

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}.$$


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This function is continuous at the origin; many books prove this as an application of the Squeeze Theorem. Because  $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$ , it follows that  $-|x| \leq \sin\left(\frac{1}{x}\right) \leq |x|$ . The Squeeze Theorem says

$$\lim_{x \rightarrow 0} -|x| \leq \lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right) \leq \lim_{x \rightarrow 0} |x|$$


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$$0 \leq \lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right) \leq 0$$

$\uparrow f(x)$

Hence  $\lim_{x \rightarrow 0} f(x) = f(0) = 0$ , so  $f$  is continuous at  $x = 0$ .

However,  $f(x)$  is *not* differentiable at  $x = 0$ . We can verify this using the limit definition of the derivative:

$$\begin{aligned} f'(0) &= \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{f(h) - 0}{h} \\ &= \lim_{h \rightarrow 0} \frac{h \sin\left(\frac{1}{h}\right)}{h} = \lim_{h \rightarrow 0} \frac{\sin\left(\frac{1}{h}\right)}{1} \\ &= \lim_{h \rightarrow 0} \sin\left(\frac{1}{h}\right) \end{aligned}$$

which does not exist. You can see this in the applet on the left. Move your mouse over the picture to the left to start the animation; you may also have to click on the picture before it starts.

※ 특경 퀴어시 '연속'하라 할지라도, 해당 퀴어시  
외분이 불가능한 경우가 존재한다. (위 예시)

※ ~~특경~~ **해심!!!**  
※ 특경 퀴어시 외분이 가능하다면, 해당 퀴어시 '연속'한 것이다.