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Q 1. The Gauss-Elimination Method is a direct method to solve a system of n linear equations with n unknowns. The method transforms the system into an upper triangular form and then applies back substitution to find the solution.

→ Limitations

(i) Round off error.

(ii) Pivoting issues. (If $\text{pivot} = 0$ or $\text{pivot near } 0$)

(iii) Computational costs: This method involves $O(n^3)$ operations, making it computationally expensive for large systems.

(iv) Static system: Cannot handle systems with dynamically changing coefficients or iterative improvements.

→ Comparison with iterative methods

	Gauss elimination	Iterative Methods
Computational costs	$O(n^3)$	$O(n^2)$
Accuracy	Sensitive to round off errors	Can achieve desired precision.
Scalability	Less efficient for large systems	Scalable and efficient for large systems

Q2.] (a) Condition for Diagonal Dominance

For a system of linear equations represented by $Ax = b$, the coefficient matrix A is diagonally dominant if for every row i :

$$|a_{ii}| \geq \sum |a_{ij}|$$

with strict inequality $|a_{ii}| > \sum |a_{ij}|$

→ How it ensures the convergence of Gauss-Seidel?

Diagonal dominance ensures that the iterative updates do not diverge. Each new estimate of the variable depends more on its coefficient and less on the other variables, promoting stability.

In Gauss-Seidel, this condition helps the system converge because the contribution of the off-diagonal terms are small compared to the diagonal terms.

(b) comparision of Jacobi and Gauss - Seidel method.

Jacobi	Gauss Seidel method
① updates variable simultaneously	Updates variables sequentially.
② Slower convergence compared to Gauss-Seidel	Faster convergence due to immediate use of updated values.
③ Strong diagonal dominance	Converges under milder conditions.