

Solving Rubik's Cube with Lego Mindstorms

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I Signed Declaration

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II Abstract

This should be two or three short paragraphs (100-150 words total), summarising the report. A suggested flow is background, project aims, and achievements to date. It should not simply be a restatement of the original project outline

III Acknowledgements

Thanks to my parents, who raised me since I was a boy. And Rik van Grol, who raised me afterwards.

IV Definitions and Notations

Table IV.1: Terminology used in this document

Abbreviation/Word	Definition
[Rubik's] Cube	A standard 3x3x3 Rubik's Cube ¹ .
Face	One of the size faces on a Cube, Denoted by position or colour. e.g. U-face
Slice	A central layer between two faces. Usually referenced by the faces it spans.
Quarter Turn	A clockwise rotation of a face or slice by 90°
Half Turn	A clockwise rotation of a face or slice by 180°
Quarter Turn Metric	When counting moves, a Quarter Turn is counted as a single move and a half is two moves.
Half Turn Metric ²	Quarter turns and half turns are both counted as single moves.
Cubie	One of the twenty-six smaller cubes that make up a Cube.
Facelet	One of the fifty-four coloured squares on the faces of the Cube.
Goal State	All the cubies on a given face match the colour of the centre cubie. i.e. a solved Cube.
Position	A Cube's state (mixed or solved).
Moveset	A set of moves, denoted with curly braces. e.g. {U D' L ² }
Move Sequence	A series of moves performed consecutively , enclosed by colons . e.g. :F D F' D ² L' B' U L D R U L' F' U L U ² :
Solve Sequence	A move sequence which leads to the goal state.
Depth n	A Cube which has been moved n times away from the goal state.

¹This Dissertation is only dealing with 3x3x3 Rubik's Cubes, and any discussion of alternative dimensions will be explicitly stated.

²For this Dissertation the Half Turn Metric is used unless explicitly stated.

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1 Introduction

Like after a nice walk when you have seen many lovely sights you decide to go home, after a while I decided it was time to go home, let us put the cubes back in order. And it was at that moment that I came face to face with the Big Challenge: What is the way home?

Ernö Rubik [1]

1.1 Background

In 1974, Ernö Rubik was struggling to create a cube with independently moving parts which remain together, regardless of how much they moved. His first attempts made use of elastic, which broke and rendered the cube unusable. Rubik persevered in his attempts to hold the blocks (now called ‘cubies’) together - eventually concluding that the best way was to have the cubes hold themselves together. He called this design ‘*The Magic Cube*’, and it would go on to be one of the world’s best-selling puzzles [2]. It was later re-branded to ‘*Rubik’s Cube*’ to overcome an oversight involving patenting and copyrighting the design.

In an unpublished manuscript [1], Rubik described first randomising his new cube,

It was wonderful to see how, after only a few turns, the colors became mixed, apparently in random fashion. Like after a nice walk when you have seen many lovely sights you decide to go home, after a while I decided it was time to go home, let us put the cubes back in order. And it was at that moment that I came face to face with the Big Challenge: What is the way home?

It took Rubik over a month to solve this first cube - he knew intuitively that there must be a method to solving the cube, but lacked the finer methodology [3]. Since Rubik devised the first method, hobbyists and mathematicians alike have been immersed in solving the Cube as quickly and efficiently as possible. Whilst many solutions are markedly successful when it comes to optimisation, others only better them in quirkiness or internet fame [4].

There is one method which, despite having been proved mathematically, is still little more than speculation: God’s Algorithm. The theory behind this algorithm states that an omniscient being would always make the most efficient moves and that they would be able to solve a Cube from any given position in a certain number of moves or less. This is referred to as God’s Number, and was finally proved to be twenty in 2010 by a group of four researchers [5].

1.2 General Objectives

The primary objective of this project is to successfully implement an algorithm to solve a Cube with a Mindstorms robot. The efficiency of this algorithm is initially non-imperative, but will ideally be improved over the project time-line. The most recent iteration of the Mindstorms line, the Lego EV3 31313, will be used in the construction of the robot. This provides a wireless connectivity via Bluetooth (or WiFi with a USB dongle), three motors, a colour sensor, and a touch sensor amongst other peripherals. A custom operating system can also be installed on a microSD card to allow for greater expandability.

Secondary objectives include implementing other algorithms from various sources, devising and refining my own algorithms, and comparing the performance, efficiency and solve-length of the algorithm. The robot will have to be of sound construction, with little-to-no room for error when manipulating the Cube.

1.3 Limitations and Constraints

The first problem which will be encountered will undoubtedly be during the design and build of the robot: the number of Lego pieces supplied in the EV3 set is quite limited at only six-hundred-and-one pieces - approximately a quarter of which are only for aesthetics. The design will be supplemented by sets sourced externally, thus allowing a robot of sufficient quality to be built.

Despite being of high quality and uniform across all sets, Lego is still fundamentally a toy - which will lead to errors with the precision of the robot [6]. An example of this is the motors: there is a somewhat significant degree of freedom/play in the motors, which means that they can't be relied on to move to accurate positions. A worm gear is the ideal solution to this problem: despite reducing the speed, it provides a considerable increase in motor accuracy.

One of the larger issues that this project will encounter is the run-time of the program to find a solution. Many programs have been known to take upwards of three days to find a solve sequence - this project currently aims to find a solution in approximately fifteen minutes or less (an arbitrary number determined only by the attention span of the author). This will require extensive optimisation of search spaces and group theory through symmetrical elimination and set covering in order to reduce the necessary coverage of the search algorithm.

Finding the optimal solve sequence for any given position has been deliberately omitted from the objectives of this project: there are over forty-three quintillion valid positions of a Cube, each requiring an extensive amount of time to find the optimal solution for. Based on current hardware capabilities and previous studies and research, an estimated timespan for finding all optimal solutions is in the order of millennia¹.

¹This is an estimation based on data presented at cube20.org which states that finding optimal solutions takes 11,800 times as long as finding sequences of twenty moves or less [7]

2 Literature Review

Please forgive me, but to give birth to a machine is wonderful progress. It's more convenient and it's quicker, and everything that's quicker means progress. Nature had no notion of the modern rate of work. From a technical point of view, the whole of childhood is quite pointless.

Mr. Fabry, in Karel Čapek's '*Rossum's Universal Robots*' [8]

2.1 Robotics

The term robot was first brought to the collective consciousness of the general public by Karel Čapek in 1921 in his play '*Rossum's Universal Robots*' [8]. It comes from the Czech for 'forced labour' and was coined by Čapek's brother, Josef, for a short story written some years earlier [9]. One of the main discussions in the field of robotics has been about ethics and morality. This discussion was started in Čapek's play, when a visiting scientist tries to destroy the robot-manufacturing business, in the hope of giving the robots a soul and making them happier. This plan goes awry when, as is the norm in fiction surrounding Artificial Intelligence (AI), the robots become sentient and start to overthrow the humans that created them. Twenty-one years later, the novelette '*Runaround*' by Isaac Asimov was featured in the magazine '*Astounding Science-Fiction*' [10]. It included his now-prominent Three Laws of Robotics, which were a turning point in the field of robotics and meta-ethics.

The twentieth century generated many ideas about the future of robotics, such as 'Kitchen units will be devised that will prepare "automeals", heating water and converting it to coffee' alongside grandiose aspirations such as 'An experimental fusion-power plant or two will already exist in 2014' [11]. Whilst some predictions have been fulfilled by the technology of the past two decades, others are still nowhere near completion and could be considered impossible for the next century. This is a clear demonstration that the field of Robotics has made vast improvements, but is still - in some respects - very much in its youth.

In the same way as the field of Robotics, AI has vast potential and we have only touched the tip of the iceberg. The most advanced commercial AI systems are intended to make people's lives easier through organisation and saving precious seconds in performing simple tasks on their smartphones and more recently in their homes. However, Google's Assistant, Apple's Siri, and Amazon's Alexa still fall short when it comes to the intelligence demonstrated in Asimov's book '*I, Robot*'. We are still far from being unable to 'differentiate between a robot and the very best of humans' [12].

2.2 Robotics in Education

There has always been a clear use of 'edutainment' style teaching in one form or another for hundreds of years. One of the earliest known uses of edutainment is in '*Poor Richard's Almanack*', which was written by Benjamin Franklin and published annually between 1732 and 1758. Alongside all the usual information contained in an Almanac, Franklin (under the pseudonym '*Poor Richard*') included maths exercises, puzzles, and aphorisms [13], [14]. Walt Disney was another pioneer of edutainment in the time immediately before, during, and following the World Wars. His first piece of edutainment was a short film, '*Tommy Tucker's Tooth*', which was commissioned by a dental institute in 1922.

In the past few decades, there has been a marked change in the form of edutainment in line with advances in technology. We have progressed from paper-based, so called ‘serious games’ in the 1970s, to early-era video games such as the infamous Oregon Trail; and then in the past decade electronic and robotic kits have made their way into classrooms.

Robotics has been described as ‘the most effective way of motivating and supporting the study of many areas’, and has also been shown to aid the development of social and teamwork skills [15]. In the past decade or so, robots and electronic kits have been a common extra-curricular activity and are slowly being used as teaching aids and for practical sessions in actual lessons.

One of the leaders of modern edutainment is the Lego Mindstorms kit. The Lego Mindstorms kit has undergone two major revisions since the launch of the first generation of the Mindstorms kit in 1998 (the RCX): in July 2006, the second-generation NXT was released, updating the sensors and adding Bluetooth amongst other improvements; then in September 2013, Lego released the EV3 Home and Education sets. This release cemented Lego’s position at the forefront of edutainment [16].

2.3 Lego Mindstorms

A study at the University of Calabria in 2009 looked at how Lego Mindstorms worked as a learning tool when used in team-building type tasks [17]. Twenty-eight students were divided into six groups, each given a Lego robot and preliminary training which covered the Lego Mindstorms and programming environment. The groups were all given the same task, and their progress studied throughout the sessions. When studying the building and programming as two separate sub-tasks, three main categories of work subdivision were found: each group member had a set role in building, but all shared the programming; both sub-tasks were equally divided amongst members; each member had a set role in either sub-task, and there was a clear leader. This closely follows real-world teamwork mechanics and scenarios, and the use of robots was found to stimulate the student into exploring and sharing critical knowledge within the group. The study’s conclusion was that Lego Mindstorms - and inherently other variable morphology robots - stimulates further process analysis, information selection, and to observe and experiment with the consequences of their actions [17].

The Lego Mindstorms’ native programming environment is makes use of drag-and-drop code blocks to form programs which can run natively on the main ‘brick’. This provides a simplistic user experience and allows novice programmers to create complex programs with relative ease. Drag-and-drop programming can drastically reduce the amount of syntax errors in a program, allowing programmers to better focus on the functionality of their program [18].

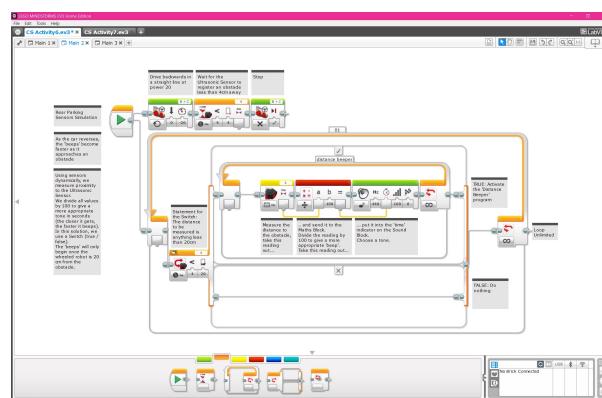


Figure 2.1: A sample program in ‘EV3 Programming Software’

2.4 Solving Puzzles with Algorithms

When it comes to playing games and solving puzzles, Google’s ‘AlphaGo Zero’ is the clear winner. Whilst Google’s first Go-playing program defeated the world champion at Go, it took months of supervised

learning from experts and reinforcement learning from self-play. The new version started *tabula rasa* and was completely self-taught - it had no interaction with any humans, and used no historical data. Through reinforced learning alone by simulating games against itself, it took three days to reach the level of AlphaGo and in forty days had surpassed any Go player's performance - human or artificial [19], [20].

When a computer solves a Cube (or any other similar puzzle), it does so by following set algorithms and rules until the solved state is achieved. Computers cannot use humanlike instincts, so we often try to provide a heuristic to make the task simpler. These heuristics can be pre-computed data tables to reduce the amount of processing at run time, or a reduction in the original data set by eliminating certain elements according to a set of rules.

In his 1997 paper on the use of pattern databases to increase solve efficiency, Richard Korf uses different heuristic functions and characterises how effective they are in reducing the number of moves in a solve sequence. His first method used an Iterative-Deepening A* algorithm, combined with the heuristic of the Manhattan distances from the edge cubies' current position and orientation to that which is desired. This reduced the time required to solve a Cube at depth-14 to three days¹, however when increased to depth-18 the time increased exponentially to two hundred and fifty years. After a re-evaluation, Korf modified the heuristic by pre-computing the Manhattan distance of each cubie from all of its possible positions and orientations and storing the table in memory at run-time. This created a table of nearly ninety-million entries - which would have been bigger but was restricted by the available memory (the table was approximately forty-two megabytes). The newer heuristic reduced the time to search at depth-18 to less than four weeks - a reduction of approximately 99.97%. Korf suggested that the speed of the algorithm used would increase linearly with the amount of memory available, meaning that if it were run today the depth-18 search would take under three hours² [21].

Table 2.1: Morwen Thistlethwaite's five groups for his algorithm [22]

Group Denotation	Mathematical Representation	Summary
G_0	$\{L\ R\ F\ B\ U\ D\}$	All valid positions
G_1	$\{L\ R\ F\ B\ U^2\ D^2\}$	All positions that can be reached with quarter turns of the L R F B faces and half turns of the U D faces
G_2	$\{L\ R\ F^2\ B^2\ U^2\ D^2\}$	Positions reachable from quarter turns of L R faces and half turns of the F B U D faces
G_3	$\{L^2\ R^2\ F^2\ B^2\ U^2\ D^2\}$	Positions only reachable from half turns of any face
G_4	$\{I\}$	The goal state

There have been many historical landmark algorithms since the inception of Rubik's Cube in 1974. One of the earliest instances was created by Morwen Thistlethwaite circa 1981 and is based on group theory. Thistlethwaite's algorithm splits all possible positions of the cube into five groups of decreasing size as seen in Table 2.1. David Singmaster, Thistlethwaite's colleague at London South Bank University (then called Polytechnic of the South Bank), describes the methodology of the algorithm: 'Once in G_i , one only uses moves in G_i to get into G_{i+1} . The ratio $\frac{|G_i|}{|G_{i+1}|}$ is called the index of G_{i+1} in G_i .' Using this algorithm, Thistlethwaite proved that the maximum number of moves required to solve any valid position is fifty-two [22]. This was the first development of God's Number, so named because it is the number of moves that an omniscient being would use to solve a Cube in the most efficient manner without failure.

¹The simulation was run on a Sun Ultra-Sparc Model 1 workstation

²This is based on the memory capacity of the Model 1 Workstation and a modern computer being 64MB and 16GB respectively.

2.5 God's Number

The lower bound for God's Number was originally recognised to be eighteen moves through analysis of the number of sequences of seventeen moves or fewer, and the discovery that there were more Cube positions than seventeen-moves-or-fewer sequences. In 1995, the lower bound was increased to twenty by Michael Reid upon discovering that the 'Superflip' position requires twenty moves to solve. He included his findings in an email to the Cube-Lovers mailing list, 'superflip is now known to require 20 face turns . . . this is the first improvement to the lower bound... given by a simple counting argument' [23].

The next refinement to the upper bound of God's Number came from Dutch mathematician Hans Kloosterman in December of 1989. Kloosterman's findings were published in a newsletter '*Cubism for Fun*' (CFF) as an article titled '*Rubik's Cube in 44 Moves*'. In this article Kloosterman discusses his existing solution to the Magic Domino, a subset of Rubik's Cube, and how he has adapted it into a more efficient Cube solution with the help of Thistlethwaite's algorithm. Kloosterman reduced the estimation of God's Number by adapting Thistlethwaite's G_3 into a subgroup of G_2 where all of the U -cubies are on the U face and all the D -cubies are similarly on the D face. This reduces the index $\frac{|G_2|}{|G_3|}$ from 15 to 8 (whilst also reducing the $(G_1 : G_2)$ index by 3 and increasing the $(G_3 : G_4)$ index by 1), creating a net decrease of 8 moves when compared to Thistlethwaite's algorithm. '*Rubik's Cube in 44 Moves*' ended with an editor's note stating that not all of the details of the algorithm were certain and more (including a more efficient algorithm) would be revealed in a later issue [24].

Sure enough, a year later Kloosterman wrote an article titled '*Rubik's Cube in 42 Moves*' in the twenty-fifth issue of CFF. This stated that the new algorithm was in fact the final version of Kloosterman's forty-four move algorithm. In the same way as its predecessor, this article is split into four stages to solve a Cube. The first three stages are based on 'solving' a Cube with only certain cubies coloured - the positions of the other remain irrelevant until a later stage. The final stage is solving the Cube in a non-particular manner, and is calculated to take no more than eighteen moves [25].

In the following twenty years, the upper bound was gradually refined to match the lower bound - meaning that God's Number is exactly twenty. The final development came from a group of four Rubik's Cube enthusiasts in July of 2010, and was only achieved through a brute force method: the four-man team first took every possible position of a Cube and reduced it from forty-three quintillion positions down to just over one quintillion - still a vast number, but a reduction by a factor of forty-three - by using rules of symmetry and mirroring. They then wrote a program to find solve sequences of length twenty or less, and ran it on a 'large number of computers at Google' to find all solve sequences in a few weeks. The parallel computation took the equivalent of thirty-five years [5].

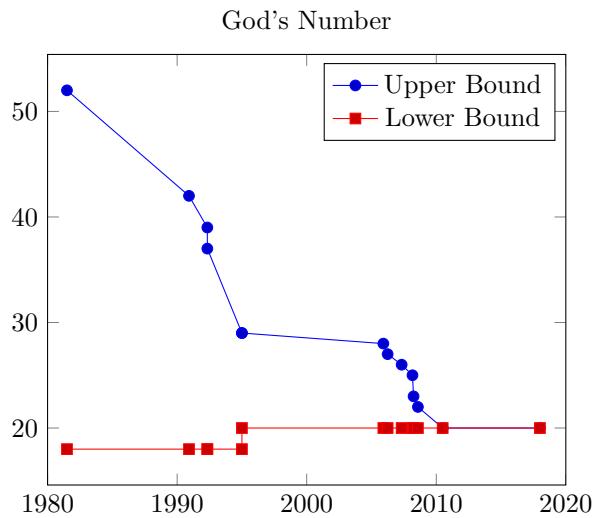


Figure 2.2: The upper bound was decreased to 20 over the course of twenty-nine years and across thirteen separate studies [5]

2.6 Python as a Scientific Scripting Language

Traditionally when performing large calculations and processes, compiled languages (C, C++, Fortran etc.) are the preferred choice over scripting languages such as Ruby, JavaScript, (pure) Python, etcetera. This is because compilation is done before run-time, allowing the program to run unhindered by on-the-fly translation/interpretation [26].

For general purpose applications, scripting languages are preferable because they allow small changes to be made without re-compiling large classes before testing. In the thirteenth issue of '*Scientific Programming*', Cai, Langtangen and Moe [26] explain how MATLAB is often a preferable choice for less intensive scientific computation due to its many features, such as: integrated simulation and visualisation tools; clear syntax; immediate feedback of commands; impressive documentation. They describe the use of MATLAB in computational science as paradoxical, as it is a scripting language which inherently is interpreted at run-time.

One of MATLAB's biggest selling points, the IDE with integrated support for matrices, graphs and other mathematical features, can be replicated in a few Python IDEs or with the addition of Python packages. Python's main core can be expanded massively by using any of the tens of thousands of available packages, which can provide any of MATLAB's advantages alongside extra flexibility and capabilities. One of the most popular packages, '*Numerical Python*' (NumPy), adds support for most scientific calculations. It expands Python's inbuilt data structures to use multi-dimensional homogeneous arrays of any Python data type. NumPy takes Python from a well-structured general-purpose language to a powerful mathematical language which can be applied to many different tasks [26], [27].

As well as the multitude of available packages, Python has another major tool in its arsenal: it can be extended with a compiled language by design. This means that processor-intensive tasks like nested for loops can be migrated to a compiled language such as C++ or Fortran, increasing run speed up to a factor of ten. This extension process is now a trivial matter thanks to automated tools such as '*f2py*', which allows for automated calls to compiled Fortran code [27].

2.7 Conclusion

Lego Mindstorms is a strong candidate for the construction of the robot: its variable morphology allows adaptation to a range of tasks; the uniformity between pieces means that the main EV3 kit can be supplemented with other kits to extend its ability; and previous studies have shown that it is ideal for stimulating analysis and observations. Instead of using a single algorithm to solve the Cube, a range will be implemented in order to find the strongest method for finding a solve sequence. This will allow cross-comparison between, and analysis of, individual algorithms with the intention of finding the algorithm best suited to this project. Tree and group based algorithms alike will be implemented.

MATLAB is incompatible with the Lego EV3, so will not be used for this project. Python's wide compatibility, clear high-level syntax, and extendability to compiled code for more complex calculations, makes it the ideal language for this project. The main program will be written in Python, with any intensive processing written in C++ to improve the speed of the algorithms.

3 Requirements and Analysis

I have yet to see any problem, however complicated, which when you looked at in the right way, did not become still more complicated.

Paul Anderson, New Scientist [28]

3.1 Introduction

This chapter covers the specific requirements needed for the project to succeed: the particular tasks to complete the project; the hardware required to build the robot and run the solving program; and the software needed to write and run the program. As well as the requirements, an in-depth analysis of the project and its goals is made, discussing their feasibility, difficulty, and how necessary they are for the success of this project.

3.2 Specific Aims and Objectives

1. Build a robot which can manipulate a Cube accurately

This is the first of the two primary objectives for this project, and is imperative to the project's success. As such it will be the first objective to be completed. This task, despite being of a medium difficulty, is fairly straightforward by nature: the robot simply needs to manipulate a Cube in any valid move.

2. Implement a system which successfully generates a solve sequence for any given position.

The second primary objective ensures that any of the forty-three quintillion positions of Rubik's Cube can be solved correctly. The time taken to generate the solve sequence and the actual length of the solve sequence will both be used when measuring an algorithm's success.

3. When the robot is provided with a move sequence...

- (a) Convert any input sequence to be robot-compatible

All of the pre-existing algorithms return a solution which a human being can follow to solve a real-world Cube. In the context of this project the manipulation is done by the robot, which cannot perform all of the standard moves, so any sequences provided must be translated to one which the robot can follow.

- (b) Correctly follow the sequence to achieve the intended output

Once a robot-compatible move sequence has been generated, the robot must move the Cube in this exact sequence with no errors or move-failures. If a single move is performed incorrectly then the entire move sequence will produce an incorrect output.

4. Ensure the runtime of the system is an acceptable length

Previous solve sequence generator algorithms have had a run-time in the order of weeks: the desired run-time of the generation by this project will be in the order of minutes. This will require careful analysis of the trade off between generator efficiency and solve sequence length.

5. Implement a program to use a range of algorithms to generate a solve sequence

In order to effectively compare the performance of several algorithms, there must be a method of running them under the same conditions and with the same overheads (e.g. transmission time to the robot). For this reason, there will be one main program with an option for the algorithm to use.

6. Compare the performance of different algorithms, especially the difference between human and robot compatible move sequences

The performance of each algorithm will be tracked and compared fairly to show each one's merits and faults. This will allow the choice of one algorithm to be improved to create the most efficient method - or the potential combination of algorithms to form a stronger one.

3.3 Analysis

3.3.1 Software

PyCharm/CLion

An Integrated Development Environment (IDE) makes development, running programs, and testing and debugging much easier than using a simple text editor and command line. PyCharm and CLion by JetBrains [29] will be used for developing in Python and C++ respectively. PyCharm and CLion provide strong code completion abilities, project-wide refactoring, and software development kit (SDK) modification amongst many other abilities. This will allow more time to be allocated to tasks of higher value rather than struggling with relatively minor issues.

Python 3.6 Runtime Environment

As discussed in the Literature Review, Python is the ideal language for this project. It requires a native runtime environment for programs to be run: this consists of the Python interpreter, libraries, and any packages used in the development process.

MinGW

C++ requires an environment to develop and compile in. MinGW (Minimalist GNU for Windows) is a popular choice when developing C++ applications.

ev3dev

ev3dev [30] is a custom operating system (OS) for the EV3 which contains a low-level framework for the peripherals of an EV3. It allows users to write their own programs in a multitude of languages (including Python) to create complex functions and models. It is installed by flashing the ev3dev OS to a microSD card which is then inserted into the EV3, avoiding affecting the EV3's original firmware.

A Python wrapper is available on GitHub [31] for the ev3dev OS, allowing the creation of Python programs with packaged support for the actuators and sensors of the EV3.

3.3.2 Software Requirements

This table shows the requirements of the software to be used in the creation of this project, and the capabilities of the computer that they will be running on. Any

Table 3.1: Software requirements for the main development computer and its capabilities

Software	Requirements/Capabilities			
	Minimum OS	Processor	Memory	
Connectivity				
PyCharm	Windows 2003 (or newer)		1GB minimum 2GB recommended	
CLion	Windows 7 64-bit (or newer)		2GB minimum	
Lego EV3 Programming Software[32]	Windows 2003 (or newer)	Dual Core 2GHz	2GB	1 USB Port
Python 3.6	Windows Vista (or newer)			
MinGW				
<i>Main Development Computer</i>	<i>Windows 10 Pro 64-bit</i>	<i>Quad Core i5 6500 3.6GHz</i>	<i>32GB</i>	<i>WiFi, Bluetooth, 12 × USB Port</i>

3.4 Feasibility Study

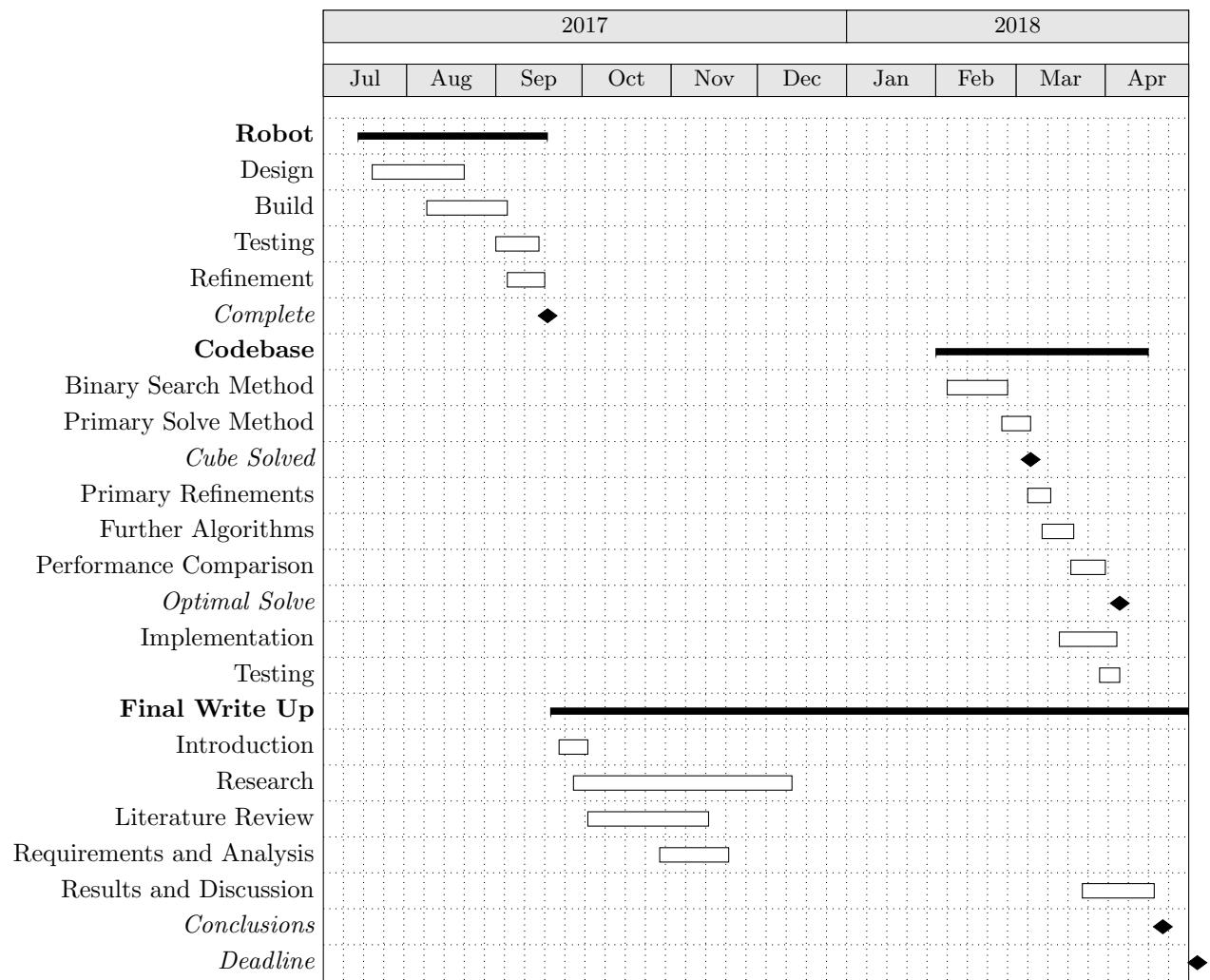
3.4.1 Optimal Solutions

3.4.2 Tree Based Methods

3.4.3 Group Based Methods

3.5 Evaluative Techniques

The progress of this project is estimated to follow the below Gantt Chart. For the duration of its lifespan, there will be a continual measurement of the actual progress against predicted.



4 Design, Implementation and Testing

There are two ways of constructing a software design: One way is to make it so simple that there are obviously no deficiencies, and the other way is to make it so complicated that there are no obvious deficiencies. The first method is far more difficult.

C.A.R. Hoare[33]

4.1 Introduction

The nature of this project lends itself to a spiral design cycle, where both the robot and the solver can be designed, implemented, tested and then optimised to re-start the cycle. This non-linearity of the Design → Implementation → Testing process means that the structure of this dissertation must follow the process. Consequentially this chapter's structure is split into two main parts - 'Robot' and 'Codebase' - each with introductory sections for fundamental components that are invariant through the different designs, and further sections to discuss each design.

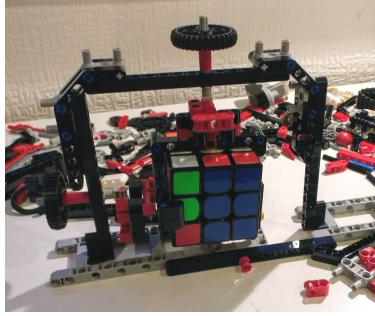
4.2 Robot

4.2.1 Sensors and Actuators

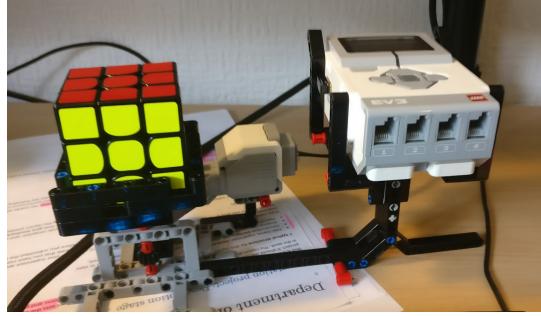
The Lego EV3 comes with three DC motors (two large (#45502) and one medium (#45503)), a colour sensor and a touch sensor amongst other peripherals. The medium motor will be used to move the colour sensor to scan the Cube, and the large motors to rotate the Cube about the horizontal and vertical axes respectively. The quantity of the motors will be the largest restriction in designing the robot: it will only be able to rotate a Cube about two of the three axes; and the colour sensor will only have one degree of freedom so will rely on the rotation of the Cube to scan all of the facelets. The touch sensor can be used to allow limited user input to the robot at runtime, such as an emergency stop if the Cube becomes dislodged, or to confirm that the Cube is in place.

4.2.2 Prototyping with Lego

The first step in designing the robot was to prototype some simple designs for holding a Cube and successfully manipulating it. The first concept design was to grip each face of the Cube, and use a large motor to power the 'grippers'. The second large motor could control which gripper was being powered at any time. This would allow a strong manipulation of the Cube and mean no delay between the movement of different faces. Figure 4.1a shows the initial prototype that was built to fit this design. Once it was built, it became apparent that it was actually an impossible design (or at least, impossible with Lego) because if one of the grippers rotated by a quarter turn, it would block two of the other faces from turning. This is shown in Figure 4.1a, where the upper gripper is blocking the right-hand one from turning. The solution to this would be to have the grippers retract when they are not in use, however this would be very difficult if not impossible to achieve with the available Lego pieces.



(a) The first prototype



(b) The model used to test the motor's torque

Figure 4.1: A cross-section of the cradle and Cube

One of the other problems with the first prototype was that the Cube was unbalanced and would often fall out of the robot. This led to the design to have a ‘cradle’ for the Cube to sit in. This is a good support mechanism for the Cube, and also handle the rotation about the Y axis. Once the cradle was built, it was a good milestone to check that the large motors had enough torque to rotate the face of a Cube against the friction between faces and slices. The motor was set to rotate indefinitely with the Cube in the cradle (as seen in Figure 4.1b) and the L-R-slice manually held in place. There was no visible reduction in the rotational velocity of the motor, so this first small test was passed.

Although the cradle was a good solution to the problem outlined above, it also meant that only the D-face could be rotated, therefore requiring a mechanism to ‘flip’ the Cube in place so different faces could sit in the cradle. The most obvious design for this was to apply a force at the upper corner to create a resultant force to pivot about the wall of the cradle. This is shown by the free body diagram in Figure 4.2: the force F will be applied by a mechanism powered by the second large motor; this will be combined with the resistive force R of the cradle wall; the resultant force (shown in red) will be about the pivot point (also in red) to rotate the Cube. Once the Cube is partially out of the cradle, it will slide back into place due to the height difference between the wall and the base of the cradle, and the low friction coefficient between the plastic of the Cube and the Lego pieces.

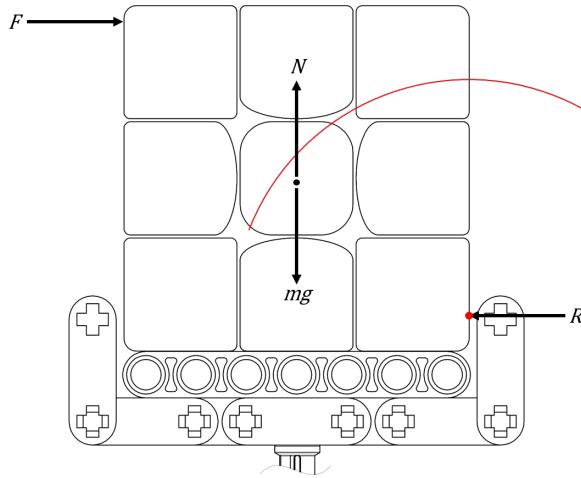


Figure 4.2: A free body diagram showing how the Cube will be rotated

4.2.3 Cube Manipulation

Following the construction and analysis of small Lego prototypes, it became apparent that there would be three fundamental components for an archetype of the robot: a cradle would be needed to hold the Cube and rotate about the Y axis; an arm to rotate the Cube about the X axis and hold it in place for performing D-moves; and a moving arm for the colour sensor to scan the Cube. The essential requirements for these components are described below.

Cradle

The Cube will sit in a cradle which rotates about the Y axis to provide the movement needed for the moveset {Y Y' Y² D D' D²}. This will have an inner dimension of 7 studs × 7 studs (56 mm × 56 mm × 56 mm) to closely match the dimensions of the Valk 3: 55.5 mm × 55.5 mm. The 0.5 mm tolerance means that the Cube doesn't have to perfectly aligned when it is rotated (i.e. the program won't fail if the cube is a degree out of line). The cradle must rotate as accurately as possible to 90° increments to ensure the success of the X rotations and avoid the Cube falling out of the cradle.

X-Move Arm

To perform an X-move, some kind of arm will need to ‘flip’ the Cube about its X axis. This should be a quick movement, with high reliability and good structural soundness. The arm must not interfere with the other components of the robot at any point. Furthermore the arm will also need to incorporate functionality to hold the L-R-slice and the U-face in place whilst the cradle rotates to perform a permutation of the D-move.

Colour Sensor

The colour sensor will be attached to the end of an extending arm which can be moved by a rack and worm gear to ensure the sensor can be accurately placed above the correct facelet. The worm gears will have to be driven by the medium sized motor, which produces less torque than the large motors but has a larger maximum rotational velocity - making it ideal for overcoming the inherent speed reduction of the worm gears. The colour sensor will be in a fixed position relative to the arm, and will be centred relative to the Cube.

4.2.4 Robot Design MkI

When designing the robot, there was a clear hierarchy of components and an inherent build order: the X-move arm relied on the cradle to rotate the cube, and the colour sensor needs the arm and the cradle to rotate the cube for the scanning process. The design process for the components comprised a real-world build and the concurrent creation of a virtual 3D model using BrickLink’s Stud.io software [34]. Rendered images of the virtual model are displayed below to show the finer details of the mechanism and as evidence of the consideration used in the design of the robot.

Cradle

The sides of the cradle will only be 1 stud high, and will have a semi-circular profile to allow the Cube to rotate out of the base of the cradle as demonstrated in Figures 4.3b and 4.3c. The curved edge will also cause the Cube to slide back into the cradle completely, rather than resting on the upper corner of the edge piece.

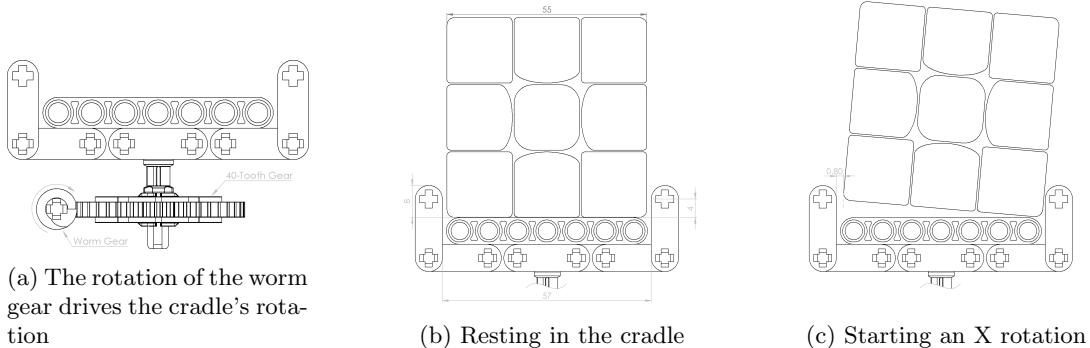


Figure 4.3: A cross-section of the cradle and Cube

X-Move Arm

To rotate the Cube about the X axis, a rotating arm will push the Cube up and out of the cradle and cause it to pivot about the edge of the cradle. The Cube will then slide back into the cradle to complete the rotation. A large motor will be directly connected to the shaft which the arm will rotate about, with no need for gears. One end of the arm will have a wheel with a rubber tire to provide enough friction to tip the Cube over, and the other side will have a set of guards which can hold the top two slices of the Cube in place when performing a D-move or D'-move. The design of the swing arm and its positioning relative to the cradle can be seen in Figure 4.4.

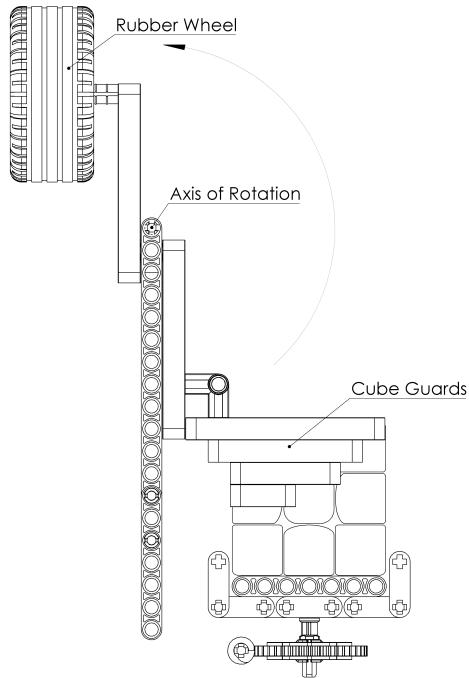


Figure 4.4: The rotating arm used to flip the Cube

The initial design for the cradle was for it to be mounted directly on the rotor of a large motor. This gave it stability from the secure mount, high rotational velocity because there would be no gearing down, and it would be accurate due to the in-built tachometer which provides ‘precise control to within one degree of accuracy’ [35]. Figure 4.5 shows the large motor, the mechanism and the framework to support them. The large black gear underneath the rotor has no mechanical use. Rather, it is used throughout the robot to flag where the motors are and to allow the user to manually rotate them to the correct positions on initialisation.

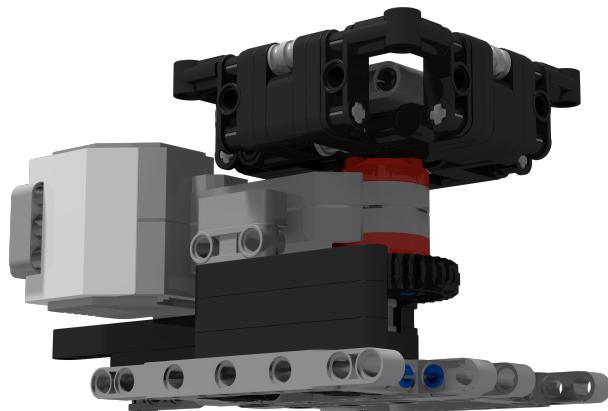


Figure 4.5: A render showing the original cradle mount design

The play in the motor will reduce the accuracy of the cradle, as it effectively introduces a zero error for each rotation. If the cradle only rotates in one direction then the zero error would only have to be offset at the very start of the program because the cradle would ‘sit’ at one end of the region of play. However, this will reduce the size of the available moveset by 50% and thus have a worst case runtime increase of 100% (Sequentially, $|\text{:D' X:}| = 2$ which would change to $|\text{:D D D X:}| = 4$).

A worm gear (#4716) will solve both of these problems: it will be combined with a large gear (#3649) to create a gear ratio of 1:40, increasing the accuracy of the motor by a factor of forty (see Figure 4.3a). This will also slow the maximum rotational velocity of the cradle to 0.209 rad s^{-1} (2RPM) at 7.5V, so a 90° turn will take 7.5 seconds¹. A compromise between maximum rotational velocity and accuracy will be found by adding more gears to reduce the ratio. The problem presented by the play in the motors will be reduced to an acceptable level due to the unidirectional nature of the gear train.

4.2.5 Colour Sensor

In order to scan a full Cube, the sequence $:\text{X X X Y X X X:}$ will be followed, and the U-face scanned every time a new face is seen as shown by algorithms 1 and 2.

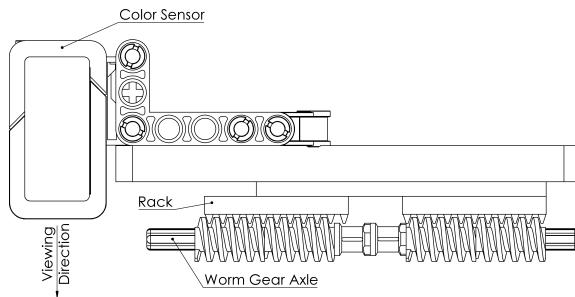


Figure 4.6: The colour sensor mounting arm is driven by a set of worm gears

Algorithm 1 Scanning the Up-Face of a Cube

```

1: function SCAN_UP_FACE
2:   facelet_list  $\leftarrow [ ]$                                  $\triangleright$  Initialise as empty list
3:   for  $i \leftarrow 0$  to 4 do                                 $\triangleright$  4 Corner-Edge pairs
4:     MOVE_COLOR_SENSOR(edge_cubie)
5:     facelet_list  $\leftarrow$  color_value
6:     ROTATE_CRADLE(45)
7:
8:     MOVE_COLOR_SENSOR(corner_cubie)
9:     facelet_list  $\leftarrow$  color_value
10:    ROTATE_CRADLE(45)
11:   end for
12:   MOVE_COLOR_SENSOR(centre_cubie)
13:   facelet_list  $\leftarrow$  color_value
14:   return facelet_list
15: end function

```

Algorithm 1 is called six times by the main scanning process, and is used to move the colour sensor to the correct position to scan each facelet in turn. In the algorithm, an empty list is declared for the nine facelets to be appended to. `move_color_sensor` takes a single parameter: the stopping point for the motor. The variables `edge_cubie`, `corner_cubie` and `centre_cubie` are all integer values which are used as these stopping points. The motor is rotated until the tachometer value is greater than or equal to the

¹This figure is the result of a measurement of the maximum rotational velocity of a large motor, and will fluctuate depending on the voltage supplied by the batteries in the EV3 brick

variable - or less than if it is rotating backwards - so that if it is rotating faster than one ‘tacho-unit’ at a time, it will not skip the stopping variable and continue indefinitely.

Similarly to `move_color_sensor`, `rotate_cradle` takes a single parameter - but not a stopping point. Rather, it is the value which is to be added to the current position of the motor in degrees. This value will then be multiplied as necessary to account for the gear ratios to create an output rotation matching the provided angle.

Algorithm 2 Scanning an entire Cube

```

1: function SCAN_CUBE
2:   faces← [ [],[],[],[],[],[]]                                ▷ Initialise as 6 empty lists
3:   for i ← 0 to LENGTH(faces) do
4:     faces[i] ← SCAN_UP_FACE
5:     if i == 3 then
6:       ROTATE_CRADLE(90)                                     ▷ Allows L-face and R-face to be scanned after X-move
7:     end if
8:     if i < 5 then
9:       X_MOVE_CUBE                                         ▷ X-move the Cube to get a new Color on the U-face
10:    end if
11:    if i == 4 then
12:      X_MOVE_CUBE                                         ▷ Extra X-move to get to D-face
13:    end if0
14:   end for
15:   return faces
16: end function

```

4.2.6 3D Model

The drawings above show how each component will work and be built out of Lego, however a design for the whole robot is difficult to represent in 2-dimensional drawings. For this reason, a 3-dimensional model of the robot was created in BrickLink’s Stud.io software [34] to create a full model representative of the final design. The components drawn above are rendered below for comparison and clarification on specific details. Some parts may be hidden in the renders to show a clearer view.

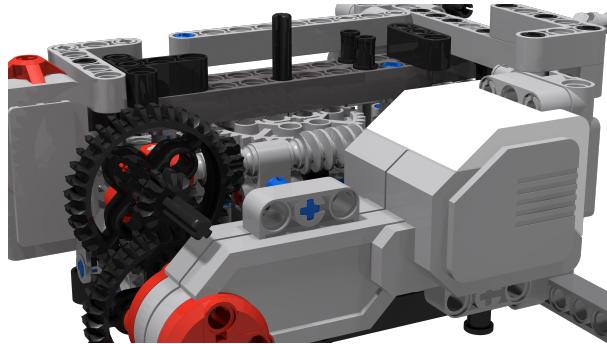


Figure 4.7: A render showing how a large motor will rotate the cradle and the gears used

Figure 4.7 shows how the cradle will be driven by a large motor. A large 36-tooth gear (#32498) is directly attached to the rotor with two small pins to reduce the need for an extended shaft which will flex when placed under strain (i.e. at the start of a rotation). This gear drives a 12-tooth gear (#32270) to triple the rotational velocity at that point of the gear train. This connection is immediately repeated and used to create a 90° turn onto the shaft with the worm gear. At this point the speed of the worm gear has a ratio of 9:1 when compared to the motor’s output, which is then decreased forty times by the large 40-tooth gear to create a final ratio of 1:4.444. This means the maximum rotational velocity of the cradle will be 1.843 rad s^{-1} (17.6 RPM) at 7.5V and the original torque is increased from 17.3Nm

to 76.8Nm, which will allow a face of the Cube to be rotated with ease. The vertical black shaft seen at the top of the image provides a mounting point for the cradle.

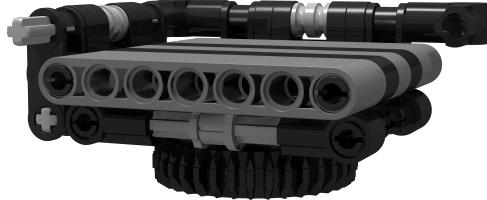


Figure 4.8: A view of the cradle with some parts removed

The cradle is a solid base of 7 studs \times 7 studs \times 2 studs (length \times breadth \times height) with a surrounding wall with a height and depth of 1 stud. This ensures that the weight of the cradle uniformly distributed and the centre of gravity is low. These are important factors in the design of the cradle because it is mounted on a singular vertical shaft, which is quite flexible under relatively low amounts of strain². The upper layer of the cradle's base has a smooth surface to allow the Cube to slide properly when moved out of the cradle for an X-move; and the lower layer provides an interface to allow mounting on the vertical shaft. It uses two pins (#2780) to connect to another of the large gears (#32498) which has the female axle-type connection for connecting to the gear train discussed above.

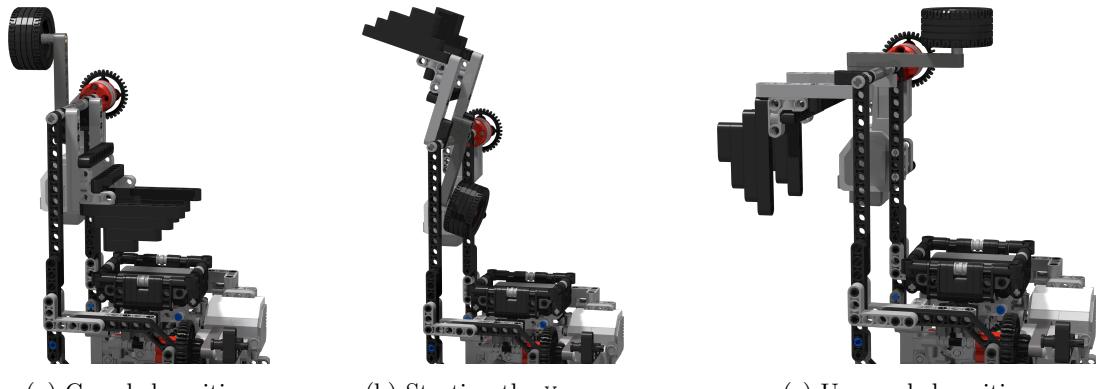


Figure 4.9: Three different positions of the X-Move Arm

Figure 4.9 shows where the arm is positioned in relation to the cradle. Figure 4.9a shows the ‘guarded’ position of the arm, where the guards are either side of the Cube to hold the L–R-slice and the U-face in place when performing a D-move, D'-move or D²-move. The rotation of the Cube will undoubtedly try to separate the guards, so two liftarms (#40490) have been attached to the upper part of the guards to hold them together. The guard assembly is attached to a central axle which is rotated by the large motor visible at the far side of the model. Also attached to the axle is a second liftarm to hold the wheel in place to rotate the Cube. The entire assembly’s weight is balanced sufficiently well to ensure that there is a smooth rotation about the axle and no unnecessary strain is placed on the motor. The play in the motor that proved an issue with the cradle, whilst still being present, is not a concern for this assembly because accuracy is of much less importance. As long as the arm moves by *at least* the correct angle to rotate the Cube or place the guard in position, the exact position is unimportant. The accuracy of the large motor’s rotor is therefore accepted for this assembly.

²The cause of any excess strain is discussed in the Implementation and Testing section



(a) A full view of the mechanics

(b) The assembly's mounting structure

Figure 4.10: The colour sensor assembly

The colour sensor is powered by the medium motor and a gear train which ends with a worm gear and rack (#3743). The rack is directly connected to a liftarm which has two long parallel shafts (#3708) mounted above it, which feed through a liftarm attached to the main frame of the robot. These shafts are the only fixed mounting points for the whole colour sensor assembly - the worm gears and rack provide lateral movement but little to no structural support.

The medium motor powers a 24-tooth gear (#3648), which then transmits power to an 8-tooth gear (#3647), which is on the same axle as the worm gears. The rotational velocity of the worm gear is 63.75 rad s^{-1} (608 RPM), which means that a large 40-tooth gear would have a rotational velocity of 1.59 rad s^{-1} (15.2 RPM). Using this rotational velocity, the surface velocity of the large gear and therefore the lateral velocity of the rack can be calculated to be $\sim 0.0662 \text{ m s}^{-1}$. Given that the range of the colour sensor assembly's moveable distance is 80mm, the time taken for the colour sensor to travel from its neutral position to above the centre facelet is 1.21s. Whilst this may seem like a relatively long time for the motor to move a light load, the assembly will only have to make the full journey between each face.

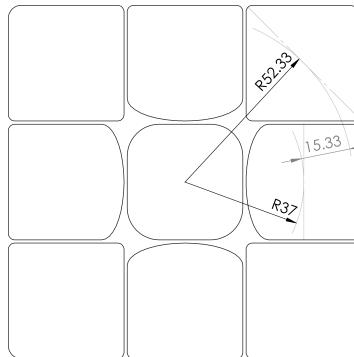


Figure 4.11: A Cube, with arcs drawn at the centre points of the cubies

Figure 4.11 shows that the radial distance between the centre points of the middle and corner cubies is 15.33mm. The colour sensor will have to travel this distance forty-two times during the scanning process, as well as the distance to the centre six times, making the total theoretical travelling time of the colour sensor $\sim 21.26\text{s}$.

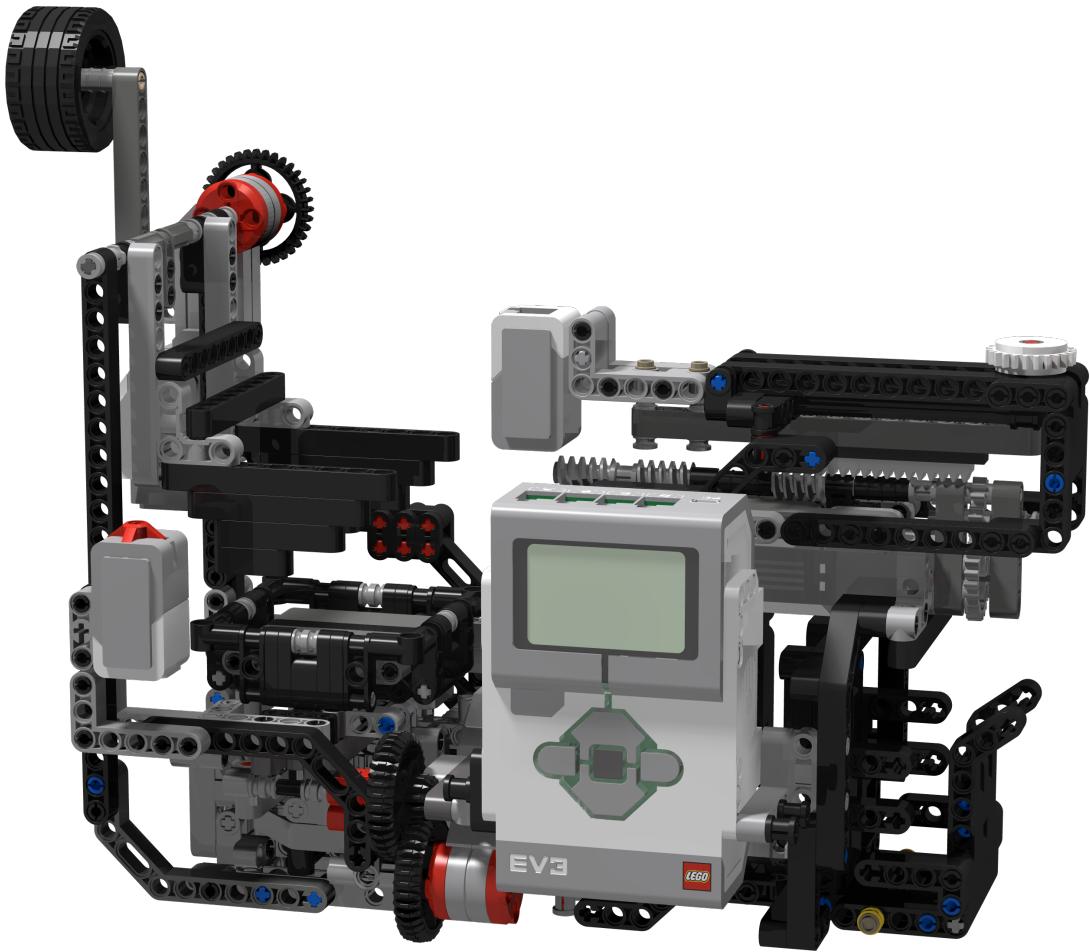


Figure 4.12: The full robot assembly

Figure 4.12 demonstrates the full assembly of the robot, including the main EV3 control brick. The 3D model was created and modified concurrently to the actual building process. This is because some of the framework was a lot easier to create with actual Lego pieces rather than working with a mouse and keyboard. As a result of this dual-methodology building process, the design was optimised as much as possible with regards to functionality, structural integrity, and efficiency to work under restrictions from piece availability.

The build of the robot is quite compact, and the design is largely influenced by the pieces available: during the modelling process, small sections of the model were built with actual Lego to check functionality translated from the virtual model to the physical model as intended. Due to the limited quantity of Lego pieces, some less-than-optimal sections arose where two parts had to be connected but the necessary liftarm was not available. This situation is visible behind the cradle, where six small red pieces (#32062) are being used to connect two right-angled liftarms. Another example of ‘design by necessity’ is in the colour sensor’s gear train: one medium and two small gears were used consecutively instead of two medium, or one large and one small. The functional impact of some of these sections is minimised by workarounds and extra pieces, however the aesthetics have unfortunately suffered minor setbacks in places.

A touch sensor (#95648) is mounted on the side of the frame that surrounds the cradle and the X-Move Arm. This adds the ability for user input when the program is running without needing to access the operating system through the control brick. This means that simple tasks can be bound to the button at each stage of the program (e.g. pause the scanning process if a move fails or confirm initialisation).

The EV3 control brick is mounted on the side of the robot to allow easy access to the controls which allow navigation of the file system and turning the brick on and off. It is held in place by several long pins (#32054) which can be easily retracted to allow quick removal of the brick so the batteries can be changed.

4.2.7 Version 2

4.3 Codebase

4.3.1 Object Classes

Cube

Position

4.3.2 Solve Methods

4.3.3 Method 1: Tree Solving

4.3.4 Method 2: Group Solving

4.3.5 Database

4.3.6 Move Sequence Translation

The algorithms considered and discussed in the Literature Review (as well as others used in determining God's Number) are designed to work with a standard moveset {L R U D F B} (as well as half turns and counter-clockwise turns). The design of the robot means that it is restricted to the moveset {X Y Y' Y² D D' D²} - and will therefore require any incompatible sequences to be translated to the robot's moveset.

When the entire Cube is rotated, the **colour** of the faces will change but the faces' **orientations** will not (analogous to when one turns on the spot: left and right have changed, but east and west have not). This means that the frame of reference for the moves will have to remain invariant throughout a sequence by compensating for any full Cube rotations.

This is best explained with an example: the first step in performing the sequence :U F: would be to do perform :X² D: in order to complete the U-move; but now that the original F-face has moved (due to the :X² :) - it must be located, the Cube must be rotated to get the original F-face into the cradle, and then the sequence can be completed. The simplest method of overcoming this complication is to convert the original move sequence to an equivalent sequence of colour-based moves: :U F: \equiv :W G:.

Once the original sequence has been converted to a colour-based sequence, a deep copy of the original Cube is made and the coloured sequence is iteratively translated and applied to this Cube. On each element of the sequence, the colour is converted back to a face move (allowing preservation of the correct frame of reference). Once this new face-based move is found, it is converted to a short move sequence that will move the face into the cradle and rotate the face correctly to match the original move. This short sequence comprises moves compatible with the robot.

Each of the moves in the short sequence is applied to the copied Cube, and appended to a list of moves which is returned once the original sequence is processed. The application of the moves to the Cube allows following moves to be translated correctly (as the Cube will have changed orientation in real life so needs to in this simulation). Continuing the previous example of :U F:, the translation is written

below. Colours are underlined and the output sequence is emboldened for clarity:

```
:U F: → :W G:
:W: → :U:
:U: → :X2 D: → :X X D:
    ⇒ :U F: → :X X D G:
:G: → :B:
:B: → :X D:
    ⇒ :U F: → :X X D X D:
```

This final move sequence will then be transmitted to the robot and used to control the motors to manipulate the Cube.

5 Results and Discussion

6 Conclusions

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