

1

What matrix E puts A into triangular form $EA = U$? Multiply by $E^{-1} = L$ to factor A into LU :

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix}.$$

2

What are L and D (the diagonal **pivot matrix**) for this matrix A ? What is U in $A = LU$ and what is the new U in $A = LDU$?

Already triangular

$$A = \begin{bmatrix} 2 & 4 & 8 \\ 0 & 3 & 9 \\ 0 & 0 & 7 \end{bmatrix}.$$

3

Tridiagonal matrices have zero entries except on the main diagonal and the two adjacent diagonals. Factor these into $A = LU$ and $A = LDL^T$:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{bmatrix}.$$

4

Find the inverses (in any legal way) of

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 \\ 0 & 0 & -\frac{3}{4} & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{bmatrix}.$$

5

There are sixteen 2 by 2 matrices whose entries are 1s and -1 s. How many are invertible?

6

Factor these symmetric matrices into $A = LDL^T$. The pivot matrix D is diagonal:

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 1 & b \\ b & c \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

7

True or false:

- (a) The block matrix $\begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix}$ is automatically symmetric.
- (b) If A and B are symmetric then their product AB is symmetric.
- (c) If A is not symmetric then A^{-1} is not symmetric.
- (d) When A, B, C are symmetric, the transpose of ABC is CBA .

8

Suppose $(x_1, x_2) + (y_1, y_2)$ is defined to be $(x_1 + y_2, x_2 + y_1)$. With the usual multiplication $c\mathbf{x} = (cx_1, cx_2)$, which of the eight conditions are not satisfied?

- (1) $\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$
- (2) $\mathbf{x} + (\mathbf{y} + \mathbf{z}) = (\mathbf{x} + \mathbf{y}) + \mathbf{z}$
- (3) There is a unique “zero vector” such that $\mathbf{x} + \mathbf{0} = \mathbf{x}$ for all \mathbf{x}
- (4) For each \mathbf{x} there is a unique vector $-\mathbf{x}$ such that $\mathbf{x} + (-\mathbf{x}) = \mathbf{0}$
- (5) 1 times \mathbf{x} equals \mathbf{x}
- (6) $(c_1c_2)\mathbf{x} = c_1(c_2\mathbf{x})$
- (7) $c(\mathbf{x} + \mathbf{y}) = c\mathbf{x} + c\mathbf{y}$
- (8) $(c_1 + c_2)\mathbf{x} = c_1\mathbf{x} + c_2\mathbf{x}$.

9

The matrix $A = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$ is a “vector” in the space \mathbf{M} of all 2 by 2 matrices. Write down the zero vector in this space, the vector $\frac{1}{2}A$, and the vector $-A$. What matrices are in the smallest subspace containing A ?

10

Describe the column spaces (lines or planes) of these particular matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 0 \end{bmatrix}.$$