ECON 204C - Macroeconomic Theory

Equilibrium with Complete Markets

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Learning Objective

- Equilibrium with Complete Markets
  - Date 0 trading: Arrow-Debreu securities
  - Sequential trading: Arrow securities
  - Recursive competitive equilibrium

Stochastic Event

In each period $t \geq 0$, there is a realization of a stochastic event $s_t \in S$. Let the history of events up and until time t be denoted by $s^t = (s_t, s_{t-1}, \cdots, s_1, s_0) \in S^t$.

- Unconditional probability of observing a particular sequence of events s^t is $\pi_t(s^t)$.
- Probability of observing s^τ conditional on the realization of s^t $\pi_\tau(s^\tau | s^t)$.

There are I agents named $i \in \mathcal{I} = \{1, \cdots, I\}$.

- Agent i owns a stochastic endowment $y_t^i(s^t)$ that depends on s^t.
- The history s^t is **publicly observable**.

Preferences

Household i purchases a **history-dependent consumption plan** $c^i = \{c_t^i(s^t)\}_{t=0}^{\infty}$.

$$U(c^i) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t^i) \right] = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u\big(c_t^i(s^t)\big) \pi_t(s^t) \qquad \text{where} \qquad \beta \in (0,1)$$

- u is an increasing, twice continuously differentiable, and strictly concave function.
- The utility function satisfies the Inada condition.

$$\lim_{c \to 0} u'(c) = \infty$$

- We are imposing identical preference across all individuals i that can be represented in terms of discounted expected utility with common discount factor β, common Bernoulli utility function u, and common probability distributions $\pi_t(s^t)$.

Date 0 Trading - Arrow-Debreu Structure

- Households trade dated history-contingent claims to consumption.
- There is a complete set of Arrow-Debreu securities.
- Trades occur at time 0, after s_0 has been realized.
  - ⋆ we assume that $\pi_0(s_0) = 1$ for the initially given value of s_0.

Date 0 Trading - Household i's UMP

Household $i \in \mathcal{I}$ purchases a **history-dependent consumption plan** $c^i = \{c_t^i(s^t)\}_{t=0}^{\infty}$.

$$\max_{\{c_t^i(s^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u\Big(c_t^i(s^t)\Big)\pi_t(s^t) \quad s.t. \quad \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) c_t^i(s^t) \leq \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) y_t^i(s^t)$$

- $q_t^0(s^t)$ denotes the price of time t consumption contingent on history s^t at time t **in terms of an abstract unit of account or numeraire**.
  - ⋆ If we assume $q_0^0(s_0) = 1$, then $c_0(s_0)$ is numeraire.
  - ⋆ $\frac{q_\tau^0(s^\tau)}{q_t^0(s^t)}$ denotes the price of time τ consumption contingent on history s^τ at time τ **in terms of time t consumption contingent on history s^t at time t**.
- All trades occur at time 0. After time 0, trades that were agreed to at time 0 are executed, but no more trades occur.

Date 0 Trading - Household i's UMP

$$\mathcal{L}^i_{AD} = \max_{\{c^i_t(s^t)\}_{t=0}^{\infty},\ \mu^i} \sum_{t=0}^{\infty}\sum_{s^t}\beta^t u\Big(c^i_t(s^t)\Big)\pi_t(s^t) + \mu^i\left(\sum_{t=0}^{\infty}\sum_{s^t} q^0_t(s^t) y^i_t(s^t) - \sum_{t=0}^{\infty}\sum_{s^t} q^0_t(s^t) c^i_t(s^t)\right)$$

$$\beta^t u'\Big(c^{i*}_t(s^t)\Big)\pi_t(s^t) = \mu^{i*} q^0_t(s^t) \qquad \text{(FOC w.r.t. } c^i_t(s^t))$$

$$\sum_{t=0}^{\infty}\sum_{s^t} q^0_t(s^t) y^i_t(s^t) = \sum_{t=0}^{\infty}\sum_{s^t} q^0_t(s^t) c^{i*}_t(s^t) \qquad \text{(FOC w.r.t. } \mu^i)$$

$$\frac{\beta^\tau u'\Big(c^{i*}_\tau(s^\tau)\Big)\pi_\tau(s^\tau)}{\beta^t u'\Big(c^{i*}_t(s^t)\Big)\pi_t(s^t)} = \frac{q^0_\tau(s^\tau)}{q^0_t(s^t)} \qquad \text{(within } i \text{ across histories)}$$

$$\frac{u'\Big(c^{i*}_t(s^t)\Big)}{u'\Big(c^{j*}_t(s^t)\Big)} = \frac{\mu^{i*}}{\mu^{j*}} \qquad \text{(given } s^t \text{ across individuals)}$$

Date 0 Trading - Household i's UMP

Given two histories s^t and s^τ, MRS of $c_t(s^t)$ for $c_\tau(s^\tau)$ is the same across individuals.

$$\underbrace{\frac{\beta^\tau u'\Big(c_\tau^{i*}(s^\tau)\Big)\pi_\tau(s^\tau)}{\beta^t u'\Big(c_t^{i*}(s^t)\Big)\pi_t(s^t)}}_{MRS^i_{s^t,\ s^\tau}(c^{i*})} = \underbrace{\frac{\beta^\tau u'\Big(c_\tau^{j*}(s^\tau)\Big)\pi_\tau(s^\tau)}{\beta^t u'\Big(c_t^{j*}(s^t)\Big)\pi_t(s^t)}}_{MRS^j_{s^t,\ s^\tau}(c^{j*})}$$

Given two individuals i and j, the ratio of $MU^i_{s^t}$ to $MU^j_{s^t}$ is the same across histories.

$$\frac{u'\Big(c_t^{i*}(s^t)\Big)}{u'\Big(c_t^{j*}(s^t)\Big)} = \frac{u'\Big(c_\tau^{i*}(s^\tau)\Big)}{u'\Big(c_\tau^{j*}(s^\tau)\Big)}$$

Date 0 Trading - Competitive Equilibrium

Definition A competitive equilibrium is a price system $\{q_t^0(s^t)\}_{t=0}^{\infty}$ and allocation $\{c^{i*}\}_{i\in\mathcal{I}}$ such that

1. Given a price system, each individaul $i \in \mathcal{I}$ solves the following problem:

$$\{c_t^{i*}(s^t)\}_{t=0}^{\infty} = arg \max_{\{c_t^i(s^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u\Big(c_t^i(s^t)\Big)\pi_t(s^t)$$

$$s.t. \quad \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) c_t^i(s^t) \leq \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) y_t^i(s^t)$$

2. On every history s^t at time t, market clears

$$\sum_{i\in\mathcal{I}} c_t^{i*}(s^t) = \sum_{i\in\mathcal{I}} y_t^i(s^t)$$

Date 0 Trading - Competitive Equilibrium

- Unknowns

$$\{q_t^0(s^t)\}_{t=0}^{\infty} \qquad \left\{\{c_t^{i*}(s^t)\}_{t=0}^{\infty}\right\}_{i\in\mathcal{I}} \qquad \{\mu^{i*}\}_{i\in\mathcal{I}}$$

- System of equations

$$\sum_{i\in\mathcal{I}} c_t^{i*}(s^t) = \sum_{i\in\mathcal{I}} y_t^i(s^t)$$

$$\beta^t u'\Big(c_t^{i*}(s^t)\Big)\pi_t(s^t) = \mu^{i*} q_t^0(s^t)$$

$$\frac{u'\Big(c_t^{i*}(s^t)\Big)}{u'\Big(c_t^{1*}(s^t)\Big)} = \frac{\mu^{i*}}{\mu^{1*}} \qquad \forall\ i \in \{2, \cdots, I\}$$

$$q_0^0(s_0) = 1$$

Sequential Trading

- New one-period markets are re-opened for trading each period.
- In time t, history-dependent wealth is properly assigned to each agent.
- At each date $t \geq 0$, but only at the history s^t actually realized, trades occur in a set of claims to **one-period-ahead state-contingent consumption**.
- We build on an insight of Arrow (1964) that **one-period securities are enough to implement complete markets**.

Sequential Trading - Household i's UMP

On every history s^t at time t, household $i \in \mathcal{I}$ purchases a **consumption plan** $c_t^i(s^t)$ and **one-period-ahead state-contingent claims** $\{a_{t+1}^i(s_{t+1}, s^t)\}_{s_{t+1} \in S}$ subject to

$$c_t^i(s^t) + \sum_{s_{t+1} \in S} a_{t+1}^i(s_{t+1}, s^t) Q_t(s_{t+1}|s^t) \leq y_t^i(s^t) + a_t^i(s^t) \qquad \text{(Budget constraint)}$$

$$a_{t+1}^i(s_{t+1}, s^t) \geq -A_{t+1}^i(s_{t+1}, s^t) \quad \forall\ s_{t+1} \in S \qquad \text{(Borrowing limit)}$$

- $a_{t+1}^i(s_{t+1}, s^t)$ denotes the claims to time $t+1$ consumption, other than its time $t+1$ endowment $y_{t+1}^i(s^{t+1})$, that household i brings into time $t+1$ in history s^{t+1}.
- $Q_t(s_{t+1}|s^t)$ is the price of one unit of time $t+1$ consumption, contingent on the realization s_{t+1} at time $t+1$.

Sequential Trading - Natural Borrowing Limit (NBL)

Let $q_\tau^t(s^\tau) = \frac{q_\tau^0(s^\tau)}{q_t^0(s^t)}$ be the Arrow-Debreu price, denominated in units of the date t, history s^t consumption good.

$$A_{t+1}^i(s_{t+1}, s^t) = \sum_{\tau=t+1}^{\infty} \sum_{s^\tau | (s_{t+1},\ s^t)} q_\tau^t(s^\tau) y_\tau^i(s^\tau)$$

- It is **the maximal value that agent i can repay starting from** $t+1$, assuming that his consumption is zero always.
- We shall require that household i at time t and history s^t cannot promise to pay more than $A_{t+1}^i(s_{t+1}, s^t)$ conditional on the realization of s_{t+1} tomorrow, because it will not be feasible to repay more.
- Household i at time t faces one such borrowing constraint for each possible realization of $s_{t+1} \in S$ tomorrow.

Sequential Trading - Household i's UMP

$$
\begin{aligned}
\mathcal{L}^i_{Seq} = \max_{\left\{c^i_t(s^t),\ \{a^i_{t+1}(s_{t+1},s^t)\}_{s_{t+1}\in S},\ \eta^i(s^t),\ \{\nu(s_{t+1},s^t)\}_{s_{t+1}\in S}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\sum_{s^t} \Bigg\{ & \beta^t u\Big(c^i_t(s^t)\Big)\pi_t(s^t) \\
& + \eta^i(s^t)\Big(y^i_t(s^t) + a^i_t(s^t) - c^i_t(s^t) - \sum_{s_{t+1}} a^i_{t+1}(s_{t+1}, s^t)Q \\
& + \sum_{s_{t+1}} \nu^i_t(s^t; s_{t+1})\Big(a^i_{t+1}(s_{t+1}, s^t) + A^i_{t+1}(s_{t+1}, s^t)\Big)\Bigg\}
\end{aligned}
$$

$$\beta^t u'\Big(\tilde{c}^i_t(s^t)\Big)\pi_t(s^t) = \eta^i_t(s^t) \qquad \text{(FOC w.r.t. } c^i_t(s^t)\text{)}$$

$$-\eta^i_t(s^t)Q_t(s_{t+1}|s^t) + \underbrace{\cancelto{0}{\nu^i_t(s^t; s_{t+1})}}_{\because \lim_{c\to 0} u'(c)=\infty} + \eta^i_{t+1}(s_{t+1}, s^t) = 0 \qquad \forall\ s_{t+1} \in S \qquad \text{(FOC w.r.t. } a^i_{t+1}(s_{t+1}, s^t)\text{)}$$

$$c^i_t(s^t) + \sum_{s_{t+1}} a^i_{t+1}(s_{t+1}, s^t)Q_t(s_{t+1}|s^t) = y^i_t(s^t) + a^i_t(s^t) \qquad \forall\ s_{t+1} \in S \qquad \text{(FOC w.r.t. } \eta^i(s^t)\text{)}$$

On every history s^t at time t, the following holds for all $s_{t+1} \in S$.

$$Q_t(s_{t+1}|s^t) = \frac{\beta^{t+1} u'\Big(\tilde{c}^i_{t+1}(s^{t+1})\Big)\pi_{t+1}(s^{t+1})}{\beta^t u'\Big(\tilde{c}^i_t(s^t)\Big)\pi_t(s^t)}$$

$$Q_t(s_{t+1}|s^t) = \beta \frac{u'\Big(\tilde{c}^i_{t+1}(s^{t+1})\Big)}{u'\Big(\tilde{c}^i_t(s^t)\Big)} \pi_t(s^{t+1}|s^t)$$

Sequential Trading - Competitive Equilibrium

Definition A competitive equilibrium is a price system $\left\{\{Q_t(s_{t+1}|s^t)\}_{s_{t+1}\in S}\right\}_{t=0}^{\infty}$, an allocation $\left\{\{\tilde{c}_t^i(s^t),\ \{\tilde{a}_{t+1}^i(s_{t+1},s^t)\}_{s_{t+1}\in S}\}_{t=0}^{\infty}\right\}_{i\in\mathcal{I}}$, an initial distribution of wealth $\{a_0^i(s_0)=0\}_{i\in\mathcal{I}}$, and a collection of natural borrowing limits $\left\{\{\{A_{t+1}^i(s_{t+1},s^t)\}_{s_{t+1}\in S}\}_{t=0}^{\infty}\right\}_{i\in\mathcal{I}}$ such that

1. Given a price system, an initial distribution of wealth, and a collection of natural borrowing limits, each individaul $i\in\mathcal{I}$ solves the following problem:

$$\{\tilde{c}_t^i(s^t),\ \{\tilde{a}_{t+1}^i(s_{t+1},s^t)\}_{s_{t+1}\in S}\}_{t=0}^{\infty} = arg \max_{\left\{c_t^i(s^t),\ \{a_{t+1}^i(s_{t+1},s^t)\}_{s_{t+1}\in S}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\sum_{s^t}\beta^t u\Big(c_t^i(s^t)\Big)\pi_t(s^t)$$

$$s.t. \qquad c_t^i(s^t) + \sum_{s_{t+1}\in S} a_{t+1}^i(s_{t+1},s^t)Q_t(s_{t+1}|s^t) \le y_t^i(s^t) + a_t^i(s^t)$$

$$a_{t+1}^i(s_{t+1},s^t) \ge -A_{t+1}^i(s_{t+1},s^t) \qquad \forall\ s_{t+1}\in S$$

2. On every history s^t at time t, markets clear.

$$\sum_{i\in\mathcal{I}}\tilde{c}_t^i(s^t) = \sum_{i\in\mathcal{I}} y_t^i(s^t) \qquad \text{(Commodity market clearing)}$$

$$\sum_{i\in\mathcal{I}}\tilde{a}_{t+1}^i(s_{t+1},s^t) = 0 \qquad \forall\ s_{t+1}\in S \qquad \text{(Asset market clearing)}$$

Equivalence of Allocations

$$Q_t(s_{t+1}|s^t) = \frac{q^0_{t+1}(s^{t+1})}{q^0_t(s^t)} \qquad \Rightarrow \qquad \beta \frac{u'\left(\tilde{c}^i_{t+1}(s^{t+1})\right)}{u'\left(\tilde{c}^i_t(s^t)\right)} \pi_t(s^{t+1}|s^t) = \beta \frac{u'\left(c^{i*}_{t+1}(s^{t+1})\right)}{u'\left(c^{i*}_t(s^t)\right)} \pi_t(s^{t+1}|s^t)$$

Guess for Portfolio

On every history s^t at time t,

$$\tilde{a}^i_{t+1}(s_{t+1}, s^t) = \sum_{\tau=t+1}^{\infty} \sum_{s^\tau | (s_{t+1}, s^t)} \frac{q^0_\tau(s^\tau)}{q^0_{t+1}(s^{t+1})} \Big(c^{i*}_\tau(s^\tau) - y^i_\tau(s^\tau) \Big) \qquad \forall \; s_{t+1} \in S$$

Value of this portfolio expressed in terms of the date t, history s^t consumption good is

$$\begin{aligned}
\sum_{s_{t+1} \in S} \tilde{a}^i_{t+1}(s_{t+1}, s^t) Q_t(s_{t+1}|s^t) &= \sum_{s_{t+1} \in S} \sum_{\tau=t+1}^{\infty} \sum_{s^\tau | (s_{t+1}, s^t)} \frac{q^0_\tau(s^\tau)}{q^0_{t+1}(s^{t+1})} \Big(c^{i*}_\tau(s^\tau) - y^i_t(s^\tau) \Big) Q_t(s_{t+1}|s^t) \\
&= \sum_{s_{t+1} \in S} \sum_{\tau=t+1}^{\infty} \sum_{s^\tau | (s_{t+1}, s^t)} \frac{q^0_\tau(s^\tau)}{\cancel{q^0_{t+1}(s^{t+1})}} \Big(c^{i*}_\tau(s^\tau) - y^i_t(s^\tau) \Big) \frac{\cancel{q^0_{t+1}(s^{t+1})}}{q^0_t(s^t)} \\
&= \sum_{\tau=t+1}^{\infty} \sum_{s^\tau | s^t} \frac{q^0_\tau(s^\tau)}{q^0_t(s^t)} \Big(c^{i*}_\tau(s^\tau) - y^i_t(s^\tau) \Big)
\end{aligned}$$

Verify Portfolio

On history $s^0 = s_0$ at time $t = 0$, assume that $a_0^i(s_0) = 0$. Then

$$\tilde{c}_0^i(s_0) + \sum_{s_1 \in S} \tilde{a}_1^i(s_1, s_0) Q_1(s_1|s_0) = y_0^i(s_0) + 0$$

$$\tilde{c}_0^i(s_0) + \sum_{\tau=1}^{\infty} \sum_{s^\tau | s_0} \frac{q_\tau^0(s^\tau)}{q_0^0(s_0)} \Big(c_\tau^{i*}(s^\tau) - y_t^i(s^\tau) \Big) = y_0^i(s_0) + 0$$

$$q_0^0(s_0) c_0^{i*}(s_0) + \sum_{\tau=1}^{\infty} \sum_{s^\tau | s_0} q_\tau^0(s^\tau) \Big(c_\tau^{i*}(s^\tau) - y_t^i(s^\tau) \Big) = q_0^0(s_0) y_0^i(s_0) \qquad \text{(if } \tilde{c}_0^i(s_0) = c_0^{i*}(s_0) \text{)}$$

$$\sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) y_t^i(s^t) = \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) c_t^{i*}(s^t)$$

Therefore, given $\tilde{c}_0^i(s_0) = c_0^{i*}(s_0)$, portfolio $\{\tilde{a}_1^i(s_1, s_0)\}_{s_1 \in S}$ is affordable.

Verify Portfolio

On history s^t at time t, assume that $\tilde{a}^i_t(s^t) = \sum_{\tau=t}^{\infty} \sum_{s^\tau \mid s^t} \frac{q^0_\tau(s^\tau)}{q^0_t(s^t)} \Big(c^{i*}_\tau(s^\tau) - y^i_\tau(s^\tau) \Big)$. Then

$$\tilde{c}^i_t(s^t) + \sum_{s_{t+1} \in S} \tilde{a}^i_{t+1}(s_{t+1}, s^t) Q_t(s_{t+1}|s^t) = y^i_t(s^t) + \sum_{\tau=t}^{\infty} \sum_{s^\tau \mid s^t} \frac{q^0_\tau(s^\tau)}{q^0_t(s^t)} \Big(c^{i*}_\tau(s^\tau) - y^i_\tau(s^\tau) \Big)$$

$$\tilde{c}^i_t(s^t) + \sum_{\tau=t+1}^{\infty} \sum_{s^\tau \mid s^t} \frac{q^0_\tau(s^\tau)}{q^0_t(s^t)} \Big(c^{i*}_\tau(s^\tau) - y^i_t(s^\tau) \Big) = y^i_t(s^t) + \sum_{\tau=t}^{\infty} \sum_{s^\tau \mid s^t} \frac{q^0_\tau(s^\tau)}{q^0_t(s^t)} \Big(c^{i*}_\tau(s^\tau) - y^i_\tau(s^\tau) \Big)$$

$$q^0_t(s^t) c^{i*}_t(s^t) + \sum_{\tau=t+1}^{\infty} \sum_{s^\tau \mid s^t} q^0_\tau(s^\tau) \Big(c^{i*}_\tau(s^\tau) - y^i_t(s^\tau) \Big) = q^0_t(s^t) y^i_t(s^t) + \sum_{\tau=t}^{\infty} \sum_{s^\tau \mid s^t} q^0_\tau(s^\tau) \Big(c^{i*}_\tau(s^\tau) - y^i_\tau(s^\tau) \Big)$$

(if $\tilde{c}^i_t(s^t) = c^{i*}_t(s^t)$)

$$\sum_{\tau=t}^{\infty} \sum_{s^\tau \mid s^t} q^0_\tau(s^\tau) \Big(c^{i*}_\tau(s^\tau) - y^i_\tau(s^\tau) \Big) = \sum_{\tau=t}^{\infty} \sum_{s^\tau \mid s^t} q^0_\tau(s^\tau) \Big(c^{i*}_\tau(s^\tau) - y^i_\tau(s^\tau) \Big)$$

Therefore, given $\tilde{c}^i_t(s^t) = c^{i*}_t(s^t)$, portfolio $\{\tilde{a}^i_{t+1}(s_{t+1}, s^t)\}_{s_{t+1} \in S}$ is affordable.

Equivalence of Allocations

- We have shown that the proposed portfolio strategy attains the same con- sumption plan as in the competitive equilibrium of the Arrow-Debreu economy.
- What precludes household i from further increasing current consumption by reducing some component of the asset portfolio?
  - ⋆ Natural borrowing limits
- These are all nice, but terribly abstract and complicated. So we impose a Markov structure, which admits a beautiful recursive structure.

Household's Recursive Problem

$$V(a,s) = \max_{c,\ \{a'(s')\}_{s'\in S}} \left\{ u(c) + \beta \underbrace{\sum_{s'\in S} V\Big(a'(s'), s'\Big)\pi(s'|s)}_{\mathbb{E}_s\left[V\left(a'(s'),\ s'\right)\right]} \right\}$$

$$\begin{aligned} s.t. \qquad & c + \sum_{s'\in S} Q(s'|s)a'(s') \le y(s) + a \\ & a'(s') \ge -A(s') \qquad \text{where} \qquad A(s') = y(s') + \sum_{s"\in S} Q(s"|s')A(s") \end{aligned}$$

$h(a,s)$ and $\{g(a,s,s')\}_{s'\in S}$ are associated policy function for consumption and Arrow securities.

Recursive Competitive Equilibrium (RCE)

Definition A recursive competitive equilibrium is a price kernel $\{Q(s'|s)\}_{s' \in S}$, sets of value functions $\{V^i(a,s)\}_{i \in \mathcal{I}}$, sets of policy functions $\left\{h^i(a,s), \{g^i(a,s,s')\}_{s' \in S}\right\}_{i \in \mathcal{I}}$, an initial distribution of wealth $\{a^i\}_{i \in \mathcal{I}}$ where $\sum_{i \in \mathcal{I}} a^i = 0$, and a collection of natural borrowing limits $\left\{\{A^i(s')\}_{s' \in S}\right\}_{i \in \mathcal{I}}$ such that

1. The state-by-state borrowing constraints satisfy the recursion.

$$A(s') = y(s') + \sum_{s" \in S} Q(s"|s')A(s")$$

2. Given a price kernel, an initial distribution of wealth, and a collection of natural borrowing limits, each individaul $i \in \mathcal{I}$'s value function and policy function solves the household's recursive problem.

3. On every state s, given a, markets clear.

$$\sum_{i \in \mathcal{I}} c^i = \sum_{i \in \mathcal{I}} y^i(s) \qquad \text{where} \qquad c^i = h(a,s) \qquad \text{(Commodity market clearing)}$$

$$\sum_{i \in \mathcal{I}} a'^i(s') = 0 \qquad \forall\ s' \in S \qquad \text{where} \qquad a'^i(s') = g(a,s,s') \qquad \text{(Asset market clearing)}$$

Reference

Ljungqvist, L., & Sargent, T. J. (2018). Recursive macroeconomic theory. MIT press.