

# Course Topics Up to the Midterm, ESE 524

March 7, 2019

- Probability Review:
  - Basic axioms of probability
  - Random variables
  - Expected Values
  - Conditional probability
  - Bayes rule
  - Transformation of Random Variables
  - Gaussian univariate and multivariate pdfs
- Estimator Performance:
  - Bias
  - Variance
  - Mean Squared Error
- Sufficient Statistics:
  - What is a sufficient statistic?
  - Factorization Theorem
  - Exponential Family of Distributions, what assumptions must be satisfied for a distribution to be in the exponential family?
- Cramer Rao Bound:
  - Fisher Information matrix - several formulas to find this.
  - CRB in scalar and vector cases.
  - Cauchy Schwartz inequality.
  - CRB and exponential family.
  - Efficient estimator - the variance reaches the CRB.
- Linear Models:

- General formulation. How can we write different things like signals or fourier transform as linear models?
- Least squares/minimum variance optimal solution.
- CRB for optimal solution.
- What to do for colored noise (non diagonal covariance matrix)
- BLUE
- Complete Sufficient Statistics
- Maximum Likelihood Estimation:
  - General formulation.
  - Asymptotic unbiasedness and efficiency.
  - Properties of MLE Estimator
  - How to maximize functions
    - \* Sometimes it can be done by looking at the function (i.e. function is monotonic one way or the other)
    - \* Set gradient to 0, check second derivative at critical points (this part is sometimes omitted).
    - \* Computing estimates - gradient descent/newton raphson
- Bayesian Estimation:
  - Bayes rule
  - Posterior distribution - likelihood \* prior
  - $\propto$  operation - get rid of constants that don't depend on  $\theta$ , figure out the normalizing factor in the end.
  - Types of prior:
    - \* Conjugate prior - posterior is same type of distribution as the prior.
    - \* Proper prior - integrates to 1
    - \* Improper - integrates to  $\infty$
    - \* Jeffrey's Prior - improper prior that leads to proper posterior, based off of fisher information
    - \* Informative Prior - based off of actual information about the parameters.
    - \* Non-informative prior - chosen as uniform distribution or as a conjugate prior for mathematical convenience.
  - Sequential Bayesian Inference - posterior is the new prior, new data comes in.
  - posterior predictive distribution
  - Bayesian Sufficient Statistics