Problem 1

(2 points each) Circle the best answer:

(i) The Mohr-failure envelope for a rock as a distinct parabolic curve, which failure model would provide the best capability to predict the failure of the rock.

(a) Hoek-Brown
(b) von Mises
(c) Mohr-Coulomb

(ii) True or False? A typical range for Poisson ratio in rocks is between $0.2 < \nu < 0.6$

(a) True
(b) False

$\nu \leq 0.5$

(iii) Which of the following *is not* a standard assumption of poroelasticity.

(a) There is an interconnected pore system uniformly saturated with fluid.
(b) The pore pressure, the total stress acting on the rock externally, and the stresses acting on the grains are statistically defined.
(c) The total volume of the pore system is large compared to the volume of the rock.

(iv) True or False? The elastic behavior of an *isotropic* solid is fully characterized by three independent constants.

(a) True
(b) False

(v) True or False? A typical value of Biot's coefficient for petroleum reservoir rocks would be 0.2.

(a) True
(b) False

(vi) Which of the following conditions is true in a triaxial compression test for principle stresses, S_1, S_2, S_3

(a) $S_1 > S_2 = S_3$
(b) $S_1 < S_2 = S_3$
(c) $S_1 = S_2 = S_3$

(vii) So-called *cap failure models* provide the ability to model

(a) inelastic effects occurring for increasing hydrostatic pressure.
(b) failure in pure shear.
(c) inelastic effects due to slip on crystallographic planes.

Problem 1 continued on next page...

(viii) Simplifying the Kirsch equations at a vertical wellbore wall, it can be shown that the difference in minimum and maximum hoop stress in the wellbore is what factor times the horizontal principle difference.

(a) 2
(b) 4
(c) 10

(ix) In a vertical wellbore, we expect breakouts to occur along the direction of

(a) S_v.
(b) S_{Hmax}.
(c) S_{hmin}.

(x) For a friction coefficient of $\mu = 0.6$ we can estimate bounds on the ratio of largest-to-smallest *in situ* principle stresses at around 3.1, this estimate derives from what idea?

(a) Frictional faulting equilibrium
(b) Conservation of mass
(c) Tectonic static equilibrium

(xi) Raising the drilling mud weight above the *frac gradient* will lead to

(a) breakouts.
(b) drilling induced tensile fractures.
(c) washouts.

(xii) Using wellbore imaging techniques such as ultrasonic logs or electrical resistivities, wellbore breakouts will have what appearance on the televiewer?

(a) vertical shaded areas
(b) horizontal shaded areas
(c) spiraling shaded areas

(xiii) True or False? A *stable wellbore* is defined as one that is absent from any breakouts.

(a) True
(b) False

(xiv) In a vertical wellbore, we expect drilling induced tensile fractures to occur along the direction of

(a) S_v.
(b) S_{Hmax}.
(c) S_{hmin}.

(xv) True or False? In a lower hemispherical projection plot associated with drilling deviated wells, the outermost concentric ring, i.e. the edge of the plot, represents a vertical well.

(a) True
(b) False

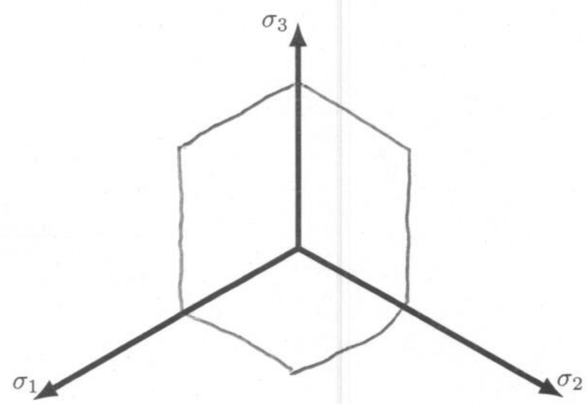
Problem 2

(4 points each) Short answer:

(i) A displacement field for a one dimensional bar is given as $u(x) = 3x^2$, what is the strain in the bar?

$$\varepsilon(x) = \frac{\partial u}{\partial x} = 6x$$

(ii) Sketch a Mohr-Coulomb failure surface in the π-plane on the figure.

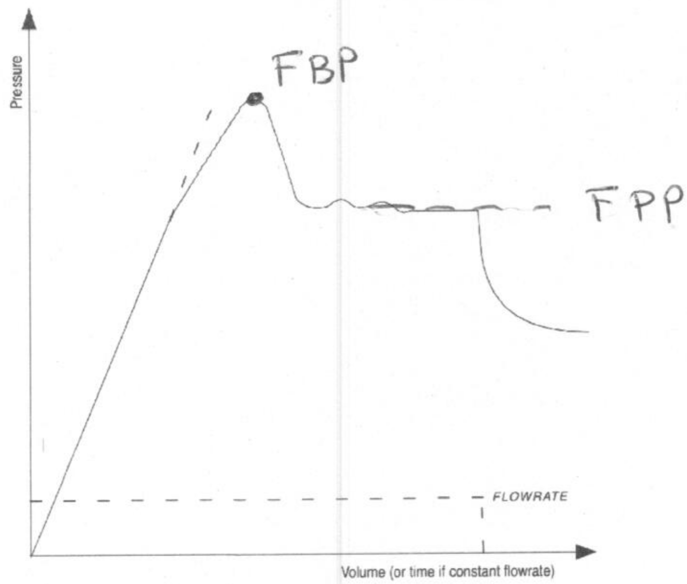


(iii) List two reasons why tensile ~~strength~~ stress is relatively unimportant in reservoir geomechanics.

1. The tensile strength of rocks is neglible,

2. The in situ stress in the earth is ~~compssive~~. compressive.

(iv) On the figure below that schematically represents an extended leakoff-test



Label the **formation breakdown pressure** and the **fracture propagation pressure**.

(v) You are performing triaxial rock strength tests in the lab. Starting with Hooke's law for an isotropic solid, express volumetric strain in terms of axial and confining stress.

$$\bar{\bar{S}} = K \varepsilon_{vol} \bar{\bar{I}} + 2\mu \left(\bar{\bar{\varepsilon}} - \frac{1}{3} \varepsilon_{vol} \bar{\bar{I}} \right) \quad \text{use } K = \frac{S_{11} + S_{22} + S_{33}}{3 \varepsilon_{vol}} = \frac{S_{vol}}{\varepsilon_{vol}}$$

$$= S_{vol} \bar{\bar{I}} + 2\mu \left(\bar{\bar{\varepsilon}} - \frac{1}{3K} S_{vol} \bar{\bar{I}} \right) \quad \text{Solve for } \bar{\bar{\varepsilon}}$$

$$\bar{\bar{\varepsilon}} = \frac{1}{2\mu} \left(\bar{\bar{S}} - S_{vol} \bar{\bar{I}} \right) + \frac{1}{3K} S_{vol} \bar{\bar{I}}$$

Now for triaxial test

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_1 & & \\ & \varepsilon_2 & \\ & & \varepsilon_2 \end{bmatrix} \qquad \bar{\bar{S}} = \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_2 \end{bmatrix}$$

$$\varepsilon_1 = \frac{1}{2\mu} (\sigma_1 - S_{vol}) + \frac{1}{3K} S_{vol} = \frac{1}{2\mu} \sigma_1 + \left(\frac{1}{3K} - \frac{1}{2\mu} \right) S_{vol}$$

$$\varepsilon_2 = \frac{1}{2\mu} (\sigma_2 - S_{vol}) + \frac{1}{3K} S_{vol} = \frac{1}{2\mu} \sigma_2 + \left(\frac{1}{3K} - \frac{1}{2\mu} \right) S_{vol}$$

This page provided for additional calculations

$$\varepsilon_{VOL} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \varepsilon_1 + 2\varepsilon_2$$

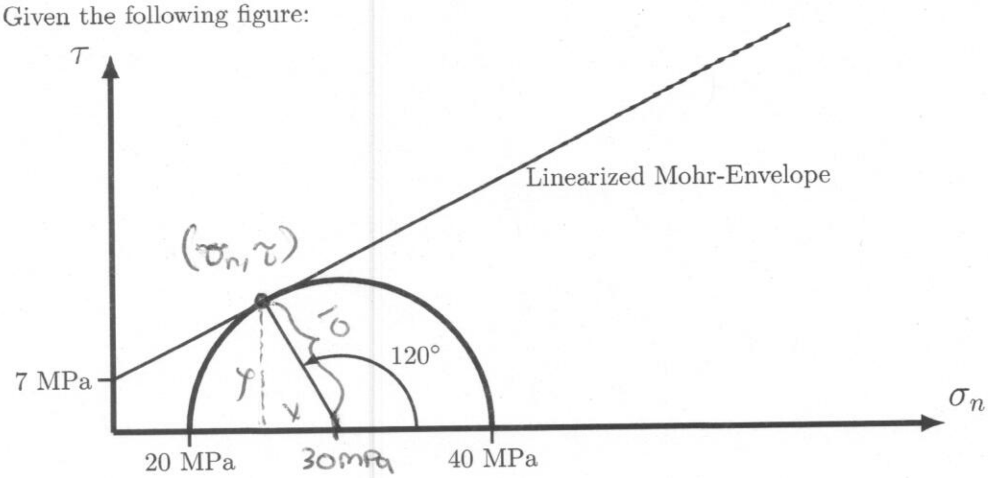
$$\varepsilon_{VOL} = \frac{1}{2\mu}(\sigma_1 + 2\sigma_2) + \left(\frac{2}{9K} + \frac{1}{3\mu}\right)(\sigma_1 + 2\sigma_2)$$

$$= \left(\frac{5}{6\mu} + \frac{2}{9K}\right)(\sigma_1 + 2\sigma_2)$$

Problem 3

(10 points)

Given the following figure:



Estimate the unconfined compressive strength of the material.

$x = 10 \cos 60 = 5$ $\quad\quad \sigma_n = 30 - 5 = 25$ MPa

$y = 10 \sin 60 = 5\sqrt{3}$ $\quad\quad \tau = 5\sqrt{3}$ MPa

$$\tau = S_0 + \sigma_n \mu_I \Rightarrow \mu_I = \frac{\tau - S_0}{\sigma_n} = \frac{5\sqrt{3} - 7}{25} \approx 0.0664$$

$$C_0 = 2(S_0)\left[\sqrt{\mu^2+1} + 0.0664\right] =$$

$$\approx 2(7)\left[\sqrt{0.0664^2+1} + 0.0664\right] \approx 14.96$$

$$\boxed{C_0 \approx 15 \text{ MPa}}$$

Problem 4

(15 points) Lab strength tests on *dry rock samples*, i.e. no pore fluid, with peak shear strength values have been fit to the linear relationship $S_1 = 22.8 \text{ MPa} + 4.12 \text{ MPa } S_3$. What is the unconfined compressive strength C_0 and internal friction coefficient μ_I for this rock.

$$\boxed{C_0 = 22.8 \text{ MPa}}$$

$$n = 4.12$$

$$\mu_I = \frac{n-1}{2\sqrt{n}} = \frac{4.12-1}{2\sqrt{4.12}} \approx 0.768557$$

$$\boxed{\mu_I \approx 0.77}$$

Problem 5

(15 points) The reservoir conditions around a vertical well are as follows: The vertical stress is 58 MPa and the minimum horizontal stress is 48 MPa. Given laboratory measurements $\lambda_p = 0.75$, $C_0 = 35$ MPa, $\mu_I = 1.0$ and tensile strength $T = 2$ MPa, determine the wellbore pressure, P_m that will cause simultaneous initiation of breakouts *and* tensile fractures. You can ignore temperature effects.

Hint: Tensile fractures will occur when the hoop stress is more tensile that the tensile strength of the rock. Recall that λ_p is the ratio of pore pressure to overburden stress.

$$\sigma_{\theta\theta}^{min} = 3\sigma_{Hmin} - \sigma_{Hmax} - \Delta P = -T \quad (\ast)$$

For breakouts in vertical wells

$$\sigma_{\theta\theta}^{max} = \sigma_1 \qquad \sigma_{\theta\theta}^{min} = \sigma_3 \qquad \text{so} \qquad \sigma_{\theta\theta}^{max} = C_0 - q\,\sigma_{\theta\theta}^{min} = C_0 + qT$$

Also

$$(\sigma_{\theta\theta}^{max} - \sigma_{\theta\theta}^{min}) = 4(\sigma_{Hmax} - \sigma_{hmin})$$

$$(C_0 + qT + T) = 4(\sigma_{Hmax} - \sigma_{hmin}) \Rightarrow \sigma_{Hmax} = \frac{1}{4}(C_0 + qT + T) + \sigma_{hmin}$$

Sub. into $(\ast)$

$$3\sigma_{Hmin} - \frac{1}{4}(C_0 + qT + T) - \sigma_{hmin} + T - P_p = -P_m$$

$$2\sigma_{Hmin} - \frac{1}{4}(C_0 + qT + T) + T - P_p = -P_m$$

$$2S_{hmin} - \frac{1}{4}(C_0 + qT + T) + T + P_p = -P_m \Rightarrow P_m = \frac{1}{4}(-C_0 - 4P_p + 8S_{hmin} + 3T - qT)$$

$$q = (\sqrt{\mu^2 - 1} + \mu)^2 \approx 5.82$$

$$P_p = \lambda_p S_1 = 0.75(58) \approx 43.5$$

$$\boxed{P_m \approx 42 \text{ MPa}}$$

	$K =$	$E =$	$\lambda =$	$G =$	$\nu =$	$M =$
$(K,\ E)$	K	E	$\frac{3K(3K-E)}{9K-E}$	$\frac{3KE}{9K-E}$	$\frac{3K-E}{6K}$	$\frac{3K(3K+E)}{9K-E}$
$(K,\ \lambda)$	K	$\frac{9K(K-\lambda)}{3K-\lambda}$	λ	$\frac{3(K-\lambda)}{2}$	$\frac{\lambda}{3K-\lambda}$	$3K-2\lambda$
$(K,\ G)$	K	$\frac{9KG}{3K+G}$	$K-\frac{2G}{3}$	G	$\frac{3K-2G}{2(3K+G)}$	$K+\frac{4G}{3}$
$(K,\ \nu)$	K	$3K(1-2\nu)$	$\frac{3K\nu}{1+\nu}$	$\frac{3K(1-2\nu)}{2(1+\nu)}$	ν	$\frac{3K(1-\nu)}{1+\nu}$
$(K,\ M)$	K	$\frac{9K(M-K)}{3K+M}$	$\frac{3K-M}{2}$	$\frac{3(M-K)}{4}$	$\frac{3K-M}{3K+M}$	M
$(E,\ \lambda)$	$\frac{E+3\lambda+R}{6}$	E	λ	$\frac{E-3\lambda+R}{4}$	$\frac{2\lambda}{E+\lambda+R}$	$\frac{E-\lambda+R}{2}$
$(E,\ G)$	$\frac{EG}{3(3G-E)}$	E	$\frac{G(E-2G)}{3G-E}$	G	$\frac{E}{2G}-1$	$\frac{G(4G-E)}{3G-E}$
$(E,\ \nu)$	$\frac{E}{3(1-2\nu)}$	E	$\frac{E\nu}{(1+\nu)(1-2\nu)}$	$\frac{E}{2(1+\nu)}$	ν	$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$
$(E,\ M)$	$\frac{3M-E+S}{6}$	E	$\frac{M-E+S}{4}$	$\frac{3M+E-S}{8}$	$\frac{E-M+S}{4M}$	M
$(\lambda,\ G)$	$\lambda+\frac{2G}{3}$	$\frac{G(3\lambda+2G)}{\lambda+G}$	λ	G	$\frac{\lambda}{2(\lambda+G)}$	$\lambda+2G$
$(\lambda,\ \nu)$	$\frac{\lambda(1+\nu)}{3\nu}$	$\frac{\lambda(1+\nu)(1-2\nu)}{\nu}$	λ	$\frac{\lambda(1-2\nu)}{2\nu}$	ν	$\frac{\lambda(1-\nu)}{\nu}$
$(\lambda,\ M)$	$\frac{M+2\lambda}{3}$	$\frac{(M-\lambda)(M+2\lambda)}{M+\lambda}$	λ	$\frac{M-\lambda}{2}$	$\frac{\lambda}{M+\lambda}$	M
$(G,\ \nu)$	$\frac{2G(1+\nu)}{3(1-2\nu)}$	$2G(1+\nu)$	$\frac{2G\nu}{1-2\nu}$	G	ν	$\frac{2G(1-\nu)}{1-2\nu}$
$(G,\ M)$	$M-\frac{4G}{3}$	$\frac{G(3M-4G)}{M-G}$	$M-2G$	G	$\frac{M-2G}{2M-2G}$	M
$(\nu,\ M)$	$\frac{M(1+\nu)}{3(1-\nu)}$	$\frac{M(1+\nu)(1-2\nu)}{1-\nu}$	$\frac{M\nu}{1-\nu}$	$\frac{M(1-2\nu)}{2(1-\nu)}$	ν	M