

1

Here is a strange fact about 2 by 2 matrices with eigenvalues $\lambda_1 \neq \lambda_2$: The columns of $A - \lambda_1 I$ are multiples of the eigenvector x_2 . Any idea why this should be?

2

Give an example to show that the eigenvalues can be changed when a multiple of one row is subtracted from another. Why is a zero eigenvalue *not* changed by the steps of elimination?

3

True or false: If the columns of S (eigenvectors of A) are linearly independent, then

- (a) A is invertible (b) A is diagonalizable
(c) S is invertible (d) S is diagonalizable.

4

This matrix is singular with rank 1. Find three λ 's and three eigenvectors:

$$A = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 4 \\ 2 & 1 & 2 \end{bmatrix}.$$

5

Factor these two matrices into $A = S\Lambda S^{-1}$:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}.$$

6

$A^k = S\Lambda^k S^{-1}$ approaches the zero matrix as $k \rightarrow \infty$ if and only if every λ has absolute value less than _____. Which of these matrices has $A^k \rightarrow 0$?

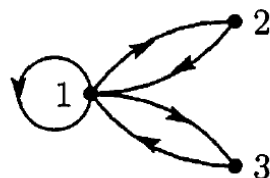
$$A_1 = \begin{bmatrix} .6 & .9 \\ .4 & .1 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} .6 & .9 \\ .1 & .6 \end{bmatrix}.$$

7

Find the eigenvalues and eigenvectors and the k th power of A . For this “adjacency matrix” the i, j entry of A^k counts the k -step paths from i to j .

1's in A show
edges between nodes

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$



8

The Hadamard matrix H has orthogonal rows. The box is a hypercube!

What is $|H| = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{vmatrix} =$ volume of a hypercube in \mathbf{R}^4 ?

9

Find an orthonormal basis for the column space of A :

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} -4 \\ -3 \\ 3 \\ 0 \end{bmatrix}.$$

Then compute the projection of \mathbf{b} onto that column space.

10

Find the height C of the best *horizontal line* to fit $\mathbf{b} = (0, 8, 8, 20)$. An exact fit would solve the unsolvable equations $C = 0, C = 8, C = 8, C = 20$. Find the 4 by 1 matrix A in these equations and solve $A^T A \hat{x} = A^T \mathbf{b}$. Draw the horizontal line at height $\hat{x} = C$ and the four errors in e .