

**Problem 1. LASSO**

The data file, *data.csv*, contains observations generated by

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} + \epsilon_i$$

You decide to investigate three regression models:

a null model  $E(y_i|x_i) = \beta_0$  with the least squares loss

a full model  $E(y_i|x_i) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5}$  with the least squares loss

a LASSO model  $E(y_i|x_i) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5}$  with the LASSO loss

You also decide to fit these models using *centered-and-normalized features* with each feature having a zero mean and a unit L2-norm.

Please code to calculate and report the following in your writeup (see the report template):

Model	$\lambda_{best}$	$ \hat{\beta} $	$RMSE_{fit}$	$R^2_{fit}$	$E(RMSE_{test})$	$E(R^2_{test})$
Null	na	na				
Full	na					
LASSO at $\lambda_{best}$ by LassoCV						
LASSO at $\lambda_{best}$ by YourCV w. rescaling						
LASSO at $\lambda_{best}$ by YourCV w/o rescaling						

Model	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$
Null						
Full						
LASSO at $\lambda_{best}$ by LassoCV						
LASSO at $\lambda_{best}$ by YourCV w. rescaling						
LASSO at $\lambda_{best}$ by YourCV w/o rescaling						

For convenience of grading,

A. Please use a 10-fold CV when estimating  $E(RMSE_{test})$  and  $E(R^2_{test})$  for the Null model and the Full model.

B.  $|\hat{\beta}| = \sum_{j=1}^5 |\beta_j|$  does NOT include the intercept.

C.  $\lambda_{best}$  by LassoCV is obtained by

`LassoCV(alphas = lambdas, cv = KFold(n_splits = 10, shuffle = True, random_state = 10))`

D.  $\lambda_{best}$  by YourCV is obtained by identifying the  $\lambda$  from  $\lambda = \text{np.linspace}(0, 0.1, 101)$  that has the smallest estimated test RMSE, using a 10-fold cross validation approach. Please use

`KFold(n_splits = 10, shuffle = True, random_state = 10)`

You may find the following resources useful.

[https://scikit-learn.org/stable/modules/generated/sklearn.linear\\_model.Lasso.html](https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.Lasso.html)

[https://scikit-learn.org/stable/modules/generated/sklearn.linear\\_model.LassoCV.html](https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LassoCV.html)