Evolution in a Food Web part 2

As in the "Food Web" section, we provide a directed graph representing the food web. Node labels are species names, and arrow labels are strength of predation. Let's say the conversion factor from prey to predator is constant k.

Then species i's dynamics is

$$\frac{dX_i}{dt} = \left(r_i + k \sum_{j \to i} f_{ji} X_j - \sum_{i \to j} f_{ij} X_j \right) X_i$$

where

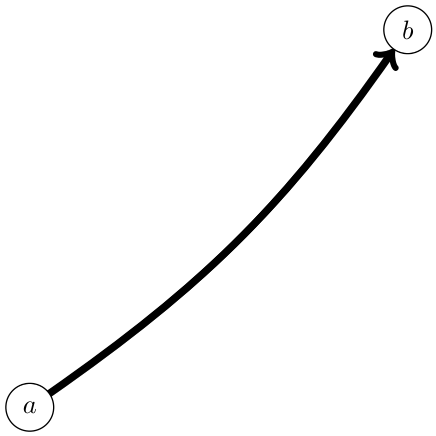
$f_{ij} = f(u_i, u_j)$ is some function of the two phenotypes controlling how well j eats i;

u_i is the phenotype of species i; and

$r_i =$ (0 if i is a predator, 1 else).

This will induce the usual dynamics of apparent competition, and adaptive dynamics of all the u_i follows.

- UPDATE: the above forces a fixed ratio of $-k$ between paired a values. We can allow that to drift a little by giving each a 'conversion efficiency' value, maybe? Use a planar $s_i = (u_i, \gamma_i)$, and instead of writing $kf(u_i, u_j)$, we use $k\gamma_i\gamma_j f(u_i, u_j)$. On the prey side just $f(u_i, u_j)$. Then the ratio is $k\gamma_i\gamma_j$. Let the γ values vary more slowly than f.
- UPDATE: Yes, but that way selection is only on the predator's γ because they only affect the predatory's fitness. Predatory's γ_i can be how much it gets from eating a prey, but what does it mean for the prey to change the conversion rate? A predation event kills less than one prey animal? I mean, maybe if it's a fungus or something. That might work for me - let $f(u_i, u_j)$ be encounter rate, predator's γ_j is how much it gets from encounter, prey's σ_i is how much it loses from encounter, so we can say $-k\gamma_j f(u_i, u_j)$ and $\sigma_i f(u_i, u_j)$ respectively, which will produce the selection I'm looking for, and ought to lead to marginal benefit for both.

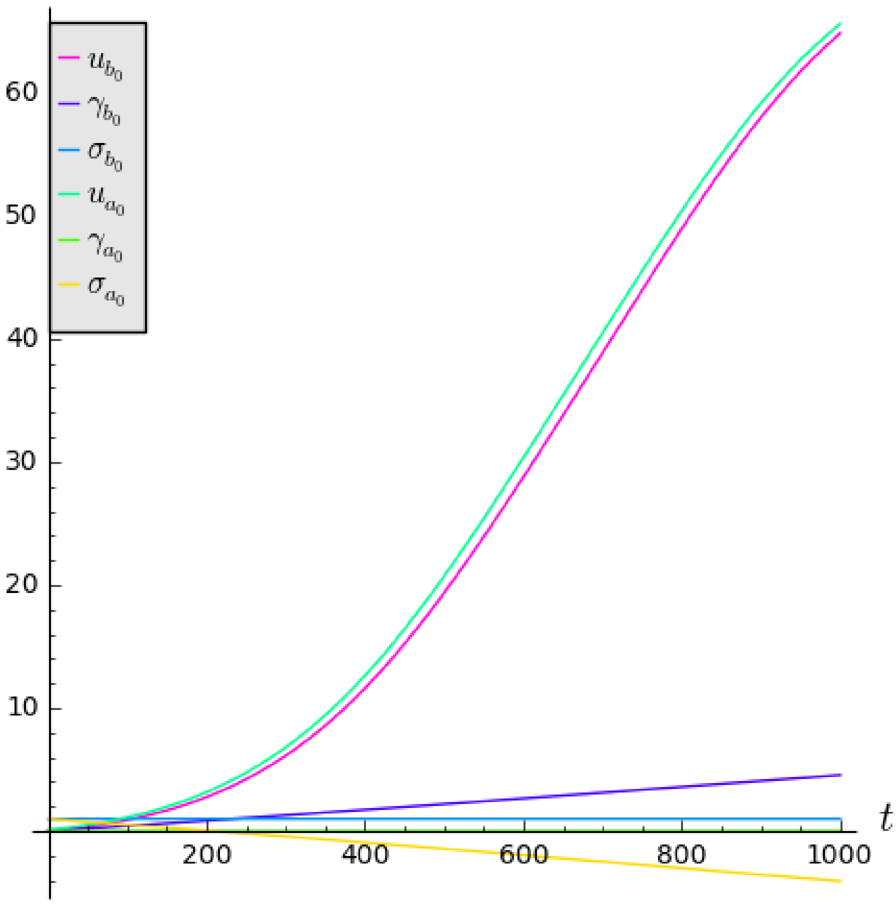


The foodweb model:

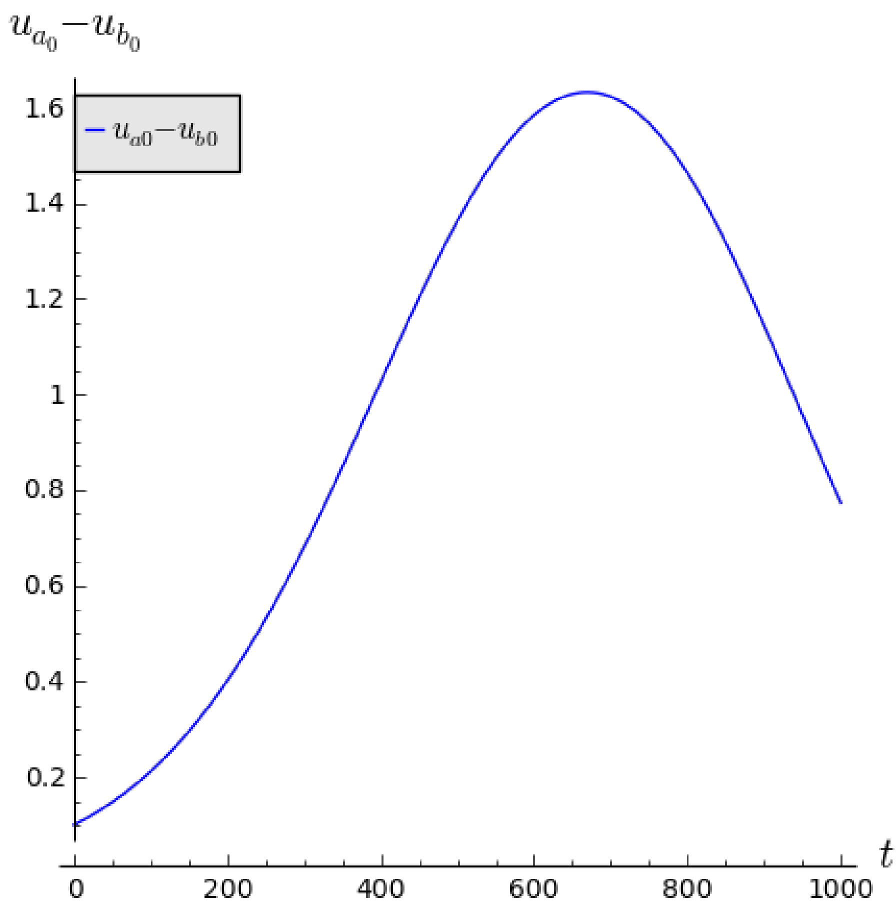
$$\frac{dX_{b0}}{dt} = \frac{9}{1250}\,X_{a0}X_{b0}(\gamma_{b0}+50)(2\,\cos\left(-u_{a0}+u_{b0}\right)+5) - X_{b0}$$
$$\frac{dX_{a0}}{dt} = -\frac{1}{125}\,X_{a0}X_{b0}(\sigma_{a0}+50)(2\,\cos\left(-u_{a0}+u_{b0}\right)+5) - X_{a0}^2 + X_{a0}$$

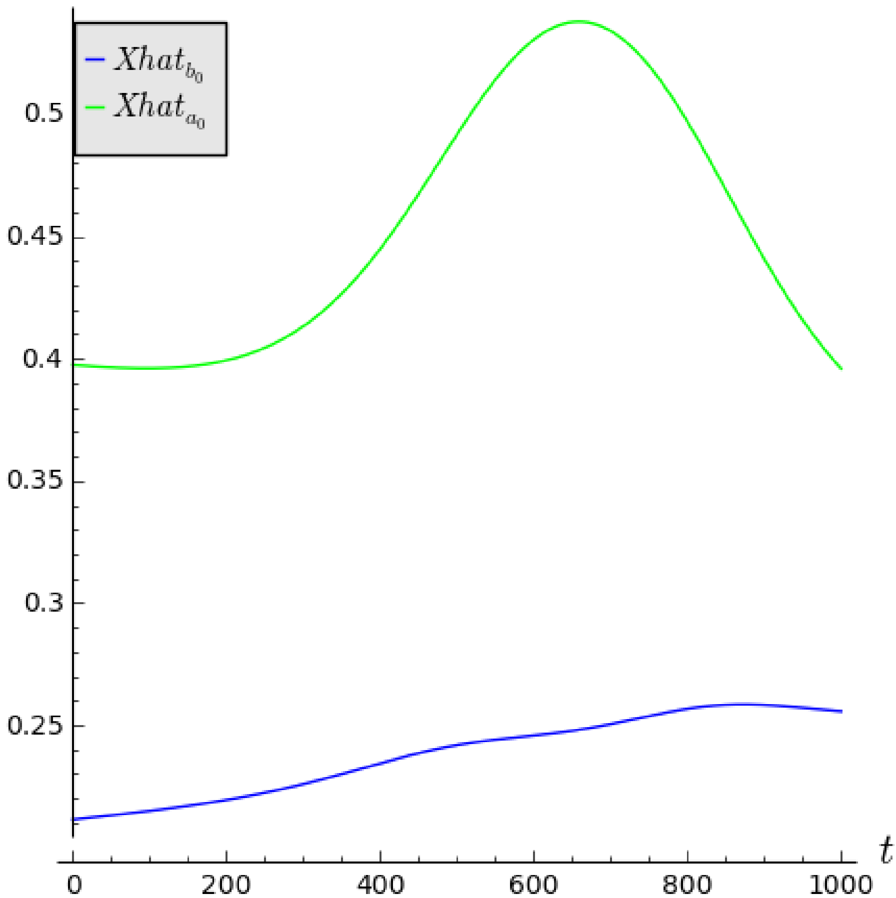
The foodweb adaptive dynamics:

$$\frac{du_{b0}}{dt} = -\frac{9}{625}\left(\hat{X}_{a_0}\gamma_{b0}\sin\left(-u_{a0}+u_{b0}\right) + 50\,\hat{X}_{a_0}\sin\left(-u_{a0}+u_{b0}\right)\right)\hat{X}_{b_0}$$
$$\frac{d\gamma_{b0}}{dt} = \frac{9}{1250}\left(2\,\hat{X}_{a_0}\cos\left(-u_{a0}+u_{b0}\right) + 5\,\hat{X}_{a_0}\right)\hat{X}_{b_0}$$
$$\frac{d\sigma_{b0}}{dt} = 0$$
$$\frac{du_{a0}}{dt} = -\frac{2}{125}\left(\hat{X}_{b_0}\sigma_{a0}\sin\left(-u_{a0}+u_{b0}\right) + 50\,\hat{X}_{b_0}\sin\left(-u_{a0}+u_{b0}\right)\right)\hat{X}_{a_0}$$
$$\frac{d\gamma_{a0}}{dt} = 0$$
$$\frac{d\sigma_{a0}}{dt} = -\frac{1}{125}\left(2\,\hat{X}_{b_0}\cos\left(-u_{a0}+u_{b0}\right) + 5\,\hat{X}_{b_0}\right)\hat{X}_{a_0}$$

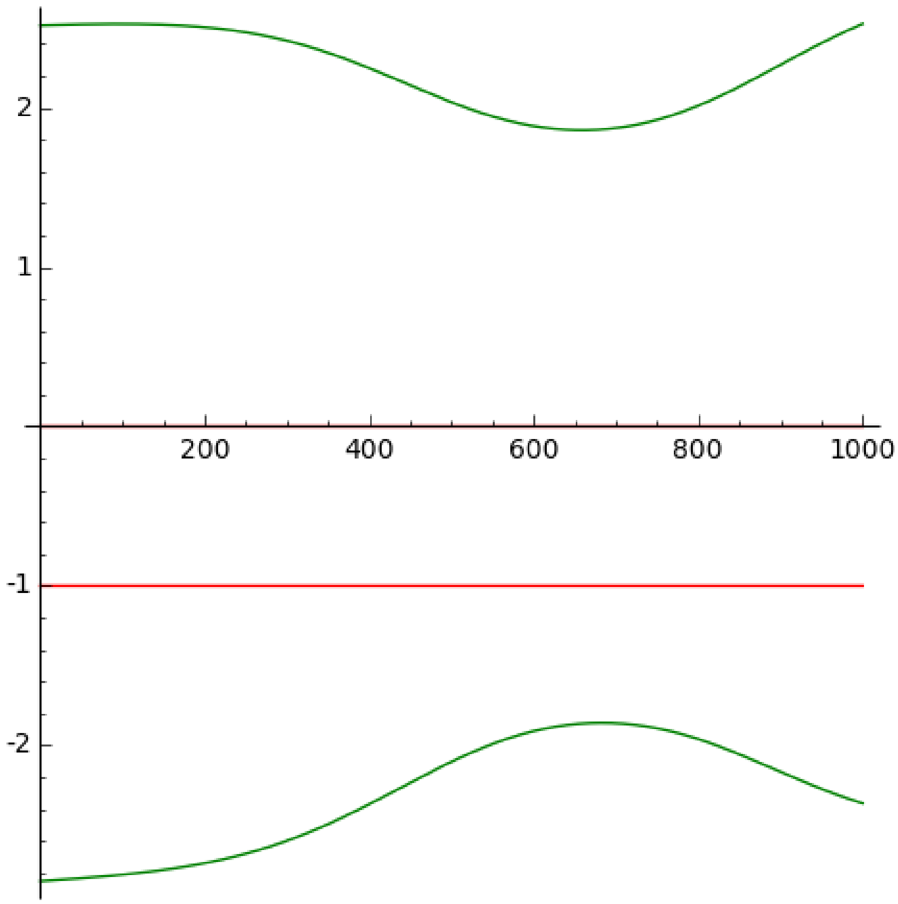


u_{b_0}
γ_{b_0}
σ_{b_0}
u_{a_0}
γ_{a_0}
σ_{a_0}
60
50
40
30
20
10
200
400
600
800
1000
t

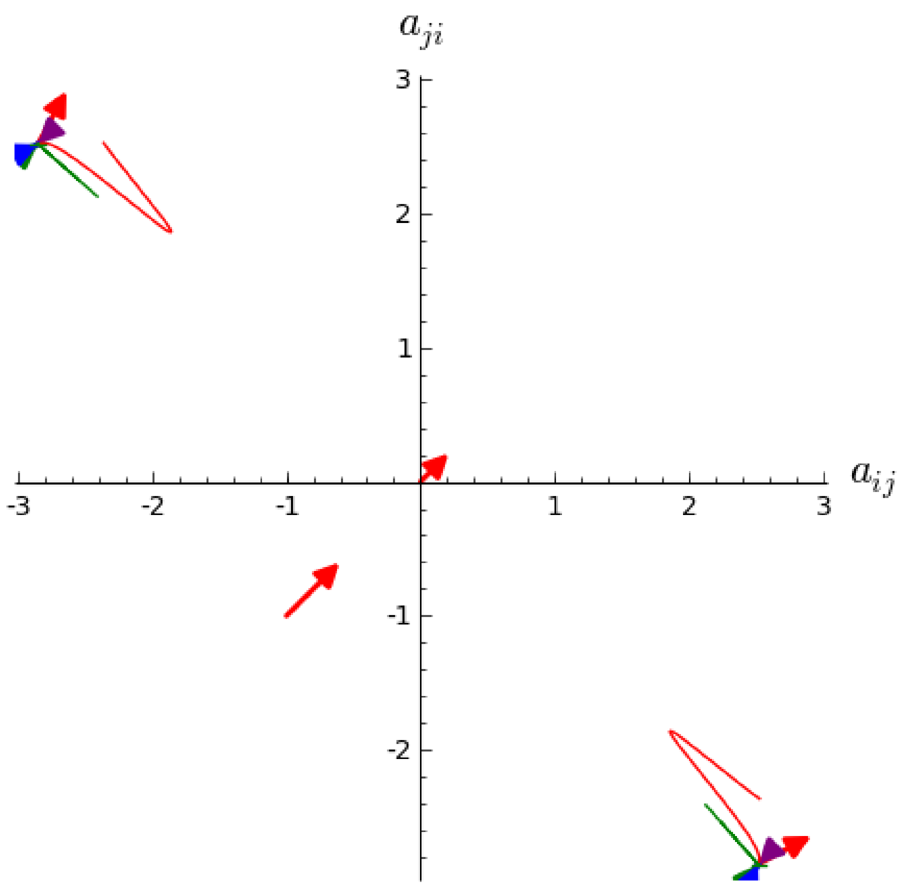

$u_{a_0} - u_{b_0}$
$u_{a0} - u_{b0}$
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
200
400
600
800
1000
t



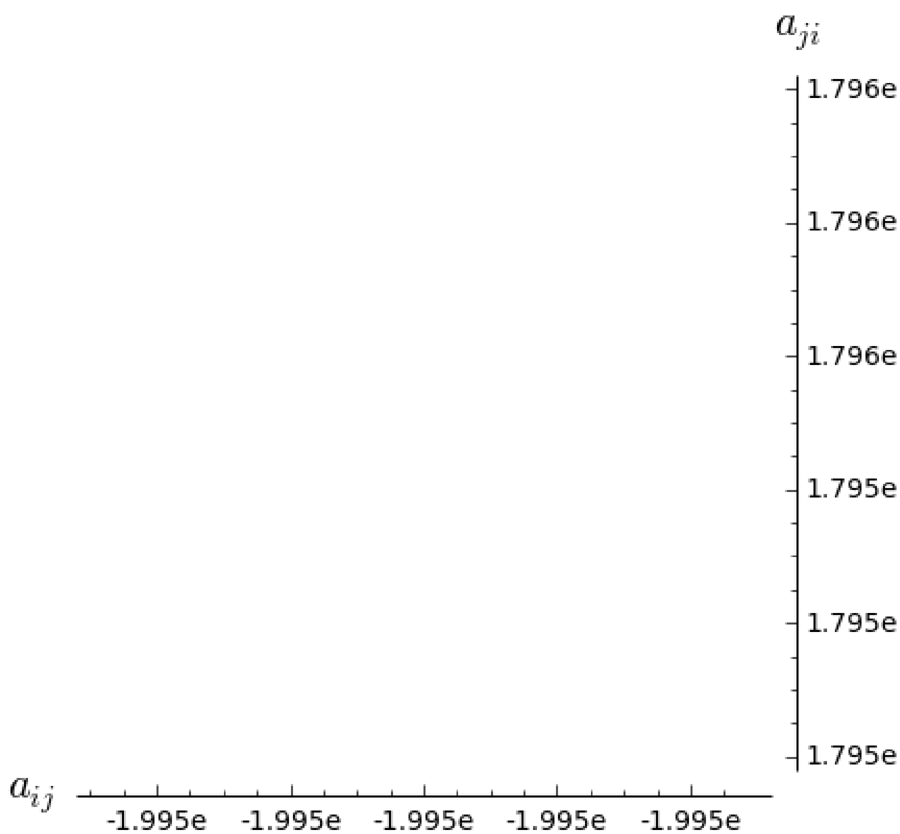
$Xhat_{b_0}$
$Xhat_{a_0}$
0.5
0.45
0.4
0.35
0.3
0.25
0
200
400
600
800
1000
t



2
1
-1
-2
200
400
600
800
1000



a_{ji}
a_{ij}
3
2
1
-1
-2
-3
-2
-1
1
2
3



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