

Project #1 : EigenFace

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- [Extended Yale Face Database B](#)

Please refer to [this link](#) for detailed information.

For this database, we simply use the cropped images (which can be downloaded [here](#)) and resize them to 32x32 pixels. This dataset now has 38 individuals and around 64 near frontal images under different illuminations per individual.

[32x32 Data File](#): contains variables 'fea' and 'gnd'. Each row of 'fea' is a face; 'gnd' is the label. [Sample images](#)

A random subset with $p(=5,10,\dots,50)$ images per individual was taken with labels to form the training set, and the rest of the database was considered to be the testing set. For each given p , there are 50 randomly splits:

[5 Train](#) | [10 Train](#) | [20 Train](#) | [30 Train](#) | [40 Train](#) | [50 Train](#) |

[Experimental results \(new!\)](#) | [old experimental results](#)

데이터는 <http://www.cad.zju.edu.cn/home/dengcai/Data/FaceData.html> 사이트에서 위의 Extended Yale Face Database B 를 사용했습니다. 이 데이터는 32 x 32 pixels이고, 38 명의 사람들을 대상으로 각각 illumination 이 다른 64개의 사진을 가지기 때문에 이번 과제의 조건에 부합한다고 생각했습니다. 이는 총 2414개의 데이터로 이 중 28명의 1774개의 데이터는 Training데이터로, 나머지 10명의 50개의 데이터는 Test 데이터로 사용했습니다.

```
def main():  
    # 파일의 데이터 가져오기  
    mat_file = io.loadmat("YaleB.mat")  
    data_set = mat_file["fea"]  
    # 사진 입력 받아서 float형태 행렬도 만들기  
    all_faces_temp = np.ndarray.flatten(data_set)[: 1024 * 1774]  
    test_faces_temp = np.ndarray.flatten(data_set)[1024 * 1774 :]  
    all_faces_temp = all_faces_temp.astype(np.float)  
    test_faces_temp = test_faces_temp.astype(np.float)  
  
    # 데이터는 1774개로 정했고, 원본 데이터의 형식을 맞추기 위해 일단 이렇게 설정, 또 테스트용 50개의 vector를 만들기  
    all_faces = all_faces_temp.reshape(1774, 1024)  
    test_faces = test_faces_temp.reshape(640, 1024)
```

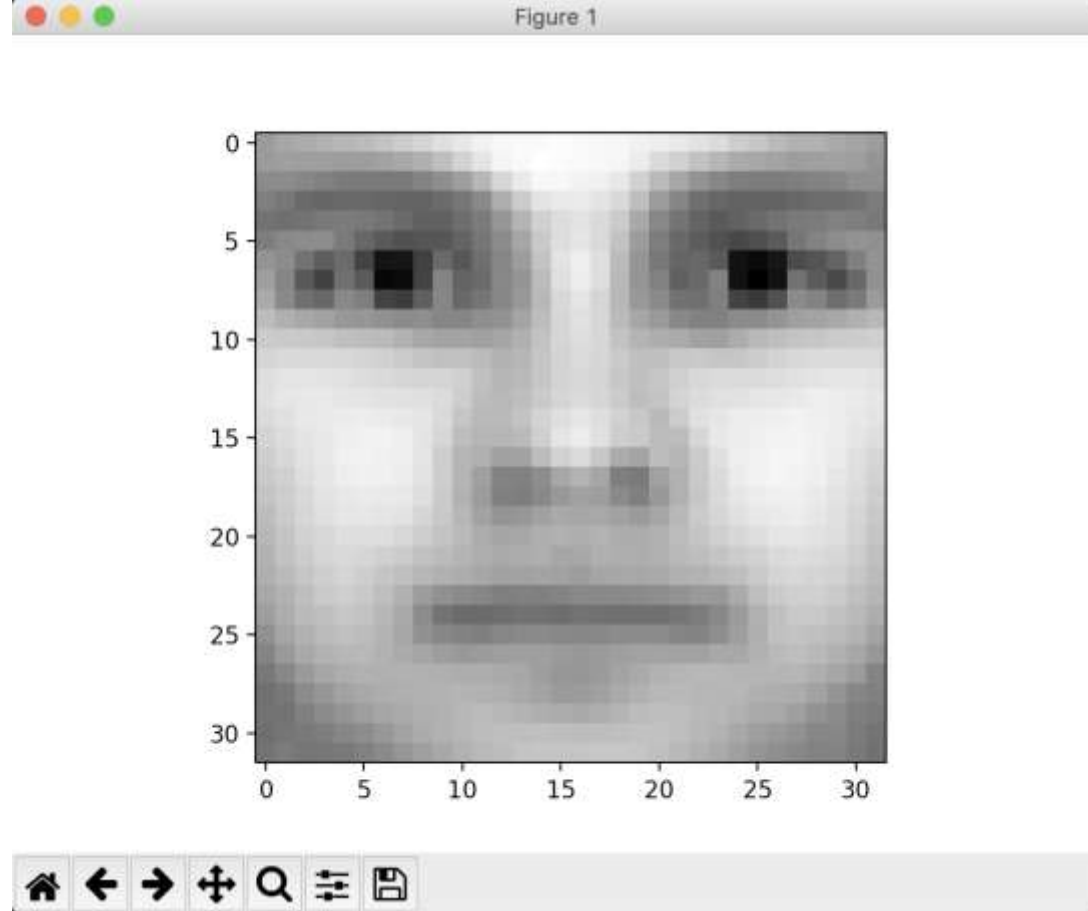
이 부분의 코드는 데이터가 저장되어있는 matlab 형식의 파일을 읽어와 1774 x 1024의 Training데이터 행렬, 640 x 1024의 테스트 데이터 행렬로 만든 코드 입니다.

```
# 입력 받은 사진이 옆으로 누워있어서 바로 세우기
for i in range(0, 1774):
    all_faces[i] = all_faces[i].reshape(32, 32).transpose().flatten()
for i in range(0, 640):
    test_faces[i] = test_faces[i].reshape(32, 32).transpose().flatten()
```

이 부분의 코드는 데이터를 입력받아 사진으로 출력했는데 사진이 옆으로 누워있어서 바로 세우도록 고쳐준 코드입니다.

```
# mean 구하기
mean_arr = []
for i in range(0, 1024):
    sum = 0
    for j in range(0, 1774):
        sum += all_faces[j][i]
    mean_arr.append(sum / 1774)
mean_face = np.array(mean_arr)
for i in range(0, 1774):
    all_faces[i] -= mean_face
```

이 부분은 데이터들의 중앙값 즉 Mean Vector를 만들기 위한 코드입니다.
이를 위해 라이브러리를 쓰지 않고 직접 구현해 봤습니다.
또 데이터가 Vector Space를 포함하기 위해 원래 Training Data Vector들에서 전부 Mean Vector를 빼줬습니다.



위의 사진이 데이터들의 중앙값 즉 Mean Vector를 사진으로 나타낸것 입니다.

```

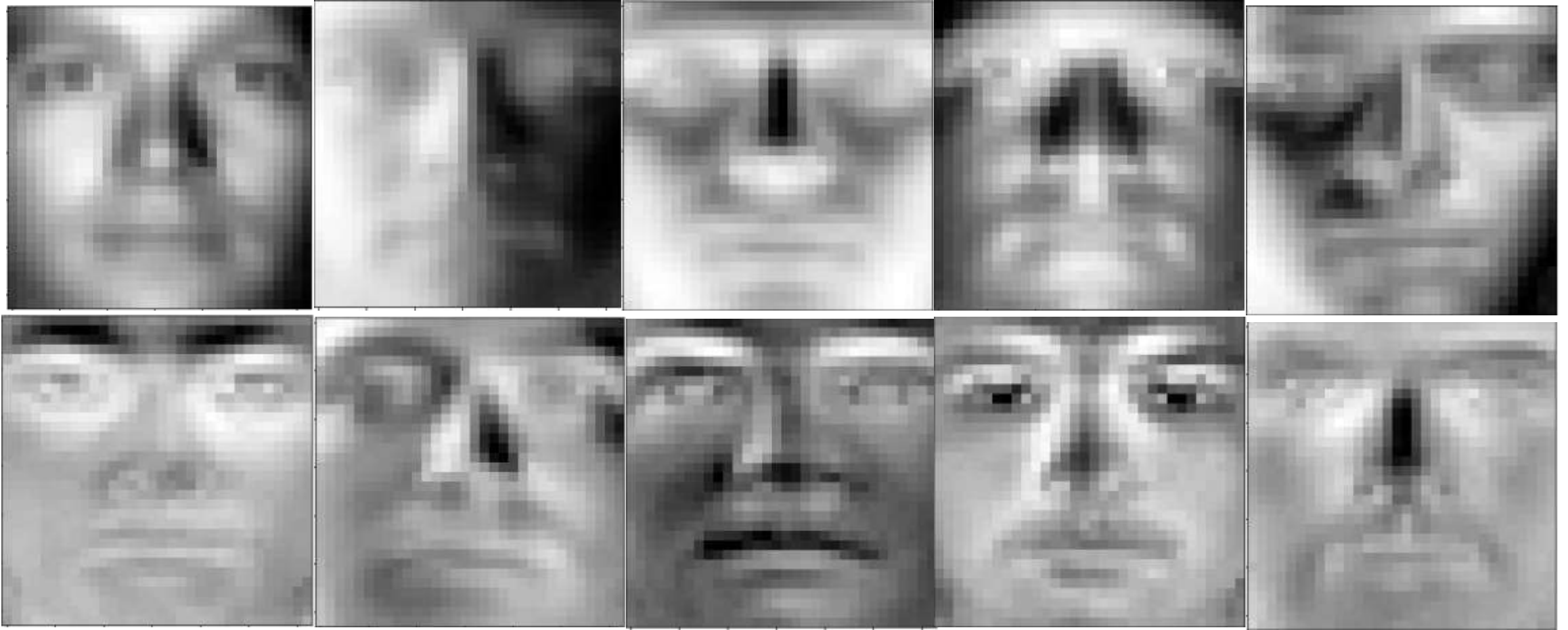
# Matrix를 svd 분할
u, s, vt = np.linalg.svd(all_faces)
# img = plt.imshow(u[:, 0].reshape(32, 32))
# img.set_cmap("gray")
# plt.show()

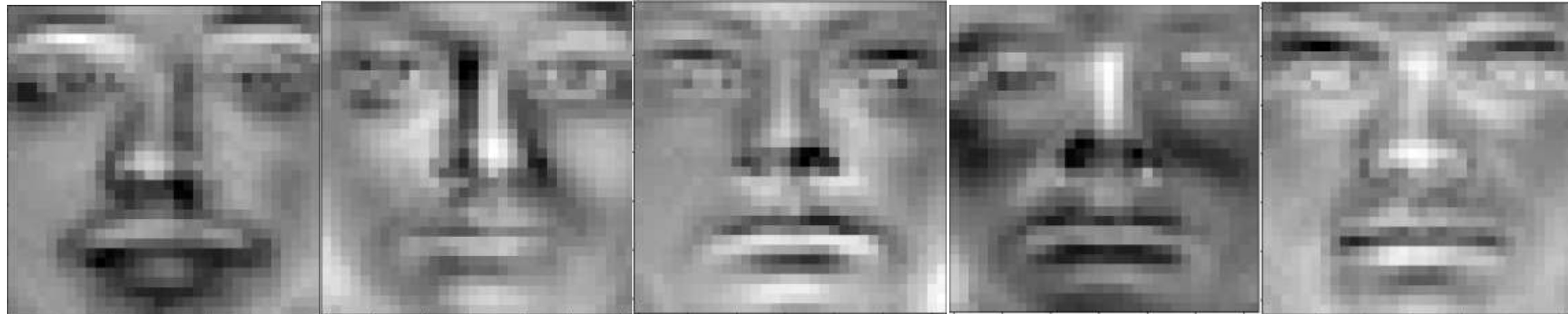
# eigen value가 큰 eigen vector를 위해 idx를 리스트로 나타내려고 하는데 자동으로 정렬되는거같다.
# eigen_idx = []
# for i in range(0, 40):
#     max_eigen = 0
#     idx = -1
#     for j in range(0, 1024):
#         if max_eigen < s[j]:
#             if j in eigen_idx:
#                 continue
#             else:
#                 idx = j
#                 max_eigen = s[j]
#     eigen_idx.append(idx)
# print(eigen_idx)

```

Numpy 라이브러리를 이용해 Training Data에 MeanVector를 뺀 Matrix를 svd 분할을 했습니다. 또 svd 분할한 값에서 EigenValue들의 크기를 이용해 EigenVector들의 Priority를 정하려고 했지만 이는 자동으로 내림차순이 되어 이를 사용했습니다.

40개의 EigenVector(EigenFace)







```

# eigen vector들을 모아놓은 matrix를 계산하기 쉽도록 transpose 시킨다.
u = u.transpose()
all_faces = all_faces.transpose()
for k in range(0, 10):
    for i in range(0, 5):
        weights = []
        # test_face = test_faces[i] - mean_face
        test_face = test_faces[k * 64 + i] - mean_face
        for j in range(0, 40):
            weights.append(u[j].transpose() @ test_face)
        print(weights)
        # print("k : ", k, " ", weights)
        face = mean_face
        for j in range(0, 40):
            face += weights[j] * u[j]
        img_origin = plt.imshow((test_faces[k * 64 + i]).reshape(32, 32))
        img_origin.set_cmap("gray")
        plt.show()

        img = plt.imshow(face.reshape(32, 32))
        img.set_cmap("gray")
        plt.show()

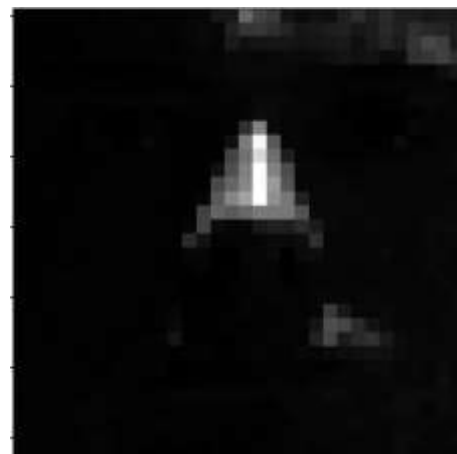
```

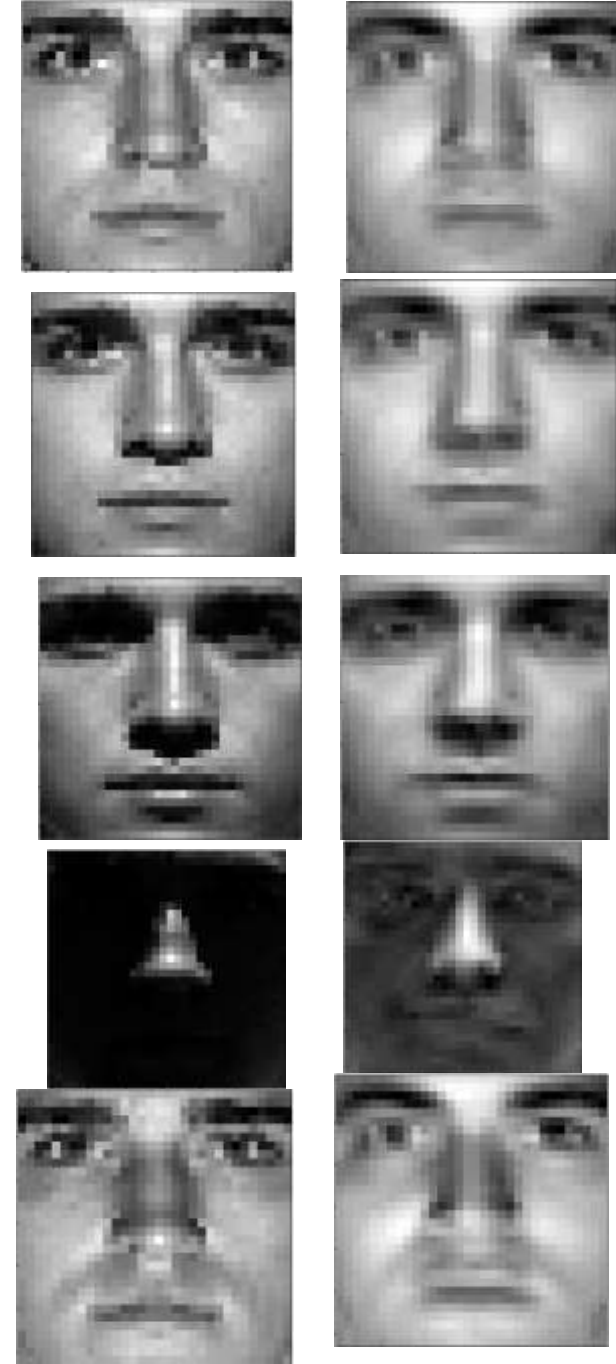
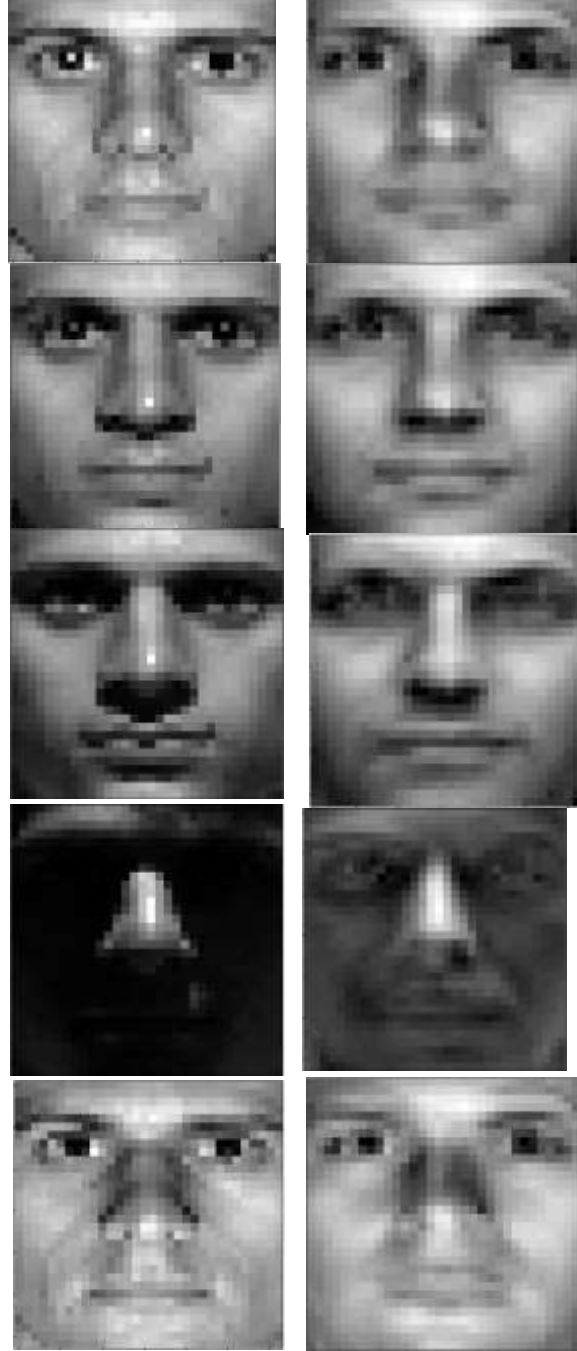
이 부분의 코드는 Test Data의 얼굴을 EigenVector, MeanVector로 만드는 부분입니다. 먼저 EigenVector들은 서로 Orthogonal하기 때문에 각각 EigenVector들의 Weight를 EigenVector와 TestData-Mean 의 innerProduct로 구했습니다. 그리고 Mean Vector에 40개의 eigenvector*weight 를 더해 비슷한 얼굴을 만듭니다.

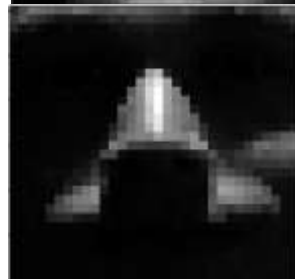
테스트 결과

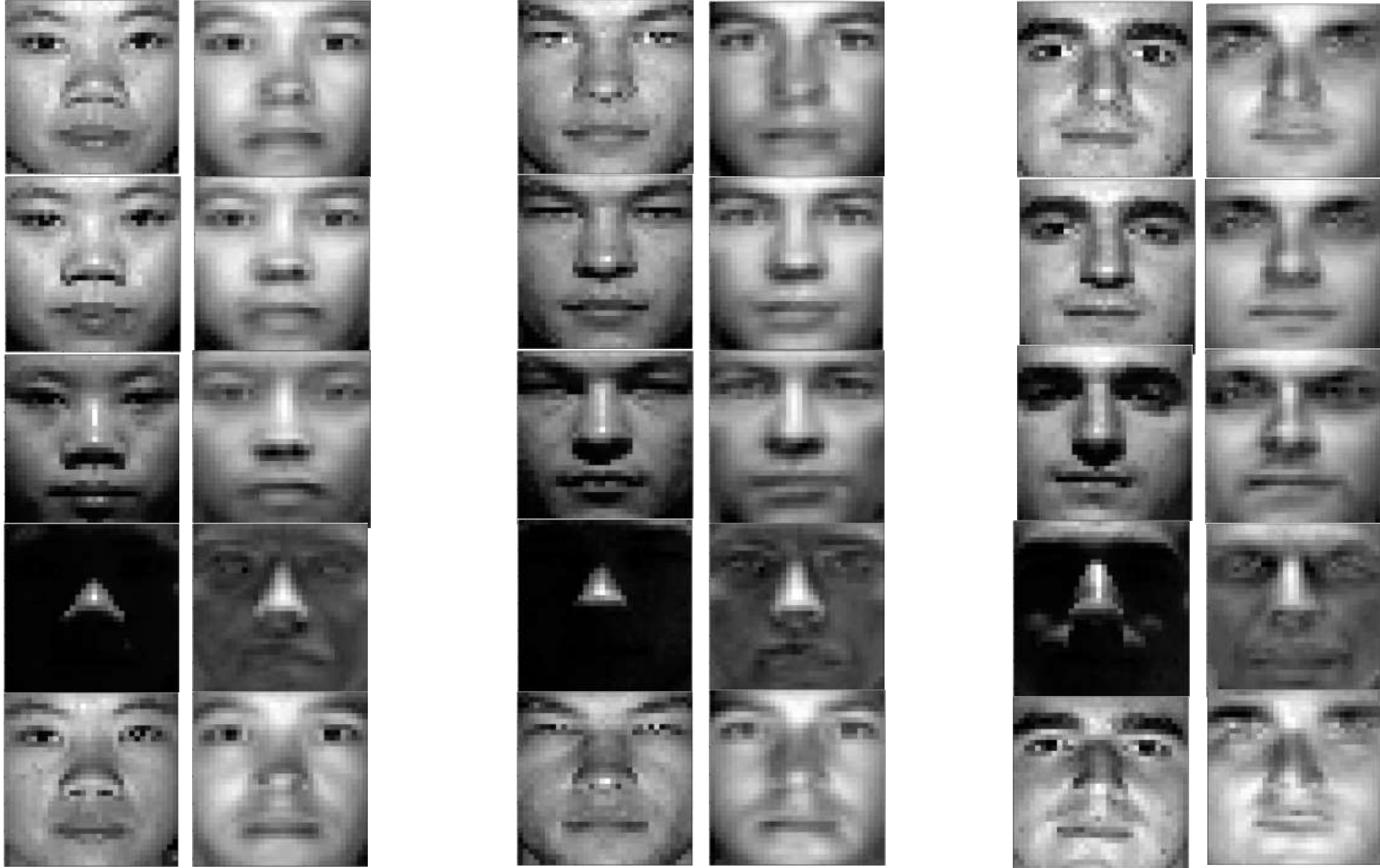
원본

만든이미지









개인별 Coefficient 분석

```
k : 0
coefficients of 0 : [76.60928077185105, 799.2522101244956, 49.99024576857568, -337.24130462230494, -23.05235655952072,
52.80621840222507, -34.671837874597415, 1.732028508852301, -189.5534406931489, -150.79383672699035, 59.558710641340326, 80
.15135584244199, -32.38763691421071, -86.96764652399949, -18.93275042178208, -159.7880682154625, -55.82455255451201, 53.24
430675038843, 24.71845884557816, 70.29667594362297, 115.95901229295038, -100.33269012176059, -11.53955528405055, 4.8498849
61869555, 29.386053379427167, -60.63452768080591, 96.84302019875719, -6.85627604306546, -42.388516489172105, -4.7579612915
73906, -17.24095937101981, 24.93202293068932, 64.81871959003027, -68.76878689444075, 49.5853441508461, 44.049377827761546,
-24.993539017716103, 72.20956164578698, -82.36530968144507, 68.1959925947901]
coefficients of 1 : [48.5612631616192, 94.64724883653011, 393.3130598691653, 174.12399658234244, -5.307797356953293, 66
.71836211532221, 16.99819011098495, 21.76361339619995, -12.741078201203022, 1.7493894161215582, -30.153023670368206, -72.2
1141934841454, 46.51029162778548, 56.673417173731586, -28.93712176153509, -45.33221850634001, 45.04856590939081, -81.59057
262163284, -11.894114165167633, 1.055700084314588, 9.688167027239178, -65.58271610933033, -31.863156552779593, -6.17319184
2204679, -49.10081882869443, -33.88548109297784, -59.1297971398137, 4.425895926980568, 38.19679777501066, 11.6463462386538
48, 15.788485164635878, -21.21086608605612, 41.81368337763989, -32.235268284358334, -3.421541265295735, 24.841532829525747
, 6.423934416119139, -62.52586047389481, 42.295062573305664, -9.733591190565392]
coefficients of 2 : [29.158290792565953, -437.9638729480985, 361.66686231292056, 130.6848315286174, -2.0991617311825657
, 162.6635795908763, 62.42819873281156, -6.533332468498777, 47.246588304522945, 51.702078850815525, -111.21570966187392, -
200.45460096681137, 8.19155685300274, 141.97797613098032, 67.89048684879397, 77.55057403179441, -16.01336909549248, -192.3
5848765641543, -62.11101489996442, -29.994308315925817, 27.778800016100604, -17.851535340324688, -32.96164233582131, -12.1
98178360697366, -43.82871110755167, 105.49422218829794, -167.66465639342863, 17.610265728493033, 32.765911359860745, 92.46
147902370564, 27.12806279983154, 30.68380847768797, -33.7813488278631, 51.832684834785276, 11.83852162405241, -7.683943652
378426, 43.73080340838611, -106.06509033268368, 71.34898760608229, -90.02275559159192]
coefficients of 3 : [-261.9868154520915, -2422.704790676488, -352.1391499213973, -444.26320229726855, 5.36484949434298,
-380.4687886232656, 126.98401777118923, -78.96611312600328, 194.36762640061912, 284.05780106184034, 95.91763443826848, 93
.56465019759386, -19.425600593499453, -236.73138333305954, -25.65965851882357, 141.62580751823924, -49.227649264365894, 18
8.95779746065767, 41.61956692022656, 26.381788145810532, -31.456020811946757, 222.05224164796527, 182.6243097256197, -31.8
68739706627554, 45.77361140649977, -43.89792146104121, 106.42870365906501, 20.131585723586205, -96.06431019962275, -75.375
23183595674, -16.122730361083153, 5.899495253970343, -60.48243895509178, 105.16087921831269, -48.55939973401942, -8.055229
754772498, 23.237940533247667, 77.64720767358962, -39.036045912712225, 16.971572359903423]
coefficients of 4 : [156.08823451983238, 2571.092051960538, -717.810669520275, 2.7637164735795245, 39.96771140323068, 7
9.9732637390504, -207.0296326300532, 88.10048821889434, -214.5356688332292, -333.697953748256, 97.69886775374978, 186.348
76307255723, -78.87424095554368, -39.33816687445772, -2.0916944681466587, -77.99225190679225, -7.218648018860193, 133.7957
07152102, 22.159110579852314, -4.47450468792034, 28.38011781899354, -154.25712608330716, -126.82493367304332, 32.377046758
98234, 75.25386390243847, 9.406997762606128, 120.49549860866432, 27.286269337047447, -9.781509375306086, -47.2983148369344
1, -11.718785971131705, 49.51143457574956, 54.90192265381688, -65.6533809317742, 16.961183880466095, -17.117655111985357,
-90.37108222568102, 150.0210658894134, -74.50536527198312, 84.67463257204247]
```


k : 1
coefficients of 0 : [150.75090310027065, 271.2527778489229, 208.87895763164494, 296.3783466913645, -37.04838789958182, 220.8331276224617, 146.83947296257844, 120.95954101778099, 184.40165624568778, 200.8132166300379, -175.1129972317196, -74.94809329383935, 63.40546892555139, 204.95165431216088, -1.4729296821624374, 21.554602683674418, 143.81979635762872, -69.16818139307439, -78.10042298020699, -113.11769523627481, -182.14360396361238, 71.5565758112645, -114.66302415597781, -20.597073478910367, -69.7532260111605, 94.16679371672637, -140.0893437774228, 32.095507885817405, 65.78917276179784, 95.635319633812, 46.27201558990846, -172.34120508487757, -109.23746555766994, -37.39333262782318, 30.359245989323316, -87.69243202366914, 113.31778385650051, -104.80996859401961, 55.80322185567725, 83.5510689460815]
coefficients of 1 : [41.26011127045483, -66.175906978737, 417.4648810855572, 40.80811218919, 6.623061581815241, 149.26423790975358, 23.50413037332458, -4.4420454767163955, -14.560357285755094, 27.55565349620004, -113.32815964060713, -79.08091629275047, -51.13921486544745, 90.1809166960468, -21.307383712208768, -15.231938827021791, 14.848540062978019, -136.05809944249322, -43.54762533386342, 18.571334433914593, -43.68407161501605, -1.9070369592691805, 29.60604986046131, 14.824364428186744, -71.56203815956727, -32.536577932533454, -81.44538781617973, 12.526697727922937, 55.13104704250533, 53.57424824304775, 84.49476718047273, 18.270539122482134, 0.29186520759344603, 19.225624287923452, 27.066424448090995, -2.401654038631479, 58.01388143325619, 37.74001118579341, 34.952781138643324, -61.22387689198797]
coefficients of 2 : [-48.16828017030011, -648.2416269325413, 364.28812646013984, -14.406538037267264, -35.78299296745734, 87.91174302132222, -6.230167383254338, -33.05843986902316, 4.972793134545583, 11.708213728664962, -148.7905391550762, -34.5483177999392, -39.96042653966396, -33.39729688360444, 37.43557291539232, 67.8823262625648, 27.379652884836688, -26.247595204751494, 32.00726539480563, 32.120786496896145, 7.605442121567766, -1.812471352807267, 76.24769322506073, 2.319779529214518, -33.75580550291116, -10.122259946744457, -44.93758908859836, -26.336130166079244, -22.296420476544142, -96.89981627697314, -63.4148689058588, 86.98455177199924, -18.158734814161143, 42.345054641903026, -5.601716074891387, -33.5831756802162, -44.106374252155426, -36.37416401453817, 57.06352254435582, -119.61126523633337]
coefficients of 3 : [-428.5766179954727, -1907.6338851842088, -219.95344496222708, -300.7511555326931, 128.9561590262528, -334.6785637909685, -76.2202274160319, -77.90539261966657, 109.03070145593026, 228.618919033496, 173.03335278681195, 60.2132381207292, 93.15543426632247, -290.74544190689414, -86.80038043585196, -14.13946758050979, -250.67150814456892, 180.83531939624578, 111.09248054748952, 93.44645903682974, 100.96305384158254, 74.39993564934902, 183.19511543815617, 105.607849459562, 89.75747395257609, -86.1276388488842, 131.68550589418243, -38.76245272732871, -62.65171003714943, -62.52880193468139, -11.881628197161174, 73.96915389594416, 56.7896362481829, -28.77986843345282, -75.12118840789695, 55.740074626867425, -98.61062838997962, -132.49853305890593, -106.94048305996549, 72.39221779996905]
coefficients of 4 : [413.8671277928472, 2554.616374813608, -938.2812010274085, 229.80877633817275, -89.15480697736837, -4.668481967890344, 16.96944942975569, 70.72699007249734, -115.65092110831819, -264.90157208816777, 155.22396170786647, 83.20445867453589, 46.391609498765575, 153.0964057989593, 92.30798533543283, -21.157952092521647, 178.76887652902232, 56.131686537779416, -147.599846994612, -184.3431877157821, -38.295798744883626, -42.926491494770374, -332.53739714035083, -76.66132524220328, 68.4796524208179, 164.40814616138516, 54.16899024466187, 71.81006971608409, 1.6747245453708164, 52.14672312598813, -62.04004076717696, -192.282420680187, -22.47834578667871, -13.364507542911923, 62.761720856987196, -5.891030583200267, 37.63667440400683, 130.48660450066717, -11.994457630068176, 107.97281620139503]

k : 2

coefficients of 0 : [-37.85963109152783, -202.53267452019767, 526.7385392465055, -176.72302179381188, 121.83121481351372, -683.9462901939326, -205.43292413253337, -178.00960815973417, -79.22652858984651, 6.291583963881763, 77.1153292450849, 20.119336718742364, 42.60251096097463, 194.4123129198702, 116.5946845885766, 203.16753135454945, 10.712342612806387, -26.73447507005436, 185.23470935999194, -46.665379247484, 8.394691045911106, 99.24777586225116, 156.04915596492492, 55.62668907576386, 42.44027801057223, -212.04658154245277, 98.42527777901779, -165.14290892263122, -12.160219461299963, -79.27233648425027, 63.25694895169641, 111.22454881436715, -8.511543854151595, 131.29867025495258, -48.57469611347537, 62.51780120563362, 38.284126494094096, -94.18498286592747, 74.95406839542113, -159.14191182961304]

coefficients of 1 : [-46.26239041674908, -233.79935222388536, 359.2047226619078, 109.49566147636091, -28.142622651300893, -71.3076757576635, -16.885020415707878, -48.8508805730113, -70.8826824390749, -14.73441978604668, 125.33808819488047, 29.912388273015104, 23.41939627776161, -138.95178336159648, 69.6694667309271, -80.31904997436175, 28.522163486371443, 23.46735137510781, -77.20439427677597, -34.63914878890832, 57.60247817310636, 37.37687710452432, 58.590152501250394, 88.46883011384794, -34.29655546558299, 31.744520785147156, 78.01868461174166, 15.068290399151408, 34.494627991345986, -74.91792551728093, -77.05471501093638, 2.986311617116808, -6.630547863056961, -23.67083393437654, -18.802658411268787, 88.7068585665331, -19.05176600797395, 92.56259079482935, -75.20535432407435, 50.298754577912675]

coefficients of 2 : [-62.73682007162883, -745.3599384567689, 353.16883769495325, 125.21074585912089, -13.28553359991877, 201.10826860881537, 44.6881310659748, -24.036452498993576, -102.74349715804418, 75.3960471199033, -53.22104964867914, -204.97949503539496, 39.90777138667198, -19.735692408322585, -136.87878783029984, -20.416746487479074, -23.133632581234288, 16.104922777091314, -34.88314026402955, -3.2396952732505127, 160.04346322238422, -44.654413810285575, -111.18118450031446, 106.68044635953763, -156.7517301399883, 52.86031286579588, -110.99553956593189, 97.11281437729227, 47.1945825609363, 151.24177757032624, 70.13671211505549, 22.027667030727553, -8.739139670242235, -80.24164097618596, 19.927721438315245, -9.588657998225967, -4.6105030067827615, 132.79269553185637, -28.507326619634085, -7.575509272881743]

coefficients of 3 : [-272.44201249792576, -1705.7619882706313, -411.3193638241736, -429.49387382310357, -39.876435699717504, 306.45105236163664, 154.34673102277876, 116.3303154466168, 274.01233324604664, -24.790772084052005, -129.25308049101722, 40.06375138229717, -103.46470440229376, -60.690780833186096, -99.65828141393656, -95.44416048663281, -69.01755633146749, -99.9312212311444, -18.189308159929457, 160.94792758820114, -166.17886018468158, -44.60816509376668, 183.7174742978395, -240.5856333126306, 92.0921274657912, -16.28261024620989, -86.27782095097581, -29.177392038859438, -53.96027728987819, -16.49320792943914, -34.646341025529175, 18.867226386942182, 16.952109028905173, 19.039697082614204, -4.733110511404479, -48.16804054569684, -63.945378976216666, -185.0508872703249, 90.14628021723475, -23.16422184056323]

coefficients of 4 : [453.7645265667798, 2944.62639336845, -617.0335152333265, 148.17326759972684, 75.81119180884279, -319.37363228104437, -258.6905472989905, -186.66827755363187, -145.76179668469328, 2.1873567790292956, 96.28483614713727, 75.47835664558382, 43.295740325953865, 259.2584563344167, 29.296864244802386, 14.35676727851746, 77.19969319978216, 76.83202532671092, 121.78133948542188, -66.46478250688504, -127.36861856812033, 44.042796599877555, -46.88894018928829, 12.980536577076258, 103.86657244261579, -15.577009423279666, 88.39141099449822, -92.72751671865035, 14.589414248265026, -15.573032681103319, 55.655185931338266, -49.779004565621975, -43.64451141779427, 144.14447636914161, -14.801548734644083, -29.936446327856242, 77.21958990917076, -43.480079969788385, 31.00989913333147, -62.46297351456286]

k : 3
coefficients of 0 : [-134.3676037818522, 44.14575212313734, 169.5608981695142, 372.9324627135503, -166.3077167806571, 3
49.01056961274406, -4.253880512393582, -95.41003255320562, 260.12160075035155, -197.18463844547165, -86.12033079468759, -7
.17072270864014, 9.919940257460068, -206.14867009936887, -3.5289768876664946, 112.33244510787239, -2.6561858740678543, -99
.21380151432449, -107.0771707886293, 6.17563874465684, 152.22310582082218, -59.68998111203233, -34.16542784355997, -107.55
773664222234, -78.77224578464146, 3.4086263220558095, -142.76328523992038, 117.4798844935151, -7.567697562578557, 38.70817
902732352, -25.883064434108583, 50.08743871286241, 115.89139080812897, -258.1399216889925, 37.24581937552381, -51.22005754
9582435, -38.02410395399357, 70.57139723579826, -95.14201026295305, 109.79530364987176]
coefficients of 1 : [-22.974171763358257, -85.34909947040819, 386.06019280051135, 139.25441647384093, -2.37973622927567
43, 20.01581592386632, -41.36625436564978, -86.21696933109101, 12.872422460209224, -20.904046877316375, -22.38816339155402
, 14.760140210600422, -176.6737540293327, 63.06802298275516, -90.57658744209266, -57.50917216898747, -16.45267434537284, -
98.14377326347521, -29.15496087635482, -34.533957259663275, 81.12666371546928, 22.598086813572976, 3.135761414086982, 48.9
11663326313615, -38.74119783775003, 0.054259856458962474, -9.75991656402864, 95.4539298942784, -18.194062209695332, 25.799
60403461722, 25.79430612877452, 14.1913861528639, -3.2680180217847514, 36.54363296044043, 33.85680244140882, -16.780147880
878985, 35.07189746101959, 46.15914449729406, 6.175128741320357, -10.302180955165095]
coefficients of 2 : [-39.634741340838765, -611.3670329300306, 384.58763157679385, 79.03903846264748, 20.64179794041289,
44.072575475659804, 24.067660330824637, 9.582526854573032, -69.99259959599638, 71.46192816330051, -43.2279369361579, 29.2
1212203394569, -148.13582208185855, 5.423631569354722, -50.525094785823754, -66.75029279974278, -42.83461818579619, -89.98
362600753583, -13.268420073915866, -47.66462474644237, 28.15408869116897, -10.320179755476829, -13.043538482124369, 52.713
09279237634, -35.77509949650979, -1.7754067528757318, 15.043452897587152, 29.396153540919265, 11.175583878122175, -60.9864
10861604185, 16.677513660944474, 12.646810160091919, -15.763532700459198, 137.65487160703958, 19.611529357611328, 31.93492
943806021, 26.050462894517988, 34.89100229017557, 82.79680952628584, -107.81005780425559]
coefficients of 3 : [-189.92430799039064, -2219.398543684677, -347.73509775075047, -654.2484668147392, 46.1657260077850
3, 6.625455324660799, 298.8966071927785, 285.15471915612835, 13.92752333228455, 184.7622507211325, 1.2669050962488733, -12
1.72343493269757, 216.86847209399417, -121.99365968669437, 62.983476218465206, -10.355849453233091, -51.50394029641265, 21
6.61486311762707, 61.78514192589505, 222.3714837794767, -135.80705651957769, -11.94156317145074, 12.545263144638382, -27.5
78476488168405, 61.74756147054971, 53.55429949614354, 84.81712989531394, -145.28313648570548, -7.681655114606346, 40.51257
9826771024, -36.859440606875395, -3.809398994894429, -30.76573058119796, -10.199195733051003, -125.47738872145707, 56.2281
6836037792, -110.51611488546182, -54.18955923291864, -23.892245172521548, 89.00052249675024]
coefficients of 4 : [253.0117314616653, 2885.260970790402, -800.4620816507947, 331.4697498331993, -41.518942310818815,
-80.6688979304522, -251.06095267580488, -142.5926481346567, 5.045541009205884, -174.86961003480027, 88.46991398369745, 42.
41097508070479, 244.5489589457436, -32.11758913622103, 111.38664416912397, 203.34025052302513, 124.08486755543521, 62.6487
1688718261, -37.99549921582981, -141.46836993197408, -23.800432615167804, 13.898132896740334, -27.315946360046603, -76.518
40479481505, -1.4829273424768372, -49.281504070865935, -80.04354331271881, -32.77432389050157, 17.98905251895514, -8.88165
6474687057, -11.63948784760826, -21.101191684693447, 8.275608604785761, -159.76789326654918, 69.84287584832461, -74.807513
21125824, 34.2135689530358, -27.06635602988436, -45.58816336507549, 14.828859656904795]

k : 4

coefficients of 0 : [233.67060454787335, 455.48548338294245, 353.10503686793294, 135.87624016446568, 173.6591185321846, 30.92350155597735, 158.35209958433592, 210.23290157141682, -169.82247865341265, 17.334656860999573, -153.7512511445026, 34.5235639873045, -252.53606535639497, 127.03575580892903, 128.38534531212926, -112.57133739358137, -53.2921749462715, -108.4141943918219, -47.21619511275738, 78.67091704404734, 54.517990783499926, -61.537229717589604, -38.52590390458417, 43.04202129156645, -49.486463488992555, -13.656812996120646, -46.90445866227536, 166.46034206769872, 110.80938169416918, 95.09527541483303, 31.787533346994046, -118.90213465394679, 66.44047291475007, 37.33934683506426, 38.72922743548427, -9.632993225019023, 0.5976680991165049, -139.81003356812982, 56.37818541903081, 78.25546549490075]

coefficients of 1 : [-11.669319880431646, -314.5525202577625, 372.63107785002694, 128.91305334689767, -8.869922937617536, -134.63072231460376, -75.02419526164698, -71.53561081075372, -9.201075754767308, 5.973265737560867, -0.777062478699531, 4.5124928920649765, -155.43292827877758, 71.12718838714987, -22.21304463113404, -65.05891056105054, -21.068045387832658, -37.34865601730138, 21.53099938847609, -51.447918042287604, 96.53059175593489, 72.93187457834347, -28.711733290250123, 68.96747179244365, -32.54622146180535, -5.60088592445029, 12.802040654583399, 24.41576563727082, -5.306690691587164, 32.39843945067197, 61.99277303226667, 67.24350224479456, -19.091524895493464, 12.60725138291197, 6.739756961847137, -3.1132353250153724, -0.27255170662194317, 86.0874211621873, -32.02142438303354, 41.24856180924543]

coefficients of 2 : [-60.06648620387154, -854.4734551108066, 343.5243281804147, 38.57878866512357, -46.27049239689264, -185.398105582955, -41.67059000699802, -80.96921941796278, 16.93253207887727, 29.801879578582678, 38.79903816488844, 24.873754675622713, -131.5302588254471, 17.423314517299264, -106.30775627464773, 33.772673808290136, 23.12049715234716, -22.814011800268922, 2.051854775535914, -91.96149416703503, 75.13507013456895, 58.23562403542148, -1.1455726149801193, 97.85830201705646, 0.5614555485540489, -19.0763934116791, 45.92706833056561, -42.18559050003894, -77.18377011130382, -123.33521670055875, -68.6434908538644, 129.5915676180768, -18.466041748799917, 74.60230186909834, 7.240289858129968, 8.777822158451155, -3.813374435416417, 130.726932356375, -41.48116362744567, -40.579013795723384]

coefficients of 3 : [-447.0863459516803, -2153.6355301956646, -369.7812411265071, -697.6500453346627, -40.291258778920735, 327.9785770666289, 173.68977819439, 166.39332165572444, 78.91107999888574, 14.414062774723494, 56.14256562998173, -83.59773198344722, 328.5128765896345, -138.8643394586241, -72.90389916246836, -58.55151392772402, -29.903718034595443, 88.37984738979293, 36.91628089336025, 161.8028927975015, -245.05339925520218, -73.14027161117832, 112.2713034088298, -116.40543181833806, 62.48930801678494, 58.56776297563202, 47.629845031663095, -92.4039888451746, 37.29724225677288, -25.503941379058215, -31.99609856650301, -88.34931264354421, -60.12191904286244, -25.932644111262825, -106.25745353046233, 45.31922248709331, -38.52914183430747, -72.07492497805814, 30.75372379058596, -90.04819026865283]

coefficients of 4 : [539.7993882006019, 3548.3190413038365, -711.3100170725584, 437.08698831680164, 94.36831394197708, 114.5719067758505, 8.867884586083932, 26.76597179164614, -112.70512684521353, -29.084042211453337, -137.5262103881916, 44.07819150235113, 106.19535673268142, 3.178590889273485, 140.44597146830827, 142.243588576857, 66.2293972162059, -26.5303438329192, -91.55218771620213, 19.146885955070594, -38.813533537388, -126.76989513973463, -70.56589199704123, -115.06804408867154, -23.94611659804302, -30.783642631233104, -120.71842601084346, 87.59586466400935, 80.05943346619722, 111.54451409614447, -2.5166869086519483, -181.70453760348204, 50.27990535918309, -81.53125077674378, 136.47101996359288, -40.977957576041035, 55.73472252351348, -192.20382894229954, 99.34093023350226, 34.46917862332615]

k : 5

coefficients of 0 : [-510.13848928422897, -1459.9791844561119, 576.0579214758632, -139.0591248524541, -261.9551832054085, 34.11827481362158, -39.7764471039713, -63.978158655496266, 190.4251977516073, 64.71916033657426, 282.4051883846115, -160.25044122600627, -123.5072064746041, 53.8761730768705, -113.85493085911334, -161.33968730330247, -76.55111217443573, 37.58051419662047, 127.96123566528357, -65.73478560819458, -68.1418289356958, 122.95371541698461, 34.42326689656872, 75.91100228872817, 22.547184396314165, 81.70356762116188, 79.32288007506327, -200.81319298827438, -126.81527782859145, -122.58598639715021, -42.808854911791244, 136.96784842262966, 14.966481703791407, 100.20749578718502, -135.43474979739392, -0.44700257783700703, -8.501903001261445, 180.30007952024363, -105.85989190401759, -56.54703387473032]

coefficients of 1 : [-34.904692590556515, -51.699034046970766, 336.1244714597068, 124.15595340716573, 11.193024834005215, 23.189646516721492, 7.600554966408143, -2.5333410746343628, -64.55828104991943, 9.322848826432471, -10.20027878101165, -25.989666185686595, 7.001475240626754, 31.3183237411068, -93.0381971080533, -55.946751587871645, 9.450103541909135, 1.984844095036922, -1.1042037135549405, -5.5114505901228785, 66.97617353260297, -56.898702802570924, -19.073573338810522, -49.82671438788092, -55.61515812040618, 7.140577924600663, -22.61910645584204, 42.69693053679361, 7.462260154413942, -8.416411162002037, 15.017425444254945, -26.311127730322063, 3.5163236904352395, 15.643313118808607, 17.97079591346921, 60.250419472905136, 20.29674836558687, 38.41580598064937, -26.301585805828964, -51.819805303854864]

coefficients of 2 : [-49.66386579825805, -444.390418742384, 297.87877187530785, 63.410040166136866, 74.78158237138648, -144.63780220436684, -14.662608750630508, 35.877448522322226, -20.719307761890448, -11.904824587311387, -22.796795696708426, -16.782888044401826, 58.75709397442289, -11.313243333425067, -52.2458281824252, 57.15998412741752, -22.79345834504663, -62.15985861178469, -46.000028387592884, -42.09664117417761, 65.14710715076659, 22.78303942449421, 50.90594299949525, -14.12973998965387, -60.934664921004895, -22.35498054222445, -8.361367617972292, 25.48461441974567, 4.822687399587881, -10.0222570832046, 17.760471667025968, -9.177509150634249, -104.82864830528902, 13.16386307364175, 19.487811599355258, 17.0988009901118, 25.57093322902402, -15.558309427678594, 18.736783946205353, -30.921265379621122]

coefficients of 3 : [42.28928604419775, -1634.6047535892797, -264.0321841521194, -396.30875777070537, 54.30907766980553, -92.86016745167993, 46.10794041282473, -12.234066032771889, 20.91240259484805, 198.18346223082932, -125.825379678171, 151.6947672004165, -67.9336945333618, -186.074048628478, 95.29025898109091, 65.42227262699888, -19.12246958520905, 65.61734410730315, 30.93017545349297, 126.06709116169321, 73.75896328700964, 37.02507701624922, 108.47825550320985, 95.64368299878385, 139.57535729136532, -20.052120985931904, 53.78696567533209, -9.823760402162268, -83.19029502042687, 47.490927491493366, 11.425697751692867, 99.13593158335468, 61.78737835659686, -48.2920447059626, 2.886795059041835, 3.0220467687052253, -42.35489030494426, -56.54815924286956, 52.92251173478027, 87.6509799207411]

coefficients of 4 : [63.99705892975048, 2074.5118450819973, -674.8256097068088, 104.05729979430242, -138.52132408814992, 236.75455891539104, -32.71129484778602, -13.902840851900514, 44.446648814721996, -176.90267726131157, 140.38548605824644, -85.5379117295654, -22.606782595592545, 146.66579235002592, 124.36984939193573, -60.850457975579204, 43.38738346710113, 28.222408841184045, 11.141733208042004, -57.79796608550244, -202.41298218506887, -8.956513555339168, -154.98310333682969, 11.22467646793046, -25.357261345008137, 52.762669733953246, 6.835517987001712, -65.79379732250703, 76.55888509924283, -47.29103925161177, -32.02352386216148, -41.99518211254913, 72.68408142385636, -7.369217956030013, 7.062071509740037, -68.48413751842284, -9.335818110512225, 88.90021623791166, -36.01134532885103, -10.615272373587741]

k : 6
coefficients of 0 : [468.4291057351699, 1518.2752716575344, -81.41009770629259, 265.2612119403775, 254.20916098214278, -360.61259024794913, -20.176850072997617, 34.16332321447774, -22.270823979034, -127.44605334174383, -322.7859682328051, 52.103583676721364, -100.47677934354171, -19.74538611068397, 60.747842986620796, 67.06029472783835, -27.430917457836824, -78.31160520176299, -31.442372635155188, 104.58305149995344, 20.768462941639534, -109.78710145265661, -29.795481570616385, -134.288613810382, 16.120789733616235, -77.92379703838955, -29.507896953834575, 170.57428366338718, 57.87165178372925, 148.21893525476708, 8.077385605426706, -139.726297347957, 49.81102583843725, -36.522561039837456, 44.416116531650985, -45.82247027947494, -14.243964949319793, -118.90702146227693, -43.17462997026004, 212.49887977165463]
coefficients of 1 : [20.941182860987716, -10.652473962352087, 472.4689113206981, 215.64522960820312, -18.061162623971615, -79.6107076018093, -188.44880087662602, -205.7363072204066, -33.269084479051536, -20.615068034956668, -25.21049365263816, 60.583247002325976, -184.55473812664667, 69.08798158683214, 53.16996337949352, -67.35760092061983, -17.91682585128123, -109.21847041033294, 0.27904181696602404, -87.65274555904224, 164.83531998465827, 51.451740517751524, 4.052358572518999, 172.47736136445332, -7.091473075503756, -43.13764004540627, 78.1567869663607, 65.18817404967963, 21.00861851341719, 22.997666879487923, 31.21060812278344, 9.381603450432108, 81.13572624240544, -13.895075310406977, -25.939063212136514, 49.16195874966718, 24.0550613086774, 7.303394166837659, -53.3131727112704, -67.44997432676067]
coefficients of 2 : [-89.29621322514744, -661.4957327059481, 625.5396469312564, 225.0603422361316, -27.690750255947194, 70.47728606075289, -48.829886068979974, -114.30551905305076, 70.55536488363501, -121.32067370858111, 101.21415453404087, 57.630031909848235, -78.98883154441948, 145.67380083647737, -262.3580516839147, -108.09845799600458, 33.34136143029977, -144.79224903724824, 52.635696959716924, -219.64284985777954, 83.75551743301205, 45.88932451909689, 134.3791375269949, 111.87438722357456, 21.39855569405832, -83.7649732227657, -109.38013018152172, -10.03111430135101, -73.35087579691883, -53.29714879495735, -22.48445490866903, 56.51157304478399, -60.770535783138634, 17.226933106389904, 15.406157180410128, -11.107629565677705, -89.87825986501525, 19.543821727320584, -11.900384738874148, -115.14842151071657]
coefficients of 3 : [-330.56444849832894, -2501.9919239950796, -144.05094933316653, -513.0067039617882, -90.33261319473162, 235.7498424907053, 188.31706067652843, 258.03058208551795, -113.82291244010949, 556.3832961058138, 13.302478027951512, 71.40220315347533, 356.7467741395783, -441.1965623667895, 141.59357196157208, 202.5397364837081, 47.002391663293906, 218.00190818926433, 42.59544140225784, 280.35995526249485, 0.2514456676294087, -63.483375224162614, 120.9922630371916, -170.13328773735583, 83.04241067439534, 144.88536867646064, -40.67532888193427, -140.44537257696868, -15.155830617409798, -69.07308515204488, 71.95685759292185, 156.20317372016996, -208.26718791475395, -20.873915385802956, -33.06555473290573, 20.339620043822702, 192.6403500677984, -46.845199243369535, 137.6972712806705, -85.24092761343735]
coefficients of 4 : [362.28312379722774, 2950.2626574633096, -1366.061189798403, -126.32701626795918, 162.31545429763008, -190.67164519875575, 120.2316082943505, 166.8722349731531, 55.4909076816443, -349.3795568897085, -54.699857516842215, -231.28070849722369, 43.83803729493134, 135.47950799129305, 8.378180883450607, -71.33248732573324, -59.78764032571588, 162.41824518358914, -105.99005352635551, 104.99688904798546, -409.9874898423932, -32.01376892802085, -290.9881284997879, -239.3344241883836, -132.06956725491824, 44.15336600627006, -2.6156220693456795, 28.665319857146756, 119.7560815014871, 61.851297530838806, -86.88807117765683, -195.26134173591754, 134.03433415309667, 64.75752130681205, 130.64527086567674, -90.18227751944667, -150.07538209033697, 59.65433641063967, -14.205167208035256, 176.6767465341078]

k : 7
coefficients of 0 : [-280.2742818255107, -339.4720997101581, 881.7957613190455, 99.87728724271291, -134.885218262446, 1
01.46302009795107, -118.1965883787424, -100.89240298989404, -125.26877596290234, -32.46119704683688, 126.13679282975357, 1
65.44612441560406, -143.98387739712277, 4.175708788505836, 123.39713698310136, -117.08494214708148, -65.33986418187682, -1
53.09868390576597, 81.2372050421024, -21.966727968597713, 198.94357050413626, 111.38483183710876, 70.147548100342, 115.384
56299522318, 55.1840473076507, -45.57708120455792, 92.20658339544532, -164.4248286440905, 22.901928001899194, -25.94902918
3367546, 9.883856180182017, 88.42671584711849, -46.20431720200844, -3.8707935430515157, -192.02581401911226, 41.5277644425
43585, -36.3553069387829, 2.729454235311742, -39.25773504527538, -120.75701258688323]
coefficients of 1 : [2.7863652926795215, -250.1870704945634, 347.4239785953181, 112.64663279417341, -3.607118248335648,
-52.030097844324615, -22.23591070250597, 0.21692856445644282, 26.44135491146982, 47.08814642848054, -14.123568050081307,
53.7000973206134, -44.12010655391973, 89.39596940259152, -19.564514502165935, -92.21983531843368, 1.7388289931179433, 6.16
7849747405176, 65.99132314798918, 30.448501395543808, -32.78817201793156, 24.157333402192023, 31.316961137904514, 25.89165
06771033, -0.3629271385496846, -17.360338735391764, 6.713543480088806, 24.041908599808785, 15.769261985609466, 5.996155829
2851596, 39.888951628499186, -9.192924512413015, 30.70501847290958, -2.6349969054603424, 13.51779470779465, 62.12335734749
489, 15.773367525087295, 22.367641223195164, 22.847259338492847, 2.2403065580301256]
coefficients of 2 : [-1.725380668721506, -746.5437445972906, 298.1685677063555, 66.85322521654581, -17.40900197743757,
158.922946052173, 6.936705529364257, -69.15137030508662, 53.80546196791167, 41.337213136856775, -57.80230436269461, -26.54
2331221940522, -185.66767332024114, 43.57239168732282, -12.084883177516652, 17.670178938068293, -38.35306564644386, 13.086
813295620676, 2.4555201809483522, 34.903071526469915, 131.69917316451819, -35.635077771347326, -5.127400910119502, 33.2266
0423215449, 0.03707615830496103, 16.668774309335568, -11.348293076887652, 91.32607536158979, -67.3087057610515, 22.5499680
8336992, 16.298805577099145, 70.81718289897563, 8.533203731730953, -15.948089014230527, 54.336989090404316, 42.52464702096
196, 43.97858215964645, 47.425277650146015, 11.839494589124657, 17.13640063627154]
coefficients of 3 : [-200.32711239851267, -1992.32357038285, -457.1394100359822, -480.9338012102842, 19.65699583485723,
-9.957317786478965, 161.91848116059373, 76.46033877390485, 132.50410267121612, 56.17423847610286, 68.45695468362996, -102
.90517816850904, 353.76203852074264, -61.10671489429393, -153.82480234851752, 123.87349689418221, 151.7290819921992, 86.87
24854752776, -126.81935740804305, -191.80139904247935, 8.157989578133192, 49.091451409118726, 43.663890875262666, 66.24421
714565965, -10.85611136584064, -25.41924928874819, 60.49085548430553, 48.049936852834165, -103.59316864234265, -73.4574218
4728729, -34.58314000656679, 42.78359814758026, -118.26470431464652, 104.4651376519113, -3.4867271561899025, 46.8428248726
4241, -22.247269132059294, -66.91811142624732, 5.310188105174143, 6.534444138840252]
coefficients of 4 : [135.67522795600027, 3128.9131582947493, -551.2450615429495, 193.97858768862966, -58.23102059822538
, -89.78318507383835, -153.0411189775083, -6.91497299536573, -241.57892470017202, -181.8729874936442, -40.947419797315916,
113.78274566913498, -64.46583069638399, -146.6998075281722, 133.75197320810057, 21.707511790761277, -100.05144736223963,
-79.357163744456, 19.036794720638653, 59.49068521050124, -86.2168053269963, 12.664195930115891, -70.0486034177764, -129.84
707435775587, -15.535847937209738, 29.72616106947614, -30.18915157759012, -200.41103385023962, 103.52801186020105, 55.5763
7603422286, -11.791372891550322, -75.29781781910069, 37.36943889259359, -77.47018104080631, -78.76342585565986, -204.45165
822054724, -12.736203535083439, 24.531064367248796, -56.74803530230239, -12.043273615337323]

k : 8
coefficients of 0 : [247.17077090159285, -599.9158510670046, 382.21004424466537, -63.89533612552948, 32.21419447809429, -19.951801127749018, 47.64693522698431, 44.94694331982944, 265.73550096677025, -0.4266026728005361, 165.9724071219777, 0.693943635669541, 76.71490296172423, 167.50090814615834, -120.94459303952604, 30.480992278326788, 50.45425543735578, 23.571452956829305, 61.54918679083713, 8.427926622354233, -57.522516672281455, -39.75949446038746, 81.6951465991414, 82.78714120699979, -38.727066089252816, -35.7393539840574, -51.32697693971844, 92.4979625167408, 4.524430666543813, -21.310981928328864, -48.79014557432441, -37.679725930762594, -8.875200442858532, -68.41107916439374, -19.294270570993202, 77.95898400036855, 27.998096670606913, 36.642535338372454, 35.97855645234489, 12.353309551853053]
coefficients of 1 : [-9.584249093251831, -56.943120368371964, 405.6038472320854, 170.796009855222, -14.374638777651942, 11.180657550834521, -10.570935252967326, -47.86634605058764, -24.239550489051307, 9.785766605469533, 0.08313320863747187, -20.353358305413693, -3.267223660816247, 24.929876786509247, -95.30184858139788, -117.0394116079517, -0.3392895556055713, -37.30496631387123, 4.880897422511514, 58.05194342605456, -36.7044762385444, 6.276650414009238, 15.157186630180616, 48.529405750813964, -60.655444617569295, 4.2111001345375545, 8.144515655595658, 40.99984467082142, 21.310248465400726, 62.213327127817415, 37.20492205253174, -2.705289846297295, -13.222151251084403, -9.034414234213127, -7.508847643792233, 0.9923213896648242, -21.627220252552945, 13.226312732328687, 19.25228913710385, 0.5568786013584444]
coefficients of 2 : [-142.937514706645, -611.3803859857056, 321.6224192755237, 154.3257829568661, 14.867543444459068, -122.1114401029684, 6.204903940739838, 43.9286250234566, 16.332830466334414, 2.12334605471494, -93.78814001629266, 91.74858437258904, 112.00867967309404, -39.77330472717463, -73.54377266381773, 49.15860857947365, 90.23351199646737, -16.430512076468705, 58.91211316306432, -34.82007346955869, -4.890817745946218, 52.070005053650206, 20.390561644092205, 55.99785634445082, -15.656190237798395, -48.3993810258453, -0.5089606150534607, 19.345344939592916, -16.670166458486392, -38.99172486296655, 81.05115801939178, 58.58185083271255, -45.55444874597784, -27.367278651670546, 11.273820915514651, -21.214920025239785, -37.575753287528016, 27.953004640760874, 3.316476456378064, 29.37694504860737]
coefficients of 3 : [-265.00857728231796, -1901.474690720579, -559.9639221658225, -464.02769376459264, 41.26767256020281, 184.5446123078168, 24.16149639841624, -9.856030002627508, -34.6431447143671, 109.71410792914605, 3.933660048283017, -126.60934930017847, -91.51080336915697, -28.01252161494343, 138.89521270884444, 48.92808455419129, 9.928053195803418, 87.91092055616137, -196.13035241481737, -85.76243620746789, 144.89691233844124, -54.00030158637675, -59.56537062119557, -31.894896953663363, 170.59290213551867, 79.50348016740816, 49.53530983387222, 1.594897348184225, -63.697090829655984, -52.284948428426226, -69.78184779465163, 46.00255976495933, 24.802960954345245, 106.9377930905625, 22.43727908806015, 91.29207269525102, 40.771537967118, -104.4958591765709, -3.1674667430144616, -12.659302262796176]
coefficients of 4 : [428.2352021286898, 2600.775661570559, -493.5323142586341, 31.305146156828567, -23.620566514811948, -49.811838106216314, -39.90307995048861, -2.1448580280084926, -32.8358439824718, -57.224461255041206, 68.35540637382987, 74.8154997717453, -74.8997667139088, 31.5009065666046, 92.83549210131301, 22.238063716694597, -93.90541787343273, -11.21741677682884, 69.91891461270768, 29.968022921370846, -89.61205652759612, -3.800951499567441, 1.456392759106727, -50.748484145869355, -135.00830213366467, -21.386000295319313, -32.27644732874366, -88.18427197503021, 80.58268496829936, 6.3884987855956386, -49.457716571001214, -109.4896009205507, 38.55879062957156, -10.36738830986252, 18.056063824155014, -44.0929741396925, 29.484514375966505, 114.17830935052447, -5.198456997902625, -61.77212308386123]

k : 9
coefficients of 0 : [-165.09811055468356, 318.0482499743945, -71.48774558696458, 320.148017568382, -134.11550333119646, 381.47652872636627, 101.0501081882057, -38.72570144704687, -127.08264021403323, -18.3453419170006, -3.877949838240326, 25.454605690034256, 192.4686339388811, -80.2888932948554, -213.25847066795416, 69.6434337027058, 189.53054642062642, -111.88491806233156, 62.08187471197759, -208.9547262896668, 148.07149137452916, 17.751178505929726, -85.47264043426753, -7.335984427013889, 77.84781986746313, -5.859381343210036, 18.127866682894783, 187.5437872241794, -172.1116397774579, -6.602936336760715, 80.80249067725791, 59.89210047394938, 69.22729232258511, -102.51923807154637, 25.092868138067743, 63.10121804274211, 117.85082825820268, 24.149329046295563, -22.778471988698712, 96.93105512894778]
coefficients of 1 : [-32.69722471877009, 158.52532580541236, 415.9710189718985, 59.68461883102384, -29.33348942096096, 199.1952027041516, 26.23915896627409, 1.4307778961319824, -37.44372206297823, 17.399357622474675, -141.82969500350458, -79.62397738454396, -34.004889153359144, 138.17417601492588, -35.87900972678137, -78.83034909151664, 23.914846607371928, -79.99311806238109, -9.59780438863411, 39.18963004611826, -65.94679961231935, -42.36003730159261, 5.715592792056002, 19.799787898523306, -90.34563589974744, -40.00336637522221, -34.66640773209619, -28.54545048935055, 39.11261944482433, 28.588033739960913, 33.82446997662263, 11.902101349584708, -27.05565895574732, -21.22031616585459, -0.8096491721329793, -45.289888236978484, 30.204162149497108, 15.196517954593723, 8.63988760577417, -35.32460590812566]
coefficients of 2 : [-115.36771172539531, -331.2451035241652, 371.65442791753173, 15.132303174610396, -18.442158373845942, 173.59255304047048, 63.72605137375689, 121.13013735485454, 2.4595651973318255, -21.763359582753807, -356.9593970320946, 42.24168254890425, -3.4630867547500683, 103.75636959279217, 120.8781928986279, -59.66855444378993, 61.34921845206657, -44.73492458454598, 99.00194883367115, 37.16493090866503, -153.11360180370463, 86.85205972549903, 68.3325675845891, 48.21190440413389, -82.83159984163872, -33.991601576422866, -9.37045222233825, 3.5005113773874834, 45.29942671620323, -136.935677838108, 10.041227827121856, 15.4855492139658, -95.4475964407476, 44.05078826330115, -68.69233556884431, -187.8828077635245, -27.827056839011394, -14.2564172848753, 38.1250680703412, -68.45104734427309]
coefficients of 3 : [-130.3822897902397, -2044.4262288816974, 121.2880602470886, -374.82788265452484, 237.51551764361704, -86.89561874207162, -103.40829909418358, 151.13986469918132, 382.97351934470043, 162.11961333273248, 145.80628392886928, -45.10851101420299, -40.92792695885491, -437.0002987957281, 52.20153908931499, 65.14776357807337, -265.1758310451961, 106.32051794463439, -110.25527149231166, 279.89414455092333, 79.01712639215384, -116.06912720538564, 68.3999484417526, 99.5736742815291, 211.41302741151048, 73.13982575508996, -48.29712444670284, -3.579500984549274, 47.369434022099284, 65.77806742866187, -51.037679578894085, 55.9658573732908, 33.29193908198437, -64.47825112312006, 8.22387950104051, 13.148709515549168, -197.66762114339724, -226.081148640347, -92.97827032795654, 56.41839247558482]
coefficients of 4 : [306.7912342922208, 2004.432054392053, -1232.2054081860374, 214.45349193840428, -188.73911317956316, -426.46966492228097, -61.46170991799933, -336.55500226623747, -307.1130499291919, -157.0324939932171, 491.22771907326717, 164.57839044153712, 81.65207254654437, 166.5705454411586, -151.70108743887124, 112.93900379866831, 177.30659619724057, 16.88195804382435, 30.817144985108825, -408.8242785639438, 238.94253807924716, 121.08473130287702, -162.84410116758806, -145.80498468650273, 41.03594343188605, 40.24933674685272, 80.3159888571391, 14.099675457301702, -162.87123696855497, 20.524319017813475, -3.686190466435887, -81.58966733061348, 101.86245674543704, 51.39461318055585, 37.36532806815417, 259.19132523204155, 151.317511723682, 239.68066571636868, 7.871846077354105, 53.18574360555915]

각 얼굴 별 Coefficient값의 표준편차 구하기

```
weights_all = np.array(weights_all)
weights_all = weights_all.astype(np.float)
weights_mean = []
weights_variance = []
for i in range(0, 10):
    temp_weight = []
    mean_weight = (
        weights_all[i * 5]
        + weights_all[i * 5 + 1]
        + weights_all[i * 5 + 2]
        + weights_all[i * 5 + 3]
        + weights_all[i * 5 + 4]
    )
    mean_weight /= 5
    weights_mean.append(mean_weight)

all_variances = []
for i in range(0, 10):
    variance = []
    for j in range(0, 40):
        var = 0
        for k in range(0, 5):
            var += (weights_all[i * 5 + k][j] - weights_mean[i][j]) * (
                weights_all[i * 5 + k][j] - weights_mean[i][j]
            )
        var /= 5
        variance.append(np.sqrt(var))
    all_variances.append(variance)
all_variances = np.array(all_variances)
print(all_variances)
```

[142.55449108	1627.66987886	427.36597948	250.43343187	20.71688787
	192.24175758	113.24793378	53.62923621	152.30842525	206.46420169
	81.32712034	136.94763451	41.76841991	128.93230334	35.93261654
	108.55820797	36.03336646	140.01236836	36.85501651	33.93498103
	48.1370919	130.45994386	101.5741213	21.16811944	49.58401349
	59.76406056	113.97528771	12.18888301	49.9184042	57.47548696
	18.32371614	24.02419574	50.7115599	69.00055746	31.99007835
	23.30108263	46.76095097	95.35372806	62.26560451	62.16905195]
[275.091713	1459.7943729	504.72689316	209.90087018	73.66909196
	193.9025303	72.18218592	71.62626961	103.77446427	176.26988586
	153.17075074	68.12537239	57.55034644	176.71734513	59.71826893
	33.61382676	150.89263868	109.39024095	89.40371327	102.16198726
	91.61121057	45.89724161	178.31266106	59.24419731	69.01490107
	90.67245861	97.21447175	40.03202666	47.9778343	74.40891705
	58.79356787	120.97189323	53.42448447	30.01105684	46.81109224
	46.63710814	75.38518447	96.01139233	61.6525947	90.06028206]
[239.74937573	1564.03566959	463.08527025	225.98338844	63.94048329
	359.14073188	154.48411333	111.70585991	151.71126762	35.12047012
	97.73694255	100.30718551	56.7745187	153.24031117	97.86320858
	107.11641095	49.12986026	58.97994085	100.40263513	82.08540692
	119.94096061	55.67738298	113.62090327	126.68412965	96.30092732
	94.0192124	92.12864817	89.73758584	36.07697026	83.71405287
	59.93223106	51.92343042	19.35697289	87.30990457	22.15519914
	53.43860937	48.4688339	117.7379061	62.39732201	69.55170642]
[152.74442667	1651.26790042	464.48001459	371.00351894	74.60891796
	146.77264141	175.80735901	153.68819061	112.33237215	145.19145572
	58.04538934	58.95163921	176.52098511	95.28117758	73.43891712
	105.14716424	63.46594282	125.22495171	54.02221471	120.79756823
	97.44691417	28.64469213	17.65370938	64.7055055	47.05332242
	32.571148	78.46227922	95.0076469	13.35769342	38.35701024
	24.09922374	23.61672123	52.15137247	141.17819759	68.25632923
	49.16395307	56.87308283	46.91951062	59.07616605	77.58945131]
[327.61153304	1906.43426597	452.59654535	377.8791918	85.9628158
	184.08787981	102.59898891	119.73762298	89.96249973	19.91203144
	88.81459562	46.13769268	210.82504634	88.90143079	102.38053947
	90.53518891	42.55480114	63.09635415	47.36362099	90.70687377
	125.56100664	78.13169928	62.85216135	92.60981943	39.03395614
	31.42401114	64.13127594	91.82146067	66.06138915	85.7552344
	45.99561348	117.38264901	47.30754776	53.62861262	77.6370342
	27.97332331	30.22645991	125.83336947	53.26633975	61.08978929]

[210.76013194	1330.2238746	456.93467718	197.37235994	128.79103932
131.64933217	31.07161315	31.89102361	86.55304297	121.32332049
142.81505286	103.16869746	62.42366215	109.46067569	98.57490043
84.34595459	39.63736927	43.28289658	57.53226954	70.64099809
108.58006638	59.31574751	89.27424381	54.57978029	73.96890272
41.09519729	38.57135801	87.80685124	72.1206114	56.07871842
25.85336979	72.24425716	63.0283213	48.51767754	59.25156866
41.46255277	24.4055997	82.74531905	53.82154634	52.6484378]
[293.62257898	1863.96191397	701.32974073	298.58497403	129.99403044
206.00882477	132.43962168	171.53476882	66.69409667	304.05145157
142.56940717	116.85339764	189.3768267	217.68879728	138.17927279
115.5214283	39.61306863	149.97867534	57.3689557	173.09989202
199.38188357	62.49958982	153.31155033	163.13546872	70.76339863
86.99538438	60.72958557	101.38573021	65.29372345	79.35264454
53.42569059	128.67209875	121.73276915	35.86119653	59.30934763
48.95269659	116.80006846	61.75946752	69.23100728	140.35713074]
[150.63497644	1702.15417072	537.80037971	243.33007143	54.26457891
93.87128133	110.4954434	61.39048999	134.50278043	89.71124141
69.96923801	96.14121285	192.36768366	82.66057936	106.09417482
87.08305185	87.4868457	82.82646219	73.55918686	90.96508017
105.67510221	48.17343829	48.50236174	82.31255184	25.46591669
27.69160593	45.75500789	118.5862818	72.92249989	43.94499659
25.34523637	59.42404772	58.29158459	58.62889638	86.83668113
101.35166516	28.75734373	39.12824411	31.06277673	50.56404505]
[253.74449629	1486.72992755	440.57867864	231.29629014	25.37428296
101.95023652	29.81552215	35.14138286	115.3268443	54.02552896
85.8506644	77.9454294	80.44059818	73.7030135	106.16482544
62.77633214	61.59397205	43.90534718	100.63635653	50.57722317
81.6347339	37.29458397	45.10078207	52.43293317	101.47962749
45.43250525	34.80468552	59.18141536	47.26025496	40.57730583
58.57314045	60.9144995	29.75596939	58.37850862	15.87805358
54.04591098	31.27370571	70.31877633	15.55331753	30.93645517]
[172.63618945	1298.55424916	603.01578219	237.388159	146.75128746
280.4133389	76.54375235	173.33382204	226.74166578	101.97332757
285.13293755	84.35642749	88.19639098	224.78960434	123.88819098
76.46950057	164.33204536	95.75784431	71.85458154	235.95658085
142.02552873	85.88546146	90.51265194	82.34381342	111.82646987
43.66819696	45.59383498	77.74865361	103.6503673	69.56452016
43.39113902	50.98487215	70.39626141	59.96411474	36.91196538
146.00968957	123.65544505	147.83606465	44.73661263	62.01428371]]

EigenValue의 값이 클수록 중요한 EigenVector이라고 생각했습니다. 현재 EigenVector를 40개 사용했지만 만약 더 사용했다면 정확도가 조금 더 떨어졌을수도 있다고 생각합니다. 왜냐하면 이후의 EigenVector은 noise로 작용할 수도 있기 때문입니다. 하지만 innerProduct 했을 때의 Coefficient값을 비교해 보면 원래 EigenValue값의 순서대로 Coefficient가 큰 것이라고는 할 수 없었습니다. 이를 보면 어떤 데이터에서는 EigenValue의 값이 작더라도 데이터가 더욱 중요하게 작용할 수 있다고 생각했습니다.

이론적으로 같은 사람의 얼굴이라면 얼굴이 비슷하게 만들어 져야 하기 때문에 Coefficient값이 비슷하게 나와야 한다고 생각합니다. 그래서 얼굴별 각 픽셀 마다의 Coefficient값의 표준 편차를 구해 보았는데 생각과는 다르게 표준편차값이 조금 크게 나왔습니다. 이는 어둡기와 같은 여러 조건들이 다른 상황에서 같은 얼굴이라도 어둡기가 다르면 픽셀의 값이 다르기 때문에 발생하는 값이라고 생각합니다. 그래서 결과 사진을 보면 어둡기는 조금씩 다르지만 얼굴형은 비슷하게 나왔습니다.