JANSEN MECHANISM FOR 4-LEGGED ROBOT

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Introduction

Jansen's linkage is a planar leg mechanism designed by Theo Jansen to generate a smooth walking motion. It is a 11 bar mechanism. We have chosen this mechanism for our project due to its simplicity. Also we got to apply many concepts from the lectures in this mechanism.

Objectives

The objectives of the project are:

1. Select link lengths and develop a system of equations.
2. Perform MATLAB based analysis and simulation.
3. Design a CAD model.
4. 3D printing and laser cutting using selected materials.
5. Assemble and perform demonstrations.

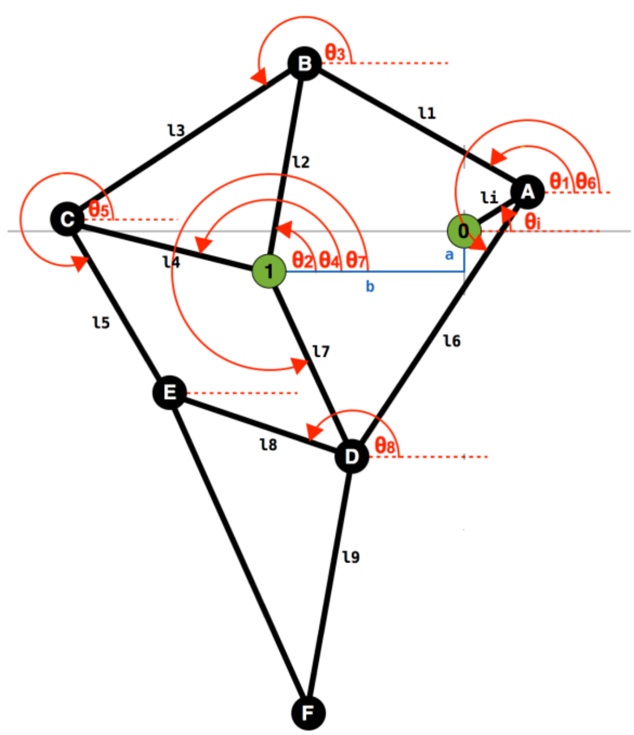
Materials

- 4mm MDF Wood has been used for the base.
- 2mm Acrylic Sheet has been used for the links, spacers and other connectors used in the legs.
- 3d printing has been done for pins and connectors (including motor input extender).
- 3mm screws have been used for connecting the legs to the base.

Pre-requisites

We have used a scale of 2.33, which is applied to every link length.

Notation	Value	Description
θ_i	Independent	Crank Angle
l_i	15	Length of link connecting joints 0 and A
l_1	50	Length of link connecting joints A and B
l_2	41.5	Length of link connecting joints 1 and B
l_3	55.8	Length of link connecting joints B and C
l_4	40.1	Length of link connecting joints 1 and C
l_5	39.4	Length of link connecting joints C and E
l_6	61.9	Length of link connecting joints A and D
l_7	39.3	Length of link connecting joints 1 and D
l_8	36.7	Length of link connecting joints E and D
a	7.8	Vertical distance from joint 1 to joint 0
b	38	Horizontal distance from joint 1 to joint 0



These are the initial values of joint angles using the above figure.

Angle	Measurement (rad)
θ_i	1.57
θ_1	2.62
θ_2	1.40
θ_3	3.73
θ_4	2.90
θ_5	5.26
θ_6	4.12
θ_7	5.14
θ_8	2.81

System of Equations

$$l_i \sin(\theta_i) + l_1 \sin(\theta_1) = -a + l_2 \sin(\theta_2)$$
$$l_i \cos(\theta_i) + l_1 \cos(\theta_1) = -b + l_2 \cos(\theta_2)$$

$$l_i \sin(\theta_i) + l_1 \sin(\theta_1) + l_3 \sin(\theta_3) = -a + l_4 \sin(\theta_4)$$
$$l_i \cos(\theta_i) + l_1 \cos(\theta_1) + l_3 \cos(\theta_3) = -b + l_4 \cos(\theta_4)$$

$$l_i \sin(\theta_i) + l_6 \sin(\theta_6) = -a + l_7 \sin(\theta_7)$$
$$l_i \cos(\theta_i) + l_6 \cos(\theta_6) = -b + l_7 \cos(\theta_7)$$

$$l_i \sin(\theta_i) + l_1 \sin(\theta_1) + l_3 \sin(\theta_3) + l_5 \sin(\theta_5) = -a + l_7 \sin(\theta_7) + l_8 \sin(\theta_8)$$
$$l_i \cos(\theta_i) + l_1 \cos(\theta_1) + l_3 \cos(\theta_3) + l_5 \cos(\theta_5) = -b + l_7 \cos(\theta_7) + l_8 \cos(\theta_8)$$

Multi Order Newton-Raphson Method

The NR method considers the mechanism as a **combined** system of 8 equations. Loop closure considers 4 sets, each set containing 2 equations. No sequential solutions are required. Loop closure requires the solution of the first 2 equations before solving third and fourth equations. The NR method has very good and fast convergence. We wanted to experiment (try out) with an approximation based method and hence chose the NR method. We boosted NR method’s performance by taking inverse of four 2x2 matrices instead of one 8x8 matrix.

$$\mathbf{x}_{n+1} = \mathbf{x}_n - (Df(\mathbf{x}_n))^{-1} f(\mathbf{x}_n)$$

$$\mathbf{x} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \end{bmatrix}$$

$$f(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ f_3(\mathbf{x}) \\ f_4(\mathbf{x}) \\ f_5(\mathbf{x}) \\ f_6(\mathbf{x}) \\ f_7(\mathbf{x}) \\ f_8(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} l_i \sin(\theta_i) + l_1 \sin(\theta_1) + a - l_2 \sin(\theta_2) \\ l_i \cos(\theta_i) + l_1 \cos(\theta_1) + b - l_2 \cos(\theta_2) \\ l_i \sin(\theta_i) + l_1 \sin(\theta_1) + l_3 \sin(\theta_3) + a - l_4 \sin(\theta_4) \\ l_i \cos(\theta_i) + l_1 \cos(\theta_1) + l_3 \cos(\theta_3) + b - l_4 \cos(\theta_4) \\ l_i \sin(\theta_i) + l_6 \sin(\theta_6) + a - l_7 \sin(\theta_7) \\ l_i \cos(\theta_i) + l_6 \cos(\theta_6) + b - l_7 \cos(\theta_7) \\ l_i \sin(\theta_i) + l_1 \sin(\theta_1) + l_3 \sin(\theta_3) + l_5 \sin(\theta_5) + a - l_7 \sin(\theta_7) - l_8 \sin(\theta_8) \\ l_i \cos(\theta_i) + l_1 \cos(\theta_1) + l_3 \cos(\theta_3) + l_5 \cos(\theta_5) + b - l_7 \cos(\theta_7) - l_8 \cos(\theta_8) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$
Df(\mathbf{x}) = \begin{bmatrix}
\frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial \theta_2} & \frac{\partial f_1}{\partial \theta_3} & \frac{\partial f_1}{\partial \theta_4} & \frac{\partial f_1}{\partial \theta_5} & \frac{\partial f_1}{\partial \theta_6} & \frac{\partial f_1}{\partial \theta_7} & \frac{\partial f_1}{\partial \theta_8} \\
\frac{\partial f_2}{\partial \theta_1} & \frac{\partial f_2}{\partial \theta_2} & \frac{\partial f_2}{\partial \theta_3} & \frac{\partial f_2}{\partial \theta_4} & \frac{\partial f_2}{\partial \theta_5} & \frac{\partial f_2}{\partial \theta_6} & \frac{\partial f_2}{\partial \theta_7} & \frac{\partial f_2}{\partial \theta_8} \\
\frac{\partial f_3}{\partial \theta_1} & \frac{\partial f_3}{\partial \theta_2} & \frac{\partial f_3}{\partial \theta_3} & \frac{\partial f_3}{\partial \theta_4} & \frac{\partial f_3}{\partial \theta_5} & \frac{\partial f_3}{\partial \theta_6} & \frac{\partial f_3}{\partial \theta_7} & \frac{\partial f_3}{\partial \theta_8} \\
\frac{\partial f_4}{\partial \theta_1} & \frac{\partial f_4}{\partial \theta_2} & \frac{\partial f_4}{\partial \theta_3} & \frac{\partial f_4}{\partial \theta_4} & \frac{\partial f_4}{\partial \theta_5} & \frac{\partial f_4}{\partial \theta_6} & \frac{\partial f_4}{\partial \theta_7} & \frac{\partial f_4}{\partial \theta_8} \\
\frac{\partial f_5}{\partial \theta_1} & \frac{\partial f_5}{\partial \theta_2} & \frac{\partial f_5}{\partial \theta_3} & \frac{\partial f_5}{\partial \theta_4} & \frac{\partial f_5}{\partial \theta_5} & \frac{\partial f_5}{\partial \theta_6} & \frac{\partial f_5}{\partial \theta_7} & \frac{\partial f_5}{\partial \theta_8} \\
\frac{\partial f_6}{\partial \theta_1} & \frac{\partial f_6}{\partial \theta_2} & \frac{\partial f_6}{\partial \theta_3} & \frac{\partial f_6}{\partial \theta_4} & \frac{\partial f_6}{\partial \theta_5} & \frac{\partial f_6}{\partial \theta_6} & \frac{\partial f_6}{\partial \theta_7} & \frac{\partial f_6}{\partial \theta_8} \\
\frac{\partial f_7}{\partial \theta_1} & \frac{\partial f_7}{\partial \theta_2} & \frac{\partial f_7}{\partial \theta_3} & \frac{\partial f_7}{\partial \theta_4} & \frac{\partial f_7}{\partial \theta_5} & \frac{\partial f_7}{\partial \theta_6} & \frac{\partial f_7}{\partial \theta_7} & \frac{\partial f_7}{\partial \theta_8} \\
\frac{\partial f_8}{\partial \theta_1} & \frac{\partial f_8}{\partial \theta_2} & \frac{\partial f_8}{\partial \theta_3} & \frac{\partial f_8}{\partial \theta_4} & \frac{\partial f_8}{\partial \theta_5} & \frac{\partial f_8}{\partial \theta_6} & \frac{\partial f_8}{\partial \theta_7} & \frac{\partial f_8}{\partial \theta_8}
\end{bmatrix} =
$$

$$
\begin{bmatrix}
l_1\cos(\theta_1)\ , & -l_2\cos(\theta_2)\ , & 0\ , & 0\ , & 0\ , & 0\ , & 0\ , & 0 \\
-l_1\sin(\theta_1)\ , & l_2\sin(\theta_2)\ , & 0\ , & 0\ , & 0\ , & 0\ , & 0\ , & 0 \\
l_1\cos(\theta_1)\ , & 0\ , & l_3\cos(\theta_3)\ , & -l_4\cos(\theta_4)\ , & 0\ , & 0\ , & 0\ , & 0 \\
-l_1\sin(\theta_1)\ , & 0\ , & -l_3\sin(\theta_3)\ , & l_4\sin(\theta_4)\ , & 0\ , & 0\ , & 0\ , & 0 \\
0\ , & 0\ , & 0\ , & 0\ , & 0\ , & l_6\cos(\theta_6)\ , & -l_7\cos(\theta_7)\ , & 0 \\
0\ , & 0\ , & 0\ , & 0\ , & 0\ , & -l_6\sin(\theta_6)\ , & l_7\sin(\theta_7)\ , & 0 \\
l_1\cos(\theta_1)\ , & 0\ , & l_3\cos(\theta_3)\ , & 0\ , & l_5\cos(\theta_5)\ , & 0\ , & -l_7\cos(\theta_7)\ , & -l_8\cos(\theta_8) \\
-l_1\sin(\theta_1)\ , & 0\ , & -l_3\sin(\theta_3)\ , & 0\ , & -l_5\sin(\theta_5)\ , & 0\ , & l_7\sin(\theta_7)\ , & l_8\sin(\theta_8)
\end{bmatrix}
$$

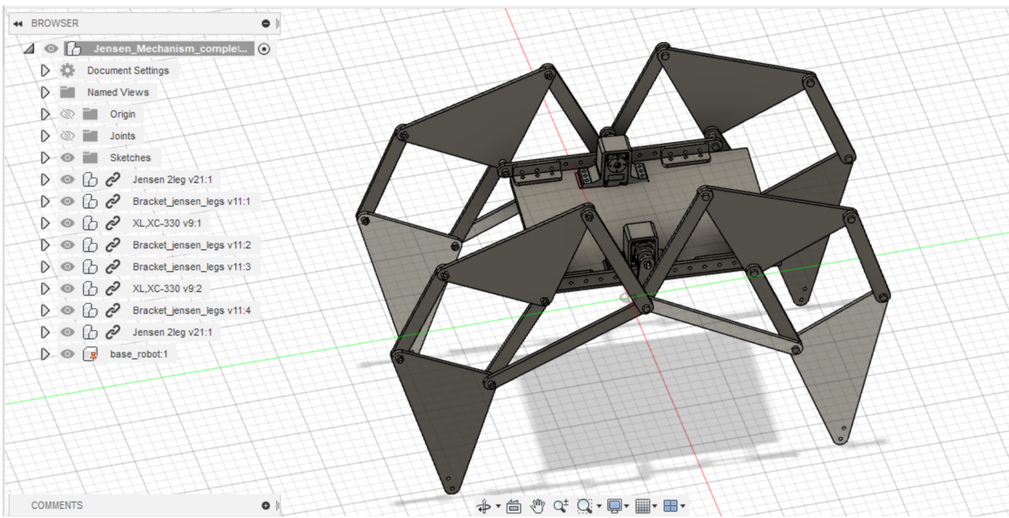
MATLAB simulation

The code is available here. Run the following command to get the required simulation:

```
[tpos,trace] = plot_one_leg(1.57,[2.62,1.4,3.73,2.9,5.26,4.12,5.14,2.81],0.19,200)
```

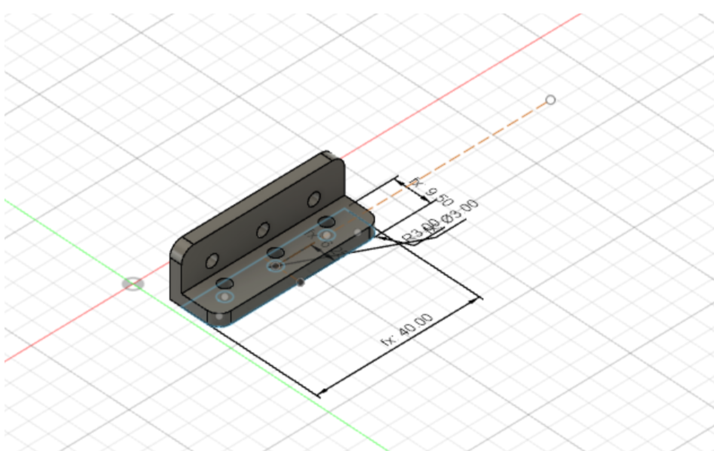
The simulation video is available here.

CAD Model

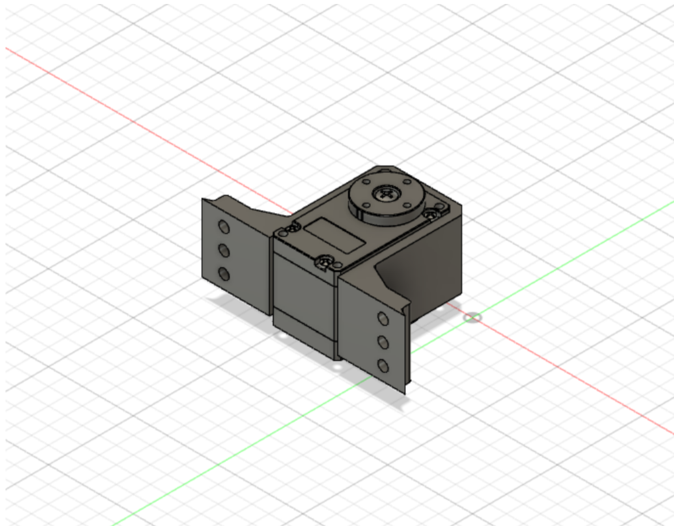


1. Each leg is scaled by a factor of 2.33 of the given jansen leg link ratios.
2. The whole design is modular and parametric, so that it's easy to go through the multiple iterations. And change sizing and thicknesses.
3. Link to the Cad Model : https://a360.co/3FdEAsD

Leg mount bracket design: 2 brackets are used per leg



Motor Mount Bracket Design:



List of Model parameters and equations:

Parameters

Parameter	Name	Unit	Expression
User Parameters			
User Parameter	Pin_Length_2	mm	link_thickness * 2 + Pin_Cut_offset + Pin_Cut_Heig
User Parameter	Pin_ShaftDiameter	mm	link_hole + Pin_ShaftDiameter_tolerance
User Parameter	Pin_HeadDiameter	mm	Pin_ShaftDiameter * 2
User Parameter	i	mm	49.00 mm
User Parameter	G	mm	36.7 mm
User Parameter	h	mm	65.7 mm
User Parameter	d	mm	40.1 mm
User Parameter	e	mm	55.8 mm
User Parameter	b	mm	41.5 mm
User Parameter	M	mm	15 mm
User Parameter	k	mm	61.9 mm
User Parameter	j	mm	50 mm
User Parameter	Ff	mm	39.4 mm
User Parameter	cc	mm	39.3 mm
User Parameter	a	mm	38.0 mm * scaling_param
User Parameter	L	mm	7.8 mm * scaling_param
User Parameter	scaling_param		2.33
User Parameter	link_width	mm	12.5 mm
User Parameter	link_thickness	mm	3 mm
User Parameter	link_hole	mm	3 mm
User Parameter	ternary_link_radius	mm	6.25 mm
User Parameter	Pin_HeadThickness	mm	-link_thickness * 0.75
User Parameter	Pin_Cut_Width	mm	(Pin_ShaftDiameter / 2) * 0.35
User Parameter	Pin_Cut_Height	mm	link_thickness
User Parameter	Pin_Cut_offset	mm	link_thickness * 0.75
User Parameter	Pin_Length_3	mm	link_thickness * 3 + Pin_Cut_offset + Pin_Cut_Heig
User Parameter	Pin_Length_tolerance	mm	0 mm
User Parameter	Pin_ShaftDiameter_to...	mm	0 mm
User Parameter	Pin_Length_4	mm	link_thickness * 4 + Pin_Cut_offset + Pin_Cut_Heig
User Parameter	Pin_Length_5	mm	link_thickness * 5 + Pin_Cut_offset + Pin_Cut_Heig
User Parameter	Pin_Length_6	mm	link_thickness * 6 + Pin_Cut_offset + Pin_Cut_Heig

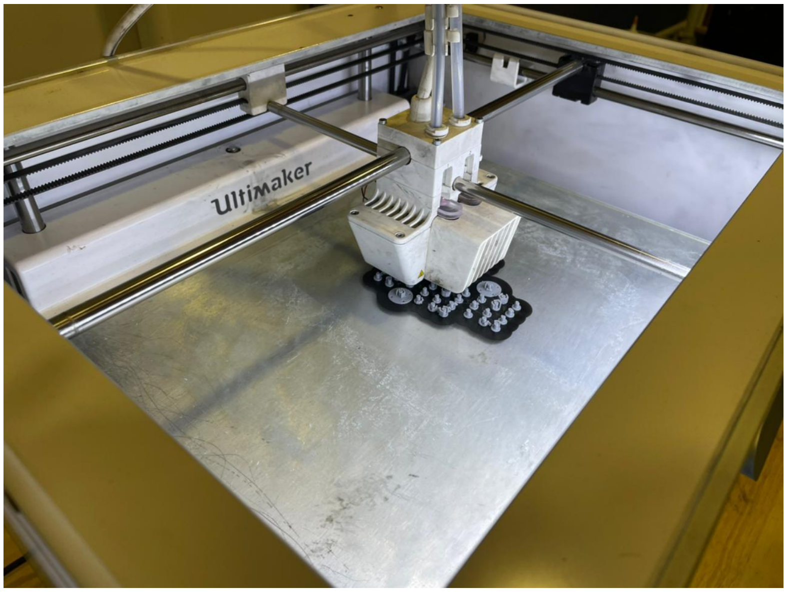
User Parameter	Pin_Length_6	mm	link_thickness * 6 + Pin_Cut_offset + Pin_Cut_Heig
Model Parameters			
jensenleg v28			
m			
k			
j			
f			
c			
pin4x_2			
Clip			
t_link1			
t_link2			
a_L			
pin2x_2			
Clip (1)			
pin2x3			
Clip (2)			
pin2x_4			
Clip (3)			
pin3x_1			
Clip (5)			
pin4x_1			
Clip (6)			
pin6x_1			
Clip (7)			
Spacer1			
Spacer2			
Spacer3			
Spacer4			
Spacer5			
Spacer6			

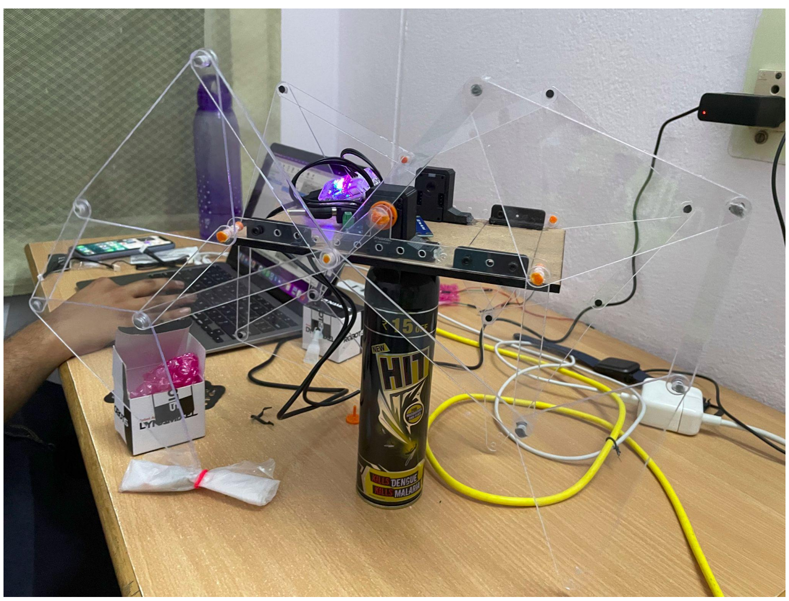
2 dynamixel XL330-M288-T servo motors are used for moving the legs. The motors are controlled using a U2D2 controller via TTL. Power is supplied using a U2D2 power hub connected to a 5V DC adapter.

For giving commands, Dynamixel Wizard 2.0 is used on a Linux machine.

3D Printing and Laser Cutting

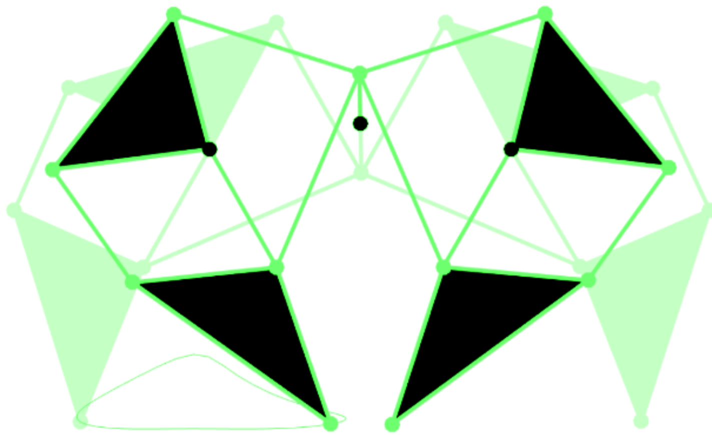
We did everything in the Maker's Lab.





Conclusion

We were able to make a 4-legged walking robot. But due to constraints on material of legs and wires attached (controller to PC), it could not walk sturdily on the ground. But the legs, when powered using the servos, generated the correct trajectory. It was a great experience learning a lot of new things. The challenges that we faced were majorly with type of material used for legs and motor dimensions (difference in datasheet and reality).



References

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- https://ocw.metu.edu.tr/pluginfile.php/3961/mod_resource/content/12/ch3/3-9.ht
- Dynamixel Motor Specifications: https://www.robotis.us/dynamixel-xl330-m288-t/