

**Problem 1. KNN**

$X_1 + \sin X_2 - 5 = 0$ , where  $X_1 \in [0, 10]$  and  $X_2 \in [0, 10]$  defines a boundary in the feature space. In addition, the true class probabilities are

$$\Pr(Y = 1 | \mathbf{x} \text{ is on the good side of the boundary}) = 0.8 + (x_1 + x_2) * 0.01$$

$$\Pr(Y = 2 | \mathbf{x} \text{ is not on the good side of the boundary}) = 0.8 + (x_1 + x_2) * 0.01$$

Feel free to define which side of the boundary is the good side.

[1] Produce a chart which shows, on the feature space  $\{(X_1, X_2)\}$ , the decision regions of the Bayes Classifier.

[2] Code to estimate the overall Bayes error rate

[3] Generate a sample, consistent with the true class probabilities, on the grid where  $X_1 \in \{0, 0.2, 0.4, 0.6, \dots, 10\}$  and  $X_2 \in \{0, 0.2, 0.4, 0.6, \dots, 10\}$ .

Produce a chart which shows, on the feature space  $\{(X_1, X_2)\}$ , the decision regions of a KNN Classifier with  $K = 5$ .

[4] Code to estimate the test error rate of the KNN Classifier with  $K = 5$ , using the 5-fold cross validation approach. Report the following

Validation Set	Estimated Test Error Rate for this Validation Set
1	
2	
3	
4	
5	
Estimated KNN(5) Error Rate	

You may find the following resources useful.

<https://scikit-learn.org/stable/modules/generated/sklearn.neighbors.KNeighborsClassifier.html>  
[https://scikit-learn.org/stable/modules/generated/sklearn.model\\_selection.KFold.html](https://scikit-learn.org/stable/modules/generated/sklearn.model_selection.KFold.html)