1.1

$$\begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{20} \begin{bmatrix} 4 & -2 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \frac{1}{20} \begin{bmatrix} 6 \\ 22 \end{bmatrix} \quad a = \frac{3}{10} \quad b = \frac{11}{10}$$

$$\boxed{y = \frac{3}{10}x + \frac{11}{10}}$$

1.2 $E_2 = \sum_i \left[ax_i + by_i - d \right]^2$

$$\frac{\partial E_2}{\partial d} = \cancel{2}\sum_i (ax_i + by_i - d) = 0$$

$$-\cancel{a} - d + 2\cancel{b} - d + \cancel{a} + 2b - d + 2\cancel{a} + b - d = 2a + 5b - 4d = 0$$

$$\boxed{d = \frac{2a+5b}{4}}$$

1.3 $E_2 = \sum_{i=1}^{4} \left(a\left(x_i - \frac{1}{2}\right) + b\left(y_i - \frac{5}{4}\right)\right)^2$

$\Downarrow$

$$\begin{bmatrix} -\frac{3}{2} & -\frac{5}{4} \\ -\frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & \frac{3}{4} \\ \frac{3}{2} & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$A^TAx = \lambda x \to$ eisen value

$x^TA^TAx = \|Ax\|^2 \to$ minimize

$$A^TA = \begin{bmatrix} 5 & \frac{3}{2} \\ \frac{3}{2} & \frac{11}{4} \end{bmatrix} \to \lambda = \underset{\text{minimum}}{2}, \frac{27}{4}$$

$\lambda = 2$ $\quad e_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $\quad a = 1, b = -2$ $\quad d = \frac{2-10}{4} = \boxed{-2}$

$ax + by = d \Rightarrow x - 2y = -2$ $\quad \boxed{y = \frac{1}{2}x + 1}$

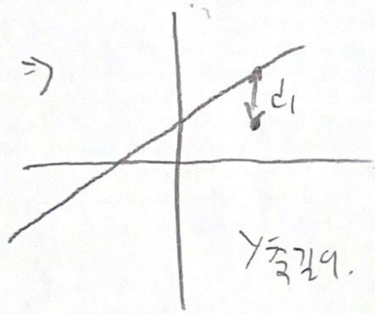
1.4 $E_1 = \sum_{i=1}^{4} \left(y_i - \frac{3}{10} x_i - \frac{11}{10} \right)^2$

$= \left(\frac{8}{10} \right)^2 + \left(\frac{9}{10} \right)^2 + \left(\frac{6}{10} \right)^2 + \left(\frac{7}{10} \right)^2 = \frac{23}{10} = \boxed{2.3}$

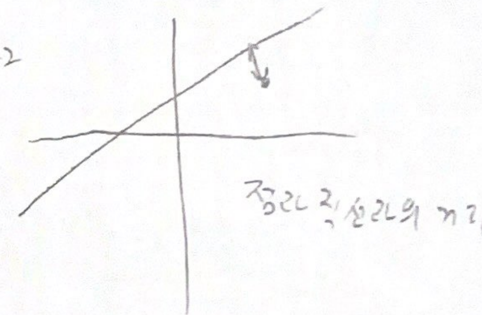
$E_2 = \sum_{i=1}^{4} (a x_i + b y_i - d)^2 \to$ min eigenvalue $= \boxed{2}$

1.5

$E_1 \Rightarrow$



E_2



2

2.1 $A = \begin{bmatrix} 2 & 5 \\ 4 & 4 \\ 4 & -2 \end{bmatrix}$

$\hat{x} = (A^TA)^{-1}A^Tb$

$P = A(A^TA)^{-1}A^T$

$A^TA = \begin{bmatrix} 2 & 4 & 4 \\ 5 & 4 & -2 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 4 & 4 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 36 & 18 \\ 18 & 45 \end{bmatrix}$

$(A^TA)^{-1} = \frac{1}{1296} \begin{bmatrix} 45 & -18 \\ -18 & 36 \end{bmatrix} = \frac{1}{144} \begin{bmatrix} 5 & -2 \\ -2 & 4 \end{bmatrix}$

$A(A^TA)^{-1}A^T = \frac{1}{144} \begin{bmatrix} 2 & 5 \\ 4 & 4 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 5 & 4 & -2 \end{bmatrix}$

$= \frac{1}{144} \begin{bmatrix} 0 & 16 \\ 12 & 8 \\ 24 & -16 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 5 & 4 & -2 \end{bmatrix} = \frac{1}{36} \begin{bmatrix} 20 & 16 & -8 \\ 16 & 20 & 8 \\ -8 & 8 & 32 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & 2 \\ -2 & 2 & 8 \end{bmatrix}$

$\boxed{P = \frac{1}{9} \begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & 2 \\ -2 & 2 & 8 \end{bmatrix}}$

2.2

밑변 : $(0,0,0)$ to $(2,4,4) = 6$

$(0,0,0)$ to $(5,4,-2) = 3\sqrt{5}$

$(2,4,4)$ to $(5,4,-2) = 3\sqrt{5}$

밑변넓이 $= 6 \times 6 \times \frac{1}{2} = 18$

높이 : $[4,-2,-3]^T$ projection to 평면

$\Rightarrow \frac{1}{9} \begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & 2 \\ -2 & 2 & 8 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 18 \\ 0 \\ -36 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$, $(4,-2,-3)$ to $(2,0,-4) = \sqrt{9} = 3$

높이

부피 : $18 \times 3 \times \frac{1}{3} = \boxed{18}$

2.3

$$q_1 = \frac{a}{\|a\|} = \frac{a}{6} = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$$

$$B = b - (q_1^T b) q_1 = \begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix} - \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -4 \end{bmatrix} \qquad q_2 = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}$$

$$C = c - (q_1^T c) q_1 - (q_2^T c) q_2 = \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix} - \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix} - \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix} \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix} + \begin{bmatrix} \frac{2}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{bmatrix} - \begin{bmatrix} \frac{8}{3} \\ \frac{4}{3} \\ -\frac{8}{3} \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$q_3 = \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$\therefore$ $q_1 = \frac{1}{3}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

$q_2 = \frac{1}{3}\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

$q_3 = \frac{1}{3}\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$ $\quad A^TA = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$$\det \begin{bmatrix} 2-\lambda & 2 \\ 2 & 2-\lambda \end{bmatrix} = (2-\lambda)^2 - 4 = 0$$

$$\lambda = 0, 4$$

1) $\lambda = 4$

$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad x_1 = x_2 \Rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix} \to \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow v_1$

2) $\lambda = 0$ $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad x_1 = -x_2 \Rightarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix} \to \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow v_2$

$v_1, v_2 \Rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Left Null space

$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 + x_2 = 0$ $\quad u_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ $\quad u_1$ is orthonormal to u_2

$\Rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = u_1$

$$u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$