Model Question Paper-I with effect from 2023-24 (CBCS Scheme)

USN | | | | | | | | | | |

Third Semester B.E. Degree Examination
Mathematics for Computer Science

TIME: 03 Hours **Max. Marks: 100**

Note: 01. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**.
02. Statistical tables and Mathematics formulae handbooks are allowed

<table>
<tr><th colspan="3">Module -1</th><th>Bloom's Taxonomy Level</th><th>Marks</th></tr>
<tr><td>Q.01</td><td>a</td><td>The probability distribution function of variate X is given by the following table;
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>P(x)</td><td>k</td><td>3k</td><td>5k</td><td>7k</td><td>9k</td><td>11k</td><td>13k</td></tr>
</table>
i) Find the value of k, ii) $P(x \geq 5)$ & iii) $P(3 < x \leq 6)$.</td><td>L2</td><td>6</td></tr>
<tr><td></td><td>b</td><td>If the probability of a bad reaction from a certain injection is 0.001, determine the chance that more than two of 2000 individuals will have a bad reaction.</td><td>L3</td><td>7</td></tr>
<tr><td></td><td>c</td><td>Find the mean and standard deviation of Poisson's distribution.</td><td>L2</td><td>7</td></tr>
<tr><td colspan="3">OR</td><td></td><td></td></tr>
<tr><td>Q.02</td><td>a</td><td>The probability of a pen manufactured by a factory be defective is 1/10. If 12 such pens are manufactured, what is the probability that i) exactly 2 are defective, ii) at least 2 are defective, iii) none of them are defective.</td><td>L3</td><td>6</td></tr>
<tr><td></td><td>b</td><td>Determine the value of k, so that the function $f(x) = k(x^2 + 4)$ for x = 0, 1, 2, 3 can serve as a probability distribution of the discrete random variable X: Also find i) $P(0 < x \leq 2)$ and ii) $P(x \geq 1)$.</td><td>L2</td><td>7</td></tr>
<tr><td></td><td>c</td><td>Find the mean and standard deviation of Binomial distribution.</td><td>L2</td><td>7</td></tr>
<tr><td colspan="3">Module-2</td><td></td><td></td></tr>
<tr><td>Q.03</td><td>a</td><td>The joint probability distribution of discrete random variables X & Y are as follows;
<table>
<tr><td>X\Y</td><td>-3</td><td>2</td><td>4</td></tr>
<tr><td>1</td><td>0.1</td><td>0.2</td><td>0.2</td></tr>
<tr><td>2</td><td>0.3</td><td>0.1</td><td>0.1</td></tr>
</table>
Then i) determine marginal distribution of X & Y, ii) show that X & Y are dependent.</td><td>L2</td><td>6</td></tr>
<tr><td></td><td>b</td><td>Determine the value of k so that the function $f(x, y) = k|x - y|$, for $x = -2, 0, 2$; $y = -2, 3$ represents joint probability distribution of the random variables X and Y. Also determine cov(X, Y).</td><td>L2</td><td>7</td></tr>
<tr><td></td><td>c</td><td>Three boys X, Y, Z are throwing a ball to each other. X always throws the ball to Y & Y always throws the ball to Z. But Z is just as likely to throw the ball to Y or as to X. Write TPM if Z is the first person to throw the ball, find the probability that X has the ball after fourth throw.</td><td>L3</td><td>7</td></tr>
<tr><td colspan="3">OR</td><td></td><td></td></tr>
</table>

Q.04

a Given the following joint distribution of the random variable X & Y,

X\Y	−2	−1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine the marginal probability distributions of X & Y. Also compute i) Expectations of X, Y & XY, ii) Covariance of X & Y, iii) Correlation of X & Y. — L2 — 6

b The joint probability distribution of random variables X & Y are as follows.

x\y	−4	2	7
1	1/8	¼	1/8
5	¼	1/8	1/8

then determine i) marginal distribution of X & Y, ii) E(X), E(Y) & E(XY), iii) COV(X,Y), iv) ρ(X,Y). — L2 — 7

c The students study habits are as follows; If he studies on one night, he is 60% sure not to study on next night. On the other hand, if he does not study on to night, he is 80% sure to study next night. Write the transition probability matrix for his chain of study. In the long run how often does he study? Suppose he studies on Monday night, what is the probability that he does not study on Friday night? — L3 — 7

Module-3

Q. No.		Question	Level	Marks
Q.05	a	A die was thrown 9000 times and throw of 5 or 6 was obtained 3240 times on the assumption of random throwing do the data indicate an unbiased die?	L3	6
	b	Before an increase in excise duty on tea 400 people out of a sample 500 persons were found to be tea drinkers. After an increase in duty 400 people were tea drinkers in a sample of 600 people. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea for 95% and 99% level of significance?	L3	7
	c	A survey was conducted in a slum locality of 2000 families by selecting a sample of size 800 families. It was revealed that 180 families were illiterate. Find the probable limits of the illiterate families in the population of 2000.	L3	7
		OR		
Q.06	a	In 324 throws a die an odd number turned up 181 times. Is it reasonable to think that the die is an unbiased one?	L3	6
	b	The mean weight obtained from a random sample of size 100 is 64gms. The standard deviation of the weight distribution of the population is 3gms. Test the statement that the mean weight of the population is 67gms at 5% level of significance. Also set up 99% confidence limits of the mean weight of the population.	L3	7
	c	In a sample of 100 people in this city, the average income was Rs. 210, with a standard deviation of Rs. 10. For another sample of 150 persons, the average income was Rs. 220, with a standard deviation of Rs. 12. The standard deviation of the incomes of the people of the city was Rs. 11. Test whether there is any significant difference between the average incomes of the localities.	L3	7

Module-4				
Q.07	a	An experiment on Pea breading the following frequency of seeds were obtained Round & Yellow: 315; Wrinkled & Yellow: 101; Round & Green: 108; Wrinkled & Green: 32; Total: 556 Theory predicts that the frequences should be in the proportions 9:3:3:1. Examine the correspondence between theory and experiment ($\chi^2_{0.05}$= 7.815).	L3	6
	b	Use the Central Limit theorem to evaluate $P[50 < \bar{X} < 56]$ where $\bar{X}$ represents the mean of a random sample of size 100 from an infinite population with mean $\mu = 53$ and variance $\sigma^2 = 400$ (Given, A(1.5) = 0.4332).	L2	7
	c	Ten individuals are chosen at random from a population and their heights in inches found to be 63, 63, 66, 67, 68, 69, 70, 70, 71 and 71. Test the hypothesis that the mean height of the universe is 66 inches. ($t_{0.05}$ = 2.262 for 9 d.f.).	L3	7
OR				
Q.08	a	The following table shows the runs scored by two batsmen can it be said that the performance of batsman A is more consistent than the performance of batsman B? Use 1% level of significance ($F_{0.01}$,4,7 = 7.85). Batsman-A: 40, 50, 35, 25, 60, 70, 65, 55 Batsman-B: 60, 70, 40, 30, 50	L3	6
	b	The following table gives the number of aircraft accidents that occurred during the various days of the week. Find whether the accidents are uniformly distributed over the week? Days: Sun, Mon, Tue, Wed, Thu, Fri, Sat, Total Accidents: 14, 16, 8, 12, 11, 9, 14, 84 Given that $\chi^2_{0.05}$= 12.59.	L3	7
	c	Consider the sample consisting of nine numbers 45, 47, 50, 52, 48, 47, 49, 53, 51. The sample is drawn from a population whose mean is 47.5. Find whether the sample mean differs significantly from the population mean at 5% level of significance ($t_{0.05}$ for 8 d.f. = 2.31).	L2	7
Module-5				
Q.09	a	A manufacturing company has purchase three new machines of different brands and wishes to determine whether one of them is faster than the others in producing a certain output, 5 hourly production figures are obtained at random from each other machine and the results are given below; (see table) Use ANOVA and determine whether the machines are significantly	L3	10

Q.07 (a):

Round & Yellow	Wrinkled & Yellow	Round & Green	Wrinkled & Green	Total
315	101	108	32	556

Q.08 (a):

Batsman-A	40	50	35	25	60	70	65	55
Batsman-B	60	70	40	30	50			

Q.08 (b):

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
Accidents	14	16	8	12	11	9	14	84

Q.09 (a):

Observation	A	B	C
1	25	31	24
2	30	39	30
3	36	38	28
4	38	42	25
5	31	35	28

		different in their mean speed. Given at 5% level $F_{2,12}$= 3.89.		
	b	Set up ana analysis of variance table for the following per acre production data for three varieties of wheat, each grown on 4 plots and state if the variety differences are significant.	L3	10

Plot of Land	Per acre production data		
	A	B	C
1	6	5	5
2	7	5	4
3	3	3	3
4	8	7	4

Use ANOVA, given at 5% level $F_{2,9} = 4.26$.

OR

Q.10	a	Set up an analysis of variance table for the following two-way design results: per acre production data of wheat in metric tons;	L3	10

Varieties of fertilizers	Varieties of seeds		
	A	B	C
W	6	5	5
X	7	5	4
Y	3	3	3
Z	8	7	4

Also state whether variety differences are significant at 5% level. Given that $F_{2,6} = 5.14$ and $F_{3,6} = 4.76$.

	b	Analyze the variance in the following table Latin square of yields in kgs of Paddy where A, B, C, D denotes the different methods of cultivation.	L3	10

D-122	A-121	C-123	B-122
B-124	C-123	A-122	D-125
A-120	B-119	D-120	C-121
C-122	D-123	B-121	A-122

Examine whether the different methods of cultivation have given significantly different is given that $F_3 = 4.76$.

Q1 (a)

w.k.t $P(x) \geq 0$ & $\sum P(x) = 1$ ie $k \geq 0$ &

$k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

(i) Hence $\boxed{k = 1/49}$

(ii) $P(x \geq 5) = P(5) + P(6) = 11k + 13k = 24k = 24/49$

(iii) $P(3 < x \leq 6) = P(4) + P(5) + P(6) = 33k = 33/49$

Q2 (b)

Given $f(x) = k(x^2+4)$ for $x = 0, 1, 2, 3$

w.k.t $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$= \int_0^3 k(x^2+4)\, dx = 1$

$= k\left[\frac{(x^2+4)^2}{2}\right]_0^3 = 1$

$= k\left[\frac{(9+4)^2}{2} - \frac{(-4)^2}{2}\right] = 1$

$= k\left(\frac{9}{2}\right) = 1$

$\Rightarrow \boxed{k = 2/9}$

A probability of a bad reaction from certain injection is 0.001. Determine the chance out of 2k individuals more than 2 will get bad reaction

→ P.D function is given by $p(x) = \frac{m^x e^{-m}}{x!}$

Mean for poisson distribution $\mu = m$

$p = 0.001 \quad n = 2000$

$\mu = np$ (binomial distribution)

$= 2000 \times 0.001$

$\mu = 2 \quad \therefore m = 2$

$p(x>2) = 1 - p(x \leq 2)$

$= 1 - (p(0) + p(1) + p(2))$

$= 1 - \{0.13533 + 0.27067 + 0.27066\}$

$= 0.32334.$

1b

1c

Poisson Distribution :-

It is regarded as the limiting for of binomial distribution where 'n' is very larger 'p' is very small. where p is the probability of success

Probability Distribution is given by $p(x) = \frac{m^x e^{-m}}{x!}$

Mean (μ), Variance (V) & Standard Deviation of Poisson Dist^n

Mean(μ) = $\sum_{x=0}^{n} x\,p(x)$

$\sum_{x=0}^{n} x \frac{m^x e^{-m}}{x!}$

$\sum_{x=0}^{n} \cancel{x} \frac{m^x e^{-m}}{\cancel{x}(x-1)!}$

$\sum_{x=0}^{n} \frac{m^1 m^{x-1} \cdot e^{-m}}{(x-1)!}$

$\mu = me^{-m} \sum_{x=1}^{n} \frac{m^{x-1}}{(x-1)!}$

$\mu = me^{-m}\left[\left[\frac{1}{1}\right] + \frac{m^1}{1!} + \frac{m^2}{2!} + \dots\right]$

$\mu = me^{-m} e^{m}$

$\boxed{\mu = m}$

Variance(V) = $\sum_{x=0}^{n} x^2 p(x) - \mu^2$ — ①

$\Rightarrow \sum x^2 p(x) \to \sum [x(x-1) + x] \cdot p(x)$

$= \sum_{x=0}^{n} x(x-1)p(x) + \sum_{x=0}^{n} x\,p(x)$

$= \sum_{x=0}^{n} x(x-1)\,p(x) + \mu$

$= \sum_{x=0}^{n} \frac{\cancel{x(x-1)}\, e^{-m} m^2 m^{x-2}}{\cancel{x(x-1)}(x-2)!} + m$

$= \sum_{x=2} e^{-m} m^2 \frac{m^{x-2}}{(x-2)!} + m$

$= \sum x^2 p(x) = m^2 e^{-m} \sum_{x=2}^{n} \frac{m^{x-2}}{(x-2)!} + m$

$= m^2 e^{-m}\left[\frac{m^0}{0!} + \frac{m^1}{1!} + \dots\right] + m$

$= m^2 e^{-m} e^{m} + m$

$\sum x^2 p(x) = m^2 + m$

On Substituting in Eq^n ①

$= m^2 + m - m^2$

$\boxed{\text{Variance (V)} = m}$

Note:

Mean and Variance Both are equal to 'm' in Poisson Distribution

2a

The probability that a pen manufactured by factory be deffective is 1/10. If 12 such pens are manufactured what is the probability of that (i) Exactly 2 are defective $P(x=2)$
(ii) atleast 2 are defective $(x\geq 2)$
(iii) No Defectives

→ Probability of pen = $p = 1/10 = 0.1$

$q = 1-p = 1-0.1 = 0.9$

we have $P(x) = {}^nC_x \, p^x q^{n-x}$ $n=12$, $p=0.1$, $q=0.9$

$= {}^{12}C_x (0.1)^x (0.9)^{12-x}$

(i) $P(x=2) = {}^{12}C_2 (0.1)^2 (0.9)^{10}$

$= 0.2301$ #

(iii) $P(x=0) = {}^{12}C_0 (0.1)^0 (0.9)^{12}$

$= 0.2824$ #

(ii) $P(x\geq 2) = 1 - P(x<2)$

$= 1 - [P(0) + P(1)]$

$= 1 - [0.2824 + 0.3765]$

$P(x\geq 2) = 0.3410$ #

Q2 (b)

W.k.t To serve as PD of the discrete random variable X

$\sum f(x) = 1$ ie $\sum_{x=0}^{3} [k(x^2+4)] = 1$

$f(0) + f(1) + f(2) + f(3) = 1$

$k(0+4) + k(1+4) + k(4+4) + k(9+4) = 1$

$k(4+5+8+13) = 1$

$30k = 1$

$k = 1/30$

(i) $P(0 < x \le 2) = f(1) + f(2)$

$= 5k + 8k$

$= 13/30$

(ii) $P(x \geq 1) = f(1) + f(2) + f(3)$

$= 5k + 8k + 13k$

$= 26/30$

Binomial distribution

If 'p' is the probability of sucess & q is the probability of faliure the probability of x-sucess out of n trial is given by

$$p(x) = {}^nC_x\, p^x q^{n-x}$$

Mean(μ), Variance(v) & Standard Deviation of Binomial Distribution

Mean(μ) $= \sum x\, p(x)$

$$= \sum_{x=0}^{n} x\, {}^nC_x\, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{x\, n!}{x!\,(n-x)!}\, p \cdot p^{x-1} q^{n-1+1-x}$$

$$= \sum_{x=0}^{n} \frac{\cancel{x}\, n\,(n-1)!}{\cancel{x}\,(x-1)!\,(n-1+1-x)!}\, p\; p^{x-1} q^{(n-1)-(x-1)}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,[(n-1)\,(x-1)]!}\, p^{x-1} q^{(n-1)-(x-1)}$$

$$\mu = np \sum_{x=1}^{n} {}^{n-1}C_{x-1}\, p^{x-1} q^{(n-1)-(x-1)}$$

$$= np\,(q+p)^{n-1}$$

$$= np\,(1)^{(n-1)}$$

$$\mu = np(1)$$

$$\therefore \boxed{\mu = Mean = np}$$

2c

Variance $= V = \sum_{x=0}^{n} x^2 p(x) - (\mu)^2$

$$\sum_{x=0}^{n} x^2 p(x) = \sum_{x=0}^{n} \{ x(x-1) + x \} p(x)$$

$$= \sum_{x=0}^{n} x(x-1)p(x) + \sum_{x=0}^{n} x\,p(x)$$

$$= \sum_{x=0}^{n} x(x-1)\,{}^{n}C_x p^x q^{n-x} + np$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x q^{n-x} + np$$

$$= \sum_{x=0}^{n} \frac{x(x-1)\, n(n-1)(n-2)!}{x(x-1)(x-2)!\,[n-2+2-x]!} p^{(x-2)} p^2 q^{n-2+2-x} + np$$

$$= \sum_{x=2}^{n} n(n-1)p^2 \frac{(n-2)!}{(x-2)!\,[(n-2)-(x-2)]!} q^{(n-2)-(x-2)} p^{(x-2)} + np$$

$$= n(n-1)p^2 (p+q)^{n-2} + np$$

$$= n(n-1)p^2 (1)^{n-2} + np$$

$$\sum_{x=0}^{n} x^2 = n(n-1)p^2 + np$$

$$V = \sum_{x=0}^{n} x^2 p(x) - \mu^2$$

$$V = n(n-1)p^2 + np - (np)^2$$

$$= (n^2-n)p^2 + np - n^2p^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= np - np^2$$

$$= np(1-p)$$

$$V = npq$$

Standard Deviation $(\sigma) = \sqrt{V}$

$$\sigma = \sqrt{npq}$$

3a

JPD of two random variable X & Y is →

$x \backslash y$	-3	2	4
1	0.1	0.2	0.2
2	0.3	0.1	0.1

Compute → Marginal distribution of X & Y

→ $E(X)$, $E(Y)$, $E(XY)$, σ_x, σ_y

→ $\mathrm{cov}(X,Y)$ → $\rho(X,Y)$ → Whether X & Y are independent ?

→

X	1	2
$f(x)$	0.5	0.5

Y	-3	2	4
$g(y)$	0.4	0.3	0.3

→ $E(X) = 0.5 + 1 = 1.5$

$E(Y) = -1.2 + 0.6 + 1.2 = 0.6$

$E(XY) = \sum x_i y_j J_{ij}$

$= -0.3 + 0.4 + 0.8 - 1.8 + 0.4\ 0.8 = 0.3$

→ Whether X & Y are independent or Not

Condition for independent is $f(x_i) \cdot g(y_j) = J_{ij}$

$f(x_1) = 0.5$ $g(y_1) = 0.4$ $J_{11} = 0.1$

$(0.5)(0.4) \neq 0.1$

Hence X & Y are not independent

OR

$E(X, Y) = E(X) \cdot E(Y)$

$0.3 \neq (1.5)(0.6)$

$cov(XY)$ should be equal to '0'

$cov(XY) = -0.6 \neq 0$

Hence X & Y are not independent.

Experiment No. :

Experiment Name :

Date :

Q3 (b)

Given $f(x,y) = k|x-y|$ for $x = -2, 0, 2$; $y = -2, 3$

$x \backslash y$	-2	3	sum
-2	0	$5k$	$5k$
0	$2k$	$3k$	$5k$
2	$4k$	k	$5k$
sum	$6k$	$9k$	$\boxed{15k}$

(i) we must have

$15k = 1$

$k = 1/15$

Marginal Probability distribution is as follows

x_i	-2	0	2
$f(x_i)$	$1/3$	$1/3$	$1/3$

y_i	-2	3
$g(y_i)$	$2/5$	$3/5$

$E(x) = 1/3 + 1/3 + 1/3 = 1$

$E(y) = 2/5 + 3/5 = 1$

$E(XY) = 2/15 + 3/15 + 2/15 + 3/15 + 2/15 + 3/15 = 1$

$cov = E(XY) - E(X).E(Y)$

$= 1 - 1$

$cov(X,Y) = 0$ $\therefore$ X & Y are independent.

3c

3 Boys A, B, C are throwing ball to each other. A always throws the ball to B. And B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball find the probability that after 3 throws (i) A has the Ball (ii) B has the Ball (iii) C has the Ball

→ The state space of the system is A, B, C

The associated TPM is,

$$P = \begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

After 3 throws means we need to compute P^3

$$P^2 = P.P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

$$P^3 = P^2.P = \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$$

Initially C has the ball the associated initial probability vector is $P^{(0)} = [0 \quad 0 \quad 1]$

$$P^{(3)} = P^{(0)}.P^3 \Rightarrow P^{(3)} = [0 \quad 0 \quad 1]\begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$$

$$P^{(3)} = [1/4 \quad 1/4 \quad 1/2]$$

Thus after 3 throws the probability that (i) A has the Ball is 1/4 #

(ii) B has the Ball is 1/4 #

(iii) C has the Ball is 1/2 #

[2] *The joint probability distribution table for two random variables X and Y is as follows.*

X \ Y	−2	−1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine the marginal probability distributions of X and Y.
Also compute,

(a) *Expectations of X, Y and X Y* **(b)** *S.Ds of X, Y*

(c) *Covariance of X and Y* **(d)** *Correlation of X and Y.*

Further verify that X and Y are dependent random variables.

Also find P (X + Y > 0). **[June 2019]**

4a

☞ Marginal distribution of X and Y are got by adding all the respective row entries and the respective column entries.

x_i	1	2
$f(x_i)$	0.6	0.4

y_j	−2	−1	4	5
$g(y_j)$	0.3	0.3	0.1	0.3

(a) $\mu_X = E(X) = \sum_i x_i f(x_i)$

$$= (1)(0.6) + (2)(0.4) = 1.4$$

$$\mu_Y = E(Y) = \sum_j y_j\, g(y_j)$$

$$=(-2)(0.3)+(-1)(0.3)+(4)(0.1)+(5)(0.3)$$

$$=-0.6-0.3+0.4+1.5=1$$

$$E(XY) = \sum_{i,j} x_i\, y_j\, J_{ij}$$

$$= (1)(-2)(0.1)+(1)(-1)(0.2)+(1)(4)(0)$$

$$+(1)(5)(0.3)+(2)(-2)(0.2)+(2)(-1)(0.1)$$

$$+(2)(4)(0.1)+(2)(5)(0)$$

$$= -0.2-0.2+0+1.5-0.8-0.2+0.8+0 = 0.9$$

Thus, $\boxed{E(X) = 1.4,\ E(Y) = 1 \text{ and } E(XY) = 0.9}$

(b) $\sigma_X^2 = E(X^2) - \mu_X^2$ and $\sigma_Y^2 = E(Y^2) - \mu_Y^2$

$$E(X^2) = \sum_i x_i^2 f(x_i)$$

$$= (1)(0.6)+(4)(0.4) = 2.2$$

$$E(Y^2) = \sum y_j^2\, g(y_j)$$

ie., $= (4)(0.3)+(1)(0.3)+(16)(0.1)+(25)(0.3) = 10.6$

$\therefore$ $\sigma_X^2 = 2.2-(1.4)^2 = 0.24$ and hence $\sigma_X = 0.49$

$\sigma_Y^2 = 10.6-(1)^2 = 9.6$ and hence $\sigma_Y = 3.1$

Thus, $\boxed{\sigma_X = 0.49 \text{ and } \sigma_Y = 3.1}$

(c) $COV(X,Y) = E(XY) - E(X)E(Y)$

$$= 0.9-(1.4)(1) = -0.5$$

$\therefore$ $\boxed{COV(X,Y) = -0.5}$

(d) Correlation of X and $Y = \rho\,(X,Y) = \dfrac{COV(X,Y)}{\sigma_X \sigma_Y}$

ie., $\rho(X,Y) = \dfrac{-0.5}{(0.49)(3.1)} = -0.3$

$\therefore$ $\boxed{\rho(X,Y) = -0.3}$

If X and Y are independent random variables we must have $f(x_i)g(y_j) = J_{ij}$

It can be seen that $f(x_1)g(y_1) = (0.6)(0.3) = 0.18$ and $J_{11} = 0.1$

ie., $f(x_1)\cdot g(y_1) \neq J_{11}$

Similarly for others also the condition is not satisfied.

Hence we conclude that X and Y are dependent random variables.

We have, $X = \{x_i\} = \{x_1, x_2\} = \{1, 2\}$ respectively.

$$Y = \{y_j\} = \{y_1, y_2, y_3, y_4\} = \{-2, -1, 4, 5\} \text{ respectively.}$$

Also, $J_{11} = 0.1$, $J_{12} = 0.2$, $J_{13} = 0$, $J_{14} = 0.3$,

$J_{21} = 0.2$, $J_{22} = 0.1$, $J_{23} = 0.1$, $J_{24} = 0$

$X + Y > 0$ is possible when (X, Y) take the values

$(x_1, y_3) = (1, 4)$; $(x_1, y_4) = (1, 5)$; $(x_2, y_2) = (2, -1)$;

$(x_2, y_3) = (2, 4)$ and $(x_2, y_4) = (2, 5)$

Hence, $P(X+Y>0) = J_{13} + J_{14} + J_{22} + J_{23} + J_{24}$

$$= 0 + 0.3 + 0.1 + 0.1 + 0 = 0.5$$

Thus, $\boxed{P(X+Y>0) = 0.5}$

Compute the following.

(a) $E(X)$ *and* $E(y)$ (b) $E(XY)$ (c) σ_X *and* σ_y

(d) $COV(X, Y)$ (e) $\rho(X, Y)$ **[June 2017, 18, Dec 18]**

☞ The distribution (*marginal distribution*) of X and Y is as follows.
This distribution is obtained by adding the all the respective row entries and also the respective column entries.

Distribution of X :

x_i	1	5
$f(x_i)$	1/2	1/2

Distribution of Y :

y_j	−4	2	7
$g(y_j)$	3/8	3/8	1/4

(a) $E(X) = \sum x_i f(x_i) = (1)(1/2) + (5)(1/2) = 3$

$$E(Y) = \sum y_j g(y_j)$$

$$= (-4)(3/8) + (2)(3/8) + (7)(1/4) = 1$$

Thus, $\boxed{\mu_X = E(X) = 3 \text{ and } \mu_Y = E(Y) = 1}$

(b) $E(XY) = \sum x_i y_j J_{ij}$

$$= (1)(-4)(1/8) + (1)(2)(1/4) + (1)(7)(1/8)$$

$$+ (5)(-4)(1/4) + (5)(2)(1/8) + (5)(7)(1/8)$$

$$= \frac{-1}{2} + \frac{1}{2} + \frac{7}{8} - 5 + \frac{5}{4} + \frac{35}{8} = \frac{3}{2}$$

Thus, $\boxed{E(XY) = 3/2}$

(c) $\sigma_X^2 = E(X^2) - \mu_X^2$ and $\sigma_Y^2 = E(Y^2) - \mu_Y^2$

Now, $E(X^2) = \sum x_i^2 f(x_i)$

ie., $E(X^2) = (1)(1/2) + (25)(1/2) = 13$

Also, $E(Y^2) = \sum y_j^2 g(y_j)$

ie., $E(Y^2) = 16(3/8) + (4)(3/8) + (49)(1/4) = 79/4$

Hence, $\sigma_X^2 = 13 - (3)^2 = 4$; $\sigma_Y^2 = (79/4) - (1)^2 = 75/4$

Thus, $\boxed{\sigma_X = 2 \quad \text{and } \sigma_Y = \sqrt{75/4} = 4.33}$

(d) $COV(X,Y) = E(XY) - \mu_X \mu_Y$

$$= (3/2) - (3)(1) = -3/2$$

$\therefore$ $\boxed{COV(X,Y) = -3/2}$

(e) $$\rho(X,Y) = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

$$= \frac{-3/2}{(2)\sqrt{75/4}} = \frac{-3}{2\sqrt{75}}$$

Thus, $\boxed{\rho(X,Y) = -0.1732}$

4c

A student study habits are as follows. If he studied 1 nght he is 70% sure not to study the next nght. On other hand if he doesn't study 1 nght he is 60% sure not to study next nght. In the long run how often does he study.

→ The state space of the system is A, B

A - Study B - Not Study. The associated TPM is $= P = \begin{array}{c|cc} & A & B \\ \hline A & 0.3 & 0.7 \\ B & 0.4 & 0.6 \end{array}$

$VP = V \quad V = [x \ \ y] \text{ where } x+y = 1$

$[x \ \ y] \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix} = [x \ \ y] \rightarrow 0.3x + 0.4y = x \qquad 0.7x + 0.6y = y$

$0.7x - 0.4y = 0 \ \text{—(1)} \qquad 0.7x - 0.4y = 0 \ \text{—(2)}$

Put $x = 1-y$ in (1) or (2)

$0.7(1-y) - 0.4y = 0$

$0.7 - 0.7y - 0.4y = 0$

$1.1y = 0.7$

$y = \frac{0.7}{1.1} = \boxed{y = \frac{7}{11}}$

$x + y = 1$

$x = 1 - y$

$x = 1 - \frac{7}{11}$

$\boxed{x = \frac{4}{11}}$

$V = \left[\frac{4}{11} \ \ \frac{7}{11}\right]$

In the long run he often studies $4/11 \approx 36.36$

36.36 % of the time

5a

A dice is thrown 9000 times and a throw of 3 or 4 (5 or 6) was observed 3240 times show that dice cannot be regarded as an unbaised one.

→ Probability of getting 3 or 4 (5 or 6) is $2/6$ ie $P = 1/3$ $q = 1-P \Rightarrow q = 2/3$

Expected no. of success $= np = 9000 \times 1/3 = 3000$

Observed no. of success $= 3240$

we have $Z = \dfrac{x - np}{\sqrt{npq}} = \dfrac{3240 - 3000}{\sqrt{9000 \times 1/3 \times 2/3}} = \dfrac{240}{\sqrt{2000}} = 5.36$

$\left.\begin{array}{l} 5.36 > 1.96 \\ 5.36 > 2.58 \end{array}\right\}$ hence the dice is baised at both 5% & 1% LOS

→ P_1 = Probability of a person to be a tea drinker before increase in excise duty

Q 5 $P_1 = 4\cancel{00}/5\cancel{00} = 4/5 = 0.8$

(b) P_2 = " " " " " after " " " "

$P_2 = 4\cancel{00}/6\cancel{00} = 2/3 = 0.6$

Given = $n_1 = 500$ $n_2 = 600$ $P_1 = 4/5$ $P_2 = 2/3$

we have $Z = \dfrac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $p = \dfrac{n_1P_1 + n_2P_2}{n_1 + n_2}$

H_0: there is no ~~difference between~~ decrease in consump^n of tea

H_1: there is ~~diff~~ " " " " "

$p = \dfrac{n_1P_1 + n_2P_2}{n_1 + n_2} = \therefore p = \dfrac{\cancel{500}^{100}(4/\cancel{5}) + 2/\cancel{3}(\cancel{600}^{200})}{500+600} = \dfrac{400+400}{1100} = \dfrac{8\cancel{00}}{11\cancel{00}}$

$p = 8/11 = 0.72 \qquad q = 0.28$

$$z = \left| \frac{0.8 - 0.6}{\sqrt{(0.72)(0.28)\left(1/500 + 1/600\right)}} \right|$$

$z = 8.16$

$$\boxed{\begin{aligned} z &> 1.9645 \\ &> 2.33 \end{aligned}}$$

↓

Hypothesis is rejected at both 95% & 99% LOS

5c

[15] *A survey was conducted in a slum locality of 2000 families by selecting a sample of size 800. It was revealed that 180 families were illiterates. Find the probable limits of the illiterate families in the population of 2000.*

☞ Probability of illiterate families $= p = \dfrac{180}{800} = 0.225 \quad \therefore \; q = 0.775$

Probable limits of illiterate families $= p \pm (2.58)\sqrt{pq/n}$

ie., $= 0.225 \pm (2.58)\sqrt{\dfrac{(0.225)(0.775)}{800}} = 0.225 \pm 0.038$

$= 0.187$ and 0.263

$\therefore$ the probable limits of illiterate families in the population of 2000 is (0.187) 2000 and (0.263) 2000

Thus, 374 to 526 are probably illiterate families.

6a

In 324 throws of a six faced dice, an odd no. turned up 181 times. Is it reasonable to think that the dice is unbaised one

→ Probability of turned up an odd is $3/6 = p = 1/2 = q$

Expected no. of success $= np = 324 \times \frac{1}{2} = 162$ #

observed no of success $= 181$ #

we have $z = \frac{x - np}{\sqrt{npq}} = \frac{181 - 162}{\sqrt{324 \times 1/2 \times 1/2}} = \frac{19}{\sqrt{81}} = \frac{19}{9} = 2.1$

$2.1 > 1.96$ hence the dice is baised at 5% LOS

$2.1 < 2.58$ hence the dice is unbaised at 1% LOS

5 or 6

Q(6) b. $n = 100$ $\bar{x} = 64$ $\sigma = 3$ $\mu = 67$?

$H_0 = \mu \neq 67$ (there is no chang in the mean weight)

$H_1 = \mu = 67$ (there is chang " " " ")

w.k.t $Z = \left|\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}\right| = \left|\frac{64-67}{3/\sqrt{100}}\right| = \left|\frac{\cancel{3} \times 10}{\cancel{3}}\right|$

$Z = 10$

$Z > 1.645$ Hence hypothesis is rejected

> 2.33 that is $\mu \neq 67$ is rejected

∴ the mean weight of the population is 67gms @ 5% LOS

99% confidence limits are

$\bar{x} \pm 2.58 \frac{\sigma}{\sqrt{n}} \rightarrow 64 \pm 2.58\left(\frac{3}{10}\right) \rightarrow 64 + 0.774$ $64 - 0.774$

$= 64.774$ $= 63.226$

6c

10. The average income of persons was ₹210 with a std deviation of ₹10 in sample of 100 people of city. For another sample of 150 persons the average income was ₹220 with SD of ₹12. The SD of incomes of the people of city was ₹11. Test whether there is any significant difference between the average income of the localities

→ H_0 : There is no difference between the avg income of 2 localities
H_1 : There is difference between the avg income of 2 localities

Given : $\bar{x}_A = 210$ $\sigma_A = 10$ $n_1 = 100$, $\bar{x}_B = 220$ $\sigma_B = 12$ $n_2 = 150$

w.k.t $z = \dfrac{\bar{x}_A - \bar{x}_B}{\sqrt{\sigma_A^2/n_1 + \sigma_B^2/n_2}} = \dfrac{210 - 220}{\sqrt{10^2/100 + 12^2/150}} = 7.14$

$7.14 > 1.96$ hypothesis is rejected @ 5% LOS of TTT

$7.14 > 2.58$ hypothesis is rejected @ 1% LOS of TTT

Hence there is significant difference between the average income of the localities

7a

→ The expected frequencies are

$\frac{9}{16} \times 556$, $\frac{3}{16} \times 556$, $\frac{3}{16} \times 556$, $\frac{1}{16} \times 556$

312.75 104.25 104.25 35

The table is as follows

O_i	315	101	108	32
E_i	313	104	104	35

we have $\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = \frac{(315-313)}{313} + \frac{(101-104)^2}{104} + \frac{(108-104)^2}{104} + \frac{(32-35)^2}{35}$

$= 0.51030 \simeq 0.51$

d.f $\gamma = n-1 = 4-1 = 3$ $\chi^2_{0.05}$ at 3df $= 7.81$

$\chi^2 = 0.51 < \chi^2_{0.05} = 7.81$

Accept the hypothesis

Q7 (b) Given $n = 100$ $\mu = 53$ $\sigma^2 = 400$

we have $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{\bar{x} - 53}{2}$ put $\bar{x} = 50$ & 56 $\dfrac{50-53}{2} = 1.5$ & $\dfrac{56-53}{2} = 1.5$

$P(50 < \bar{x} < 56) \simeq P(-1.5 < z < 1.5)$

$= 2P(0 \leq z < 1.5)$

$= 2\phi(1.5)$

$= 2(0.4332)$

$P(50 < \bar{x} < 56) = 0.8664$

7c

10 individuals are choosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 71, 71, 70. Test the hypothesis that the mean height of the universe is 66 inches ($t_{0.05}$ 9 df = 2.262)

→ Given data 63, 63, 66, 67, 68, 69, 70, 71, 71, 70

$\mu = 66$ $\quad n = 10$ $\quad H_0 : \mu = 66$ $\quad H_1 : \mu \neq 66$

$\bar{x} = \frac{\sum x}{n} = \frac{678}{10} = 67.8$

$|t| = \frac{\bar{x} - \mu}{s} \sqrt{n}$

$s^2 = \frac{1}{(n-1)} \left\{ \sum_{i=1}^{n} x_i^2 - \frac{1}{n} \sum (x_i)^2 \right\}$

$= \frac{67.8 - 66}{3.011} \sqrt{10}$

$= \frac{1}{9} \left\{ 46,050 - \frac{1}{10} (459684) \right\}$

$|t| = 1.89$

$s^2 = 9.067$

$|t| = 1.89 < t_{0.05} = 2.262$

$s = 3.011$

Calculated Value Table Value

Accept the Hypothesis. That is the mean height of the universe is 66 inches at 5% Level of Significance (LOS)

Experiment No. :

Date :

Experiment Name :

Q 8 (a)

H_0 : There is no difference in the performance of A & B

H_1 : A > B

x =	Batsman A	40	50	35	25	60	70	65	55
y =	Batsman B	60	70	40	30	50			

$\bar{x} = \frac{\Sigma x}{n_1} = \frac{400}{8} = 50$ $\quad \bar{y} = \frac{\Sigma y}{n_2} = 50$ $\quad n_1 = 8 \quad n_2 = 5$

$$\Sigma(x-\bar{x})^2 = (40-50)^2 + (50-50)^2 + (35-50)^2 + (25-50)^2 + (60-50)^2 + (70-50)^2 + (65-50)^2 + (55-50)^2$$

$$= 1700$$

$$\Sigma(y-\bar{y})^2 = (60-50)^2 + (70-50)^2 + (40-50)^2 + (30-50)^2 + (50-50)^2$$

$$= 1000$$

w.k.t $S^2 = \frac{1}{n_1+n_2-2}\{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2\}$

$$= \frac{1}{11}(1700 + 1000)$$

$S^2 = 245.45$

$S = 15.6$

$$t = \frac{\bar{x}-\bar{y}}{S\sqrt{1/n_1 + 1/n_2}} = \frac{50-50}{15.6\sqrt{1/8 + 1/5}}$$

$t = 0$

degrees of freedom for 2 sample is $n_1+n_2-2 = 11$

$(t_{0.01} = 7.85)$ $\quad 0 < 7.85$

Accept the null hypothesis $\therefore$ There is no difference between the performance of Batsman A & B

Q 8 (b)

Total accidents = 84 Number of days = 7

Expected accidents per day = $84/7 = 12$

w.k.t $\chi^2 = \Sigma\frac{(O_i - E_i)^2}{E_i}$

$$= \frac{(14-12)^2}{12} + \frac{(16-12)^2}{12} + \frac{(8-12)^2}{12} + \frac{(12-12)^2}{12} + \frac{(11-12)^2}{12} + \frac{(9-12)^2}{12} + \frac{(14-12)^2}{12}$$

$\chi^2 = 3.165 < \chi^2_{0.05} = 12.59$

Hence we accept the null hypothesis ie accidents are uniformly distributed over the week

Q8 (c) Given data : 45, 47, 50, 52, 48, 47, 49, 53, 51

$\mu = 47.5$ $n = 9$ $H_0 : \mu = 47.5$ $H_1 : \mu \neq 47.5$

$\bar{x} = \frac{\Sigma x}{n} = \frac{442}{9} = 49.1$

$|t| = \frac{\bar{x} - \mu}{s} \sqrt{n}$

$= \frac{49.1 - 47.5}{2.61} \sqrt{9}$

$|t| = 1.83$

$|t| = 1.83 < t_{0.05} = 2.31$

Accept the null hypothesis

i.e. $\mu = 47.5$

$S^2 = \frac{1}{(n-1)} \left\{ \Sigma x_i^2 - \frac{1}{n} \Sigma (x_i)^2 \right\}$

$= \frac{1}{8} \left(21,762 - \frac{1}{9}(195364) \right)$

$= \frac{1}{8} (21,762 - 21,707.1)$

$= \frac{54.9}{8}$

$S^2 = 6.8625$ $\therefore S = 2.61$

09 (a)

Observation	A	B	C
1	25	31	24
2	30	39	30
3	36	38	28
4	38	42	25
5	31	35	28

→ (i) $H_0 : \bar{X}_A = \bar{X}_B = \bar{X}_C$

$H_1 : \bar{X}_A \neq \bar{X}_B \neq \bar{X}_C$

(i) Calculate the variance between the samples (SSC)

(a) calculate mean of each sample

$\bar{x}_A = \frac{160}{5} = 32$ $|||^{ly}$ $\bar{x}_B = 37$ $\bar{x}_C = 27$

(b) Calculate the average of these means

$(\bar{\bar{x}}) = (32 + 37 + 27)/3 = 96/3 = 32$

(c) Take the difference b/w mean of various sample and $(\bar{\bar{x}})$ & sq it.

$= \Sigma (\bar{x}_A - \bar{\bar{x}})^2 + \Sigma (\bar{x}_B - \bar{\bar{x}})^2 + \Sigma (\bar{x}_C - \bar{\bar{x}})^2$

$= 0 + 125 + 125$

SSC = 250

Experiment No. :   Date :

Experiment Name :

(iii) Calculate the variance within the samples (SSE)

(a) $\bar{x}_A = 32 \quad \bar{x}_B = 37 \quad \bar{x}_C = 27$

(b) $\Sigma(A-\bar{x}_A)^2 = (25-32)^2 + (30-32)^2 + (36-32)^2 + (38-32)^2 + (31-32)^2$
$= 106$

$\Sigma(B-\bar{x}_B)^2 = (31-37)^2 + (39-37)^2 + (38-37)^2 + (42-37)^2 + (35-37)^2$
$= 70$

$\Sigma(C-\bar{x}_C)^2 = (24-27)^2 + (30-27)^2 + (28-27)^2 + (25-27)^2 + (28-27)^2$
$= 24$

$SSE = \Sigma(A-\bar{x}_A)^2 + \Sigma(B-\bar{x}_B)^2 + \Sigma(C-\bar{x}_C)^2$
$= 106 + 70 + 24$
$= 200$

(iv) Calculate F

Source of Var^n	Sum of Sq^s	d.f	MSS	F
	$SSC = 250$	$\gamma_1 = C-1$ $= 3-1$ $= 2$	$MSC = \frac{SSC}{C-1}$ $= 125$	$F = \frac{MSC}{MSE}$
	$SSE = 200$	$\gamma_2 = n-C$ $= 15-3$ $= 12$	$MSE = \frac{SSE}{n-C}$ $= 16.6$	$= \frac{125}{16.6}$ $\boxed{F = 7.53}$

Given $F_{2,12} = 3.89$ $\quad F_{cal} > F_{tab}$
$7.53 > 3.89$

Reject the Null Hypothesis. Hence machines are significantly different on their mean speed @ 5% LOS

Q9 (b) This is similar to the above example only values are different rest all method is same ☺

10a

Set up ANOVA for the following data. 2-way design verify the difference @ 5% LOS

Varities of seeds Fertilizers	A	B	C
W	6	5	5
X	7	5	4
Y	3	3	3
Z	8	7	4

Given that

$F(2,6) = 5.14$

$F(3,6) = 4.76$

→ Given problem is 2-way design without repeated values

	A	B	C	Total	A^2	B^2	C^2
W	6	5	5	16	36	25	25
X	7	5	4	16	49	25	16
Y	3	3	3	9	9	9	9
Z	8	7	4	19	64	49	16
Total	24	20	16	60 → T	158	108	66

$= 332 \rightarrow \Sigma x_{ij}^2$

(i) Correction factor $= \frac{T^2}{n} = \frac{60^2}{12} = 300$

(ii) Total SS $= \Sigma x_{ij}^2 - T^2/n = 332 - 300 = 32$

(iii) SSB columns $= (24^2 + 20^2 + 16^2)/4 - 32 = 8$

(iv) SSB rows $= (16^2 + 16^2 + 9^2 + 19^2)/3 - 32 = 18$

(v) SSR or Error $=$ Total SS $-$ [SSB col^{ms} + SSB rows]

$= 32 - (8+18)$

$= 6$

Source of Variation	SS	d.f	Mean Sum of Squares	F-test (F-ratio)
Between Columns	8	$c-1 = 2$	MSB col $= \frac{\text{SSB col}}{c-1} = 4$	$\frac{\text{MSB col}}{\text{MSR}} = \frac{4}{1} = 4 < 5.14$
Between Rows	18	$r-1 = 3$	MSB rows $= \frac{\text{SSB row}}{r-1} = 6$	$\frac{\text{MSB row}}{\text{MSR}} = \frac{6}{1} = 6 > 4.76$
Residual or Error	6	$(c-1)(r-1) = 6$	MSR $= \frac{\text{SSR}}{(c-1)(r-1)} = 1$	

From the above table we conclude that varities of seeds are insignificant at 5% LOS and varities of fertilizer are significant at 5% LOS according to F ratio test.

Analyze the variance in the following Latin-Square of yield of paddy where A, B, C, D denotes different method of cultivation. Examine wheather the different methods of cultivation have given different yields significanty given that $F(3,6) = 4$ ...

D 122	A 121	C 123	B 122
B 124	C 123	A 122	D 125
A 120	B 119	D 120	C 121
C 122	D 123	B 121	A 122

10b

→ H_0 – (i) There is not difference between rows

(ii) There is not difference between collumns

(iii) There is not difference between Letters

(i) Using coding method substract by 120

D 2	A 1	C 3	B 2	8	4	1	9	4	
B 4	C 3	A 2	D 5	14	16	9	4	25	
A 0	B -1	D 0	C 1	0	0	1	0	1	
C 2	D 3	B 1	A 2	8	4	9	1	4	
8	6	6	10	30 — T	24	20	14	34	92 — Σx_{ij}^2

correction factor $T^2/n = 30^2/16 = 56.25$

(ii) $SST = \Sigma x_{ij}^2 - T^2/n = 92 - 56.25 = 35.75$

(iii) $SSR = (8^2 + 14^2 + 0^2 + 8^2)/4 - 56.25 = 24.75$

(iv) $SSC = (8^2 + 6^2 + 6^2 + 10^2)/4 - 56.25 = 2.75$

(v) $SSL = \left[(\Sigma A)^2 + (\Sigma B)^2 + (\Sigma C)^2 + (\Sigma D)^2\right]/n - 56.25 = 4.25$

$(5^2 + 6^2 + 9^2 + 10^2)/4 - 56.25 = 4.25$

(vi) $SSE = SST - (SSR + SSC + SSL)$

$SSE = 4$

Source of Var^n	S.S	d.f	MSS	F-ratio
Rows	SSR = 24.75	r−1	MSR = SSR/d.f	F_R = MSR/MSE
		= 4−1 = 3	= 8.25	= 12.31 > 4.76
Columns	SSC = 2.75	c−1	MSC = SSC/d.f	F_C = MSC/MSE
		4−1 = 3	= 0.92	= 1.37 < 4.76
Letters	SSL = 4.25	L−1	MSL = SSL/d.f	F_L = MSL/MSE
		4−1 = 3	= 1.42	2.12 < 4.76
Error	SSE = 4	(c−1)(r−2)	MSE = SSE/d.f	
		3×2 = 6	= 0.67.	

Hypothesis is accepted for Columns & Letters & is rejected for rows