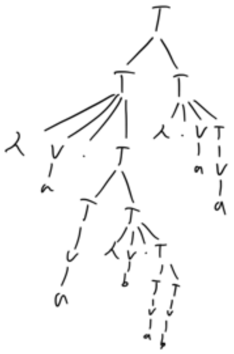
CSE216 HW3

1. $(\lambda a.\ a \lambda b.\ a b)\ \lambda a.\ a$

   1. Parse tree:



   2. Reduction to normal form:

$(\lambda a.\underbrace{(a(\lambda b.(ab)))}_{M})\ (\underbrace{\lambda a'.a'}_{N})$ $a' \to c$ ∵ α

$= ((\lambda c.c)(\lambda b.((\lambda c.\underbrace{c}_{M})\underbrace{b}_{N})))$ $a \to \lambda c.c$ ∵ β

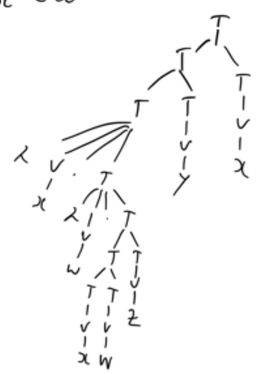
$= ((\lambda c.c)(\lambda b.(b)))$ $c \to b$ ∵ β

$= ((\lambda c.\underbrace{c}_{M})\ (\underbrace{\lambda b.b}_{N}))$ remove currying

$= \lambda b.b$ ~~$c \to \lambda b.b$~~ ∵ β

2. $(((\lambda x.(\lambda w.(x\ w)\ z))\ y)\ x)$

   1. Parse tree:



   2. Reduction to normal form:

$(\lambda x.(\lambda w.(x\ w)\ z))\ y\ x$

$= (((\lambda x.\underbrace{(\lambda w.(x\ w)\ z))}_{M}\ \underbrace{y}_{N})\ x)$

$= ((\lambda w.\underbrace{(y\ w)\ z}_{M})\ \underbrace{x}_{N})$ $x \to b$ ∵ β

$= (y\ x)\ z$

3. TRUE - $\lambda x.\lambda y.x$

   FALSE - $\lambda x.\lambda y.y$

1. TRUE TRUE TRUE

$(\lambda x.\underbrace{(\lambda y.x)}_{M})\ \underbrace{(\lambda x.(\lambda y.x))}_{N}\ (\lambda x.(\lambda y.x))$

no redex; y returns constant M $\quad x \to (\lambda x.(\lambda y.x)) \because \beta$

$= (\lambda y.\underbrace{(\lambda x.(\lambda y.x))}_{M})\ \underbrace{(\lambda x.(\lambda y.x))}_{N}$

$= \lambda x.(\lambda y.x) \equiv$ TRUE.

2. TRUE TRUE FALSE

$(\lambda x.\underbrace{\lambda y.x}_{M})\ \underbrace{(\lambda x.\lambda y.x)}_{N}\ (\lambda x.\lambda y.y)$

$= (\lambda y.\underbrace{(\lambda x.\lambda y.x)}_{M})\ \underbrace{(\lambda x.\lambda y.y)}_{N} \quad x \to (\lambda x.\lambda y.x) \because \beta.$

no bound instances of y.

$= (\lambda x.\lambda y.x) \equiv$ TRUE. $\quad (\lambda x.\lambda y.x) \to (\lambda x.\lambda y.x) \because \beta.$

3. TRUE FALSE TRUE

$(\lambda x.\underbrace{\lambda y.x}_{M})(\underbrace{\lambda x.\lambda y.y}_{N})(\lambda x.\lambda y.x)$

$= (\lambda y.\underbrace{(\lambda x.\lambda y.y)}_{M})(\underbrace{\lambda x.\lambda y.x}_{N}) \quad x \to (\lambda x.\lambda y.y) \because \beta.$

no bound instances of y.

$= (\lambda x.\lambda y.y) \equiv$ FALSE. $\quad (\lambda x.\lambda y.y) \to (\lambda x.\lambda y.y) \because \beta.$

4. TRUE FALSE FALSE

$(\lambda x.\underbrace{\lambda y.x}_{M})\ (\underbrace{\lambda x.\lambda y.y}_{N})\ (\lambda x.\lambda y.y)$

$= (\lambda y.\underbrace{(\lambda x.\lambda y.y)}_{M})\underbrace{(\lambda x.\lambda y.y)}_{N} \quad x \to (\lambda x.\lambda y.y) \because \beta.$

$= \lambda x.\lambda y.y \equiv$ FALSE. $\quad (\lambda x.\lambda y.y) \to (\lambda x.\lambda y.y) \because \beta$

5. FALSE TRUE TRUE

$(\lambda x.\underbrace{\lambda y.y}_{M})\ \underbrace{(\lambda x.\lambda y.x)}_{N}\ (\lambda x.\lambda y.x)$

no bound instances of x. $\therefore$ returns $\lambda y.y$ as a constant $\quad \lambda y.y \to \lambda y.y \because \beta$

$(\lambda y.\underbrace{y}_{M})\ (\underbrace{\lambda x.\lambda y.x}_{N})$

Identity function. foo(x): return x

$= (\lambda x.\lambda y.x) \equiv$ TRUE. $\quad y \to (\lambda x.\lambda y.x) \because \beta$

6. FALSE FALSE TRUE

$(\lambda x.\underbrace{\lambda y.y}_{M})(\underbrace{\lambda x.\lambda y.y}_{N})(\lambda x.\lambda y.x)$

$= (\lambda y.\underbrace{y}_{M})(\underbrace{\lambda x.\lambda y.x}_{N}) \quad \lambda y.y \to \lambda y.y \because \beta$

$= (\lambda x.\lambda y.x) \equiv$ TRUE $\quad y \to (\lambda x.\lambda y.x) \because \beta$

7. FALSE TRUE FALSE

$(\lambda x.\lambda y.y)(\lambda x.\lambda y.x)(\lambda x.\lambda y.y)$

$$(\lambda x.\underbrace{\lambda y.\lambda y}_{M})(\underbrace{\lambda x.\lambda y.x}_{N})(\lambda x.\lambda y.y)$$

$$= (\lambda y.\underbrace{y}_{M})(\underbrace{\lambda x.\lambda y.y}_{N}) \qquad \lambda y.y \to \lambda y.y \quad \because \beta$$

$$= (\lambda x.\lambda y.y) \qquad y \to (\lambda x.\lambda y.y) \quad \because \beta$$

$$\equiv \underline{\text{FALSE}}.$$

4.1. $(\lambda x.\underbrace{(\lambda y.(x\ y))}_{M})\ \underbrace{a}_{N}\ b$

$(\lambda y.\underbrace{(a\ y)}_{M})\ \underbrace{b}_{N}$ $\quad x \to a \ \because \beta$

$= a\ b$ $\quad y \to b \ \because \beta$

2. $(\lambda x.\underbrace{x}_{M}(\lambda y.(y\ x)))\ \underbrace{(\lambda z.z)}_{N}$

$= (\lambda z.\underbrace{z}_{M})\underbrace{(\lambda y.(y\ (\lambda z.z)))}_{N}$

$= (\lambda y.(y(\lambda z.z)))$

3. $(\lambda x.\underbrace{(\lambda y.(x\ (\lambda z.(y\ z))))}_{M})\ \underbrace{a}_{N}\ b$

$= \lambda y.\underbrace{(a\ (\lambda z.(y\ z)))}_{M}\underbrace{b}_{N}$ $\quad x \to a \ \because \beta$

$= (a(\lambda z.(b\ z)))$ $\quad y \to b \ \because \beta$

4. $((((\lambda x.\underbrace{(\lambda y.(\lambda z.(x\ z)(y\ z)))}_{M})\ \underbrace{a}_{N})\ b)\ c)$

$(((\lambda y.\underbrace{(\lambda z.(a\ z)(y\ z))}_{M}))\ \underbrace{b}_{N})\ c)$ $\quad x \to a \ \because \beta$

$((\lambda z.((a\ z)(b\ z)))\ c)$ $\quad y \to b \ \because \beta$

$= (a\ c)(b\ c)$ $\quad z \to c \ \because \beta$

5. $(\lambda x.x\ (\lambda y.y))\ (\lambda x'.x')$

$= (\lambda x.\underbrace{x(\lambda y.y)}_{M})\ \underbrace{(\lambda z.z)}_{N}$ $\quad x' \to z \ \because \alpha$

$= (\lambda z.\underbrace{z}_{M})\underbrace{(\lambda y.y)}_{N}$ $\quad x \to \lambda z.z \ \because \beta$

$= \lambda y.y$ $\quad z \to y \ \because \beta$