Question ID fc3d783a

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: fc3d783a

In the xy-plane, a line with equation $2y = 4.5$ intersects a parabola at exactly one point. If the parabola has equation $y = -4x^2 + bx$, where b is a positive constant, what is the value of b?

ID: fc3d783a Answer

Correct Answer: 6

Rationale

The correct answer is 6. It's given that a line with equation $2y = 4.5$ intersects a parabola with equation $y = -4x^2 + bx$, where b is a positive constant, at exactly one point in the xy-plane. It follows that the system of equations consisting of $2y = 4.5$ and $y = -4x^2 + bx$ has exactly one solution. Dividing both sides of the equation of the line by 2 yields $y = 2.25$. Substituting 2.25 for y in the equation of the parabola yields $2.25 = -4x^2 + bx$. Adding $4x^2$ and subtracting bx from both sides of this equation yields $4x^2 - bx + 2.25 = 0$. A quadratic equation in the form of $ax^2 + bx + c = 0$, where a, b, and c are constants, has exactly one solution when the discriminant, $b^2 - 4ac$, is equal to zero. Substituting 4 for a and 2.25 for c in the expression $b^2 - 4ac$ and setting this expression equal to 0 yields $b^2 - 4\left(4\right)\left(2.25\right) = 0$, or $b^2 - 36 = 0$. Adding 36 to each side of this equation yields $b^2 = 36$. Taking the square root of each side of this equation yields $b = \pm 6$. It's given that b is positive, so the value of b is 6.

Question Difficulty: Hard

Question ID 4661e2a9

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 4661e2a9

$x - y = 1$

$x + y = x^2 - 3$

Which ordered pair is a solution to the system of equations above?

A. $(1 + \sqrt{3}, \sqrt{3})$

B. $(\sqrt{3}, -\sqrt{3})$

C. $(1 + \sqrt{5}, \sqrt{5})$

D. $(\sqrt{5}, -1 + \sqrt{5})$

ID: 4661e2a9 Answer

Correct Answer: A

Rationale

Choice A is correct. The solution to the given system of equations can be found by solving the first equation for x, which gives $x = y + 1$, and substituting that value of x into the second equation which gives $y + 1 + y = (y + 1)^2 - 3$. Rewriting this equation by adding like terms and expanding $(y + 1)^2$ gives $2y + 1 = y^2 + 2y - 2$. Subtracting $2y$ from both sides of this equation gives $1 = y^2 - 2$. Adding to 2 to both sides of this equation gives $3 = y^2$. Therefore, it follows that $y = \pm\sqrt{3}$. Substituting $\sqrt{3}$ for y in the first equation yields $x - \sqrt{3} = 1$. Adding $\sqrt{3}$ to both sides of this equation yields $x = 1 + \sqrt{3}$. Therefore, the ordered pair $(1 + \sqrt{3}, \sqrt{3})$ is a solution to the given system of equations.

Choice B is incorrect. Substituting $\sqrt{3}$ for x and $-\sqrt{3}$ for y in the first equation yields $\sqrt{3} - (-\sqrt{3}) = 1$, or $2\sqrt{3} = 1$, which isn't a true statement. Choice C is incorrect. Substituting $1 + \sqrt{5}$ for x and $\sqrt{5}$ for y in the second equation yields $(1 + \sqrt{5}) + \sqrt{5} = (1 + \sqrt{5})^2 - 3$, or $1 + 2\sqrt{5} = 2\sqrt{5} + 3$, which isn't a true statement. Choice D is incorrect. Substituting $\sqrt{5}$ for x and $(-1 + \sqrt{5})$ for y in the second equation yields $\sqrt{5} + (-1 + \sqrt{5}) = (\sqrt{5})^2 - 3$, or $2\sqrt{5} - 1 = 2$, which isn't a true statement.

Question Difficulty: Hard

Question ID f65288e8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: f65288e8

$$\frac{1}{x^2+10x+25}=4$$

If x is a solution to the given equation, which of the following is a possible value of $x+5$?

A. $\frac{1}{2}$

B. $\frac{5}{2}$

C. $\frac{9}{2}$

D. $\frac{11}{2}$

ID: f65288e8 Answer

Correct Answer: A

Rationale

Choice A is correct. The given equation can be rewritten as $\frac{1}{(x+5)^2}=4$. Multiplying both sides of this equation by $(x+5)^2$ yields $1=4(x+5)^2$. Dividing both sides of this equation by 4 yields $\frac{1}{4}=(x+5)^2$. Taking the square root of both sides of this equation yields $\frac{1}{2}=x+5$ or $-\frac{1}{2}=x+5$. Therefore, a possible value of $x+5$ is $\frac{1}{2}$.

Choices B, C, and D are incorrect and may result from computational or conceptual errors.

Question Difficulty: Hard

Question ID f2f3fa00

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: f2f3fa00

During a 5-second time interval, the average acceleration *a*, in meters per second squared, of an object with an initial velocity of 12 meters per second is defined by the equation $a=\frac{v_f-12}{5}$, where v_f is the final velocity of the object in meters per second. If the equation is rewritten in the form $v_f = xa + y$, where x and y are constants, what is the value of x ?

ID: f2f3fa00 Answer

Rationale

The correct answer is 5. The given equation can be rewritten in the form $v_f=xa+y$, like so:

$$a=\frac{v_f-12}{5}$$

$$v_f-12=5a$$

$$v_f=5a+12$$

It follows that the value of x is 5 and the value of y is 12.

Question Difficulty: Hard

Question ID 6ce95fc8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 6ce95fc8

$2x^2 - 2 = 2x + 3$

Which of the following is a solution to the equation above?

A. 2

B. $1 - \sqrt{11}$

C. $\frac{1}{2} + \sqrt{11}$

D. $\frac{1 + \sqrt{11}}{2}$

ID: 6ce95fc8 Answer

Correct Answer: D

Rationale

Choice D is correct. A quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are constants, can be solved using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Subtracting $2x + 3$ from both sides of the given equation yields $2x^2 - 2x - 5 = 0$. Applying the quadratic formula, where $a = 2$, $b = -2$, and $c = -5$, yields $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-5)}}{2(2)}$. This can be rewritten as $x = \frac{2 \pm \sqrt{44}}{4}$. Since $\sqrt{44} = \sqrt{2^2(11)}$, or $2\sqrt{11}$, the equation can be rewritten as $x = \frac{2 \pm 2\sqrt{11}}{4}$. Dividing 2 from both the numerator and denominator yields $\frac{1 + \sqrt{11}}{2}$ or $\frac{1 - \sqrt{11}}{2}$. Of these two solutions, only $\frac{1 + \sqrt{11}}{2}$ is present among the choices. Thus, the correct choice is D.

Choice A is incorrect and may result from a computational or conceptual error. Choice B is incorrect and may result from using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{a}$ instead of $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ as the quadratic formula. Choice C is incorrect and may result from rewriting $\sqrt{44}$ as $4\sqrt{11}$ instead of $2\sqrt{11}$.

Question Difficulty: Hard

Question ID 1fe32f7d

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 1fe32f7d

$$-x^2 + bx - 676 = 0$$

In the given equation, b is a positive integer. The equation has no real solution. What is the greatest possible value of b?

ID: 1fe32f7d Answer

Correct Answer: 51

Rationale

The correct answer is 51. A quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants, has no real solution if and only if its discriminant, -$4ac + b^2$, is negative. In the given equation, $a =$ - 1 and $c =$ - 676. Substituting -1 for a and -676 for c in this expression yields a discriminant of b^2 - 4-1-676, or b^2 - $2{,}704$. Since this value must be negative, b^2 - $2{,}704 < 0$, or $b^2 < 2{,}704$. Taking the positive square root of each side of this inequality yields $b < 52$. Since b is a positive integer, and the greatest integer less than 52 is 51, the greatest possible value of b is 51.

Question Difficulty: Hard

Question ID c303ad23

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: c303ad23

If $3x^2 - 18x - 15 = 0$, what is the value of $x^2 - 6x$?

ID: c303ad23 Answer

Correct Answer: 5

Rationale

The correct answer is 5. Dividing each side of the given equation by 3 yields $x^2 - 6x - 5 = 0$. Adding 5 to each side of this equation yields $x^2 - 6x = 5$. Therefore, if $3x^2 - 18x - 15 = 0$, the value of $x^2 - 6x$ is 5.

Question Difficulty: Hard

Question ID 7bd10ef3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 7bd10ef3

$2x^2 - 4x = t$

In the equation above, t is a constant. If the equation has no real solutions, which of the following could be the value of t ?

A. -3

B. -1

C. 1

D. 3

ID: 7bd10ef3 Answer

Correct Answer: A

Rationale

Choice A is correct. The number of solutions to any quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are constants, can be found by evaluating the expression $b^2 - 4ac$, which is called the discriminant. If the value of $b^2 - 4ac$ is a positive number, then there will be exactly two real solutions to the equation. If the value of $b^2 - 4ac$ is zero, then there will be exactly one real solution to the equation. Finally, if the value of $b^2 - 4ac$ is negative, then there will be no real solutions to the equation.

The given equation $2x^2 - 4x = t$ is a quadratic equation in one variable, where t is a constant. Subtracting t from both sides of the equation gives $2x^2 - 4x - t = 0$. In this form, $a = 2$, $b = -4$, and $c = -t$. The values of t for which the equation has no real solutions are the same values of t for which the discriminant of this equation is a negative value. The discriminant is equal to $(-4)^2 - 4(2)(-t)$; therefore, $(-4)^2 - 4(2)(-t) < 0$. Simplifying the left side of the inequality gives $16 + 8t < 0$. Subtracting 16 from both sides of the inequality and then dividing both sides by 8 gives $t < -2$. Of the values given in the options, -3 is the only value that is less than -2. Therefore, choice A must be the correct answer.

Choices B, C, and D are incorrect and may result from a misconception about how to use the discriminant to determine the number of solutions of a quadratic equation in one variable.

Question Difficulty: Hard

Question ID 17d0e87d

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 17d0e87d

$$\frac{14x}{7y} = 2\sqrt{w+19}$$

The given equation relates the distinct positive real numbers w, x, and y. Which equation correctly expresses w in terms of x and y?

A. $w = \sqrt{\frac{x}{y}} - 19$

B. $w = \sqrt{\frac{28x}{14y}} - 19$

C. $w =$ msup $- 19$

D. $w =$ msup $- 19$

ID: 17d0e87d Answer

Correct Answer: C

Rationale

Choice C is correct. Dividing each side of the given equation by 2 yields $\frac{14x}{14y} = \frac{2\sqrt{w+19}}{2}$, or $\frac{x}{y} = \sqrt{w+19}$. Because it's given that each of the variables is positive, squaring each side of this equation yields the equivalent equation $\frac{x}{y}^2 = w + 19$. Subtracting 19 from each side of this equation yields $\frac{x}{y}^2$ - $19 = w$, or $w = \frac{x}{y}^2$ - 19.

Choice A is incorrect. This equation isn't equivalent to the given equation.

Choice B is incorrect. This equation isn't equivalent to the given equation.

Choice D is incorrect. This equation isn't equivalent to the given equation.

Question Difficulty: Hard

Question ID 66bce0c1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 66bce0c1

$$\sqrt{2x+6}+4=x+3$$

What is the solution set of the equation above?

A. $\{-1\}$

B. $\{5\}$

C. $\{-1, 5\}$

D. $\{0, -1, 5\}$

ID: 66bce0c1 Answer

Correct Answer: B

Rationale

Choice B is correct. Subtracting 4 from both sides of $\sqrt{2x+6}+4=x+3$ isolates the radical expression on the left side of the equation as follows: $\sqrt{2x+6}=x-1$. Squaring both sides of $\sqrt{2x+6}=x-1$ yields $2x+6=x^2-2x+1$. This equation can be rewritten as a quadratic equation in standard form: $x^2-4x-5=0$. One way to solve this quadratic equation is to factor the expression x^2-4x-5 by identifying two numbers with a sum of -4 and a product of -5. These numbers are -5 and 1. So the quadratic equation can be factored as $(x-5)(x+1)=0$. It follows that 5 and -1 are the solutions to the quadratic equation. However, the solutions must be verified by checking whether 5 and -1 satisfy the original equation, $\sqrt{2x+6}+4=x+3$. When $x=-1$, the original equation gives $\sqrt{2(-1)+6}+4=(-1)+3$, or $6=2$, which is false. Therefore, -1 does not satisfy the original equation. When $x=5$, the original equation gives $\sqrt{2(5)+6}+4=5+3$, or $8=8$, which is true. Therefore, $x=5$ is the only solution to the original equation, and so the solution set is $\{5\}$.

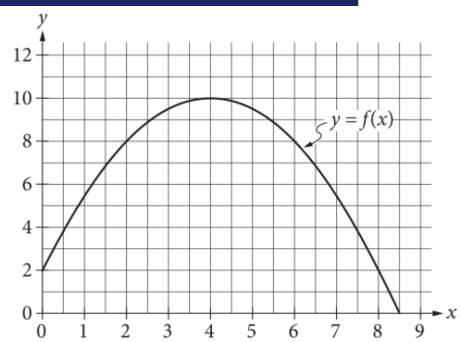
Choices A, C, and D are incorrect because each of these sets contains at least one value that results in a false statement when substituted into the given equation. For instance, in choice D, when 0 is substituted for x into the given equation, the result is $\sqrt{2(0)+6}+4=(0)+3$, or $\sqrt{6}+4=3$. This is not a true statement, so 0 is not a solution to the given equation.

Question Difficulty: Hard

Question ID 97e50fa2

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 97e50fa2



The graph of the function *f*, defined by $f(x) = -\frac{1}{2}(x-4)^2 + 10$, is shown in the *xy*-plane above. If the function *g* (not shown) is defined by $g(x) = -x + 10$, what is one possible value of *a* such that $f(a) = g(a)$?

ID: 97e50fa2 Answer

Rationale

The correct answer is either 2 or 8. Substituting $x = a$ in the definitions for f and g gives $f(a) = -\frac{1}{2}(a-4)^2 + 10$ and $g(a) = -a + 10$, respectively. If $f(a) = g(a)$, then $-\frac{1}{2}(a-4)^2 + 10 = -a + 10$. Subtracting 10 from both sides of this equation gives $-\frac{1}{2}(a-4)^2 = -a$. Multiplying both sides by -2 gives $(a-4)^2 = 2a$. Expanding $(a-4)^2$ gives $a^2 - 8a + 16 = 2a$. Combining the like terms on one side of the equation gives $a^2 - 10a + 16 = 0$. One way to solve this equation is to factor $a^2 - 10a + 16$ by identifying two numbers with a sum of -10 and a product of 16. These numbers are -2 and -8, so the quadratic equation can be factored as $(a-2)(a-8) = 0$. Therefore, the possible values of a are either 2 or 8. Note that 2 and 8 are examples of ways to enter a correct answer.

Alternate approach: Graphically, the condition $f(a) = g(a)$ implies the graphs of the functions $y = f(x)$ and $y = g(x)$ intersect at $x = a$. The graph $y = f(x)$ is given, and the graph of $y = g(x)$ may be sketched as a line with y-intercept 10 and a slope of -1 (taking care to note the different scales on each axis). These two graphs intersect at $x = 2$ and $x = 8$.

Question Difficulty: Hard

Question ID 3d12b1e0

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 3d12b1e0

$$-16x^2 - 8x + c = 0$$

In the given equation, c is a constant. The equation has exactly one solution. What is the value of c?

ID: 3d12b1e0 Answer

Correct Answer: -1

Rationale

The correct answer is -1. A quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are constants, has exactly one solution when its discriminant, b^2 - $4ac$, is equal to 0. In the given equation, -$16x^2$ - $8x + c = 0$, $a =$ - 16 and $b =$ - 8. Substituting -16 for a and -8 for b in b^2 - $4ac$ yields -8^2 - 4-$16c$, or $64 + 64c$. Since the given equation has exactly one solution, $64 + 64c = 0$. Subtracting 64 from both sides of this equation yields $64c =$ - 64. Dividing both sides of this equation by 64 yields $c =$ - 1. Therefore, the value of c is -1.

Question Difficulty: Hard

Question ID 71014fb1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 71014fb1

$$(x-1)^2=-4$$

How many distinct real solutions does the given equation have?

A. Exactly one

B. Exactly two

C. Infinitely many

D. Zero

ID: 71014fb1 Answer

Correct Answer: D

Rationale

Choice D is correct. Any quantity that is positive or negative in value has a positive value when squared. Therefore, the left-hand side of the given equation is either positive or zero for any value of x. Since the right-hand side of the given equation is negative, there is no value of x for which the given equation is true. Thus, the number of distinct real solutions for the given equation is zero.

Choices A, B, and C are incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID e9349667

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: e9349667

$$y = x^2 + 2x + 1$$
$$x + y + 1 = 0$$

If (x_1, y_1) and (x_2, y_2) are the two solutions to the system of equations above, what is the value of $y_1 + y_2$?

A. -3

B. -2

C. -1

D. 1

ID: e9349667 Answer

Correct Answer: D

Rationale

Choice D is correct. The system of equations can be solved using the substitution method. Solving the second equation for y gives $y = -x - 1$. Substituting the expression $-x - 1$ for y into the first equation gives $-x - 1 = x^2 + 2x + 1$. Adding $x + 1$ to both sides of the equation yields $x^2 + 3x + 2 = 0$. The left-hand side of the equation can be factored by finding two numbers whose sum is 3 and whose product is 2, which gives $(x + 2)(x + 1) = 0$. Setting each factor equal to 0 yields $x + 2 = 0$ and $x + 1 = 0$, and solving for x yields $x = -2$ or $x = -1$. These values of x can be substituted for x in the equation $y = -x - 1$ to find the corresponding y-values: $y = -(-2) - 1 = 2 - 1 = 1$ and $y = -(-1) - 1 = 1 - 1 = 0$. It follows that $(-2, 1)$ and $(-1, 0)$ are the solutions to the given system of equations. Therefore, $(x_1, y_1) = (-2, 1)$, $(x_2, y_2) = (-1, 0)$, and $y_1 + y_2 = 1 + 0 = 1$.

Choice A is incorrect. The solutions to the system of equations are $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (-1, 0)$. Therefore, -3 is the sum of the x-coordinates of the solutions, not the sum of the y-coordinates of the solutions. Choices B and C are incorrect and may be the result of computation or substitution errors.

Question Difficulty: Hard

Question ID b03adde3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: b03adde3

If $u-3=\frac{6}{t-2}$, what is t in terms of u ?

A. $t=\frac{1}{u}$

B. $t=\frac{2u+9}{u}$

C. $t=\frac{1}{u-3}$

D. $t=\frac{2u}{u-3}$

ID: b03adde3 Answer

Correct Answer: D

Rationale

Choice D is correct. Multiplying both sides of the given equation by $t-2$ yields $(t-2)(u-3)=6$. Dividing both sides of this equation by $u-3$ yields $t-2=\frac{6}{u-3}$. Adding 2 to both sides of this equation yields $t=\frac{6}{u-3}+2$, which can be rewritten as $t=\frac{6}{u-3}+\frac{2(u-3)}{u-3}$. Since the fractions on the right-hand side of this equation have a common denominator, adding the fractions yields $t=\frac{6+2(u-3)}{u-3}$. Applying the distributive property to the numerator on the right-hand side of this equation yields $t=\frac{6+2u-6}{u-3}$, which is equivalent to $t=\frac{2u}{u-3}$.

Choices A, B, and C are incorrect and may result from various misconceptions or miscalculations.

Question Difficulty: Hard

Question ID 1ce9ffcd

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 1ce9ffcd

$$-9x^2 + 30x + c = 0$$

In the given equation, c is a constant. The equation has exactly one solution. What is the value of c?

A. 3

B. 0

C. -25

D. -53

ID: 1ce9ffcd Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the equation $-9x^2 + 30x + c = 0$ has exactly one solution. A quadratic equation of the form $ax^2 + bx + c = 0$ has exactly one solution if and only if its discriminant, $-4ac + b^2$, is equal to zero. It follows that for the given equation, $a = -9$ and $b = 30$. Substituting -9 for a and 30 for b into $b^2 - 4ac$ yields $30^2 - 4 \cdot -9c$, or $900 + 36c$. Since the discriminant must equal zero, $900 + 36c = 0$. Subtracting $36c$ from both sides of this equation yields $900 = -36c$. Dividing each side of this equation by -36 yields $-25 = c$. Therefore, the value of c is -25.

Choice A is incorrect. If the value of c is 3, this would yield a discriminant that is greater than zero. Therefore, the given equation would have two solutions, rather than exactly one solution.

Choice B is incorrect. If the value of c is 0, this would yield a discriminant that is greater than zero. Therefore, the given equation would have two solutions, rather than exactly one solution.

Choice D is incorrect. If the value of c is -53, this would yield a discriminant that is less than zero. Therefore, the given equation would have no real solutions, rather than exactly one solution.

Question Difficulty: Hard

Question ID 30281058

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 30281058

In the *xy*-plane, the graph of $y = x^2 - 9$ intersects line *p* at $(1, a)$ and $(5, b)$, where *a* and *b* are constants. What is the slope of line *p* ?

A. 6

B. 2

C. -2

D. -6

ID: 30281058 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the graph of $y = x^2 - 9$ and line p intersect at $(1, a)$ and $(5, b)$. Therefore, the value of y when $x = 1$ is the value of a, and the value of y when $x = 5$ is the value of b. Substituting 1 for x in the given equation yields $y = (1)^2 - 9$, or $y = -8$. Similarly, substituting 5 for x in the given equation yields $y = (5)^2 - 9$, or $y = 16$. Therefore, the intersection points are $(1, -8)$ and $(5, 16)$. The slope of line p is the ratio of the change in y to the change in x between these two points: $\frac{16 - (-8)}{5 - 1} = \frac{24}{4}$, or 6.

Choices B, C, and D are incorrect and may result from conceptual or calculation errors in determining the values of a, b, or the slope of line p.

Question Difficulty: Hard

Question ID 5910bfff

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 5910bfff

$$D = T - \frac{9}{25}(100 - H)$$

The formula above can be used to approximate the dew point *D*, in degrees Fahrenheit, given the temperature *T*, in degrees Fahrenheit, and the relative humidity of *H* percent, where $H > 50$. Which of the following expresses the relative humidity in terms of the temperature and the dew point?

A. $H = \frac{25}{9}(D - T) + 100$

B. $H = \frac{25}{9}(D - T) - 100$

C. $H = \frac{25}{9}(D + T) + 100$

D. $H = \frac{25}{9}(D + T) - 100$

ID: 5910bfff Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that $D = T - \frac{9}{25}(100 - H)$. Solving this formula for H expresses the relative humidity in terms of the temperature and the dew point. Subtracting T from both sides of this equation yields $D - T = -\frac{9}{25}(100 - H)$. Multiplying both sides by $-\frac{25}{9}$ yields $-\frac{25}{9}(D - T) = 100 - H$. Subtracting 100 from both sides yields $-\frac{25}{9}(D - T) - 100 = -H$. Multiplying both sides by -1 results in the formula $\frac{25}{9}(D - T) + 100 = H$.

Choices B, C, and D are incorrect and may result from errors made when rewriting the given formula.

Question Difficulty: Hard

Question ID 1697ffcf

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 1697ffcf

In the *xy*-plane, the graph of $y = 3x^2 - 14x$ intersects the graph of $y = x$ at the points $(0, 0)$ and (a, a). What is the value of *a* ?

ID: 1697ffcf Answer

Rationale

The correct answer is 5. The intersection points of the graphs of $y = 3x^2 - 14x$ and $y = x$ can be found by solving the system consisting of these two equations. To solve the system, substitute x for y in the first equation. This gives $x = 3x^2 - 14x$. Subtracting x from both sides of the equation gives $0 = 3x^2 - 15x$. Factoring $3x$ out of each term on the left-hand side of the equation gives $0 = 3x(x - 5)$. Therefore, the possible values for x are 0 and 5. Since $y = x$, the two intersection points are $(0,0)$ and $(5,5)$. Therefore, $a = 5$.

Question Difficulty: Hard

Question ID 5edc8c98

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 5edc8c98

$$64x^2 - (16a + 4b)x + ab = 0$$

In the given equation, a and b are positive constants. The sum of the solutions to the given equation is $k(4a + b)$, where k is a constant. What is the value of k?

ID: 5edc8c98 Answer

Correct Answer: .0625, 1/16

Rationale

The correct answer is $\frac{1}{16}$. Let p and q represent the solutions to the given equation. Then, the given equation can be rewritten as $64x - px - q = 0$, or $64x^2 - 64p + q + pq = 0$. Since this equation is equivalent to the given equation, it follows that $-16a + 4b = -64p + q$. Dividing both sides of this equation by -64 yields $\frac{16a + 4b}{64} = p + q$, or $\frac{1}{16}4a + b = p + q$. Therefore, the sum of the solutions to the given equation, $p + q$, is equal to $\frac{1}{16}4a + b$. Since it's given that the sum of the solutions to the given equation is $k4a + b$, where k is a constant, it follows that $k = \frac{1}{16}$. Note that 1/16, .0625, 0.062, and 0.063 are examples of ways to enter a correct answer.

Alternate approach: The given equation can be rewritten as $64x^2 - 44a + bx + ab = 0$, where a and b are positive constants. Dividing both sides of this equation by 4 yields $16x^2 - 4a + bx + \frac{ab}{4} = 0$. The solutions for a quadratic equation in the form $Ax^2 + Bx + C = 0$, where A, B, and C are constants, can be calculated using the quadratic formula, $x = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$ and $x = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$. It follows that the sum of the solutions to a quadratic equation in the form $Ax^2 + Bx + C = 0$ is $\frac{-B + \sqrt{B^2 - 4AC}}{2A} + \frac{-B - \sqrt{B^2 - 4AC}}{2A}$, which can be rewritten as $\frac{-B + -B + \sqrt{B^2 - 4AC} - \sqrt{B^2 - 4AC}}{2A}$, which is equivalent to $\frac{-2B}{2A}$, or $-\frac{B}{A}$. In the equation $16x^2 - 4a + bx + \frac{ab}{4} = 0$, $A = 16$, $B = -4a + b$, and $C = \frac{ab}{4}$. Substituting 16 for A and $-4a + b$ for B in $-\frac{B}{A}$ yields $-\frac{-4a + b}{16}$, which can be rewritten as $\frac{1}{16}4a + b$. Thus, the sum of the solutions to the given equation is $\frac{1}{16}4a + b$. Since it's given that the sum of the solutions to the given equation is $k4a + b$, where k is a constant, it follows that $k = \frac{1}{16}$.

Question Difficulty: Hard

Question ID ff2e5c76

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: ff2e5c76

$$x^2 - 40x - 10 = 0$$

What is the sum of the solutions to the given equation?

A. 0

B. 5

C. 10

D. 40

ID: ff2e5c76 Answer

Correct Answer: D

Rationale

Choice D is correct. Adding 10 to each side of the given equation yields $x^2 - 40x = 10$. To complete the square, adding $\left(\frac{40}{2}\right)^2$, or 20^2, to each side of this equation yields $x^2 - 40x + 20^2 = 10 + 20^2$, or $(x - 20)^2 = 410$. Taking the square root of each side of this equation yields $x - 20 = \pm\sqrt{410}$. Adding 20 to each side of this equation yields $x = 20 \pm \sqrt{410}$. Therefore, the solutions to the given equation are $x = 20 + \sqrt{410}$ and $x = 20 - \sqrt{410}$. The sum of these solutions is $20 + \sqrt{410} + 20 - \sqrt{410}$, or 40.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 2c5c22d0

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 2c5c22d0

$$y = x^2 + 3x - 7$$
$$y - 5x + 8 = 0$$

How many solutions are there to the system of equations above?

A. There are exactly 4 solutions.

B. There are exactly 2 solutions.

C. There is exactly 1 solution.

D. There are no solutions.

ID: 2c5c22d0 Answer

Correct Answer: C

Rationale

Choice C is correct. The second equation of the system can be rewritten as $y = 5x - 8$. Substituting $5x - 8$ for y in the first equation gives $5x - 8 = x^2 + 3x - 7$. This equation can be solved as shown below:

$$x^2 + 3x - 7 - 5x + 8 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)^2 = 0$$

$$x = 1$$

Substituting 1 for x in the equation $y = 5x - 8$ gives $y = -3$. Therefore, $(1, -3)$ is the only solution to the system of equations.

Choice A is incorrect. In the xy-plane, a parabola and a line can intersect at no more than two points. Since the graph of the first equation is a parabola and the graph of the second equation is a line, the system cannot have more than 2 solutions. Choice B is incorrect. There is a single ordered pair (x, y) that satisfies both equations of the system. Choice D is incorrect because the ordered pair $(1, -3)$ satisfies both equations of the system.

Question Difficulty: Hard

Question ID fc3dfa26

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: fc3dfa26

$$\frac{4x^2}{x^2-9}-\frac{2x}{x+3}=\frac{1}{x-3}$$

What value of x satisfies the equation above?

A. -3

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. 3

ID: fc3dfa26 Answer

Correct Answer: C

Rationale

Choice C is correct. Each fraction in the given equation can be expressed with the common denominator x^2-9. Multiplying $\frac{2x}{x+3}$ by $\frac{x-3}{x-3}$ yields $\frac{2x^2-6x}{x^2-9}$, and multiplying $\frac{1}{x-3}$ by $\frac{x+3}{x+3}$ yields $\frac{x+3}{x^2-9}$. Therefore, the given equation can be written as $\frac{4x^2}{x^2-9}-\frac{2x^2-6x}{x^2-9}=\frac{x+3}{x^2-9}$. Multiplying each fraction by the denominator results in the equation $4x^2-(2x^2-6x)=x+3$, or $2x^2+6x=x+3$. This equation can be solved by setting a quadratic expression equal to 0, then solving for x. Subtracting $x+3$ from both sides of this equation yields $2x^2+5x-3=0$. The expression $2x^2+5x-3$ can be factored, resulting in the equation $(2x-1)(x+3)=0$. By the zero product property, $2x-1=0$ or $x+3=0$. To solve for x in $2x-1=0$, 1 can be added to both sides of the equation, resulting in $2x=1$. Dividing both sides of this equation by 2 results in $x=\frac{1}{2}$. Solving for x in $x+3=0$ yields $x=-3$. However, this value of x would result in the second fraction of the original equation having a denominator of 0. Therefore, $x=-3$ is an extraneous solution. Thus, the only value of x that satisfies the given equation is $x=\frac{1}{2}$.

Choice A is incorrect and may result from solving $x+3=0$ but not realizing that this solution is extraneous because it would result in a denominator of 0 in the second fraction. Choice B is incorrect and may result from a sign error when solving $2x-1=0$ for x. Choice D is incorrect and may result from a calculation error.

Question Difficulty: Hard

Question ID a54753ca

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: a54753ca

In the *xy*-plane, the graph of the equation $y = -x^2 + 9x - 100$ intersects the line $y = c$ at exactly one point. What is the value of c?

A. $-\frac{481}{4}$

B. -100

C. $-\frac{319}{4}$

D. $-\frac{9}{2}$

ID: a54753ca Answer

Correct Answer: C

Rationale

Choice C is correct. In the *xy*-plane, the graph of the line $y = c$ is a horizontal line that crosses the *y*-axis at $y = c$ and the graph of the quadratic equation $y = -x^2 + 9x - 100$ is a parabola. A parabola can intersect a horizontal line at exactly one point only at its vertex. Therefore, the value of c should be equal to the *y*-coordinate of the vertex of the graph of the given equation. For a quadratic equation in vertex form, $y = ax - h^2 + k$, the vertex of its graph in the *xy*-plane is h, k. The given quadratic equation, $y = -x^2 + 9x - 100$, can be rewritten as $y = -x^2 - 2\frac{9}{2}x + \frac{9}{2}^2 + \frac{9}{2}^2 - 100$, or $y = -x - \frac{9}{2}^2 + -\frac{319}{4}$. Thus, the value of c is equal to $-\frac{319}{4}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 58b109d4

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 58b109d4

$$x^2 + y + 7 = 7$$
$$20x + 100 - y = 0$$

The solution to the given system of equations is (x, y). What is the value of x?

ID: 58b109d4 Answer

Correct Answer: -10

Rationale

The correct answer is -10. Adding y to both sides of the second equation in the given system yields $20x + 100 = y$. Substituting $20x + 100$ for y in the first equation in the given system yields $x^2 + 20x + 100 + 7 = 7$. Subtracting 7 from both sides of this equation yields $x^2 + 20x + 100 = 0$. Factoring the left-hand side of this equation yields $x + 10x + 10 = 0$, or $x + 10^2 = 0$. Taking the square root of both sides of this equation yields $x + 10 = 0$. Subtracting 10 from both sides of this equation yields $x =$ - 10. Therefore, the value of x is -10.

Question Difficulty: Hard