ECON 204C - Macroeconomic Theory

Sovereign Defaults

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Learning Objective

- Long-duration bonds and sovereign defaults (Hatchondo and Martinez, 2009)
- Review

The Environment

- There is a single tradable good. The economy receives a stochastic endowment stream of this good y_t where

$$\ln y_t = (1-\rho)\mu + \rho \ln y_{t-1} + \varepsilon_t \quad \text{with} \quad |\rho| < 1, \quad \text{and} \quad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

- The government's objective is to maximize the present expected discounted value of future utility flows of the representative agent in the economy

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right] \quad \text{where} \quad u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$$

The Environment

Each period, the government makes two decisions.

1. The government decides whether to default, which implies repudiating all current and future debt obligations contracted in the past. The default cost is represented by an endowment loss of $\phi(y)$ in the default period.

2. The government decides the number of bonds that it purchases or issues in the current period.

    ⋆ Assume that each period the government can choose any debt level for the following period, anticipating that the price at which it can issue or purchase bonds is such that lenders make zero profits in expectation.

    ⋆ Lenders can borrow or lend at the risk-free rate r, and have perfect information regarding the economy's endowment.

Coupon Structure

- A bond issued in period t promises an infinite stream of coupons, which decreases at a constant rate δ.
  - ⋆ A bond issued in period t promises to pay 1 unit of the good in period $t+1$ and $(1-\delta)^{s-1}$ units inperiod $t+s$, with $s \geq 2$.
- Note that the coupon structure allows us to consider debt with durations longer than one period without increasing the dimensionality of the state space.
  - ⋆ The government default and issuance decisions in period t are influenced by **its debt obligations for that period and by its current debt obligations for every future period.**
  - ⋆ Since we assume that bonds issued in all previous periods promise a sequence of coupons that decrease at the same constant rate, **current coupon obligations are sufficient to predict the stream of future debt obligations derived from past issuances.**

Law of Motion

$$b' = b(1-\delta)(1-d) - i$$

b: the number of outstanding coupon claims at the beginning of the current period

- ⋆ A negative value of b implies that the government was a net issuer of bonds in the past.

b': the number of outstanding coupon claims at the beginning of next period

d: the current-period default decision

- ⋆ d is equal to 1 if the government defaulted in the current period and is equal to 0 if it did not.

i: the current-period issuance level

1. If $b' = b(1-\delta)(1-d)$, then it is neither issuing nor buying bonds.
2. If $b' < b(1-\delta)(1-d)$, then it is issuing new bonds.
3. If $b' > b(1-\delta)(1-d)$, then it is purchasing bonds.

Recursive Formulation

- Let $V(b, y)$ denote the government's value function at the beginning of a period, that is, before the default decision is made.
- Let $\tilde{V}(d, b, y)$ denote its value function after the default decision has been made.
- Let $F(y'|y)$ denote the conditional cdf of the next-period endowment y'.

For any bond price function $q(b', y)$, the function $V(b, y)$ satisfies the following functional equation,

$$V(b, y) = \max_{d \in \{0,1\}} \left\{ d\tilde{V}(1, b, y) + (1-d)\tilde{V}(0, b, y) \right\} \tag{1}$$

$$\tilde{V}(d, b, y) = \max_{b' \leq 0} \left\{ u(c) + \beta \int V(b', y') F(dy'|y) \right\} \tag{2}$$

$$\text{where} \quad c = y - d\phi(y) + \underbrace{(1-d)b}_{\text{Current Obligation}} - q(b', y)\underbrace{[b' - (1-d)(1-\delta)b]}_{=-i}$$

Zero-profit Condition

$$c' = y' - d'\phi(y') + (1-d')\boldsymbol{b'} - q(b'', y')\Big[b'' - (1-d')(1-\delta)\boldsymbol{b'}\Big]$$

- $h(b, y)$ denote the future default rules that **lenders expect the government to follow**.
- $g(d, b, y)$ denote the future borrowing rules that **lenders expect the government to follow.**

$$q^{ZP}(b', y) = \underbrace{\frac{1}{1+r}\int [1 - h(b', y')]F(dy'|y)}_{(A)}$$
$$+ \underbrace{\frac{1}{1+r}\int [1 - h(b', y')](1-\delta)q^{ZP}\Big(g\big(h(b', y'), b', y'\big),\ y'\Big)F(dy'|y)}_{(B)} \tag{3}$$

- (A) equlas the expected value of the next-period coupon payment promised in a bond.
- (B) equals the expected value of all other future coupon payments, which is summarized by the expected price at which the bond could be sold next period.

MPE

Definition A Markov Perfect Equilibrium is characterized by

- a set of value function $\tilde{V}(d, b, y)$ and $V(b, y)$,
- a default rule $h(b, y)$ and a borrowing rule $g(d, b, y)$,
- a bond price function $q^{ZP}(b', y)$,

such that:

1. given $h(b, y)$ and $g(d, b, y)$, $v(b, y)$ and $\tilde{V}(d, b, y)$ satisfy functional equations (1) and (2) when the government can trade bonds at $q^{ZP}(b', y)$;
2. given $h(b, y)$ and $g(d, b, y)$, the bond price function $q^{ZP}(b', y)$ offered to the government satisfies lenders' zero-profit condition implicit in equation (3); and
3. the default rule $h(b, y)$ and borrowing rule $g(d, b, y)$ solve the dynamic programming problem defined by equation (1) and (2) when the government can trade bonds at $q^{ZP}(b', y)$.

Review

- Sequential Problem and Recursive Problem
  - ⋆ Dynamic Programming - Guess & Verify / Value Function Iteration
- Uncertainty - History and Markov Process (Transition)
- Merton Portfolio Choice Model in Discrete Time
- Equilibrium with Complete Markets (Date-0/Sequential/Recursive)
  - ⋆ Definition of complete market
  - ⋆ Borrowing Constraints, No Ponzi Scheme
  - ⋆ Consistency / Rational Expectation / Big K - little k Analysis
- Welfare Theorem
  - ⋆ Planner's Problem and Decentralization
  - ⋆ Pareto Efficiency
  - ⋆ Negish Algorithm
- Aggregation
- Constrained Inefficiency and Pecuniary Externality
- Calibration

Review

- HAM and Recursive Stationary Competitive Equilibrium
  - Hugget (1993), Aiyagari (1994), Krusell and Smith (1998)
- Consumption-based Asset Pricing Model
  - Zero vs. Positive Asset Supply
  - Recursion and No Bubble
  - Price-Dividend Ratio
  - Stochastic Discount Factor
- Equity Premium Puzzle and Solution
- The Hotelling Rule
- Optimal Taxes on Fossil Fuel in General Equilibrium
- The Basic New Keynesian Model
  - Lucas Critique and Microfoundation
  - Log-linearization around Steady State
  - Calvo Fairy and Price Rigidities
- Sovereign Defaults

Review

- Euler Equation - Equimarginal Principle (MB=MC)

$$u'(c^*) = \beta u'(c'^*)[1 + r]$$

- Envelope Theorem

  At the optimum, choice variables are already optimized. An infinitesimal change in parameter does not alter these choices. Thus, at the optimum, the effect of parameter on objective depends only on its direct effect and does not depend on its indirect effect due to re-optimization.

- Shadow Price

  It is the value of relaxing the constraint by one unit. This only holds when choice variables are optimized.

Reference

Hatchondo, J. C., & Martinez, L. (2009). Long-duration bonds and sovereign defaults. Journal of international Economics, 79(1), 117-125.