① $\int \log_a x \, dx$

- $t = \log_a x \iff x = a^t$

- $\cancel{dt} \times \dfrac{dx}{\cancel{dt}} = a^t \cdot \ln a \quad \times dt$

- $dx = a^t \cdot \ln a \, dt$

- $\int \log_a x \, dx = \int t \cdot a^t \cdot \ln a \, dt$

- $\ln a \cdot \underline{\int t \cdot a^t \, dt}$

부분적분 적용.

- $\int t \cdot a^t \, dt \rightarrow \cdot f(t) = t, \; g'(t) = a^t, \; g(t) = \dfrac{a^t}{\ln a}$

- $\int t \cdot a^t \, dt = t \times \dfrac{a^t}{\ln a} - \int \dfrac{a^t}{\ln a} \, dt$

$= t \times \dfrac{a^t}{\ln a} - \dfrac{1}{\ln a} \int a^t \, dt$

$= t \times \dfrac{a^t}{\ln a} - \dfrac{1}{\ln a} \times \dfrac{a^t}{\ln a}$

$$= \frac{}{\ln a}\left(t - \frac{1}{\ln a}\right) + C$$

$$\therefore \ln a \cdot \int t \cdot a^{t}\, dt = \cancel{\ln a} \times \frac{a^{t}}{\cancel{\ln a}}\left(t - \frac{1}{\ln a}\right)$$

$$= a^{t}\left(t - \frac{1}{\ln a}\right) + C$$

$$\therefore \int \log_a x \, dx = a^{\log_a x}\left(\log_a x - \frac{1}{\ln a}\right)$$

$$= \boxed{x\left(\log_a x - \frac{1}{\ln a}\right) + C}$$

② $$\int \log_e x \, dx = x \cdot \left(\log_e x - 1\right) + C$$

$$= \boxed{x \cdot \log_e x - x + C}$$

③ $\int_1^n \log_e x \, dx = n \cdot \log_e n - n - (-1)$

$= \boxed{n \cdot \log_e n - n + 1}$