Evolution in a Food Web

Evolution in a food web – is it arms-race-like?

We provide a directed graph representing the food web. Node labels are species names, and arrow labels are strength of predation. Let's say the conversion factor from prey to predator is constant c.

Then species i's dynamics is

$\frac{dX_i}{dt} = (r_i + k\sum_{j\to i} f_{ji}X_j - \sum_{i\to j} f_{ij}X_j)X_i$

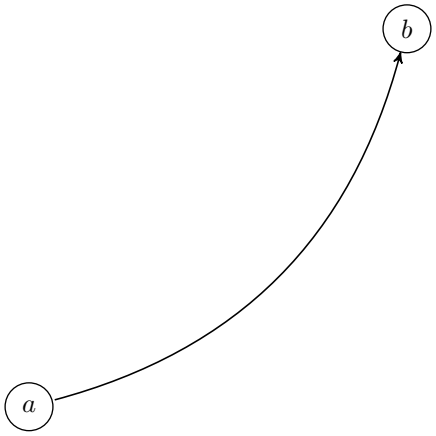
where

$f_{ij} = f(u_i, u_j)$ is some function of the two phenotypes controlling how well j eats i;

u_i is the phenotype of species i; and

$r_i =$ (0 if i is a predator, 1 else).

This will induce the usual dynamics of apparent competition, and adaptive dynamics of all the u_i follows.



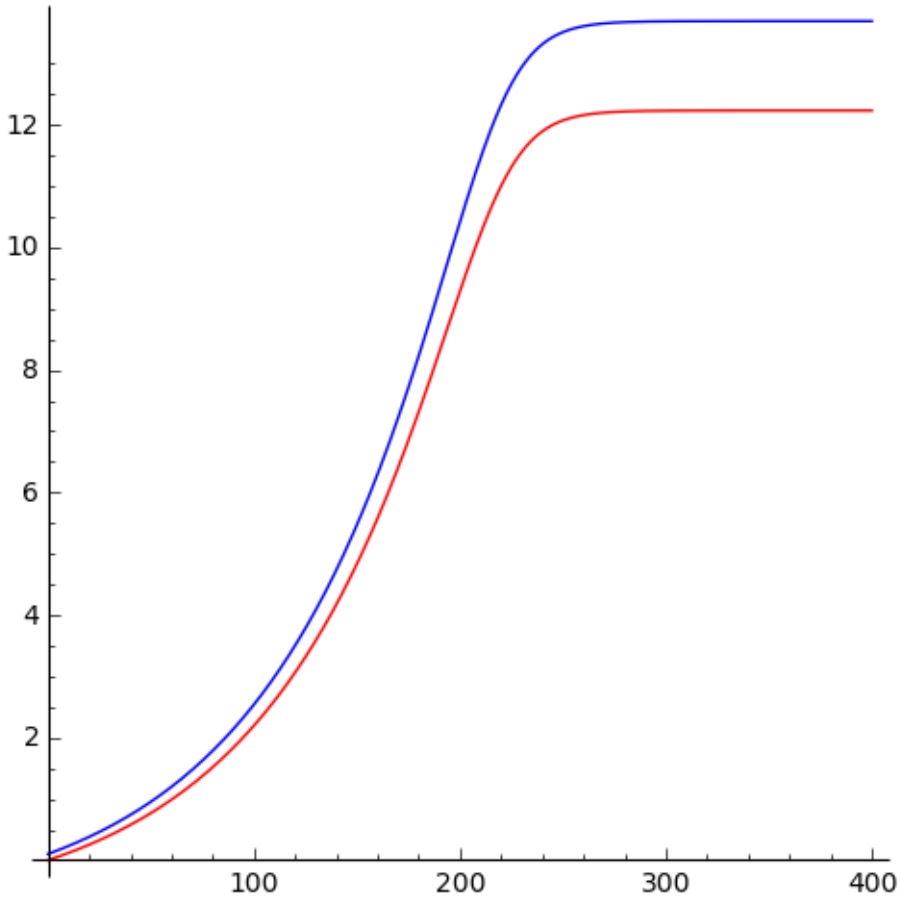
The foodweb model:

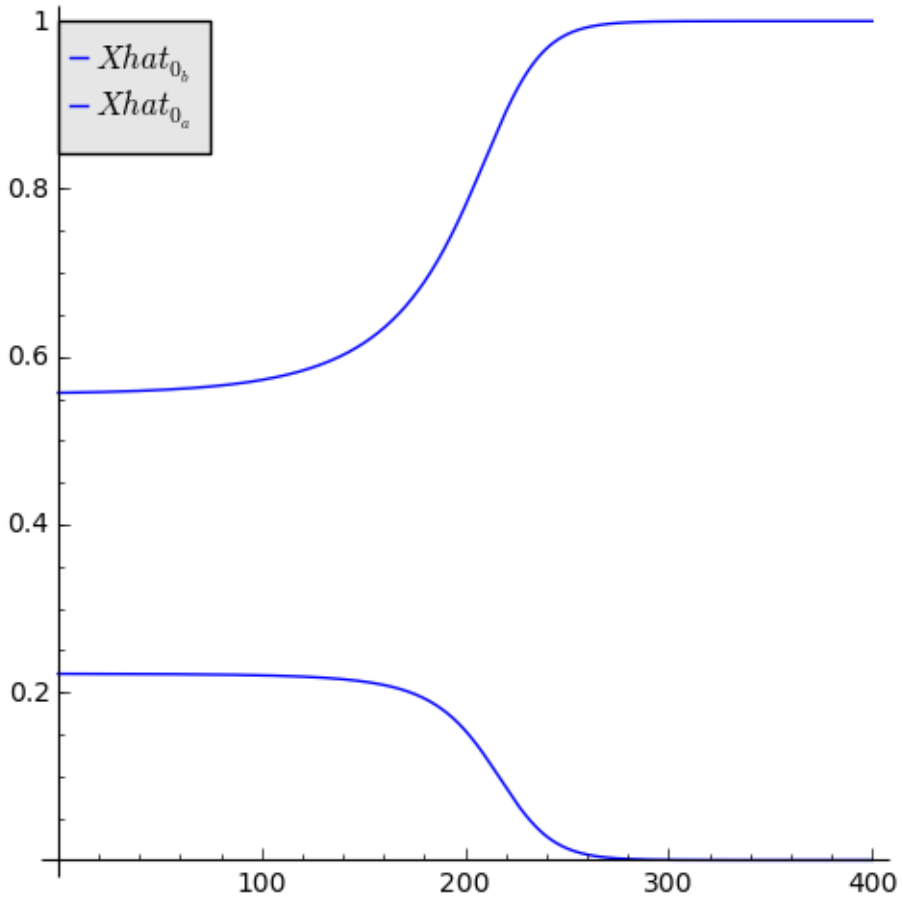
$$\frac{dX_{0b}}{dt} = \frac{9}{10} X_{0a} X_{0b} (\cos(-u_{0a} + u_{0b}) + 1) - X_{0b}$$
$$\frac{dX_{0a}}{dt} = -X_{0a} X_{0b} (\cos(-u_{0a} + u_{0b}) + 1) - X_{0a}^2 + X_{0a}$$

Adaptive dynamics of model:

$$\frac{du_{0a}}{dt} = -\frac{10\,(9\,\cos(-u_{0a} + u_{0b}) - 1)\sin(-u_{0a} + u_{0b})}{81\,(\cos(-u_{0a} + u_{0b}) + 1)^3}$$
$$\frac{du_{0b}}{dt} = -\frac{(9\,\cos(-u_{0a} + u_{0b}) - 1)\sin(-u_{0a} + u_{0b})}{9\,(\cos(-u_{0a} + u_{0b}) + 1)^3}$$

flow at $\begin{pmatrix} 0.100000000000000 \\ 0 \end{pmatrix}$: $\begin{pmatrix} 0.0123481451481798 \\ 0.0111133306333618 \end{pmatrix}$


12
10
8
6
4
2
100
200
300
400


$Xhat_{0_b}$
$Xhat_{0_a}$
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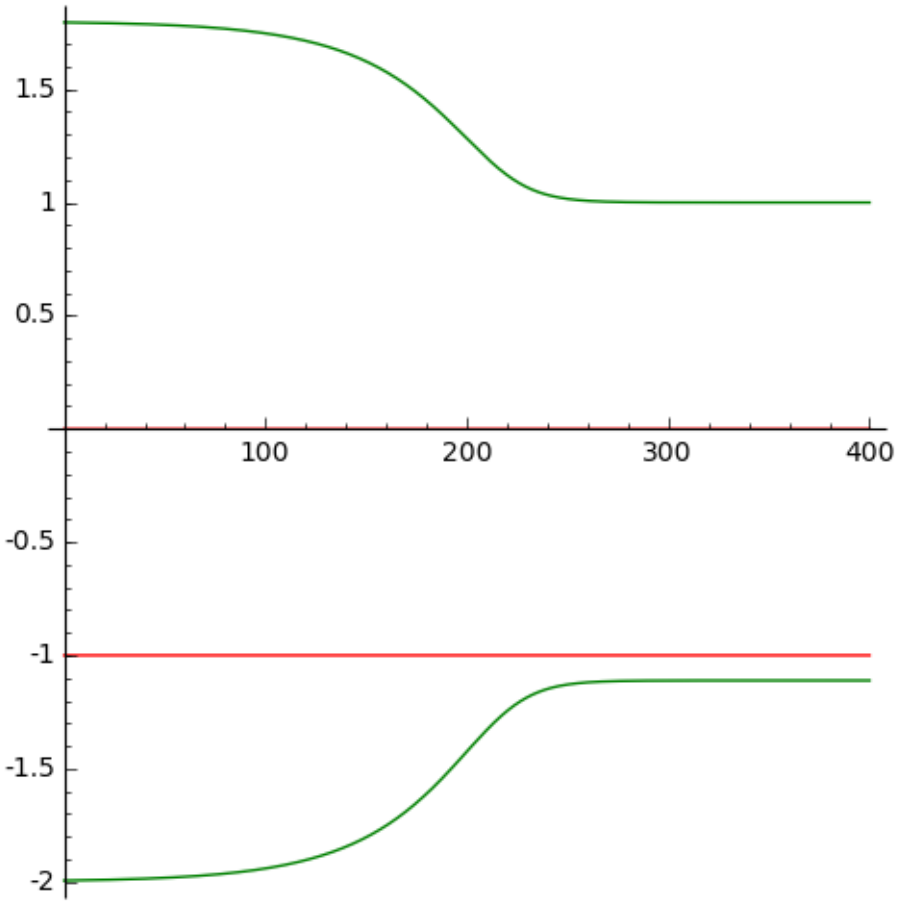
0.8
0.6
0.4
0.2
100
200
300
400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0.5
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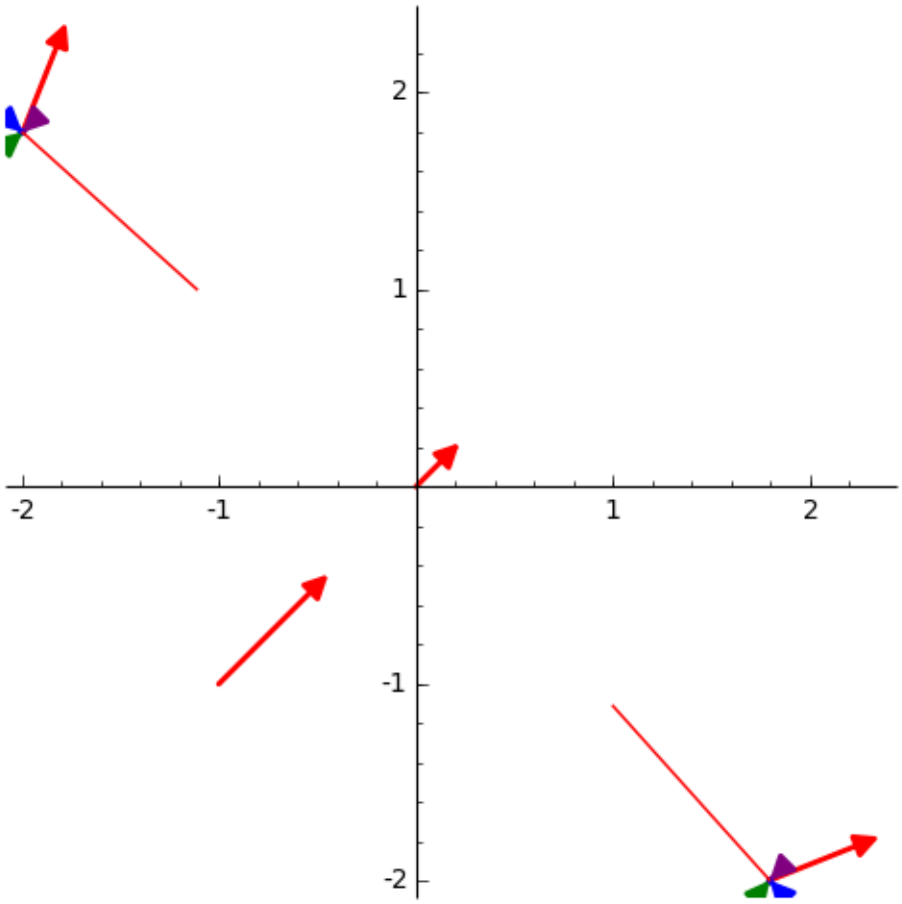
-1
-1.5
-2


2
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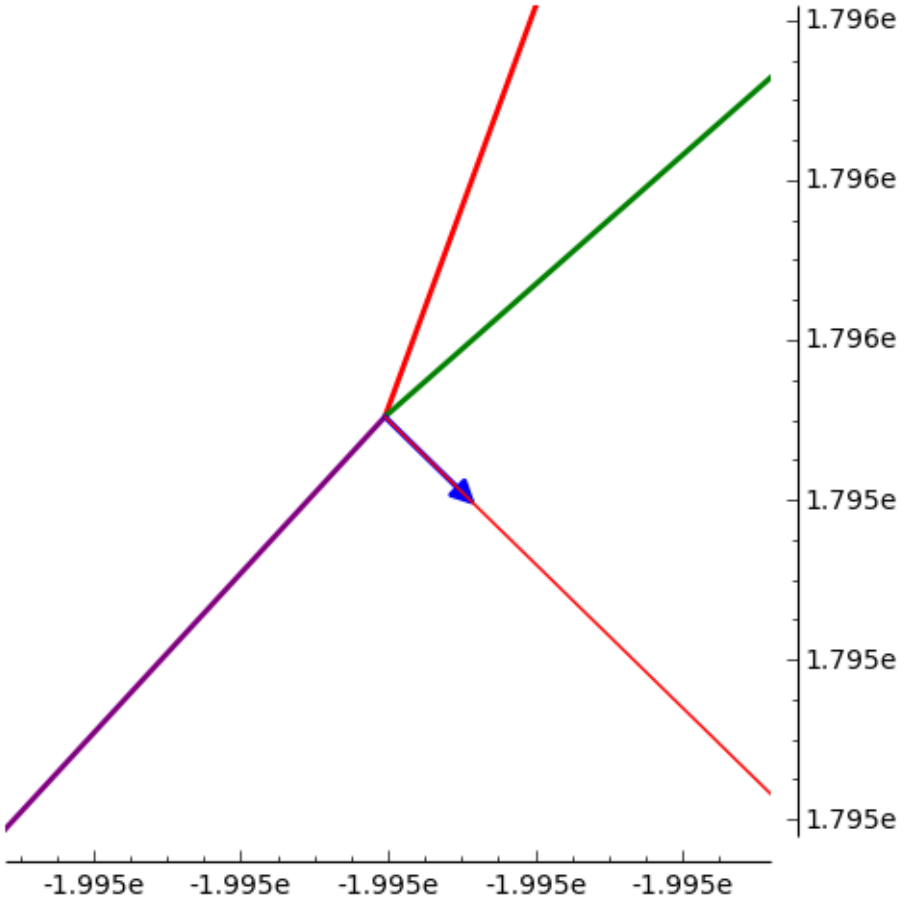
-2
-1
1
2
-1
-2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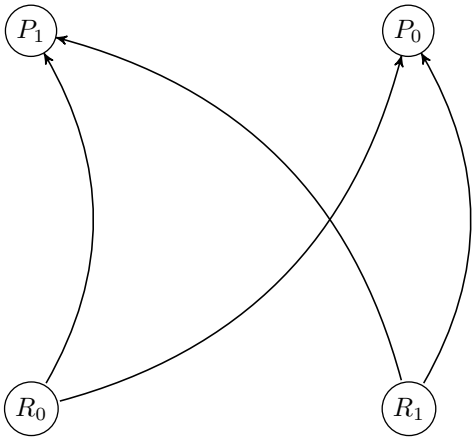

The foodweb model:

$$\frac{dX_{0R_0}}{dt} = -X_{0P_0}X_{0R_0}(\cos(u_{0P_0} - u_{0R_0}) + 1) - X_{0P_1}X_{0R_0}(\cos(u_{0P_1} - u_{0R_0}) + 1) - X_{0R_0}^2 - X_{0R_0}X_{0R_1} + X_{0R_0}$$
$$\frac{dX_{0P_1}}{dt} = \frac{9}{10}X_{0P_1}X_{0R_0}(\cos(u_{0P_1} - u_{0R_0}) + 1) + \frac{9}{10}X_{0P_1}X_{0R_1}(\cos(u_{0P_1} - u_{0R_1}) + 1) - X_{0P_1}$$
$$\frac{dX_{0P_0}}{dt} = \frac{9}{10}X_{0P_0}X_{0R_0}(\cos(u_{0P_0} - u_{0R_0}) + 1) + \frac{9}{10}X_{0P_0}X_{0R_1}(\cos(u_{0P_0} - u_{0R_1}) + 1) - X_{0P_0}$$
$$\frac{dX_{0R_1}}{dt} = -X_{0P_0}X_{0R_1}(\cos(u_{0P_0} - u_{0R_1}) + 1) - X_{0P_1}X_{0R_1}(\cos(u_{0P_1} - u_{0R_1}) + 1) - X_{0R_0}X_{0R_1} - X_{0R_1}^2 + X_{0R_1}$$

Adaptive dynamics of model:

$$\frac{du_{0P_1}}{dt} = \left(\hat{X}_{0R_0}kD[0](f)(u_{0P_1}, u_{0R_0}) + \hat{X}_{0R_1}kD[0](f)(u_{0P_1}, u_{0R_1})\right)\hat{X}_{0P_1}$$
$$\frac{du_{0P_0}}{dt} = \left(\hat{X}_{0R_0}kD[0](f)(u_{0P_0}, u_{0R_0}) + \hat{X}_{0R_1}kD[0](f)(u_{0P_0}, u_{0R_1})\right)\hat{X}_{0P_0}$$
$$\frac{du_{0R_1}}{dt} = -\left(\hat{X}_{0P_0}D[1](f)(u_{0P_0}, u_{0R_1}) + \hat{X}_{0P_1}D[1](f)(u_{0P_1}, u_{0R_1})\right)\hat{X}_{0R_1}$$
$$\frac{du_{0R_0}}{dt} = -\left(\hat{X}_{0P_0}D[1](f)(u_{0P_0}, u_{0R_0}) + \hat{X}_{0P_1}D[1](f)(u_{0P_1}, u_{0R_0})\right)\hat{X}_{0R_0}$$

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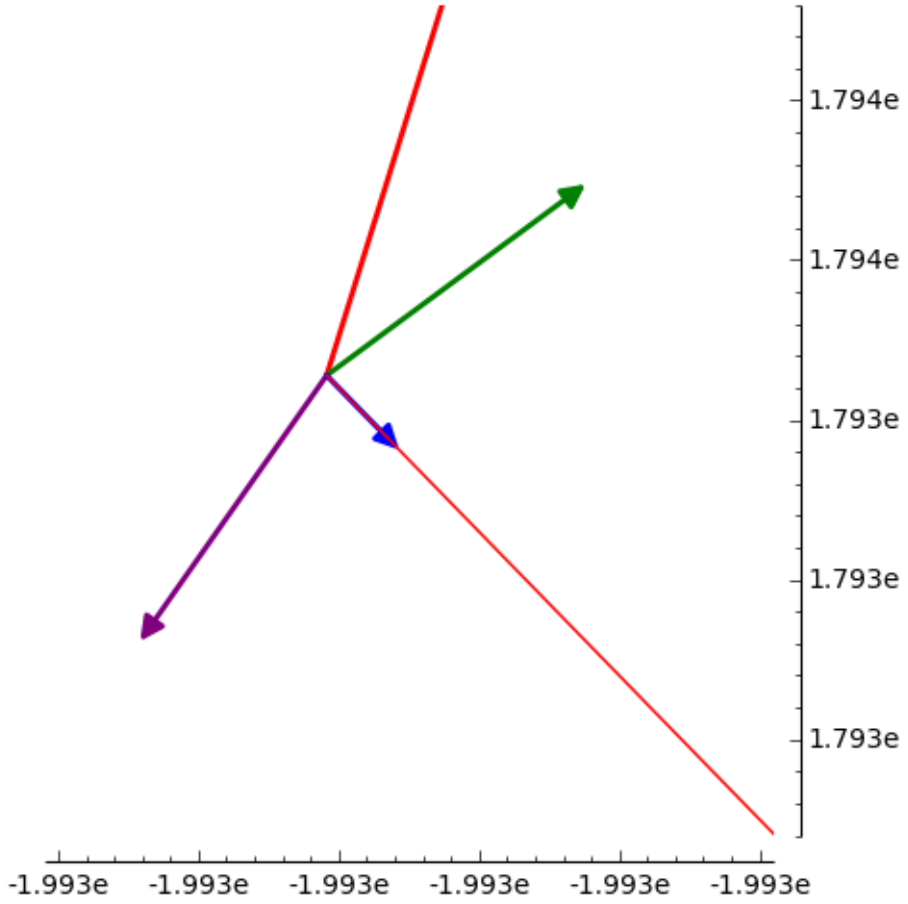

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