"Pick up your damn sufferings, BEAR IT and try to be a good person."

10/10/2022

I Should Know Tables From 1 to 20

11	12	13	14	15	16	17	18	19	20
22	24	26	28	30	32	34	36	38	40
33	36	39	42	45	48	51	54	57	60
44	48	52	56	60	64	68	72	76	80
55	60	65	70	75	80	85	90	95	100
66	72	78	84	90	96	102	108	114	120
77	84	91	98	105	112	119	126	133	140
88	96	104	112	120	128	136	144	152	160
99	108	117	126	135	144	153	162	171	180
110	120	130	140	150	160	170	180	190	200

1 - 1	11 - 121	21 - 441	1 - 1
2 - 4	12 - 144	22 - 484	2 - 8
3 - 9	13 - 169	23 - 529	3 - 27
4 - 16	14 - 196	24 - 576	4 - 64
5 - 25	15 - 225	25 - 625	5 - 125
6 - 36	16 - 256	26 - 676	6 - 216
7 - 49	17 - 289	27 - 729	7 - 343
8 - 64	18 - 324	28 - 784	8 - 512
9 - 81	19 - 361	29 - 841	9 - 729
10 - 100	20 - 400	30 - 900	10 - 1000

25 [1] 2 3 5 7 11 13 17 19 23 29 31 37 41 43 47
53 59 61 67 71 73 79 83 89 97

21 101 103 107 109 113 127 131 137 139 149 151 157 163 167
⋮ 173 179 181 191 193 197 199
100

★ Tables:

60	1	61	59
120	2	122	118
180	3	183	177
240	4	244	236
300	5	305	295
360	6	366	41354
420	7	427	473
480	8	488	472
540	9	549	531
600	10	610	590

Homework

83
47
92
38

Imp/Useful Tables

37
48

70	2	72	68
140	4	144	136
210	6	216	204
280	8	288	272
350	10	360	340
420	12	432	408
490	14	504	476
560	16	576	544
630	18	648	612
700	20	720	680

★ Squares:

1. Base 50 - (i) Get difference and square it ← Fill Last 2 positions
   (ii) If num <50 : sub (25 - num)
   num >50 : add (25 + diff)

Ex.

43^2	47^2	54^2	62^2
1849	2209	2916	3844
			1

Note: Carry will always be added at the end.

2. Base 100 : ① Get difference & square it.
② If num <100 : sub (num - diff) * 1
num > 100 : add (num + diff) * 1

3. Base 200 : ① Get difference and square it.
② If num < 200 : sub (num - diff) * 2
num > 200 : add (num + diff) * 2

Ex.

93^2	88^2	124^2	113^2
8649	7744 (1)	15376 (5)	12769 (1)

The only 4 digit perfect square with pattern xxyy.

204^2	211^2	195^2	188^2
41616	44521 (1)	38025	35344 (1)

11/10/2022

4. Base 150: Similar for all odd multiples of 50.
① Get difference and square it
② If num > 150 : add (num + diff) × 3/2 (1.5 = 3/2)
num < 150 : sub (num - diff) × 3/2

★
$1^2 = 1$
$11^2 = 121$
$111^2 = 12321$
$1111^2 = 1234321$
$11111^2 = 123454321$
1111111111^2 ?? 12345679009876543 21

★ Square Roots:

Last digit of a Perfect Square -

Never: 2 3 7 8

Always: 0 1 4 5 6 9

2nd Last digit of a Perfect Square -

1. □6 → Odd
2. □5 → 2
3. □ 1/4/9 → even
4. □ 00 → even pair of zeros → perfect square

Q1 Which of the foll. is a perfect square?

a. 98182
b. 99748
c. 96263
d. 98596

Q2 Which of the following is a perfect square?

a. 49166
b. 48686
c. 47126
d. 46656

★ Calculate Square root:

$\sqrt{5184}$

5100

4900 6400

70 80

4 → 2,8

ie. 72/78

↳ nearest

$\sqrt{6889}$

6800

6400 8100

80 90

9 → 3,7

ie. 83/87

$\sqrt{41209}$

40000 → 20 | 44100 → 21

3,7

ie. 203 / 207

$\sqrt{23104}$

22500 → 15 | 25600 → 16

2,8

152 / 158

412.09 | 231.04 | 53.29 | 0.003721 | 4.1209

20.3 | 15.2 | 7.3 | 0.061 | 2.03

Q3 $\sqrt{1471369}$

a. 1213 b. 1223 c. 1233 d. 1243 e. 1187

Note: If last two digits of a square are 69, then that square root will be of the type $(50x \pm 13)$.

Q4 $\sqrt{6.4009}$

a. 2.53
b. 2.55
c. 2.56
d. 2.59

Q. $\sqrt{906.01}$

a. 31.11
b. 30.11
c. 30.1
d. 31.1

★ Cubes:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$13^3 = 2197$

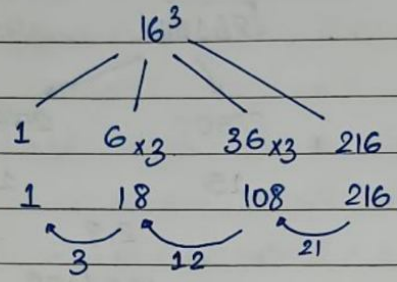
a^3 a^2b b^2a b^3

1 3x3 9x3 27

1 9 27 27

1 2 2

16^3 = 4096



24^3

8 48 96 64

13824

44^3

64 192 192 64

85184

53^3

125 225 135 27

148859

81^3

512 192 24 1

531441

63^3

216 324 162 27

250047

62^3

216 216 72 8

238328

12/10/2022

☆ Number Systems:

1. Natural nos - 1, 2, 3, 4, ...
2. Whole nos - 0, 1, 2, 3, ...
3. Integer nos - -3, -2, -1, 0, 1, 2, ...
4. Rational nos - $\{ p/q \mid p, q \in I, q \neq 0 \}$

Note: Zero is the most powerful Rational No., as it is divisible by all integers.

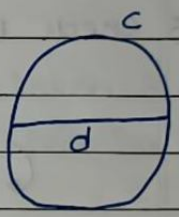
5. Irrational nos - Not rational. ie. either p or q will not belong to integers.
eg. $\frac{\sqrt{2}}{1}$ or $\frac{2}{\sqrt{2}}$

6. Real nos - Union set of Rational & Irrational nos.

★ GYAAN:

$\frac{\text{Circumference}(c)}{\text{diameter}(d)} = \pi$ (pie)

↳ holds true for all circles.



★ Cube Roots:

You will always find all numbers from (0 to 9) at units place of some or the other numbers cube.

$\overline{13824}$ → $4^3 = 64$

ignore

8,000 → 20

27,000 → 30

4

24

$\overline{175616}$ → $6^3 = 216$

125,000 → 50

216,000 → 60

6

56

★ Natural Numbers:

(i) Zero is even → it is divisible by 2.

(ii) A num having exactly 2 positive factors is prime.

(iii) more than 2 composite.

(iv) 1 is nither prime nor composite

(v) We have 25 prime nos of 0 to 100 range, out of which 2 is the only prime no which is even.

(vi) Except 2 & 3 every prime number is of the range $(6x-1 / 6x+1)$. But the converse is not true.

Check if a number is prime.

157 → Nearest square → 169 (13^2)

↳ so check divisibility by all prime nos under 13 if passed by 2, 3, 5, 7, 11 then 157 is prime.

★ Divisibility Tests

[2] → units place divisible by 2

[3] → sum of nos divisible by 3

[4] → last 2 digits divisible by 4

[5] → units place 0 or 5

[6] → 2 & 3

[8] → last 3 digits divisible by 8

↳ 16 → 2^4 → last 4

32 → 2^5 → last 5

[9] → sum divisible by 9

→ 9 is a seed number ie. consecutive addition will always give 9.

Eg. 648 → 18 → 9

[11] → Difference btw 2 groups of alternative digits should be either zero or a multiple of 11.

Eg. (i) $\underline{7}\bar{1}\underline{7}\bar{2}$ ⇒ 14 - 3 = 11

(ii) $\bar{1}\underline{8}\bar{7}$ ⇒ 8 - 8 = 0

(iii) $\underline{9}\bar{1}\underline{8}\bar{0}\underline{7}\bar{1}$ ⇒ 24 - 2 = 22

12 → 3 & 4

15 → 3 & 5

7 → Difference of two groups, taken 3 digits at a time should be either zero or a multiple 7.

→ For a 3 digit number 'abc',
if (ab + 5c)%7 = 0, then abc is divisible by 7

Eg.

$\underline{370}\overline{356}$ = 370 - 356 = 14

7370356 = 370 - (356 + 7) = 7

$\overline{550}\underline{500}\overline{006}$
= 556 - 500
= 56

$\underline{23}\overline{100}$ = 100 - 23 = 77

392 = 39 + (5x2) = 49

14 → 2 & 7

13 → Same test as of 7, just final diff should be divisible by 13.

→ (ab + 4c)%13 = 0

Eg. $\underline{210}\overline{184}$ = 210 - 184 = 26

403 = 40 + 12 = 52

17 → $(ab - 5c)\%17$

Eg. 8544 → 54 − 20 → 34

1887 → 188 − 35 → 153 → 15 − 15 = 0

19 → $(ab + 2c)\%19$

Eg.

551 → 55 + 2 → 57

★ GYAAN :

720720 = Smallest natural number which is divisible by 1st 16 natural numbers.

★ Proof of divisibility for 3 and 9

$$xyz = 100x + 10y + z$$
$$= 99x + 9y + (x + y + z)$$

↳ sum of numbers.

★ Puzzle:

$$\frac{abcabc}{7 \times 11 \times 13} = \frac{abcabc}{1001} = \frac{abc \times 1001}{1001}$$

★ Proof of divisibility for 11.

$$abcd = 1000a + 100b + 10c + d$$
$$= 1001a - a + 99b + b + 11c - c + d$$
$$= 1001a + 99b + 11c + (b + d - (a + c))$$

$$abc = 100a + 10b + c$$
$$= 99a + a + 11b - b + c$$
$$= 99a + 11b + (a + c - b).$$

★ Proof for divisibility of 4.

$abc = 100a + bc$ → check bc

$abcd = 1000a + 100b + cd$ → check cd

★ Proof for divisibility of 8

$abcd = 1000a + bcd$ → check only bcd

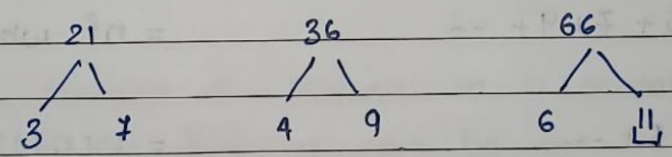
★ Least Perfect Square: (LPS).

Q6 Find LPS which is divisible by 21, 36 and 66.

a. 213444 b. 214434

c. 214344 d. 231444

Soln



→ largest, so first check divisibility for 11

11 ✓ 4 ✓ 213444 11 ✓ 4 × 214343

11 × 214344 11 × 231444

Q7 Find 'x' in 765432x such that the num is divisible by 9.

Soln 0 or 9 (Using divisibility test.)

Q8 765432x by 18 : Soln 0.

(Apply divisibility tests for 2 and 9)

Q9 Find x in x765432 such that the 7 digit number is divisible by 9.

a. 0 c. 9

b. 3 d. 1

Q10 54321x should be divisible by 12

a. 0 c. 6

b. 3 d. 9

★ Number series formulas:

1. $1 + 2 + 3 + 4 + \ldots = n(n+1)/2$
2. $1^2 + 2^2 + 3^2 + 4^2 + \ldots = n(n+1)(2n+1)/6$
3. $1^3 + 2^3 + 3^3 + 4^3 + \ldots = [n(n+1)/2]^2$
4. $1 + 3 + 5 + 7 + 9 + \ldots = n^2$ where $n = \frac{LN+1}{2}$
5. $2 + 4 + 6 + 8 + \ldots = n(n+1)$ $n = \frac{LN}{2}$

Q11. $21 + 22 + 23 + \ldots + 40 =$

Q12. $11 + 13 + 15 + \ldots + 29 =$

Q13. $2^2 + 4^2 + 6^2 + \ldots + 20^2 =$

Q14. In a party of 25 each person shakes hand with all other persons exactly once. Find total handshakes.

Q15. In a kabbadi league 12 teams have participated. Each team will play against all other teams exactly twice. Find total number of leagued matches.

Q16. In a knockout tennis tournament of 64 players find total no. of matches to be played.

Q17 Count the number of squares & rectangles on a chess board.

13-10-2022

★ We have 21 prime numbers btw 100 & 200
16 200 & 300

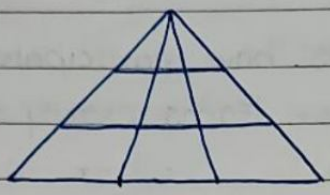
Q.18 How many numbers btw 100 and 200 have exactly 2 factors.?
Q19 How many numbers from 101 to 200 have atleast 2 factors?

Q20. Total number of students in a school is a prime num. Which of the following can be the ratio of girls and boys in school?

a. 59 : 60 b. 70 : 73
c. 80 : 81 d. 90 : 91

Q21. 4 6 12 18 30 42 60 72 102 108 ? ?
3 5 11 17 29 41 59 71 101 107 ? ?

★ Twin Prime numbers are consecutive prime numbers which are also consecutive odd numbers.

Q22.  Count the numbers of triangles.

★ GYAAN: 'You Are Defined By The Choices You Make'.

Q23. Let x & y be two natural numbers such that $(x+y)=20$. Find max and min values of xy.

Note: xy is max at center & min at extremes.

Q24. $x+y = 20$. Find max and min of x^2+y^2.

Note: x^2+y^2 is max at extremes and min at center.

↳ For all x^n+y^n

Q25. Salman and Jackie marry & have x children of their own. But salman already has y children from his 1st wife. Jackie also has z children from her 1st husband.

$$x+y+z = 9$$

Find max number of fights that can take place among kids, if for a fight 2 children are required. and no 2 children have same parents.

Also calculate combinations if $x \neq y \neq z$

★ HCF - Highest Common Factor

↳ HCF $\leq$ The numbers itself.

Q. Find HCF of 56, 84

Tradition method:

2	56	84
2	28	42
7	14	21
	2	3

54 90 126

2	54	90	126
3	27	45	63
3	9	15	21
	3	5	7

=20.

Difference method:

259, 407
259, 148
111, 148
111, 37
74, 37
37, 37

203, 319
203, 116
87, 116
87, 29
58, 29
29, 29

The above method does not work for co-prime numbers eg. 92, 145

★ 1. If we do not get any common element then the HCF is one, and the numbers are said to be co-prime.

ong

2. All prime numbers are always co-prime but the converse is not true.

3. Consecutive natural nos are always co-prime. ie. HCF = 1.
4. Consecutive odd natural nos are also always coprime. Hence their HCF = 1.
5. HCF of consecutive even natural numbers is 2.

Q26. Find biggest num such that 42 and 98 are divisible by that number.

Q27. Find largest num such that when 43, 65 & 109 are divided by that number we get remainders as 1, 2 and 4 respectively.

Q28. Find highest 53, 101, 173 such that we get a same remainder in each case. Find that common remainder

Q29. Find HCF of 1111... (35 times) & 111..(60 times).

Q30. 333... (60 times) and 333... (100 times).

Q31. Find HCF of $3^{45}-1$ and $3^{120}-1$.

Q32. Find HCF of $7^{20}-1$ and $7^{20}+1$.

★ Note: $(odd)^{even/odd} = odd$.

Q33. Find HCF of $8^{20}-1$ and $8^{20}+1$

★ Note: $(even)^{pow} = even$

Q34. The total number of admission for science, commerce and arts streams were 48, 84 and 72 respectively. The principal wants to arrange these students into separate classrooms such that each class has an equal number of students of same stream. Find the minimum number of classrooms required.

a. 12 b. 15 c. 17 d. 18

Q35. A rectangular floor with length and 84 and bream 48 feet is to be tiled using only square shaped tiles. Find the minimum number of tiles needed. ie. area of tile = max.

Q36 How many pairs of positive integers x and y exists such that $x+y = 156$ and $HCF(x,y) = 13$

a. 2 b. 4 c. 5 d. 6

Soln

$x = 13a \qquad y = 13b$

$13(a+b) = 156$

$a+b = 12$

1 11 & co-prime factors

5 7

$1 \cdot 13 , \quad 11 \cdot 13 \Rightarrow 13, 143$

$5 \cdot 13 , \quad 7 \cdot 13 \Rightarrow 65 \quad 91$ & 2 pairs.

Q37. How many pairs of positive integers x and y exists such that $x+y = 80$ and $HCF(x,y) = 5$.

Q38 The product of two numbers is 4107 and their $HCF(x,y) = 37$

Soln

$x = 37a \qquad y = 37b$

$37a \times 37b = 4107$

$a \times b = 3$

$1 \times 3 = 3$

37, 111.

★ Least Common Multiple (LCM)

LCM ≥ The number itself.

Find LCM of 36 and 48 : Traditional method

2	36	48
2	18	24
2	9	12
2	9	6
3	9	3
3	3	1
	1	1

Q39 Find the LCM of 20, 24 and 30.

Q40 For the numbers 259 and 407, find LCM.
203 and 319

★ (i) For any two numbers : HCF × LCM = Product of Nos.

(ii) For co-prime pairs like 92 and 145 the product is LCM itself, as their HCF is 1.

Q41. Find the least number which when divided by 12, 16 & 20 leaves a remainder of 7 in each case.

↳ First calculate LCM

↳ If Remainder is common, ans = LCM + common remainder

Q42. Find smallest num, which when divided by 24, 36 and 45 leaves remainder as 17, 24 & 38.

$24)17^{7}$ $36)24^{7}$ $45)38^{7}$ → If difference is common, ans = LCM − common diff

Q43. Find the smallest number which when divided by 4, 5 and 7 leaves remainder as 2, 3 and 1.

a. 392 c. 218 ✓

b. 358 ✓ passing divisibility tests d. None

Soln

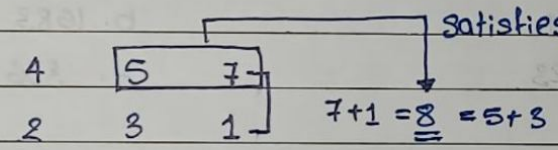
358 − 218 = 140

218 − 140 = 78

Note: If we add or substract the LCM, the basic property of the numbers do not change.

Here we are actually trying to get closer to LCM.

Shortcut soln



LCM of 5 and 7 = 35

Add 35+8 until the number satisfies all conditions.

i.e 35 + 8 = 43

43 + 35 = 78

Q44. Find smallest no. which when divided by 7, 8 and 10 leaves remainder and as 1, 4 and 2 respectively.

Q45. Find the least number which when divided by 5, 6 & 7 leaves remainder as 3, 4 and 2 respectively.

Q46 While running on a circular track 3 runners A, B and C reach the starting point after every 12, 15, 24 and 40 seconds. After how many seconds will A, B, C meet at starting point for the 1st time.

In half an hour, how many times have A, B and C will meet at starting point.

Q47. Find the least number which when divided by 5, 6, 7 and 8 leaves the remainder by 3 in each case but when divided by 9 leaves no remainder.

a. 1677 b. 1683
c. 2523 d. 336

Q48. Find lowest num 16, 18, 20, 25 leaves remainder of 4 in each case but when divided by 7 leaves zero remainder.

a. 3604 b. 1015
c. 1022 d. 18004

Q49 Find the smallest multiple of 23 which when divided by 18, 21, 24 gives remainder as 7, 10 & 13.

a. 3002 b. 3013
c. 3024 d. 3036

Note: If the divisor is even and the remainder is odd then the orignal number is also odd.

Q49 ⇒ Ans = b. 3013

Q50. Find the LCM & HCF of (i) $\frac{12}{25}, \frac{16}{25}$ (ii) $\frac{7}{22}, \frac{14}{11}$

HCF (fractions) = $\frac{\text{HCF}}{\text{LCM}}$, LCM = $\frac{\text{LCM}}{\text{HCF}}$

Q51. Find the LCM of 3, 2.7, 0.09.

a. 0.027 b. 0.27

c. 2.7 d. 27

* Q52. Which of the following is the greatest number:

a. $2^{1/2}$ b. $3^{1/3}$

c. $4^{1/4}$ d. $6^{1/6}$

Answers:

Q1. d. 98596

Q2. d. 46656

Q3 $\sqrt{1471369} \Rightarrow (50x \pm 13) \Rightarrow$ either a. 1213 or e 1187.

But as $1100^2 = 1210000$ & $1200^2 = 1440000$

Ans = a. 1213.

Q4. a. 2.53

Q5. c. 30.1

Q9 c. 9

Q10. c. 6

Q11. $21 + 22 + 23 + \dots + 40$ $\quad n(n+1)/2$

$40(40+1)/2 - 20(20+1)/2 = 820 - 210 = 610$

Q12. $11 + 13 + 15 + \dots + 29$ $\quad n^2 \Rightarrow n = (LN+1)/2$

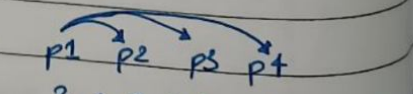
$[(29+1)/2]^2 - [(9+1)/2]^2 = 15^2 - 5^2 = 200$

Q13. $2^2 + 4^2 + 6^2 + 8^2 + \dots + 20^2$

$2^2 [1^2 + 2^2 + 3^2 + \dots + 10^2] = 4 \times [10(10+1)(20+1)]/6$

$= 1540$

Q14. Take a small sample set Eg. 4.



$\therefore$ for 4, $= 3+2+1$.

$= n(n+1)/2$, $n = 24$.

$= 300$ hand-shakes.

Q15. $2 \times n(n+1)/2$

$2 \times 11(12)/2 = 132.$

Q16. For any knockout tournament, no. of matches $= (n-1)$.

$\therefore$ Ans $= 64-1 = \underline{63}$.

Q17. Number of squares $= \Sigma n^2 = n(n+1)(2n+1)/6$

Number of rectangles $= \left({}^{n+1}C_2\right)^2 = \left(\frac{(n+1)!}{2!\,(n+1-2)!}\right)^2 = \left(\frac{(n+1)!}{2\,(n-1)!}\right)^2$

formula for ${}^nC_r = \frac{n!}{r!\,(n-r)!}$ $= \left(\frac{n(n+1)}{2}\right)^2$ squares included.

Q18. 21 prime numbers

Q19. All 100 numbers

Q20.

59 : 60	119	} not-prime numbers
70 : 73	143	
80 : 81	161	
Ans = 90 : 91	181	

Q21 4 6 12 18 30 42 60 72 102 108 — —

The series is Average of Twin Prime Numbers.

Eg. Average of 3 & 5 is 4

5 & 7 is 6

Next twin prime no's after 108 are 137 and 139 = Avg 138

149 and 151 = 150

3 5 11 17 29 41 59 71 101 107 ____

Twin prime numbers : 1st number of the pair.

∴ Next two no's are 137 and 149

Q22 Ans = 18

Q23. $xy_{max} = 200$, $x=10$, $y=10$

$xy_{min} = 19$, $x=1$, $y=19$

Q24. $(x^2+y^2)_{max} = 362$, $x=1$, $y=19$

$(x^2+y^2)_{min} = 200$, $x=10$, $y=10$

Q25. $x+y+z=9$

equation for flights (total) = $xy+yz+xz$ → max at center

∴ $x=y=z=3$

$xy+yz+xz = 3(3)+3(3)+3(3) = \underline{\underline{27}}$.

for $x \neq y \neq z$, $x=4$, $y=3$, $z=2$

$xy+yz+xz = 4(3)+3(2)+4(2) = \underline{\underline{26}}$

Q26. 42 & 98 difference method.

42 —— 98-42 = 56

42 —— $\underline{\underline{14}}$

Q27. No's 43 65 109 } New No's → 43-1 65-2 109-4

remainders 1 2 4 } 42 63 105

Now apply difference method.

42 $\underline{\underline{21}}$ 63

Q28. 53 101 173.

let remainder = x.

New Nos = $53-x$ $101-x$ $173-x$

difference method:

$101-x-(53-x) = 48$

$173-x-(101-x) = 72$

again difference method on 48 & 72, HCF = 24.

Find x: $53-x$ → closest multiple of 24 is 48.

$\therefore 53-x = 48$

$x = 5$

Q29. HCF of 35 & 60 = 5

$\therefore$ Ans = 11111 (five 1's).

Q30. HCF of 60 & 100 = 20

$\therefore$ Ans = 20 3's.

Q31. $3^{45}-1$ & $3^{120}-1$ HCF of 45 & 120 = 15

Ans: $3^{15}-1$.

Q32. $7^{20}-1$ & $7^{20}+1$. $\text{odd}^{\text{pow}} = \text{odd}$. $\therefore 7^{20} = \text{odd}$.

odd - 1 & odd + 1

i.e. even & even ⇒ Two consecutive even numbers always HCF as 2

Q33. $8^{20}-1$ & $8^{20}+1$ $\text{even}^{\text{pow}} = \text{even}$.

odd & odd. ⇒ Two consecutive odd numbers always have HCF = 1.

Q34. Minimum class rooms = Max. students in One class.

∴ HCF of 48, 84 and 72 = 12.

no. of class rooms = 48/12 + 84/12 + 72/12

= 17

Q35. Minimum tiles = max area for each tile.

HCF of 48 & 84 = 12.

No. of tiles = $\frac{\text{floor area}}{\text{tile area}} = \frac{48 \times 84}{12 \times 12} = 28$

Q36. $5(a+b) = 80$

$a+b = 16$

co-prime factors for 16 : 5 11

3 13

-1 17

-2 19

Ans: 4 pairs.

Q39 120.

Q40 LCM of 259 and 407 = $(259 \times 407)/37 = 2849$

203 and 319 = $(203 \times 319)/29 = 2233$.

Q41. LCM of 12, 16 & 20 = 240.

Ans = 240 + 7 = 247

Q42. LCM of 24, 56 and 45 = 360

Ans = 360 - 7 = 353

Q44. 7 | 8 10 | → 8+4 = 12 = 10+2

1 4 2

LCM of 8 and 10 = 40.

40 + 12 = 52

52 + 40 = 92 → satisfies all conditions

Q45. 5 | 6 7 | → 7+2+7 = 16 = 6+4+6

3 4 2

LCM of 6 and 7 = 42

42 + 16 = 58 → Ans.

Q46. LCM of 15, 24 and 40 = 120.

Ans = 120s = 2min

Total meets in half hour = 15.

Q47. LCM of 5, 6, 7 and 8 is 840.

remainder = 3.

∴ Actual number = 840(k) + 3

↳ variable.

840(k) + 3 should be completely divisible by 9.

k = 2 satisfies the condition.

840(2) + 3 = 1683.

Q48. Eliminate options by checking divisibility tests.

	3604	1015	1022	18004 ← Ans.
20	✓	×	×	✓
7	×			✓

Q51 $3 \quad 2.7 \quad 0.09 \Rightarrow \frac{3}{1} \quad \frac{27}{10} \quad \frac{9}{100}$

$$LCM = \frac{\text{LCM of } (3, 27, 9)}{\text{HCM of } (1, 10, 100)} = \frac{27}{1} = 27$$

Q52
$2^{1/2} = 2^{6(1/12)} = 64^{1/12}$
$3^{1/3} = 3^{4(1/12)} = 81^{1/12}$ → largest.
$4^{1/4} = 4^{3(1/12)} = 64^{1/12}$
$6^{1/6} = 6^{2(1/12)} = 36^{1/12}$

14-11-2022

★ Power Cycles: End digits for consecutive multiplications.

2 = 2, 4, 8, 6

3 = 3, 9, 7, 1

4 = 4, 6

5 = 5

6 = 6

7 = 7, 9, 3, 1

8 = 8, 4, 2, 6

9 = 9, 1

0 = 0

1 = 1

Q Find the last digit of 2^{17}.

17/4 = 1 = 1st digt of 2's power cycle = 2

More questions: $2^{42} = 4$, $2^{63} = 8$, $8^{22} = 4$, $8^{72} = 6$, $3^{35} = 3$, $3^{46} = 9$, $7^{21} = 7$, $7^{83} = 3$, $4^{25} = 4$, $9^{n} = 9$, $5^{n} = 5$, $6^{n} = 6$, $0^{n} = 0$, $1^{n} = 1$.

Q Last digit of 72^{23} = similar as 2^{23} ⇒ 8

Q $2^{21} + 3^{41} = 2+3 = 5$

Q $9^{11} + 5^{6} = 9+5 = 4$

Q $9^{11} - 5^{6} = 9-5 = 4$

Q $9^{12} - 5^{6} = 1 - 5 = 11-5 = 6$

Q $7^{41} \times 2^{28} = 7 \times 8 = 56 = 6$

Q $7^{27} \div 10$ = } Find remainder = 3

Q $7^{27} \div 100$ = } = 43

★ Special powercycle for 7 (07, 49, 43, 01).

★ Q. Find the remainder when $7^{27}/8$.

⇒ Use 1st 4 powers of 7.

$7^1/8$	$7^2/8$	$7^3/8$	$7^4/8$
⇒ 7/8	49/8	345/8	2401/8

remainders

↳ 7 1 7 1

odd even odd even

→ odd

$7^{27}/8$ ⇒ remainder = 7.

Q53. Find remainder for $4^{33}/5$.

Q54. $4^{96}/6$.

Q55. $5^{37}/4$

Q56 $4^{93}/3$

★ Whenever any power of $(x+1)$ is divided by x, then the remainder is always 1.

Examples: Find remainder

(i) $5^{18}/24 \Rightarrow (5^2)^9/24 = 25^9/24 = 1.$

(ii) $7^{84}/342 \Rightarrow (7^3)^{28}/342 = 1$

(iii) $2^{256}/17 \Rightarrow (2^4)^{64}/17 = 1$

↓

★ Whenever any power of $(x-1)$ is divided by x, then the remainder is $(x-1)$ for odd powers and the remainder is 1 for even powers.

Example: Find remainder

$12^{51}/13 \Rightarrow$ rem = 12

GYAAN: "Make the world that put you here, happy that you were put here."

★ Fermat's Little Theorem:

Let 'a' be a natural number & 'p' a prime number such that a is not divisible by p, then the remainder when a^{p-1}/p is always 1.

Example: (i) $12^{36}/19 \Rightarrow (12^{18})^2/19 \Rightarrow$ rem = 1.

(ii) $8^{30}/11 \Rightarrow (8^{10})^3/11 \Rightarrow$ rem = 1.

★ Factorials.

1! = 1
2! = 2
3! = 6
4! = 24
5! = 120
6! = 720
7! = 5040
8! = 40320
9! = 362880
10! = 3628800

- 2 factorial onwards every factorial is even
- 5 factorial onwards every factorial ends in zero.
- Number of zeros at the end of a factorial = number of fives present in the factorial.

Ex. 5 10 15 20 25
1 2 3 4 (5+1)

↳ 25/5 = 5 ← add
↓
5/5 = 1

Q Calculate number of zero's at the end of 123!

Ans: 123/5 = 24 ⇒ 24/5 = 4

Zero's = 24 + 4 = 28.

Q57. Find number of zeros at end of 160!

15-11-22

Q Find the minimum power of 3 that will divide 24! completely.

Ans: $24/3 \Rightarrow 8/3 \Rightarrow 2$

$8+2 = 10.$

ie. 3^{10}

Q58. Find max. pow. of 2 that will divide 10! completely.

In both of the above questions, denominators were prime numbers.

Q. Find max power of 6 that will divide 18! completely.

Ans: 6 is not prime, factorize it. (3,2).

Always solve for biggest prime numbers possible.

$18/3 \Rightarrow 6/3 \Rightarrow 2$

ie. 6^8

Q59. Find max. power of 35 that will divide 56! completely.

Q. Find remainder for $(1! + 2! + 3! + \cdots + 10!)/24$

Ans:

$1! + 2! + 3! + (4! + 5! + \cdots + 10!)/24$

This block is divisible by 24 as every factorial consists of 24.

$\therefore$ Remainder $= 1! + 2! + 3! = 9$

Q60. Find remainder for $(1! + 2! + 3! + \dots 100!) / 20$

Q61. Find the remainder when $(2! + 4! + 6! + \dots + 100!) / 8$

Q62. Use exactly 5 zeros & any mathematical operations to get 120.

Q63. Use exactly 4 zeros & any mathematical operators to get 120.

Q64. Use the numbers 1, 1, 5 and 8 along with the 4 basic operators to generate 10.

★ Decimal & Fractions.

1/2	0.5	0.7 = 7/10
1/3	0.333...	0.45 = 9/20
1/4	0.25	0.275 = 11/40
1/5	0.2	0.444... = 4/9
1/6	0.1666...	0.777... = 7/9
★ 1/7	0.142857	0.2727... = 3/11
1/8	0.125	0.6363... = 7/11
1/9	0.111	0.4747... = 47/99
1/11	0.0909...	
1/12	0.08333...	0.aaa = a/9
1/15	0.0666...	0.abab = ab/99
1/16	0.0625	0.abcabc = abc/999

Q Represent 0.91666 in p/q form.

Ans:

$$0.91666 = 0.91\bar{6} = \frac{916-91}{900} = \frac{825}{900} = \frac{11}{12}$$

(916 → without bar num.; 900 → no. of digits without b; no. of bar digits.)

Q65. 0.68181... in p/q form.

Q66. 0.61666... in fractions.

★ Comparing Fractions:

If the difference between the numerator & denominator is constant then the fraction with highest numerator is the greatest (for fractions < 1).

For the fractions > 1, if difference is constant then the fraction with smallest numerator is biggest.

Q Find the biggest fraction.

$\frac{8}{11}$ $\frac{14}{17}$ $\frac{19}{22}$ $\frac{23}{26}$

Q Find the biggest fraction.

$\frac{7}{5}$ $\frac{11}{9}$ $\frac{13}{11}$ $\frac{19}{17}$

Q. Arrange in ascending order.

Soln: $\frac{4}{7}$ $\frac{7}{8}$ $\frac{9}{11}$ $\frac{3}{5}$

Diff → 3 1 2 2 ⇒ LCM = 6.

$\frac{4}{7} \times \frac{6}{3}$ $\frac{7}{8} \times \frac{6}{1}$ $\frac{9}{11} \times \frac{6}{2}$ $\frac{5}{5} \times \frac{6}{2}$

The fraction with highest numerator value will be first in the ascending order.

$$\frac{7}{8} > \frac{9}{11} > \frac{5}{5} > \frac{4}{7}$$

* 142857 = Cyclic Number. If multiplied by 2, 3, 4, 5 or 6 the answer will be a cyclic permutation of itself.

$$1/7 = 0.\overline{142857}$$

$142857 \times 2 = 285714$

$\times 3 = 428571$

$\times 4 = 571428$

$\times 5 = 714285$

$\times 6 = 857142$

$\times 7 = 999,999$

Q If abcdef $\times 6$ = defabc. Find a+b+c.

Ans: abcdef = 142857

a+b+c = 1+4+2 = 7

Q In above code, how will you write dac?

Q67. A number x is of the form 0.abab... Which of the following numbers is definitely an integer.

a. 9x b. 11x c. 33x d. 297x

Q68. Ram spends 1/3rd of his income on travelling & 1/2 of his income on rent and saves 1000. Find his income.

Q69. Sita spends 1/3rd of her income on cosmetics & 1/4th of the remaining on cloths & saves 18000. Find her income.

Q70. Ravan spends 1/6th of his income on entertainment, 1/4th of remaining on shaving and 2/3rd of remaining on education. Finally Ravan saves 20,000₹. Find his actual income.

Q71. Bali spends 1/4th of income on Gym, 1/3rd of remaining on protein suppliments and half of the remaining on Angat's education & still saves 18,000₹. Find his income.

* Q72. Hanuman has some apples with him. He gives half of his apples +1 extra to Laxman. Then Hanuman gives half of the remaining +1 extra apple to Jambavan. Hanuman then repeats the same process separately for Nal, Neela and Sugriva. Finally Hanuman saves 5 apples for himself. Find total number of apples.

Note: When we have repetation of any operation, backtrack the operations in sequence, followed by reversing operations.

Q73. Meghnam defeats Indra and takes away his diamonds. While leaving swarga, Meghnam gives half of these diamonds +2 extra diamonds to Urvashi. Then he repeats same process separately for Rambha & Menaka. Then he manages to save 1 diamond for his wife

★ (i) The difference between a 2 digit number and its reverse is always a multiple of 9.

(ii) The sum of a 2 digit number and its reverse is always a multiple of 11.

(iii) The difference between a 3 digit number and its reverse is always a multiple of 99.

16-11-2022

★ Averages:

For an Arithmatic Progression, $avg = \frac{1st\ term + last\ term}{2}$

Q. Find avg of all numbers between 100 and 900.

$$avg = \frac{101+899}{2} = 500$$

Q74. Two sets x and y are such that

$x = \{1, 3, 5, \ldots, 2007\}$

$y = \{2, 4, 6, \ldots, 2008\}$

Let A and B be the averages of set x & y. Find (B-A).

Q75. Let $x_1, x_2, x_3, \ldots x_9$ be nine consecutive natural numbers such that the avg of first 7 numbers is x. What would be the average of all nine numbers?

Q76. Same above question for seven consecutive natural numbers. Avg of first 5 = x. Find avg of all 7.

★ Q77. 10 years ago the total of the ages of 8 members in a family was 232 years. 3 years later the from this point (7 years ago) a family member aged 60 years died. The next day a baby was born in the family.

3 years later another family member aged 60 dies and the next day a baby was born in the family. Find the present average of family members.

Q78. The average weight of 30 students is 58 kg. Two students with weight 49 and 52 kg are replaced by two new students 80 and 81 weight reopectively. Find new average.

Tip: Average is a number that is distributed equally among all the members.

Q79. The average marks of a class of 84 students were 61. Two students with 90 and 94 marks were replaced by 60 and 82.

Q80. While entering DBT marks by mistake Kishori Madam entered Ashutosh's marks as 83 insted of orignal 13 marks. After the correction the average score decreased by 0.5. How many students wrote the retest.

Q81 The average age of students, in a class of 30 students is 9 years. When the teachers age is also added the avg age becomes 10. Find teachers age.

Q82. The average marks of class of 97 students were 51. When a new student was added the average became 51.5. Find the marks of newly added student.

★ Q83 After 37 matches Virats batting average was x, in next match the player scored 162 runs & got out. Thereby increasing his batting average by 3. Find virats new batting average.

a. 45 b. 48 c. 51 d. 54

★ Answers

Q53. $4^{33}/5$

	$4^1/5$	$4^2/5$	$4^3/5$	$4^4/5$
rem →	4	1	4	1
	odd	even	odd	even

→ odd

remainder of $4^{33}/5$ = 4.

Q54. $4^{96}/6$

$4^1/6$	$4^2/6$	$4^3/6$	$4^4/6$
4	4	4	4

remainder of $4^{96}/6$ is 4.

Q55. $5^{37}/4$

$5^1/4$	$5^2/4$	$5^3/4$	$5^4/4$
1	1	1	1

remainder of $5^{37}/4$ is 1

Q56. $4^{93}/3$

$4^1/3$	$4^2/3$	$4^3/3$	$4^4/3$
1	1	1	1

remainder of $4^{93}/3$ is 1.

Q57. $160/5 \Rightarrow 32/5 \Rightarrow 6/5 \Rightarrow 1$

no. of zeros = 32 + 6 + 1 = 39.

Q58. $10/2 \Rightarrow 5/2 \Rightarrow 2/2 \Rightarrow 1$

Ans: 2^8

Max power = 8

Q59. 35 is not prime. Factors are (7,5).

$56/7 \Rightarrow 8/7 \Rightarrow 1$

max power = 8+1 = 9.

Q60. remainder for $1! + 2! + 3! + 4! + (5! + \cdots + 100!) / 20$

↑ divisible part.

Ans = $1! + 2! + 3! + 4! = \underline{33}$.

Q61. remainder for $2! + (4! + 6! + \cdots + 100!) / 3$

↳ Ans = $\underline{2}$

Q62. $0! = 1$.

Ans = $(0! + 0! + 0! + 0! + 0!)! = 120$

Q63. Ans = $[(0! + 0! + 0!)! - 0!]! = 120$.

Q64. Ans = $\dfrac{8}{1 - 1/5} = \dfrac{8}{4} \times 5 = \dfrac{40}{4} = \underline{10}$.

Q65. $0.\overline{681} \Rightarrow \dfrac{681 - 6}{990} \Rightarrow \dfrac{675}{990} \Rightarrow \underline{\dfrac{15}{22}}$.

↳ without bar number

↓ ↳ number of digits without bar

number of digits with a bar

Q66. $0.61\overline{6} \Rightarrow \dfrac{616 - 61}{900} = \underline{\dfrac{37}{60}}$

Q67. $0.\overline{abab} \Rightarrow ab/99$ Ans

(a) $\dfrac{9 \times ab}{99} = \dfrac{ab}{11}$ (b) $\dfrac{11 \times ab}{99} = \dfrac{ab}{9}$ (c) $\dfrac{33 \times ab}{99} = \dfrac{ab}{3}$ (d) $\dfrac{297 \times ab}{99} = 3ab$

Q68. Total income = x.

$$x - \frac{x}{3} - \frac{x}{2} = 4000$$

$$x - \frac{5x}{6} = 4000$$

$$\frac{x}{6} = 4000 \Rightarrow x = 24,000/-$$

Q69. Total = x

$$x - \frac{x}{3} = \frac{2x}{3} \Rightarrow \frac{2x}{3} - \frac{2x}{3} \times \frac{1}{4} = \frac{x}{2}$$

$$\Rightarrow \frac{x}{2} = 18,000 \Rightarrow x = 36,000$$

Shortcut: given fractions = 1/3, 1/4.

convert into 2/3, 3/4.

$$\Rightarrow x \times \frac{2}{3} \times \frac{3}{4} = 18,000$$

$$x = 36,000$$

Q70

$$x \times \frac{5}{6} \times \frac{3}{4} \times \frac{1}{3} = 20,000$$

$$x = 96,000$$

Q71.

$$x \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = 18,000$$

$$x = 72,000$$

*Q72. Use back-tracking, reverse the operations.
Hanuman gives away half apples +1. ie. ÷2 −1.
Reverse the operations & the sequence: +1 ×2.

Apples remaining at the end = 5.
(+1 ×2) → (5 + 1) × 2 = 12
(12 + 1) × 2 = 26
(26 + 1) × 2 = 54
(54 + 1) × 2 = 110
(110 + 1) × 2 = 222 → Ans

Q73. ÷2 −2 → +2 ×2
Diamonds left = 1.
(1 + 2) × 2 = 6
(6 + 2) × 2 = 16
(16 + 2) × 2 = 36.

Q74. $B - A = \frac{2008+2}{2} - \frac{2007+1}{2} = 1005 - 1004 = 1$.

Q75. Avg $(x_1 + x_2 + \cdots + x_7) = x$
Avg $(x + x_8 + x_9) = ?$ = Avg $(x_1 + x_2 + \cdots + x_9)$.
$= [x + (x+1) + (x+2)]/3$
consecutive numbers.
$= (3x+3)/3 = x+1$

Q76. Avg $(x_1 + x_2 + \cdots + x_5) = x$
Avg $(x_1 + x_2 + \cdots + x_7)$ = Avg $(x + x_6 + x_7) = ?$
$= [x + (x+1) + (x+2)]/3$
$= x+1$.

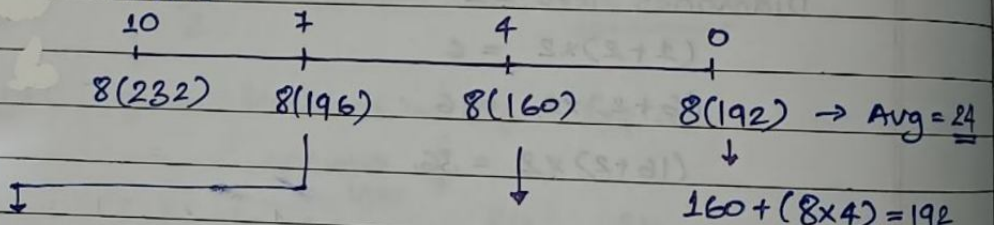
Q78. Total weigh increased = (80-49) + (81-52)
= 60

60/30 = 2 ie. Avg increased by 2.
New average = 60.

Q79. (60 - 90) + (82 - 94) = -30 - 12 = -42

-42/84 = -0.5
New avg = 61 - 0.5 = 60.5

Q77. Timeline:



= 232 + (8×3) - 60
= 196

= 196 + (8×3) - 60
= 160

Shortcut:

old avg = 232/8 = 29
age profit = 8 × 10 = 80
age loss = 60 × 2 = 120] Diff = 40 loss

$$29 - \frac{40}{8} = 24$$

Q80. Avg = $\frac{\text{total marks}}{\text{total students}}$ ⇒ total std = $\frac{\text{total marks}}{\text{Avg.}}$

$= \frac{33 - 13}{0.5} = 40$ students

Q81.

newly added value = old avg ± (num ± 1) × (Diff)

profit/loss; added/removed; num: old number of students; Diff: difference in avg.

$= 9 + (30 + 1) \times (1)$

$= 9 + 31 = \underline{40}.$

Q82.

$= 51 + (97 + 1) \times (0.5)$

$= 51 + 49 = \underline{100}$

* Q83.

$\text{old avg} = x = \frac{\text{total runs}(t)}{37} \Rightarrow 37x = t \quad \text{—①}$

$\text{new avg} = x + 3 = \frac{\text{total runs}\ (t + 162)}{38}$

$\Rightarrow 38x + 38(3) = t + 162 \quad \text{—②}$

② − ① ⇒ $162 = x + 38(3)$

$x = \underline{48}$

New avg $= x + 3 = \underline{51}.$

16-11-2022

★ Mixtures and Alligations

Q 20 kg of rice of 62 Rs/kg is mixed with 10 kg of rice priced 56 Rs/kg. Find price per kg of mixture.

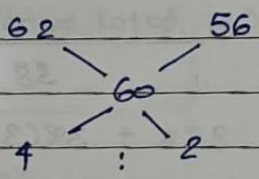
Ans:

$$\text{Group avg} = \frac{A_1N_1 + A_2N_2}{N_1 + N_2}$$

$$\frac{20 \times 62 + 10 \times 56}{10+20} = \frac{1240+560}{30} = \frac{180}{3} = 60$$

Q In what average should we mix rice at 62₹/kg with rice at 56₹/kg to get a new variety priced at 60₹/kg.

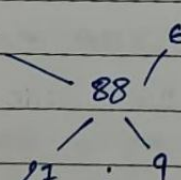
Ans:



2 : 1

Q In what ratio shall we mix a 97% acid solution to a 61% acid solution to get 88% solution.

Ans:



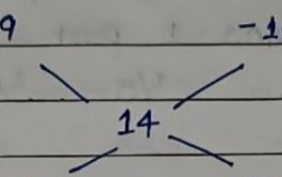
3 : 1

★ Maintain the order mentioned in question.

Q A farmer purchases a bullock & a cart for a total of 13000. He sells the bullock at 29% profit and

the cart at 10% loss. Thereby making a net profit of 14%. Find CP of bullock.

Ans:



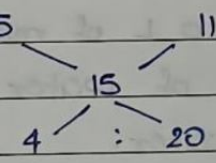
8 : 5

CP of bullock $= \frac{8}{13} \times 13000$

$= 8000.$

Q. In a company the average salary of 21 officers is 35,000 while avg sal of non-officers is 11,000. If avg sal of entire company is 15,000. Find total no. of employees.

Ans:



1 : 5

Total emp $= x$.

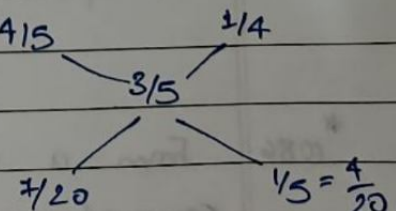
$\frac{1}{6} x = 21$

$x = 21 \times 6 = 126$

Q Two mixtures A and B contains milk and water in the ratio 4:1 and 1:3 respectively. In what ratio shall we mix A and B to get new mixture C containing milk and water in the ratio 3:2.

	Ratios	Milk	Water
A	4:1	4/5	1/5
B	1:3	1/4	3/4
C	3:2	3/5	2/5

For milk:



7 : 4

Q84: A container contains a liquid which is 5 parts syrup and 3 parts water. What part of this liquid must be removed and replaced by pure water so that final mixture contains 1 part syrup & 1 part water.

1/3 1/4 1/5 1/7.

* From a container containing xL of milk if yL of mixture is removed and replaced by water n number of times then the final ratio $\frac{\text{Milk}}{\text{Mix}} = \frac{(x-y)^n}{(x)^n}$

	Milk	Water	Mix	Ratio = Milk/Mix
(i)	100	0	100	$1 = (9/10)^0$
(ii)	90	10	100	$90/100 = (9/10)^1$

Next time we remove 10 L of mixture, we are removing 9L of milk and 1L of water, which will be then replaced by pure water.

(iii)	81	9+10	100	$81/100 = (9/10)^2$

& so on.

Q.85 From a cask containing wine 8L of wine is removed and replaced by water. The same process was repeated 3 more times. Finally the ratio of wine & water was $\frac{16}{65}$. Find quantity of wine initially present.

24 18 32 42

*Q86: From a container containing 40L of pure milk, 4L is removed and replaced by water 3 times. Find final quantity of milk present in the container.

26.34 27.18 28 29.16

***

Q87. Container A contains 100L of pure milk & B contains 100L of pure water. 2 buckets of milk from A is removed & poured into B. Now the same 2 buckets from mixture B is poured into A. Let the % of water in container A be x & % of ~~water~~ milk in container B be y.

Then: $x > y$ $x < y$ $x = y$ Depends on volume of bucket.

GYAAN: "Never Take ANYTHING For Granted, GOD Will Take It Away From You"

Answers:

Q.84 Syrup = $\frac{5}{8}$, water = $\frac{3}{8}$. Pick any one.

water : 3/8 — 1/2 — 1/2, 1/8 — 4:1

or syrup: 5/8 — 1/2 — 1/2, 1/8 — 4:1

∴ The fraction of mixture to be removed = $\frac{1}{5}$.

Shortcut solution:

Assume liquid to be 8L of mixture.

	8L	
initial ratio	5S	3W
final ratio	4S	4W

(1 unit of syrup was replaced by water.)

Ans = 1/5.

Q.85. wine : water = 16 : 65

∴ wine : mixture = 16 : 81

using formula,

$$\left(\frac{x-8}{x}\right)^4 = \frac{16}{81} = \left(\frac{2}{3}\right)^4$$

$$2x = 3x - 24$$

$$x = 24$$

Q86. milk : mixture ratio $= \left(\frac{40-4}{40}\right)^3 = \left(\frac{9}{10}\right)^3$

Final quantity of milk $= \left(\frac{9}{10}\right)^3 \times 40 = \frac{729}{1000} \times 40 = \frac{729}{25} = 29.16.$

Q87. Note: In above question, the quantity of milk removed will be less than 12 L in total, as with every successive removal, ratio of milk is reduced.
So we can directly eliminate first three options.

Q87. Ans: $x = y$.

★ **Mean**

$$AM = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

$$GM = \sqrt[n]{x_1 * x_2 \times x_3 \times \cdots \times x_n}$$

$$HM = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

Q For two numbers x and y AM is $\frac{x+y}{2}$, GM is $\sqrt{xy}$ and HM is $\frac{2}{\frac{1}{x}+\frac{1}{y}} = \frac{2xy}{x+y}$.

Calculate AM, GM and HM for $x = 20$ and $y = 80$.

AM = 50 GM = 40 HM = 32

Calculate AM, GM and HM for $x = 10$ and $y = 10$.

AM = GM = HM = 10.

Note: (i) For same two numbers AM = GM = HM.

(ii) For any two numbers: $GM^2 = AM \times HM$.

Q While going from pune to aurangabad Amrutas speed is 40 km/hr. While comming back to pune her speed is 60km/hr. Find her average speed through the journey.

⇒ Ans: 48

If the distance is same, avg speed = HM.

$$\text{Avg speed} = \frac{\text{Total distance}}{\text{Total time}}$$

Q While going from pune to indore Chancals speed is 40km/hr for 1st five hours and 60 km/hr for next 5 hours. Find avg speed throughout journey.

⇒ Ans: 50 km/hr

If time is same, avg. speed = AM.

Q A car owner buys petrol at 7.5Rs, 8Rs and 8.5Rs per liter for three successive years. What approximately is the average cost per liter of petrol if he spends 4000Rs every year.

7.98 8 8.5 9

⇒

Tip: Had the consumption been same then it was an AM problem, then the ansnswer would have been 8.

This is a classic harmonic mean problem.

$$\text{HM for 3 nos} = \frac{3xyz}{xy+yz+xz}$$

$$\text{HM for 2 nos} = \frac{2xy}{x+y}$$

★ Median

Q. Find the median for 6, 1, 10, 3, 9.

Ans: 1 3 6 9 20

Q Find the median for 12, 20, 24, 10, 32, 64.

Ans: 10 12 20 24 32 64

$$\frac{20+24}{2} = 22$$

★ **Mode:** The number occuring with the highest frequency is called mode.

Q 1, 3, 7, 1, 7, 3, 1 ⇒ mode = 1.

Q 2, 4, 6, 8, 10, 12, 14 ⇒ mode = ? Every number is a mode.

If every number occurs with the same highest frequency, it is called multimodal distribution.

Relation between Mean, Median and Mode:

$$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$$

or

$$\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$

★Q Siddhart while going from Pune to Bhillai, his speed is 20 km/hr. While comming back to Pune his speed is x km/hr. If his avg speed throughout the journey is 40 km/hr find x.

★ Percentage

Fractions	%
1/2	50
1/4	25
1/5	20
1/10	10
1/8	12.5
3/4	75
4/5	80
1/3	33.33
2/3	66.66
1/6	16.66
5/6	83.33
6/5	120
5/4	125
4/3	133.33
3/2	150
1/7	14.28
1/9	11.11
1/11	9.09
1/12	8.33
1/15	6.66
1/16	6.25

Q A no x is 83.33% of y. Find y% of x.

→ $x = \frac{5y}{6} \Rightarrow y = \frac{6x}{5} = 120\%$.

Q A no x is 125% of y. Therefore y is —% of x.

→ $x = \frac{5}{4}y \quad y = \frac{4}{5}x \Rightarrow 80\%$.

Q. A no. x is 25% less than y. Therefore y is ___% more than x.

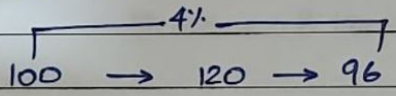
→ $x = \frac{3}{4}y \Rightarrow y = \frac{4}{3}x = 133.33\%$

Q. A no. x is 80% of y, y is 83.33% of z and z is 66.66% of w. If $w = 3600$, find x, y & z.

→ $x = \frac{4}{5}y$, $y = \frac{5}{6}z$, $z = \frac{2}{3}w$.

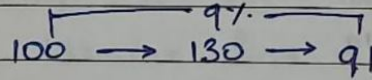
Q. Increase a number by 20%. Now decrease the new number by 20%. Find net percentage change.

⇒



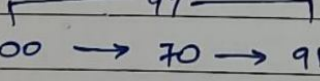
Q. Increase by 30%, decrease new number by 30%. Find net % change.

→



Q. Decrease by 30%, increase by 30%.

⇒



Note: If a number is increased by x% followed by x% decrease, then the net change is always a loss of $\left(\frac{x}{10}\right)^2$%.

Same is for x% decrease followed by x% increase

Q. The length of rectangle increases by 30% while the breadth increases by 10%. Find % change in area.

⇒

$l = 100 \rightarrow 130$

$b = 100 \rightarrow 110$

area = 10000 area = 14300 ⇒ 43%.

Q Length & breadth of a room increase by 25 & 40% respectively, while height decreases by 20%. Find the percentage change in volume.

⇒ Ans: 40% change.

★ Complementary % pairs:

25↑ & 20↓ 25↓ & 33↑

★ Q The price of sugar is increased by 33% because of which a man is able to purchase 5kg less for total of 240 Rs. Find orignal price per kg before increase.

(10, 12, 16, 20).

⇒ x = orignal price per kg ; y = quantity.

$240 = x \times y = 1.33x \times (y-5)$

$xy = 1.33 \cdot x \cdot (y-5)$

$y = 1.33(y-5)$

$y = 20$

$240 = x \times 20$

$x = 12$ → Ans.

★ Q 600 kg watermelon contains 99% water, rest is solid. This watermelon is kept in sunlight. After some time it contains 98% water of its new weight. Find new total weight.

⇒

600
99% / \ 1%
594 6

new weight = x
98% / \ 2%
? 6

2%(6) → 100% (6×50)

x = 300

Q. After some more time in sunlight, it contains 97% water of its new total weight. Find new total weight.

⇒

x
97% / \ 3%
? 6

3% (6) → 100% (6 × 33.33)

x = 200

Note: If the percentage of the constant quantity (generally solid) is multiplied by k, then total weight will be divided by k.

Eg. for 2% ⇒ $\frac{600}{2} = 300$

3% = $\frac{600}{3} = 200$

4% = $\frac{600}{4} = 150$.

Q. Fresh grapes contain 90% water. Dried grapes contain 20% water. How many kg of dried grapes can we make from 40kg of fresh grapes.

⇒

40		x		
90 / \ 10		20 / \ 80		80% (4) → 100% (5)
36	4	?	4	∴ x = 5 kg

Using Formula :

$$80 \text{ to } 10 = 8\% = \frac{40}{8} = 5$$

Q In an election between two candidates every voter voted. 6% of total voters supported NOTA. 63% supported DM who defeated YS by 960 votes.

Find (i) Total voters

(ii) No. of votes received by YS

(iii) No. of NOTA votes

⇒

100%

DM / | YS \ NOTA

63% 31% 6%

$$\frac{x \times 63}{100} - \frac{x \times 31}{100} = 960$$

$x = 30,00$ → total voters

★ Profit & Loss:

CP = Cost Price , Profit SP > CP

SP = Selling Price Loss CP > SP.

Q. SP = 400, profit = 25%. Find CP.

⇒ 1.25 CP = SP = 400

CP = 320

Q. Swanand purchases 2 books for 300 Rs each. Sells 1st book at 20% profit and 2nd at 20% loss. Find net profit% or loss %.

⇒ 300 → 20% ↑ → 360

300 → 20% ↓ → 240

600 → No profit, no loss.

Q Suraj sells 2 books for 300 Rs each, on 1st he makes a profit of 20%. On 2nd he suffers a loss of 20% Find net profit loss %.

⇒

$x \longrightarrow 1.2x = 300 \quad x = 250$

$y \longrightarrow 0.8y = 300 \quad y = 375$

625.

$\text{loss \%} = \frac{25}{625} \times 100 = 4\%$

Note: If profit and loss % for two products is same:

(i) If CP is same → no profit, no loss.

(ii) If SP is same → loss of $\left[\frac{x}{10}\right]^2$ %, x = profit / loss %

Q. A carpenter sells 77 chairs at cost of 88 chairs. Find his profit/loss %.

⇒

$$CP \times 88 = SP \times 77$$

$$\frac{SP}{CP} = \frac{88}{77} \Rightarrow SP > CP \Rightarrow \text{profit.}$$

$$\text{profit \%} = \frac{88-77}{77} = \frac{1}{7} \Rightarrow \underline{\underline{14.28\%}}$$

Q. A book seller sells 96 books at cost of 88 books. Find his net profit % or loss %.

⇒

$$SP \times 96 = CP \times 88$$

$$\frac{SP}{CP} = \frac{88}{96} \Rightarrow SP < CP \Rightarrow \text{loss}$$

$$\text{loss \%} = \frac{96-88}{96} = \frac{11}{12} \Rightarrow \underline{\underline{8.33\%}}$$

Q A fruit vendor sells 800g of apple insed of 1 kg at the cost price. Find his profit/loss %.

⇒

let, CP for 1 kg = 100

∴ CP for 0.8 kg = 80

but, SP for 0.8 kg = 100.

$$\text{profit \%} = \frac{SP-CP}{CP} = \frac{100-80}{80} = \frac{20}{80} \Rightarrow \underline{\underline{25\%}}$$

Q. A fruit vendor sells grapes at 8% loss but he uses the weight of 800g insted of 1 kg. Find profit %.

⇒ let, CP for 1kg = 100

CP for 0.8kg = 80

but SP for 0.8 kg = 8% loss on CP of 1 kg

= 92

$$\text{profit \%} = \frac{92-80}{80} = \frac{12}{80} \Rightarrow 15\%$$

Q. A fruit vendor cheats wholeseller as well as the customers by 10% each. If he sells the fruits at CP, find his profit %.

⇒

Summary: vendor pays for 10 apples, received 11.

vendor sells 9 apples, gets paid for 10.

$$\text{profit \%} = \frac{11-9}{9} = \frac{2}{9} = 22.22\%$$

★ Discounts

Q. What a single discount is equal to 2 successive discount of 20% and 10%.

⇒

$$100 \xrightarrow{-20\%} 80 \xrightarrow{-10\%} 72$$

Ans: 28%

Q Which of the following successive discount options, is the best deal for customer:

30, 20 25, 25 40, 10, all are same.

Note: If the sum of successive discount pairs is same, then the one with biggest first discount is best.

Q. A shopkeeper sells a watch at 20% profit. One day he gets a discount of 10% from the wholeseller. But he increases the selling price by 72 Rs, thereby making a net profit of 40% today. Find the price of watch before discount was offered.

192 240 800 1200.

⇒

$$x \longrightarrow 1.2x$$

$$0.9x \longrightarrow (1.2x + 72) = \left(0.9 + 0.9 \times \frac{40}{100}\right)x$$

$$1.2x + 72 = 1.26x$$

$$72 = 0.06x$$

$$x = \frac{7200}{6} = \underline{\underline{1200}}.$$

★ Simple & Compound Interest.

$$SI = \frac{P \times N \times R}{100}, \quad \text{Amount} = \underbrace{P}_{\text{constant.}} + I$$

SI is an example of AP.

Q. Using SI a bank offers 7.5%, 7.75% and 9.75% for 1st, 2nd and 3rd year respectively on the principal of 4000 Rs. Find SI.

→

$$\begin{array}{r} 7.5 \\ +\ 7.75 \\ 9.75 \\ \hline 25.00\% \end{array}$$

$$SI = \frac{4000 \times 25 \times 1}{100} = \underline{\underline{1000}}$$

★ Q Using CI the principal doubles in 9 years. In how many years will the principal become (i) 4 times (ii) 8 times.

★ Compound Interest.

$$CI = A - P \quad \Rightarrow \quad A = P\left(1 + \frac{R}{100}\right)^N$$

CI is an example of GP.

Ans: (i) 4 times = 9+9

$100 \xrightarrow{9} 200$

$200 \xrightarrow{9} 400$

(ii) 8 times = 9+9+9

$400 \xrightarrow{9} 800$

Q The difference between CI and SI for 2 years on certain principal at 20% per annum is 84 Rs. Find CI and SI.

→ $SI = x \longrightarrow 1.2x \longrightarrow 1.4x$

$CI = x \longrightarrow 1.2x \longrightarrow 1.44x$

$CI - SI = (1.44 - 1.40)x = (0.04)x = 84$

$x = 2100$

Q Total SI and CI for 2 years on a certain principal are 360 Rs and 378 Rs. Find the rate of interest.

→

SI = 360 = 180 per year

CI = 378 = 180 (1st year), 378 − 180 = 198 (2nd year)

198 = (180 + 18)

18 = 10% of 180

★ Work and Time.

Set 1:

Q Two workers A and B can finish a job in 20 and 30 days.

(i) In how many hours days will job be done if A & B work togeather?

(ii) A and B start working togeather, after how many days should A leave the job if total work is completed in 24 days.

(iii) A and B start working togeather, after how many days should B leave the job if total week is completed in 18 days?

(iv) A starts working alone. After how many days should B join him if total work done in 14 days.

⇒

Assume total work ~~done~~ is LCM of 20 & 30 = 60 units ~~do~~.

∴ To finish 60 units of work,

A does 3 units of work in 1 day. ($^{60}/_{20} = 3$)

B 2 units . ($^{60}/_{30} = 2$)

(i) A+B ⇒ $^{60}/_{5}$ ⇒ 12 days

(ii) Total days = 24.

B works for all 24 days. Hence units of work done by B in 24 days = 2 × 24 = 48 units.

remaining 12 units work is done by A.

$12/3$ = 4 days.

(iii) A ⇒ 18 × 3 = 54 units

rem ⇒ $6/2$ = 3days

(iv) A ⇒ 14×3 = 42 units

rem ⇒ 18/2 = 9 days.

14 - 9 = 5 days.

Hence B should join after 5 days.

Set 2:

Q 3 workers A, B & C can finish a job in 12, 15 and 20 days respectively.

(i) In how many days will the job be done if A, B and C work togeather.

(ii) After the job was completed, they were paid a total of 3600. Find the share of A.

(iii) A, B and C work togeather for 2 days. Now C leaves the job. In how many days will A and B togeather finish the job?

(iv) A works alone for 3 days. Then B works alone for 6 days. In how many days will C finish remaining work?

(v) A, B and C work togeather for 3 days. Now B leaves the job. In how many days will job be done if A and C work on alternate days?

★ (vi) A, B and C work togeather for 4 days. Now B is leaves the job. In how many exact days will remaining work be done if A & C work alternately starting with A.

(vii) In the above question, what if C and A work alternately starting with C.

⇒ Total work = 60 units.

per day work done : A = 5, B = 4, C = 3.

(i) $(A+B+C) \Rightarrow 60/12 = 5$ days.

(ii) share of A $= 3600 \times \frac{5}{12} = 1500$.

(iii) After 2 days $\Rightarrow 60 - (12 \times 2) = 36$.

rem $\Rightarrow 36/(A+B) \Rightarrow 36/9 = 4$ days.

(iv) A works alone for 3 days $\Rightarrow 60 - (5 \times 3) = 45$.

B alone 6 days $\Rightarrow 45 - (4 \times 6) = 21$.

rem $= 21/C = 21/3 = 7$ days.

(v) $(A+B+C)$ for 3 days $\Rightarrow 60 - (12 \times 3) = 24$.

A & C $= 5+3 = 8$ units in 2 days.

rem $\Rightarrow 24/8 = 3 \Rightarrow 3 \times 2$ days $\Rightarrow$ 6 days

(vi) $(A+B+C)$ for 4 days $= 60 - (12 \times 4) = 12$.

A & C, alternate, starts with A.

$12/8 \Rightarrow$ not perfectly divisible.

$12 \xrightarrow{-5} 7 \xrightarrow{-3} 4 \xrightarrow{-4/5} 0$

day 1 day 2 4/5 day 3 $= 0.8$

Ans = 2.8 days

(vii) This time C starts first:

$12 \xrightarrow{-3} 9 \xrightarrow{-5} 4 \xrightarrow{-3} 1 \xrightarrow{-1/5} 0$ (0.2)

Ans = 3.2 days.

GYAAN: "Genius is about knowing when to stop."

★ **Boats and Rivers:**

x = speed of boat

y = speed of river current.

$x \geq y$.

Downstream speed $= x + y$

Upstream speed $= x - y$.

Q. How much time will Sahil take to row 60 km downstream & back if speed of boat & river are 10 km/hr and 5km/hr respectively. Ans: 16.

Q. Arbaz rows to a place 55 km away & comes back in 16 hrs. Find speed of boat if speed of river is 3km/hr. Ans: 8

Q Raju rows to a place 96km away & comes back in 20 hrs. Find speed of boat if speed of the river is 2km/hr. Ans: 10.

Q. Shubham travels 84 km downstream and 33 km upstream in 7hrs. The next day he travels 63 km downstream & 55 km upstream in 8 hrs. Find speed of boat & river current.

⇒

$$\frac{84}{x+y} + \frac{33}{x-y} = 7$$

$(84, 63)$ HCF $= 21$

$(33, 55)$ HCF $= 11$

$$\frac{63}{x+y} + \frac{55}{x-y} = 8$$

$x+y = 21$ & $x = 16$

$x-y = 11$ $\quad y = 5$

Q. A boat takes 16hrs for travelling ~~downstairs~~ downstream from point A to point B & comming back to a point C located in middle of A and B. If the speed of the boat is 14km/hr & the river is 4km/hr. Find the distance between A and B.

→ (160 180 200 220.)

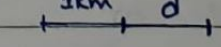
$$\frac{d}{18} + \frac{d/2}{10} = 19 \Rightarrow \frac{d}{18} + \frac{d}{20} = 19$$

$$38d = 19 \times 18 \times 20$$

$$d = 180.$$

Q. A swimmer jumps from the bridge over a canal & swimmers 1 km. After that 1st km he passes a floating tork. He continues swimming for ~~that~~ half hour and then turns around and swims back to the bridge. The swimmer & the cork reach bridge at same time. Find the speed of canal.

(1 2 3 4).



⇒ Swimmer speed = x, current speed = y.

$$\frac{1}{y} = \frac{1+D}{x+y} + \frac{1}{2}$$

For $y = 2$, $\frac{1}{2} = \frac{1}{2} + \frac{1+D}{x+y} \Rightarrow \frac{1+D}{x+y} = 0$ ✗ invalid condition, return time cannot be zero

For $y > 2$, (Eg $y = 4$).

$$\frac{1}{4} = \frac{1}{2} + \frac{D+1}{x+y} \Rightarrow \frac{D+1}{x+y} = -\frac{1}{4}$$

← invalid, time cannot be -ve

For $y < 2$, (Eg $y = 1$).

$$\frac{1}{1} = \frac{1}{2} + \frac{D+1}{x+y} \Rightarrow \frac{1}{2} = \frac{D+1}{x+y}$$

✓ valid.

Now the only option for $y < 2$ is $\underline{y = 1}$.

★ Race

Q In a 100m race A beats B by 20m. By how many meters will A beat B in a race of 200m, 500m and 50m.

⇒ 200m → beats by 40m.
500m → 100m.
50m → 10m.

Q In a 100m race A beats B by 20m. In same race B beats C by 10m. Therefore A beats C by how many meters?

⇒

A	B	C
100	80	x
	100	90

$100x = 80 \times 90$

$\underline{x = 72m}$

A beats C by $(100 - 72)m = 28m$.

Q. In a 100m race A beats B by 20m while B beats C by 20m. A beats C by how many meters?

⇒

A	B	C	
100	80	x	$x = 64$
100	80		$100 - 64 = \underline{\underline{36m}}$

Q In a 100m race A beats B by 20s, while B beats C by 10s. Therefore A beats C by — sec.

⇒ For time related comparisons, directly add.

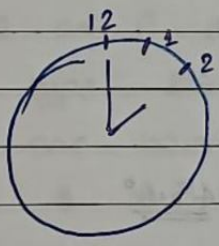
$20 + 10 = \underline{\underline{30s}}$.

★ Clocks:

60 minute spaces.

Speed of minute hand = 60 min spaces / hr

= 1 min space / min.



Hr hand speed → 5 m.s. / hr = $\frac{5}{60} = \frac{1}{12}$ m.s/min

Relative speed = $1 - \frac{1}{12} = \frac{11}{12}$ m.s/min

- The two hands coincide 11 times in 12 hr period ie. 22 times in a day.

- The two hands are in opposite direction 11 times in 12 hr period ie. 22 times a day.

- The two hands make 22 right angles in 12 hours period ie. 44 right angles in a day.

Q At what time between 2 and 3 the two hands coincide.

→ $\frac{10}{1\frac{1}{12}} = 10 \times \frac{12}{11} \rightarrow 10 + \frac{10}{11}$ → ie. 2 hr $10 + \frac{10}{11}$ min

(Speed)

Q. Between 4 and 5 when will the two hands coincide?

→ $20 \times \frac{12}{11} \rightarrow \frac{240}{11} \rightarrow 21\frac{9}{11}$ ie. 4 hr $21\frac{9}{11}$ min.

Q Between 3 and 4 when will two hands come in opposite direction.

⇒ $45 \times \frac{12}{11}$

★ The angle between two hands at any time H:M is $30H - 5.5M$.

Q Angle at 7:30.

⇒ $30(7) - 30(5.5) = \underline{\underline{45.0°}}$

Q Angle at 6:20

⇒ $30(6) - 20(5.5) = 70°$

★ A pendulum clock takes 7sec to strike 7. In how many seconds will it strike 10?

★ Work and Time continued

Q 2 workers can finish a job in 16 and 12 days respectively

A → 48/16 = 3 unit/day

B → 48/12 = 4 unit/day.

↓ LCM

In how many exact days will job be done if A & B work on alternate days starting with A.

⇒ 48/(3+4) ⇒ q = 6, rem = 6

6 × 2 = 12 whole days.

$6 \xrightarrow{-3} 3 \xrightarrow{-3/4} 0$; $\frac{-3}{4} = 0.75$

13th day

Ans: 13.75 days.

Q. P, Q and R are 3 typists who are working simultaneously can type 216 pages in 4 hours.

① [In one hour R can type as many pages more than Q as Q can type more than P.] In 5 hours R can type as many pages as P can type in 7 hours. How many pages does each of them type per hour?

(A) 14 17 20 (C) 15 18 21

(B) 15 17 22 (d) 16 18 22

⇒ P+Q+R = 216/4 = 54 pages/hour.

Statement 1 tells us it is an AP.

The only two options with AP are (A) & (C).

Only (C) option has its addition equal to 54.

Ans: C.

Q. 2 workers A and B togeather can finish a job in 12 days while B and C togeather takes 16 days. C & A togeather takes 24 days. In how many days can A alone finish total work?

A & B	B & C	C & A	
12	16	24	⇒ 48.

$A+B = 48/12 = 4$

$B+C = 48/16 = 3$

$C+A = 48/24 = 2$

$2A+2B+2C = 9$

$A+B+C = 4.5$

$A+(3) = 4.5$

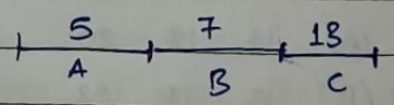
$A = 1.5$

Q. A and B togeather can finish a job in 12 days while B and C togeather take 15 days. A starts working alone once for 5 days & leaves the job. Then b starts working alone for 7 days and leaves. Finally C works alone for 18 days and finishes the job. In how many days can C alone finish total work?

$A+B = 12$

$B+C = 16$] LCM = 48

$A+2B+C = 28$



$5A+7B+13C = 48$

$5A+5B+2B+2C+11C = 48$

$5(12)+2(16)+11C = 48$

$5(4)+2(3)+11C = 48$

$C = 2$

$48/2 = 24$ days

★ **Efficiency.**

Efficiency and time are inversly proportional.
Efficiency and work are directly proportional.

Q. A is twice as efficient as B. Working alone A can finish a job 11 days before B can do it. In how many days can B finish it alone?

⇒

	A		B
Eff:	2	:	1
Time:	1	:	2
	11		22

Q A is twice as efficient as B working togeather they can finish a job in 30 days. In how many days can A finish it alone.

⇒

Eff:	2	:	1
Time:	1	:	2
Work:	2	:	1

A & B working togeather cover 3 units of job.

∴ In 30 days, job done = 3×30 = 90 units.

Days = $\frac{90}{2}$ = 45 days (for A).

Q. A is 30% more efficient by B. In how many days A alone finishes the job which A & B togeather can finish in $13\frac{1}{3}$ days

⇒

	A		B
Eff:	1.3	:	1

A & B togeather → total 2.3 units of job

Total job done by A & B in 13 days = 2.3×13

A alone = $\frac{2.3 \times 13}{1.3}$ = 23 days

Q. Two grapes crushers take 4 days to crush a certain amount of grapes. If one of them crushed half the grapes of and other crushed the other half they complete the job in 9 days. In how many days will the slower crusher complete its job. alone?

14 15 16 None.

⇒

LCM of 4 8 9 : 36.

Assume total weight of grapes to be 36 kg.

Crusher A & B can crush $\frac{36}{4}$ kg of grapes in 1 day.

$A + B = 9$ kg/day.

Both crushers crush half the amount of grapes in 9 days.

$\therefore \quad \frac{18}{A} + \frac{18}{B} = 9 \quad \Rightarrow \quad \frac{2}{A} + \frac{2}{B} = 1.$

$2A + 2B = A \cdot B$

$2(A+B) = AB$

$AB = 2(9) = 18.$

$A + B = 9 \quad \& \quad A \cdot B = 18$

by factorizing we get $A = 6, B = 3.$

Day required by slower crusher $= \frac{36}{3} = 12$ days.

Ans: (d) None.

Pipes

Q. An inlet pipe A can fill a tank in 24 hours while the outlet pipe can empty the same tank in 60 hours. In how many hours will the tank be filled if A & B work togeather?

⇒ LCM (24, 60) → 120 = capacity of tank.

A = 120/24 = 5 hours to fill tank

B = 120/60 = -2 hours to empty tank.

A+B ⇒ 120/(5-2) = 40 hours.

Q Two inlet pipes A and B can fill a tank in 8 and 12 hours respectively. Outlet pipe C can empty the tank in 6 hours. Total hours to fill tank = ?

⇒ LCM (8, 6, 12) ⇒ 24.

A = 24/8 = 3

B = 24/12 = 2

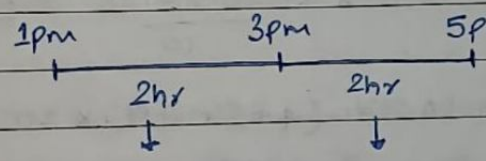
C = 24/6 = -4.

24/(3+2-4) = 24 hours.

Follow-up question:

*Q At 1pm only pipe A is opened, at 3pm pipe B is also opened. At 5pm C is also opened. At what exact time will the tank be full? (11pm, 12Mn., 1am, 2am).

⇒



total capacity of tank = 24.

= A x 2 = 3x2 = 6 units

(A+B) x 2 = 5x2 = 10 units

→ tank is 16 units full, 8 units empty.

To fill the rest 8 units, all 3 pipes work.

A+B+C = 3+2-4 = 1 unit/hr. = total 8 hours to fill rest of the tank.

8 hours from 5pm = 1am

Q 3 inlet pipes A, B and C can fill a tank in 20, 40 and 50 hours respectively. Initially all 3 pipes were opened togeather.
After 4 hours pipe C was closed. After 3 more hours pipe B was closed. Pipe A worked till end. Find % of tank filled by pipe A from start to end.

⇒ LCM(20, 40, 50) = 200.

A = 200/20 = 10 units/hr

B = 5

C = 4.

4hr	3hr	xhr
19	15	10

After 4hr + 3hr, tank filled = 19×4 + 15×3.

Rest is filled by pipe A.

∴ 200 − (19×4 + 15×3) = 10×x

$$x = \frac{200 - 121}{10} = \underline{\underline{7.9 \text{ hr}}}$$

Tank filled by pipe A = (4 + 3 + 7.9) × 10

= 149 units.

$$\% = \frac{149}{200} = \underline{\underline{74.5\%}}$$

★★ An inlet pipe A can fill a tank in 15 hours, while an outlet pipe B can empty the same tank in 25 hours. In how many hours will tank be full if both the pipe work in alternate hours, starting with pipe A?

⇒ LCM (15, 25) = 75 units → tank capacity

A = 75/15 = 5 units/hr

B = 75/25 = −3 units/hr.

A+B = 2 units in 2 hours → alternate. → 1 unit/hr

go upto 70 units to fill tank, as the pipe A will make last iteration to fill remaining 5 units.

To go upto 70 units (A+B) need 70 hours & last 1 hour for pipe A.

Ans = 71

★ Chain Rule

For direct proportions keep the ratio as it is, but for inverse proportions, flip the second ratio.

Q. 24 engineers can finish the project in 12 days. In how many days can 32 engineers finish the same project?

⇒

more engineers, less days = Inverse proportion.

$$\therefore \frac{24}{32} = \frac{x}{12} \Rightarrow x = 9$$

Q. A merchant employeed a servent promising him annual salary of 9000Rs cash and one gold chain. However the servant worked for 9 months only and received 6500 Rs cash and the gold chain. Find the value of gold chain.

9000 + gold chain (G) = 12 months

6500 + G = 9 months.

$$1 \text{ month} = \frac{9000}{12} + \frac{G}{12}$$

$$6500 + G = \left(\frac{9000}{12} + \frac{G}{12}\right) \times 9$$

$$81000 + 9G = 12G + 78,000$$

$$3G = 3000$$

$$\underline{\underline{G = 1000}}$$

Q. On a fort there is enough food so that 240 soldiers can eat for 50 days. After 10 days x soldiers left the fort. Now remaining food lasted for 50 more days. Find x. (40 48 60 80).

⇒

Let total food be F.

F lasts for 240 soldiers for 50 days.

$$\therefore \text{food ate by 1 soldier on one day} = \frac{F}{240 \times 50}$$

$$\text{After 10 days, food left} = F - \frac{F}{240 \times 50} \times (240 \times 10)$$

$$= F - \frac{F}{5} = \underline{\underline{\frac{4F}{5}}}$$

x soldiers left & food lasted for 50 next days.

$$\therefore \frac{4F}{5} = \frac{F}{240 \times 50} \times (240 - x) \times 50$$

$$\frac{4F}{5} = \frac{F}{240}(240 - x)$$

$$\frac{4F}{5} = F - \frac{Fx}{240}$$

$$\frac{x}{240} = \frac{1}{5}$$

$$x = 48$$

Q 32 painters working 6 hrs a day can paint 1 building in 5 days. In how many days can 45 painters working 4 hrs a day paint 3 buildings?

⇒

$$\frac{32 \times 6 \times 5}{45 \times 4 \times ?} : \frac{1}{3} \Rightarrow x = 16$$

Q 36 workers working 4 hrs a day can construct 1 bunglow in 5 days. How many workers are required to build 5 such bunglows if they work 6 hrs a day for 10 days.

→

$$\frac{36 \times 4 \times 5}{x \times 6 \times 12} : \frac{1}{5} \Rightarrow x = 50$$

*Q A contractor employeed 175 workers to construct a bridge in 19 days. However after 12 days only 1/4 th of bridge was built. How many extra workers should he employ so that the bridge is built as per the schedule.

525 725 900 1100.

→

$175 \times 12 \longrightarrow 1/4\ B$

$(175 + x) \times 7 \longrightarrow 3/4\ B$

$x = 25 \times 12 \times 3 - 175$

$x = 725$

GYAAN : "Freedom is Always on the other side of Discipline"

Q 8 men or 12 women can construct a wall in 33 days. In how many days can 14 men and 15 women construct the same wall.

⇒

$$\text{Work} = 8M \times 33 = 12W \times 33$$

$$\text{Same work} = (15W + 14M).x$$

↳ no of days.

$$\left(15 \times \frac{\text{Work}}{12 \times 33} + 14 \times \frac{\text{Work}}{8 \times 33}\right) x = \text{Work}$$

$$\left(\frac{7}{4 \times 33} + \frac{5}{4 \times 33}\right) x = 1$$

$$\frac{1}{11} x = 1$$

$$x = 11 \text{ days}$$

Q. 7 men and 9 boys can paint a building in 12 days. In how many days can 6 men and 11 boys paint same building?

⇒

$7M + 9B \longrightarrow 12$
$6M + 11B \longrightarrow x$

} Data insufficient, as there is no "OR" clause mentioned

⇒ Shortcut solution for previous question:

(i) Compare the efficiencies and convert the problem into units of Men or Women.

$8M = 12W \Rightarrow 1M = 1.5W.$

$12W = 33$ days
$14M + 15W = x$ days } inversely proportional.

$$\frac{12}{14(1.5) + 15} = \frac{x}{33} \Rightarrow x = 11$$

Q 12 Men & 16 Boys can construct a wall in 5 days, 13 Men and 24 Boys can construct the same wall in 4 days. Find how many days does 9 Men and 7 Boys require to construct the same wall.

⇒

$12M + 16B = 5$
$13M + 24B = 4$
$9M + 7B = x$

$(12M + 16B)5 = (13M + 24B)4$
$M = 2B$

$(12M + 16B)5 = (9M + 7B)x$
$((12 \times 2)B + 16B)5 = (9 \times 2B + 7B)x$
$40 \times 5B = 25B \times x$
$x = 8$ days.

★ Speed - Time - Distance.

$$1\ km/hr = \frac{5}{18}\ m/s$$

Note: Since time and speed are inversly proportional, if we multiply speed by some constant 'k', we must divide time by k.

Q. While going from pune to mumbai, if I increase my speed to 4/3 times my orignal speed, I reach 40min early. Usually in how much time do I reach the destination?

⇒

$40m = \frac{2}{3}\ hr$

$$s_1 \times t_1 = \frac{4}{3} s_1 \times \left(t_1 - \frac{2}{3}\right)$$

$$t_1 = \frac{4}{3} t_1 - \frac{8}{9}$$

$$\frac{8}{9} = \frac{1}{3} t_1 \Rightarrow t_1 = \frac{8}{3} = 2.66\ hrs$$

Q. While comming back to pune if I reduce my speed to 3/4 my orignal speed, I get late by 60min. Find orignal time taken.

⇒

$$s_1 t_1 = \frac{3}{4} s_1 \cdot (t_1 + 1)$$

$$t_1 = \frac{3}{4} t_1 + \frac{3}{4} \Rightarrow t_1 = 3\ hours = 180\ min$$

★ Trains

Q. How much time will a 360m long train running at 36 km/hr will take to cross a electric pole of height 100m.

⇒

$$\frac{360}{36 \times 5/18} = 36s$$

Q How much time will same train take to cross bridge of length 400m.

⇒

$$\frac{360 + 400}{36 \times 5/18} = 76s$$

Q. How much time same train take to cross another train 240m long, running at 18km/hr

⇒ (i) In same direction:

$$\frac{360 + 240}{(36-18) \times \frac{5}{18}} = 120s$$

(ii) In opposite direction:

$$\frac{360 + 240}{(36+18) \times \frac{5}{18}} = 40s$$

Q How much time the above mentioned train take to cross Anil who is sitting in another train whose length is 360m running at 18 km/hr in opposite direction.

⇒ Anil is point object, hence length of 2nd train doesn't matter.

$$\frac{360}{(36+18) \times \frac{5}{18}} = 24s.$$

★ Q A train overtakes 2 person who are walking at 2km/hr and 4km/hr in 9s and 10s respectively. Find the length of train.

⇒

$$\frac{l_t}{S_t - \frac{2\times5}{18}} = 9 \qquad \frac{l_t}{S_t - \frac{4\times5}{18}} = 10$$

$$9S_t - \frac{9\times2\times5}{18} = 10S_t - \frac{10\times4\times5}{18}$$

$$S_t = \frac{55}{9}\ m/s$$

$$l_t = \frac{9\times55}{9} - \frac{9\times2\times5}{9} = \frac{9\times50}{9} = \underline{\underline{50m}}$$

Note: In 99% cases,
(i) The length of train is a multiple of 10.
(ii) speed of train is a multiple of 9.

Q 2 trains having different length can cross man standing on railway platform in 27s and 17s respectively. While they can cross eachother in 23s in opposite direction. Find the ⇒ ratio of their speeds. (3:2, 2:3, 4:3, 3:4)

$$S_1 = \frac{l_1}{27} \qquad S_2 = \frac{l_2}{17} \qquad S_1 + S_2 = \frac{l_1 + l_2}{23}$$

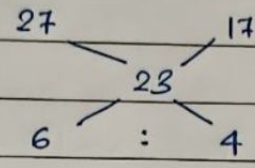
$$\frac{l_1}{27} + \frac{l_2}{17} = \frac{l_1 + l_2}{23} \Rightarrow \frac{17l_1 + 27l_2}{27\times17} = \frac{l_1 + l_2}{23}$$

$$\Rightarrow 34l_1 = 81l_2 \Rightarrow \frac{l_1}{l_2} = \frac{81}{34}$$

$$\frac{S_1}{S_2} = \frac{17}{27} \times \frac{l_1}{l_2} = \frac{17}{27} \times \frac{81}{34} = \frac{3}{2}$$

$\therefore S_1 : S_2 = 3:2$

Shortcut :



3:2

Q Garibrath leaves Pune for Amravati at 5:40 pm at 56 Km/hr. Anzadhind express leaves Amravati for Pune at 7:40pm at 44 km/hr. If distance between Pune & Amravati is 612 Km, at what exact time will the two trains cross?

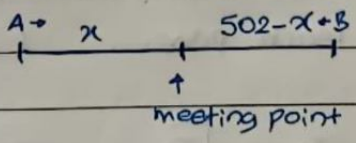
⇒

train A → 5:40 pm
train B → 7.40 pm
] 2 hours.

Distance covered by A in 2 hrs = 56×2 = 110 km.

New distance btw A & B at 7.40pm = 612 - 110 = 502 km

at 7.40pm ⇒



$$t = \frac{x}{56} = \frac{502-x}{44}$$

$100x = 502 \times 56 \qquad x = 5.02 \times 56$

$$t = \frac{5.02 \times 56}{56} = 5.02 \text{ hrs.} \rightarrow \text{past } 7.40\text{pm.}$$

Ans: 12.40 pm

★Q Shatabdi express leaves pune at 6am and reaches hydrabad at 10 am. Another train leaves hydrabad at 6am and reaches pune at 12 noon. at what exact time will the two trains start crossing eachother?

8am 8:24 am 8:48am Cannot be Determined

⇒

$$S_1 = \frac{d}{4} \qquad S_2 = \frac{d}{6}$$

Both train collectively cover d distance at crossing point. ie

$$d = (S_1 + S_2)\, t$$

$$d = \left(\frac{d}{4} + \frac{d}{6}\right) t$$

$$d = \frac{10d}{24} \times t \quad \Rightarrow \quad t = \frac{24}{10} = 2.4 \text{ hrs}$$

$$= 2\text{hrs } 24\text{min}$$

Ans: 8:24am

Q Duranto express leaves pune for ahmedabad at 9:30pm at 72 km/hr. Ahimsa express leaves ahemadabad for pune at 930pm, at x km/hr. After crossing, 1st train takes 4hr to reach ahemadabad & other train takes 9hrs to reach pune. (32, 40, 48, 58). Find x.

⇒

$$\frac{S_1}{S_2} = \sqrt{\frac{T_2}{T_1}}$$

← For questions involving time after crossing point.

$$\frac{72}{S_2} = \sqrt{\frac{9}{4}}$$

$$S_2 = \frac{72 \times 2}{3} = 48 \text{ km/hr}$$

Q. Saloni & Lakshmi start walking towards eachother on a 120 km long road with speeds 5 & 3 km/hr respectively. At the same time a bird sitting on saloni's hat starts flying towards Lakshmi, touches her hat flies back to Saloni & so on. Find total distance travelled by bird if the speed of bird is 18 km/hr.

⇒

$$\text{time} = \frac{120}{5+3} = \frac{120}{8}$$

$$\text{distance} = \frac{120}{8} \times 18 = 270 \text{ km}.$$