Question ID 6d99b141

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Lines, angles, and triangles	■ ■ ■
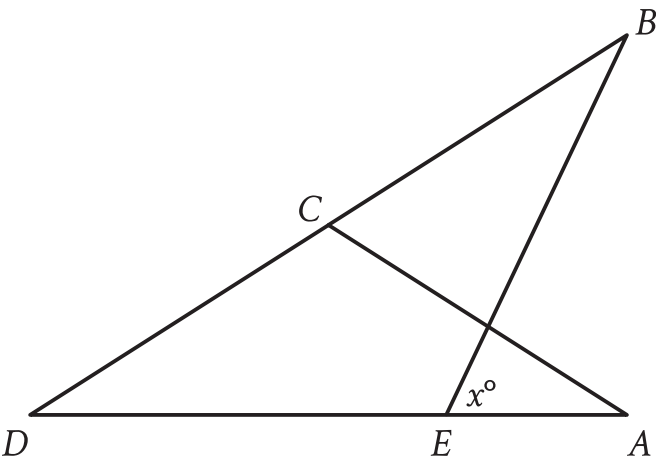
Note: Figure not drawn to scale.

In the figure, $AC = CD$. The measure of angle EBC is $45°$, and the measure of angle ACD is $104°$. What is the value of x?

Correct Answer: 83

Rationale

The correct answer is 83. It's given that in the figure, $AC = CD$. Thus, triangle ACD is an isosceles triangle and the measure of angle CDA is equal to the measure of angle CAD. The sum of the measures of the interior angles of a triangle is 180°. Thus, the sum of the measures of the interior angles of triangle ACD is 180°. It's given that the measure of angle ACD is 104°. It follows that the sum of the measures of angles CDA and CAD is 180 - 104°, or 76°. Since the measure of angle CDA is equal to the measure of angle CAD, the measure of angle CDA is half of 76°, or 38°. The sum of the measures of the interior angles of triangle BDE is 180°. It's given that the measure of angle EBC is 45°. Since the measure of angle BDE, which is the same angle as angle CDA, is 38°, it follows that the measure of angle DEB is 180 - 45 - 38°, or 97°. Since angle DEB and angle AEB form a straight line, the sum of the measures of these angles is 180°. It's given in the figure that the measure of angle AEB is $x°$. It follows that $97 + x = 180$. Subtracting 97 from both sides of this equation yields $x = 83$.

Question Difficulty: Hard

Question ID e10d8313

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Lines, angles, and triangles	■ ■ ■
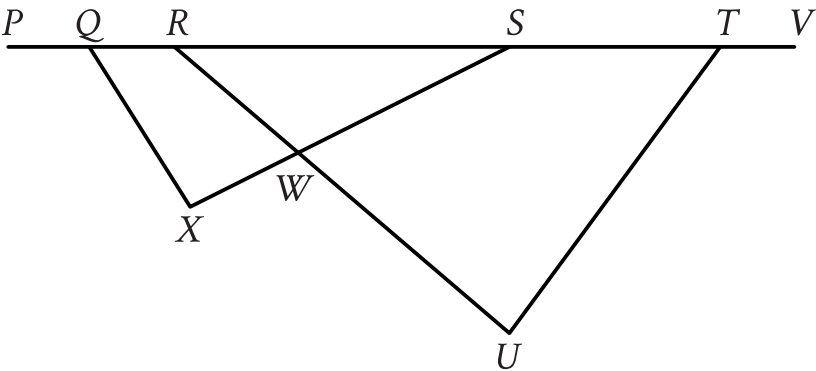
Note: Figure not drawn to scale.

In the figure shown, points Q, R, S, and T lie on line segment PV, and line segment RU intersects line segment SX at point W. The measure of $\angle SQX$ is $48°$, the measure of $\angle SXQ$ is $86°$, the measure of $\angle SWU$ is $85°$, and the measure of $\angle VTU$ is $162°$. What is the measure, in degrees, of $\angle TUR$?

Correct Answer: 123

Rationale

The correct answer is 123. The triangle angle sum theorem states that the sum of the measures of the interior angles of a triangle is 180 degrees. It's given that the measure of $\angle SQX$ is 48° and the measure of $\angle SXQ$ is 86°. Since points S, Q, and X form a triangle, it follows from the triangle angle sum theorem that the measure, in degrees, of $\angle QSX$ is 180 - 48 - 86, or 46. It's also given that the measure of $\angle SWU$ is 85°. Since $\angle SWU$ and $\angle SWR$ are supplementary angles, the sum of their measures is 180 degrees. It follows that the measure, in degrees, of $\angle SWR$ is 180 - 85, or 95. Since points R, S, and W form a triangle, and $\angle RSW$ is the same angle as $\angle QSX$, it follows from the triangle angle sum theorem that the measure, in degrees, of $\angle WRS$ is 180 - 46 - 95, or 39. It's given that the measure of $\angle VTU$ is 162°. Since $\angle VTU$ and $\angle STU$ are supplementary angles, the sum of their measures is 180 degrees. It follows that the measure, in degrees, of $\angle STU$ is 180 - 162, or 18. Since points R, T, and U form a triangle, and $\angle URT$ is the same angle as $\angle WRS$, it follows from the triangle angle sum theorem that the measure, in degrees, of $\angle TUR$ is 180 - 39 - 18, or 123.
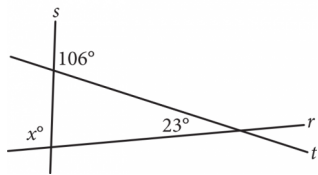
Question Difficulty: Hard

Question ID f88f27e5

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Lines, angles, and triangles	■■■

ID: f88f27e5

Intersecting lines r, s, and t are shown below.

What is the value of x ?

ID: f88f27e5 Answer

Rationale

The correct answer is 97. The intersecting lines form a triangle, and the angle with measure of $x°$ is an exterior angle of this triangle. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles of the triangle. One of these angles has measure of $23°$ and the other, which is supplementary to the angle with measure $106°$, has measure of $180° - 106° = 74°$. Therefore, the value of x is $23 + 74 = 97$.

Question Difficulty: Hard

Question ID 947a3cde

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Lines, angles, and triangles	■ ■ ■
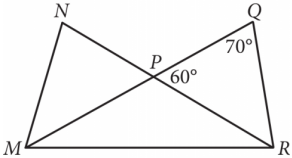
In the figure above, $\overline{MQ}$ and $\overline{NR}$ intersect at point P, $NP = QP$, and $MP = PR$. What is the measure, in degrees, of $\angle QMR$? (Disregard the degree symbol when gridding your answer.)

Rationale

The correct answer is 30. It is given that the measure of $\angle QPR$ is $60°$. Angle MPR and $\angle QPR$ are collinear and therefore are supplementary angles. This means that the sum of the two angle measures is $180°$, and so the measure of $\angle MPR$ is $120°$. The sum of the angles in a triangle is $180°$. Subtracting the measure of $\angle MPR$ from $180°$ yields the sum of the other angles in the triangle MPR. Since $180 - 120 = 60$, the sum of the measures of $\angle QMR$ and $\angle NRM$ is $60°$. It is given that $MP = PR$, so it follows that triangle MPR is isosceles. Therefore $\angle QMR$ and $\angle NRM$ must be congruent. Since the sum of the measure of these two angles is $60°$, it follows that the measure of each angle is $30°$.

An alternate approach would be to use the exterior angle theorem, noting that the measure of $\angle QPR$ is equal to the sum of the measures of $\angle QMR$ and $\angle NRM$. Since both angles are equal, each of them has a measure of $30°$.

Question Difficulty: Hard

Question ID a0369739

ID: a0369739

In triangle ABC, the measure of angle B is $90°$ and $\overline{BD}$ is an altitude of the triangle. The length of $\overline{AB}$ is 15 and the length of $\overline{AC}$ is 23 greater than the length of $\overline{AB}$. What is the value of $\frac{BC}{BD}$?

A. $\frac{15}{38}$

B. $\frac{15}{23}$

C. $\frac{23}{15}$

D. $\frac{38}{15}$

ID: a0369739 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that in triangle ABC, the measure of angle B is 90° and BD is an altitude of the triangle. Therefore, the measure of angle BDC is 90°. It follows that angle B is congruent to angle D and angle C is congruent to angle C. By the angle-angle similarity postulate, triangle ABC is similar to triangle BDC. Since triangles ABC and BDC are similar, it follows that $\frac{AC}{AB} = \frac{BC}{BD}$. It's also given that the length of $\bar{AB}$ is 15 and the length of $\bar{AC}$ is 23 greater than the length of $\bar{AB}$. Therefore, the length of $\bar{AC}$ is 15 + 23, or 38. Substituting 15 for AB and 38 for AC in the equation $\frac{AC}{AB} = \frac{BC}{BD}$ yields $\frac{38}{15} = \frac{BC}{BD}$. Therefore, the value of $\frac{BC}{BD}$ is $\frac{38}{15}$.

Choice A is incorrect. This is the value of $\frac{BD}{BC}$.

Choice B is incorrect and may result from conceptual or calculation errors.
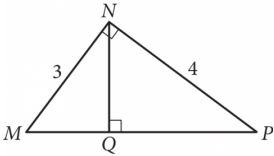
Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 740bf79f

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Lines, angles, and triangles	■ ■ ■

In the figure above, what is the length of $\overline{NQ}$?

A. 2.2

B. 2.3

C. 2.4

D. 2.5

Correct Answer: C

Rationale

Choice C is correct. First, $\overline{MP}$ is the hypotenuse of right $\triangle MNP$, whose legs have lengths 3 and 4. Therefore, $(MP)^2 = 3^2 + 4^2$, so $(MP)^2 = 25$ and $MP = 5$. Second, because $\angle MNP$ corresponds to $\angle NQP$ and because $\angle MPN$ corresponds to $\angle NPQ$, $\triangle MNP$ is similar to $\triangle NQP$. The ratio of corresponding sides of similar triangles is constant, so $\frac{NQ}{MN} = \frac{NP}{MP}$. Since $MP = 5$ and it's given that $MN = 3$ and $NP = 4$, $\frac{NQ}{3} = \frac{4}{5}$. Solving for NQ results in $NQ = \frac{12}{5}$, or 2.4.
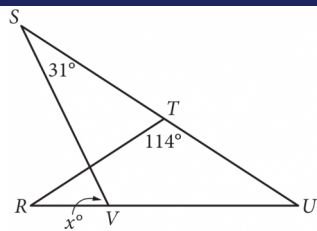
Choices A, B, and D are incorrect and may result from setting up incorrect ratios.

Question Difficulty: Hard

Question ID bd7f6e30

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Lines, angles, and triangles	■ ■ ■

ID: bd7f6e30

In the figure above, $RT = TU$.

What is the value of x?

A. 72

B. 66

C. 64

D. 58

ID: bd7f6e30 Answer

Correct Answer: C

Rationale

Choice C is correct. Since $RT = TU$, it follows that $\triangle RTU$ is an isosceles triangle with base RU. Therefore, $\angle TRU$ and $\angle TUR$ are the base angles of an isosceles triangle and are congruent. Let the measures of both $\angle TRU$ and $\angle TUR$ be $t°$. According to the triangle sum theorem, the sum of the measures of the three angles of a triangle is $180°$. Therefore, $114° + 2t° = 180°$, so $t = 33$.

Note that $\angle TUR$ is the same angle as $\angle SUV$. Thus, the measure of $\angle SUV$ is $33°$. According to the triangle exterior angle theorem, an external angle of a triangle is equal to the sum of the opposite interior angles. Therefore, $x°$ is equal to the sum of the measures of $\angle VSU$ and $\angle SUV$; that is, $31° + 33° = 64°$. Thus, the value of x is 64.

Choice B is incorrect. This is the measure of $\angle STR$, but $\angle STR$ is not congruent to $\angle SVR$. Choices A and D are incorrect and may result from a calculation error.

Question Difficulty: Hard

Question ID 5b4757df

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Lines, angles, and triangles	■ ■ ■

ID: 5b4757df

In triangle RST, angle T is a right angle, point L lies on $\overline{RS}$, point K lies on $\overline{ST}$, and $\overline{LK}$ is parallel to $\overline{RT}$. If the length of $\overline{RT}$ is 72 units, the length of $\overline{LK}$ is 24 units, and the area of triangle RST is 792 square units, what is the length of $\overline{KT}$, in units?

ID: 5b4757df Answer

Correct Answer: 14.66, 14.67, 44/3

Rationale

The correct answer is $\frac{44}{3}$. It's given that in triangle RST, angle T is a right angle. The area of a right triangle can be found using the formula $A = \frac{1}{2}l_1l_2$, where A represents the area of the right triangle, l_1 represents the length of one leg of the triangle, and l_2 represents the length of the other leg of the triangle. In triangle RST, the two legs are $\bar{RT}$ and $\bar{ST}$. Therefore, if the length of $\bar{RT}$ is 72 and the area of triangle RST is 792, then $792 = \frac{1}{2}72ST$, or $792 = 36ST$. Dividing both sides of this equation by 36 yields $22 = ST$. Therefore, the length of $\bar{ST}$ is 22. It's also given that point L lies on $\bar{RS}$, point K lies on $\bar{ST}$, and $\bar{LK}$ is parallel to $\bar{RT}$. It follows that angle LKS is a right angle. Since triangles RST and LSK share angle S and have right angles T and K, respectively, triangles RST and LSK are similar triangles. Therefore, the ratio of the length of $\bar{RT}$ to the length of $\bar{LK}$ is equal to the ratio of the length of $\bar{ST}$ to the length of $\bar{SK}$. If the length of $\bar{RT}$ is 72 and the length of $\bar{LK}$ is 24, it follows that the ratio of the length of $\bar{RT}$ to the length of $\bar{LK}$ is $\frac{72}{24}$, or 3, so the ratio of the length of $\bar{ST}$ to the length of $\bar{SK}$ is 3. Therefore, $\frac{22}{SK} = 3$. Multiplying both sides of this equation by SK yields $22 = 3SK$. Dividing both sides of this equation by 3 yields $\frac{22}{3} = SK$. Since the length of $\bar{ST}$, 22, is the sum of the length of $\bar{SK}$, $\frac{22}{3}$, and the length of $\bar{KT}$, it follows that the length of $\bar{KT}$ is 22 - $\frac{22}{3}$, or $\frac{44}{3}$. Note that 44/3, 14.66, and 14.67 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 17912810

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Lines, angles, and triangles	■ ■ ■
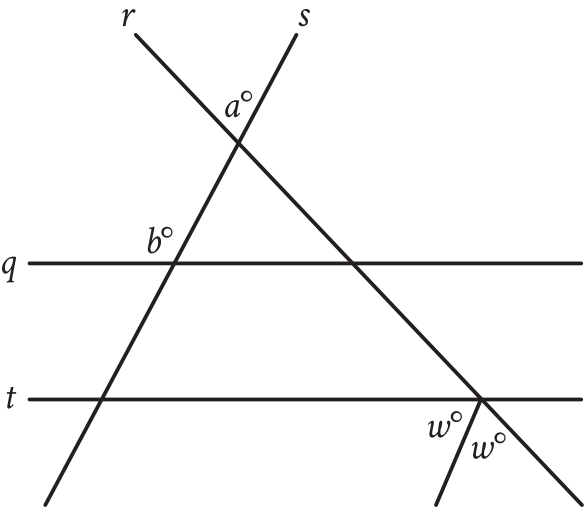
Note: Figure not drawn to scale.

In the figure, parallel lines q and t are intersected by lines r and s. If $a = 43$ and $b = 122$, what is the value of w?

Correct Answer: 101/2, 50.5

Rationale

The correct answer is $\frac{101}{2}$. In the figure, lines q, r, and s form a triangle. One interior angle of this triangle is vertical to the angle marked $a°$; therefore, the interior angle also has measure $a°$. It's given that $a = 43$. Therefore, the interior angle of the triangle has measure 43°. A second interior angle of the triangle forms a straight line, q, with the angle marked $b°$. Therefore, the sum of the measures of these two angles is 180°. It's given that $b = 122$. Therefore, the angle marked $b°$ has measure 122° and the second interior angle of the triangle has measure 180 - 122°, or 58°. The sum of the interior angles of a triangle is 180°. Therefore, the measure of the third interior angle of the triangle is 180 - 43 - 58°, or 79°. It's given that parallel lines q and t are intersected by line r. It follows that the triangle's interior angle with measure 79° is congruent to the same side interior angle between lines q and t formed by lines t and r. Since this angle is supplementary to the two angles marked $w°$, the sum of 79°, $w°$, and $w°$ is 180°. It follows that $79 + w + w = 180$, or $79 + 2w = 180$. Subtracting 79 from both sides of this equation yields $2w = 101$. Dividing both sides of this equation by 2 yields $w = \frac{101}{2}$. Note that 101/2 and 50.5 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 6a3fbec3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Lines, angles, and triangles	■ ■ ■

ID: 6a3fbec3
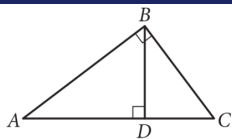
Note: Figure not drawn to scale.

In the figure above, $BD = 6$ and $AD = 8$.

What is the length of $\overline{DC}$?

## ID: 6a3fbec3 Answer

Rationale

The correct answer is 4.5. According to the properties of right triangles, BD divides triangle ABC into two similar triangles, ABD and BCD. The corresponding sides of ABD and BCD are proportional, so the ratio of BD to AD is the same as the ratio of DC to BD. Expressing this information as a proportion gives $\dfrac{6}{8} = \dfrac{DC}{6}$. Solving the proportion for DC results in $DC = 4.5$. Note that 4.5 and 9/2 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID eeb4143c

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Lines, angles, and triangles	■ ■ ■

ID: eeb4143c
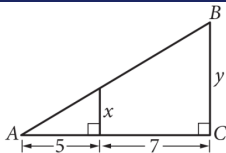
Note: Figure not drawn to scale.

The area of triangle *ABC* above is at least 48 but no more than 60. If *y* is an integer, what is one possible value of *x* ?

ID: eeb4143c Answer

Rationale

The correct answer is either $\frac{10}{3}$, $\frac{15}{4}$, or $\frac{25}{6}$. The area of triangle ABC can be expressed as $\frac{1}{2}(5+7)y$ or $6y$. It's given that the area of triangle ABC is at least 48 but no more than 60. It follows that $48 \le 6y \le 60$. Dividing by 6 to isolate y in this compound inequality yields $8 \le y \le 10$. Since y is an integer, $y = 8, 9, \text{ or } 10$. In the given figure, the two right triangles shown are similar because they have two pairs of congruent angles: their respective right angles and angle A. Therefore, the following proportion is true: $\frac{x}{y} = \frac{5}{12}$. Substituting 8 for y in the proportion results in $\frac{x}{8} = \frac{5}{12}$. Cross multiplying and solving for x yields $\frac{10}{3}$. Substituting 9 for y in the proportion results in $\frac{x}{9} = \frac{5}{12}$. Cross multiplying and solving for x yields $\frac{15}{4}$. Substituting 10 for y in the proportion results in $\frac{x}{10} = \frac{5}{12}$. Cross multiplying and solving for x yields $\frac{25}{6}$. Note that 10/3, 15/4, 25/6, 3.333, 3.75, 4.166, and 4.167 are examples of ways to enter a correct answer.

Question Difficulty: Hard