Evolution in a Food Web

Evolution in a food web – is it arms-race-like?

We provide a directed graph representing the food web. Node labels are species names, and arrow labels are strength of predation. Let's say the conversion factor from prey to predator is constant c.

Then species i's dynamics is

$\frac{dX_i}{dt} = (r_i + k \sum_{j \to i} f_{ji} X_j - \sum_{i \to j} f_{ij} X_j) X_i$

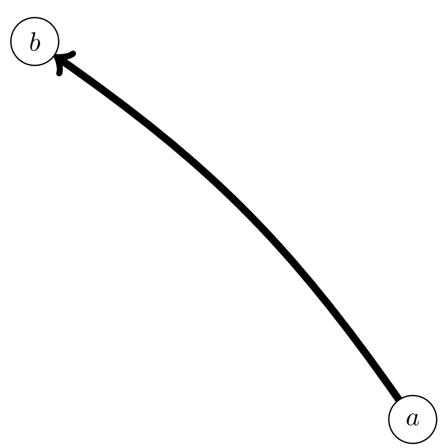
where

$f_{ij} = f(u_i, u_j)$ is some function of the two phenotypes controlling how well j eats i;

u_i is the phenotype of species i; and

$r_i =$ (0 if i is a predator, 1 else).

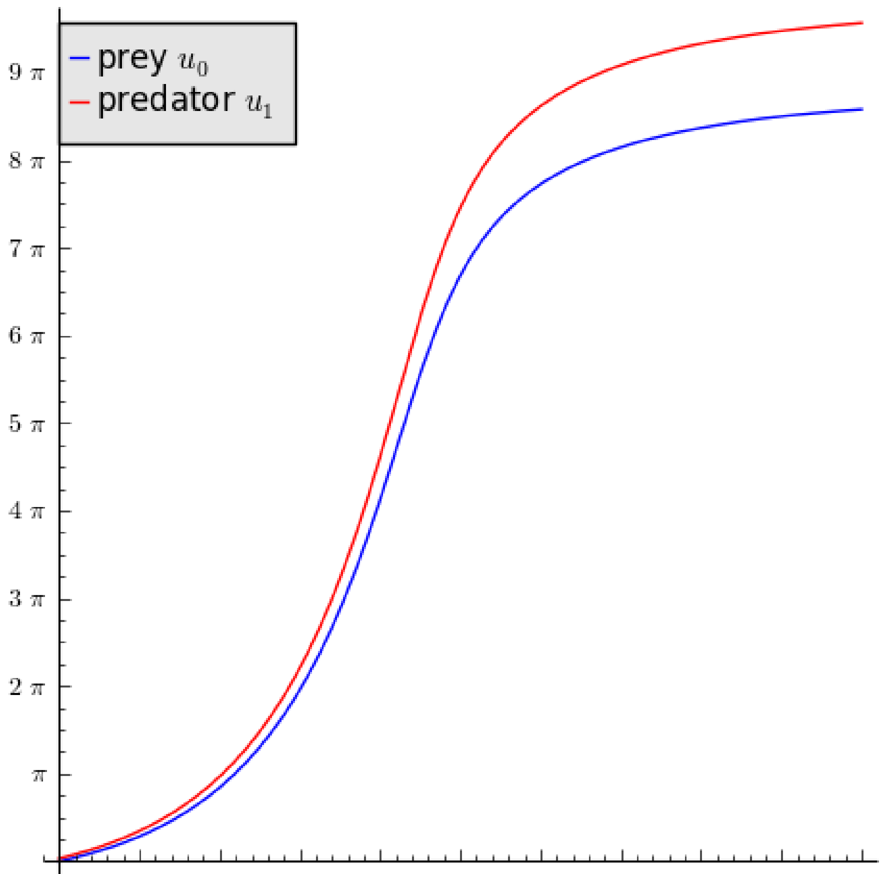
This will induce the usual dynamics of apparent competition, and adaptive dynamics of all the u_i follows.

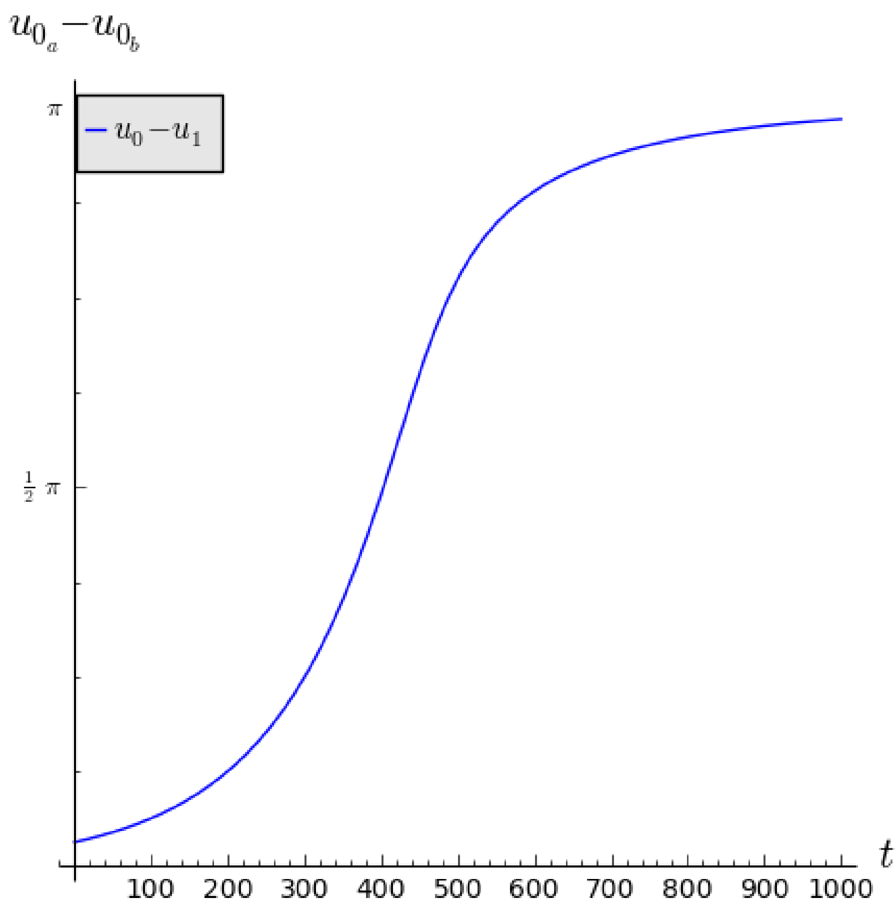


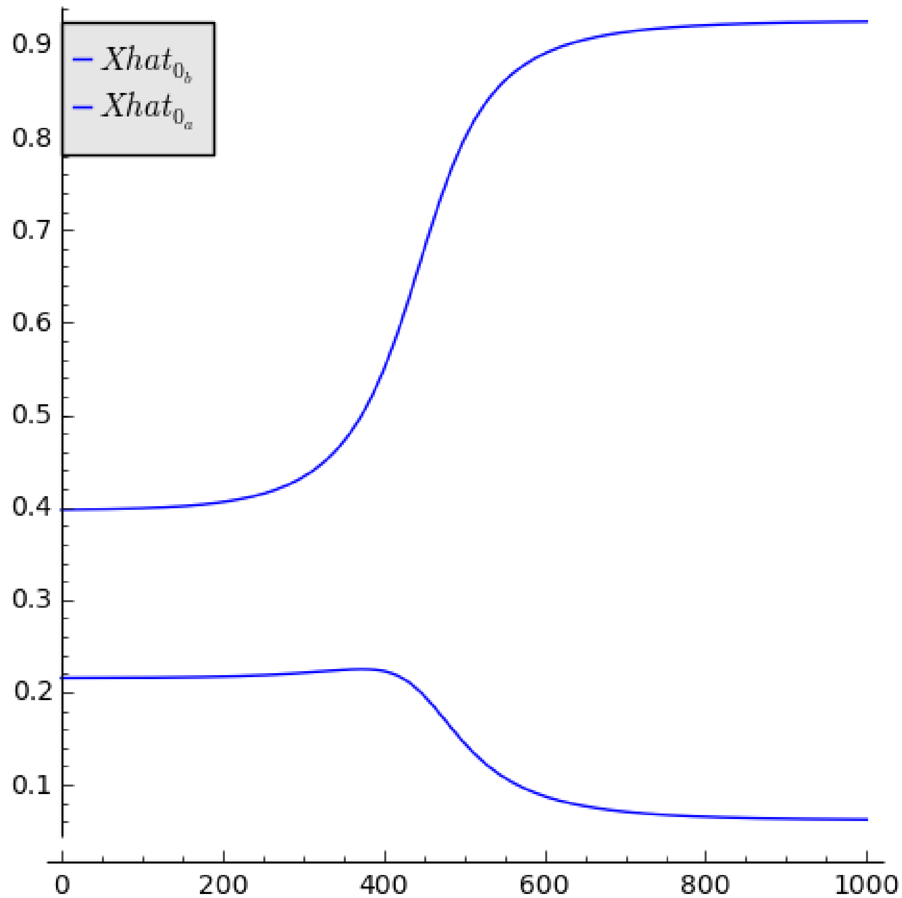
The foodweb model:

$$\frac{dX_{0b}}{dt} = \frac{9}{25}\,X_{0a}X_{0b}(2\,\cos\left(-u_{0a} + u_{0b}\right) + 5) - X_{0b}$$

$$\frac{dX_{0a}}{dt} = -\frac{2}{5}\,X_{0a}X_{0b}(2\,\cos\left(-u_{0a} + u_{0b}\right) + 5) - X_{0a}^{2} + X_{0a}$$




$u_{0_a} - u_{0_b}$
$u_0 - u_1$
π
$\frac{1}{2}\pi$
t
100 200 300 400 500 600 700 800 900 1000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$Xhat_{0_a}$
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0.4
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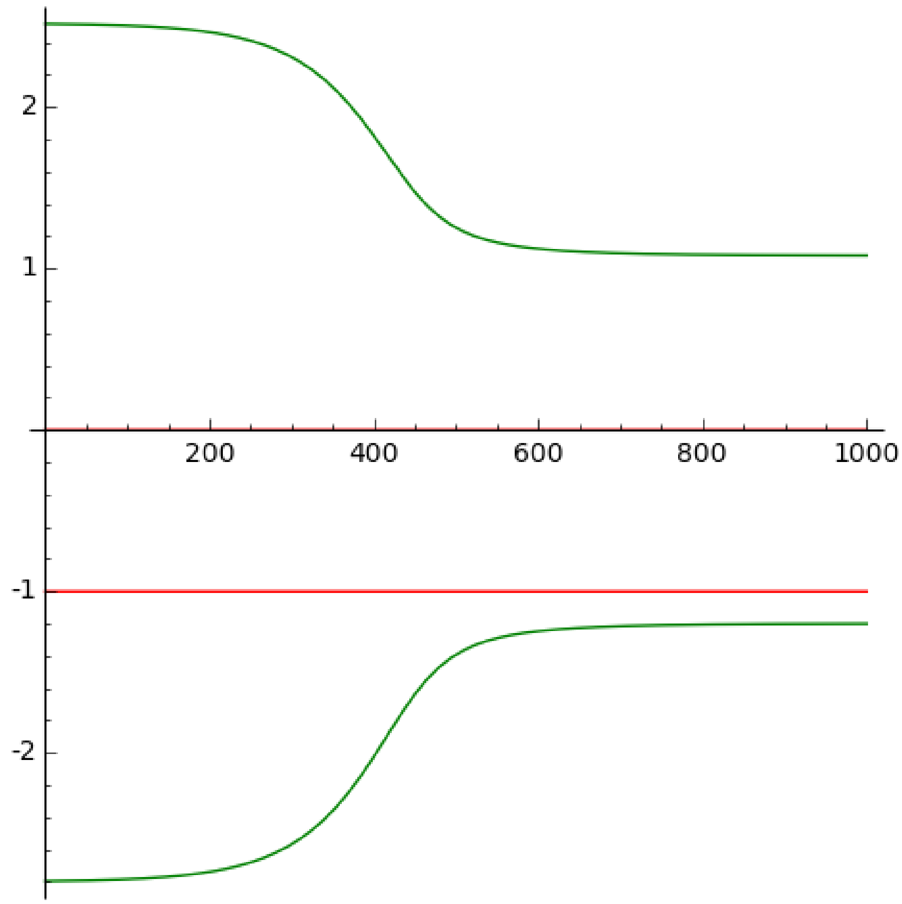
0.1
0
200
400
600
800
1000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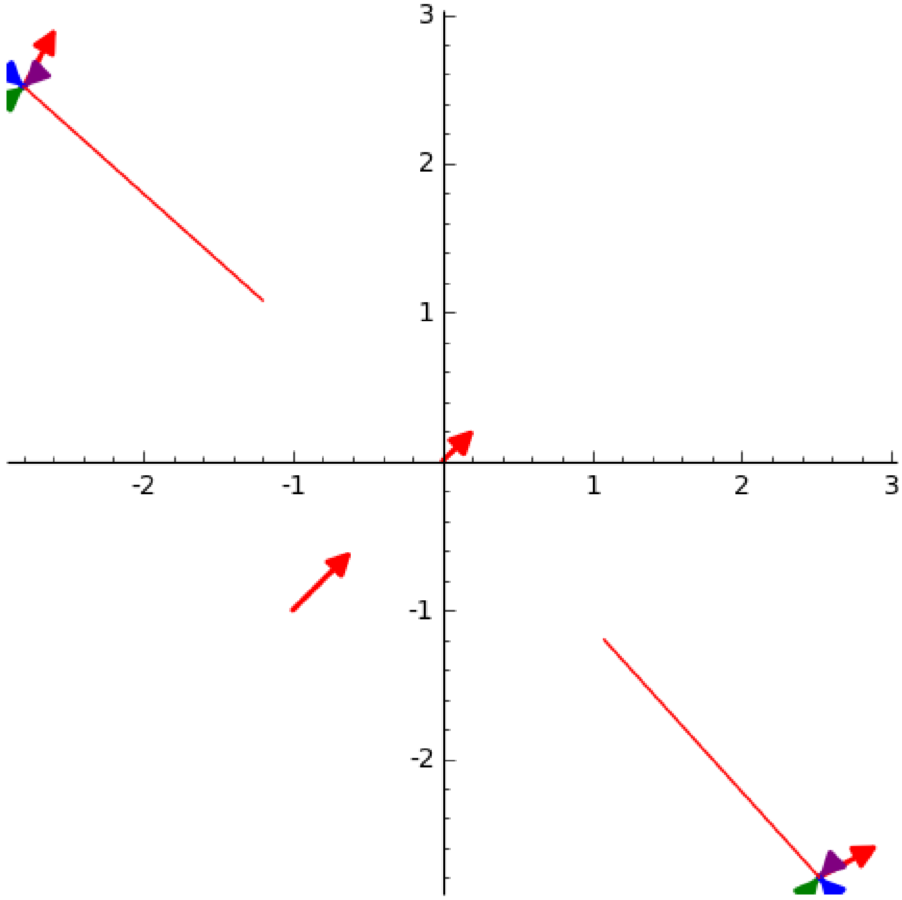
800
1000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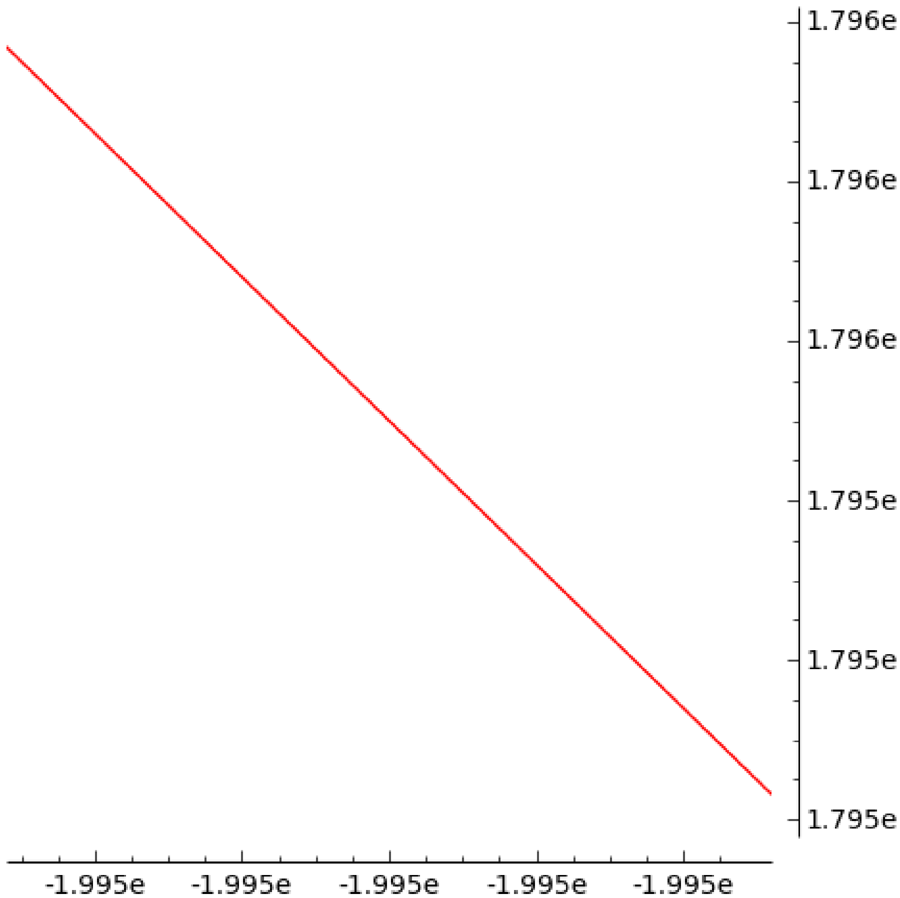
-1
1
2
3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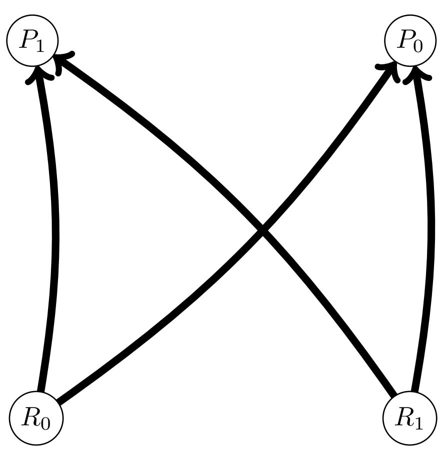
-1.995e


P_1
P_0
R_0
R_1

The foodweb model:

$$
\begin{aligned}
\frac{dX_{0R_0}}{dt} &= -X_{0P_0} X_{0R_0} (\cos(u_{0P_0} - u_{0R_0}) + 1) \\
&\quad - X_{0P_1} X_{0R_0} (\cos(u_{0P_1} - u_{0R_0}) + 1) - X_{0R_0}^2 - X_{0R_0} X_{0R_1} + X_{0R_0} \\
\frac{dX_{0P_1}}{dt} &= \frac{9}{10} X_{0P_1} X_{0R_0} (\cos(u_{0P_1} - u_{0R_0}) + 1) \\
&\quad + \frac{9}{10} X_{0P_1} X_{0R_1} (\cos(u_{0P_1} - u_{0R_1}) + 1) - X_{0P_1} \\
\frac{dX_{0P_0}}{dt} &= \frac{9}{10} X_{0P_0} X_{0R_0} (\cos(u_{0P_0} - u_{0R_0}) + 1) \\
&\quad + \frac{9}{10} X_{0P_0} X_{0R_1} (\cos(u_{0P_0} - u_{0R_1}) + 1) - X_{0P_0} \\
\frac{dX_{0R_1}}{dt} &= -X_{0P_0} X_{0R_1} (\cos(u_{0P_0} - u_{0R_1}) + 1) \\
&\quad - X_{0P_1} X_{0R_1} (\cos(u_{0P_1} - u_{0R_1}) + 1) - X_{0R_0} X_{0R_1} - X_{0R_1}^2 + X_{0R_1}
\end{aligned}
$$

Adaptive dynamics of model:

$$
\begin{aligned}
\frac{du_{0R_0}}{dt} &= -\left(\hat{X}_{0P_0} D[1](f)(u_{0P_0}, u_{1R_0}) + \hat{X}_{0P_1} D[1](f)(u_{0P_1}, u_{1R_0})\right) \hat{X}_{0R_0} \\
\frac{du_{0P_1}}{dt} &= \left(\hat{X}_{0R_0} k D[0](f)(u_{1P_1}, u_{0R_0}) + \hat{X}_{0R_1} k D[0](f)(u_{1P_1}, u_{0R_1})\right) \hat{X}_{0P_1} \\
\frac{du_{0P_0}}{dt} &= \left(\hat{X}_{0R_0} k D[0](f)(u_{1P_0}, u_{0R_0}) + \hat{X}_{0R_1} k D[0](f)(u_{1P_0}, u_{0R_1})\right) \hat{X}_{0P_0} \\
\frac{du_{0R_1}}{dt} &= -\left(\hat{X}_{0P_0} D[1](f)(u_{0P_0}, u_{1R_1}) + \hat{X}_{0P_1} D[1](f)(u_{0P_1}, u_{1R_1})\right) \hat{X}_{0R_1}
\end{aligned}
$$

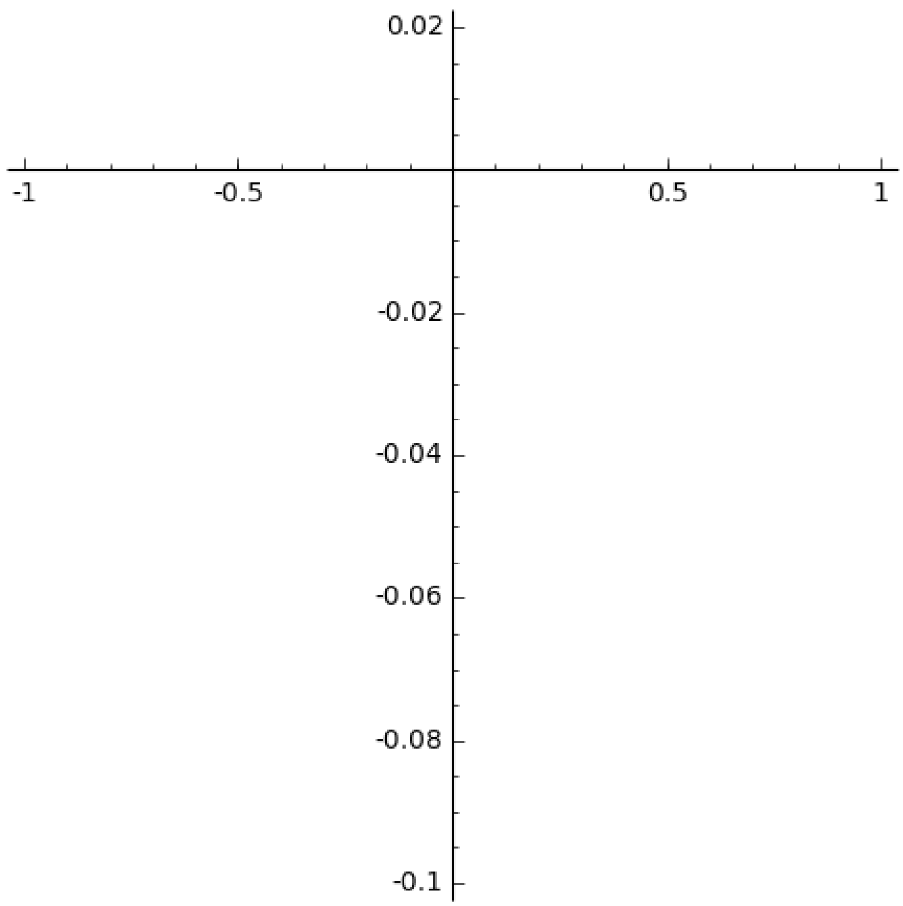
$$A\left(\hat{X}_{0R_0}, \hat{X}_{0P_1}\right) = (a_{0R_0 0P_1},\ a_{0P_1 0R_0})$$

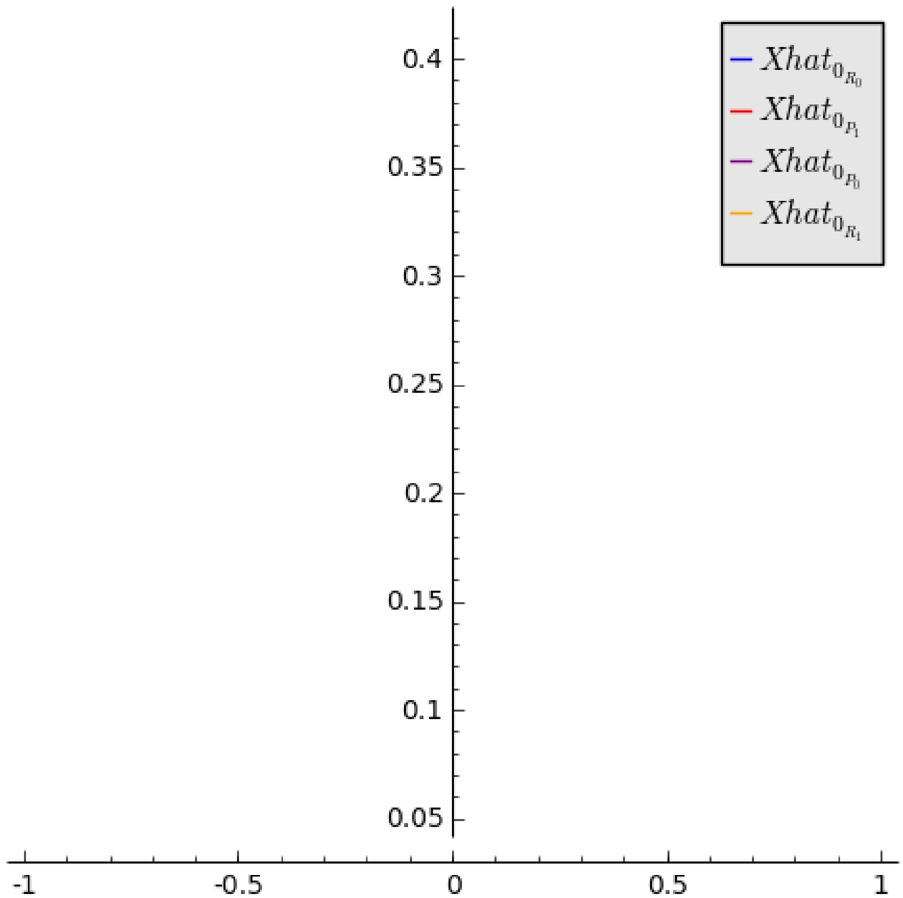
$$S\left(\hat{X}_{0R_0}, \hat{X}_{0P_1}\right) = \left(\hat{X}_{0P_1},\ \hat{X}_{0R_0}\right)$$

$$
\begin{aligned}
D\left(\hat{X}_{0R_0}, \hat{X}_{0P_1}\right) &= (-D[0](f)(u_{0R_0}(t), u_{0P_1}(t))\, D[0](u_{0R_0})(t),\ f(u_{0R_0}(t), u_{0R_0}(t))\, D[0](u_{0P_1})(t)) \\
&= \Big(\hat{X}_{0P_0} \hat{X}_{0R_0} \gamma D[0](f)(u_{0R_0}, u_{0P_0})\, D[0](f)(u_{0R_0}(t), u_{0P_1}(t)) + \hat{X}_{0P_1} \hat{X}_{0R_0} \gamma D[0](f)(u_{0R_0}, u_{0
\end{aligned}
$$

$$I\left(\hat{X}_{0R_0},\hat{X}_{0P_1}\right) = \left(-D[1]\left(f\right)\left(u_{0R_0}\left(t\right),u_{0P_1}\left(t\right)\right)D[0]\left(u_{0P_1}\right)\left(t\right),\ \left(D[0]\left(f\right)\left(u_{0R_0}\left(t\right),u_{0R_0}\left(t\right)\right)+D[1]\left(f\right)\left(u_{0R_0}\left(t\right.\right.\right.$$
$$= \left(-\hat{X}_{0P_1}\hat{X}_{0R_0}\gamma f\left(u_{0R_0},u_{0_{R_0}}\right)D[1]\left(f\right)\left(u_{0R_0}\left(t\right),u_{0P_1}\left(t\right)\right)-\hat{X}_{0P_1}\hat{X}_{0R_1}\gamma f\left(u_{0R_1},u_{0_{R_1}}\right)D[1]\left(f\right.\right.$$

$$\frac{dA}{dt}\left(\hat{X}_{0R_0},\hat{X}_{0P_1}\right) = \left(-D[0]\left(f\right)\left(u_{0P_1}\left(t\right),u_{0R_0}\left(t\right)\right)D[0]\left(u_{0P_1}\right)\left(t\right)-D[1]\left(f\right)\left(u_{0P_1}\left(t\right),u_{0R_0}\left(t\right)\right)D[0]\left(u_{0R_0}\right)\left(t\right),\right.$$
$$= \left(-\hat{X}_{0P_1}\hat{X}_{0R_0}\gamma f\left(u_{0R_0},u_{0_{R_0}}\right)D[0]\left(f\right)\left(u_{0P_1}\left(t\right),u_{0R_0}\left(t\right)\right)-\hat{X}_{0P_1}\hat{X}_{0R_1}\gamma f\left(u_{0R_1},u_{0_{R_1}}\right)D[0]\right.$$





$Xhat_{0_{R_0}}$
$Xhat_{0_{P_1}}$
$Xhat_{0_{P_0}}$
$Xhat_{0_{R_1}}$
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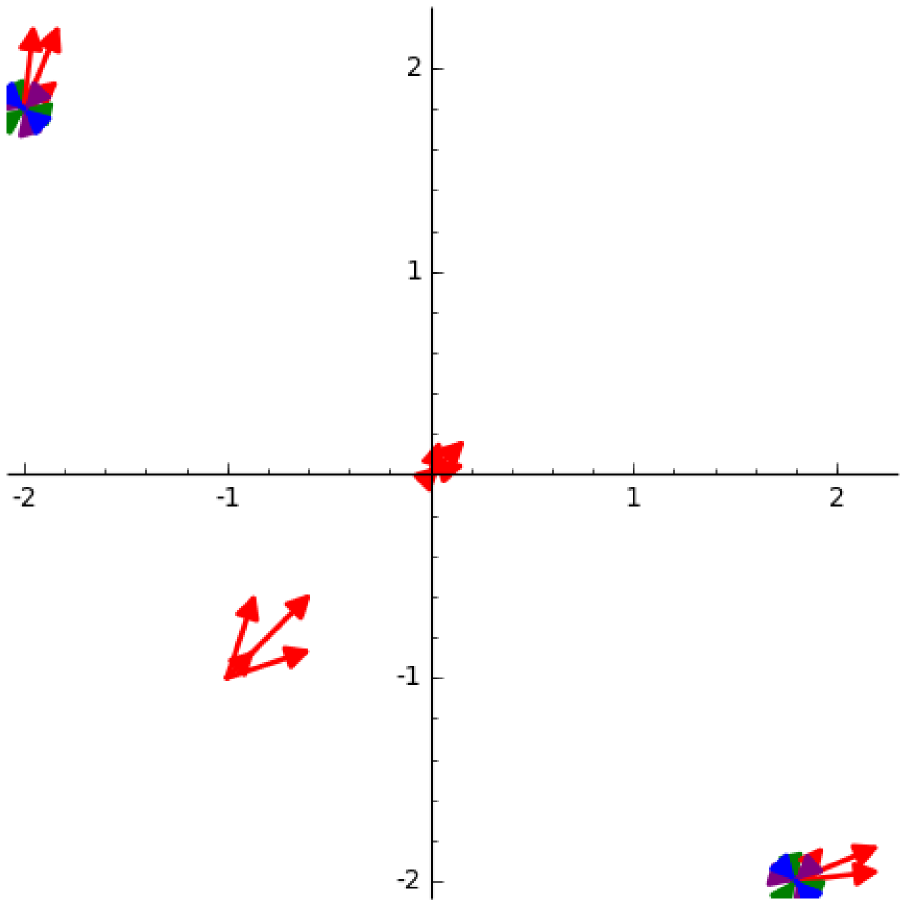
0.15
0.1
0.05
-1
-0.5
0
0.5
1



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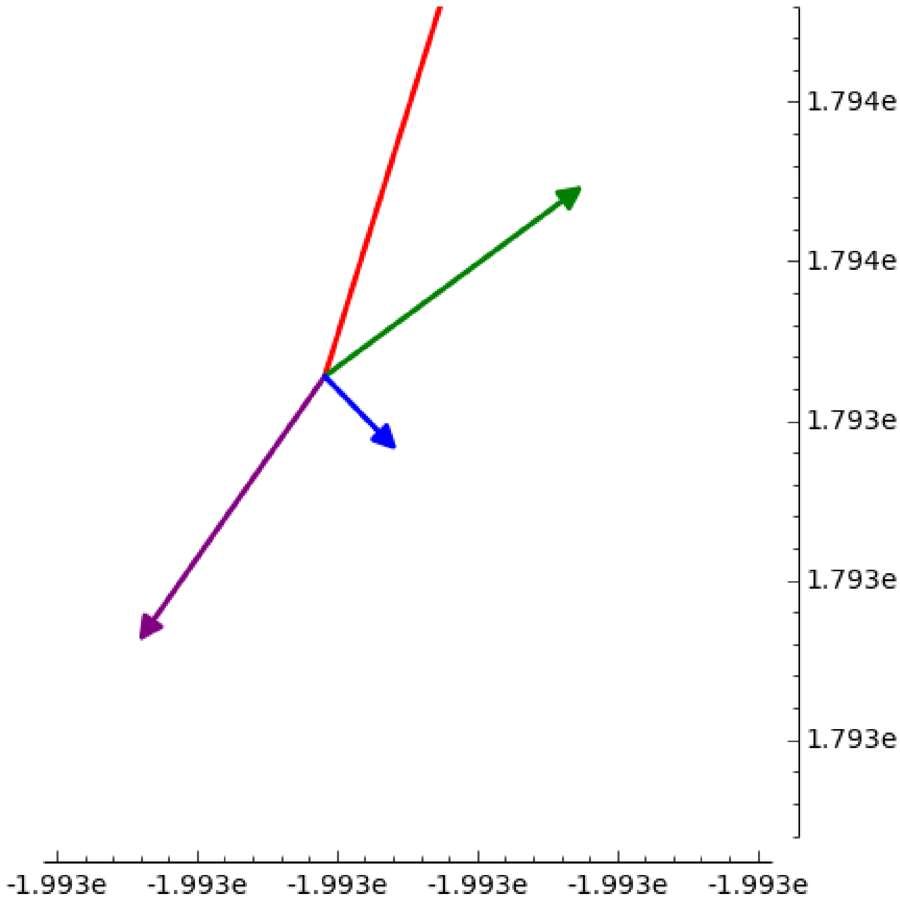
2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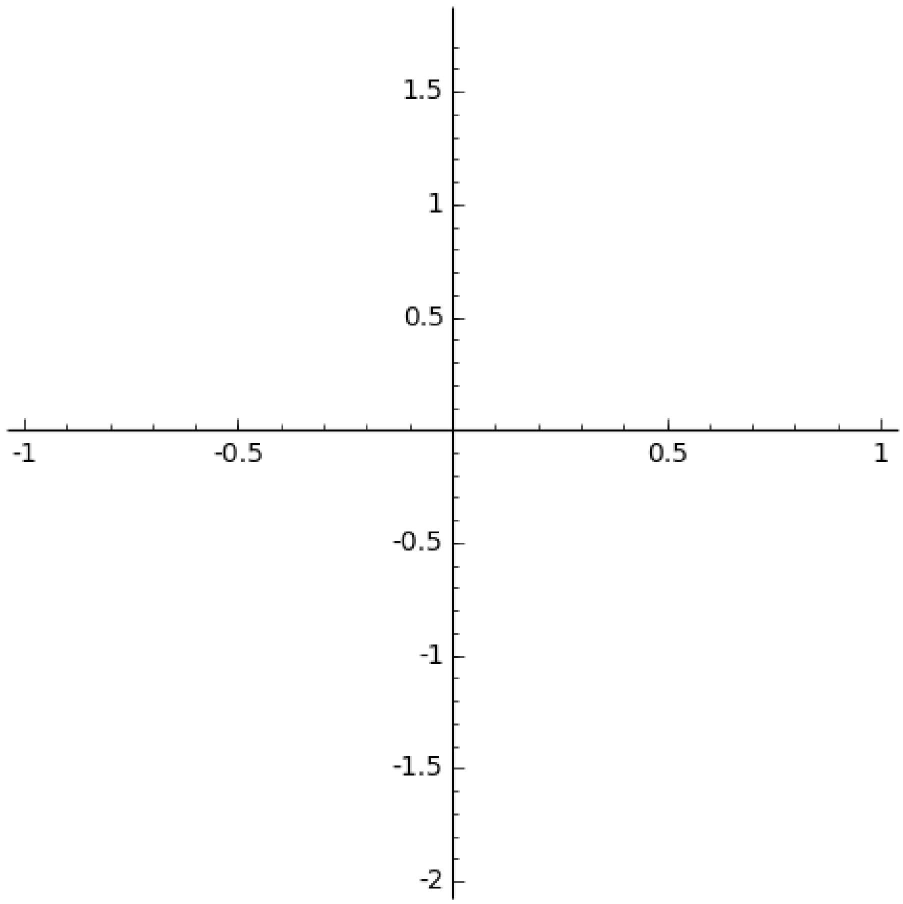
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