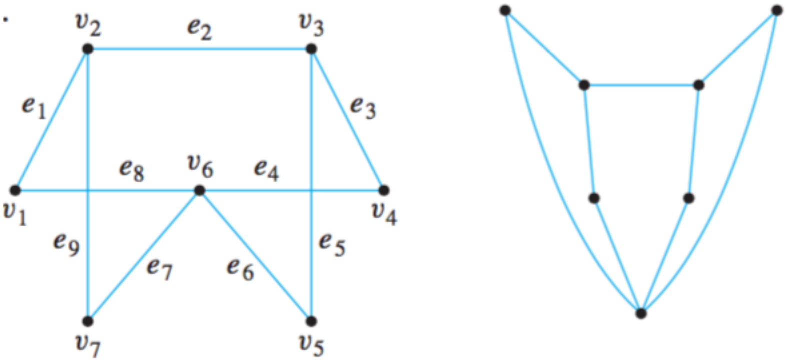
Homework 12. Graph Theory I

1. Show that there could exist a graph with n vertices with each vertex having same degree m, for all n, m $< n$. Or disprove this statement.

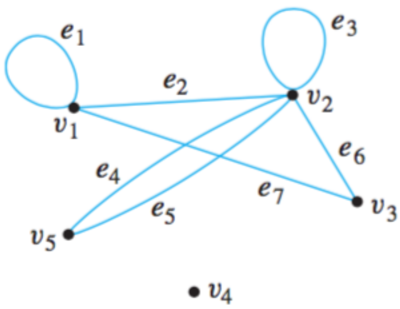
2. Can there be a simple graph that has n vertices all of different degrees? Show valid proofs.

3. In a simple graph what is the maximum number of degree value ? Show valid proofs.

4. Show that two drawings are equivalent by proper vertexes and edges labeling:



5. For the following graph :



(i) Find all edges that are incident on v_1.

(ii) Find all vertices that are adjacent to v_3.

(iii) Find all edges that are adjacent to e_1.

(iv) Find all loops.

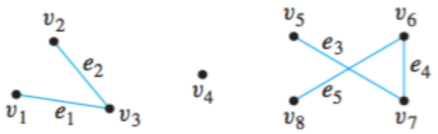
(v) Find all parallel edges.

(vi) Find all isolated vertices.

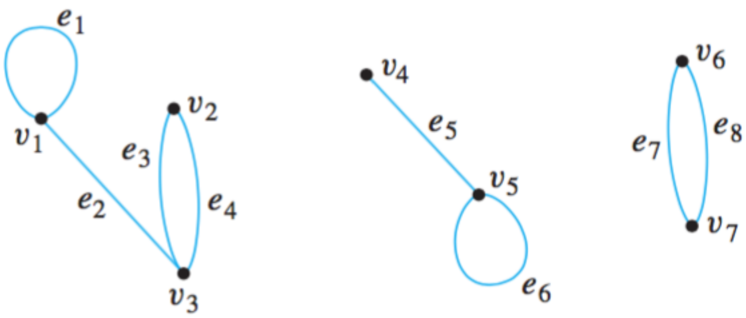
(vii) Find the degree of v_3.

(viii) Find the total degree of the graph.

8. Find all connected components in a following graph



9. For the following graph:



a) Find the adjacency matrix for the graph
b) Calculate the number of paths with a given length L = 3. Hint: use matrix multiplication

10. The following are adjacency matrices for graphs. In each case determine whether the graph is connected by analyzing the matrix without drawing the graph.

a. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

11. In the following exercises either draw a graph with the specified properties or explain why no such graph exists.

a) Graph with four vertices of degrees 1, 1, 1, and 4.

b) Graph with four vertices of degrees 1, 2, 3, and 4.

c) Graph with ten vertices of degrees 1, 1, 2, 2, 2, 3, 4, 4, 4, and 6

d) Simple graph with five vertices of degrees 1, 1, 1, 2, and 3.

e) Simple graph with five vertices of degrees 2, 3, 3, 3, and 5.

12. Let K_i – is a complete graph with i vertices. What is the maximum number of i when the complete graph is a planar graph. Give a proof using Euler's Formula for planar graph. Draw this graph.

**Extra task:

13. Graph colouring:

In this exercise a graph is used to help solve a scheduling problem. Twelve faculty members in a mathematics depart- ment serve on the following committees:

Undergraduate Education: Tenner, Peterson, Kashina, Cohen Graduate Education: Gatto, Yang, Cohen, Catoiu Colloquium: Sahin, McMurry, Ash Library: Cortzen, Tenner, Sahin

Hiring: Gatto, McMurry, Yang, Peterson

Personnel: Yang, Wang, Cortzen

The committees must all meet during the first week of classes, but there are only three time slots available. Find a schedule that will allow all faculty members to attend the meetings of all committees on which they serve. To do this, represent each committee as the vertex of a graph, and draw an edge between two vertices if the two committees have a common member. Find a way to color the vertices using only three colors so that no two committees have the same color, and explain how to use the result to schedule the meetings.