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When you write down the solutions using induction, it is always a great idea to think about this template.

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- while in **strong induction**, we assume that the particular statement holds at all the steps from the **base case to k^{th} step**.

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- In the exam, many of you will have to struggle in this part. Please pay close attention to how this suggested inductive step uses induction hypothesis for reasoning.

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- Prove by induction that every ***integer greater than or equal to*** ***2*** can be ***factored into primes.*** The statement ***P(n)*** is that an integer ***n*** ***greater than or equal to 2*** can be ***factored into primes***.

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- Therefore, the statement that every integer greater than or equal to 2 can be factored into primes holds for all such integers.

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- It works, but does not fit into the notion of inductive proof that we wanted you to learn. For inductive step in inductive proof, you must reason your argument based on induction hypothesis you yourself state.

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- Not really; those numbers are dealt in the **Case 1** in the inductive step.
- If you try to prove all these possible prime number cases, you need to do so with brute-force scheme, which means you need to prove the statement on every single prime number greater than 2.

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- Check you used induction hypothesis appropriately for inductive step.

Example

http://www.mathblog.dk/strong-induction/

- I have stolen this example from Hammack since I think it brilliantly shows when strong induction is better to use. But lets first see what happens if we try to use weak induction on the following:
- **Proposition**: if $n \in \mathbb{N}$, then $12 | (n^4 - n^2)$

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$$(k+1)^4 –(k+1)^2 = k^4+4k^3+6k^2 +4k +1–(k^2 + 2k + 1) =$$
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- How do we proceed from there? I don’t have a clue. If you can show me how to continue of this road I would be glad to hear it.

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- **Base case:**

  n = 1: $12|(1^4 – 1^2) = 12|(1- 1) = 0 = 0*12$

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- So far it fits really well.

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- **Proposition**: if $n \in \mathbb{N}$, then $12|(n^4 - n^2)$

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- So in order to prove it for ***n = k+1 = 7*** we need ***n = k-5 = 1*** to prove it, but that is handled in the base case, same goes for all the ***n = 8,9,10,11,12*** and then we start to rely on the fact that we can prove ***n= 8*** through the induction step.

Summary

- Please do not forget to read the book Chapter and the provided links for further reading about Mathematical Inductions.
- http://www.purplemath.com/modules/inductn3.htm
- http://www.mathcentre.ac.uk/resources/uploaded/mathcentre-proof2.pdf
- http://home.scarlet.be/math/volledige_inductie.htm
- https://cims.nyu.edu/~kiryl/teaching/aa/review1.pdf
- http://www.math.illinois.edu/~ajh/213/inductionsampler.pdf
- http://web.maths.unsw.edu.au/~jim/proofsch8.pdf
- https://www.mathsisfun.com/algebra/mathematical-induction.html
- http://www.socialresearchmethods.net/kb/dedind.php
- http://deborahgabriel.com/2013/03/17/inductive-and-deductive-approaches-to-research
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