(MV) Optimal Risky Portfolio

$$\boldsymbol{x} = \arg\max_{\boldsymbol{x}} \frac{E(R_p) - r_f}{\sigma_P}$$

subject to

$$\sum_i x_i = 1$$

and other applicable constraints

- Numerical solution
  - e.g. Excel Solver

(MV) Optimal Risky Portfolio

- Analytical solution

[1] Solve $E(\boldsymbol{R}) - \boldsymbol{r}_f = \boldsymbol{\Sigma x}$

[2] Scale $\boldsymbol{x}$ such that $\sum_i x_i = 1$

Derivation (1)

- Maximize $f(\boldsymbol{x}) = \ln SR_P = \ln\left[E(R_P) - r_f \sum_i x_i\right] - \frac{1}{2}\ln \sigma_P^2$

subject to $\sum_i x_i = 1$

- Set $\frac{\partial f}{\partial x_i} = \frac{E(R_i)-r_f}{E(R_P)-r_f} - \frac{\sigma_{iP}}{\sigma_P^2} = 0$

$$E(R_i) - r_f = \left[\frac{E(R_P) - r_f \sum_i x_i}{\sigma_P^2}\right]\sigma_{iP} = k\sigma_{iP}$$

- Easy to show $f(k\boldsymbol{x}) = \ln \frac{kE(R_P)-r_f \sum_i kx_i}{k\sigma_P} = f(\boldsymbol{x})$

Derivation (2)

$$E(R_i) - r_f = k\sigma_{iP}$$

- WLOG, set $k = 1$. Solve for $\boldsymbol{x}$ and scale

$$E(R_i) - r_f = \sigma_{iP} = \sum_j x_j \sigma_{ij}\,, \qquad i = 1, \dots, n$$

$$E(\boldsymbol{R}) - \boldsymbol{r}_f = \boldsymbol{\Sigma x}$$

That was