

DTMC

Chance

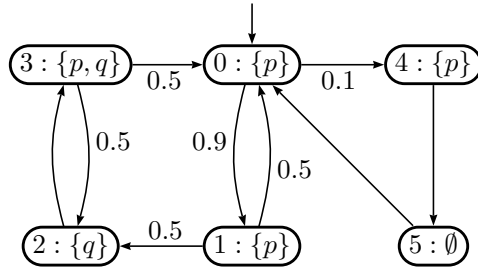
1. PCTL formulas are ϕ with the following syntax:

$$\begin{aligned}\phi &::= \mathbf{true} \mid p \mid \phi_1 \wedge \phi_2 \mid \neg\phi \mid \mathbf{P}_{\sim p}[\psi] \\ \psi &::= \mathbf{X}\phi \mid \phi_1 \mathbf{U}^{\leq k} \phi_2 \mid \phi_1 \mathbf{U} \phi_2\end{aligned}$$

where p is an atomic proposition, and \sim is one of $\leq, <, \geq, >, =$.

- (a) Give the formal semantic of PCTL, interpreted over DTMC.
 - (b) Let's introduce some derived operators with the 'usual' meaning: \forall, \rightarrow , and \mathbf{F} (or \Diamond). Propose a definition for them.
 - (c) Developer Peter Parker claims that $\neg\mathbf{P}_{>c}\phi$ is equivalent to saying $\mathbf{P}_{\leq c}\phi$. Is Peter right?
 - (d) The 'usual' way to define \mathbf{G} is: $\mathbf{G}\phi = \neg\mathbf{F}\neg\phi$. So, for example $\mathbf{P}_{\geq 0.8}[\mathbf{G}a]$ can be expressed as $\mathbf{P}_{\geq 0.8}[\neg\mathbf{F}\neg a]$. Unfortunately, the syntax of PCTL does not have negation over the temporal formula (we only have negation over proposition and probabilistic formulas). So, the formula " $\mathbf{P}_{\geq 0.8}[\neg\mathbf{F}\neg a]$ " is not allowed in PCTL. How do you then propose to define $\mathbf{P}_{\geq p}[\mathbf{G}\phi]$?
2. Explain why in PCTL (e.g. interpreted over DTMC) $\mathbf{P}_{\geq 1}[\mathbf{F}\phi]$ is not the same as CTL $\mathbf{AF}\phi$ (the former is weaker than the latter). Give an example that shows the difference. Note: $\mathbf{F}\phi$ is defined as $\mathbf{true} \mathbf{U} \phi$.

3. Consider the following DTMC M modelling some system:



Consider the following PCTL properties to check:

- $\mathbf{P}_{\geq 0.82} [p \mathbf{U} q]$
- $\mathbf{P}_{< 0.82} [p \mathbf{U} q]$
- $\mathbf{P}_{=?} [p \mathbf{U} q]$

What do these formula say? Then describe the procedure to check them (or at least one of them) on M .

4. Consider again the DTMC M in No. 3. On which states of M the following properties hold?

- $\mathbf{P}_{\leq 0} [p \mathbf{U} \neg(p \vee q)]$.
- $\mathbf{P}_{\geq 1} [p \mathbf{U} \neg(p \vee q)]$.
- $\mathbf{P}_{\geq 0.18} [p \mathbf{U} \neg(p \vee q)]$.
- $\mathbf{P}_{< 0.18} [p \mathbf{U} \neg(p \vee q)]$.

5. Consider again the DTMC M in No. 3. Which states satisfy $\mathbf{P}_{\leq 0.5} [p \mathbf{U} \mathbf{P}_{\geq 1}[q \mathbf{U} p]]$? (Well, obviously state 5, but which other states, if any, satisfy the property?)