Comparison of Logistic Growth and Resource Competition Models

In the monomorphic Macarthur-Levins model the population dynamics is

$\frac{dX}{dt} = b(u)X(c(u)w(K - \frac{c(u)X}{r}) - m(u))$

so that

$$\mathbf{A}(u) = \begin{pmatrix} r \\ a_{00} \end{pmatrix} = \begin{pmatrix} b(u)\,(c(u)wK - m(u)) \\ b(u)\frac{c(u)^2 w}{r} \end{pmatrix}.$$

As we've seen, for some, though, not all, choices of functions b, m, and c, adaptive increase in c drives the competition term a_{00} away from zero in the negative direction. This is because adaptation in this system tends to increase the Lotka-Volterra intrinsic growth term at the expense of the competition term.

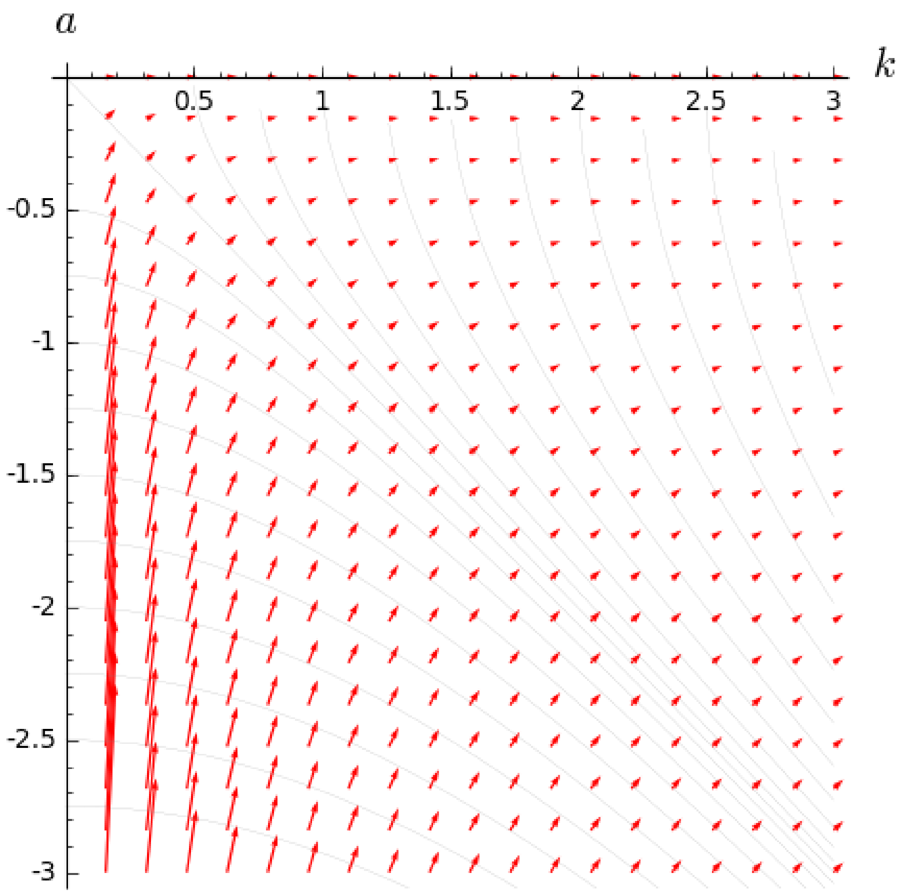
It should be possible to analyze this situation geometrically, in terms of motion of a point on a curve in the $\mathbf{A}$ plane, in which the motion is driven by the k component of the selection gradient, against the a component.

I would like to compare this to the dynamics of K selection in the logistic growth model, in which selection on the K character of a population tends to *decrease* competition.

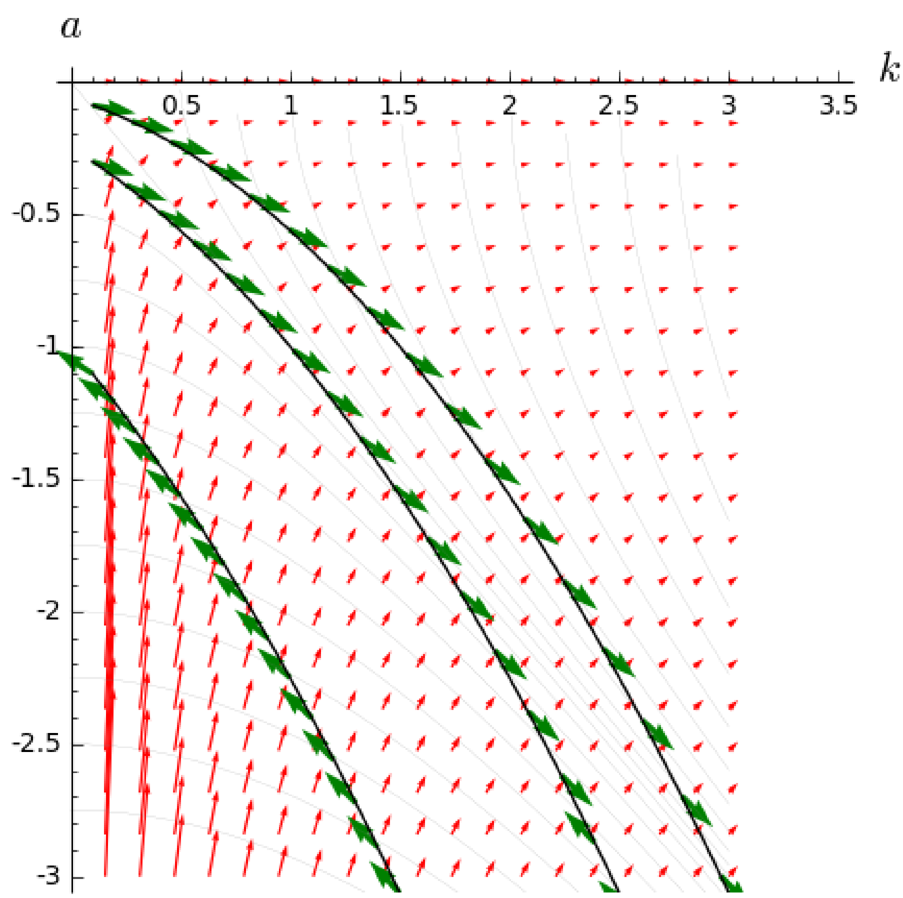
The interaction selection gradient $\mathbf{S}(\mathbf{A})$ is the same for all monomorphic Lotka-Volterra models, concerned only with selection on the Lotka-Volterra coefficients k and a regardless of their functional form in terms of population phenotypes. The interaction selection gradient vector field is

$$\mathbf{S}(\mathbf{A}) = \begin{pmatrix} 1 \\ -a/k \end{pmatrix}.$$

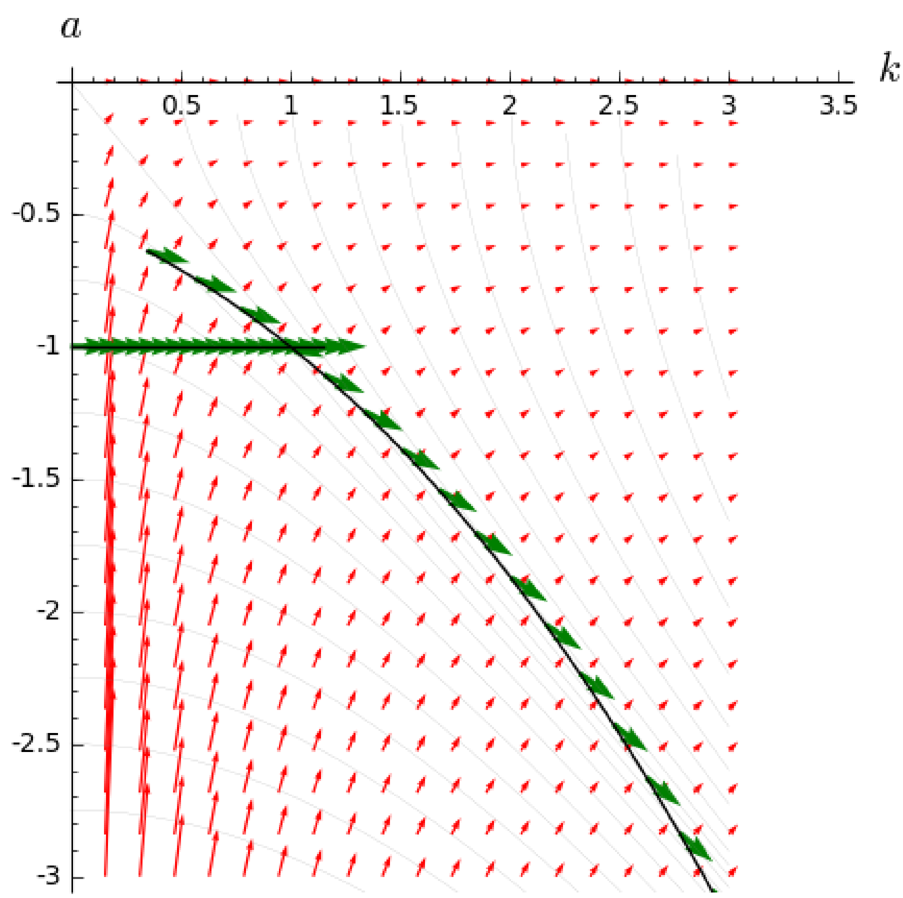
The selection gradient is perpendicular to a neutral-invasion-exponent curve that divides the plane into viable and inviable halves at each point.



Here is the (k, a) curve for the monomorphic resource competition model, with b and m fixed, showing the motion through the plane due to change in c.



And some other choices of c, m, b:



This appears to be a case of competition worsening due to a tragedy of the commons; specifically, where adaptation is driven by the vector $\partial_1 \mathbf{A}$ pointing into the viable half plane while the resultant direction of motion $d\mathbf{A}/dt$ is near but below the line, and drags the population into worsening conditions.

TODO here: verify that

- $\partial_1 \mathbf{A}$ is as shown, and above the gray line in all places
- It's very close to neutral
- The marginal-viability measure k/a is actually decreasing where it looks like it is

K selection in the logistic model

This model is

$\frac{dX}{dt} = r(u)X(1 - \frac{X}{K(u)})$,

so that

$$\mathbf{A}(u) = \begin{pmatrix} r(u) \\ -\frac{r(u)}{K(u)} \end{pmatrix},$$

with

$$\partial_1 \mathbf{A}(u) = \begin{pmatrix} r'(u) \\ \frac{r(u)K'(u) - r'(u)K(u)}{K(u)^2} \end{pmatrix}.$$

See paper.html for logistic model details.

In this model, the direction of invasion (?) $\partial_1 \mathbf{A}$ is always collinear with $d\mathbf{A}/dt$, so there is no tragedy and adaptation always increases k/a.