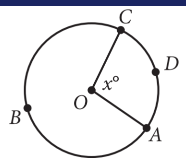
Question ID c8345903

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: c8345903



The circle above has center O, the length of arc $\overset{\frown}{ADC}$ is 5π, and $x = 100$. What is the length of arc $\overset{\frown}{ABC}$?

A. 9π

B. 13π

C. 18π

D. $\frac{13}{2}\pi$

ID: c8345903 Answer

Correct Answer: B

Rationale

Choice B is correct. The ratio of the lengths of two arcs of a circle is equal to the ratio of the measures of the central angles that subtend the arcs. It's given that arc $\overset{\frown}{ADC}$ is subtended by a central angle with measure 100°. Since the sum of the measures of the angles about a point is 360°, it follows that arc $\overset{\frown}{ABC}$ is subtended by a central angle with measure $360° - 100° = 260°$. If s is the length of arc $\overset{\frown}{ABC}$, then s must satisfy the ratio $\frac{s}{5\pi} = \frac{260}{100}$. Reducing the fraction $\frac{260}{100}$ to its simplest form gives $\frac{13}{5}$. Therefore, $\frac{s}{5\pi} = \frac{13}{5}$. Multiplying both sides of $\frac{s}{5\pi} = \frac{13}{5}$ by 5π yields $s = 13\pi$.

Choice A is incorrect. This is the length of an arc consisting of exactly half of the circle, but arc $\overset{\frown}{ABC}$ is greater than half of the circle. Choice C is incorrect. This is the total circumference of the circle. Choice D is incorrect. This is half the length of arc $\overset{\frown}{ABC}$, not its full length.

Question Difficulty: Hard

Question ID 2266984b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: 2266984b

$$x^2 + 20x + y^2 + 16y = -20$$

The equation above defines a circle in the *xy*-plane. What are the coordinates of the center of the circle?

A. $(-20, -16)$

B. $(-10, -8)$

C. $(10, 8)$

D. $(20, 16)$

ID: 2266984b Answer

Correct Answer: B

Rationale

Choice B is correct. The standard equation of a circle in the xy-plane is of the form $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) are the coordinates of the center of the circle and r is the radius. The given equation can be rewritten in standard form by completing the squares. So the sum of the first two terms, $x^2 + 20x$, needs a 100 to complete the square, and the sum of the second two terms, $y^2 + 16y$, needs a 64 to complete the square. Adding 100 and 64 to both sides of the given equation yields $(x^2 + 20x + 100) + (y^2 + 16y + 64) = -20 + 100 + 64$, which is equivalent to $(x+10)^2 + (y+8)^2 = 144$. Therefore, the coordinates of the center of the circle are $(-10, -8)$.

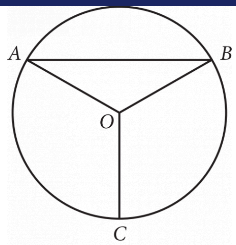
Choices A, C, and D are incorrect and may result from computational errors made when attempting to complete the squares or when identifying the coordinates of the center.

Question Difficulty: Hard

Question ID 69b0d79d

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: 69b0d79d



Point O is the center of the circle above, and the measure of $\angle OAB$ is 30°. If the length of $\overline{OC}$ is 18, what is the length of arc $\overset{\frown}{AB}$?

A. 9π

B. 12π

C. 15π

D. 18π

ID: 69b0d79d Answer

Correct Answer: B

Rationale

Choice B is correct. Because segments OA and OB are radii of the circle centered at point O, these segments have equal lengths. Therefore, triangle AOB is an isosceles triangle, where angles OAB and OBA are congruent base angles of the triangle. It's given that angle OAB measures 30°. Therefore, angle OBA also measures 30°. Let x° represent the measure of angle AOB. Since the sum of the measures of the three angles of any triangle is 180°, it follows that $30^\circ + 30^\circ + x^\circ = 180^\circ$, or $60^\circ + x^\circ = 180^\circ$. Subtracting 60° from both sides of this equation yields $x^\circ = 120^\circ$, or $\frac{2\pi}{3}$ radians. Therefore, the measure of angle AOB, and thus the measure of arc $\overset{\frown}{AB}$, is $\frac{2\pi}{3}$ radians. Since $\overline{OC}$ is a radius of the given circle and its length is 18, the length of the radius of the circle is 18. Therefore, the length of arc $\overset{\frown}{AB}$ can be calculated as $\left(\frac{2\pi}{3}\right)(18)$, or 12π.

Choices A, C, and D are incorrect and may result from conceptual or computational errors.

Question Difficulty: Hard

Question ID ebbf23ae

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: ebbf23ae

A circle in the *xy*-plane has a diameter with endpoints $(2, 4)$ and $(2, 14)$. An equation of this circle is $(x-2)^2 + (y-9)^2 = r^2$, where r is a positive constant. What is the value of r?

ID: ebbf23ae Answer

Correct Answer: 5

Rationale

The correct answer is 5. The standard form of an equation of a circle in the *xy*-plane is $x - h^2 + y - k^2 = r^2$, where h, k, and r are constants, the coordinates of the center of the circle are h, k, and the length of the radius of the circle is r. It's given that an equation of the circle is $x - 2^2 + y - 9^2 = r^2$. Therefore, the center of this circle is $2, 9$. It's given that the endpoints of a diameter of the circle are $2, 4$ and $2, 14$. The length of the radius is the distance from the center of the circle to an endpoint of a diameter of the circle, which can be found using the distance formula, $\sqrt{x_1 - x_2{}^2 + y_1 - y_2{}^2}$. Substituting the center of the circle $2, 9$ and one endpoint of the diameter $2, 4$ in this formula gives a distance of $\sqrt{2 - 2^2 + 9 - 4^2}$, or $\sqrt{0^2 + 5^2}$, which is equivalent to 5. Since the distance from the center of the circle to an endpoint of a diameter is 5, the value of r is 5.

Question Difficulty: Hard

Question ID b0a72bdc

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: b0a72bdc

What is the diameter of the circle in the *xy*-plane with equation $(x-5)^2+(y-3)^2=16$?

A. 4

B. 8

C. 16

D. 32

ID: b0a72bdc Answer

Correct Answer: B

Rationale

Choice B is correct. The standard form of an equation of a circle in the *xy*-plane is $x - h^2 + y - k^2 = r^2$, where the coordinates of the center of the circle are h, k and the length of the radius of the circle is r. For the circle in the *xy*-plane with equation $x - 5^2 + y - 3^2 = 16$, it follows that $r^2 = 16$. Taking the square root of both sides of this equation yields $r = 4$ or $r = -4$. Because r represents the length of the radius of the circle and this length must be positive, $r = 4$. Therefore, the radius of the circle is 4. The diameter of a circle is twice the length of the radius of the circle. Thus, 24 yields 8. Therefore, the diameter of the circle is 8.

Choice A is incorrect. This is the radius of the circle.

Choice C is incorrect. This is the square of the radius of the circle.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID ab176ad6

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: ab176ad6

The equation $(x+6)^2+(y+3)^2=121$ defines a circle in the *xy*-plane. What is the radius of the circle?

ID: ab176ad6 Answer

Rationale

The correct answer is 11. A circle with equation $(x-a)^2+(y-b)^2=r^2$, where a, b, and r are constants, has center (a,b) and radius r. Therefore, the radius of the given circle is $\sqrt{121}$, or 11.

Question Difficulty: Hard

Question ID 3e577e4a

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: 3e577e4a

A circle in the *xy*-plane has its center at $(-4, -6)$. Line k is tangent to this circle at the point $(-7, -7)$. What is the slope of line k?

A. -3

B. $-\frac{1}{3}$

C. $\frac{1}{3}$

D. 3

ID: 3e577e4a Answer

Correct Answer: A

Rationale

Choice A is correct. A line that's tangent to a circle is perpendicular to the radius of the circle at the point of tangency. It's given that the circle has its center at $-4, -6$ and line k is tangent to the circle at the point $-7, -7$. The slope of a radius defined by the points q, r and s, t can be calculated as $\frac{t - r}{s - q}$. The points $-7, -7$ and $-4, -6$ define the radius of the circle at the point of tangency. Therefore, the slope of this radius can be calculated as $\frac{-6 - -7}{-4 - -7}$, or $\frac{1}{3}$. If a line and a radius are perpendicular, the slope of the line must be the negative reciprocal of the slope of the radius. The negative reciprocal of $\frac{1}{3}$ is -3. Thus, the slope of line k is -3.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is the slope of the radius of the circle at the point of tangency, not the slope of line k.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 24cec8d1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: 24cec8d1

A circle has center O, and points R and S lie on the circle. In triangle ORS, the measure of $\angle ROS$ is 88°. What is the measure of $\angle RSO$, in degrees? (Disregard the degree symbol when entering your answer.)

ID: 24cec8d1 Answer

Correct Answer: 46

Rationale

The correct answer is 46. It's given that O is the center of a circle and that points R and S lie on the circle. Therefore, $\bar{OR}$ and $\bar{OS}$ are radii of the circle. It follows that $OR = OS$. If two sides of a triangle are congruent, then the angles opposite them are congruent. It follows that the angles $\angle RSO$ and $\angle ORS$, which are across from the sides of equal length, are congruent. Let x° represent the measure of $\angle RSO$. It follows that the measure of $\angle ORS$ is also x°. It's given that the measure of $\angle ROS$ is 88°. Because the sum of the measures of the interior angles of a triangle is 180°, the equation $x^\circ + x^\circ + 88^\circ = 180^\circ$, or $2x + 88 = 180$, can be used to find the measure of $\angle RSO$. Subtracting 88 from both sides of this equation yields $2x = 92$. Dividing both sides of this equation by 2 yields $x = 46$. Therefore, the measure of $\angle RSO$, in degrees, is 46.

Question Difficulty: Hard

Question ID 9e44284b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: 9e44284b

In the *xy*-plane, the graph of $2x^2 - 6x + 2y^2 + 2y = 45$ is a circle. What is the radius of the circle?

A. 5

B. 6.5

C. $\sqrt{40}$

D. $\sqrt{50}$

ID: 9e44284b Answer

Correct Answer: A

Rationale

Choice A is correct. One way to find the radius of the circle is to rewrite the given equation in standard form, $(x-h)^2 + (y-k)^2 = r^2$, where (h,k) is the center of the circle and the radius of the circle is r. To do this, divide the original equation, $2x^2 - 6x + 2y^2 + 2y = 45$, by 2 to make the leading coefficients of x^2 and y^2 each equal to 1: $x^2 - 3x + y^2 + y = 22.5$. Then complete the square to put the equation in standard form. To do so, first rewrite $x^2 - 3x + y^2 + y = 22.5$ as $(x^2 - 3x + 2.25) - 2.25 + (y^2 + y + 0.25) - 0.25 = 22.5$. Second, add 2.25 and 0.25 to both sides of the equation: $(x^2 - 3x + 2.25) + (y^2 + y + 0.25) = 25$. Since $x^2 - 3x + 2.25 = (x-1.5)^2$, $y^2 + y + 0.25 = (y+0.5)^2$, and $25 = 5^2$, it follows that $(x-1.5)^2 + (y+0.5)^2 = 5^2$. Therefore, the radius of the circle is 5.

Choices B, C, and D are incorrect and may be the result of errors in manipulating the equation or of a misconception about the standard form of the equation of a circle in the xy-plane.

Question Difficulty: Hard

Question ID 9acd101f

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	

ID: 9acd101f

The equation $x^2 + (y-1)^2 = 49$ represents circle A. Circle B is obtained by shifting circle A down 2 units in the *xy*-plane. Which of the following equations represents circle B?

A. msup $+ (y-1)^2 = 49$

B. $x^2 +$ msup $= 49$

C. msup $+ (y-1)^2 = 49$

D. $x^2 +$ msup $= 49$

ID: 9acd101f Answer

Correct Answer: D

Rationale

Choice D is correct. The graph in the *xy*-plane of an equation of the form $x - h^2 + y - k^2 = r^2$ is a circle with center h, k and a radius of length r. It's given that circle A is represented by $x^2 + y - 1^2 = 49$, which can be rewritten as $x^2 + y - 1^2 = 7^2$. Therefore, circle A has center $0, 1$ and a radius of length 7. Shifting circle A down two units is a rigid vertical translation of circle A that does not change its size or shape. Since circle B is obtained by shifting circle A down two units, it follows that circle B has the same radius as circle A, and for each point x, y on circle A, the point $x, y - 2$ lies on circle B. Moreover, if h, k is the center of circle A, then $h, k - 2$ is the center of circle B. Therefore, circle B has a radius of 7 and the center of circle B is $0, 1 - 2$, or $0, -1$. Thus, circle B can be represented by the equation $x^2 + y + 1^2 = 7^2$, or $x^2 + y + 1^2 = 49$.

Choice A is incorrect. This is the equation of a circle obtained by shifting circle A right 2 units.

Choice B is incorrect. This is the equation of a circle obtained by shifting circle A up 2 units.

Choice C is incorrect. This is the equation of a circle obtained by shifting circle A left 2 units.

Question Difficulty: Hard

Question ID ca2235f6

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: ca2235f6

A circle has center O, and points A and B lie on the circle. The measure of arc AB is $45°$ and the length of arc AB is 3 inches. What is the circumference, in inches, of the circle?

A. 3

B. 6

C. 9

D. 24

ID: ca2235f6 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that the measure of arc AB is $45°$ and the length of arc AB is 3 inches. The arc measure of the full circle is $360°$. If x represents the circumference, in inches, of the circle, it follows that $\frac{45°}{360°} = \frac{3 \text{ inches}}{x \text{ inches}}$. This equation is equivalent to $\frac{45}{360} = \frac{3}{x}$, or $\frac{1}{8} = \frac{3}{x}$. Multiplying both sides of this equation by $8x$ yields $1(x) = 3(8)$, or $x = 24$. Therefore, the circumference of the circle is 24 inches.

Choice A is incorrect. This is the length of arc AB.

Choice B is incorrect and may result from multiplying the length of arc AB by 2.

Choice C is incorrect and may result from squaring the length of arc AB.

Question Difficulty: Hard

Question ID 9d159400

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	

ID: 9d159400

Which of the following equations represents a circle in the *xy*-plane that intersects the *y*-axis at exactly one point?

A. msup $+ (y - 8)^2 = 16$

B. msup $+ (y - 4)^2 = 16$

C. msup $+ (y - 9)^2 = 16$

D. $x^2 +$ msup $= 16$

ID: 9d159400 Answer

Correct Answer: C

Rationale

Choice C is correct. The graph of the equation $x - h^2 + y - k^2 = r^2$ in the *xy*-plane is a circle with center h, k and a radius of length r. The radius of a circle is the distance from the center of the circle to any point on the circle. If a circle in the *xy*-plane intersects the *y*-axis at exactly one point, then the perpendicular distance from the center of the circle to this point on the *y*-axis must be equal to the length of the circle's radius. It follows that the *x*-coordinate of the circle's center must be equivalent to the length of the circle's radius. In other words, if the graph of $x - h^2 + y - k^2 = r^2$ is a circle that intersects the *y*-axis at exactly one point, then $r = h$ must be true. The equation in choice C is $x - 4^2 + y - 9^2 = 16$, or $x - 4^2 + y - 9^2 = 4^2$. This equation is in the form $x - h^2 + y - k^2 = r^2$, where $h = 4$, $k = 9$, and $r = 4$, and represents a circle in the *xy*-plane with center $4, 9$ and radius of length 4. Substituting 4 for r and 4 for h in the equation $r = h$ yields $4 = 4$, or $4 = 4$, which is true. Therefore, the equation in choice C represents a circle in the *xy*-plane that intersects the *y*-axis at exactly one point.

Choice A is incorrect. This is the equation of a circle that does not intersect the *y*-axis at any point.

Choice B is incorrect. This is an equation of a circle that intersects the *x*-axis, not the *y*-axis, at exactly one point.

Choice D is incorrect. This is the equation of a circle with the center located on the *y*-axis and thus intersects the *y*-axis at exactly two points, not exactly one point.

Question Difficulty: Hard

Question ID 981275d2

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: 981275d2

$$(x-6)^2+(y+5)^2=16$$

In the *xy*-plane, the graph of the equation above is a circle. Point *P* is on the circle and has coordinates $(10,-5)$. If $\overline{PQ}$ is a diameter of the circle, what are the coordinates of point Q ?

A. $(2,-5)$

B. $(6,-1)$

C. $(6,-5)$

D. $(6,-9)$

ID: 981275d2 Answer

Correct Answer: A

Rationale

Choice A is correct. The standard form for the equation of a circle is $(x-h)^2+(y-k)^2=r^2$, where (h,k) are the coordinates of the center and r is the length of the radius. According to the given equation, the center of the circle is $(6,-5)$. Let (x_1,y_1) represent the coordinates of point Q. Since point P $(10,-5)$ and point Q (x_1,y_1) are the endpoints of a diameter of the circle, the center $(6,-5)$ lies on the diameter, halfway between P and Q. Therefore, the following relationships hold: $\frac{x_1+10}{2}=6$ and $\frac{y_1+(-5)}{2}=-5$. Solving the equations for x_1 and y_1, respectively, yields $x_1=2$ and $y_1=-5$. Therefore, the coordinates of point Q are $(2,-5)$.

Alternate approach: Since point P $(10,-5)$ on the circle and the center of the circle $(6,-5)$ have the same y-coordinate, it follows that the radius of the circle is $10-6=4$. In addition, the opposite end of the diameter $\overline{PQ}$ must have the same y-coordinate as P and be 4 units away from the center. Hence, the coordinates of point Q must be $(2,-5)$.

Choices B and D are incorrect because the points given in these choices lie on a diameter that is perpendicular to the diameter $\overline{PQ}$. If either of these points were point Q, then $\overline{PQ}$ would not be the diameter of the circle. Choice C is incorrect because $(6,-5)$ is the center of the circle and does not lie on the circle.

Question Difficulty: Hard

Question ID 89661424

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: 89661424

A circle in the *xy*-plane has its center at $(-5, 2)$ and has a radius of 9. An equation of this circle is $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. What is the value of c?

ID: 89661424 Answer

Correct Answer: -52

Rationale

The correct answer is -52. The equation of a circle in the *xy*-plane with its center at h, k and a radius of r can be written in the form $x - h^2 + y - k^2 = r^2$. It's given that a circle in the *xy*-plane has its center at $-5, 2$ and has a radius of 9. Substituting -5 for h, 2 for k, and 9 for r in the equation $x - h^2 + y - k^2 = r^2$ yields $x - -5^2 + y - 2^2 = 9^2$, or $x + 5^2 + y - 2^2 = 81$. It's also given that an equation of this circle is $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. Therefore, $x + 5^2 + y - 2^2 = 81$ can be rewritten in the form $x^2 + y^2 + ax + by + c = 0$. The equation $x + 5^2 + y - 2^2 = 81$, or $x + 5x + 5 + y - 2y - 2 = 81$, can be rewritten as $x^2 + 5x + 5x + 25 + y^2 - 2y - 2y + 4 = 81$. Combining like terms on the left-hand side of this equation yields $x^2 + y^2 + 10x - 4y + 29 = 81$. Subtracting 81 from both sides of this equation yields $x^2 + y^2 + 10x - 4y - 52 = 0$, which is equivalent to $x^2 + y^2 + 10x + -4y + -52 = 0$. This equation is in the form $x^2 + y^2 + ax + by + c = 0$. Therefore, the value of c is -52.

Question Difficulty: Hard

Question ID fb58c0db

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: fb58c0db

Points A and B lie on a circle with radius 1, and arc $\overset{\frown}{AB}$ has length $\frac{\pi}{3}$. What fraction of the circumference of the circle is the length of arc $\overset{\frown}{AB}$?

ID: fb58c0db Answer

Rationale

The correct answer is $\frac{1}{6}$. The circumference, C, of a circle is $C = 2\pi r$, where r is the length of the radius of the circle. For the given circle with a radius of 1, the circumference is $C = 2(\pi)(1)$, or $C = 2\pi$. To find what fraction of the circumference the length of arc $\overset{\frown}{AB}$ is, divide the length of the arc by the circumference, which gives $\frac{\pi}{3} \div 2\pi$. This division can be represented by $\frac{\pi}{3} \cdot \frac{1}{2\pi} = \frac{1}{6}$. Note that 1/6, .1666, .1667, 0.166, and 0.167 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID acd30391

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	■■■

ID: acd30391

A circle in the *xy*-plane has equation $(x+3)^2+(y-1)^2=25$. Which of the following points does NOT lie in the interior of the circle?

A. $(-7, 3)$

B. $(-3, 1)$

C. $(0, 0)$

D. $(3, 2)$

ID: acd30391 Answer

Correct Answer: D

Rationale

Choice D is correct. The circle with equation $(x+3)^2+(y-1)^2=25$ has center $(-3,1)$ and radius 5. For a point to be inside of the circle, the distance from that point to the center must be less than the radius, 5. The distance between $(3,2)$ and $(-3,1)$ is $\sqrt{(-3-3)^2+(1-2)^2}=\sqrt{(-6)^2+(-1)^2}=\sqrt{37}$, which is greater than 5. Therefore, $(3,2)$ does NOT lie in the interior of the circle.

Choice A is incorrect. The distance between $(-7,3)$ and $(-3,1)$ is $\sqrt{(-7+3)^2+(3-1)^2}=\sqrt{(-4)^2+(2)^2}=\sqrt{20}$, which is less than 5, and therefore $(-7,3)$ lies in the interior of the circle. Choice B is incorrect because it is the center of the circle. Choice C is incorrect because the distance between $(0,0)$ and $(-3,1)$ is $\sqrt{(0+3)^2+(0-1)^2}=\sqrt{(3)^2+(1)^2}=\sqrt{8}$, which is less than 5, and therefore $(0,0)$ in the interior of the circle.

Question Difficulty: Hard