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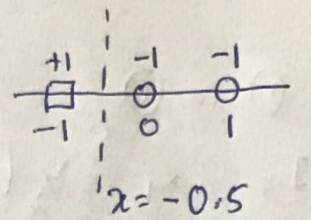
1.)

$(-1,+1),\ (0,-1)\ ;\ (+1,-1)$

a.) To write the primal problem.

Data is in 1-D, Linearly separable



Decision boundary $\Rightarrow x = -0.5$

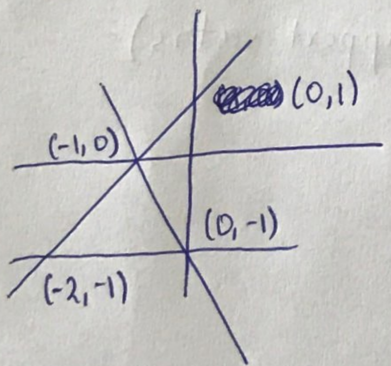
Support vectors $\Rightarrow x = -1,\ x = 0$

For primal, min $\dfrac{\|w\|}{2}$ st $y_i(w^T x + b) \geq 1$

So for each point constraints come out to be

$$-w + b \geq 1$$
$$-b \geq 1$$
$$-w - b \geq 1$$

Plotting all these



So we get

$w^* = -2$

$b^* = -1$ //

for decision boundary $-2x - 1 = 0 \Rightarrow x = -0.5$ //

b) Dual objective

$$J = \max \sum_{i=1}^{N} \alpha_i - \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j x_i^T x_j$$

$N = 3$ here

Solving,

$$J = (\alpha_1 + \alpha_2 + \alpha_3) - \frac{1}{2}(\alpha_1^2 + \alpha_3^2 - (2\alpha_1\alpha_3)(-1))$$

$$= (\alpha_1 + \alpha_2 + \alpha_3) - \frac{1}{2}(\alpha_1 + \alpha_3)^2 \quad - (1)$$

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Since $\sum \alpha_i y_i = 0 \Rightarrow \alpha_1 - \alpha_2 - \alpha_3 = 0$

$\Rightarrow \alpha_2 = \alpha_1 - \alpha_3$ — (2)

Put (2) in (1)

$\Rightarrow J = 2\alpha_1 - \frac{1}{2}(\alpha_1 + \alpha_3)^2$

To maximize this, $\alpha_3 = 0$

$\Rightarrow \alpha_2 = \alpha_1$

$\Rightarrow J = 2\alpha_1 - \frac{\alpha_1^2}{2}$. Maximizing this we get

$\alpha_1 = \alpha_2 = 2$

$\alpha_3 = 0$ //

c) $w = \sum \alpha_i y_i x_i$

$= 2(-1)(+1) + 2(0)(-1) + 0(1)(-1) = -2$

$b = -1$ (Substitute in support vectors)

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2.)

$(-1,+1),\ (0,-1),\ (+1,+1)$

a) Dual objective $J = \sum_{i=1}^{N} \alpha_i - \sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j K(x_i, x_j)$

$K = (p^T q + 1)^2 \qquad K = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 4 \end{bmatrix}$

$J = \alpha_1 + \alpha_2 + \alpha_3 - \frac{1}{2}(4\alpha_1^2 + 2\alpha_1\alpha_2(-1) + \alpha_2^2 + 2\alpha_2\alpha_3 + 4\alpha_3^2)$

$= \alpha_1 + \alpha_2 + \alpha_3 - \frac{1}{2}(4\alpha_1^2 + \alpha_2^2 + 4\alpha_3^2 - 2\alpha_1\alpha_2 + 2\alpha_2\alpha_3) \quad - (1)$

b) From $\sum \alpha_i y_i = 0$

$\Rightarrow \alpha_1 - \alpha_2 + \alpha_3 = 0$

$\Rightarrow \alpha_2 = \alpha_1 + \alpha_3 \quad - (2)$

Put (2) in (1)

$J = 2\alpha_1 + 2\alpha_3 - (2\alpha_1^2 + 2\alpha_3^2 - \frac{1}{2}(\alpha_1 + \alpha_3)^2)$

c) Differentiating wrt α_1,

$2 - (4\alpha_1 - (\alpha_1 + \alpha_3)) = 0$

$\Rightarrow 2 - 3\alpha_1 + \alpha_3 = 0 \quad - (3)$

Differentiating wrt α_3,

$2 + \alpha_1 - 3\alpha_3 = 0 \quad - (4)$

Using (3), (4) we get,

$\alpha_1 = 1,\ \alpha_2 = 2,\ \alpha_3 = 1$

d) $\text{sign}\left(\sum_i \alpha_i y_i K(x_i, x) + b\right)$

$b = -1$ is assumed.

$\Rightarrow$ Decision boundary is $\sum \alpha_i y_i K(x_i, x) + b = 0$

$\Rightarrow K(-1, x) - 2K(0, x) + K(1, x) - 1 = 0$

$\Rightarrow (-x+1)^2 - 2 + (x+1)^2 - 1 = 0$

$\Rightarrow 2x^2 - 1 = 0 \quad \Rightarrow$ Decision boundary is $2x^2 - 1 = 0$

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3.)

x_1	x_2	y
-1	-1	-1
-1	$+1$	$+1$
$+1$	-1	$+1$
$+1$	$+1$	-1

a) $K = (p^T q + 1)^2$

$$K = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 9 & 1 & 1 & 1 \\ 1 & 9 & 1 & 1 \\ 1 & 1 & 9 & 1 \\ 1 & 1 & 1 & 9 \end{bmatrix} \end{array}$$

b) Dual objective

$$J = \sum_{i=1}^{N} \alpha_i - \sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j K(x_i, x_j)$$

$$J = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - \tfrac{1}{2}\left(9\alpha_1^2 + 9\alpha_2^2 + 9\alpha_3^2 + 9\alpha_4^2 - 2\alpha_1\alpha_2 - 2\alpha_1\alpha_3 \right.$$
$$\left. + 2\alpha_1\alpha_4 + 2\alpha_2\alpha_3 - 2\alpha_3\alpha_4 - 2\alpha_2\alpha_4\right)$$

c) Differentiating wrt α_1

$$0 = 1 - \tfrac{1}{2}(8\alpha_1 - 2\alpha_2 - 2\alpha_3 + 2\alpha_4)$$

$$\Rightarrow 9\alpha_1 - \alpha_2 - \alpha_3 + \alpha_4 = 1 \quad - \ (1)$$

Differentiating wrt α_2

$$\Rightarrow 9\alpha_2 - \alpha_3 + \alpha_4 - \alpha_1 = 1 \quad - \ (2)$$

Differentiating wrt α_3

$$\Rightarrow 9\alpha_3 - \alpha_4 - \alpha_1 + \alpha_2 = 1 \quad - \ (3)$$

Differentiating wrt α_4

$$\Rightarrow 9\alpha_4 + \alpha_1 - \alpha_2 - \alpha_3 = 1 \quad - \ (4)$$

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d) After solving ①, ②, ③, ④,

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1/8$$

e) $\phi = \begin{bmatrix} 1 \\ x_1^2 \\ \sqrt{2}x_1x_2 \\ x_2^2 \\ \sqrt{2}x_1 \\ \sqrt{2}x_2 \end{bmatrix}$

$w = \sum \alpha_i y_i \phi(x_i)$

$$= 1/8 \left(\begin{bmatrix} -1 \\ -1 \\ -\sqrt{2} \\ -1 \\ \sqrt{2} \\ \sqrt{2} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ -\sqrt{2} \\ 1 \\ -\sqrt{2} \\ \sqrt{2} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ -\sqrt{2} \\ 1 \\ \sqrt{2} \\ -\sqrt{2} \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ -\sqrt{2} \\ -1 \\ -\sqrt{2} \\ -\sqrt{2} \end{bmatrix} \right)$$

$$= 1/8 \begin{pmatrix} 0 \\ 0 \\ -4\sqrt{2} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1/\sqrt{2} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

This solves exor on the basis of $x_1 x_2$

x_1	x_2	y	$w^T\phi(x)$
-1	-1	-1	$\sqrt{2}(-1/\sqrt{2}) = -1$
-1	$+1$	$+1$	$-\sqrt{2}(-1/\sqrt{2}) = +1$
$+1$	-1	$+1$	$-\sqrt{2}(-1/\sqrt{2}) = +1$
$+1$	$+1$	-1	$\sqrt{2}(-1/\sqrt{2}) = -1$

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4. a) If set of 3 cannot shatter the set, but the set of 2 can shatter.

$\Rightarrow$ VC dimension $= 2$

b) Set of 4 can shatter and set of 5 cannot

$\Rightarrow$ VC dimension $= 4$

c) Take any set of points, convex polygon will shatter

$\Rightarrow$ VC dimension $= \infty$

d) VC Dimension $= \infty$

5. $$L(w, b, \alpha, \xi) = \frac{1}{2} w^T w + \frac{C}{2} \sum_{i=1}^{N} \xi_i^2 - \sum_{i=1}^{N} \alpha_i (y_i (w^T x_i + b) - 1 + \xi_i)$$

Deriving Dual objective,

$$W(\alpha) = \min_{w, b, \xi} L(w, b, \xi, \alpha)$$

$$= \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} (\alpha_i y^{(i)} x^{(i)})^T (\alpha_j y^{(j)} x^{(j)}) + \frac{1}{2} \sum_{i=1}^{m} \frac{\alpha_i}{\xi_i} \xi_i^2 - \sum_{i=1}^{m} \alpha_i \left[y^{(i)} \left(\left(\sum_{j=1}^{m} \alpha_j y^{(j)} x^{(j)} \right)^T x^{(i)} + b \right) - 1 + \xi_i \right]$$

$$= -\frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i \alpha_j y^{(i)} y^{(j)} (x^{(i)})^T x^{(j)} + \frac{1}{2} \sum_{i=1}^{m} \alpha_i \xi_i - \left(\sum_{i=1}^{m} \alpha_i y^{(i)} \right) b + \sum_{i=1}^{m} \alpha_i - \sum_{i=1}^{m} \alpha_i \xi_i$$

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$$W(\alpha) = \sum_{i=1}^{m} \alpha_i - \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i \alpha_j y^{(i)} y^{(j)} (x^{(i)})^T x^{(j)} - \frac{1}{2} \sum_{i=1}^{m} \alpha_i \xi_i$$

$$= \sum_{i=1}^{m} \alpha_i - \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i \alpha_j y^{(i)} y^{(j)} (x^{(i)})^T x^{(j)} - \frac{1}{2} \sum_{i=1}^{m} \alpha_i^2 / C$$

Dual formulation is

$$\max_{\alpha} \sum_{i=1}^{m} \alpha_i - \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i \alpha_j y^{(i)} y^{(j)} (x^{(i)})^T x^{(j)} - \frac{1}{2} \sum_{i=1}^{m} \alpha_i^2 / C$$

Such that $\alpha_i \geq 0 \quad \forall i = 1 \text{ to } m$

$$\sum_{i=1}^{m} \alpha_i y^{(i)} = 0$$

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