1

Let $A = \{3, 4, 5\}$ and $B = \{4, 5, 6\}$ and let S be the "divides" relation. That is

$$\forall (x, y) \in A \times B, \quad x \; S \; y \Leftrightarrow x \mid y$$

State explicitly which ordered pairs are in S and S^{-1}

2

Let $A = \{-1, 1, 2, 4\}$ and $B = \{1, 2\}$ and define relations R and S from A to B as follows: $\forall (x, y) \in A \times B$

$$x \; R \; y \Leftrightarrow |x| = |y|$$

$$x \; S \; y \Leftrightarrow x - y \text{ is even}$$

.

State explicitly which ordered pairs are in $A \times B$, R, S, $R \cup S$, $R \cap S$

$$R \cup S = \{(x, y) \in A \times B \mid (x, y) \in R \text{ or } (x, y) \in S\}$$

$$R \cap S = \{(x, y) \in A \times B \mid (x, y) \in R \text{ and } (x, y) \in S\}$$

3

Assume O is the relation defined on $\mathbf{Z}$ as follows:

$$\forall m, n \in \mathbf{Z}, m \; O \; n \Leftrightarrow m - n \text{ is odd}$$

Explain whether relation O is reflexive, symmetric, transitive, or none of these. Justify your answer.

4

Let A be the set of all lines in the plane. A relation R is defined on A as follows:

$$\forall l_1, l_2 \in A, \quad l_1 \; R \; l_2 \Leftrightarrow l_1 \text{ is parallel to } l_2$$

Assume that a line is parallel to itself. Determine whether R is reflexive, symmetric, transitive, or none of these.

5

Assume that R and S are relations on a set A. Prove or disprove that if R and S are transitive, $R \cap S$ is also transitive.

6

Assume R is an equivalence relation on the set $A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$. R is defined on A as follows:

$$\forall (m, n) \in A, \quad m \; R \; n \Leftrightarrow 5 \mid (m^2 - n^2)$$

Find the distinct equivalence classes of R.

7

Define Q on the set $R \times R$ as follows:

$$\forall (w, x), (y, z) \in R \times R, \quad (w, x) \; Q \; (y, z) \Leftrightarrow x = z$$

Prove that relation Q is an equivalence relation and describe the distinct equivalence classes.