Econometric Methods I

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Lecture from MGTECON 603(Lecturer: Guido Imbens)

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1 Variance Estimation using Bootstrap and Resampling Methods

1.1 Variance Estimation under Randomization

Upto now, we have been focusing on the estimator $\hat{\tau} = \overline{Y}_T - \overline{Y}_C$ that is unbiased under randomization. Depending on the source of randomness, we had two different variance definitions:

$$\mathbb{V}_{\text{Neyman}}(\hat{\tau}) = \frac{1}{N_1}S_1^2 + \frac{1}{N_0}S_0^2 - \frac{1}{N}S_{01}^2 \tag{1.1.1}$$

$$\mathbb{V}_{\text{OLS}}(\hat{\tau}) = \frac{\sigma^2}{\sum_{i=1}^N (W_i - \overline{W})^2} = \frac{\sigma^2}{N\frac{N_1}{N}\left(1 - \frac{N_1}{N}\right)} \tag{1.1.2}$$

In the Neyman variance, S_1^2 and S_0^2 are the population[1] variances of $Y_i(1)$ and $Y_i(0)$, respectively; the remaining term S_{01}^2 is **the population variance of the unit-level treatment effect**. In the OLS variance, σ^2 is the population variance of the normal error term(ε_i), and $\overline{W}$ is the mean of the treatment indicator among our experimental units.

The corresponding estimators for (1.1.1) and (1.1.2) are given by:

$$\hat{\mathbb{V}}_{\text{Neyman}}(\hat{\tau}) = \frac{1}{N_1}\hat{S}_1^2 + \frac{1}{N_0}\hat{S}_0^2 \tag{1.1.3}$$

$$\hat{\mathbb{V}}_{\text{OLS}}(\hat{\tau}) = \frac{\hat{\sigma}^2}{\sum_{i=1}^N (W_i - \overline{W})^2} = \frac{\hat{\sigma}^2}{N\frac{N_1}{N}\left(1 - \frac{N_1}{N}\right)} \tag{1.1.4}$$

where $\hat{S}_1^2, \hat{S}_0^2$ are the sample variances of the observed outcomes in the treated and the control groups, respectively; $\hat{\sigma}^2 = \frac{1}{N-2}\sum_{i=1}^N \hat{\varepsilon}_i^2$ is the unbiased estimator of σ^2.

Additionally, allowing *heteroscedasticity* in the normal error term, we can derive the following variance estimand and the corresponding estimator:

$$\mathbb{V}_{\text{robust}}(\hat{\tau}) = \frac{1}{\left(N\overline{W}(1-\overline{W})\right)^2}\sum_{i=1}^N \left(\sigma_i^2 (W_i - \overline{W})^2\right) \tag{1.1.5}$$

$$\hat{\mathbb{V}}_{\text{robust}}(\hat{\tau}) = \frac{1}{\left(N\overline{W}(1-\overline{W})\right)^2}\sum_{i=1}^N \left(\hat{\varepsilon}_i^2 (W_i - \overline{W})^2\right) \tag{1.1.6}$$

In this section, we discuss additional class of variance estimators based on **resampling methods**. By the end of this section, you will be able to:

- Understand the intuition behind resampling methods.
- Understand the sense of richness of these ideas.
- Understand how to implement the methods in practice.

[1] Note that the Neyman world views the experiment participants as the population, and the source of randomness is the permutation of the treatment indicator.

1.2 One-Sample Example

Suppose we have a random sample $Y_1, \ldots, Y_N$ from a distribution with mean μ and variance σ^2, and we want to estimate the population mean μ. This is a simpler problem than estimating the average treatment effect τ in the randomized experiment, but it still conveys the essence of the resampling methods. The natural estimator of μ is the sample mean $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$. The variance of this estimator is given by $\mathbb{V}(\overline{Y}) = \frac{\sigma^2}{N}$, which we typically estimate with:[2]

$$\hat{\mathbb{V}}(\overline{Y}) = \frac{\hat{\sigma}^2}{N} = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \overline{Y}\right)^2 \tag{1.2.1}$$

Here's the key point: **The variance of any function of N random variables can be characterized in terms of the joint distribution of those random variables**, where the distribution can be written in terms of, CDFs($F_Y(y) = Pr(Y \leq y)$) or moment generating functions($M_Y(t) = \mathbb{E}(e^{tY})$), for example. In our case with a *random sample*, 1.2.1 can be written as:

$$\frac{\hat{\sigma}}{\sqrt{N}} = \frac{1}{N} \tag{1.2.2}$$

where $\overline{Y}$ is the sample mean of the random sample $Y_1, \ldots, Y_N$.

1.3 Bootstrap Method

[2] The OLS variance estimator is essentially a generalization of this estimator to the regression case, but in the mean case we do not have to worry about heteroscedasticity.