WUCT121

Discrete Mathematics

Logic

Tutorial Exercises

Section 1: Logic

Question1 For each of the following collections of words:

(a) Determine if it is a statement.
(b) If it is a statement, determine if it is true or false.
(c) Where possible, translate the statement into symbols, using the connectives presented in lectures.

(i) If $x=3$, then $x<2$.
(ii) If $x=0$ or $x=1$, then $x^2=x$.
(iii) There exists a natural number x for which $x^2=-2x$.
(iv) If $x\in\mathbb{N}$ and $x>0$, then if $\sqrt{x}>1$ then $x>1$.
(v) $xy=5\Rightarrow x=1$ and $y=5$ or $x=5$ and $y=1$.
(vi) $xy=0\Rightarrow x=0$ or $y=0$.
(vii) If x and y are real numbers, $xy=yx$.
(viii) There is a unique even prime number.

Question2 Translate into symbols the following compound statements and give the form of the compound statement. In each case, list the statements p, q, r …

(a) If x is odd and y is odd then $x+y$ is even.
(b) It is not both raining and hot.
(c) It is neither raining nor hot.
(d) It is raining but it is hot.
(e) $-1\le x\le 2$.

Question3 Let P be the statement "Mathematics is easy" and Q be the statement "I do not need to study". Write down in words the following statements, and simplify if possible:

(a) $P\vee Q$
(a) $P\wedge Q$
(b) $\sim Q$
(c) $\sim\sim Q$
(d) $\sim P$
(e) $\sim P\wedge Q$
(f) $P\Rightarrow Q$

Question4 Let p and q be statements.

(a) Write down the truth tables for $(\sim p\vee q)\wedge q$ and $(\sim p\wedge q)\vee q$.
What do you notice about the truth tables?
Based on this result, a creative student concludes that you can always interchange $\vee$ and $\wedge$ without changing the truth table. Is the student right?
(b) Write down the truth tables for $(\sim p\vee q)\wedge p$ and $(\sim p\wedge q)\vee p$.
What do you think of the rule formulated by the student in 4(a)?

Question5

(a) Construct truth tables for the compound statements $p \vee \sim p$ and $p \wedge \sim p$.

(b) What do you notice about each of the statements in part (a)?

(c) Determine the truth-value of the compound statements $(p \vee \sim p) \vee q$ and $(p \wedge \sim p) \wedge q$. What do you notice?

Question6

(a) Construct truth tables for the compound statements $(p \vee \sim p) \wedge (q \vee r)$ and $q \vee r$. What do you notice?

(b) Construct truth tables for the compound statements $(p \wedge \sim p) \vee (q \wedge r)$ and $q \wedge r$. What do you notice?

Question7 Determine which of the following statements are tautologies using the quick method where possible.

(a) $(p \Rightarrow q) \vee (p \Rightarrow \sim q)$

(b) $\sim (p \Rightarrow q) \vee (q \Rightarrow p)$

(c) $(p \wedge q) \Rightarrow (\sim r \vee (p \Rightarrow q))$

Question8 Using Logical Equivalences and Substitution of Equivalence, write the following expressions using only $\vee$, $\wedge$ and $\sim$. Further, write the expression in the simplest form.

(a) $(p \wedge q) \Rightarrow r$

(b) $p \Rightarrow (p \vee q)$

Question9 Let *p*, *q* and *r* be statements. Using Logical Equivalences, Substitution and Substitution of Equivalence, prove the following.

(g) $\sim (p \Rightarrow q) \equiv (p \wedge \sim q)$

(h) $((p \wedge \sim q) \Rightarrow r) \equiv (p \Rightarrow (q \vee r))$

Question10 In each case, decide whether the proposition is True or False. Give some reasons.

(a) If *x* is a positive integer and $x^2 \leq 3$ then $x = 1$.

(b) $(\sim (x > 1) \vee \sim (y \leq 0)) \Leftrightarrow \sim ((x \leq 1) \wedge (y > 0))$.

Question11 Using Logical Equivalences and Substitution of Equivalence, write the following logical expressions using $\vee$ and $\wedge$ only (even without ~).

(a) $\sim (x > 1) \Rightarrow \sim (y \leq 0)$

(b) $(y \leq 0) \Rightarrow (x > 1)$.

Question12 Simplify the expression $\sim (\sim (p \vee q) \wedge \sim q)$, using Logical Equivalences

Section 2 :Predicate Logic

Question1 Write each of the following statements in words. Write down whether you think the statement is true or false.

(a) $\forall x \in \mathbb{R}, (x \neq 0 \Rightarrow (x > 0 \vee x < 0))$

(b) $\forall x \in \mathbb{N}, \sqrt{x} \in \mathbb{N}$

(c) $\forall$ students *s* in WUCT121, $\exists$ an assigned problem *p*, *s* can correctly solve *p*.

Question2 Write each of the following statements using logical quantifiers and variables. Write down whether you think the statement is true or false.

(a) If the product of two numbers is 0, then both of the numbers are 0.
(b) Each real number is less than or equal to some integer.
(c) There is a student in WUCT121 who has never laughed at any lecturer's jokes.

Question3 Translate each of the following statements into the notation of predicate logic and simplify the negation of each statement. Which statements do you think are true?

(a) *P*: Someone loves everybody.
(b) *P*: Everybody loves everybody.
(c) *P*: Somebody loves somebody.
(d) *P*: Everybody loves somebody.
(e) *P*: All rational numbers are integers.
(f) *P*: Not all natural numbers are even.
(g) *P*: There exists a natural number that is not prime.
(h) *P*: Every triangle is a right triangle.

Question4 Are the following statements true or false? Give brief reasons why.

(a) $\forall x \in \mathbb{R}, (x > 1 \Rightarrow x > 0)$

(b) $\forall x \in \mathbb{R}, (x > 1 \Rightarrow x > 2)$

(c) $\exists x \in \mathbb{R}, (x > 1 \Rightarrow x^2 > x)$

(d) $\exists x \in \mathbb{R}, \left(x > 1 \Rightarrow \dfrac{x}{x^2+1} < \dfrac{1}{3}\right)$

(e) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, x^2 + y^2 = 9$

(f) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x^2 < y + 1$

(g) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x^2 + y^2 \geq 0$

(h) $\exists x \in \mathbb{R}, \exists y \in \mathbb{R}, (x < y \Rightarrow x^2 < y^2)$

Question5 For each of the following statements,

(a) Write down the negation of the statement,
(b) Write down whether the statement or its negation is false, and
(c) THINK about how you would disprove it.

(i) $\forall \xi > 0, \exists x \neq 0, |x| < \xi$

(ii) $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, y < x^2$

(iii) $\forall y \in \mathbb{R}, \forall x \in \mathbb{R}, x < y \Rightarrow x < \dfrac{x+y}{2} < y$

<u>Question6</u> Write down the negations of the following statements.

(a) $P: \quad \exists x \in \mathbb{Q}, x^2 = 2$

(b) $Q: \quad \forall x \in \mathbb{R}, x^2 + 1 \geq 2x$

<u>Question7</u> Write the following statements using quantifiers. Find their negations and determine in each case whether the statement or its negation is true, giving a brief reason

(a) P: For each real number, there is a smaller real number

(b) Q: Every real number is either positive or negative

<u>Question8</u> Write down the negations of the following statements. In each case decide whether the statement or its negation is true

(a) $\forall x \in \mathbb{R}, x \geq 0$

(b) $\exists z \in \mathbb{Z}, (z \text{ is odd}) \vee (z \text{ is even})$

(c) $\exists n \in \mathbb{N}, (n \text{ is even} \wedge \sqrt{n} \text{ is prime})$

(d) $\forall y \in \mathbb{R}, \left(y \neq 0 \Rightarrow \dfrac{y+1}{y} < 1 \right)$

(e) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, xy = 1$

(f) $\forall n \in \mathbb{N}, \exists p \in \mathbb{N}, n = 2p$

(g) $\forall \varepsilon \in \mathbb{R}, \forall x \in \mathbb{Z}, \exists y \in \mathbb{Q}, (\varepsilon > 0 \Rightarrow |x - y| < \varepsilon)$

<u>Question9</u> Write down the negations of the following statements. In each case decide whether the statement or its negation is false, giving a brief reason

(a) $\forall y \in \mathbb{R}, (y > -1 \Rightarrow y^2 > 1)$

(b) $\exists x \in \mathbb{R}, x^2 + 1 = 0$

(c) $\forall x, y, z \in \mathbb{R}, x - (y - z) \neq (x - y) - z$

(d) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x + y = 0$

<u>Question10</u> Write the following statements using quantifiers. Find their negations and determine in each case whether the statement or its negation is false, giving brief reason where possible.

(a) P: For each natural number there is a smaller natural number.

(b) P: The square of any real number is non-negative.

(c) P: Some dogs are vegetarians.

(d) P: There is a real number that is rational.

(e) P: Every student likes at least one Mathematics subject.

Section 3: Proofs

Question1 Each of the following demonstrates the Rule of Modus Ponens, Modus Tollens or the Law of Syllogism. In each case, answer the question or complete the sentence and indicate which of the logical rules is being demonstrated.

(a) If Peter is unsure of an address, then he will phone. Peter is unsure of John's address. What does Peter do?

(b) If $x^2 - 3x + 2 = 0$, then $(x-2)(x-1) = 0$. If $(x-2)(x-1) = 0$, then $x - 2 = 0$ or $x - 1 = 0$. If $x - 2 = 0$ or $x - 1 = 0$, then $x = 2$ or $x = 1$. Therefore, if $x^2 - 3x + 2 = 0$, then …

(c) We know that if *x* is a real number, then its square is positive or zero. If $y^2 = -1$, what do we know about *y*?

Question2 Prove or disprove the following statements

(a) For all $n \in \mathbb{N}$, the expression $n^2 + n + 29$ is prime.
(b) $\exists x \in \mathbb{Q},\ \forall y \in \mathbb{Q},\ xy \neq 1$
(c) $\forall a, b \in \mathbb{R},\ (a+b)^2 = a^2 + b^2$
(d) The average of any two odd integers is odd

Question3 Find the mistakes in the following "proofs".

(a) *Result*: $\forall k \in \mathbb{Z},\ k > 0 \Rightarrow k^2 + 2k + 1$ is not prime.

Proof: For $k = 2,\ k^2 + 2k + 1 = 9$, which is not prime. Therefore the result is true.

(b) *Result*: The difference between any odd integer and any even integer is odd.
Proof: Let *n* be any odd integer and *m* be any even integer. By definition of odd $n = 2k + 1,\ k \in \mathbb{Z}$, and by definition of even $m = 2k,\ k \in \mathbb{Z}$. Then $n - m = (2k+1) - 2k = 1.$ But 1 is odd. Therefore the result holds.

Question4 Prove each of the following results using a direct proof:

(a) For $x \in \mathbb{R},\ x^2 + 1 \geq 2x$

(b) For $n \in \mathbb{N},$ if *n* is odd, n^2 is odd.
(c) The sum of any two odd integers is even.
(d) If the sum of two angles of a triangle is equal to the third angle, then the triangle is a right angled triangle

Question5 Prove that if *x* is a negative real number, then $(x-2)^2 > 4$.

Question6 Prove that there is an integer $n > 5$, such that $2^n - 1$ is prime.

Question7 Prove that for each integer *n* such that $1 \leq n \leq 10,\ n^2 - n + 41$ is a prime number.

Question8 Prove that if *n* is an odd integer, then $(-1)^n = -1$.

Question9 Prove, by contraposition, that if n^2 is even, then *n* is even.

Question10 Prove by cases, if *m* is an integer, then $m^2 + m + 1$ is always odd.

Question11 Disprove the statement: $\forall a, b \in \mathbb{Z},\ a \neq 0, b \neq 0,\ \dfrac{1}{a+b} = \dfrac{1}{a} + \dfrac{1}{b}$. Are there any values for *a*, *b* that make the statement true? Explain.

Question12 Prove or disprove this statement: For all integers, *a*, *b* if $a < b$, then $a^2 < b^2$.

Question13 Prove if n^2 is odd, then *n* is odd.

Question14 Prove there is no smallest positive real number.

Question15 Prove each of the following using proof by cases

(a) If $x = 4, 5,$ or $6,$ then $x^2 - 3x + 21 \neq x.$

(b) $\forall x \in \mathbb{Z},\ \ x \neq 0 \Rightarrow 2^x + 3 \neq 4$

Question16 Prove there is a perfect square that can be written as the sum of two other perfect squares. (Note an integer *n* is a perfect square if and only if $\exists k \in \mathbb{Z}, n = k^2$)

Question17 Prove that the product of two odd integers is also an odd integer

Question18 Prove or disprove the following statements:

(a) The difference between any two odd integers is also an odd integer
(b) For any integer *n*, $3 \mid n(6n+3)$.
(c) The cube of any odd integer is an odd integer.
(d) For any integers *a*, *b*, *c*, if $a \mid c$, then $ab \mid c$.
(e) There is no largest even integer.
(f) For all integers *a*, *b*, *c*, if $a \nmid bc$, then $a \nmid b$.
(g) For all integers *n*, $4(n^2 + n + 1) - 3n^2$ is a perfect square.
(h) For any integers *a*, *b*, if $a \mid b$ then $a^2 \mid b^2$.
(i) For all integers *n*, $n^2 - n + 41$ is prime.
(j) For all integers, *n* and *m*, if $n - m$ is even, then $n^3 - m^3$ is even.

Question19 Prove that the product of any four consecutive numbers, increased by one, is a perfect square?

Section 4: Set Theory

Question1 Let $U = \mathbb{R}$.

Let $A = \{1\}$, $B = (0,1) = \{x \in \mathbb{R} : 0 < x < 1\}$ and $C = [0,1] = \{x \in \mathbb{R} : 0 \le x \le 1\}$.

Write down the following sets:

(a) $A \cup B$
(b) $A \cap B$
(c) $B \cap C$
(d) $A \cup C$
(e) $A \cap C$
(f) $\overline{A}$
(g) $\overline{C}$
(h) $C - A$
(i) $C - B$
(j) $A - C$

Are any of the pairs of sets *A*, *B* and *C* disjoint?

Question2 Let $U = \mathbb{N}$.

Let $A = \{x \in \mathbb{N} : x \text{ is odd}\}$, $B = \{x \in \mathbb{N} : x \text{ is even}\}$ and $P = \{x \in \mathbb{N} : x \text{ is prime}\}$.

Write down the following sets:

(a) $A \cup B$
(b) $A \cap B$
(c) $B \cap P$
(d) $A \cup P$
(e) $A \cap P$
(f) $\overline{A}$
(g) $\overline{P}$
(h) $P - A$
(i) $B - P$
(j) $A - B$

Are *A* and *B* disjoint? Is $P \subseteq A$?

Question3 Let $X = \{1, 2, 3, 4, 5\}$.

(a) Write down the set $\mathcal{P}(X)$ by listing its elements.

(b) How many elements in $\mathcal{P}(X)$?

(c) Is $\emptyset \in \mathcal{P}(X)$?

(d) Is $\{\emptyset\} \subseteq \mathcal{P}(X)$?

Question4 Write down the set $\mathcal{P}(\emptyset)$ by listing its elements.

How many elements in $\mathcal{P}(\emptyset)$?

Question5 Let $X = \{1, 2, 3, \ldots, n\}$, that is, X is a finite set with n elements.

How many elements does $\mathcal{P}(X)$ have?

Question6 Let $X = \{1, 2, 3\}$. For each of the following statements, write down whether it is True or False. Give reasons.

(a) $\forall B \in \mathcal{P}(X), \forall C \in \mathcal{P}(X), (B \subseteq C \vee C \subseteq B)$

(b) $\exists B \in \mathcal{P}(X), \forall C \in \mathcal{P}(X), B \subseteq C$.

(c) $\exists B \in \mathcal{P}(X), \forall C \in \mathcal{P}(X), C \subseteq B$

(d) The number of proper subsets of X is $2^3 - 1$.

Question7 Let $X = \{1,2\}$. Write down the set $\mathcal{P}(\mathcal{P}(X))$ by listing its elements.
If $Y = \{1,2,3\}$, how many elements would be in the set $\mathcal{P}(\mathcal{P}(Y))$? Write down two elements of $\mathcal{P}(\mathcal{P}(Y))$.

Question8 Can you write down two elements of $\mathcal{P}(\mathbb{R})$? Can you list the elements of $\mathcal{P}(\mathbb{R})$? What can you say about the set $\mathcal{P}(\mathbb{R})$?
Is $[-1,1] = \{x \mid x \in \mathbb{R} \wedge -1 \le x \le 1\} \in \mathcal{P}(\mathbb{R})$

Question9 Let $X = \{1,2,3\}$. Draw the Hasse Diagram for $\mathcal{P}(X)$.

Question10 Let $X = \{1,2,3,4\}$. Try to draw the Hasse Diagram for $\mathcal{P}(X)$.

Question11 Using the Principle of Mathematical Induction, prove that if $X = \{1,2,3,\ldots,n\}$, that is, *X* is a finite set with *n* elements, then the number of elements in $\mathcal{P}(X)$ is 2^n. (A procedure for determining the number of sets is sufficient for the inductive step.)

Question12 Give examples to demonstrate the following results.

(a) $\mathcal{P}(X) \cup \mathcal{P}(Y) \subseteq \mathcal{P}(X \cup Y)$ but $\mathcal{P}(X) \cup \mathcal{P}(Y) \ne \mathcal{P}(X \cup Y)$

(b) $\mathcal{P}(X) \cap \mathcal{P}(Y) = \mathcal{P}(X \cap Y)$

Question13 Let *U* be the universal set and let *A*, *B* and *C* be subsets of *U*.
Use a typical element argument to prove the following set theoretic results.

(a) $A \subseteq B \Rightarrow A \cup C \subseteq B \cup C$

(b) $(A \cup B) \cap B = B$

Question14 Let *U* be the universal set and let *A*, *B* and *C* be subsets of *U*.
Using properties of union, intersection and complement and known set laws, simplify the following:

(a) $\overline{(A \cap \overline{B})} \cap A$

(b) $(C \cup B) \cup \overline{C}$

(c) $(A \cap \emptyset) \cap U$

(d) $(A \cap U) \cup \overline{A}$

Question15 Prove or disprove:

$$\{0,1\} = \left\{ n \in \mathbb{Z} : \exists k \in \mathbb{Z}, \left(n = \frac{1+(-1)^k}{2} \right) \right\}$$

Question16 Let $U = \mathbb{Z}$. Let $A = \{n \in \mathbb{Z} : \exists k \in \mathbb{Z}, n = 2k-1\}$ and let $B = \{m \in \mathbb{Z} : \exists p \in \mathbb{Z}, m = 3p+2\}$. Write down 4 elements from each set.
Show that $t \in A \cap B \Leftrightarrow \exists w \in \mathbb{Z}, (w \text{ is odd } \wedge t = 3w+2)$.

Question17 Let *U* be the universal set and let *A*, *B* and *C* be subsets of *U*.
Use a typical element argument to prove the following set theoretic results.

(a) $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$

(b) $A \cap (B - C) = (A \cap B) - C$

<u>Question18</u> Let U be the universal set and let A, B and C be subsets of U.
Using properties of union, intersection and complement and known set laws, simplify the following:

(a) $(C \cap U) \cup \overline{C}$

(b) $\overline{(A \cap U)} \cup \overline{A}$

(c) $\overline{(C \cup \emptyset) \cup C}$

(d) $(A \cap B) \cap \overline{A}$

<u>Question19</u> Let $O = \{n \in \mathbb{Z} : n \text{ is odd}\}$ and let $T = \{n \in \mathbb{Z} : n^2 \text{ is odd}\}$.
Prove or disprove:

(a) $T \subseteq O$ **(b)** $O \subseteq T$ **(c)** $T = O$

<u>Question20</u> Let $T = \{n \in \mathbb{Z} : \exists x, y \in \mathbb{Z}, z = x^2 + y^2\}$ and let $E = \{z \in \mathbb{Z} : z \text{ is even}\}$.
Prove or disprove $T \subseteq E$.

<u>Question21</u> Let U be the universal set and let A, B and C be subsets of U.
Prove or disprove the following:

(a) $(A \cap B) = A \Leftrightarrow A \subseteq B$

(b) $(A \cap B) = (A \cap C) \Rightarrow B = C$

<u>Question22</u> Determine if the following statements are true or false:

(a) $A \cap B = \emptyset \Rightarrow A \subseteq \overline{B}$

(b) $(A \subseteq \overline{B} \wedge \overline{A} \subseteq \overline{B}) \Rightarrow B = \emptyset$

(c) A and $B - A$ are disjoint.

Section 5: Relations and Functions

Question1 Let $A = \{1,2\}$, $B = \{0,2,3\}$ and $C = \{a,b\}$.

(a) List the elements and sketch the graphs in $\mathbb{R}^2$ of :
(i) $A \times B$ **(ii)** $A \times A$ **(iii)** $B \times A$

(b) Is $A \times B \subseteq B \times B$

(c) List the elements of $(A \cup B) \times C$ and $(A \times C) \cup (B \times C)$. What do you notice?

(d) List the elements of $(A \times B) \times C$ and $C \times (A \times A)$.

Question2 Let $A = \{1,2\}$ and $D = \{a,b\}$. Write down $D \times A$. Will this be the same as $A \times D$?

Question3 Let $A = \{x \in \mathbb{R} : 0 < x < 1\}$, $B = \{x \in \mathbb{R} : -1 < x < 3\}$ and $C = \{x \in \mathbb{R} : 0 \le x \le 1\}$.

(a) Sketch the graph of $A \times B$ in $\mathbb{R}^2$.

(b) Sketch the graph of $C \times C$ in $\mathbb{R}^2$. Note: $C \times C$ is called the until square in $\mathbb{R}^2$.

(c) Sketch the graph of $C \times \mathbb{R}$ in $\mathbb{R}^2$.

Question4 Let $A = \{a_1, a_2, \ldots a_n\}$ and $B = \{b_1, b_2, \ldots b_m\}$.

(a) How many elements in $A \times B$?

(b) Write out the elements of $A \times B$.

Question5 Let *A*, *B* and *C* be elements of $\mathcal{P}(U)$. Prove that $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Question6 Sketch the graphs of the following relations in $\mathbb{R}^2$.

(a) $R_1 = \{(x,y) : x = y \wedge x = -y\}$.

(b) $R_2 = \{(x,y) : x^2 - y = 0\}$.

(c) $R_3 = \{(x,y) : y^2 = 2 + x\}$.

(d) $R_4 = \{(x,y) : (x^2 - y)(x - y) = 0\}$.

Question7 Sketch the graphs of the following relations in $\mathbb{R}^2$.

(a) $R_1 = \{(x,y) : |x| = |y|\}$

(b) $R_2 = \{(x,y) : (x^2 - y)(4x^2 + 9y^2 - 36) = 0\}$

Question8 Let $A = \{2,3,4\}$ and $B = \{6,8,10\}$ and define the relation *R* from *A* to *B* as follows: $R = \{(x,y) : x \text{ is a factor of } y\}$.

(a) List the elements of *R*.

(b) Graph $A \times B$ and circle the elements of *R*.

(c) True or false?
(i) $4R6$
(ii) $4R8$
(iii) $(3,8) \in R$
(iv) $(2,10) \in R$
(v) $(4,12) \in R$

Question9 Define the relations *R* and *S* on $\mathbb{R}$ as follows: $R = \{(x,y) : y = |x|\}$ $S = \{(x,x) : x = 0\}$. Find simple expressions for the relations:

(a) $R \cup S$ on $\mathbb{R}$. **(b)** $R \cap S$ on $\mathbb{R}$.

Question10 Write down the domain and range of the relation *R* on the given set *A*.
$A = \{h : h \text{ is a human being}\}$, $R = \{(h_1, h_2) : h_1 \text{ is the sister of } h_2\}$

Question11 Let $A = \{3, 4, 5\}$ and $B = \{4, 5, 6\}$ and define the relation *R* from *A* to *B* as follows: $R = \{(x, y) : x < y\}$. Write down *R* and R^{-1} by listing their elements.

Question12 For the relation *T* on $\mathbb{R}$ given by $T = \{(x, y) : \frac{x^2}{4} + \frac{y^2}{9} = 1\}$, find and expression for the inverse relation T^{-1}. Sketch both *T* and T^{-1}.

Question13 Determine whether or not the given relation is reflexive, symmetric or transitive. Give a counterexample in each case in which the relation does not satisfy the property.

(a) R_1 on the set $A = \{h : h \text{ is a human being}\}$ given by
$R_1 = \{(h_1, h_2) : h_1 \text{ is the sister of } h_2\}$

(b) R_2 on the set $A = \{a, b, c, d\}$ given by
$R_2 = \{(a,a), (a,b), (b,a), (b,b), (b,c), (c,b), (c,c), (d,d)\}$

Question14 Determine whether or not the following relation is an equivalence relation.
R on $A = \{0, 1, 2, 3\}$ given by $R = A \times A$.

Question15 Show that the relation *R* on the set $A = \{0, 1, 2, 3, 4\}$ given by
$R = \{(0,0), (0,4), (1,1), (1,3), (2,2), (3,1), (3,3), (4,0), (4,4)\}$ is an equivalence relation.
Find all the classes of *R*.

Question16 Is the following relation a function? Give brief reason.
R on $[-2, 2] = \{x \in \mathbb{R} : -2 \le x \le 2\}$, where
$R = \{(x, y) : y = \sqrt{1-(x-1)^2} \vee y = 1 - \sqrt{1-(x+1)^2}\}$.

Question17 Determine whether or not the following functions are:

(a) one-to-one , give brief reasons

(b) onto. Give brief reasons.

(i) Let $A = \{1, 5, 9\}$ and $B = \{3, 4, 7\}$. $F_1 \subseteq A \times B$ and $F_1 = \{(1,7), (5,3), (9,4)\}$

(ii) F_2 on $\mathbb{Z}$ and $F_2 = \{(x, y) : y = 2x\}$

Question18 Let $A = \{4, 5, 6\}$ and $B = \{5, 6, 7\}$ and define the relations *S* and *T* from *A* to *B* as follows: $S = \{(x, y) : x - y \text{ is even}\}$ and $T = \{(4,6), (6,5), (6,7)\}$.

(a) Find expressions for S^{-1} and T^{-1}

(b) Which of *S*, *T*, S^{-1} and T^{-1} are functions?

Question19 Simplify the following:

(a) $(1\ 3\ 4)(3\ 2\ 4)$

(b) $(1\ 3\ 4)^{-1}$

(c) $(2\ 5\ 4\ 1)^{-1}$

(d) $(3\ 2)(3\ 2\ 4)(3\ 1)(4\ 2)$