

II B. Tech I Semester Regular Examinations, March - 2021
VECTOR CALCULUS & FOURIER TRANSFORMS
(Mechanical Engineering)

Time: 3 hours

Max. Marks: 75

Answer any **FIVE** Questions each Question from each unit
All Questions carry **Equal** Marks

- 1 a) Find the directional derivative of $f(x, y, z) = xy^3 + yz^3$ at $(2, -1, 1)$ in the direction of a vector $i + 2j + 2k$. [6M]

- b) Prove that $\text{div}(r^n \bar{r}) = (n+3)r^n$. Hence show that $\frac{\bar{r}}{r^3}$ is solenoidal. [9M]

Or

- 2 Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2dy]$, where C is bounded by $y = x$ and $y = x^2$. [15M]

- 3 a) Find the Laplace transform of (i). $\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^3$. [8M]

(ii). $2t' + \frac{\cos 2t - \cos 3t}{t} + t \sin t$.

- b) Find the inverse transforms of $\frac{5s+3}{(s-1)(s^2+2s+5)}$. [7M]

Or

- 4 a) Evaluate (i) $\int_0^\infty t e^{-2t} \sin t dt$ (ii). $L\left\{\int_0^t \frac{e^t \sin t}{t} dt\right\}$. [8M]

- b) Using Laplace transform, solve $(D^2 + n^2)x = a \sin(nt + \alpha)$, $x = Dx = 0$ at $t = 0$. [7M]

- 5 a) Obtain the Fourier series of $f(x) = \left(\frac{\pi-x}{2}\right)^2$ in the interval $0 < x < 2\pi$. Deduce [8M]

that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$.

- b) Find the Fourier sine and cosine transform of $f(x) = e^{-ax}$, $a > 0, x > 0$. [7M]

Or

- 6 a) Express $f(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq \pi \\ 0 & \text{for } x > \pi \end{cases}$ as a Fourier sine integral and hence [8M]

evaluate $\int_0^\infty \frac{1 - \cos(\pi\lambda)}{\lambda} \sin(x\lambda) d\lambda$.

- b) Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$. [7M]

- 7 a) Find one independent solution of the partial differential equation [8M]
 $(mz - ny)p + (nx - lz)q = ly - mx.$

- b) Find the complete solution of $p(1 + q) = qz.$ [7M]

Or

- 8 a) Form the partial differential equation by eliminating the arbitrary function from [5M]
the relation $z = f_1(x)f_2(y).$

- b) Find the general solution of the partial differential equation [5M]
 $(x^2 - y^2 - z^2)p + 2xyq = 2zx.$

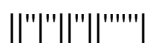
- c) Solve the partial differential equation: $p^2 + q^2 = x^2 + y^2.$ [5M]

- 9 a) Solve $2\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 5\sin(2x + y).$ [8M]

- b) Solve the by the method of separation of variables [7M]
 $4u_x + u_y = 3u$ and $u(0, y) = e^{-5y}.$

Or

- 10 An insulated rod of length L has its ends A and B maintained at 0°C and [15M]
 100°C respectively until steady state conditions prevail. If B is suddenly
reduced to 0°C and maintained at 0°C , find the temperature at a distance x
from A at time $t.$



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1 a) Prove that  $\text{div}(\phi \bar{F}) = \text{grad} \phi \cdot \bar{F} + \phi \text{div} \bar{F}$ . [7M]

b) Prove that  $\text{curl}(\bar{F} \times \bar{G}) = \bar{F} \cdot \text{div} \bar{G} - \bar{G} \cdot \text{div} \bar{F} + (\bar{G} \cdot \nabla) \bar{F} - (\bar{F} \cdot \nabla) \bar{G}$ . [8M]

Or

2 Verify Stoke's theorem for  $\bar{F} = (x^2 + y^2)\bar{i} - 2xy\bar{j}$  taken around the rectangle [15M]  
bounded by the lines  $x = \pm a$  and  $y = \pm b$ .

3 a) Find the Laplace transform of i)  $e^{-3t}(2 \cos 5t - 3 \sin 5t)$  ii)  $t \cos at$  [8M]  
ii)  $\frac{e^t \sin^2 t}{t}$

b) If  $L\{f(t)\} = F(s)$  then prove that  $L\{f(t-a)H(t-a)\} = e^{-as}F(s)$ . [7M]

Or

4 a) Find the inverse transforms of  $\frac{s}{s^4 + 4a^4}$ . [8M]

b) Solve the D.E.  $y'' + 2y' + 5y = e^{-t} \sin t$ ,  $y(0) = 0$ ,  $y'(0) = 1$ . using Laplace [7M]  
transforms.

5 a) Find the Fourier series for the function  $f(x) = \begin{cases} x & , 0 \leq x \leq \pi \\ 2\pi - x & , \pi \leq x \leq 2\pi \end{cases}$ . [8M]

Also deduce that  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$ .

b) Obtain the Fourier expansion of  $f(x) = x \sin x$  as a cosine series in  $(0, \pi)$ . [7M]

Or

6 a) Express the function  $f(x) = \begin{cases} 1 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| \geq 1 \end{cases}$  as a Fourier integral. Hence [8M]

evaluate  $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$ .

b) Find the Fourier cosine transform of the function  $f(x) = \begin{cases} x & , 0 < x < 1 \\ 2-x & , 1 < x < 2 \\ 0 & , x > 2 \end{cases}$  [7M]

7 a) Solve  $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ . [8M]

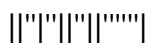
b) Solve  $p^2 + q^2 = x + y$ . [7M]

Or

- 8 a) Derive the partial differential equation by eliminating the constants from the equation  $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ . [5M]
- b) Solve the partial differential equation  $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ . [5M]
- c) Solve the partial differential equation  $z^2 = 1 + p^2 + q^2$ . [5M]
- 9 a) Solve  $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$ . [8M]
- b) Solve the by the method of separation of variables  $3u_x + 2u_y = 0$  and  $u(x, 0) = 4e^{-x}$ . [7M]

Or

- 10 A tightly stretched string with fixed end points  $x = 0$  and  $x = L$  is initially in a position given by  $y = y_0 \sin^3\left(\frac{\pi x}{L}\right)$  if it is released from rest from this position, find the displacement  $y(x, t)$ . [15M]



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- 1 a) Find a unit normal vector to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1, 2, -1)$. [6M]
b) Show that $r^n \bar{r}$ is an irrotational vector for any value of n but is solenoidal if $n + 3 = 0$, where $\bar{r} = xi + yj + zk$ and r is the magnitude of \bar{r} . [9M]

Or

- 2 Verify divergence theorem for $\bar{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ taken over the rectangular parallelepiped $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$. [15M]

- 3 a) Find the Laplace transform of i) $\cos 3t \cos 2t \cos t$ ii) $e^{3t} t^{\frac{7}{2}}$ [8M]
ii) $\frac{\cos 6t - \cos 4t}{t}$.

- b) Find the inverse Laplace transform of i) $\frac{5s-2}{s^2+4s+8}$ ii) $\log\left(\frac{s+b}{s+a}\right)$. [7M]

Or

- 4 a) If $L\{f(t)\} = F(s)$ then prove that $L\left\{\int_0^t f(u) du\right\} = \frac{F(s)}{s}$. [8M]

- b) Using Laplace transform, solve $y'' + 7y' + 10y = 4e^{-3t}$, $y(0) = 0$, $y'(0) = -1$. [7M]

- 5 a) Find a Fourier series to represent $f(x) = x - x^2$ in $-\pi \leq x \leq \pi$. Hence show [8M]
that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$.

- b) Expand $f(x) = \begin{cases} \frac{1}{4} - x, & \text{if } 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \text{if } \frac{1}{2} < x < 1 \end{cases}$ as a Fourier series of sine terms. [7M]

Or

- 6 a) Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ hence evaluate [8M]

$$\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx.$$

- b) Find the Fourier sine transform of the function $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$. [7M]

7 a) Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$. [8M]

b) Solve $z^2(p^2 + q^2) = x^2 + y^2$. [7M]

Or

8 a) Form the partial differential equation by eliminating the arbitrary constants a, b from $z = a \log \left[\frac{b(y-1)}{1-x} \right]$. [5M]

b) Solve the partial differential equation $\left(\frac{y^2 z}{x} \right) p + xzq = y^2$. [5M]

c) Find the complete integral of $p + q = pq$. [5M]

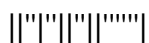
9 a) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = \cos(2x + y)$. [8M]

b) Solve the partial differential equation by the method of separation of variables $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$ and $u(0, y) = 8e^{-3y}$. [7M]

Or

10 Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy -plane, [15M]

$0 \leq x \leq a$ and $0 \leq y \leq b$ satisfying the following boundary condition
 $u(0, y) = 0, u(a, y) = 0, u(x, b) = 0$ and $u(x, 0) = f(x)$.



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- 1 a) What is the directional derivative of  $\phi = xy^2 + yz^3$  at  $(2, -1, 1)$  in the direction of the normal to the surface  $x \log z - y^2 = -4$  at  $(-1, 2, 1)$ ? [8M]  
b) Prove that  $\text{curl}(\phi \bar{F}) = \text{grad} \phi \times \bar{F} + \phi \text{curl} \bar{F}$ . [7M]  

Or
- 2 Verify Green's theorem for  $\int_C [(3x - 8y^2)dx + (4y - 6xy)dy]$  where C is the boundary of the region bounded by  $x = 0$ ,  $y = 0$  and  $x + y = 1$ . [15M]
- 3 a) Find the Laplace transform of i)  $e^{-2t} \sin^3 t$  ii)  $e^2 \cos 2t$  [8M]  
iii)  $f(t) = \begin{cases} \sin t, & t > \pi \\ \cos t, & t < \pi \end{cases}$   
b) Find the inverse Laplace transform of i).  $\frac{3s-14}{s^2-4s+8}$  ii).  $\tan^{-1}\left(\frac{2}{s}\right)$ . [7M]  

Or
- 4 a) Find the Laplace transform of  $e^{-4t} \int_0^t \frac{\sin 3u}{u} du$ . [7M]  
b) Using Laplace transform, solve  $y'' - 8y' + 15y = 9te^{2t}$ ,  $y(0) = 5$ ,  $y'(0) = 10$ . [8M]
- 5 a) Given that  $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ . Find the Fourier series for  $f(x)$ . Also deduce that  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$ . [8M]  
b) Obtain the half range sine series for  $f(x) = e^x$  in  $0 < x < 1$ . [7M]  

Or
- 6 a) Find the Fourier transform of  $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$  hence evaluate  $\int_0^\infty \frac{\sin x}{x} dx$ . [8M]  
b) Find the finite Fourier sine and cosine transforms of  $f(x) = 2x$ ,  $0 < x < 4$ . [7M]

- 7 a) Form the partial differential equation by elimination the arbitrary function  $f$  [5M]  
from the relation  $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$ .
- b) Solve the partial differential equation [5M]  
 $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ .
- c) Find the complete integral of  $pe^y = qe^x$ . [5M]

Or

- 8 a) Form the partial differential equation by eliminating the arbitrary function  $\phi$  [5M]  
from the  $\phi(x^2 + y^2 + z^2, lx + my + nz) = 0$ .
- b) Solve the partial differential equation [5M]  
 $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$ .
- c) Find the complete integral of  $pqz = p^2(xq + p^2) + q^2(yp + q^2)$ . [5M]
- 9 a) Solve  $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x$ . [8M]
- b) Solve the by the method of separation of variables  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} = 0$ . [7M]

Or

- 10 Solve the Laplace's equation  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  in a rectangle in the  $xy$ -plane, [15M]  
 $0 \leq x \leq a$  and  $0 \leq y \leq b$  satisfying the following boundary  
condition  $u(x, 0) = 0, u(x, b) = 0, u(0, y) = 0$  and  $u(a, y) = f(y)$ .

