RFIC Homework 1

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2012/09/24

Problem 1

(a)

Verify the above equation for at least three stages.

For stage 1
$$N_1 = G_1 N_i + N_{n1} \tag{1}$$
For stage 2
$$N_2 = G_2 N_1 + N_{n2} \tag{2}$$
For stage 3
$$N_3 = G_3 N_2 + N_{n3} \tag{3}$$
Combine the above 3 equations
$$N_3 = G_1 G_2 G_3 N_i + G_2 G_3 N_{n1} + G_3 N_{n2} + N_{n3} \tag{4}$$
If we view the whole 3 stages as 1 stage
$$N_3 = G_1 G_2 G_3 N_i + N_{n123} \tag{5}$$
Compare equation 5 with 4, we can derive that
$$N_{n123} = G_2 G_3 N_{n1} + G_3 N_{n2} + N_{n3} \tag{6}$$

For stage 1
$$F_1 = 1 + \frac{N_{n1}}{GNi} \tag{7}$$
which is equivalent to
$$N_{n1} = (F_1 - 1) G_1 N_i \tag{8}$$
For stage 2
$$N_{n2} = (F_2 - 1) G_2 N_i \tag{9}$$
For stage 3
$$N_{n3} = (F_3 - 1) G_3 N_i \tag{10}$$

So if we view the whole 3 stages as 1 stage

$$N_{n123} = (F_{total} - 1)G_1G_2G_3N_i \tag{11}$$

Combine equation 11 with 6

$$G_2G_3N_{n1} + G_3N_{n2} + N_{n3} = (F_{total} - 1)G_1G_2G_3N_i \tag{12}$$

Combine equation 12 with 8,9,10

$$G_2G_3(F_1-1)G_1N_i+G_3(F_2-1)G_2N_i+(F_3-1)G_3N_i = (F_{total}-1)G_1G_2G_3N_i \tag{13}$$

divide both sides by $G_1G_2G_3N_i$ we will get

$$F_1 - 1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} = F_{total} - 1 \tag{14}$$

which is equivalent to

$$F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} = F_{total} \tag{15}$$

(b)

Calculate the overall Noise Figure of the three cascaded amplifiers shown in Figure 1.

$$\begin{aligned}
F_{total} &= F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} \\
&= 10^{\frac{3}{10}} + \frac{10^{\frac{7}{10}} - 1}{10^{\frac{7}{10}}} + \frac{10^{\frac{15}{10}} - 1}{10^{\frac{7}{10}} * 10^{\frac{10}{10}}} \\
&= 2.00 + 0.80 + 0.61 \\
&= 3.41 \\
&= 5.32dB
\end{aligned}$$

(c)

If you would like to design an amplifier with a power gain of 50dB from two separate amplifiers, each with the characteristics depicted in Figure , find the optimal order and explain why.

To achieve a low NF F_{total}, G1 should be larger but the dominant NF F_1 should be smaller.

$$F_{total} = F_1 + \frac{F_2 - 1}{G_1} \tag{16}$$

case 1

stage 1 G_1F_1 stage 2 G_2F_2

$$\begin{aligned} F_{total1} &= F_1 + \frac{F_2 - 1}{G_1} \\ &= 10^{\frac{1.7}{10}} + \frac{10^{\frac{1.3}{10}} - 1}{10^{\frac{30}{10}}} \\ &= 1.48 \\ &= 1.70dB \end{aligned}$$

case 2

stage 1 G_2F_2 stage 2 G_1F_1

$$\begin{aligned} F_{total2} &= F_2 + \frac{F_1 - 1}{G_2} \\ &= 10^{\frac{1.3}{10}} + \frac{10^{\frac{1.7}{10}} - 1}{10^{\frac{20}{10}}} \\ &= 1.35 \\ &= 1.31dB \end{aligned}$$

$F_{total1} > F_{total2}$, so the optimal order is stage 1 G_2F_2 and stage 2 G_1F_1

Problem 2

A_{1dB}

$$\begin{aligned} x(t) &= A\cos\omega t \\ y(t) &= a_1x(t) + a_2x^2(t) + a_3x^3(t) \\ &= a_1A\cos\omega t + a_2A^2\cos^2\omega t + a_3A^3\cos^3\omega t \end{aligned}$$

the third-order part of output

$$\begin{aligned} a_3A^3\cos^3\omega t &= a_3A^3\cos\omega t * \frac{1}{2}(1+\cos 2\omega t) \\ &= a_3A^3(\frac{1}{2}\cos\omega t + \frac{1}{2}\cos\omega t * \cos 2\omega t) \\ &= a_3A^3(\frac{1}{2}\cos\omega t + \frac{1}{4}\cos\omega t + \frac{1}{4}\cos 3\omega t) \\ &= a_3A^3(\frac{3}{4}\cos\omega t + \frac{1}{4}\cos 2\omega t) \end{aligned}$$

the equivalent first-order part of output is $a_3A^3(\frac{3}{4}\cos\omega t)(a_3 < 0)$
A_{1dB} is the point when the nominal first-order part minus real first-order equals $1dB$

$$\begin{aligned} (a_1A\cos\omega t + \frac{3}{4}a_3A^3\cos\omega t) - A\cos\omega t &= -1dB \\ 20\log(1 + \frac{3}{4}\frac{a_3}{a_4}A^2) &= -1dB \end{aligned}$$

thus

$$A_{1db} = \sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|} * \sqrt{0.109} \tag{17}$$

A_{IP3}

$$\begin{aligned} x(t) &= A\cos\omega_1 t + A\cos\omega_2 t \\ y(t) &= a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) \end{aligned}$$

the third-order part of output

$$\begin{aligned} a_3 x^3(t) &= a_3(A\cos\omega_1 t + A\cos\omega_2 t)^3 \\ &= a_3(A^3\cos^3\omega_1 t + A^3\cos^3\omega_2 t + 3A^3\cos^2\omega_1 t\cos\omega_2 t + 3A^3\cos\omega_1 t\cos^2\omega_2 t) \end{aligned}$$

only consider the $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ component

$$\begin{aligned} 3a_3A^3\cos^2\omega_1 t\cos\omega_2 t &= \frac{3}{2}a_3A^3(1+\cos 2\omega_1 t)\cos\omega_2 t \\ &= \frac{3}{2}a_3A^3(\cos\omega_2 t + \frac{1}{2}(\cos(2\omega_1+\omega_2) + \cos(2\omega_1-\omega_2))) \end{aligned}$$

thus the $2\omega_1-\omega_2$ and $2\omega_2-\omega_1$ components are $\frac{3}{4}a_3A^3\cos(2\omega_1+\omega_2)$, $\frac{3}{4}a_3A^3\cos(2\omega_1-\omega_2)$, $\frac{3}{4}a_3A^3\cos(2\omega_2+\omega_1)$ and $\frac{3}{4}a_3A^3\cos(2\omega_2-\omega_1)$

the ω_1 and ω_2 components are $a_1A\cos\omega_1 t$ and $a_1A\cos\omega_2 t$

A_{IP3} is the point when the $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ components equal the ω_1 and ω_2 components

$$\frac{3}{4}a_3A_{IP3}^3 = a_1A_{IP3} \tag{18}$$

thus

$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|} \tag{19}$$

thus the relation between A_{1db} and A_{IP3} is

$$A_{1db} = A_{IP3} * \sqrt{0.109} \tag{20}$$