MACHINE LEARNING

SUPPORT VECTOR MACHINES

Support vector machine

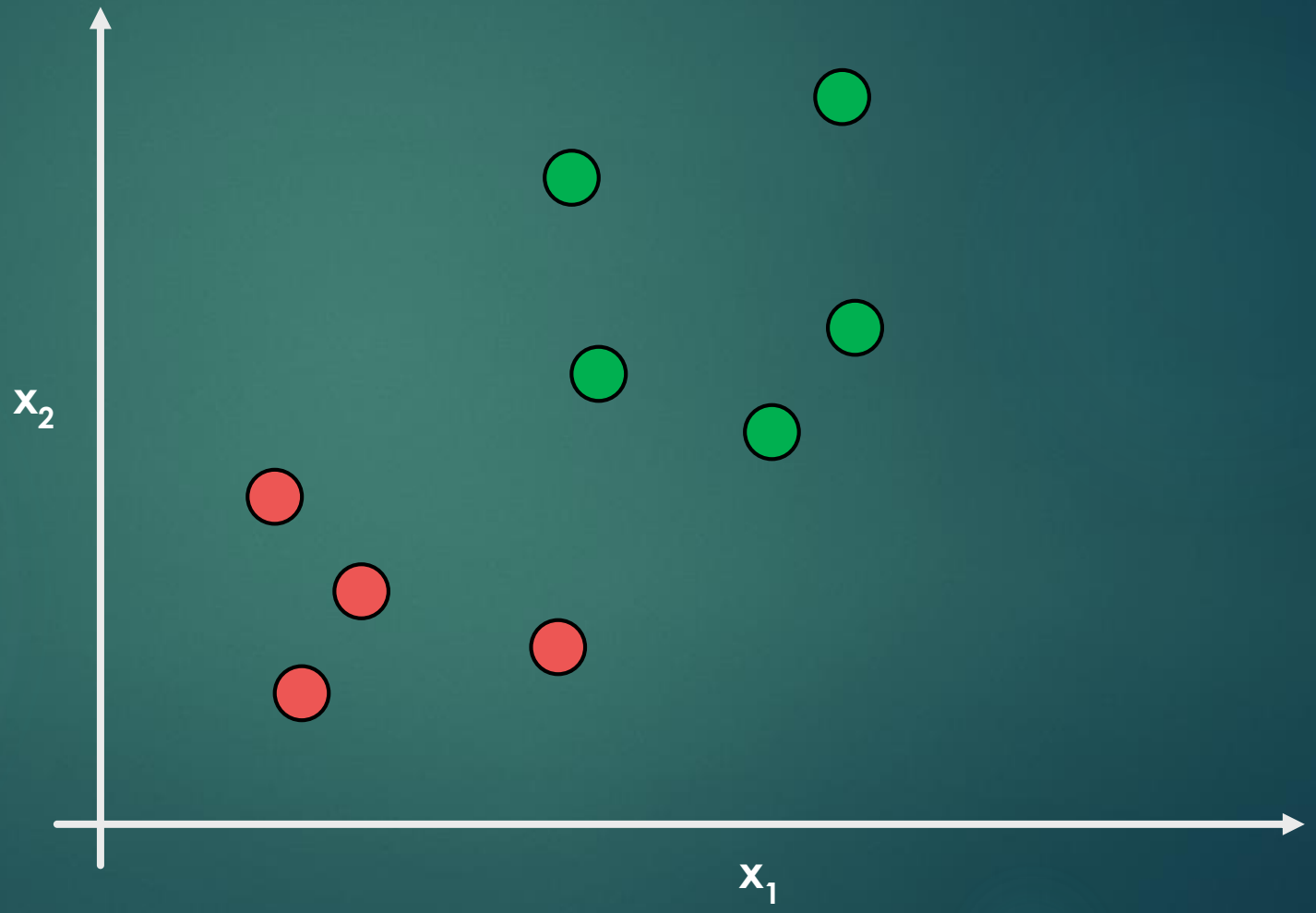
- Very popular and widely used supervised learning classification algorithm
- The great benefit: it can operates even in infinite dimensions !!!
- It defines a margin / boundary → between the data points in multidimensional space
- Goal: find a flat boundary („hyperplane") that leads to a homogeneous partition of the data
- A good separation is achieved by the hyperplane that has the largest distance to the nearest training-data point of any class since in general the larger the margin the lower the generalization error of the classifier
- So we have to maximize the margin

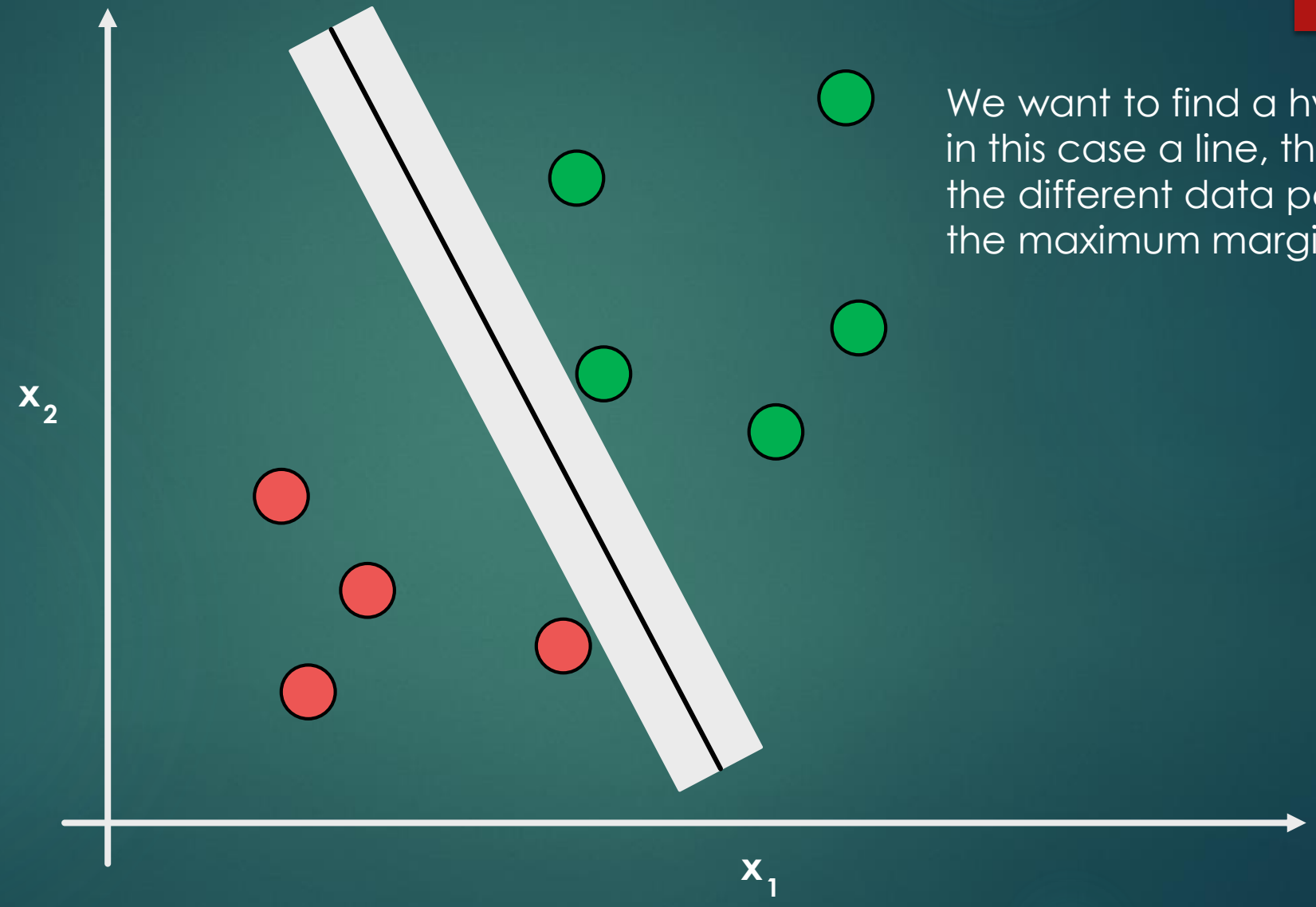
Support vector machine

- Can be applied to almost everything
- Classifications or numerical predictions
- Widely used in pattern recognition
  - Identify cancer or genetic diseases
  - Text classification: classify texts based on the language
  - Detecting rare events: earthquakes or engine failures

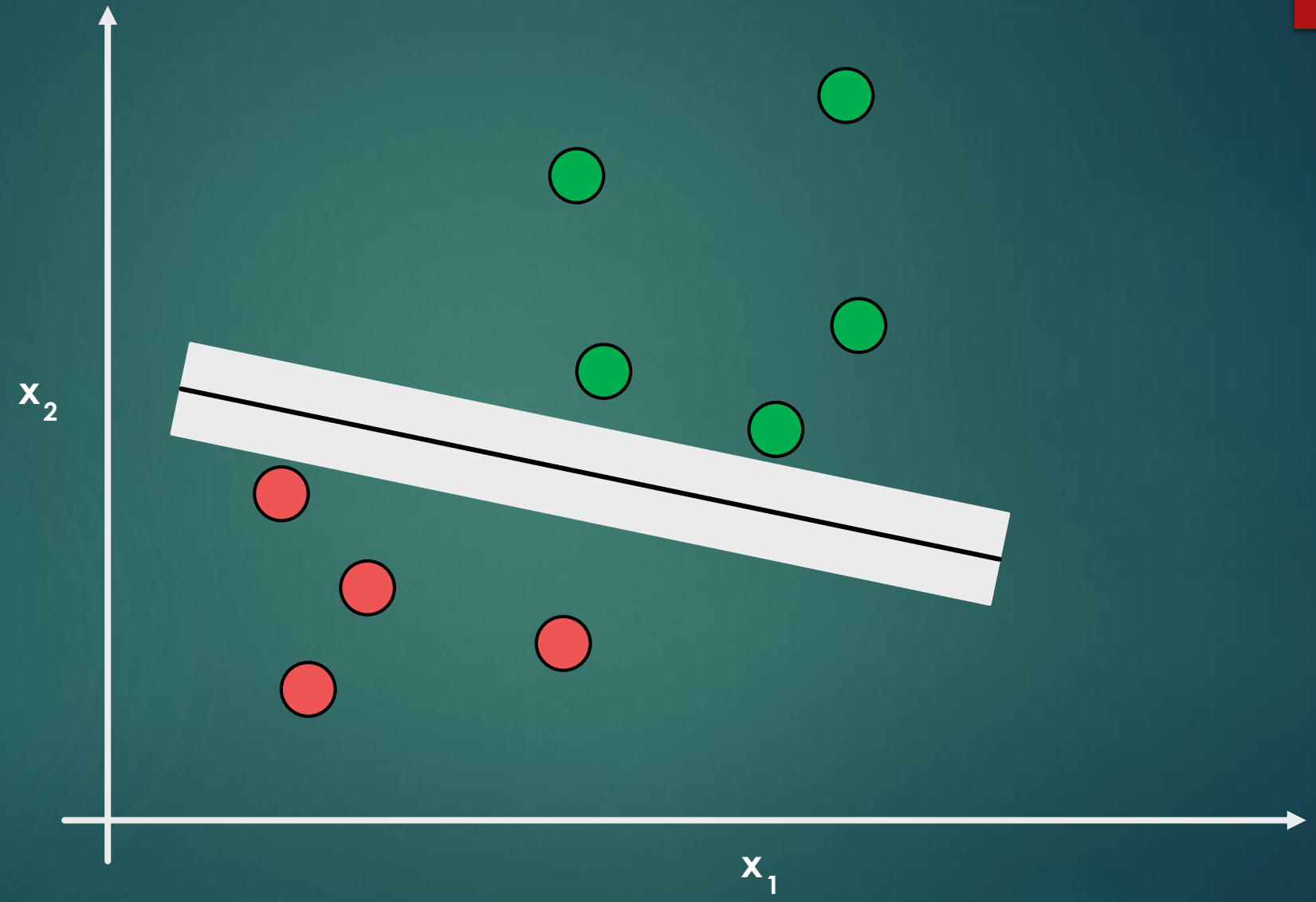
Linearly separable problem

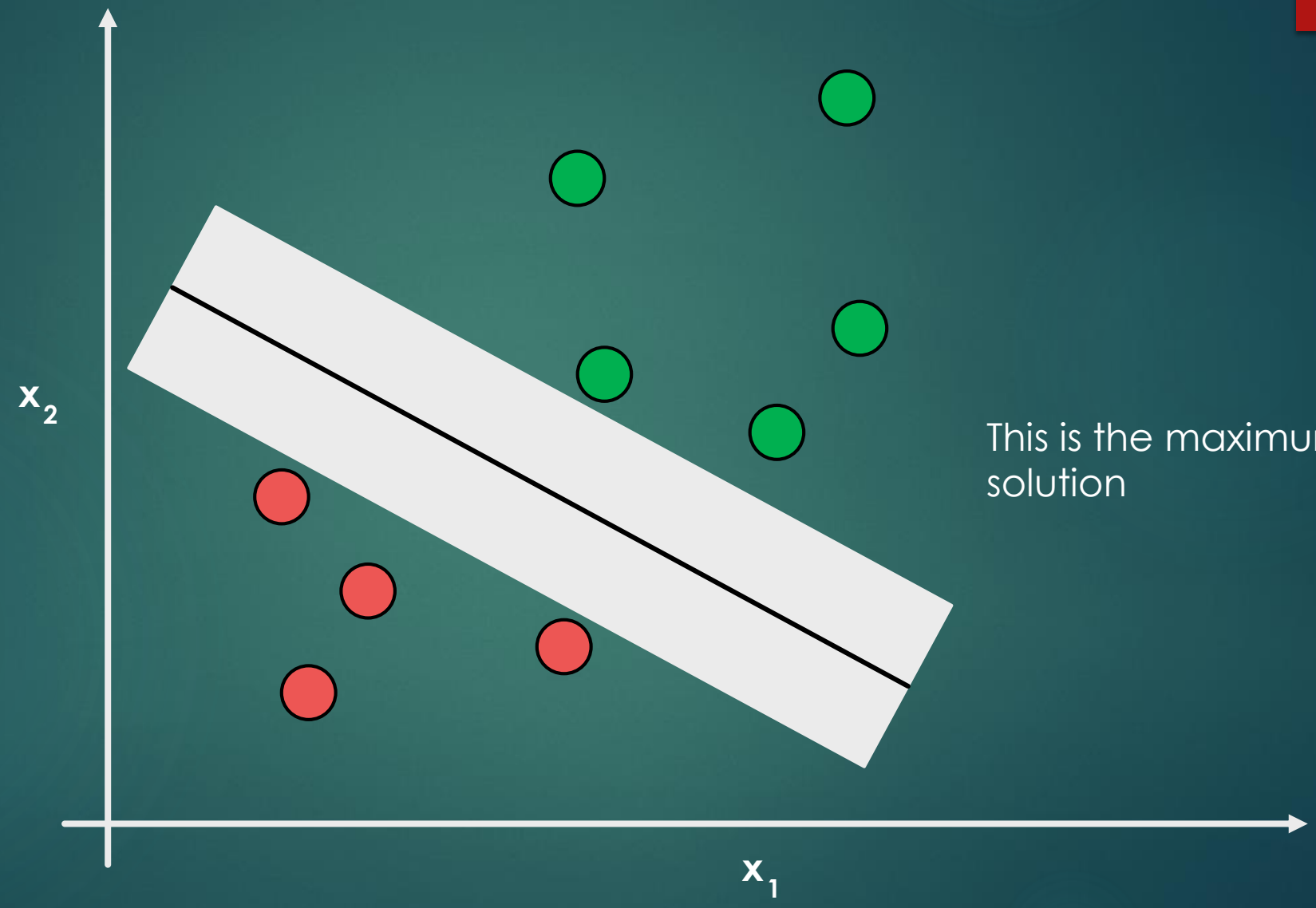
We have two features(x_2 , x_1) and some data points





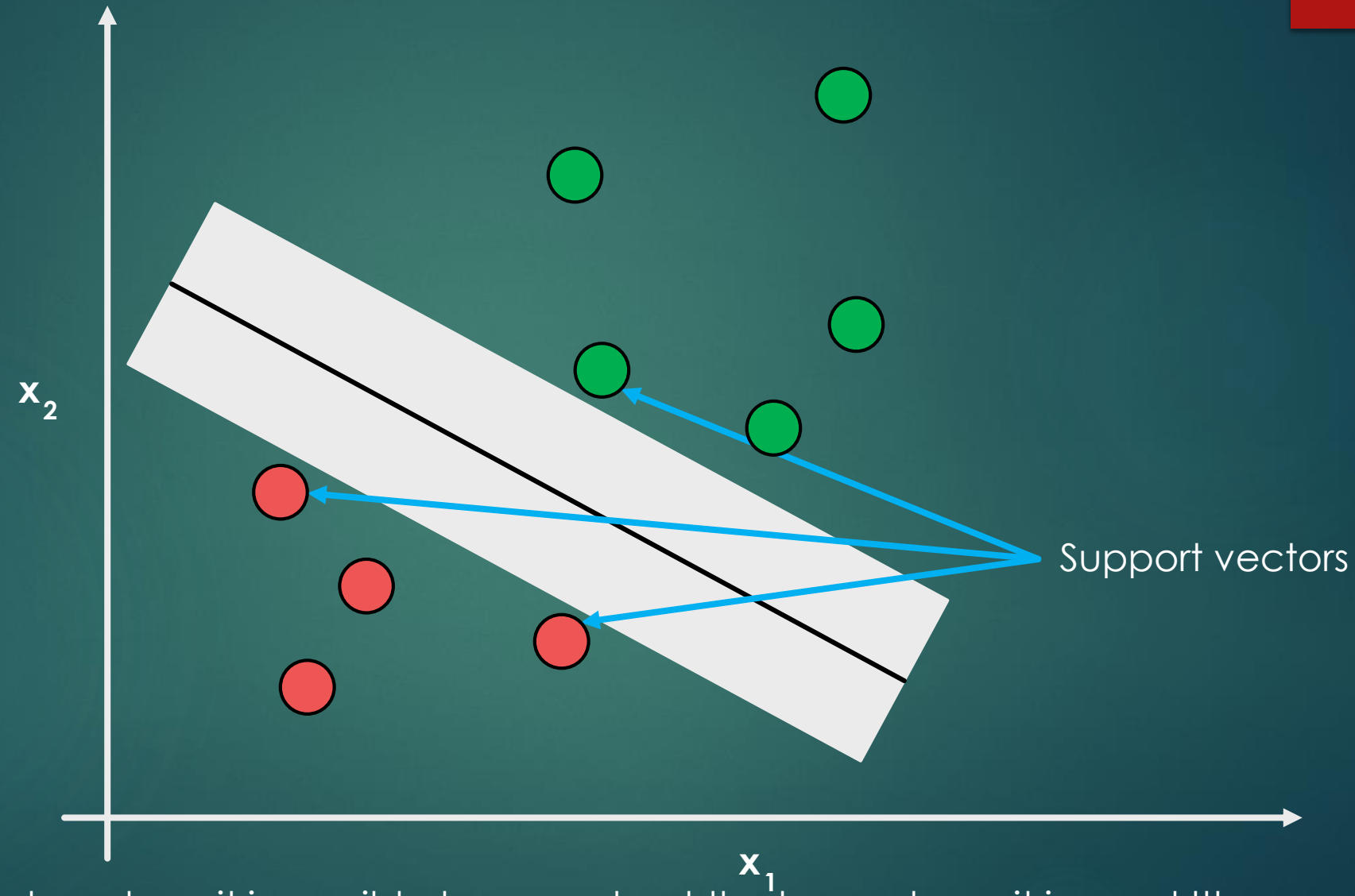
We want to find a hyperplane, in this case a line, that separates the different data points with the maximum margin


x_2
x_1



This is the maximum margin solution

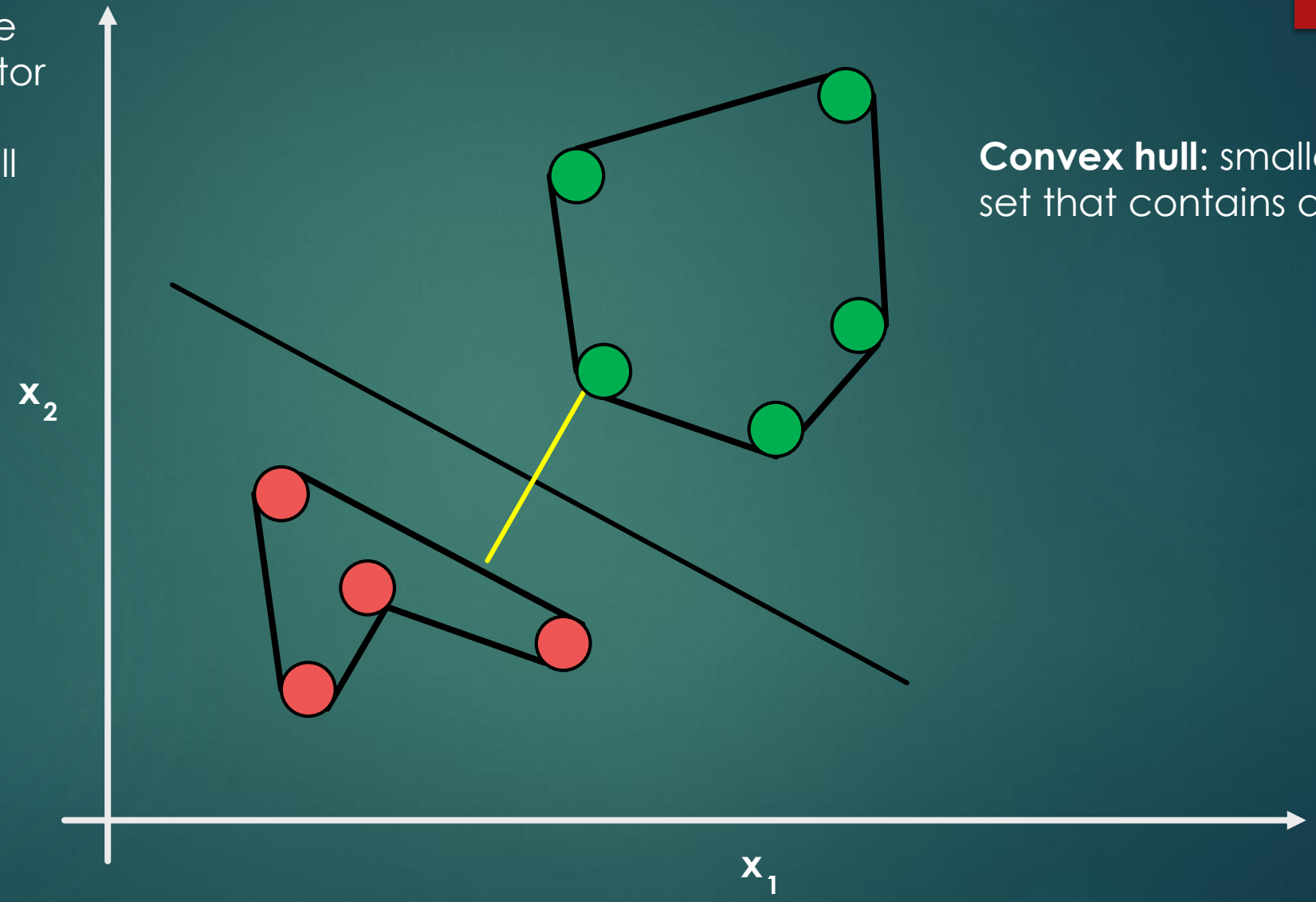
Support vectors: the points from each class that are closest to the maximum margin hyperplane // each class have at least **1** support vector



With the support vectors alone it is possible to reconstruct the hyperplane: it is good !!!
We can store the classification model even when we have millions of features

How to find the hyperplane when the problem is linearly separable? With convex hulls

The hyperplane is the perpendicular bisector of the shortest line between the two hull



Convex hull: smallest convex set that contains all the points

Mathematical approach

$$\vec{w} * \vec{x} + b = 0$$ the equation of a hyperplane in **n**-dimensions

In **2D**: **y = m*x + b**

w_1 w_2 … w_n we have the so called weights

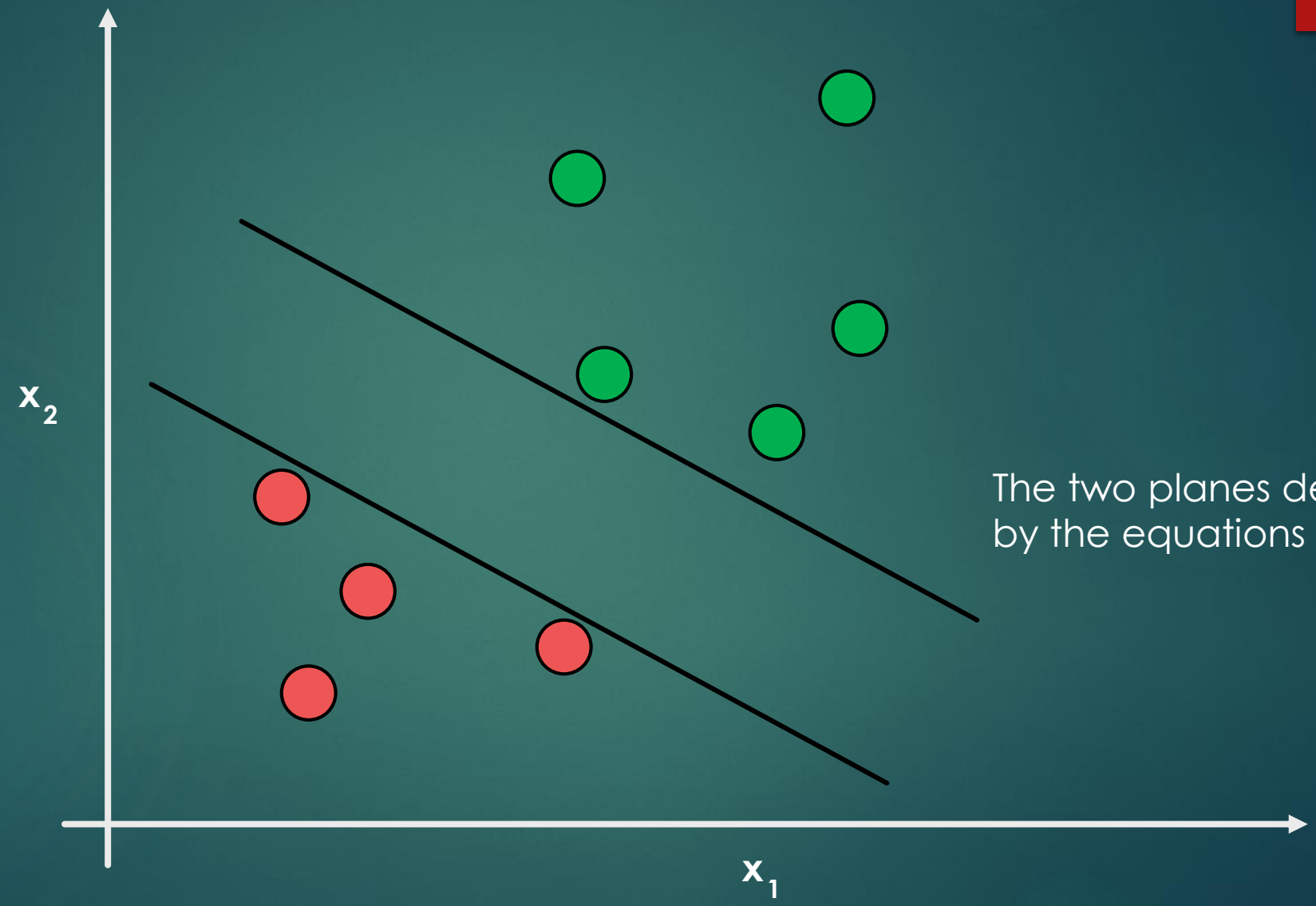
x_1 x_2 … x_n

The aim of the **SVM** algorithm is to find the w_n weights so that the data points will be separated accordingly:

$$w * x + b \geq +1$$

$$w * x + b \leq -1$$

How to find the hyperplane in **2D**? With convex hulls



The two planes defined by the equations

Mathematical approach

Vector geometry defines, that the distance between the two planes:

$$\frac{2}{||\vec{w}||}$$

Euclidean-norm (distance from 0)

We want to make the distance as large as possible → so we want to minimize the norm of the w

We usually minimize:

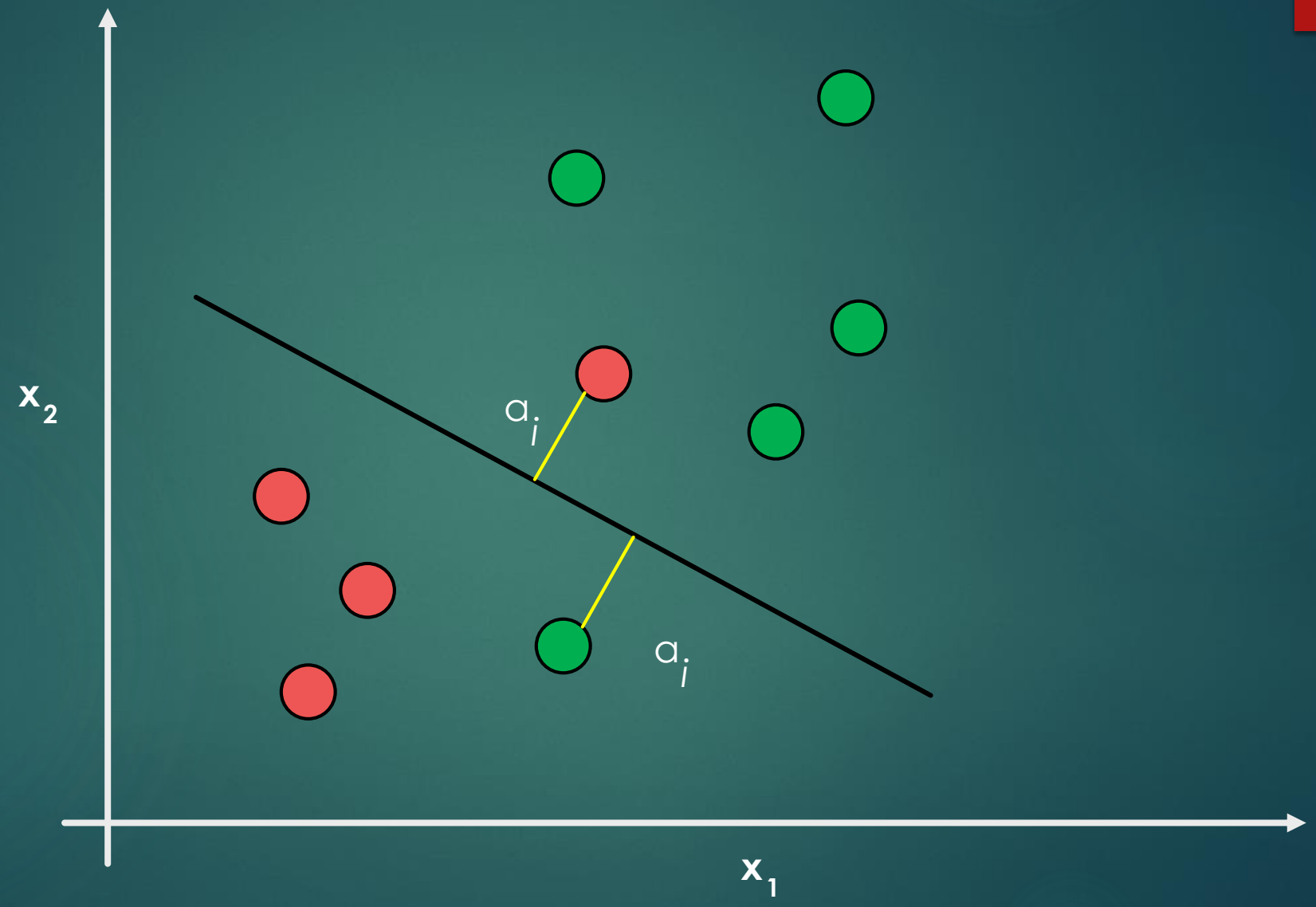
$$\frac{1}{2}||\vec{w}||^2$$

Quadratic optimization solve this problem !!!

Non-linear spaces

- In many real-world applications, the relationships between variables are non-linear
- A key feature of **SVMs** is their ability to map the problem into a higher dimensional space using a process known as the ***„kernel trick"***
- Non-linear relationship may suddenly appears to be quite linear

We have to use slack variables, it is a non-linearly separable problem



Mathematical approach

We minimize:

$$\frac{1}{2}\,||\vec{w}||^2 + C\sum_i a_i$$

C: cost parameter to all points that violate the constraints

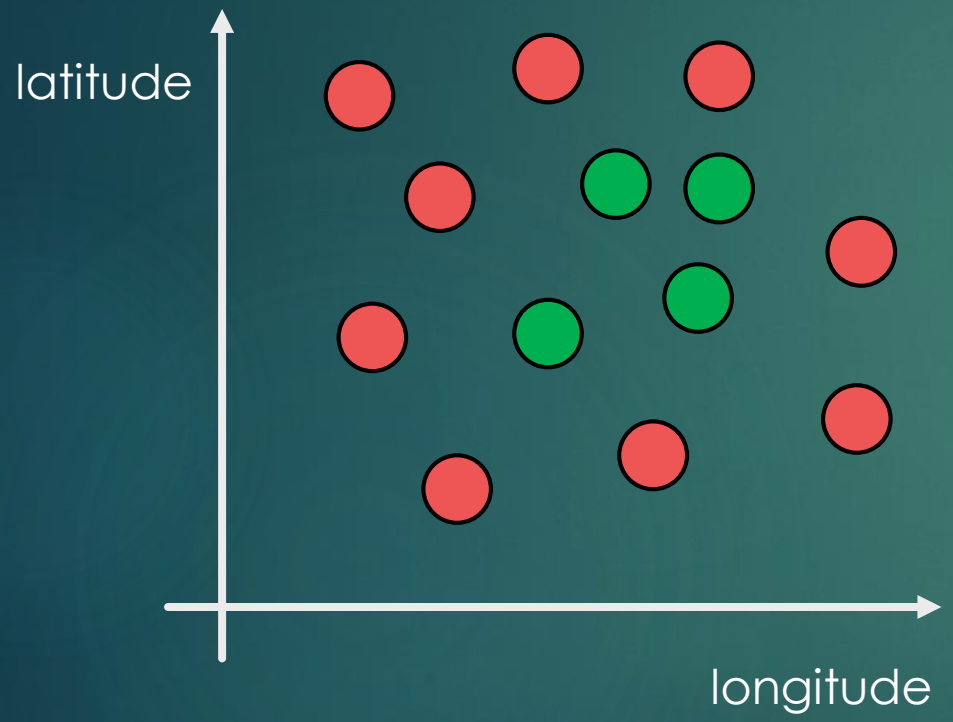
We make our optimization on this cost function

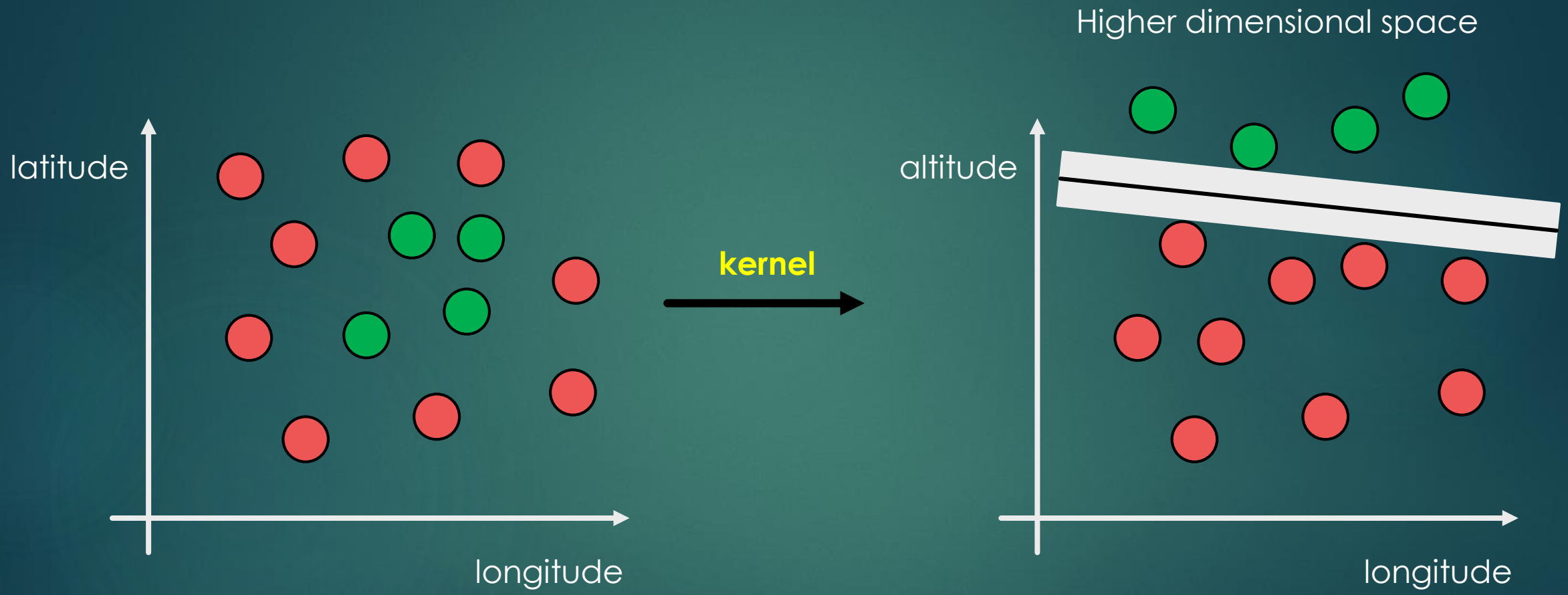
We can tune the C parameter: we can modify the penalty for the data points that are misclassified

C is very large → the algorithm tries to find a **100%** separation
C is low → wider overall margin is allowed with more misclassified data points

Kernels

It can be weather classes: sunny and snowy





With the ***kernel function*** we can transform the problem into linearly separable one !!! (slack variable: altitude)

SVM learns concepts that were not explicitly measured in the original data !!!

Kernel functions

Φ(x) „phi function”

This is the mapping of data **x** into an other space

$K(\underline{x}_i, \underline{x}_j)$ this is the kernel function

$K(\underline{x}_i, \underline{x}_j) = \underline{x}_i * \underline{x}_j$ **linear kernel**: does not transform the data

$K(\underline{x}_i, \underline{x}_j) = (\underline{x}_i * \underline{x}_j + 1)^d$ **polynomial kernel**

$K(\underline{x}_i, \underline{x}_j) = \exp \frac{-\| \underline{x}_i - \underline{x}_j \|^2}{2*\sigma^2}$ gaussian **RBF** kernel

Support vector machines

- **SVMs** with non-linear kernels add additional dimensions to the data in order to create separation in this way
- Kernel trick → process of adding new features that express mathematical relationships between measured characteristics
- This allows the **SVM** to learn concepts that were not explicitly measured in the original data

Advantages

- **SVM** can be used for regression problems as well as for classifications
- Not overly influenced by noisy data
- Easier to use than neural networks

Disadvantages

- Finding the best model requires testing of various combinations of kernels and model parameters
- Quite slow → especially when the input dataset has a large number of features
- Black box model: very hard to understand !!!