Convergence or Catch-Up?

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Online Appendix for Let's Talk Development Blog

This appendix formally shows how the distribution of growth rates across countries drives changes over time in average per capita GDP relative to the US, as well as the mean log deviation. This illustrates how both differences in average growth relative to the US and the distribution of growth rates across countries contribute to changes in the two measures over time.

Let y_i denote per capita GDP in country i, $\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ denote average per capita GDP, and let $R = \frac{\bar{y}}{y^*}$ denote average GDP per capita relative to the per capita GDP of the United States, y^*. Using a first-order Taylor series approximation, the growth rate of average GDP per capita relative to the United States, $g_R = \frac{\Delta R}{R}$, can be written as a weighted average of each country's growth in per capita GDP, $g_i = \frac{\Delta y_i}{y_i}$, relative to per capita GDP growth in the United States, $g^* = \frac{\Delta y^*}{y^*}$:

$$g_R = \frac{1}{N}\sum_{i=1}^{N} \frac{y_i}{\bar{y}}(g_i - g^*)$$

This expression has an obvious part and a not-so-obvious part. The obvious part is that growth in any country that is higher than growth in the United States, i.e. $g_i > g^*$, will contribute to an increase in average incomes relative to the United States, i.e. $g_R > 0$. The not-so-obvious part is that the distribution of growth rates matters as well. Faster-than-in-the-US growth in a richer country with per capita GDP above the mean, i.e. $\frac{y_i}{\bar{y}} > 1$, will contribute more to growth in average incomes relative to the United States than the same growth in a poorer country with per capita GDP below the mean, i.e. $\frac{y_i}{\bar{y}} < 1$. The reason for this is simple: a given growth rate applied to a larger base (in a richer country) results in a larger absolute increase in income, which in turn increases average incomes by more than if the same growth had been applied to a smaller base (in a poor country). The bottom line: catch-up happens when growth on average is high relative to the United States, and when growth is tilted towards richer countries.

We can do the same for the mean log deviation, $M = \frac{1}{N}\sum_{i=1}^{N} \ln\left(\frac{y_i}{\bar{y}}\right)$. The growth rate of the mean log deviation, $g_M = \frac{\Delta M}{M}$, is also a weighted average of the growth rates across countries:

$$g_M = \frac{1}{MLD}\frac{1}{N}\sum_{i=1}^{N}\left(\frac{y_i}{\bar{y}} - 1\right) g_i$$

This expression also has a simple interpretation: growth in countries with per capita income above the mean, i.e. $\frac{y_i}{\bar{y}} > 1$, raises inequality as measured by the mean log deviation, while growth in countries with per capita income below the mean, i.e. $\frac{y_i}{\bar{y}} < 1$, lowers the mean log deviation. The bottom line:

dispersion in per capita incomes across countries increases when growth is tilted towards rich countries (i.e. divergence), and it decreases when growth is tilted towards poor countries (i.e. convergence).