

$$1. \sum_{k=1}^n (a_k + b_k) = \sum_{k=1}^n a_k + \sum_{k=1}^n b_k$$

$$2. \sum_{k=1}^n (a_k - b_k) = (a_1 - b_1) + (a_2 - b_2) + \dots + (a_n - b_n)$$

$$\sum_{k=1}^n a_k - \sum_{k=1}^n b_k = (a_1 + a_2 + \dots + a_n) - (b_1 + b_2 + \dots + b_n)$$

$$\therefore, \sum_{k=1}^n (a_k - b_k) = \sum_{k=1}^n a_k - \sum_{k=1}^n b_k$$

$$3. \sum_{k=1}^n ca_k = c \sum_{k=1}^n a_k \text{ (단, } c \text{는 상수)}$$

4. 마지막이에요! 이번엔 일반항에 상수가 있다면 어떻게 될까요?

$$\sum_{k=1}^n c = c + c + c + \dots + c = cn$$

ex)

$$\begin{aligned} & \sum_{k=1}^n (2k + 3^{k-1} - 3n) \\ &= 2 \sum_{k=1}^n k + \sum_{k=1}^n 3^{k-1} - \sum_{k=1}^n 3n \\ &= 2 \frac{n(n+1)}{2} + \frac{1(3^n - 1)}{3-1} - 3n^2 \\ &= -2n^2 + n + \frac{3^n - 1}{2} \end{aligned}$$

Handwritten notes in the image:
- A blue arrow points from $2k$ to $\sum_{k=1}^n k$.
- A red arrow points from 3^{k-1} to $\sum_{k=1}^n 3^{k-1}$.
- A pink arrow points from $-3n$ to $\sum_{k=1}^n 3n$.
- A red handwritten note "등비수열의 합" (Sum of geometric series) points to the $\sum_{k=1}^n 3^{k-1}$ term.

시그마 기본 공식

(1) $\sum_{k=1}^n k = \frac{n(n+1)}{2}$: 자연수 합의 공식

(2) $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$: 제곱 수 합의 공식

(3) $\sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2} \right)^2 = \frac{1}{4} n^2 (n+1)^2$: 세제곱 수

(4) $\sum_{k=1}^n k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

(5) $\sum_{k=1}^n k^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$

(6) $\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + a_4 + \dots + a_n$