1.

Suppose you know that the 3 by 4 matrix A has the vector $s = (2, 3, 1, 0)$ as the only special solution to $Ax = \mathbf{0}$.

(a) What is the *rank* of A and the complete solution to $Ax = \mathbf{0}$?

(b) What is the exact row reduced echelon form R of A?

2.

Find matrices A and B with the given property or explain why you can't:

(a) The only solution of $Ax = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is $x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(b) The only solution of $Bx = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

3.

Find a basis for each of the four subspaces associated with A depends on c:

$$A = \begin{matrix} 1 & 1 & 5 & 3 \\ 2 & 2 & c & 6 \\ 2 & 1 & 8 & 4 \\ 1 & 0 & 3 & 1 \end{matrix}$$

4.

The 3x3 matrix A reduces to the matrix U by the following three row operations:

- E_{21}: subtract 2*(row 1) from row 2;
- E_{31}: subtract 3*(row 1) from row 3;
- E_{32}: subtract 2*(row 2) from row 3;

Compute A^{-1}. Find L in $A = LU$.

$$U = \begin{matrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{matrix}$$

5.

Under what condition on b_1, b_2, b_3 is this system solvable?
Find all solutions when that condition holds:

$$\begin{aligned} x + 2y - 2z &= b_1 \\ 2x + 5y - 4z &= b_2 \\ 4x + 9y - 8z &= b_3. \end{aligned}$$

6.

Find the complete solution (also called the *general solution*) to

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

7.

Describe the smallest subspace of the 2 by 2 matrix space **M** that contains

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(c) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$.

(d) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$.

8.

Forward elimination changes $Ax = b$ to a row reduced $Rx = d$: the complete solution is

$$x = \begin{bmatrix} 2 \\ 7 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} -2 \\ -4 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -3 \\ -5 \\ 0 \\ 1 \end{bmatrix}$$

What is the 4 by 4 reduced row echelon matrix R and what is d?

9.

For every c, find R and the special solutions to $Ax = 0$:

$$A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 1-c & 2 \\ 0 & 2-c \end{bmatrix}.$$

10.

Choose the number q so that (if possible) the ranks are (a) 1, (b) 2, (c) 3:

$$A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 3 & 1 & 3 \\ q & 2 & q \end{bmatrix}.$$