Solving Rubik's Cube with Lego Mindstorms

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Signed Declaration

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Abstract

This should be two or three short paragraphs (100-150 words total), summarising the report. A suggested flow is background, project aims, and achievements to date. It should not simply be a restatement of the original project outline

Acknowledgements

Thanks to my parents, who raised me since I was a boy. And Rik van Grol, who raised me afterwards.

Abbreviations, Definitions and Notations

Basic Definitions

Abbreviation/Word	Definition
Rubik's Cube Cube	A standard 3x3x3 Rubik's Cube[1].
Slice	A central layer between two faces. Usually referenced by the faces it spans.
Quarter Turn	A clockwise rotation of a face or slice by 90°
Half Turn	A clockwise rotation of a face or slice by 180°
Quarter Turn Metric	When counting moves, a Quarter Turn is counted as a single move and a half is two moves.
Half Turn Metric[2]	Quarter turns and half turns are both counted as single moves.
Cubie	One of the twenty-six smaller cubes that make up a Cube.
Goal State	All the cubies on a given face match the colour of the centre cubie. i.e. a solved Cube.
Position	A Cube's state (mixed or solved).
Valid Position	A position that can be achieved with a real-world Cube without dismantling it.
Move Sequence	A series of moves performed consecutively. e.g. $F\ D\ F'\ D2\ L'\ B'\ U\ L\ D\ R\ U\ L'\ F'\ U\ L\ U2$
Solve (Sequence)	A move sequence which leads to the goal state.
Depth n	A Cube which has been moved n times away from the goal state.

Notation

<table>
<tr><th>Symbol Notation</th><th>Meaning</th><th></th></tr>
<tr><td>L</td><td>Left</td><td rowspan="6">This notation is used to show a quarter turn of a face. It can have a single quote or a '2' appended to show a counter-clockwise quarter turn or a half turn respectively.</td></tr>
<tr><td>R</td><td>Right</td></tr>
<tr><td>F</td><td>Front</td></tr>
<tr><td>B</td><td>Back</td></tr>
<tr><td>U</td><td>Up (Top)</td></tr>
<tr><td>D</td><td>Down (Bottom)</td></tr>
<tr><td>X
Y
Z</td><td>Clockwise 90° rotation of a Cube about the relevant axis.</td><td></td></tr>
</table>

[1] This Dissertation is only dealing with 3x3x3 Rubiks Cubes, and any discussion of alternative dimensions will be explicitly stated.

[2] For this Dissertation the Half Turn Metric is used unless explicitly stated.

Contents

1 Introduction

1.1 Background

In 1974, Ernö Rubik was struggling to create a cube with independently moving parts which remain together, regardless of how much they moved. His first attempts made use of elastic, which broke and rendered the cube unusable. Rubik persevered in his attempts to hold the blocks (now called "cubies") together - eventually concluding that the best way was to have the cubes hold themselves together. He called this design "The Magic Cube", and it would go on to be one of the world's best-selling puzzles [1]. It was later re-branded to "*Rubik's Cube*" to overcome an oversight involving patenting and copyrighting the design.

In an unpublished manuscript [2], Rubik described first randomising his new cube,

> It was wonderful to see how, after only a few turns, the colors became mixed, apparently in random fashion. Like after a nice walk when you have seen many lovely sights you decide to go home, after a while I decided it was time to go home, let us put the cubes back in order. And it was at that moment that I came face to face with the Big Challenge: What is the way home?

It took Rubik over a month to solve this first cube - he knew intuitively that there must be a method to solving the cube, but lacked the finer methodology [3]. Since Rubik devised the first method, hobbyists and mathematicians alike have been immersed in solving the Cube as quickly and efficiently as possible. Whilst many solutions are markedly successful when it comes to optimisation, others only better them in quirkiness or internet fame [4].

There is one method which is mere speculation, despite having been proved mathematically: God's Algorithm. God's Algorithm states that an omniscient being would always make the most efficient moves and that they would be able to solve a Cube from any given position in a certain number of moves or less. This is referred to as God's Number, and was finally proved to be twenty in 2010 by a group of 4 researchers [5].

1.2 General Objectives

The primary objective of this project is to successfully implement an algorithm to solve a Cube with a Mindstorms robot. The efficiency of this algorithm is initially non-imperative, but will ideally be improved over the project time-line. The most recent iteration of the Mindstorms line, the Lego EV3 31313, will be used in the construction of the robot. This provides a wireless connectivity via Bluetooth (or WiFi with a USB dongle), three motors, a colour sensor, and a touch sensor amongst other peripherals. A custom operating system can also be installed on a microSD card to allow for greater expandability.

Secondary objectives include implementing other algorithms from various sources, devising and refining my own algorithms, and comparing the performance, efficiency and solve-length of the algorithm. The robot will have to be of sound construction, with little-to-no room for error when manipulating the Cube.

The objectives for this project are as follows:

1. Build a robot which can move a Cube to a sufficient degree of accuracy
2. Write a program which takes a move sequence as its input and moves a Cube to match that sequence
3. Implement a system which successfully generates a solve sequence for any given position
4. Ensure the runtime of the system is an acceptable length
5. Implement a program to use other algorithms to generate a solve sequence
6. Compare the performance of different algorithms, especially the difference between human-compatible and robot-compatible move sequences

1.3 Limitations and Constraints

The scope of this project is relatively local and therefore it lacks major limitations. Instead, it has several minor limitations which each present their own problems and solutions. The first problem which will be encountered will undoubtedly be during the design and build of the robot: the number of Lego pieces supplied in the EV3 set is quite limited at only six-hundred-and-one pieces - approximately a quarter of which are only for aesthetics. The design will be supplemented by sets sourced externally, thus allowing a robot of sufficient quality to be built.

Despite being of high quality and uniform across all sets, Lego is still fundamentally a toy - which will lead to errors with the precision of the robot[6]. An example of this is the motors: there is a somewhat significant degree of freedom/play in the motors, which means that they cant be relied on to move to accurate positions. A worm gear is the ideal solution to this problem: despite reducing the speed, it provides a considerable increase in motor accuracy.

One of the larger issues that this project will encounter is the run-time of the program to find a solution. Many programs have been known to take upwards of three days to find a solve sequence - this project currently aims to find a solution in approximately ten minutes or less. This will require extensive optimisation of search spaces and group theory through symmetrical elimination and set covering in order to reduce the necessary coverage of the search algorithm.

Finding the optimal solve sequence for any given position has been deliberately omitted from the objectives of this project: there are over forty-three quintillion valid positions of a Cube, each requiring an extensive amount of time to find the optimal solution for. Based on current hardware capabilities and previous studies and research, an estimated timespan for finding all optimal solutions is in the order of millennia[3].

[3] See cube20.org table of 20 move or less vs optimal and explain

List of Figures

List of Tables

References

[1] O. Waxman, "The 13 Most Influential Toys of All Time," 2014.

[2] E. Rubik, "The Perplexing Life of Erno Rubik," *Discover Magazine*, p. 81, mar 1986.

[3] Rubik's Cube, "Rubik's UK Website," 2017.

[4] C. Chan, "This Guy Can Solve 3 Different Rubik's Cube While Juggling Them at the Same Time," 2016.

[5] T. Rokicki, H. Kociemba, M. Davidson, and J. Dethridge, "cube20," 2010.

[6] R. T. Cook and S. Bacharach, *Lego and Philosophy: Constructing Reality Brick by Brick.* 2017.