Tragedy of the Commons Model

Comparison of ToC model to R* model. Is the resource competition situation a tragedy or not?

Review of ToC situation

Common good is quality of pasture, $q = q_0 - dhX$

Strategy is rancher's head count of cattle, h

Selection gradient is gradient of ***utility***, $U(h'|h) = h'q$ – so that gradient wrt h' is positive.

Analysis of derivatives:

Utility of h' strategy when h is resident:

$$U(h'|h) = h'(q_0 - dhX)$$
$$\partial_1 U = q \quad = \quad q_0 - dhX$$

Quality decreases when h increases:

$$\frac{dq}{dh} = -dX$$

Total change in utility when $h' > h$ replaces h:

$$\begin{aligned}\frac{dU(h|h)}{dh} &= \frac{d}{dh}(q_0 h - dXh^2)\\ &= q_0 - 2dhX\\ &= q - dhX\end{aligned}$$

Utility can either increase or decrease depending on whether it's influenced more by the growing head count or by the loss of pasture quality.

Review of 1-resource competition

Common good is presumably the resource availability, R^*

Strategy is let's say the phenotypic uptake efficiency c.

The dynamics selects one monomorphic population and equilibrates at $cwR^* = m$, or $R^*(c) = m/cw$.

The invasion speed for variant c' is

$$\mathcal{I}(c'|c) = b_i c' w R^*(c) - m$$

Selection gradient is

$$\begin{aligned} \partial_1 \mathcal{I}(c'|c) &= b_i w R^*(c) \\ &= b_w m/c. \end{aligned}$$

The strategy is to increase uptake efficiency c, and resource availability R^* decreases.

A quantity analogous to the utility question would be rate of uptake per capita:

$$\begin{aligned} \frac{d}{dt}(cR^*(c)) = \frac{d}{dt}(m/w) \\ = 0, \end{aligned}$$

so interestingly, that remains constant!

This would be quite similar to the Hardin model if we identify $h = c$, $q = R^* = m/cw$, $U = cq = m/w$. The functional form of q is different (such that quality can't become negative) but the tragedy is the same.

One difference is that in the Tilman model, the population size (= number of ranchers) varies as the strategy changes. This drives the regulation that holds the per capita utility constant. Constant utility is characteristic of ecological evolution, and not as common in microeconomics and game theory.

Conclusion: though we don't generally think of resource competition as a tragedy of the commons, if the limiting resource is considered a common good it does have the characteristics that

- individuals have fitness incentive to use more of it per capita
- increased use leads to decreased availability

The ambiguous change in overall utility in both scenarios is potentially interesting. I believe Hardin focuses on the decrease in benefit per head of cattle, which is not U but q; analogous in the Tilman model is R^* rather than $cR^* = m/w$.

I am thinking of proposing to define a tragedy by:

- Fitness or utility is a function of a common good q
- Variants whose utility when rare is greater are selected
- When they establish, the common good is degraded.

This covers both these models, without regard for whether fitness or utility increases or decreases.

The common good is distinct from the fitness or utility.

Thus there's a value judgement in this definition, as it's not clear what can and can not be labeled a common good and what changes in it are to be considered degradation (i.e. bad, rather than good).

Of course, and very relevant to the paper, the interaction strength a has some relation to a common good, or at least is conventionally used as a measure of cooperation, so I will want to apply this definition to it as well.

Questions

(For another time/project)

- Divide up competition $c_{i\ell}$ into incentive and impact parts, as in the Masel project for space competition.
- Do this kind of direct and indirect effect analysis on the space model
- Is this analysis of resource and space competition a separate paper prior to the $a()$ paper?
- Maybe flesh out lit review on these two things

Notes on direct and indirect effects.

In the older text on selection on $a(i, j)$, direct effect is change in a due to selection on a? Is that somehow clear?

It's like this:

- There's an isomorphism (via a sort of sharp-flat relationship) between the gradient of fitness and the direction of adaptive change in the bottom-level character u_i.
- In some sense (clarify this!) that adaptive change is *because* it's the direction of marginal increase in invasion fitness.
- Direct change is change that happens because it is up the fitness gradient (?).
- This is change in the first-partial direction of $a(\cdot, \cdot)$.

- Change in the second partial direction is because of a different fitness gradient (on a different population).

So how to use this on these models:

- In the resource model, there are direct and indirect effects on the quantity $\mathcal{I}(c'|c) = b_i c' w R^*(c) - m$, which is the analog of the commons model's $U(h'|h)$. Is this quantity anything besides the invasion speed? We can say it's fitness in the $R^*(c)$ environment.
- I'm hesitant at looking at the dynamics of fitness because of the way it always returns to 0, but…?
  - By definition, I guess there must always be exactly balancing direct and indirect effects of adaptation on invasion fitness…
- Above, I talk about the uptake rate per capita, which also is constant.
- But it wouldn't be with b or m adapting, I think. It has interesting direct and indirect effects.
- In the paper I define direct and indirect effects of adaptation on a such that their sum recovers $\dot{a}$.
- By analogy: direct effect of adaptation on this uptake rate – call it $\bar{U}(c'|c) = c'R^*(c)$? – is $\partial_1 \bar{U}(c|c)\dot{c}$? Indirect effect is $\partial_2 \bar{U}(c|c)\dot{c}$? What does that get us?
- Direct effect is $R^*\dot{c}$, and $\dot{c}$ is proportional to $\gamma \hat{X} \partial_1 \bar{U}$ so direct effect is always positive, per capita uptake rate selected for increase in this direction.
- Indirect effect is $c'\partial_1 R^*(c)\dot{c}$. With $R^* = m/cw$, we have $\partial_1 R^* = -m/c^2 w < 0$, so this is counter to the direct effect.
- Check that these are balancing correctly? $(\partial_1 + \partial_2)\bar{U}\dot{c} = m/cw - c(m/c^2 w) = 0$, check.
- Direct effect is the benefit that drives adaptation: increased uptake of the resources that are in the environment.
- Indirect effect is the impact on the environment: depletion of the resource pool, which acts counter to the direct effect, reducing or even reversing the gain in fitness once the change becomes common.
- To do: direct and indirect effects with variation in b and m.
- Direct effect of … economic actors' response to incentive … on the utility function $U(h'|h) = h'q(h)$ is $\propto \partial_1 U(h'|h) = q(h)$. This is the incentive we recognize - the marginal benefit of increasing head count, without regard to the impact on the commons.

- A notable consequence is that if q ever becomes negative, the incentive will reverse, favoring *reducing* the head count on the commons.

- Indirect effect is $\propto \partial_2 U = h' \partial_1 q(h)$, which is the impact of rising exploitation on the quality of the pasture (as expressed in the utility to any one rancher). Just as it should be.