- 주어진 데이터: (1,1) (2,3) (3,2)
- 직선 회귀식: $\hat{y} = \beta x + \alpha$
- True result value: $Y = \beta x + \alpha + e$

  $\Leftrightarrow Y = \hat{y} + e$

※ $\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i$

$\Leftrightarrow \bar{x} \cdot n = \sum_{i=1}^{n} x_i$

- $SSE = \sum_{i=1}^{n} (e_i)^2 = \sum_{i=1}^{n} (Y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} (Y_i - \beta x_i - \alpha)^2$

- SSE는 β와 α를 변수로 가지는 이변수 함수임!

- $\frac{\partial SSE}{\partial \beta} = 2 \cdot \sum_{i=1}^{n} (Y_i - \beta x_i - \alpha) \times (-x_i) = 0$

- $\frac{\partial SSE}{\partial \alpha} = 2 \cdot \sum_{i=1}^{n} (Y_i - \beta x_i - \alpha) \times -1 = 0$

- $\sum_{i=1}^{n} \left(-x_i Y_i + \beta \cdot (x_i)^2 + \alpha \cdot x_i \right) = 0$

①

$\Leftrightarrow \beta \cdot \sum_{i=1}^{n} (x_i)^2 + \alpha \cdot \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i \cdot Y_i$

- $\sum_{i=1}^{n} (-Y_i + \beta \cdot x_i + \alpha) = 0$

$$\Leftrightarrow \beta \sum_{i=1}^{n} x_i + \alpha \cdot n = \sum_{i=1}^{n} Y_i$$ (양변에 $\bar{x}$를 곱하면...)

$$\Leftrightarrow \underline{\beta \bar{x} \cdot \sum_{i=1}^{n} x_i + \alpha \sum_{i=1}^{n} x_i} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} Y_i$$

$$= \underline{n \cdot \bar{x} \cdot \bar{Y}} \leftarrow ②$$

- ① - ② : $\beta \left(\sum_{i=1}^{n} x_i^2 - \bar{x} \cdot \sum_{i=1}^{n} x_i \right) = \sum_{i=1}^{n} x_i \cdot Y_i - n \cdot \bar{x} \cdot \bar{Y}$

$$\Leftrightarrow \beta = \frac{\sum_{i=1}^{n} x_i Y_i - n \cdot \bar{x} \cdot \bar{Y}}{\sum_{i=1}^{n} x_i^2 - n \cdot \bar{x}^2}$$

- ② 번 양변에 $\frac{1}{n}$을 곱하면

$$\beta \cdot \bar{x}^2 + \alpha \cdot \bar{x} = \bar{x} \cdot \bar{Y}$$

$$\Leftrightarrow \beta \bar{x} + \alpha = \bar{Y}$$

$$\Leftrightarrow \alpha = \bar{Y} - \beta \cdot \bar{x}$$

- $$\beta = \frac{\sum_{i=1}^{n} x_i Y_i - n \cdot \bar{x} \cdot \bar{Y}}{\sum_{i=1}^{n} x_i^2 - n \cdot \bar{x}^2}, \quad \alpha = \bar{Y} - \beta \cdot \bar{x}$$

- $x = [1, 2, 3]$
- $Y = [1, 3, 2]$
- $\bar{x} = \frac{1+2+3}{3} = 2, \quad \bar{Y} = \frac{1+2+3}{3} = 2$

- $$\beta = \frac{(1\times 1 + 2\times 3 + 3\times 2) - 3\times 2\times 2}{(1+4+9) - 3\times 4} = \frac{1}{2}$$

- $\alpha = 2 - \frac{1}{2}\times 2 = 1$

· $\boxed{\text{회귀식 : } \hat{y} = \frac{1}{2}x + 1}$