Systems, Networks, and Strategies **Name:** ____________________

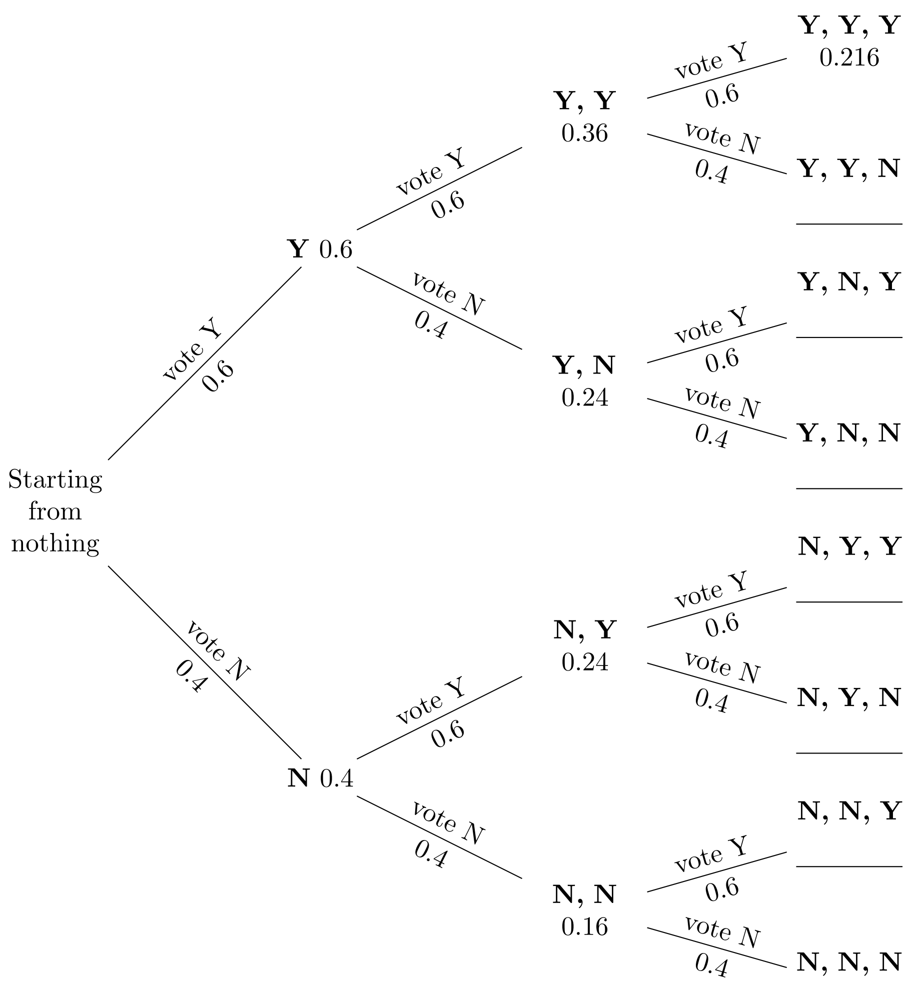
Homework due April 23, 2015

Reading: Ed Yong, "How the Science of swarms can help us fight cancer and predict the future," *Wired*, March 19, 2013.

1. Condorcet's Jury Theorem was established in 1785 by the Marquis de Condorcet, a philosopher, mathematician and political scientist who held several government positions after the French Revolution, who advocated the abolition of slavery and did research into the dynamics of voting and democratic practices.

His Jury Theorem concerns a group trying to make a decision by majority vote. If each person has probability p of voting for the "correct" decision, and their votes are not influenced by each other, and the probability p is greater than $1/2$, then the probability that the group votes for the correct decision is greater than p. That is, the group is a better decision maker than each person in the group. Bigger groups are better decision makers, and if we consider larger and larger groups the probability of the correct decision approaches 1.

Here we explore why, using this diagram.



In this example, we call the "correct" decision **Y**, and we suppose that p is 0.6, the probability that a given person chooses **Y**. So the probability of choosing **N** is 0.4, since they must add to 1. And the probability that the first person chooses **Y** and the second person chooses **Y** is 0.6 times 0.6, which is 0.36. If you follow along the very top of this diagram from left to right you'll see that 0.36 there.

The probability that the first three people choose **Y, Y, Y** is also found by multiplying: the first two choose **Y, Y** with probability 0.36, and the third chooses **Y** with probability 0.6, and 0.36 times 0.6 is 0.216. That's the value at the top right.

(a) Fill in the blanks in that diagram for the other choices that three people can make.

(b) In a group of two people, the probability of a tie vote is found by adding: the two ways it can happen are **Y, N** and **N, Y**. The sum of those probabilities is $0.24 + 0.24 = 0.48$.

In a group of three people, list the different ways they can come out to exactly two **Y** votes and one **N** vote.

(c) Using the numbers you added to the diagram, what is the total probability of getting two **Y** votes and one **N** vote?

(d) What is the probability of three **Y** votes?

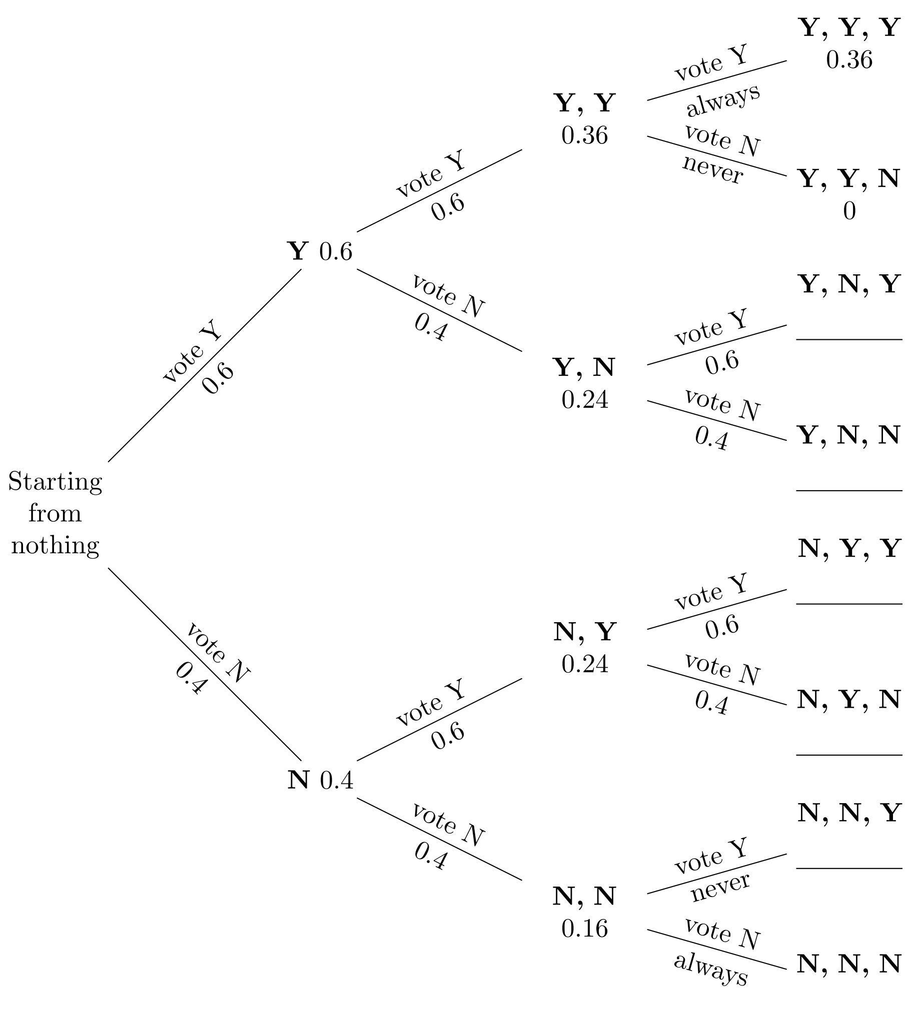
(e) Adding them together, what is the probability that in a group of three people a majority votes for **Y**?

The probability of a majority vote for **Y** should come out larger than 0.6, meaning that the group is more likely to make the "correct" decision than any one person. It becomes *much* more likely when more people vote.

2. When people don't decide independently Unfortunately, Condorcet's theorem only works when people make their choices *independently*, without influencing each other. When people influence each other, some of the benefit of the "wisdom of the crowd" can be lost.

Here we look just a little at this situation. As before, suppose each person prefers the "correct" decision, **Y**, with probability 0.6. But before they vote, they look at all the information they know about – their own preference, and the people who have already voted – and if their preference is outnumbered they vote with the majority. That is, if your preference is **N** and the two people before you voted **Y**, you go with the crowd and vote **Y**. That means you vote **Y** in that case regardless of your preference.

Because of that, this diagram is different from the last one in just a few places. If someone is outnumbered, they go with the majority no matter what they prefer, so it's marked "always" and it happens with probability 1, that is, they always make that choice. So the probability of **Y, Y, Y** is the same as the probability of **Y, Y** because after **Y, Y** the third person always votes **Y**. And **Y, Y, N** never happens, so its probability is 0.



(a) Fill in the blanks in that diagram.

(b) If the first two people vote **N**, what does the third person do?

(c) If the first two people voted **N**, given what the third person does, what will the fourth person do?

(d) And after that, what will the fifth and sixth person do?

(e) If the first four people vote **N, Y, N, N**, what will the fifth person do?

(f) In these cases, will the majority vote for **Y** or **N**?

This is called an *opinion cascade* or *information cascade.* Once a cascade starts, a lot of people's insight is unused, and the group can no longer be trusted to make the right choice, even if it's a large group. It's worth watching out for opinion cascades – sometimes when a lot of people are making the same choice, it's actually just because they're mostly following along, not because it's a good idea.