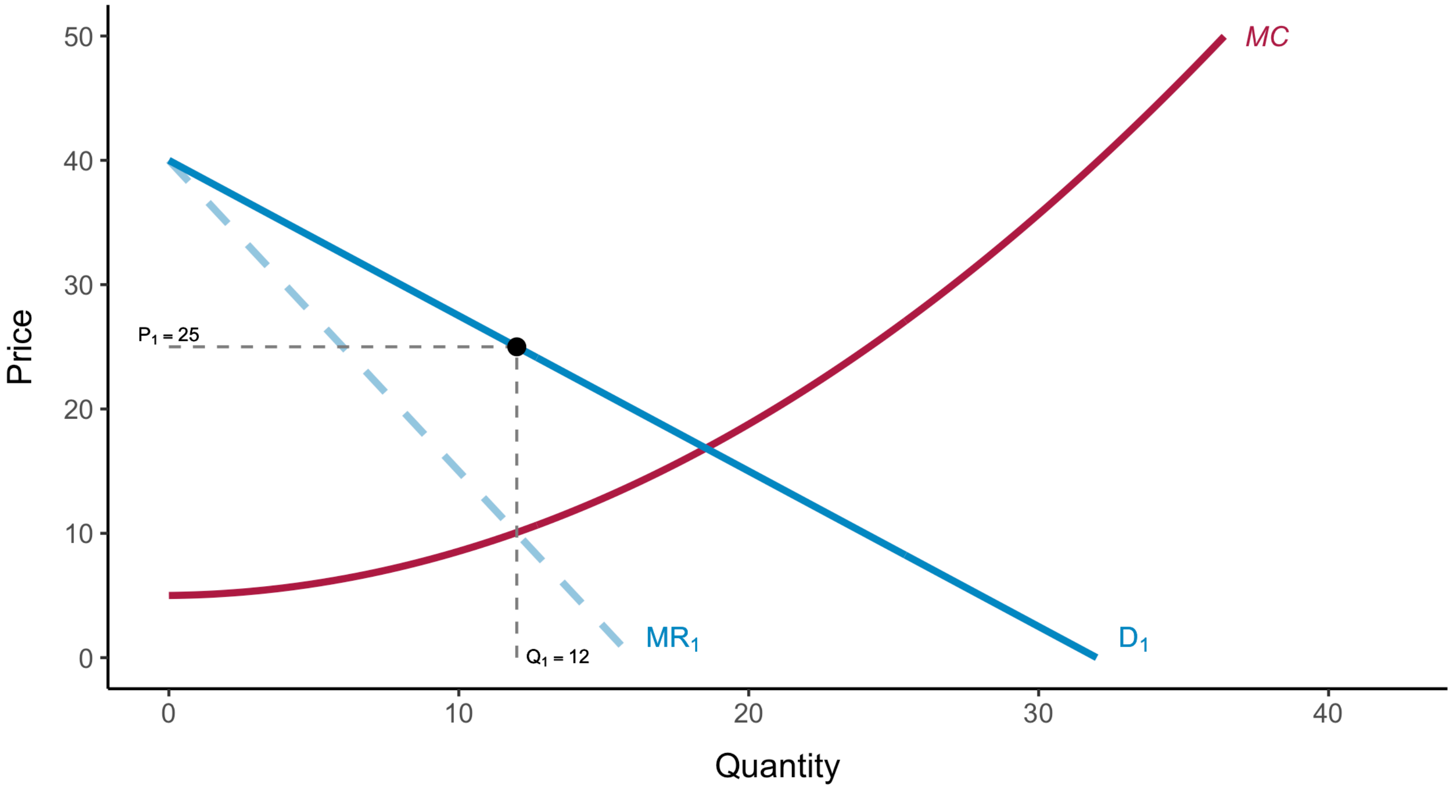
Peak-load Pricing

D_1: Demand during non-peak period.



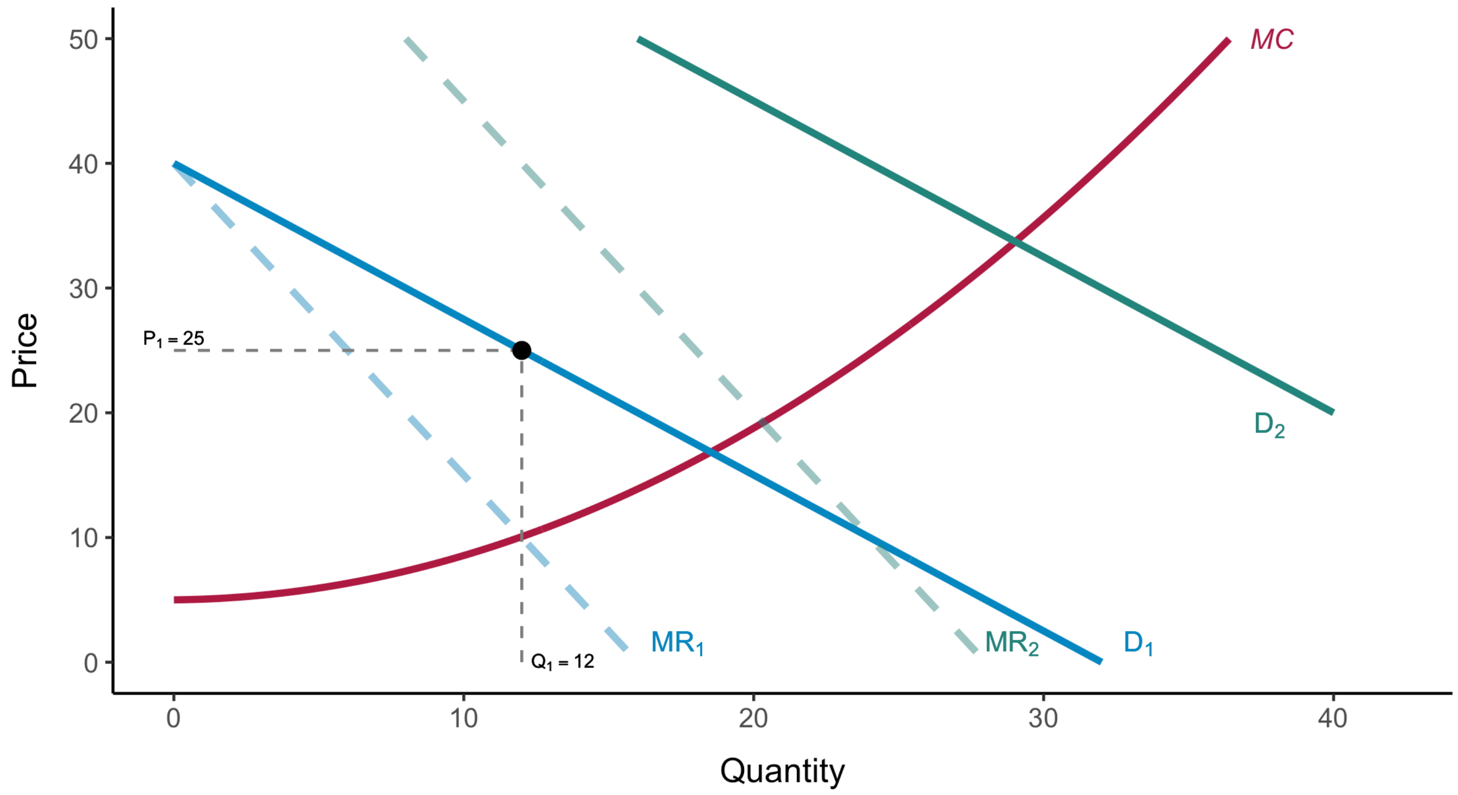
Peak-load Pricing

Set quantity Q_1 and price P_1 with market power for non-peak period.



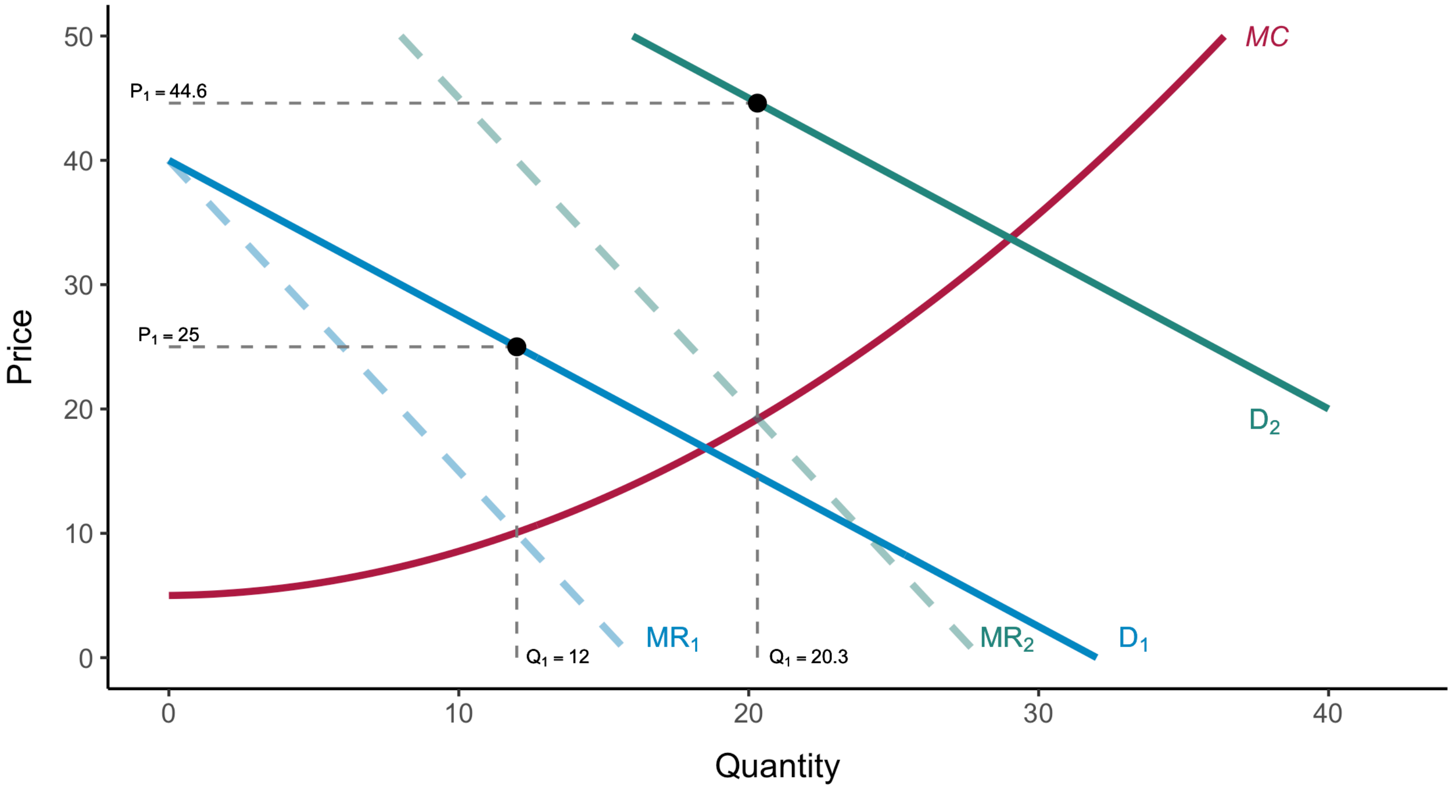
Peak-load Pricing

D_2: Demand during peak period. Note there is a shift in demand, not different preferences and elasticities.



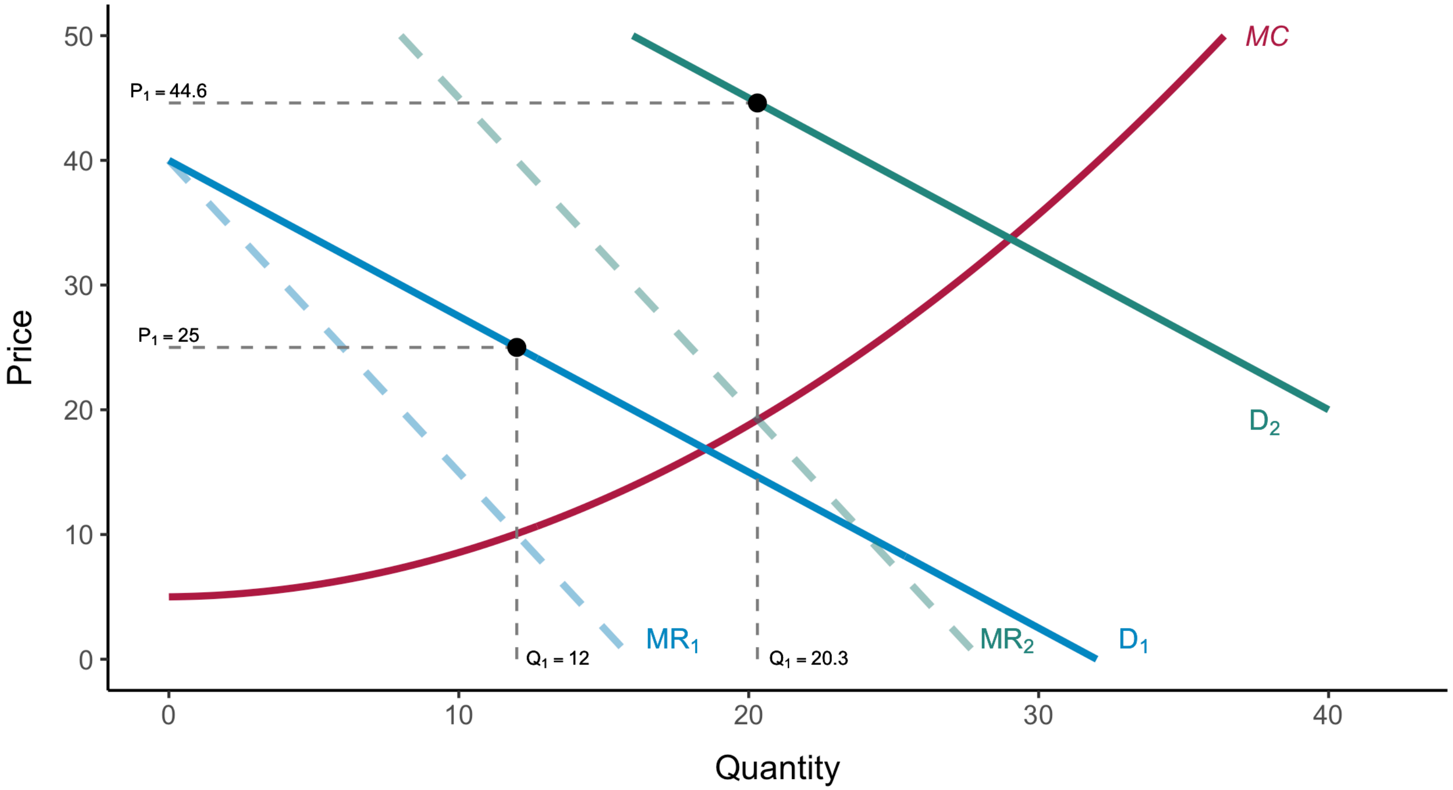
Peak-load Pricing

Set quantity Q_2 and price P_2 with market power for peak period.



Peak-load Pricing

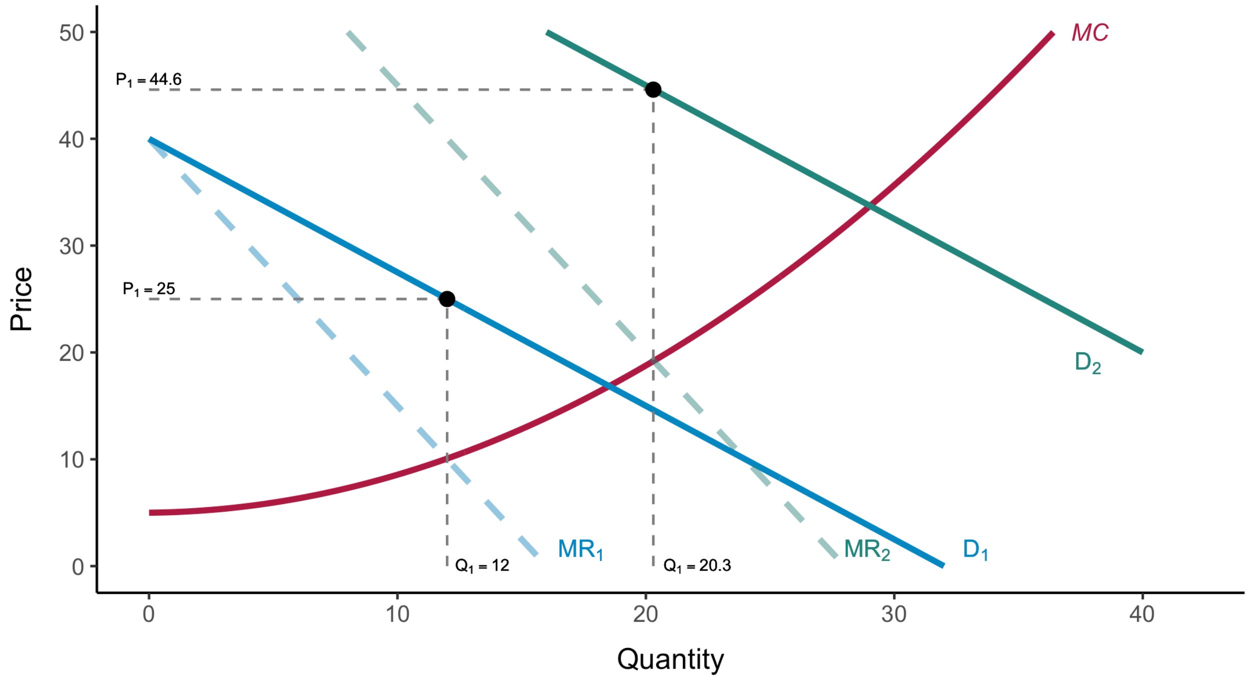
- Increase firm profits
- More efficient: sum of producer and consumer surplus is greater
- If regulated, $MC = P_1 = P_2$: consumers realize the entire efficiency gain.



Peak-load Pricing

Peak-load pricing $\neq$ 3rd-degree price discrimination

- 3rd-degree price discrimination: prices must be set so that $MR_1 = MR_2 = MC$ since serving one group affects the cost of serving another (e.g., airline tickets)
- Peak-load pricing: Selling more during off-peak times does not impact costs during peak times.



The Two-Part Tariff

The Two-Part Tariff

A pricing strategy where consumers pay an upfront fee (entry fee) plus an additional per-unit fee (usage fee).

- Phone services (monthly fee + extra charges)
- Sport clubs or gyms (membership + court fees/extra classes)
- Vélib (subscription fee + per-use fee)
- Museum memberships (membership for unlimited visits + extra fees for temporary exhibitions)
- Clubs (entrance fee + prices per drinks)

Firms must choose:

1. Entree fee T
2. Usage fee P

Pricing Dilemma:

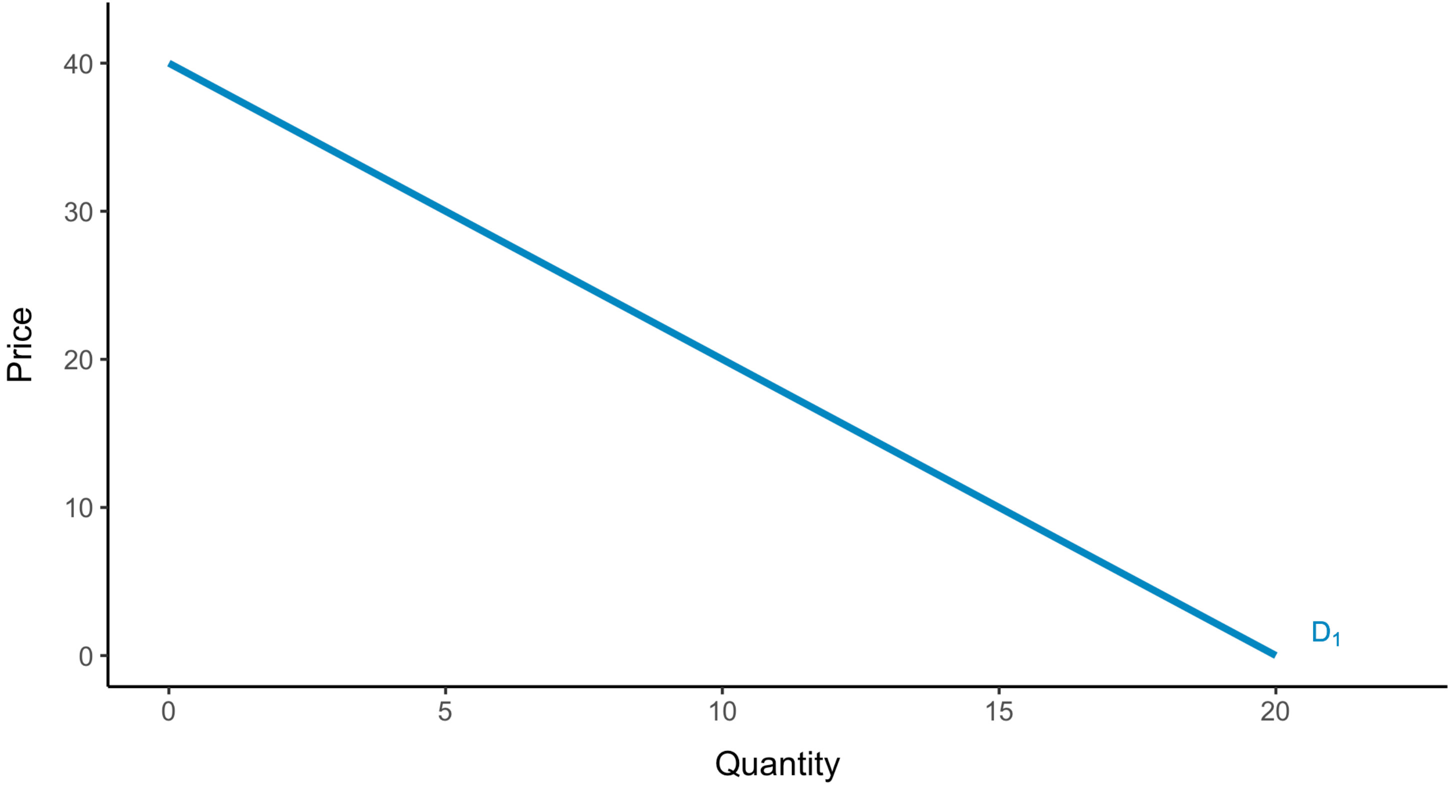
1. High entry fee + low usage fee
2. Low entry fee + high usage fee

The Two-Part Tariff

Artificial but simple case.

One consumer on the market (or many consumers with identical demand curves).

$P = 40 - 2 \cdot Q.$

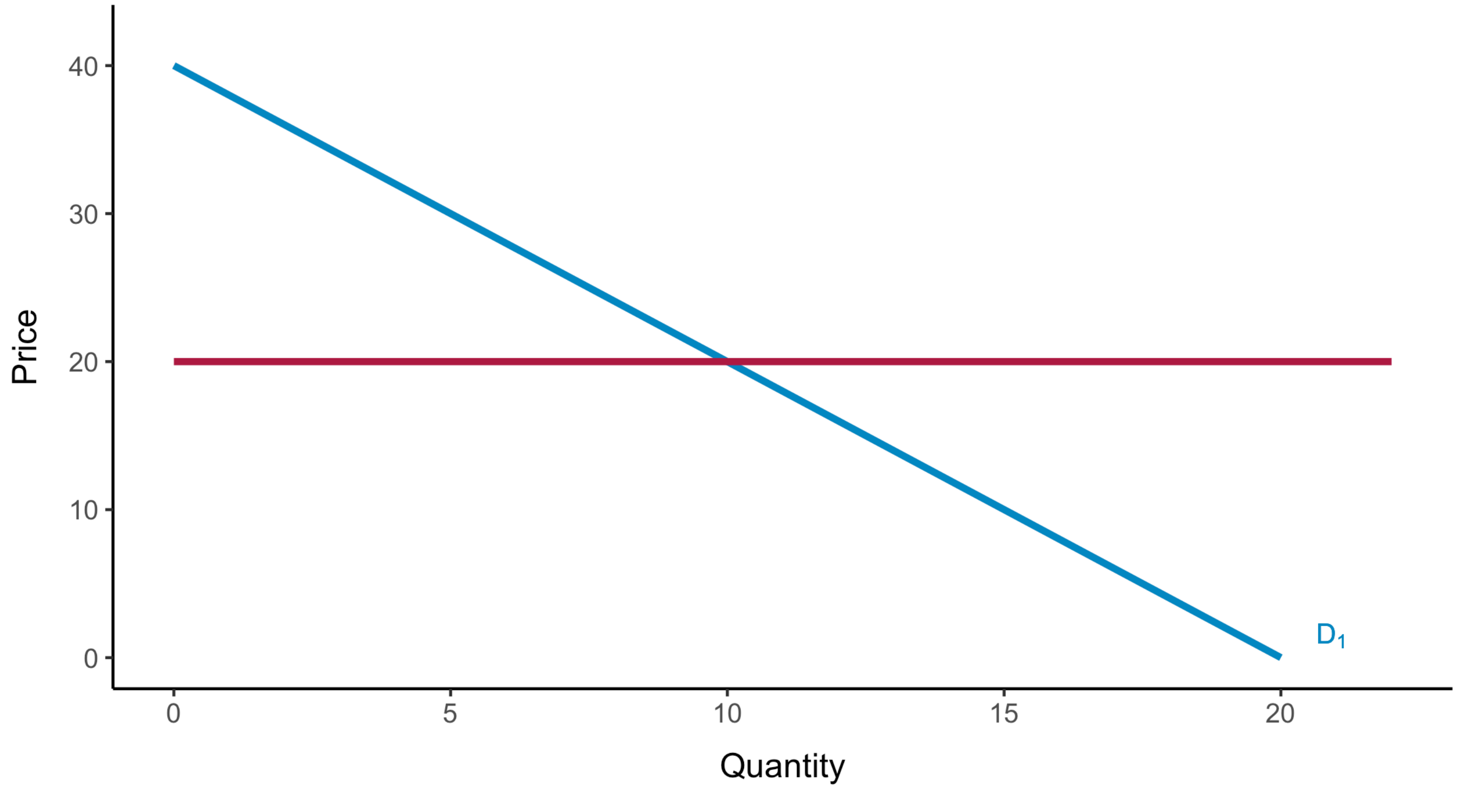


The Two-Part Tariff

Artificial but simple case.

Suppose also that the firm knows this consumer's demand curve.

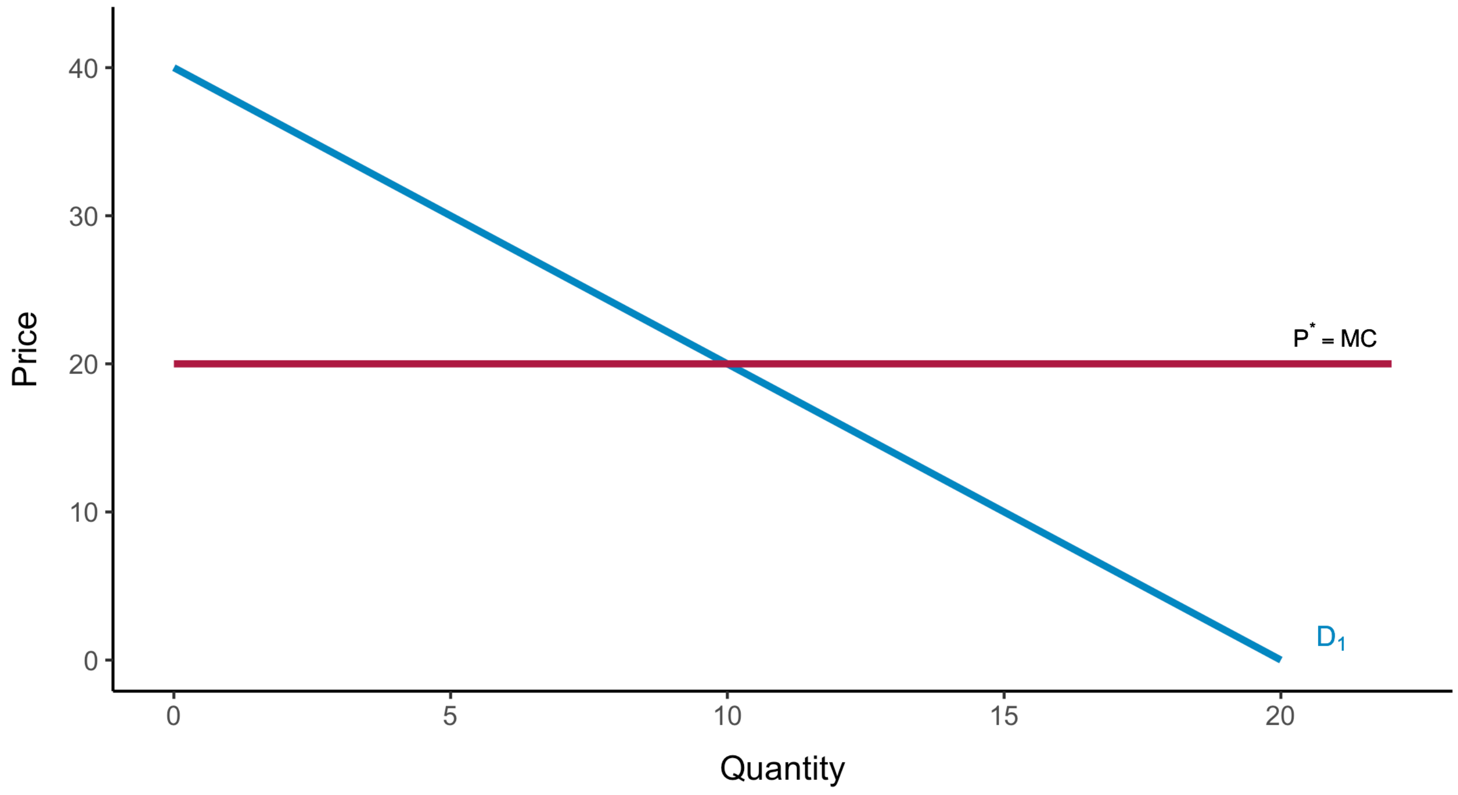
$MC = 20$



The Two-Part Tariff

Artificial but simple case.

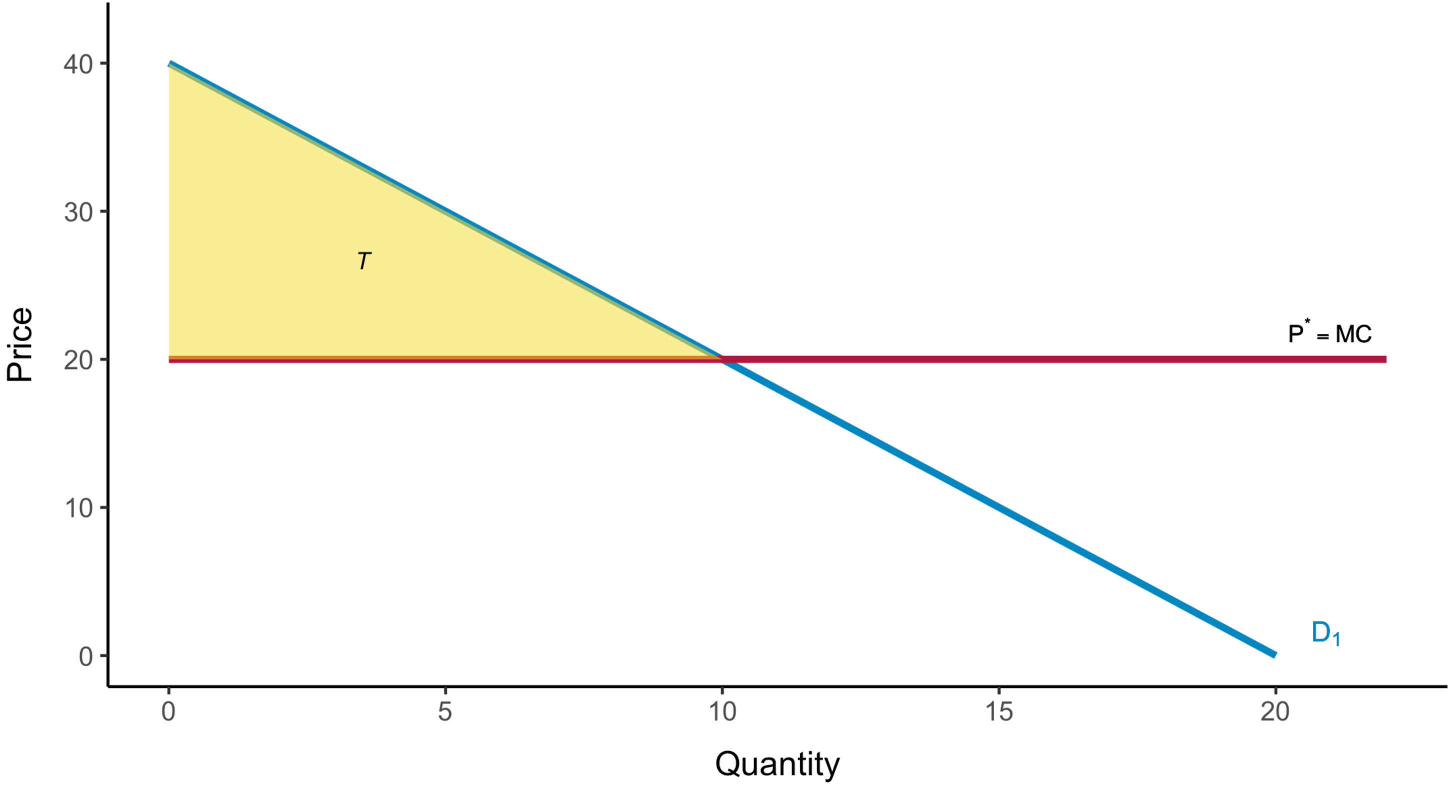
If the firm wants to capture as much consumer surplus as possible, what is P and what is T? Remember P is the usage or extra fee price per unit consumed.



The Two-Part Tariff

Artificial but simple case.

If the firm wants to capture as much consumer surplus as possible, what is P and what is T? Thus, T will be equal to the consumer surplus.

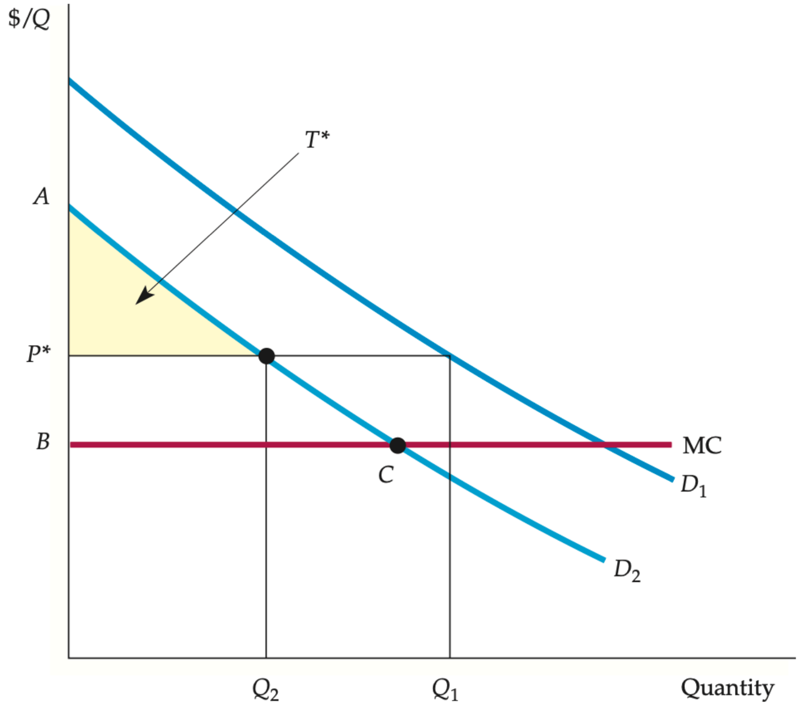


The Two-Part Tariff

Now suppose that there are two groups of consumers. The firm, however, can set only one entry fee and one usage fee.

The Two-Part Tariff

If the usage fee, P, equals MC. Thus, T would be the consumer surplus of the first group of consumers: if the firms charge a higher T, they loose D_1



P^* and T^* such that

$$\pi = 2 \cdot T^* + (P^* - MC)(Q_1 + Q_2)$$

Summary

Summary

- Firms with **market power** can earn high profits, but success depends on pricing strategy. Understanding demand elasticity is crucial for both single and multi-price strategies.
- Pricing strategies aim to **maximize consumer surplus capture** by setting **multiple prices to attract more customers**.
- Ideally, firms would charge each customer their **reservation price**, but this is impractical. Instead, **imperfect price discrimination** methods help increase profits.
- **Two-Part Tariff**: A strategy to capture consumer surplus where customers pay an entry fee plus a per-unit price. Most effective when customer demand is homogeneous.

TD

Excercise 10

As the owner of the only tennis club in an isolated wealthy community, you must decide on membership dues and fees for court time.

There are two types of tennis players.

Serious players

$$Q_1 = 10 - P$$

Serious players

$$Q_2 = 4 - 0.25 \cdot P$$

where Q is court hours per week and P is the fee per hour for each player.

There are 1000 players of each type, and they look alike (you must charge the same prices).

Because plenty of courts, the marginal cost of court time is zero. And the weekly fixed costs are 10,000.

a) Maximizing Profits with Only Serious Players

Suppose that to maintain a “professional” atmosphere, you want to limit membership to serious players.

How should you set the annual membership dues and court fees (assume 52 weeks per year) to maximize profits, keeping in mind the constraint that only serious players choose to join?

What would profits be (per week)?

Excercise 10

a) Maximizing Profits with Only Serious Players

First, note $C = 10,000 \;\Rightarrow MC = 0$. The optimal usage fee of the court per week would be $P = 0$.

Now, to determine weekly membership fees T we want to capture the consumer surplus of the serious players:

$$T = \int_0^{10} (10 - P)dP = \left[10P - \frac{P^2}{2}\right]_0^{10}$$

$$= (100 - 50) - (0 - 0) = 50$$

Thus, weekly $T_{weekly} = 50$ and $T_{annual} = 50 \times 52 = 2600$.

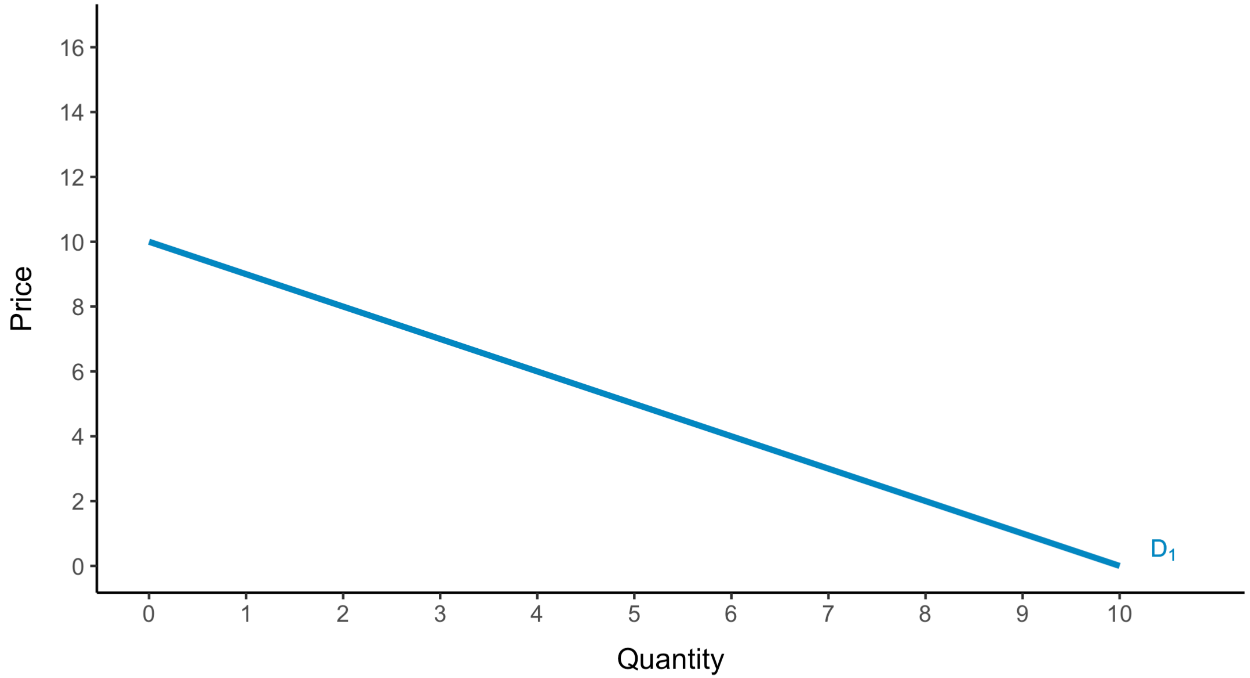
Finally

$$\pi_{weekly} = R_{membership} + R_{courts} - C = (50 \times 1000) + (0 \times 1000) - 10000 = 40000$$

Excercise 10

a) Maximizing Profits with Only Serious Players

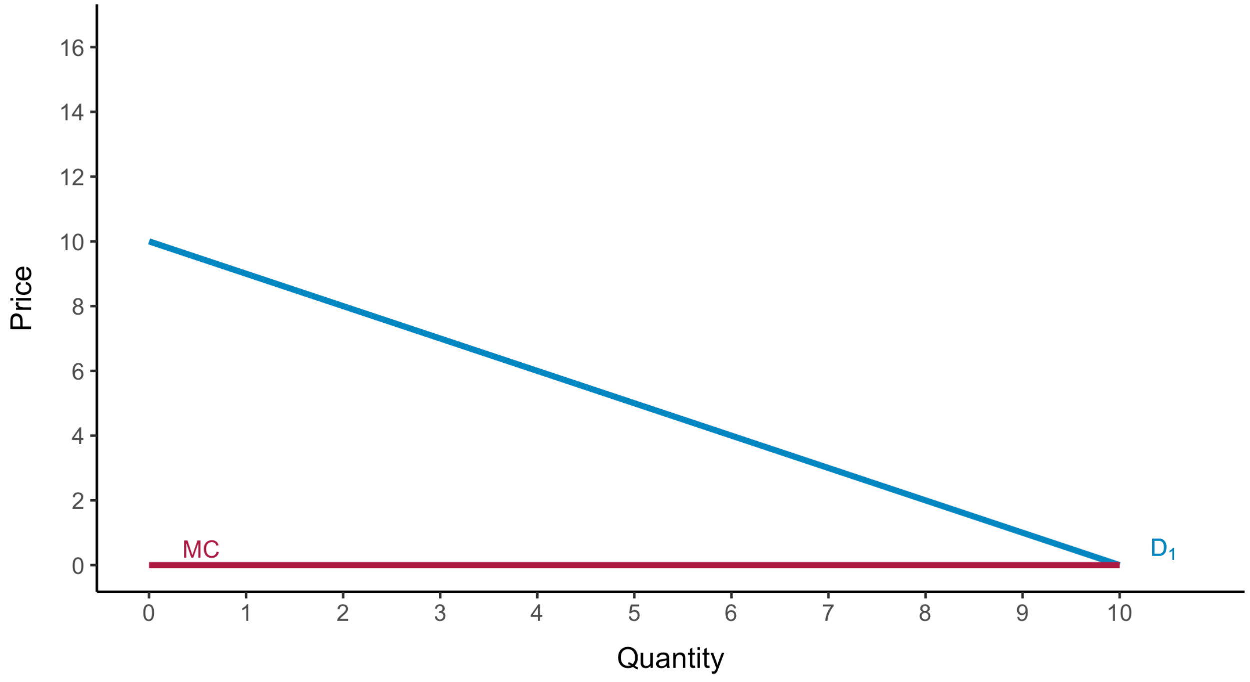
Graphically: $Q_1 = 10 - P \iff P = 10 - Q_1$



Excercise 10

a) Maximizing Profits with Only Serious Players

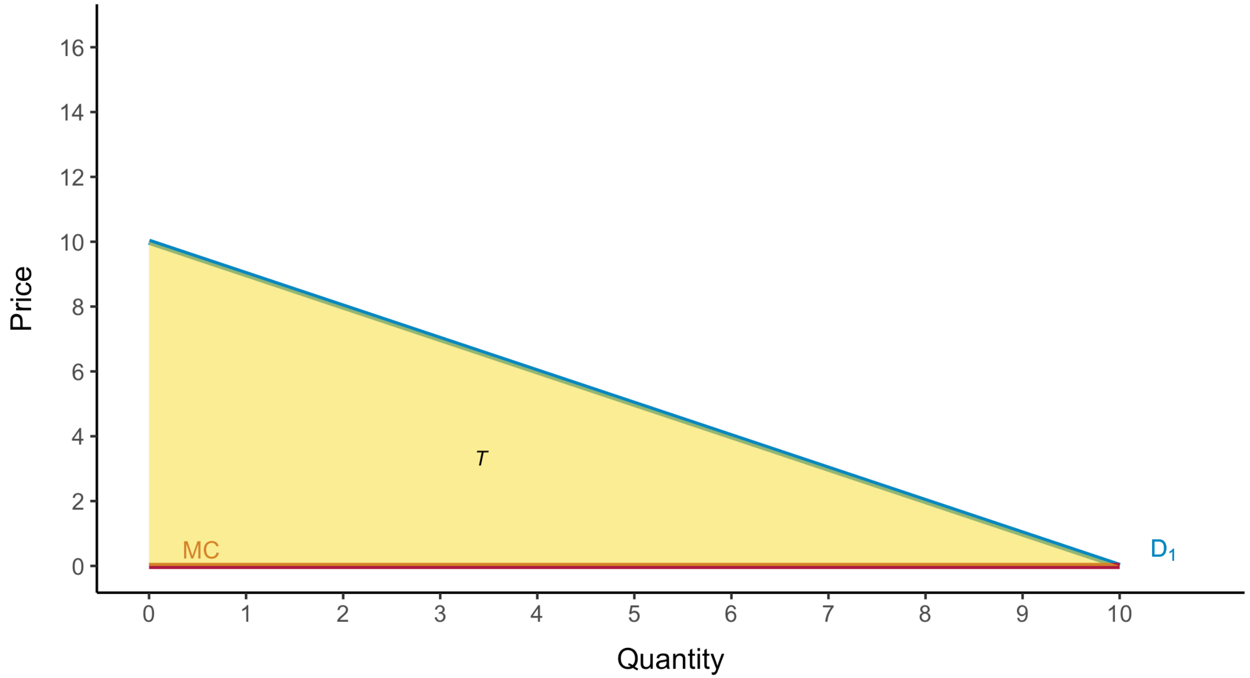
Graphically: $Q_1 = 10 - P \iff P = 10 - Q_1$



Excercise 10

a) Maximizing Profits with Only Serious Players

Graphically: $Q_1 = 10 - P \iff P = 10 - Q_1$



$$T = \int_0^{10} (10 - P)dP = \left[10P - \frac{P^2}{2}\right]_0^{10} = (100 - 50) = 50$$

Excercise 10

b) Maximizing Profits with Both Types of Players

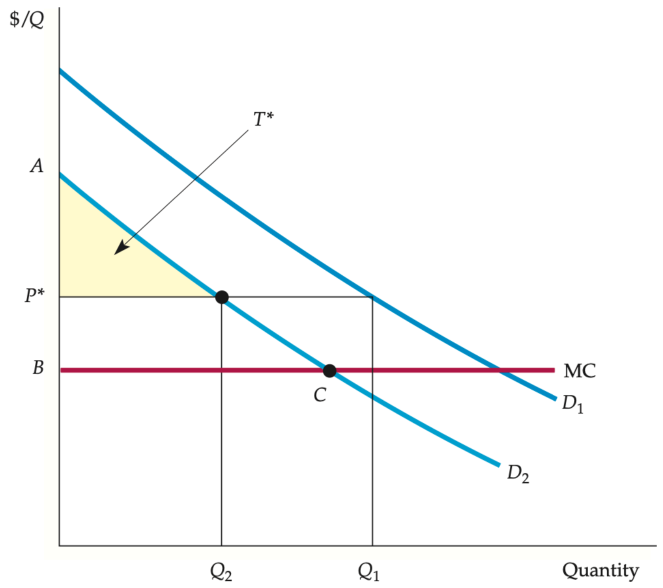
A friend tells you that you could make greater profits by encouraging both types of players to join. Is your friend right?

What annual dues and court fees would maximize weekly profits? What would these profits be?

When serving two classes of customers, the club owner maximizes profits by:

1) Setting court fees above marginal cost to extract revenue from usage.

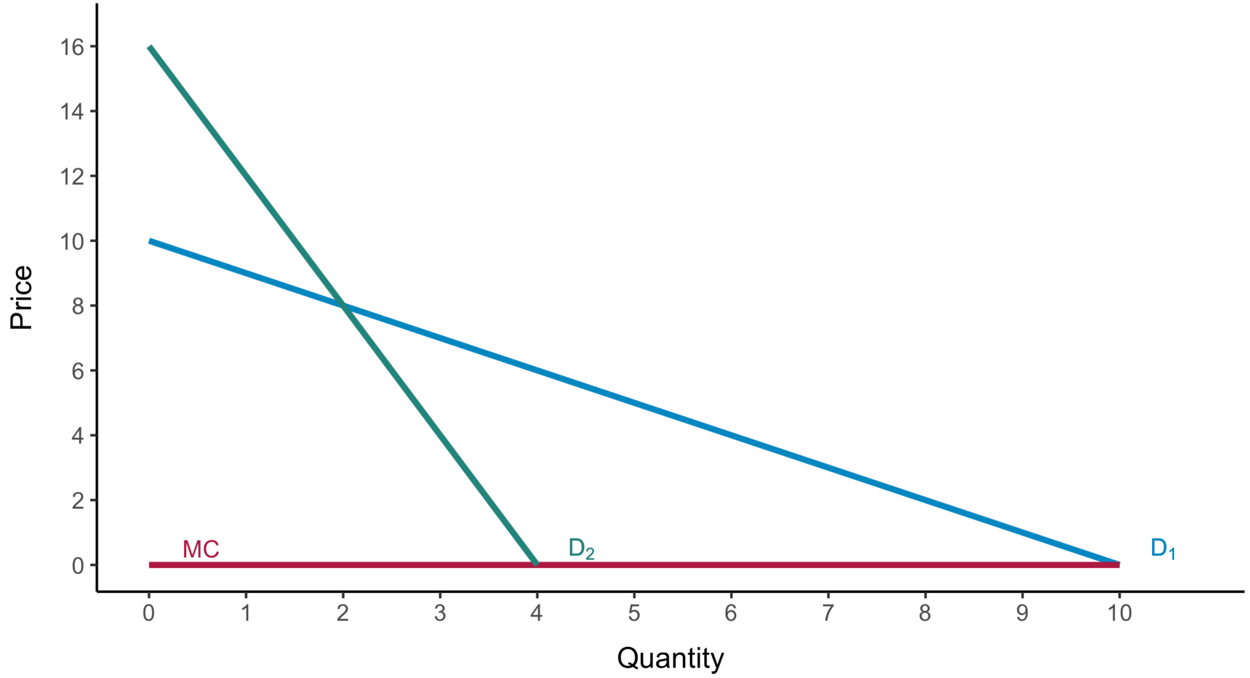
2) Charging an entry fee equal to the remaining consumer surplus of the consumer with the smaller demand (the occasional player) to ensure both groups participate while maximizing total revenue.



Excercise 10

b) Maximizing Profits with Both Types of Players

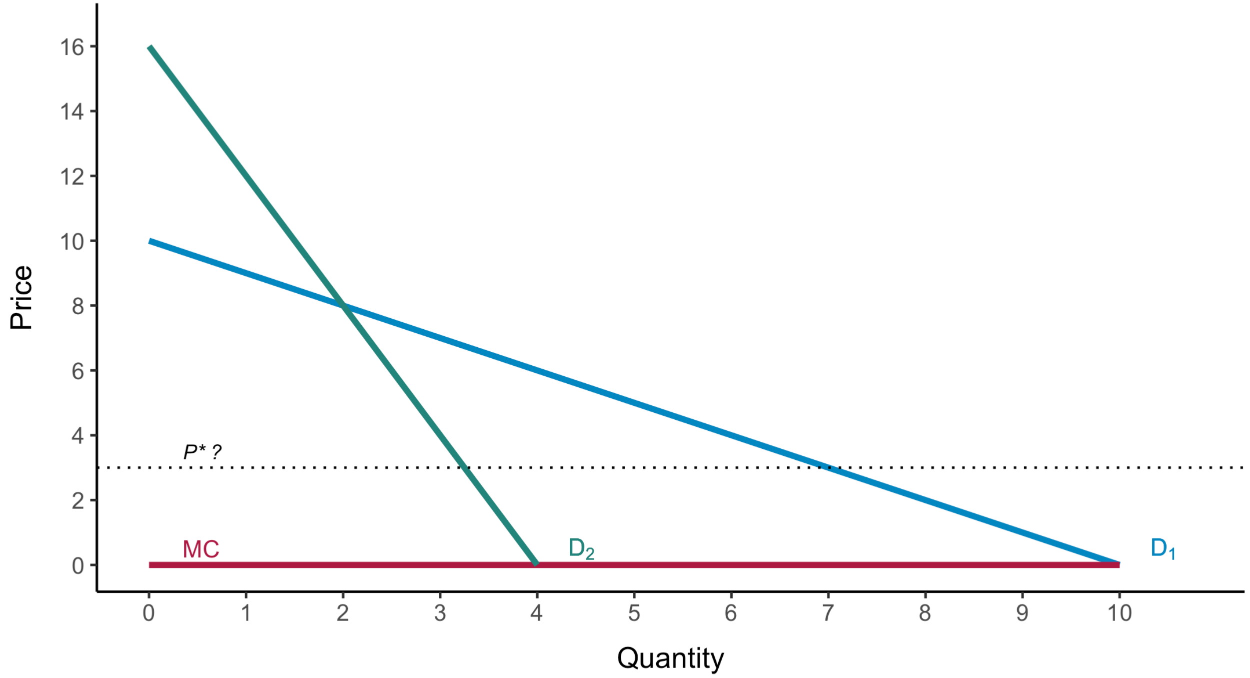
Graphically: $Q_2 = 4 - (1/4) \cdot P \iff P = 16 - 4 \cdot Q_2$



Excercise 10

b) Maximizing Profits with Both Types of Players

Graphically: $Q_2 = 4 - (1/4) \cdot P \iff P = 16 - 4 \cdot Q_2$



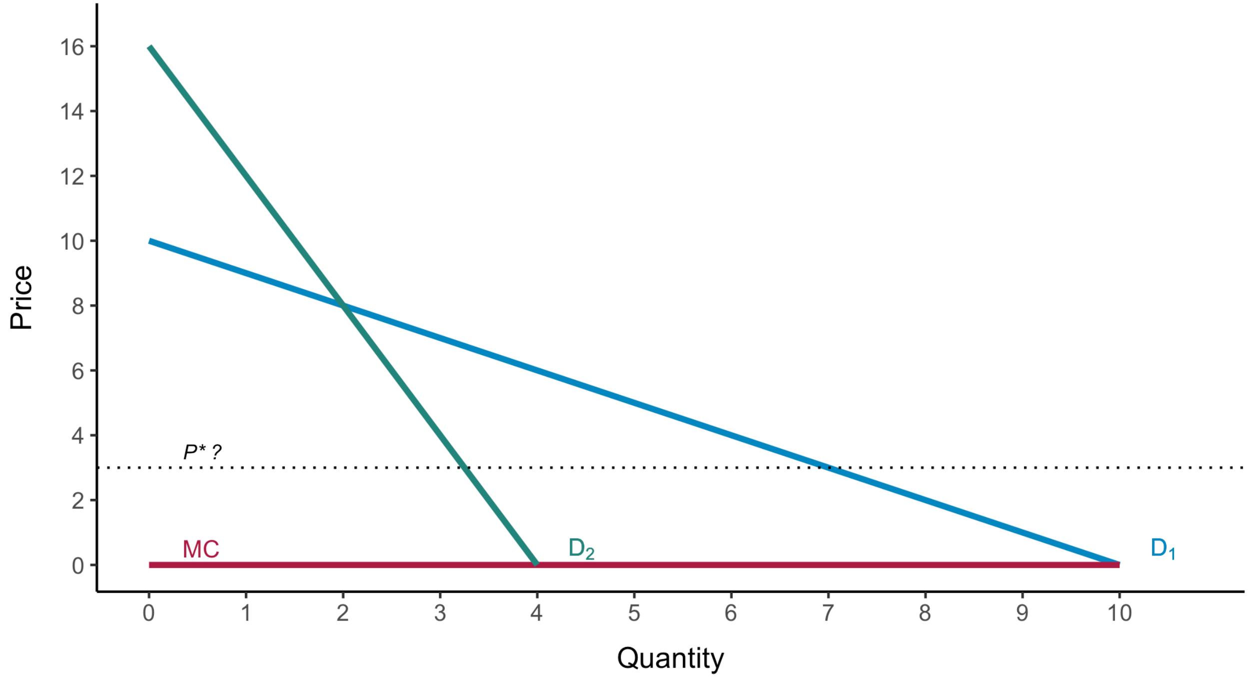
Then

$$T^* = \frac{(16 - P^*) \cdot (Q_2 - 0)}{2}$$

Excercise 10

b) Maximizing Profits with Both Types of Players

Graphically: $Q_2 = 4 - (1/4) \cdot P \iff P = 16 - 4 \cdot Q_2$



Then

$$T^* = \frac{(16 - P^*) \cdot (Q_2 - 0)}{2} = \frac{(16 - P^*) \cdot (4 - (1/4) \cdot P^*)}{2} = 32 - 4P^* + P^{*2}/8$$

Excercise 10

b) Maximizing Profits with Both Types of Players

If the two types of players join, 1000 each type, then

$$R_{membership} = 2000 \cdot (32 - 4P^* + P^{*2}/8)$$

$$R_{courts} = P^* \cdot (Q_1 + Q_2) = P^*[1000 \cdot (10 - P^*) + 1000 \cdot (4 - P^*/4)] = 14,000 \cdot P^* - 1,250 \cdot P^{*2}$$

$$\Rightarrow TR = R_{membership} + R_{courts}$$

$$TR = 2,000 \cdot (32 - 4P^* + P^{*2}/8) + 14,000 \cdot P^* - 1,250 \cdot P^{*2}$$

$$= 64,000 - 8,000 \cdot P^* + 250 \cdot P^{*2} + 14,000 \cdot P^* - 1,250 \cdot P^{*2}$$

$$= 64,000 + 6,000 \cdot P^* - 1,000 \cdot P^{*2}$$

$$\Rightarrow \Delta TR/\Delta P = 6,000 - 2,000 \cdot P^* = 0 = MC$$

$$\Rightarrow 6,000 = 2,000 \cdot P^* \iff P^* = 3$$

$$\Rightarrow TR^* = 73,000$$

$$\Rightarrow \pi^*_{weekly} = 73,000 - 10,000 = 63,000$$

Excercise 10

c) Change in composition of players

Suppose now there are 3000 serious players and 1000 occasional players. Would it still be profitable to cater to the occasional player?

What would be the profit-maximizing annual dues and court fees? What would profits be per week?

1) Weekly profits for only serious players

$$\pi_{weekly,serious} = R_{membership} + R_{courts} - C = (50 \times 3,000) + (0 \times 1,000) - 10,000 = 140,000$$

2) Weekly profits with both type of players

$$R_{membership} = 4,000 \cdot (32 - 4P^* + P^{*2}/8)$$

$$R_{courts} = P^* \cdot (Q_1 + Q_2) = P^*[3,000 \cdot (10 - P^*) + 1,000 \cdot (4 - P^*/4)] = 34,000 \cdot P^* - 3,250 \cdot P^{*2}$$

$$TR = 128,000 + 18,000 \cdot P^* - 2,750 \cdot P^{*2}$$

$$\Rightarrow \Delta TR/\Delta P = 18,000 - 5,500 \cdot P^* = 0 \quad \Rightarrow P^* = 3.27$$

$$\Rightarrow TR^* = 157,455 \quad \Rightarrow \pi^*_{weekly,both} = 157,455 - 10,000 = 147,455$$