Statements, rules, formulas and theorems:

L4P3

- **Multiplication Rule**

  $P(A_1 \cap A_2 \cap \dots \cap A_n) =$
  $= P(A_1) * \; P(A_2 | A_1) * P(A_3 | A_1 \cap A_2) * \dots * P(A_n | A_1 \cap A_2 \cap \dots \cap A_{(n-1)})$

L5P2

- **Multiplication Rule for a pair of independent events**:
  $P(A \cap B) = P(A)*P(B)$.

- **Multiplication Rule for mutually independent events:**
  $P(A_1 \cap A_2 \cap \dots \cap A_n) = P(A_1) * P(A_2) * \dots * P(A_n)$

L5P3

- **Total Probability Formula/формула полной вероятности**
  Let a sample space be partitioned: $\Omega = \cup_{n \in [0..m]} H_n$;
  then for every event $A \subseteq \Omega$
  $P(A) = P(\cup_{n \in [0..m]}(A \cap H_n)) =$
  $= \Sigma_{n \in [0..m]} P(A \cap H_n) = \Sigma_{n \in [0..m]} P(A | H_n) * P(H_n)$.

- **Bayes' Formula/формула Байеса**
  Let a sample space be partitioned: $\Omega = \cup_{n \in [0..m]} H_n$;
  then for every event A and hypothesis Hk the ***posterior probability/апостериорная вероятность*** can be computed using ***prior probabilities/априорные вероятности*** as follows:
  $P(H_k | A) = P(A | H_k) * P(H_k) / P(A) =$
  $= P(A | H_k) * P(H_k) / (\Sigma_{n \in [0..m]} P(A | H_n) * P(H_n))$.

L7P3

- ***Total Expectation Formula/формула полного мат. ожидания***

  If $H_1, \dots H_n$ is a partition then $P(A)= \Sigma_{k\in[1..n]}P(A|H_k)*P(H_k)$

  and hence $P_X(x)= \Sigma_{k\in[1..n]}P_{X|Hk}(x)*P(H_k)$, then

  $$\mathbf{M(X) = \Sigma_{k\in[1..n]}M(X|H_k)*P(H_k)}$$

L9P1

- ***Poisson Limit Theorem (law of rare events)/теорема Пуассона***

  Assume we are given an infinite series of trials $X_1, \dots X_n, \dots$ where $X_n= B(n,p_n)$; $p_n= \mu/n$ (where μ is a constant).

  For every n>0 let $P_n(m)$ be the probability of m positive outcomes in the n-th trial; then this sequence converges to $e^{-\mu} *(\mu^m/m!)$

L9P2

- ***Local Moivre-Laplace theorem/локальная теорема Муавра-Лапласа***

  Assume we are given an infinite series of trials $X_1, \dots Xn$, where X_n= binomial(n, p) and 0<p<1, q = 1−p.

  For every n>0 let $P_n(m)$ be the probability of m positive outcomes in the n^{th} trial; then

  $P_n(m) = \dfrac{\phi_0(t_n)}{\sqrt{npq}}(1+\alpha_n)$

  where $\phi_0(t_n) = \frac{1}{\sqrt{2\pi}}e^{-t_n^2/2}$, $t_n = \frac{m-np}{\sqrt{npq}}$

  and $|\alpha_n| < \frac{c}{\sqrt{n}}$

- ***De Moivre-Laplace theorem/теорема Муавра-Лапласа***

  Assume we are given an infinite series of trials $X_1, \dots X_n, \dots$ where X_n= binomial(n, p) and 0<p<1, q = 1−p.

  For every n>0 let $P_n(m)$ be the probability of m positive outcomes in the n^{th} trial and $P_n(m_1,m_2) = P_n(m_1 \le m \le m_2)$; then

  $P_n(m_1, m_2) \approx \phi_0(x_{m_2}) - \phi_0(x_{m_1})$,

  $x_{m_1} = \frac{m_1-np}{\sqrt{npq}}$, $x_{m_2} = \frac{m_2-np}{\sqrt{npq}}$

L10P1

- ***Bernoulli' law/теорема Бернулли***
Assume we are given an infinite series of trials $X_1, \dots X_n, \dots$,
where $X_n = B(n,p)$, $0<p<1$. Then for every $\varepsilon > 0$:
$$\lim_{n\to\infty} P(|{}^{m}/_{n} - p| \le \varepsilon) = 1$$

- ***Chebyshev's inequality/неравенство Чебышёва***
Let X be a random variable with a finite expectation μ and finite non-zero deviation σ. Then for any real number $k>0$
$$P(|X - \mu| \ge k\,\sigma) \le 1/k^2.$$

- ***Khintchin's (weak) law/слабый закон больших чисел***
Let $X_1, X_2, \dots$ be an infinite sequence of *independent and identically distributed* (i.i.d. or IID) random variables with same sets of outcomes, finite expectation μ and finite non-zero deviation σ.
Let $\underline{\mathbf{X}}_{\mathbf{n}}$ be $(X_1 + \dots + X_n)/n$
Then for every $\varepsilon > 0$ the probability
$$P(|\underline{\mathbf{X}}_{\mathbf{n}} - \mu| > \varepsilon) \to 0 \text{ (as } n \to \infty)$$

L11P3

- ***Central Limit Theorem/центральная предельная теорема***
Let $X_1, X_2, \dots$ be an infinite sequence of IID (i.e. independent and identically distributed) random variables with a finite expectation μ and finite non-zero deviation σ, $S_n = X_1 + \dots + X_n$.
Then $\frac{S_n - n\mu}{\sigma\sqrt{n}}$ *converges in distribution* to N(0,1) (as $n \to \infty$).

- ***68-95-99.7 rule/правило одного-двух-трёх сигм*** is a shorthand to remember the percentage of values that lie within a band around the mean in a normal distribution:
–68.27%, 95.45% and 99.73% of the values lie within distance σ, 2σ and 3σ of the mean μ.