Week 9 Graph Sketching & Kinematics Lecture Note

Notebook:	Computational Mathematics		
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Cornell Notes	**Topic:** Graph Sketching & Kinematics	Course: BSc Computer Science Class: Computational Mathematics[Lecture] Date: May 22, 2020

Essential Question:

What is a function and what are its applications to kinematics (simple motion)?

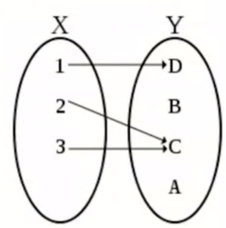
Questions/Cues:

- What is the definition of a function?
- What are surjective, injective and bijective functions?
- What are Cartesian Coordinates?
- What is the Distance Formula used to find the distance between two points P and Q?

Notes

What is a function?

A function $f(x)$ links elements x,y of two sets X and Y



$f(x)$ tells you what to do with an input x:

ex. $f(x)=2x+4$ multiply by 2 and add 4

$f(3)=2\times3+4=10$ $\quad f(-1)=2\times(-1)+4=2$ $\quad f(13)=2\times13+4=143$

Domain of a function: elements of X on which f is defined

ex. $-4<x<4$ or $(-4,4)$: all values between -4 and 4 excluding -4,4

$-4\leq x\leq 4$ or $[-4,4]$ includes -4,4

$-4<x\leq 4$ or $(-4,4]$,includes 4 not -4

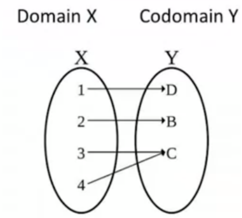
$-4\leq x<4 \cup 6<x<8$ or $[-4,4) \cup (6,8)$ etc…

Codomain of a function: elements of Y linked by f to X

(codomain also called range or image)

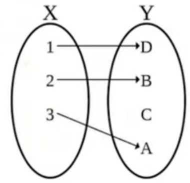
- the function f(x) = 2x+4 maps a real number to a real number; the domain is the set of all real number
- The domain and range can also be restricted and written in interval notation by inequalities or braces like above

What is a function?

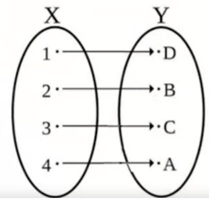


Surjective function: to each y∈Y →at least one x∈X

Injective function:
to each x∈X → only one distinct y∈Y



Bijective function: Injective+Surjective



- A bijective function has one-to-one correspondence, it's a one-to-one function

Cartesian Coordinates

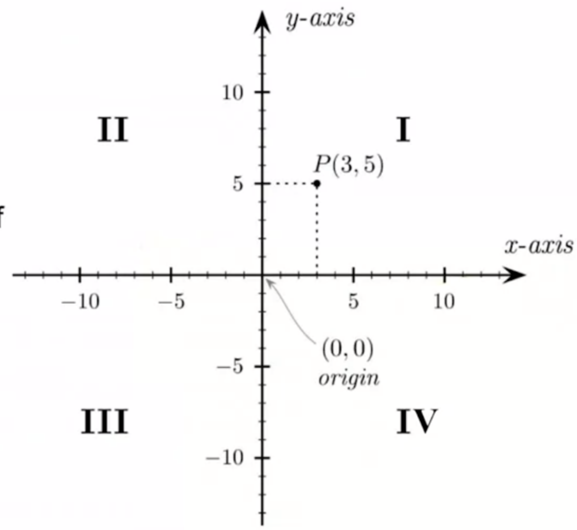
System of two perpendicular axes, *x,y*, to map and label points on the plane:

each point P is labeled by a pair of numbers (x,y)

x is the length of the projection of the point on the *x*-axis

and y is the length of the projection of the point on *y*-axis

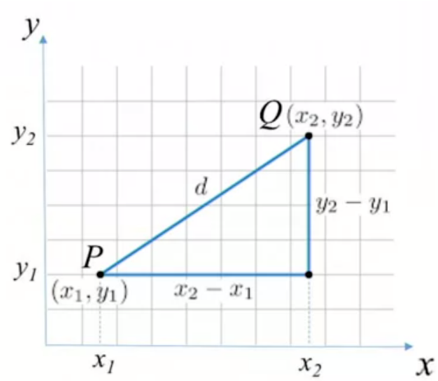
Generic point on *y*-axis P(0,y)
Generic point on *x*-axis P(x,0)



Distance between P and Q:

Using Pythagoras theorem

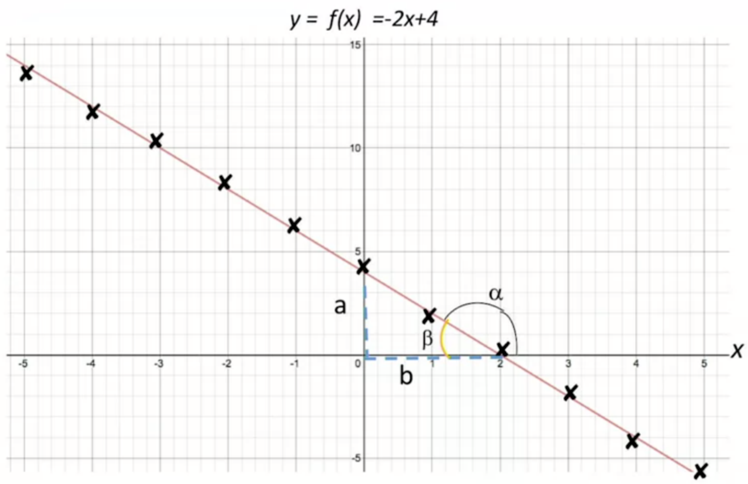
$$d_{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Examples: $f(x)= -2x+4$ Domain R

x	-5	-4	-3	-2	-1	0
f(x)	=-2(-5)+4=14	=12	=10	=8	=6	=4
coordinates	(-5, 14)	(-4,12)	(-3,10)	(-2,8)	(-1,6)	(0,4)

x	1	2	3	4	5
f(x)	=-2(1)+4=2	=0	=-2	=-4	
coordinates	(1,2)	(2,0)	(3,-2)	(4,-4)	



Note: $\beta=180-\alpha$
$\tan(\beta)=a/b=4/2=2$

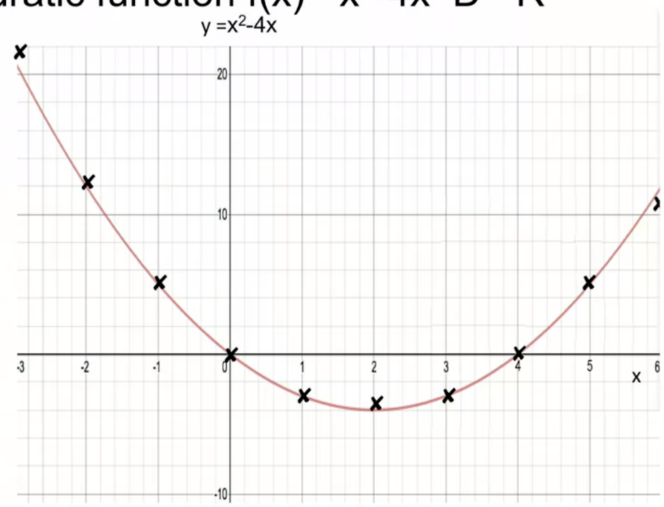
In general for a straight line
$y=mx+n$

with $m=\tan(\alpha)$

In our case n=4
$m=\tan(\alpha)=-\tan(\beta)=-2$

Intersection with y-axis $\rightarrow x=0 \rightarrow y_0=f(0)=-2(0)+4=4$
Intersection with x-axis $\rightarrow y=0 \rightarrow f(x_0)=0$
Solve $-2x_0+4=0 \rightarrow 2x_0=4 \rightarrow x_0=2$

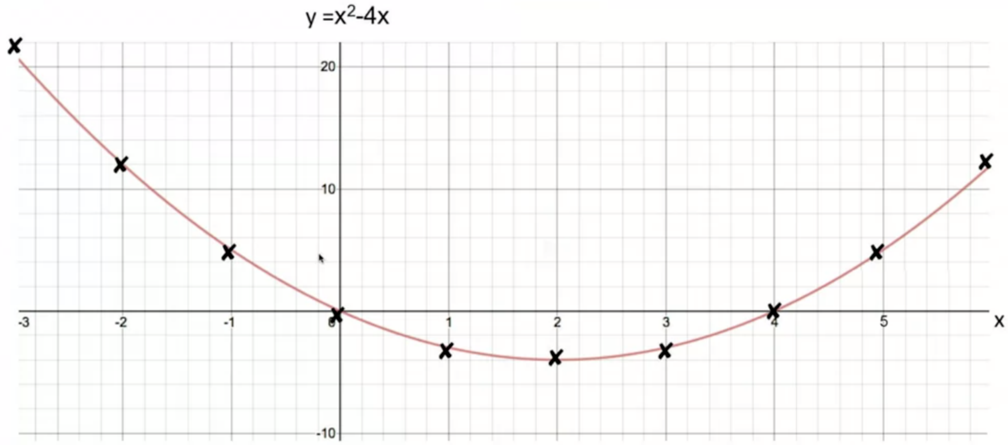
Examples: quadratic function $f(x)= x^2-4x$ D= R



x	-3	-2	-1
f(x)	$=(-3)^2+12=21$	=12	=5
coordinates	(-3,21)	(-2,12)	(-1,5)

x	0	1	2
f(x)	=0	=-3	=-4
coordinates	(0,0)	(1,-3)	(2,-4)

x	3	4	5	6
f(x)	$=(3)^2-12=-3$	=0	=5	=12
coordinates	(1,-3)	(4,0)	(5,5)	(6,12)



Intersection with y-axis $\rightarrow x=0 \rightarrow y_0=f(0)=(0)^2-4(0)=0$
Intersection with x-axis $\rightarrow y=0 \rightarrow f(x_0)=x_0{}^2-4x_0=0$

Solve $x_0{}^2-4x_0=x_0(x_0-4)=0 \rightarrow x_0=0, x_0=4$

Generic quadratic function $f(x)= ax^2+bx+c$

Intersection with y-axis $\rightarrow x=0 \rightarrow y_0=f(0)=a(0)^2+b(0)+c=c$
Intersection with x-axis $\rightarrow y=0 \rightarrow f(x_0)=ax_0{}^2+bx_0+c=0$

Solve $ax_0{}^2+bx_0+c=0 \rightarrow x_0=(-b\pm\sqrt{(b^2-4ac)})/(2a)$

Cubic function $f(x)= ax^3+bx^2+cx+d$ Ex: x^3-4x D=R

x	-4	-3	-2	-1	0	1
f(x)	$=(-4)^3-4(-4)=-48$	$=(-3)^3-4(-3)=-15$	$=(-2)^3-4(-2)=0$	=3	=0	=-3
coordinates	(-4,-48)	(-3,-15)	(-2,0)	(-1,3)	(0,0)	(1,-3)

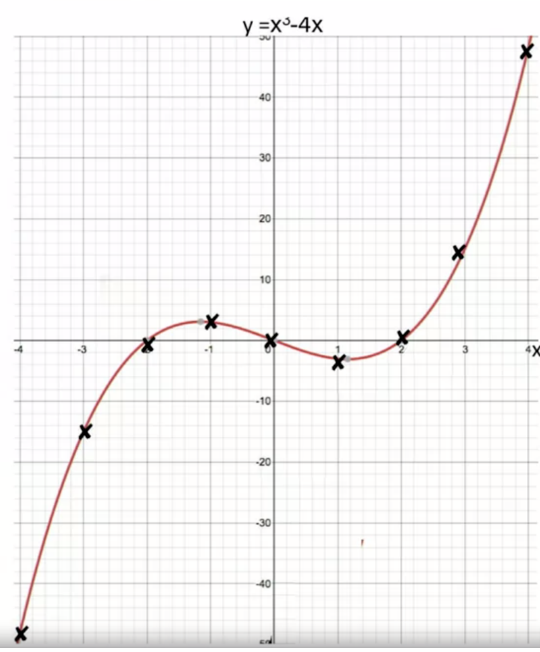
x	2	3	4
f(x)	$=(2)^3-4(2)=0$	$=(3)^3-4(3)=15$	=48
coordinates	(2,0)	(3,15)	(4,48)

Intersection with y-axis $\rightarrow x=0$
$\rightarrow y_0=f(0)=(0)^3-4(0)=0$

Intersection with x-axis $\rightarrow y=0$
$\rightarrow f(x_0)=x_0{}^3-4x_0=0$

Solve $x_0{}^3-4x_0=x_0(x_0{}^2-4)=0$
$\rightarrow x_0=0, x_0=\pm2$

Note: a vertical line intersects the curve in <u>only one point</u>
$\rightarrow$ single-valued functions



Summary

In this week, we learned about what a function, surjective/injective functions, the Cartesian coordinate system and distance formula.