Rubik's Cube in 44 Moves
by Hans Kloosterman

Introduction
As long as I have a *Magic Domino* I was fascinated about it, because:

- it is an (attractive) subset of the Cube
- it has a reasonable low number of different patterns (406425600) *and*
- it has only seven different moves (R^2, B^2, L^2, F^2, U, U^2 and U')

The last two arguments are attractive for finding, with the aid of a computer, a solution for the Magic Domino. Later on I used my results for the Magic Domino to develop a short solution (in terms of maximum number of moves) for the Rubik's Cube.

Magic Domino in 25 Moves
The shortest solution I knew for the Magic Domino was 30 moves. It is obtained by applying stages 3 and 4 of *M.B.Thistlethwaite's* well-known four-stage algorithm for solving the Cube, while taking account of the improvements found by students of *D.E.Knuth,* by which these stages 3 and 4 now each require 15 moves.

My algorithm for the Magic Domino uses the following two stages:
Stage 1
Put *U-edges and U-corners in the U-face*
Implicitly all D-edges and D-corners are then put in the D-face. This means that each face has its right colour. A few years ago I generated all different processes for this stage with the aid of a computer.
This stage requires at most 8 moves.
Stage 2
Restore the Magic Domino.
This means that we have to permute all cubies of the U-face and all cubies of the D-face as well. This year I generated all minimum solutions for this stage.
This stage requires at most 17 moves.

Rubik's Cube in 44 Moves

Thinking about a short solution for the Cube I observed that stages 3 and 4 of Thistlethwaite's algorithm were in the same class of moves as a Magic Domino solution. A Magic Domino solution is a subset of these stages 3 and 4. In case of the Cube we also have to restore the edges of the U-D-slice. A possible solution is to restore the permutation of the edges of the U-D-slice after restoring the U- and D- faces. Then we have the following number of moves for the solution of these stages: 8 + (17 + 4 + 2) = 31 moves.

The 8 is the length of stage 1 of my Magic Domino solution. The *17* is the pure length of the permutation of the U- and D-faces, the *4* is the length needed for restoring the edges of the U-D- slice and the *2* is for correction moves (U and D moves).

An attentive reader may notice that we need *6* moves to restore the permutation of the edges of the U-D- slice (process $R^2L^2UD'F^2B^2$). However we can start with all the four non U-D moves R^2, B^2, L^2 or F^2, so the first move of the process is always cancelled against the last move of stage 2 of the Magic Domino solution.

I was not satisfied with this solution. So I started to integrate the permutation of the edges from the U-D-slice in the stage in which the U-face and D-face are restored. Most of these processes were not difficult to find, because I didn't use D-moves in stage 2 of the Magic Domino. By replacing some U-moves by D-moves or U-D-moves it is possible to get different permutations of the edges in the U-D-slice, e.g. we can change $R^2UR^2U^2B^2UR^2$ in $R^2UR^2U^2B^2DB^2$ or $R^2UR^2U^2B^2D'U^2F^2$.

The algorithm now becomes:

<u>Stage 1</u>

Reduce group G0=⟨R, L, F, B, U, D⟩ *to group* G1=⟨R, L, F^2, B^2, U, D⟩.

This means that we have to orient all 12 edges. GOOD.

This stage requires at most **7 moves.**

<u>Stage 2</u>

Reduce group G0=⟨R, L, F^2, B^2, U, D⟩ *to group* G1=⟨R^2, L^2, F^2, B^2, U, D⟩.

This means that we have to orient all corners and put the edges from the U-D-slice in this slice. The moves used are in group **G1**, which means that the orientation of the edges doesn't change.

This stage requires at most **10 moves.**

Stage 3

Put all U-edges and U-corners in the U-face.

Implicitly, after this process all D-cubies are in the D-face and all U-D-slice cubies are in the U-D-slice. In this stage only moves from the group G2 are used.

This stage requires as most **8 moves**.

Stage 4

Restore the Cube.

In other words: permute the U-face, permute the D-face and permute the U-D-slice. In this stage only moves from the group G2 are used.

This stage requires at most **19 moves**.

Concluding Remarks

Because I reduced the number of moves for stage 2 of Thistlethwaite's algorithm from 13 to 10 moves, this algorithm now needs at most 47 moves. (I only looked at stages 1 and 2. I have not investigated stages 3 and 4 of this algorithm).

For my search process I used in the beginning an IBM 4.77 MHz 8088 machine (an 'XT'). Later on I used an IBM-compatible 10 MHz 80286 machine (an 'AT'). I used *Turbo Pascal* with a few machine language procedures to implement my program.

Hans Kloosterman

Editor's Note. We suppose that, like the editor, many of our readers are anxious to know more about the algorithms outlined above. Before providing all details, Hans Kloosterman prefers to work out further his system, which is still in a stage of lively development. Since we received the above article Hans has already improved his system, in such a way that now it is almost for sure that by means of his algorithm the Cube can be solved in at most **42 moves!**

In next issue of CFF we'll certainly come back to his achievements in much more detail.