## Hometask #6. Sequences and Induction.

- 1. Write the first four terms of sequences defined by the formulas in a-c ([] integer part):
  - a)  $c_i = \frac{(-1)^i}{3^i}$ , for all integers  $i \ge 0$
  - b)  $e_n = \left\lceil \frac{n}{2} \right\rceil \cdot 2$ , for all integers  $n \ge 0$
  - c)  $f_n = \left\lceil \frac{n}{4} \right\rceil \cdot 4$ , for all integers  $n \ge 1$
- 2. Compute the summations:

a) 
$$\sum_{k=1}^{5} (k+1)$$

b) 
$$\sum_{k=-1}^{1} (k^2 + 3)$$

c) 
$$\sum_{m=0}^{3} \frac{1}{2^m}$$

d) 
$$\sum_{n=1}^{10} \left( \frac{1}{n} - \frac{1}{n+1} \right)$$

3. Compute the products

a) 
$$\prod_{k=2}^{4} k^2$$

b) 
$$\prod_{j=0}^{4} (-1)^j$$

$$c) \qquad \prod_{k=2}^{2} \left( 1 - \frac{1}{k} \right)$$

d) 
$$\prod_{i=2}^{5} \frac{i(i+2)}{(i-1)(i+1)}$$

4. Write as single summation or product:

a) 
$$\sum_{i=1}^{k} i^3 + (k+1)^3$$

b) 
$$\sum_{k=1}^{m} \frac{k}{k+1} + \frac{m+1}{m+2}$$

c) 
$$\sum_{m=0}^{n} (m+1)2^{m} + (n+2)2^{n+1}$$

d) 
$$2 \cdot \sum_{k=1}^{n} (3k^2 + 4) + 5 \cdot \sum_{k=1}^{n} (2k^2 - 1)$$

e) 
$$\left(\prod_{k=1}^{n} \frac{k}{k+1}\right) \cdot \left(\prod_{k=1}^{n} \frac{k+1}{k+2}\right)$$

5. Write, using summation or product notation:

a) 
$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2$$

b) 
$$(2^2-1)(3^2-1)(4^2-1)$$

c) 
$$\frac{2}{3 \cdot 4} - \frac{3}{4 \cdot 5} + \frac{4}{5 \cdot 6} - \frac{5}{6 \cdot 7} + \frac{6}{7 \cdot 8}$$

d) 
$$(1-t)(1-t^2)(1-t^3)(1-t^4)$$

6. Transform by making the change of variable j = i - 1

a) 
$$\sum_{i=1}^{n-1} \frac{i}{(n-i)^2}$$

b) 
$$\prod_{i=n}^{2n} \frac{n-i+1}{n+i}$$

7. Prove using mathematical induction

$$4^3 + 4^4 + 4^5 + ... + 4^n = \frac{4(4^n - 16)}{3}$$
 for all  $n \ge 3$ ,  $n \in \mathbb{Z}$ 

8. Prove using mathematical induction

$$\prod_{i=0}^{n} \left( \frac{1}{2i+1} \cdot \frac{1}{2i+2} \right) = \frac{1}{(2n+2)!} \text{, for all integers } n \ge 1$$