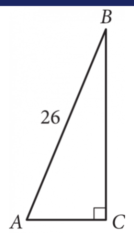
Question ID bd87bc09

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	■■■

ID: bd87bc09



Triangle ABC above is a right triangle, and $\sin(B) = \frac{5}{13}$.

What is the length of side $\overline{BC}$?

ID: bd87bc09 Answer

Rationale

The correct answer is 24. The sine of an acute angle in a right triangle is equal to the ratio of the length of the side opposite the angle to the length of the hypotenuse. In the triangle shown, the sine of angle B, or $\sin(B)$, is equal to the ratio of the length of side $\overline{AC}$ to the length of side $\overline{AB}$. It's given that the length of side $\overline{AB}$ is 26 and that $\sin(B) = \frac{5}{13}$. Therefore, $\frac{5}{13} = \frac{AB}{26}$. Multiplying both sides of this equation by 26 yields $AC = 10$.

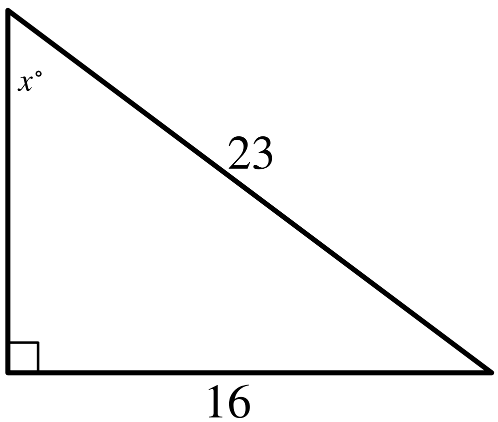
By the Pythagorean Theorem, the relationship between the lengths of the sides of triangle ABC is as follows: $26^2 = 10^2 + BC^2$, or $676 = 100 + BC^2$. Subtracting 100 from both sides of $676 = 100 + BC^2$ yields $576 = BC^2$. Taking the square root of both sides of $576 = BC^2$ yields $24 = BC$.

Question Difficulty: Hard

Question ID 1429dcdf

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	■■■

ID: 1429dcdf



Note: Figure not drawn to scale.

In the triangle shown, what is the value of $\sin x^\circ$?

ID: 1429dcdf Answer

Correct Answer: .6956, .6957, 16/23

Rationale

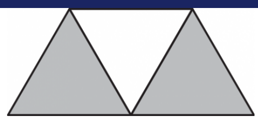
The correct answer is $\frac{16}{23}$. In a right triangle, the sine of an acute angle is defined as the ratio of the length of the side opposite the angle to the length of the hypotenuse. In the triangle shown, the length of the side opposite the angle with measure x° is 16 units and the length of the hypotenuse is 23 units. Therefore, the value of $\sin x^\circ$ is $\frac{16}{23}$. Note that 16/23, .6956, .6957, 0.695, and 0.696 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 4c95c7d4

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	■■■

ID: 4c95c7d4



A graphic designer is creating a logo for a company. The logo is shown in the figure above. The logo is in the shape of a trapezoid and consists of three congruent equilateral triangles. If the perimeter of the logo is 20 centimeters, what is the combined area of the shaded regions, in square centimeters, of the logo?

A. $2\sqrt{3}$

B. $4\sqrt{3}$

C. $8\sqrt{3}$

D. 16

ID: 4c95c7d4 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the logo is in the shape of a trapezoid that consists of three congruent equilateral triangles, and that the perimeter of the trapezoid is 20 centimeters (cm). Since the perimeter of the trapezoid is the sum of the lengths of 5 of the sides of the triangles, the length of each side of an equilateral triangle is $\frac{20}{5} = 4$ cm. Dividing up one equilateral triangle into two right triangles yields a pair of congruent 30°-60°-90° triangles. The shorter leg of each right triangle is half the length of the side of an equilateral triangle, or 2 cm. Using the Pythagorean Theorem, $a^2 + b^2 = c^2$, the height of the equilateral triangle can be found. Substituting $a = 2$ and $c = 4$ and solving for b yields $\sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3}$ cm. The area of one equilateral triangle is $\frac{1}{2}bh$, where $b = 2$ and $h = 2\sqrt{3}$. Therefore, the area of one equilateral triangle is $\frac{1}{2}(4)(2\sqrt{3}) = 4\sqrt{3}$ cm^2. The shaded area consists of two such triangles, so its area is $(2)(4)\sqrt{3} = 8\sqrt{3}$ cm^2.

Alternate approach: The area of a trapezoid can be found by evaluating the expression $\frac{1}{2}(b_1 + b_2)h$, where b_1 is the length of one base, b_2 is the length of the other base, and h is the height of the trapezoid. Substituting $b_1 = 8$, $b_2 = 4$, and

$h = 2\sqrt{3}$ yields the expression $\frac{1}{2}(8+4)(2\sqrt{3})$, or $\frac{1}{2}(12)(2\sqrt{3})$, which gives an area of $12\sqrt{3}$ cm^2 for the trapezoid. Since two-thirds of the trapezoid is shaded, the area of the shaded region is $\frac{2}{3} \times 12\sqrt{3} = 8\sqrt{3}$.

Choice A is incorrect. This is the height of the trapezoid. Choice B is incorrect. This is the area of one of the equilateral triangles, not two. Choice D is incorrect and may result from using a height of 4 for each triangle rather than the height of $2\sqrt{3}$.

Question Difficulty: Hard

Question ID a4bd60a3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	■■■

ID: a4bd60a3

The perimeter of an equilateral triangle is 624 centimeters. The height of this triangle is $k\sqrt{3}$ centimeters, where k is a constant. What is the value of k?

ID: a4bd60a3 Answer

Correct Answer: 104

Rationale

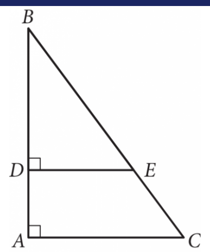
The correct answer is 104. An equilateral triangle is a triangle in which all three sides have the same length and all three angles have a measure of $60°$. The height of the triangle, $k\sqrt{3}$, is the length of the altitude from one vertex. The altitude divides the equilateral triangle into two congruent 30-60-90 right triangles, where the altitude is the side across from the $60°$ angle in each 30-60-90 right triangle. Since the altitude has a length of $k\sqrt{3}$, it follows from the properties of 30-60-90 right triangles that the side across from each $30°$ angle has a length of k and each hypotenuse has a length of $2k$. In this case, the hypotenuse of each 30-60-90 right triangle is a side of the equilateral triangle; therefore, each side length of the equilateral triangle is $2k$. The perimeter of a triangle is the sum of the lengths of each side. It's given that the perimeter of the equilateral triangle is 624; therefore, $2k + 2k + 2k = 624$, or $6k = 624$. Dividing both sides of this equation by 6 yields $k = 104$.

Question Difficulty: Hard

Question ID 55bb437a

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	■■■

ID: 55bb437a



In the figure above, $\tan B = \frac{3}{4}$. If $BC = 15$ and $DA = 4$, what is the length of $\overline{DE}$?

ID: 55bb437a Answer

Rationale

The correct answer is 6. Since $\tan B = \frac{3}{4}$, $\triangle ABC$ and $\triangle DBE$ are both similar to 3-4-5 triangles. This means that they are both similar to the right triangle with sides of lengths 3, 4, and 5. Since $BC = 15$, which is 3 times as long as the hypotenuse of the 3-4-5 triangle, the similarity ratio of $\triangle ABC$ to the 3-4-5 triangle is 3:1. Therefore, the length of $\overline{AC}$ (the side opposite to $\angle B$) is $3 \times 3 = 9$, and the length of $\overline{AB}$ (the side adjacent to $\angle B$) is $4 \times 3 = 12$. It is also given that $DA = 4$. Since $AB = DA + DB$ and $AB = 12$, it follows that $DB = 8$, which means that the similarity ratio of $\triangle DBE$ to the 3-4-5 triangle is 2:1 ($\overline{DB}$ is the side adjacent to $\angle B$). Therefore, the length of $\overline{DE}$, which is the side opposite to $\angle B$, is $3 \times 2 = 6$.

Question Difficulty: Hard

Question ID 568d66a7

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	

ID: 568d66a7

An isosceles right triangle has a perimeter of $94 + 94\sqrt{2}$ inches. What is the length, in inches, of one leg of this triangle?

A. 47

B. $47\sqrt{2}$

C. 94

D. $94\sqrt{2}$

ID: 568d66a7 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that the right triangle is isosceles. In an isosceles right triangle, the two legs have equal lengths, and the length of the hypotenuse is $\sqrt{2}$ times the length of one of the legs. Let l represent the length, in inches, of each leg of the isosceles right triangle. It follows that the length of the hypotenuse is $l\sqrt{2}$ inches. The perimeter of a figure is the sum of the lengths of the sides of the figure. Therefore, the perimeter of the isosceles right triangle is $l + l + l\sqrt{2}$ inches. It's given that the perimeter of the triangle is $94 + 94\sqrt{2}$ inches. It follows that $l + l + l\sqrt{2} = 94 + 94\sqrt{2}$. Factoring the left-hand side of this equation yields $1 + 1 + \sqrt{2}l = 94 + 94\sqrt{2}$, or $2 + \sqrt{2}l = 94 + 94\sqrt{2}$. Dividing both sides of this equation by $2 + \sqrt{2}$ yields $l = \frac{94 + 94\sqrt{2}}{2 + \sqrt{2}}$. Rationalizing the denominator of the right-hand side of this equation by multiplying the right-hand side of the equation by $\frac{2 - \sqrt{2}}{2 - \sqrt{2}}$ yields $l = \frac{94 + 94\sqrt{2}\,2 - \sqrt{2}}{2 + \sqrt{2}\,2 - \sqrt{2}}$. Applying the distributive property to the numerator and to the denominator of the right-hand side of this equation yields $l = \frac{188 - 94\sqrt{2} + 188\sqrt{2} - 94\sqrt{4}}{4 - 2\sqrt{2} + 2\sqrt{2} - \sqrt{4}}$. This is equivalent to $l = \frac{94\sqrt{2}}{2}$, or $l = 47\sqrt{2}$. Therefore, the length, in inches, of one leg of the isosceles right triangle is $47\sqrt{2}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is the length, in inches, of the hypotenuse.

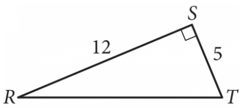
Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 6933b3d9

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	■■■

ID: 6933b3d9



In triangle RST above, point W (not shown) lies on $\overline{RT}$. What is the value of $\cos(\angle RSW) - \sin(\angle WST)$?

ID: 6933b3d9 Answer

Rationale

The correct answer is 0. Note that no matter where point W is on $\overline{RT}$, the sum of the measures of $\angle RSW$ and $\angle WST$ is equal to the measure of $\angle RST$, which is $90°$. Thus, $\angle RSW$ and $\angle WST$ are complementary angles. Since the cosine of an angle is equal to the sine of its complementary angle, $\cos(\angle RSW) = \sin(\angle WST)$. Therefore, $\cos(\angle RSW) - \sin(\angle WST) = 0$.

Question Difficulty: Hard

Question ID 6ab30ce3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	

ID: 6ab30ce3

Triangle ABC is similar to triangle DEF, where A corresponds to D and C corresponds to F. Angles C and F are right angles. If $\tan(A) = \sqrt{3}$ and $DF = 125$, what is the length of $\overline{DE}$?

A. $125\frac{\sqrt{3}}{3}$

B. $125\frac{\sqrt{3}}{2}$

C. $125\sqrt{3}$

D. 250

ID: 6ab30ce3 Answer

Correct Answer: D

Rationale

Choice D is correct. Corresponding angles in similar triangles have equal measures. It's given that triangle ABC is similar to triangle DEF, where A corresponds to D, so the measure of angle A is equal to the measure of angle D. Therefore, if $\tan A = \sqrt{3}$, then $\tan D = \sqrt{3}$. It's given that angles C and F are right angles, so triangles ABC and DEF are right triangles. The adjacent side of an acute angle in a right triangle is the side closest to the angle that is not the hypotenuse. It follows that the adjacent side of angle D is side DF. The opposite side of an acute angle in a right triangle is the side across from the acute angle. It follows that the opposite side of angle D is side EF. The tangent of an acute angle in a right triangle is the ratio of the length of the opposite side to the length of the adjacent side. Therefore, $\tan D = \frac{EF}{DF}$. If $DF = 125$, the length of side EF can be found by substituting $\sqrt{3}$ for $\tan D$ and 125 for DF in the equation $\tan D = \frac{EF}{DF}$, which yields $\sqrt{3} = \frac{EF}{125}$. Multiplying both sides of this equation by 125 yields $125\sqrt{3} = EF$. Since the length of side EF is $\sqrt{3}$ times the length of side DF, it follows that triangle DEF is a special right triangle with angle measures $30°$, $60°$, and $90°$. Therefore, the length of the hypotenuse, $\overline{DE}$, is 2 times the length of side DF, or $DE = 2DF$. Substituting 125 for DF in this equation yields $DE = 2125$, or $DE = 250$. Thus, if $\tan A = \sqrt{3}$ and $DF = 125$, the length of $\overline{DE}$ is 250.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is the length of $\overline{EF}$, not $\overline{DE}$.

Question Difficulty: Hard

Question ID 7c25b0dc

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	■■■

ID: 7c25b0dc

The length of a rectangle's diagonal is $3\sqrt{17}$, and the length of the rectangle's shorter side is 3. What is the length of the rectangle's longer side?

ID: 7c25b0dc Answer

Correct Answer: 12

Rationale

The correct answer is 12. The diagonal of a rectangle forms a right triangle, where the shorter side and the longer side of the rectangle are the legs of the triangle and the diagonal of the rectangle is the hypotenuse of the triangle. It's given that the length of the rectangle's diagonal is $3\sqrt{17}$ and the length of the rectangle's shorter side is 3. Thus, the length of the hypotenuse of the right triangle formed by the diagonal is $3\sqrt{17}$ and the length of one of the legs is 3. By the Pythagorean theorem, if a right triangle has a hypotenuse with length c and legs with lengths a and b, then $a^2 + b^2 = c^2$. Substituting $3\sqrt{17}$ for c and 3 for b in this equation yields $a^2 + 3^2 = 3\sqrt{17}^2$, or $a^2 + 9 = 153$. Subtracting 9 from both sides of this equation yields $a^2 = 144$. Taking the square root of both sides of this equation yields $a = \pm\sqrt{144}$, or $a = \pm 12$. Since a represents a length, which must be positive, the value of a is 12. Thus, the length of the rectangle's longer side is 12.

Question Difficulty: Hard

Question ID c6dff223

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	■■■

ID: c6dff223

Triangle ABC is similar to triangle DEF, where angle A corresponds to angle D and angles C and F are right angles. The length of $\overline{AB}$ is 2.9 times the length of $\overline{DE}$. If $\tan A = \frac{21}{20}$, what is the value of $\sin D$?

ID: c6dff223 Answer

Correct Answer: .7241, 21/29

Rationale

The correct answer is $\frac{21}{29}$. It's given that triangle ABC is similar to triangle DEF, where angle A corresponds to angle D and angles C and F are right angles. In similar triangles, the tangents of corresponding angles are equal. Therefore, if $\tan A = \frac{21}{20}$, then $\tan D = \frac{21}{20}$. In a right triangle, the tangent of an acute angle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle. Therefore, in triangle DEF, if $\tan D = \frac{21}{20}$, the ratio of the length of $\overline{EF}$ to the length of $\overline{DF}$ is $\frac{21}{20}$. If the lengths of $\overline{EF}$ and $\overline{DF}$ are 21 and 20, respectively, then the ratio of the length of $\overline{EF}$ to the length of $\overline{DF}$ is $\frac{21}{20}$. In a right triangle, the sine of an acute angle is the ratio of the length of the leg opposite the angle to the length of the hypotenuse. Therefore, the value of $\sin D$ is the ratio of the length of $\overline{EF}$ to the length of $\overline{DE}$. The length of $\overline{DE}$ can be calculated using the Pythagorean theorem, which states that if the lengths of the legs of a right triangle are a and b and the length of the hypotenuse is c, then $a^2 + b^2 = c^2$. Therefore, if the lengths of $\overline{EF}$ and $\overline{DF}$ are 21 and 20, respectively, then $21^2 + 20^2 = DE^2$, or $841 = DE^2$. Taking the positive square root of both sides of this equation yields $29 = DE$. Therefore, if the lengths of $\overline{EF}$ and $\overline{DF}$ are 21 and 20, respectively, then the length of $\overline{DE}$ is 29 and the ratio of the length of $\overline{EF}$ to the length of $\overline{DE}$ is $\frac{21}{29}$. Thus, if $\tan A = \frac{21}{20}$, the value of $\sin D$ is $\frac{21}{29}$. Note that 21/29, .7241, and 0.724 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 92eb236a

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	■■■

ID: 92eb236a

In a right triangle, the tangent of one of the two acute angles is $\frac{\sqrt{3}}{3}$. What is the tangent of the other acute angle?

A. $-\frac{\sqrt{3}}{3}$

B. $-\frac{3}{\sqrt{3}}$

C. $\frac{\sqrt{3}}{3}$

D. $\frac{3}{\sqrt{3}}$

ID: 92eb236a Answer

Correct Answer: D

Rationale

Choice D is correct. The tangent of a nonright angle in a right triangle is defined as the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle. Using that definition for tangent, in a right triangle with legs that have lengths a and b, the tangent of one acute angle is $\frac{a}{b}$ and the tangent for the other acute angle is $\frac{b}{a}$. It follows that the tangents of the acute angles in a right triangle are reciprocals of each other. Therefore, the tangent of the other acute angle in the given triangle is the reciprocal of $\frac{\sqrt{3}}{3}$ or $\frac{3}{\sqrt{3}}$.

Choice A is incorrect and may result from assuming that the tangent of the other acute angle is the negative of the tangent of the angle described. Choice B is incorrect and may result from assuming that the tangent of the other acute angle is the negative of the reciprocal of the tangent of the angle described. Choice C is incorrect and may result from interpreting the tangent of the other acute angle as equal to the tangent of the angle described.

Question Difficulty: Hard

Question ID 2be01bd9

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	■■■

ID: 2be01bd9

Triangle ABC is similar to triangle DEF, where angle A corresponds to angle D and angle C corresponds to angle F. Angles C and F are right angles. If $\tan(A) = \frac{50}{7}$, what is the value of $\tan(E)$?

ID: 2be01bd9 Answer

Correct Answer: .14, 7/50

Rationale

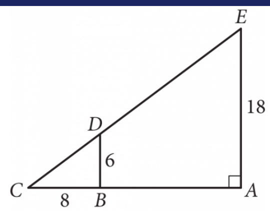
The correct answer is $\frac{7}{50}$. It's given that triangle ABC is similar to triangle DEF, where angle A corresponds to angle D and angle C corresponds to angle F. In similar triangles, the tangents of corresponding angles are equal. Since angle A and angle D are corresponding angles, if $\tan A = \frac{50}{7}$, then $\tan D = \frac{50}{7}$. It's also given that angles C and F are right angles. It follows that triangle DEF is a right triangle with acute angles D and E. The tangent of one acute angle in a right triangle is the inverse of the tangent of the other acute angle in the triangle. Therefore, $\tan E = \frac{1}{\tan D}$. Substituting $\frac{50}{7}$ for $\tan D$ in this equation yields $\tan E = \frac{1}{\frac{50}{7}}$, or $\tan E = \frac{7}{50}$. Thus, if $\tan A = \frac{50}{7}$, the value of $\tan E$ is $\frac{7}{50}$. Note that 7/50 and .14 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID dba6a25a

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	■■■

ID: dba6a25a



In the figure above, $\overline{BD}$ is parallel to $\overline{AE}$.

What is the length of $\overline{CE}$?

ID: dba6a25a Answer

Rationale

The correct answer is 30. In the figure given, since $\overline{BD}$ is parallel to $\overline{AE}$ and both segments are intersected by $\overline{CE}$, then angle BDC and angle AEC are corresponding angles and therefore congruent. Angle BCD and angle ACE are also congruent because they are the same angle. Triangle BCD and triangle ACE are similar because if two angles of one triangle are congruent to two angles of another triangle, the triangles are similar. Since triangle BCD and triangle ACE are similar, their corresponding sides are proportional. So in triangle BCD and triangle ACE, $\overline{BD}$ corresponds to $\overline{AE}$ and $\overline{CD}$ corresponds to $\overline{CE}$. Therefore, $\frac{BD}{CD} = \frac{AE}{CE}$. Since triangle BCD is a right triangle, the Pythagorean theorem can be used to give the value of CD: $6^2 + 8^2 = CD^2$. Taking the square root of each side gives $CD = 10$. Substituting the values in the proportion $\frac{BD}{CD} = \frac{AE}{CE}$ yields $\frac{6}{10} = \frac{18}{CE}$. Multiplying each side by CE, and then multiplying by $\frac{10}{6}$ yields $CE = 30$. Therefore, the length of $\overline{CE}$ is 30.

Question Difficulty: Hard

Question ID 25da87f8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	■■■

ID: 25da87f8

A triangle with angle measures 30°, 60°, and 90° has a perimeter of $18+6\sqrt{3}$.

What is the length of the longest side of the triangle?

ID: 25da87f8 Answer

Rationale

The correct answer is 12. It is given that the triangle has angle measures of 30°, 60°, and 90°, and so the triangle is a special right triangle. The side measures of this type of special triangle are in the ratio $2:1:\sqrt{3}$. If x is the measure of the shortest leg, then the measure of the other leg is $\sqrt{3}x$ and the measure of the hypotenuse is 2x. The perimeter of the triangle is given to be $18+6\sqrt{3}$, and so the equation for the perimeter can be written as $2x+x+\sqrt{3}x=18+6\sqrt{3}$. Combining like terms and factoring out a common factor of x on the left-hand side of the equation gives $(3+\sqrt{3})x=18+6\sqrt{3}$. Rewriting the right-hand side of the equation by factoring out 6 gives $(3+\sqrt{3})x=6(3+\sqrt{3})$. Dividing both sides of the equation by the common factor $(3+\sqrt{3})$ gives x = 6. The longest side of the right triangle, the hypotenuse, has a length of 2x, or 2(6), which is 12.

Question Difficulty: Hard