ECON 204C - Macroeconomic Theory **Incomplete Markets Model**

TA: Woongchan Jeon Week 4 - April 24, 2020

These practice problems are pulled from Ljungqvist and Sargent (2018).

1. Random discount factor Consider a Huggett-style incomplete markets model where individual households face discount factor shocks. A household has preferences over consumption of a single good ordered by a value function defined recursively by

$$v(\beta_t, a_t, s_t) = u(c_t) + \beta_t \mathbb{E}_t \Big[v(\beta_{t+1}, a_{t+1}, s_{t+1}) \Big]$$

where $\beta_t \in (0, 1)$ is the time t value of a discount factor, and a_t is time t holding of a single asset. Here v is the discounted utility for a consumer with asset holding a_t , discount factor β_t, and employment state s_t . The discount factor evolves according to a three-state Markov chain with transition probabilities $P(\beta'|\beta) = Prob(\beta_{t+1} = \beta'|\beta_t = \beta)$. The discount factor and employment state at t are both known. The household faces the sequence of budget constraints

$$c_t + qa_{t+1} \leq a_t + s_t$$

where s_t evolves according to an n-state Markov chain with transition probability $Q(s'|s) = Prob(s_{t+1} = s'|s_t = s)$. The household faces the borrowing constraint $a_{t+1} \geq -\phi$ for all t. Formulate Bellman equations for the household's problem and define a stationary equilibrium for this economy.

2. Unemployment There is a continuum of workers with identical probabilities λ of being fired each period when they are employed. With probability $\mu \in (0, 1)$, each unemployed worker receives one offer to work at wage w drawn from the cumulative distribution function F with the support $[0, B]$. If he accepts the offer, the worker receives the offered wage each period until he is fired. With probability $1 - \mu$, an unemployed worker receives no offer this period. The probability μ is determined by the function $\mu = f(U)$, where U is the unemployment rate, and $f'(U) < 0, f(0) = 1, f(1) = 0$. A worker's utility is given by $\mathbb{E}_{t=1}^{\infty} = \beta^t y_t$, where $\beta \in (0, 1)$ and y_t is income in period t, which equals the wage if employed and zero otherwise. There is no unemployment compensation. Each worker regards U as fixed and constant over time in making his decisions.

a. For fixed U, write the Bellman equation for the worker. Argue that his optimal policy has the reservation wage property.

b. Given the typical worker's policy (i.e., his reservation wage), display a difference equation for the unemployment rate. Show that a stationary unemployment

$$\lambda(1-U) = Uf(U)[1-F(\bar{w})],$$

where $\bar{w}$ is the reservation wage.

c. Define a stationary equilibrium.