Module-5. Design of Experiments & ANOVA.

1. principles of experimentation in design.
2. Analysis of completly randomized design.
3. Randomized block design.
4. The ANOVA Technique,
5. Basic principle of ANOVA,
6. One-way ANOVA.
7. two-way ANOVA.
8. Latin-square Design. and Analysis of Co-Variance.

Design of expeeiments (DOE) is defined as a branch of applied statistica that deals with planning, conducting, analyzing and interpreting controlled tests to evaluate the factors that control the value of a parameter or group of parameter

ANOVA uses F-tests to statistically test the equality of means

Variance measures variability from the average or mean.

Analysis of Completly Randomized design.

A Completely randomized design (CRD) is one where the treatments are assigned completely at random so that each experimental unit has the same chance of receiving any one treatment.

this is suitable only for the experiments such as laboratory experiments or greenhouse studies etc, where the experiment material is homogenous and not for heterogeneous studies.

(primary factors are of 3 no. CRDS) factors, levels, replications.

fixed vs Random Effect in complete random design (CRD)

1. ANOVA assumes the independent variable is fixed in fixed effect, while in Random effect, it assumes an independent variable is random.
2. fixed effects probably produce smaller standard errors while the random effect produces larger standard errors.
3. The fixed effect has a large number of parameters whereas the random effect has the small number of parameters.

Principles of Experimentation in design:

All experiments involve three factors.

* 1. **Randomization.**

Every experimental unit will have the same chance of receiving any treatment.

* 2. **Replication.**

Repetition of experiment under identical conditions.

Number of experimental units under the same treatment.

* 3. **Local Control.**

Control of all factors except the ones about which we are investigation.

The Above three way is to

1. To increase the accuracy of experiment.
2. To provide a valid test of significance.

Experimental Designs:

Three basic types are

* 1. **Completely randomized design (CRD)**

Analysis of the variance for the one-way classification.

* 2. <u>Randomized complete block design (RCBD)</u>

Analysis of the variance for two-way classification

* 3. <u>Latin Square design (LSD)</u>

Analysis of the variance for three-way classification

☆ Basic principle of ANOVA.

The priciple of the ANOVA is to compare the differences in the different means of population. by examining the amount of variations within the samples, relative to the amount of Variation between the samples.

While estimating the ANOVA, we need two estimates of population Variance:

1. Based on between the Sample variance.
2. Based on within sample variance.

The value of the above two estimates of the population are compared with the F-tests.

☆. Techniques of ANOVA:

1. Variance which occurs due to the One-variable
2. Variance which occurs due to the two-variable.

Experimental unit :-

For Conducting an Experiment, the Experimental material is divided into Smaller parts and each part is referred to as an Experimental units. The Experimental unit is randomly assigned to. treatment is the Experimental unit. The phrase "randomly assigned" is very important in this definition.

Experiment :-

A way of getting an answer to a question which the Experimenter wants to know.

Treatment :

Different objects or procedures which are to be Compared in an Experiments are Called treatments.

Sampling unit :

The object that is measured in an Experiment is Called the Sampling unit. This may be diffrent from the Experimental unit.

Factor:

A factor is a variable defining a categorization. A factor can be fixed or random in nature. A factor is termed as a random factor if all the levels of interest are not included in the experiment and those that are can be considered to be randomly chosen from all the levels of interest.

Replication:

It is the repetition of the experimental situation by replicating the experimental unit.

Experimental error:

The unexplained random part of the variation in any experiment is termed as experimental error. An estimate of experimental error can be obtained by replication.

Treatment design:

A treatment design is the manner in which the levels of treatment are arranged in an experiment.

ANOVA

Analysis of variance (ANOVA) is an analysis tool used in statistics that splits an observed aggregate variability found inside a data set into two parts: systematic factors and random factors. The Systematic factors have a statistical influence on the given data set. while the random factors do not.

ANOVA stands for Analysis of variance. It is a statistical method used to analyse the differences between the means of two or more groups or treatments. It is often used to determine whether there are any statistically significant differences between the means of different groups.

There are two main types of ANOVA: one-way (or unidirectional) and two-way. There also variations of ANOVA.

ANOVA

- One way ANOVA → For one independent variable
- Two way ANOVA and MANOVA → For Two or more independent variable

Real Life Applications of ANOVA

* In Social Sciences, ANOVA tests can be used to study the statistical significance of various study environments on test scores. Medical research, the ANOVA test can be used to identify the

relationship between various types or brands of medications on individuals with migraines or depression

* We can use the ANOVA test to compare different suppliers and select the best available. ANOVA (Analysis of Variance) is used when we have more than two sample groups and determine whether there are any statistically significant differences between the means of two or more independent sample groups.

CRD: A completely randomized design (CRD) is one where the treatments are assigned completely at random so that each experimental unit has the same chance of receiving any one treatment.

RBD: A randomized block design is a restricted randomized design, in which experimental units are first organised into homogeneous blocks and then treatments are assigned at random to these units within these blocks. The main advantage of this design is, if done properly, it provides more precise results.

LSD: The Latin Square Design gets its name from the fact that we can write it as a square with Latin letters to correspond to the treatments. The treatment factor levels are the Latin letters in the Latin square design. The number of rows and columns has to correspond to the number of treatment levels.

One way classification

* Define the problem for different treatments.

Verities					Sum	Squares
x_{11}	x_{12}	x_{13}	....	x_{1n_1}	T_1	T_1^2
x_{21}	x_{22}	x_{23}	.....	x_{2n_2}	T_2	T_2^2
x_{31}	x_{32}	x_{33}	.....	x_{3n_3}	T_3	T_3^2
...	...	...	.....	....	....	....
x_{K1}	x_{K2}	x_{K3}	......	x_{Kn_K}	T_K	T_K^2

* Define the null hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3 = \dots = \mu_n$ for the level of significance.
* Find the sum of all the verities (Row wise) find the sum of all the contents of N varities, say T.
* Find the correction factor $CF = \frac{T^2}{N}$
* Find the sum of squares of individual items $TSS = \sum_i \sum_j x_{ij}^2 - CF$
* Find the sum of the squares of between the treatment $SST = \sum_i \frac{T_i^2}{n_i} - CF$
* Find the sum of squares with in the class or sum of squares due to error by subtraction $SEE = TSS - SST$

* Here K represents total number of verities, N represents the total number of observations.

* plot the ANOVA table.

Sources variation	d.f (Degree of freedom)	SS (Sum of Squares)	MSS (Mean Average Sum of squares)	F Ratio (The ratio of the b/w group variance to the within group variance)
Between treatments	$K-1$	SST → Total	$MST = \frac{SST}{K-1}$ Mean sq b/w the treatments	$F = \frac{MST}{MSE}$
Error	$N-K$	SSE - error	$MSE = \frac{SSE}{N-K}$ error	
Total	$N-1$	-	-	

Note :- * If Calculated value < given tabulated value it is accepted

Problems :- * If calculated value > tabulated value then it is reject

1) Three processes A, B and C are tested to see whether their outputs are equivalent. The following observations of outputs are made:

A	10	12	13	11	10	14	15	13
B	9	11	10	12	13	-	-	-
C	11	10	15	14	12	13	-	-

Carry out the analysis of variance and state your Conclusion.

Solution :-

To Carry out the analysis of variance, we form the following tables.

									Total	Squares
A	10	12	13	11	10	14	15	13	$T_1 = 98$	$T_1^2 = 9604$
B	9	11	10	12	13				$T_2 = 55$	$T_2^2 = 3025$
C	11	10	15	14	12	13			$T_3 = 75$	$T_3^2 = 5625$
Total T									228	—

The squares are as follows.

									Sum of Squares
A	100	144	169	121	100	196	225	169	1224
B	81	121	100	144	169				615
C	121	100	225	196	144	169			955
Grand Total - $\sum_i \sum_j x_{ij}^2$									2794

Set the null hypothesis $H_0: \mu_1 = \mu_2 = \mu_3$

Correction Factor $CF = \frac{T^2}{N} = \frac{(228)^2}{19} = 2736$

Therefore, Total sum of Squares $TSS = \sum_i \sum_j x_{ij}^2 - CF$

$\Rightarrow TSS = 2794 - 2736$

$\Rightarrow TSS = 58$

Sum of the Squares of between the treatment

$$SST = \sum_i \frac{T_i^2}{n_i} - CF$$

$$SST = \frac{9604}{8} + \frac{3025}{5} + \frac{5625}{6} - 2736$$

$\Rightarrow SST = 1200.5 + 605 + 937.5 - 2736$

$\Rightarrow SST = 2743 - 2736$

$\Rightarrow SST = 7$

$\therefore$, Sum of squares due to error

$SSE = TSS - SST$

$\Rightarrow SSE = 58 - 7$

$\Rightarrow SSE = 51$

Sources Variation	d.f	SS	MSS	F Ratio
Between treatments	$3-1=2$	$SST=7$	$MST = \frac{7}{2} = 3.5$	$F = \frac{3.5}{3.1875} = 1.0980$
Error	$19-3=16$	$SSE=51$	$MSE = \frac{51}{16} = 3.1875$	
Total	$19-1=18$	—	—	

Since evaluated value $1.0980 < 3.63$ for $F_{(2, 16)}$ (K-1, n-K) at 5% level of significance
Hence the null hypothesis is accepted, there is no significances between the three process.

2) A test was given to five students taken at random from the fifth class of three schools of a town. The individual scores are.

School I X_1	9	7	6	5	8
School II X_2	7	4	5	4	5
School III X_3	6	5	6	7	6

Carry out the analysis of variance.

Solution :-

To carry out the analysis of variance, we form the following tables.

						Total	Squares
S_1	9	7	6	5	8	$T_1 = 35$	$T_1^2 = 1225$
S_2	7	4	5	4	5	$T_2 = 25$	$T_2^2 = 625$
S_3	6	5	6	7	6	$T_3 = 30$	$T_3^2 = 900$
Total T =						90	-

The Squares are as follows

						Sum of Squares
S_1	81	49	36	25	64	225
S_2	49	16	25	16	25	131
S_3	36	25	36	49	36	182
Grand total - $\sum_i \sum_j x_{ij}^2$						568

Set the null hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3$

Correction Factor $CF = \frac{T^2}{N} = \frac{(90)^2}{15} = 540$

$\therefore$, total Sum of Squares $TSS = \sum_i \sum_j x_{ij}^2 - CF$

$\Rightarrow TSS = 568 - 540$

$TSS = 28$

Sum of the Squares of between the treatment

$$SST = \sum_i \frac{T_i^2}{n_i} - CF$$

$$SST = \frac{1225}{5} + \frac{625}{5} + \frac{900}{5} - 540$$

$\Rightarrow SST = 245 + 125 + 180 - 540$

$\Rightarrow SST = 550 - 540$

$SST = 10$

$\therefore$ Sum of Squares due to error $SEE = TSS - SST$

$\Rightarrow SEE = 28 - 10$

$SEE = 18$

Sources variation	d.f	SS	MSS	F Ratio
Between treatments	$3-1=2$	$SST=10$	$MST=\frac{10}{2}=5$	$F=\frac{5}{1.5}=3.33$
Error	$15-3=12$	$SSE=18$	$MSE=\frac{18}{12}=1.5$	
Total	$15-1=14$	—	—	

Since evaluated value $3.33 < 3.89$ for $(2, 12)$ at 5% level of significance

Hence the null hypothesis is accepted, there is no significance between the three process.

3) Three different kinds of food are tested on three groups of rats for 5 weeks. The objective is to check the difference in mean weights (in grams) of the rats per week. Apply one-way ANOVA using a 0.05 significance level to the following data.

Food 1	8	12	19	8	6	11
Food 2	4	05	04	6	9	7
Food 3	11	8	7	13	7	9

Solution :- To carry out the analysis of variance, we form the following tables

							Total	Squares
F_1	8	12	19	8	6	11	$T_1 = 64$	$T_1^2 = 4096$
F_2	4	5	4	6	9	7	$T_2 = 35$	$T_2^2 = 1225$
F_3	11	8	7	13	7	9	$T_3 = 55$	$T_3^2 = 3025$
Total T							154	—

The squares are as follows

							Sum of Squares
F_1	64	144	361	64	36	121	790
F_2	16	25	16	36	81	49	223
F_3	121	64	49	169	49	81	533
Grand Total $- \sum_i \sum_j x_{ij}^2$							1546

Set the null hypotheses $H_0: \mu_1 = \mu_2 = \mu_3$

Correction Factor $CF = \frac{T^2}{N} = \frac{(154)^2}{18} = \frac{23716}{18} = 1317.55$

$\therefore$ Total sum of squares $TSS = \sum_i \sum_j x_{ij}^2 - CF$

$\Rightarrow TSS = 1546 - 1317.55$

$\Rightarrow TSS = 228.45$

Sum of the squares of between the treatment

$$SST = \sum_i \frac{T_i^2}{n_i} - CF$$

$$SST = \frac{4096}{6} + \frac{1225}{6} + \frac{3025}{6} - 1317.55$$

$\Rightarrow SST = 682.66 + 204.166 + 504.166 - 1317.55$

$\Rightarrow SST = 1391 - 1317.55$

$\Rightarrow SST = 73.45$

$\therefore$, Sum of squares due to error $SEE = TSS - SST$

$\Rightarrow SSE = 228.45 - 73.45 \Rightarrow SEE = 155$

Sources variation	d.f	SS	MSS	F Ratio
Between treatments	$3-1=2$	$SST = 73.45$	$MST = \frac{73.45}{2} = 36.725$	$F = \frac{36.725}{10.33} = 3.55$
Error	$18-3=15$	$SSE = 155$	$MSE = \frac{155}{15} = 10.33$	
Total	$18-1=17$	–	–	

Since evaluated value $3.55 < 3.68$ for $F(2,15)$ at 5% level of significance, Hence the null hypothesis is accepted. there is no significance between the three process.

4) Three types of fertilizers are used on three groups of plants for 5 weeks. We want to check if there is a difference in the mean growth of each group. Using the data given below apply a one-way ANOVA test at 0.05 significant level.

Fertilizer 1	6	8	4	5	3	4
Fertilizer 2	8	12	9	11	6	8
Fertilizer 3	13	9	11	8	7	12

Solution :-

To carry out the analysis of variance, we form the following table

							Total	Squares
F_1	6	8	4	5	3	4	$T_1 = 30$	$T_1^2 = 900$
F_2	8	12	9	11	6	8	$T_2 = 54$	$T_2^2 = 2916$
F_3	13	9	11	8	7	12	$T_3 = 60$	$T_3^2 = 3600$
Total T							144	—

The squares are as follows

							Sum of Squares
F_1	36	64	16	25	9	16	166
F_2	64	144	81	121	36	64	510
F_3	169	81	121	64	49	144	628
Grand Total - $\sum_i \sum_j x_{ij}^2$							1304

Set the null hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3$

Correction Factor $CF = \frac{T^2}{N} = \frac{(144)^2}{18} = \frac{20736}{18} = 1152$

$\therefore$ Total Sum of Squares $TSS = \sum_i \sum_j x_{ij}^2 - CF$

$\Rightarrow TSS = 1304 - 1152$

$\Rightarrow TSS = 152$

Sum of the squares of between the treatment

$$SST = \sum_i \frac{T_i^2}{n_i} - CF$$

$$SST = \frac{900}{6} + \frac{2916}{6} + \frac{3600}{6} - 1152$$

$\Rightarrow SST = 150 + 486 + 600 - 1152$

$\Rightarrow SST = 1236 - 1152$

$\Rightarrow SST = 84$

$\therefore$ Sum of Squares due to error $SSE = TSS - SST$

$\Rightarrow SSE = 152 - 84 \Rightarrow SSE = 68$

Sources variation	d.f	SS	MSS	F Ratio
Between treatments	$3-1=2$	$SST = 84$	$MST = \frac{84}{2} = 42$	$F = \frac{42}{4.533} = 9.2653$
Error	$18-3=15$	$SSE = 68$	$MSE = \frac{68}{15} = 4.533$	
Total	$18-1=17$	–	–	

Since evaluated value $9.2653 > 3.68$ for $F(2, 15)$ at 5% level of significance Hence the null hypothesis is rejected, there is significance between the three process.

5) Set an analysis of variance table for the following data.

A	6	7	3	8
B	5	5	3	7
C	5	4	3	4

Solution :-

To carry out the analysis of variance, we form the following tables.

					Total	Squares
A	6	7	3	8	$T_1 = 24$	$T_1^2 = 576$
B	5	5	3	7	$T_2 = 20$	$T_2^2 = 400$
C	5	4	3	4	$T_3 = 16$	$T_3^2 = 256$
Total T					60	—

The squares are as follows.

					Total squares
A	36	49	9	64	158
B	25	25	9	49	108
C	25	16	9	16	66
Grand total $\sum_i \sum_j x_{ij}^2$					332

Set the null hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3$

Correction Factor $CF = \frac{T^2}{N} = \frac{3600}{12} = 300$

$\therefore$ Total Sum of Squares $TSS = \sum_i \sum_j x_{ij}^2 - CF$

$\Rightarrow TSS = 332 - 300$ $\Rightarrow TSS = 32$

Sum of the Squares of between the treatment

$$SST = \sum_i \frac{T_i^2}{n_i} - CF$$

$$SST = \frac{576}{4} + \frac{400}{4} + \frac{256}{4} - 300$$

$SST = 308 - 300$

$SST = 8$

$\therefore$ Sum of Squares due to error $SSE = TSS - SST$

$\Rightarrow SSE = 32 - 8$

$SSE = 24$

Sources Variation	d.f	SS	MSS	F Ratio
Between treatments	$3-1=2$	$SST = 8$	$MST = \frac{8}{2} = 4$	$F = \frac{4}{2.66} = 1.5037$
Error	$12-3=9$	$SSE = 24$	$MSE = \frac{24}{9} = 2.66$	
Total	$12-1=11$	—	—	

Since evaluated value $1.5037 < 4.26$ for $F(2,9)$ at 5% level of significance Hence the null hypothesis is accepted, there is no significance between the three process.

6) A trial was run to check the effects to different diets. positive numbers indicate weight loss and negative numbers indicate weight gain. Check if there is an average difference in the weight of people following different diets using an ANOVA Table.

Low Fat	Low Calorie	Low protein	Low Carbohydrate
8	2	3	2
9	4	5	2
6	3	4	-1
7	5	2	0
3	1	3	3

Solution :-

To carry out the analysis of variance, we form the following tables.

	Low Fat	Low Calorie	Low protein	Low Carbohydrate	
	8	2	3	2	
	9	4	5	2	
	6	3	4	-1	
	7	5	2	0	
	3	1	3	3	
T	33	15	17	6	71
T^2	1089	225	289	36	-

The Squares are as follows

	Low Fat	Low Calorie	Low protein	Low Carbohydrate.	
	64	4	9	4	
	81	16	25	4	
	36	9	16	1	
	49	25	4	0	
	9	1	9	9	
$\sum_i \sum_j x_{ij}^2$	239	55	63	18	375

Set the null hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3$

$$CF = \frac{T^2}{N} = \frac{5041}{20} = 252$$

$$\therefore TSS = \sum_i \sum_j x_{ij}^2 - CF$$

$$= 375 - 252$$

$$TSS = 123$$

$$SST = \sum_i \frac{T_i^2}{n_i} - CF$$

$$SST = \frac{1089}{5} + \frac{225}{5} + \frac{289}{5} + \frac{36}{5} - 252$$

$$= 327.8 - 252$$

$$SST = 75.80$$

$\therefore SSE = TSS - SST$

$= 123 - 75.80$

$SSE = 47.2$

Sources Variation	df	SS	MSS	F Ratio
Between treatments	$4-1=3$	$SST = 75.80$	$MST = \frac{75.80}{3}$ $= 25.26$	$F = \frac{25.26}{2.95}$ 8.56
Error	$20-4=16$	$SSE = 47.20$	$MSE = \frac{47.20}{16}$ $= 2.95$	
Total	$20-1=19$	—	—	

Since evaluated value $8.56 > 3.24$ for $F_{(3,16)}$ at 5% level of significance Hence the null hypothesis is reject, there is significance between the four process.

Two way classification

* Define the problem for different varieties and different treatments

	Verities					Sum	Squares
	x_{11}	x_{12}	x_{13}	...	x_{1n_1}	T_1	T_1^2
	x_{21}	x_{22}	x_{23}	...	x_{2n_2}	T_2	T_2^2
	x_{31}	x_{32}	x_{33}	...	x_{3n_3}	T_3	T_3^2
	—	....	....	....	....	....	....
	...	...	...	...	....	......	......
	x_{K1}	x_{K2}	x_{K3}	....	x_{Kn_K}	T_K	T_K^2
Sum	P_1	P_2	P_3	...	P_K	$= G$	
Squares	P_1^2	P_2^2	P_3^2	...	P_K^2		

* Define null hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3 = \dots = \mu_3$ for the level of significance.
* Find the sum of all the verities (Row wise) find the sum of all the observations of N varieties, say T
* Find the Correction factor $CF = \frac{T^2}{N}$
* Find the sum of squares of individual items
  $TSS = \sum_i \sum_j x_{ij}^2 - CF$
* Find the sum of squares of rows $SSR = \sum_i \frac{T_i^2}{n_i} - CF$
* Find the sum of the squares of columns $SSC = \sum_i \frac{P_i^2}{n_i} - CF$
* Find the sum of squares with in the class or sum of squares due to error by subtraction
  $SSE = TSS - SSR - SSC$
* plot the ANOVA table.

Sources variation	d.f	SS	MSS	F Ratio
Rows	$r-1$	SSR	$MSR = \frac{SSR}{r-1}$	$F_r = \frac{MSR}{MSE}$
Columns	$c-1$	SSC	$MSC = \frac{SSC}{c-1}$	
Error	$(r-1)(c-1)$	SSE	$MSE = \frac{SSE}{(r-1)(c-1)}$	$F_c = \frac{MSC}{MSE}$
Total	$N-1$	—		

1) Set up an analysis of variance table for the following per acre production data for three varieties of wheat, each grown on 4 plots and state it the variety differences are significant at 5% significant level.

Per acre production data			
Plot of land	variety of wheat		
	A	B	C
1	6	5	5
2	7	5	4
3	3	3	3
4	8	7	4

Solution:-

To carry out the analysis of variance, we form the following tables.

Scanned with ACE Scanner

Per acre production data				T	T^2
Plot of land	Variety				
	A	B	C		
1	6	5	5	16	256
2	7	5	4	16	256
3	3	3	3	9	81
4	8	7	4	19	361
P	24	20	16	= 60	—
P^2	576	400	256		

The Squares are as follows

Variety			
A	B	C	
36	25	25	
49	25	16	
09	09	09	
64	49	16	Grand Total
158	108	66	$\sum_i \sum_j x_{ij}^2 = 332$

Set the null hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3$, $N = 12$

Correction pactor $CF = \frac{T^2}{N} = \frac{(60)^2}{12} = \frac{3600}{12} = 300$

$\therefore$ Total sum of Squares $TSS = \sum_i \sum_j x_{ij}^2 - CF$

$TSS = 332 - 300 = 32$

Sum of the row Squares $= \sum_i \frac{T_i^2}{n_i} - CF$

$$SSR = \frac{256}{3} + \frac{256}{3} + \frac{81}{3} + \frac{361}{3} - 300$$

$$\Rightarrow SSR = 85.33 + 85.33 + 27 + 120.33 - 300$$

$$SSR = 318 - 300$$

$$\Rightarrow SSR = 18$$

Sum of the Column Squares $SSC = \sum_i \frac{P_i^2}{n_i} - CF$

$$SSC = \frac{576}{4} + \frac{400}{4} + \frac{256}{4} - 300$$

$$= 144 + 100 + 64 - 300$$

$$= 308 - 300$$

$$SSC = 8$$

$$\therefore SSE = TSS - SSR - SSC$$

$$SSE = 32 - 18 - 8$$

$$SSE = 6$$

Sources variation	d.f	SS	MSS	F Ratio
Rows	$4-1=3$	$SSR=18$	$MSR = \frac{18}{3} = 6$	$F_r = \frac{6}{1} = 6$
Columns	$3-1=2$	$SSC=8$	$MSC = \frac{8}{2} = 4$	$F_c = \frac{4}{1} = 4$
Error	$3 \times 2 = 6$	$SSE = 6$	$MSE = \frac{6}{6} = 1$	
Total	$12-1=11$	—		

$F_R = 6 > F(3,6) = 4.76$ and

$F_C = 4 < F(2,6) = 19.33$

2) Three varieties of Coal were analysed by four Chemists and the ash-Content in the varieties was found to be as under.

Varieties	Chemists			
	1	2	3	4
A	8	5	5	7
B	7	6	4	4
C	3	6	5	4

Carry out the analysis of variance.

Solution:-

To Carry out the analysis of variance, we form the following tables.

	Chemists				T	T^2
Variety	1	2	3	4		
A	8	5	5	7	25	625
B	7	6	4	4	21	441
C	3	6	5	4	18	324
P	18	17	14	15	=64	-
P^2	324	289	196	225		-

The Squares are as follows.

Chemists

1	2	3	4
64	25	25	49
49	36	16	16
9	36	25	16

Grand Total - $\sum_i \sum_j x_{ij}^2 = 366$

Set the null hypothesis $H_0: \mu_1 = \mu_2 = \mu_3$, $N = 12$

Correction factor $= \frac{T^2}{N} = \frac{(64)^2}{12} = \frac{4096}{12} = 341.33$

$\therefore$ Total Sum of Squares $TSS = \sum_i \sum_j x_{ij}^2 - CF$

$\Rightarrow TSS = 366 - 341.33$

$\Rightarrow TSS = 24.67$

Sum of the row Squares $SSR = \sum_i \frac{T_i^2}{n_i} - CF$

$SSR = \frac{625}{4} + \frac{441}{4} + \frac{324}{4} - 341.33$

$\Rightarrow SSR = 156.25 + 110.25 + 81 - 341.33$

$\Rightarrow SSR = 6.17$

Sum of the Column Squares $SSC = \sum_i \frac{P_i^2}{n_i} - CF$

$SSC = \frac{324}{3} + \frac{289}{3} + \frac{196}{3} + \frac{225}{3} - 341.33$

$\Rightarrow SSC = 108 + 96.33 + 65.33 + 75 - 341.33$

$\Rightarrow SSC = 344.66 - 341.33$

$\Rightarrow SSC \quad 3.33$

$\therefore\ SSE = TSS - SSR - SSC$

$SSE = 24.67 - 6.17 - 3.33$

$SSE = 15.17$

Sources Variation	d.f	SS	MSS	F Radio
Rows	$3-1=2$	$SSR = 6.17$	$MSR = \frac{6.17}{2} = 3.085$	$F_R = \frac{3.085}{2.53} = 1.22$
Columns	$4-1=3$	$SSC = 3.33$	$MSC = \frac{3.33}{3} = 1.11$	
Error	$3\times 2 = 6$	$SSE = 15.17$	$MSE = \frac{15.17}{6} = 2.53$	$F_C = \frac{2.53}{1.11} = 2.28$

$F_R = 1.22 < F_{(2,6)}$ and

$F_C = 2.28 < F_{(3,6)}$

3) Perform ANOVA and test at 0.05 level of significant whether there are differences in the detergent or in the engines for the following data.

Detergent	Engine		
	I	II	III
A	45	43	51
B	47	46	52
C	48	50	55
D	42	37	49

Soln :-

Given the data

Detergent	Engine		
	I	II	III
A	45	43	51
B	47	46	52
C	48	50	55
D	42	37	49

Subtract 45 from all the observations

Detergent	Engine			T	T^2
	I	II	III		
A	0	−2	6	4	16
B	2	1	7	10	100
C	3	5	10	18	324
D	−3	−8	4	−7	49
P	2	−4	27	2	25
P^2	4	16	729	4	-

The Squares are

Detergent	Engine			Sum
	I	II	III	
A	0	4	36	40
B	4	1	49	54
C	9	25	100	134
D	9	64	16	89
Grand Total - $\sum_i \sum_j x_{ij}^2 =$				317

Set the null hypothesis $H_0: \mu_1 = \mu_2 = \mu_3$

$N = 12$

$$CF = \frac{T^2}{N} = \frac{625}{15} = 52.08$$

$$\therefore TSS = \sum_i \sum_j x_{ij}^2 - CF$$

$$= 317 - 52.08$$

$$TSS = 264.92$$

$$SSR = \sum_i \frac{T_i^2}{n_i} - CF$$

$$= \frac{16}{3} + \frac{100}{3} + \frac{324}{3} + \frac{49}{3} - 52.08$$

$$= 163 - 52.08$$

$$SSR = 110.92$$

$$SSC = \sum_i \frac{P_i^2}{n_i} - CF$$

$$SSC = \frac{4}{4} + \frac{16}{4} + \frac{\cancel{7}29}{4} - 52.08$$

$$= 187.25 - 52.08$$

$$SSC = 135.17$$

$$\therefore \quad SSE = TSS - SSR - SSC$$

$$SSE = 264.92 - 110.92 - 135.17$$

$$SSE = 18.83$$

Sources Variation	df	SS	MSS	F Ratio
Rows	$4-1 = 3$	$SSR = 110.92$	$MSR = \frac{110.92}{3} = 36.97$	$F_r = \frac{36.97}{3.14} = 11.77$
Columns	$3-1=2$	$SSC = 135.17$	$MSC = \frac{135.17}{2} = 67.58$	
Error	$3\times2=6$	$SSE = 18.83$	$MSE = \frac{18.83}{6} = 3.14$	$F_c = \frac{67.58}{3.14} = 21.52$
Total	$2-1=11$	–	–	

$$F_r = 11.77 > F(3,6) \quad \& \quad F_c = 21.52 > F(2,6)$$

Since the null hypothesis is rejected and there is a significance between Detergent and Engine.

Problems on Latin Square design.

1) Analyze and interpret the following statistics Concerning output of wheat for field obtained as result of Experiment Conducted to test for four Varieties of wheat viz. A, B, C and D under Latin Square design.

C 25	B 23	A 20	D 20
A 19	D 19	C 21	B 18
B 19	A 14	D 17	C 20
D 17	C 20	B 21	A 15

Solution :-

Given observations are

C 25	B 23	A 20	D 20
A 19	D 19	C 21	B 18
B 19	A 14	D 17	C 20
D 17	C 20	B 21	A 15

Null hypothesis H_0: There is no significant difference between rows, Columns and treatment. Code the data by subtracting 20 from each value, we get

					T	T^2
	C 5	B 3	A 0	D 0	8	64
	A -1	D -1	C 1	B -2	-3	9
	B -1	A -6	D -3	C 0	10	100
	D -8	C 0	B 1	A -5	-7	49
P	0	-4	-1	-7	= -12	
P^2	0	16	1	49	-	-

The Squares are as follows

C 25	B 9	A 0	D 0	
A 1	D 1	C 1	B 4	
B 1	A 36	D 9	C 0	
D 9	C 0	B 1	A 25	
36	46	11	29	$\sum_i \sum_j x_{ij}^2 = 122$

Correction Factor $CF = \frac{T^2}{N} = \frac{(-12)^2}{16} = 9$

$\therefore$ total Sum of Squares

$TSS = \sum_i \sum_j x_{ij}^2 - CF$

$\Rightarrow TSS = 122 - 9$

$\Rightarrow TSS = 113$

Sum of the row Squares

$$SSR = \sum_i \frac{T_i^2}{n_i} - CF$$

$$SSR = \frac{64}{4} + \frac{9}{4} + \frac{100}{4} + \frac{49}{4} - 9$$

$$SSR = 55.5 - 9$$

$$\Rightarrow SSR = 46.5$$

Sum of the Column Squares $SSC = \sum_i \frac{P_i^2}{n_i} - CF$

$$SSC = 0 + \frac{16}{4} + \frac{1}{4} + \frac{49}{4} - 9$$

$$\Rightarrow SSC = 4 + 0.25 + 12.25 - 9 = 16.5 - 9$$

$$\Rightarrow SSC = 7.5$$

To find the Sum of the treatments

observations					$= Q \sum$ (observations)	Q^2
A	0	-1	-6	-5	-12	144
B	3	-2	-1	1	1	1
C	5	1	0	0	6	36
D	0	-1	-3	-3	-7	49

Sum of the Squares of treatment

$$SST = \sum_i \frac{Q_i^2}{n_i} - CF$$

$$SST = \frac{144}{4} + \frac{1}{4} + \frac{36}{4} + \frac{49}{4} - 9$$

$SST = 36 + 0.25 + 9 + 12.25 - 9$

$SST = 57.50 - 9$

$\Rightarrow SST = 48.50$

$\therefore SSE = TSS - SSR - SSC - SST$

$= 113 - 46.5 - 7.5 - 48.50$

$= 10.5$

We know that $F(3,6) = 4.76$

Sources variation	df	SS	MSS	F Ratio	Conclusion
Rows	$4-1=3$	$SSR = 46.5$	$MSR = \frac{46.5}{3} = 15.5$	$F_R = \frac{15.5}{1.75} = 8.85$	$F_R > F(3,6)$ Ho-rejected
Columns	$4-1=3$	$SSC = 7.5$	$MSC = \frac{7.5}{3} = 2.5$	$F_C = \frac{2.5}{1.75} = 1.428$	$H_C < F(3,6)$ Ho-accepted
Treatments	$4-1=3$	$SST = 48.5$	$MST = \frac{48.5}{3} = 16.16$	$F_T = \frac{16.16}{1.75}$ 9.234	$F_T > F_{(3,6)}$ 4.76 Ho-Rejected
Error	$3 \times 2 = 6$	$SSE = 10.5$	$MSE = \frac{10.5}{6} = 1.75$	-	-
Total	$25-1 = 24$	-	-	-	-

2) Five varieties of paddy A, B, C, D and E are tried. The plan, the varieties shown in each plot and yields obtained in kg are given in the following table (LSD)

B 95	E 85	C 139	A 117	D 97
E 90	D 89	B 75	C 146	A 87
C 116	A 95	D 92	B 89	E 74
A 85	C 130	E 90	D 81	B 77
D 87	B 65	A 99	E 89	C 93

Test whether there is a significant difference b/w row & columns at 5% LOS

Solution :-

Given observations are

B 95	E 85	C 139	A 117	D 97
E 90	D 89	B 75	C 146	A 87
C 116	A 95	D 92	B 89	E 74
A 85	C 130	E 90	D 81	B 77
D 87	B 65	A 99	E 89	C 93

Null hypothesis H_0: There is no significant difference between rows, columns and treatment code the data by substracting 100 from each value, we get

						T	T^2
	B -5	E -15	C 39	A 17	D -3	33	1089
	E -10	D -11	B -25	C 46	A -13	-13	169
	C 16	A -5	D -8	B -11	E -26	-34	1156
	A -15	C 30	E -10	D -19	B -23	-37	1369
	D -13	B -35	A -1	E -11	C -7	-67	4489
P	-27	-36	-5	22	-72	= -118	
P^2	729	1296	25	484	5184	-	-

The squares are as follows

B 25	E 225	C 1521	A 289	D 9	
E 100	D 121	B 625	C 2116	A 169	
C 256	A 25	D 64	B 121	E 676	
A 225	C 900	E 100	D 361	B 529	
D 169	B 1225	A 1	E 121	C 49	
775	2496	2311	3008	1432	$\sum_i \sum_j x_{ij}^2 = 10022$

Correction factor $CF = \frac{T^2}{N} = \frac{(-118)^2}{25} = \frac{13924}{25}$

$= 557$

$\therefore$ total sum of squares $TSS = \sum_i \sum_j x_{ij}^2 - CF$

$\Rightarrow TSS = 10022 - 557$

$TSS = 9465$

Sum of the row squares $SSR = \sum_i \frac{T_i^2}{n_i} - CF$

$SSR = \frac{1089}{5} + \frac{169}{5} + \frac{1156}{5} + \frac{1369}{5} + \frac{4489}{5} - 557$

$SSR = 217.8 + 33.8 + 231.2 + 273.8 + 897.8 - 557$

$SSR = 1654.4 - 557$

$SSR = 1097.4$

Sum of the column squares $SSC = \sum_i \frac{P_i^2}{n_i} - CF$

$SSC = \frac{729}{5} + \frac{1296}{5} + \frac{25}{5} + \frac{484}{5} + \frac{5184}{5} - 557$

$\Rightarrow SSC = 145.8 + 259.2 + 5 + 96.8 + 1036.8 - 557$

$= 1543.6 - 557$

$SSC = 986.6$

To find the sum of treatments

	Observations					Q = Σ(Observations)	Q^2
A	17	-13	-5	-15	-1	-17	289
B	-5	-25	-11	-23	-35	-99	9801
C	39	46	16	30	-7	124	15376
D	-3	-11	-8	-19	-13	-54	2916
E	-15	-10	-26	-10	-11	-72	5184

Sum of the Squares of treatment

$$SST = \sum_i \frac{Q_i^2}{n_i} - CF$$

$$SST = \frac{289}{5} + \frac{9801}{5} + \frac{15376}{5} + \frac{2916}{5} + \frac{5184}{5} - 557$$

$\Rightarrow SST = 57.8 + 1960.2 + 3075.2 + 583.2 - 557$

$\Rightarrow SST = 6713.2 - 557$

$\Rightarrow SST = 6156.2$

$\therefore SSE = TSS - SSR - SSC - SST$

$= 9465 - 1097.4 - 986.6 - 6156.2$

$\Rightarrow SSE = 1224.8$

WKT $F(4,12) = 3.26$

Sources variation	d.f	SS	MSS	F Ratio	Conclusion
Rows	$5-1=4$	$SSR=1097.4$	$MSR = \frac{1097.4}{4} = 274.3$	$F_r = \frac{274.3}{102.66} = 2.672$	$H_r < F_{(4,12)}$ Ho-accepted
Columns	$5-1=4$	$SSC=986.6$	$MSC = \frac{986.6}{4} = 246.65$	$F_c = \frac{246.65}{102.66} = 2.4026$	$F_c < F_{(4,12)}$ Ho-accepted
Treatments	$5-1=4$	$SST=6156.2$	$MST = \frac{6156.2}{4} = 1539.05$	$F_T = \frac{1539.05}{102.66} = 15$	$F_T > F_{(4,12)}$ Ho-Rejected
Error	$4\times3=12$	$SSE=1224.8$	$MSE \frac{1224.8}{12} = 102.66$		
Total	$25-1 = 24$	—	—	—	

3) Present your conclusions after doing analysis of variance to the following results of the Latin-Square design Experiment conducted in respect of fine fertilizers which were used on plots of different fertility.

A 16	B 10	C 11	D 9	E 9
E 10	C 9	A 14	B 12	D 11
B 15	D 8	E 8	C 10	A 18
D 12	E 6	B 13	A 13	C 12
C 13	A 11	D 10	E 7	B 14

Solution :- Given observations are

A 16	B 10	C 11	D 9	E 9
E 10	C 9	A 14	B 12	D 11
B 15	D 8	E 8	C 10	A 18
D 12	E 6	B 13	A 13	C 12
C 13	A 11	D 10	E 7	B 14

Null hypothesis H_0 : There is no significant difference between rows, columns and treatment.
Code the data by subtracting 10 from each value we get

$\therefore SSC = \sum_i \frac{P_i^2}{n_i} - CF$

$SSC = \frac{256}{5} + \frac{36}{5} + \frac{36}{5} + \frac{1}{5} + \frac{196}{5} - 38.44$

$= 105 - 38.44$

$SSC = 66.56$

To find the sum of treatments

observations					$Q = \sum(\text{observations})$	Q^2
A	6	4	8	3	1	22	484
B	0	2	5	3	4	14	196
C	1	-1	0	2	3	5	25
D	-1	1	-2	2	0	0	0
E	-1	0	-2	-4	-3	-10	100

$SST = \sum_i \frac{Q_i^2}{n_i} - CF$

$SST = \frac{484}{5} + \frac{196}{5} + \frac{25}{5} + \frac{0}{5} + \frac{100}{5} - 38.44$

$= 161 - 38.44$

$SST = 122.56$

$\Rightarrow SSE = TSS - SSR - SSC - SST$

$SSE = 5.28.$

A 6	B 0	C 1	D 1	E 1	
E 0	C 1	A 16	B 4	D 1	
B 25	D 4	E 4	C 0	A 64	
D 4	E 16	B 9	A 9	C 4	
C 9	A 1	D 0	E 9	B 16	
74	22	30	23	86	$\sum_i \sum_j x_{ij}^2 = 235$

Correction factor $CF = \frac{T^2}{N} = \frac{(31)^2}{25} = \frac{961}{25} = 38.44$

$\therefore \quad TSS = \sum_i \sum_j x_{ij}^2 - Cf$

$= 235 - 38.44$

$= 196.56$

$SSR = \sum_i \frac{T_i^2}{n_i} - Cf$

$= \frac{25}{5} + \frac{36}{5} + \frac{81}{5} + \frac{36}{5} + \frac{25}{5} - 38.44$

$= 40.60 - 38.44$

$SSR = 2.16$

Sources variation	df	S S	MSS	F Ratio	Conclusion
Rows	$5-1=4$	$SSR=2.16$	$MSR=\frac{2.16}{4}$ $=0.54$	$F_r=\frac{0.54}{0.44}$ $=1.227$	$F_r < F_{(4,12)}$ Ho - accepted.
Columns	$5-1=4$	$SSC=66.56$	$MSC=\frac{66.56}{4}$ $=16.64$	$F_c=\frac{16.64}{0.44}$ $=37.81$	$F_c > F_{(4,12)}$ Ho - Rejected
Treatments	$5-1=4$	$SST=122.56$	$MST=\frac{122.56}{4}$ $=30.64$	$F_T=\frac{30.64}{0.44}$ $=69.63$	$F_r > F_{(4,12)}$ Ho - Rejected
Error	$4\times3=12$	$SSE=5.28$	$MSE=\frac{5.28}{12}$ $=0.44$	-	-
Total	$25-1=24$	-	-	-	-

If two samples

Step 1 :- Sum and Sum Squares

Step 2 :- The Squares are as follows

Step 3 :- $CF = \frac{T^2}{N}$

Step 4 :- TSS

Step 5 :- SSR

Step 6 :- SSC

Step 7 :- Mean.

Step 8 :- Sum of Squares within treatments/sample (SSW)

Step 9 :- SSI (Sum of Squares due to interaction

$$SSI = TSS - SSR - SSC - SSW.$$

Step 10 :- Sources variation table

1) Set up ANOVA table for the following information relating to three drugs testing to judge the effectiveness in reducing blood pressure for three different groups of people

Group of people	Drug		
	X	Y	Z
A	14 15	10 9	11 11
B	12 11	7 8	10 11
C	10 11	11 11	8 7

Do the drugs act differently?
Are the different groups of people affected of people affected differently? Is the interaction term significant? Answer the above questions taking a significant level of 5%

Solution :-

Given observations from different people (A, B, C) to the different drug (X, Y, Z) are as

	Group of people	Drug X	Drug Y	Drug Z	Sum (T)	T^2
	A	14 15	10 9	11 11	$T_1 = 70$	$T_1^2 = 4900$
	B	12 11	7 8	10 11	$T_2 = 59$	$T_2^2 = 3481$
	C	10 11	11 11	8 7	$T_3 = 58$	$T_3^2 = 3364$
Sum	P	73	56	58	= 187	-
Squares	P^2	5329	3136	3364	-	-

The squares are as follows

Group of people	Drug X	Drug Y	Drug Z	Sum of Squares
A	196 225	100 81	121 121	844
B	144 121	49 64	100 121	599
C	100 121	121 121	64 49	576

Grand total $\sum_i \sum_j x_{ij}^2 = 2019$

Set the null hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3$

Correction factor (CF) $= \frac{T^2}{N} = \frac{(187)^2}{18} = \frac{34969}{18}$

$= 1942.722$

Total Sum of Squares (TSS) $= \sum_i \sum_j x_{ij}^2 - CF$

$= 2019 - 1942.722$

$= 76.28$

Sum of Squares of Rows $SSR = \sum_i \frac{T_i^2}{n_i} - CF$

$= \frac{4900}{6} + \frac{3481}{6} + \frac{3364}{6} - 1942.722$

$SSR = 14.78$

Sum of Squares of Columns $SSC = \sum_i \frac{P_i^2}{n_i} - CF$

$= 888.16 + 522.66 + 560.67 - 1942.722$

$SSC = 28.77$

Mean $= \frac{14+15}{2} = 14.5$, $\frac{10+9}{2} = 9.5$, $\frac{11+11}{2} = 11$

$\frac{11+12}{2} = 11.5$, $\frac{7+8}{2} = 7.5$, $\frac{10+5}{2} = 10.5$

Sum of Squares within Treatments / Samples (SSW)

$(14-14.5)^2 + (15-14.5)^2 + (10-9.5)^2 + (9-9.5)^2 +$
$(11-11)^2 + (11-11)^2 + (12-11.5)^2 + (11-11.5)^2 + (7-7.5)^2$
$+ (8-7.5)^2 + (10-10.5)^2 + (11-10.5)^2 + (10-10.5)^2 +$
$(11-10.5)^2 + (11-11)^2 + (11-11)^2 + (8-7.5)^2 + (7-7.5)^2$

SSW = 3.50

$\therefore$ SSI (Sum of Squares due to interaction)
SSI = TSS − SSR − SSC − SSW
SSI = 76.28 − 14.78 − 28.77 − 3.50 = 29.23

Sources variation	d-f	SS	MSS	F-Ratio	Conclusion
Rows	$r-1$ $3-1=2$	SSR = 14.78	$MSR = \frac{14.78}{2}$ $= 7.39$	$F_R = \frac{7.39}{0.389}$ $= 19$	$F(2,9) = 4.26$ $F_R > F(2,9)$ Ho rejected
Columns	$C-1$ $3-1=2$	SSC = 28.77	$MSC = \frac{28.77}{2}$ $= 14.385$	$F_C = \frac{14.385}{0.389}$ $= 37$	$F_C > F_{(2,9)}$ Ho rejected
Due to Interaction	$(r-1)(C-1)$ $2\times2=4$	SSI = 29.23	$MSI = \frac{29.23}{4}$ $= 7.33$	$F_I = \frac{7.33}{0.389}$ $= 18.84$	$F_I > F_{(4,9)}$ Ho rejected
Within treatment or Error	(N − ~~total~~ mean) $18-9$ $=9$	SSW = 3.50	$MSW = \frac{3.50}{9}$ $= 0.389$	—	—
Total	$N-1$ $18-1=17$	-	-	-	-

$\therefore F(2,9) = 4.26$

$F(4,9) = 3.63$

∴ The drugs act differently.

* The different groups of the people are affected differently.

* Interaction term is significant

Because from the above table, all the three F-ratioes are significant at 5% level of significance.

2) Set up ANOVA table for the following

Students	Low Noise	Medium Noise	Loud Noise
Male Students	10 12 11 9	7 9 8 12	4 5 6 5
Female Students	12 13 10 13	13 15 12 12	6 6 4 4

Does Noise has an effect on the marks of a student scores? Answer :- yes

Does Gender has an effect on the marks of a Student scores? Answer :- yes

Does Gender effect how a Students react the Noise? Answer :- yes

Solution :-

Students	Low Noise	Medium Noise	Loud Noise	Sum (T)	T^2
Male Students	10 12 11 9	7 9 8 12	4 5 6 5	$T_1 = 98$	$T_1^2 = 9604$
Female Students	12 13 10 13	13 15 12 12	6 6 4 4	$T_2 = 120$	$T_2^2 = 14400$
P	90	88	40	= 218	–
P^2	8100	7744	1600	–	–

Squares are as follows

Student	Low Noise	Medium Noise	Loud Noise	Sum of Squares
Male Student	100 144 121 81	49 81 64 144	16 25 36 25	886
Female Students	144 169 100 169	169 225 144 144	36 36 16 16	1368
Grand total	$\sum_i \sum_j x_{ij}^2$		=	2254

Set null hypothesis $H_0: \mu_1 = \mu_2 = \mu_3$

$$CF = \frac{T^2}{N} = \frac{(218)^2}{24} = \frac{47524}{24} = 1980.17$$

$$TSS = \sum_i \sum_j x_{ij}^2 - CF = 2254 - 1980.17$$
$$= 273.83$$

$SSR = 20.16$

$SSC = 200.33$

SSW = Sum of squares within treatments

$$\text{Mean} = \frac{10+12+11+9}{4} = 10.5, \quad \frac{7+9+8+12}{4} = 9,$$

$$\frac{4+5+6+5}{4} = 5, \quad \frac{12+13+10+13}{4} = 12, \quad \frac{13+15+12+12}{4} = 13,$$

$$\frac{6+6+4+4}{4} = 5$$

$\therefore$ Total no of mean = 6

$$SSW = (10-10.5)^2 + (12-10.5)^2 + (11-10.5)^2 + (9-10.5)^2 + (7-9)^2 + (9-9)^2 + (8-9)^2 + (12-9)^2 + (4-5)^2 + (5-5)^2 + (6-5)^2 + (5-5)^2 + (12-12)^2 + (13-12)^2 + (10-12)^2 + (13-12)^2 + (13-13)^2 + (15-13)^2 + (12-13)^2 + (12-13)^2 + (6-5)^2 + (6-5)^2 + (4-5)^2 + (4-5)^2$$

$SSW = 37$

$$SSI = TSS - SSR - SSC - SSW$$

$$= 273.83 - 20.16 - 200.33 - 37$$

$$SSI = 16.34$$

$$\left\{ \begin{array}{l} F_{(1,8)} = 4.41 \\ F_{(2,8)} = 3.56 \end{array} \right\}$$

Sources Variation	df	SS	MSS	F-Ratio	Conclusion F_C
Rows	$r-1$ $2-1=1$	$SSR =$ 20.16	$MSR = \frac{20.16}{1}$ $= 20.16$	$F_R = \frac{20.16}{2.056}$ $= 9.8054$	$F_R > F_{(1,8)}$ Ho rejected
Columns	$C-1$ $3-1=2$	$SSC =$ 200.33	$MSC =$ $\frac{200.33}{2}$ $= 100.165$	$F_C = \frac{100.165}{2.056}$ $= 48.7184$	$F_C > F_{(2,18)}$ Ho rejected
Due to Interaction	$(r-1)(c-1)$ $= 1 \times 2$ $= 2$	$SSI =$ 16.34	$MSI =$ $\frac{16.34}{2}$ $= 8.17$	$F_I = \frac{8.17}{2.056}$ $= 3.9737$	$F_I > F_{(2,18)}$ Ho rejected
Within treatments (Error)	$N-T$ $= (24-6)$ $= 18$	$SSW =$ 37	$MSW =$ $\frac{37}{18}$ $= 2.056$	—	—
Total	$N-1$ $= 24-1$ 23	—	—	—	—