EQUENTIAL LOGIC DESI

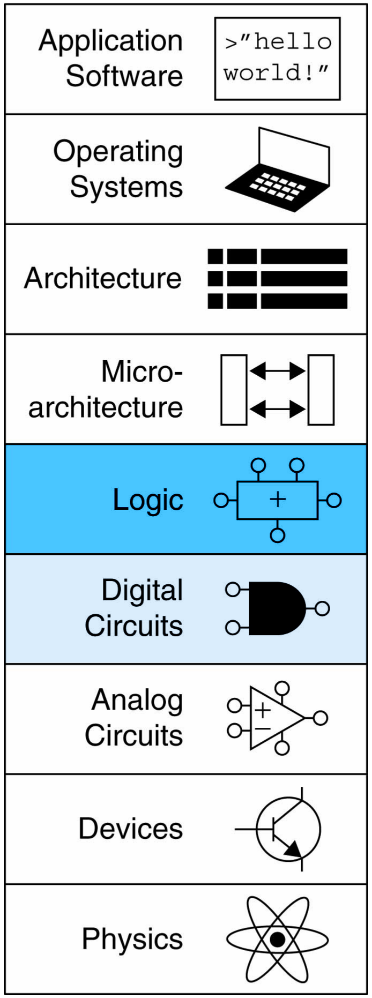
Chapter 3

Digital Design and Computer Architecture, 2nd Edition

David Money Harris and Sarah L. Harris

Chapter 3 :: Topics

- **Introduction**
- **Latches and Flip-Flops**
- **Synchronous Logic Design**
- **Finite State Machines**
- **Timing of Sequential Logic**
- **Parallelism**



Introduction

- Outputs of sequential logic depend on current *and* prior input values – it has ***memory***.
- Some definitions:
  - **State:** all the information about a circuit necessary to explain its future behavior
  - **Latches and flip-flops:** state elements that store one bit of state
  - **Synchronous sequential circuits:** combinational logic followed by a bank of flip-flops

Sequential Circuits

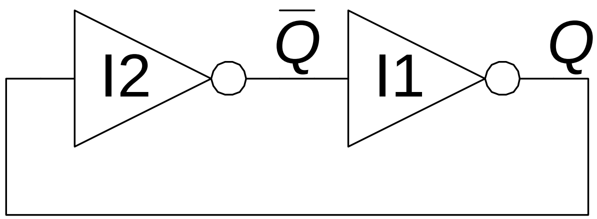
- Give sequence to events
- Have memory (short-term)
- Use feedback from output to input to store information

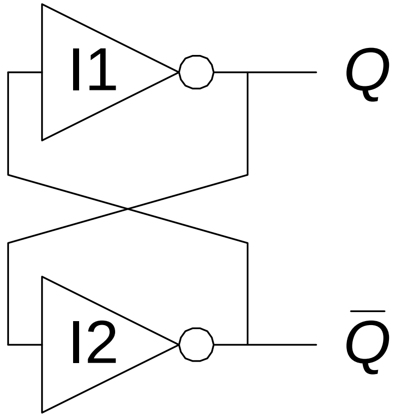
State Elements

- The state of a circuit influences its future behavior
- State elements store state
  - Bistable circuit
  - SR Latch
  - D Latch
  - D Flip-flop

Bistable Circuit

- Fundamental building block of other state elements
- Two outputs: Q, $\overline{Q}$
- No inputs

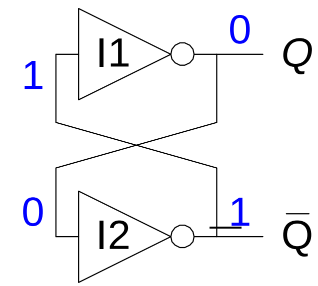




Bistable Circuit Analysis

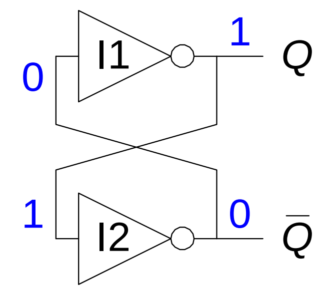
- Consider the two possible cases:
  - **$Q = 0$:**

    then $\bar{Q} = 1$, $Q = 0$ (consistent)



  - **$Q = 1$:**

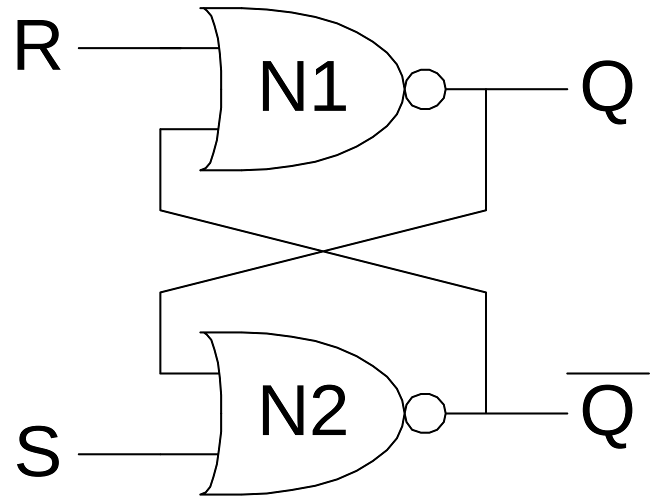
    then $\bar{Q} = 0$, $Q = 1$ (consistent)



- Stores 1 bit of state in the state variable, Q (or $\bar{Q}$)
- But there are **no inputs to control the state**

SR (Set/Reset) Latch

- SR Latch

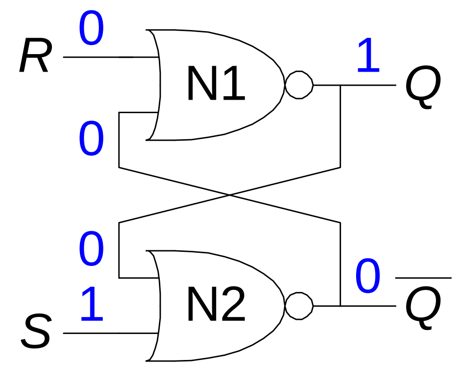


- Consider the four possible cases:
  - $S = 1$, $R = 0$
  - $S = 0$, $R = 1$
  - $S = 0$, $R = 0$
  - $S = 1$, $R = 1$

SR Latch Analysis

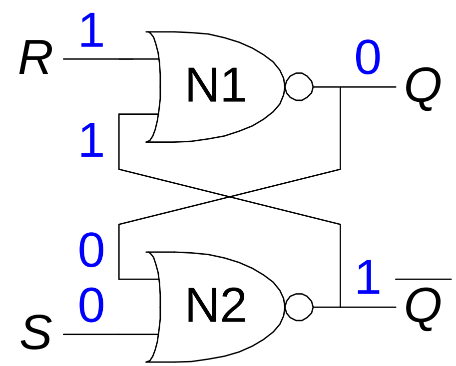
- $S = 1, R = 0$:

  then $Q = 1$ and $\overline{Q} = 0$



- $S = 0, R = 1$:

  then $Q = 1$ and $\overline{Q} = 0$

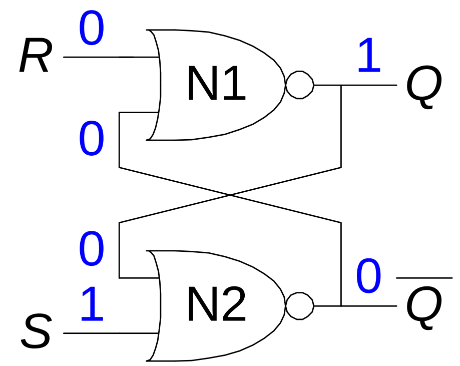


SR Latch Analysis

– **$S = 1$, $R = 0$:**

then $Q = 1$ and $\overline{Q} = 0$

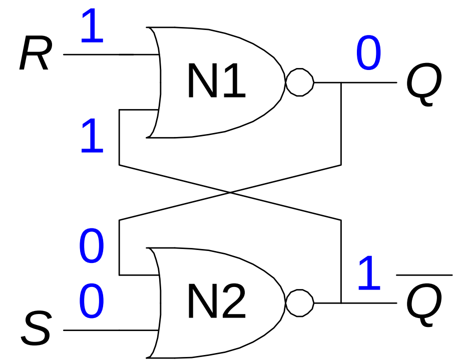
***Set* the output**



– **$S = 0$, $R = 1$:**

then $Q = 1$ and $\overline{Q} = 0$

***Reset* the output**



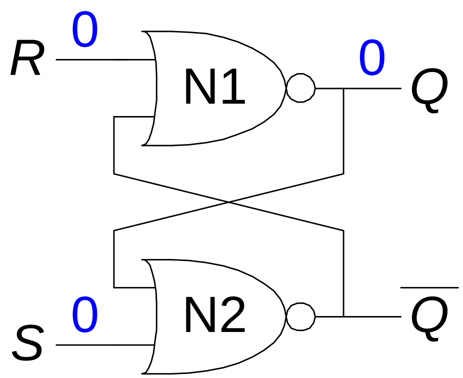
ELSEVIER

SR Latch Analysis

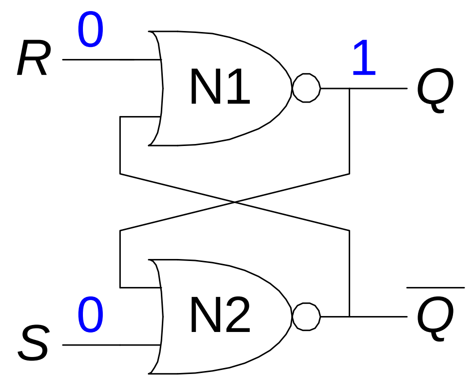
– ***S* = 0, *R* = 0:**

then $Q = Q_{prev}$

$Q_{prev} = 0$

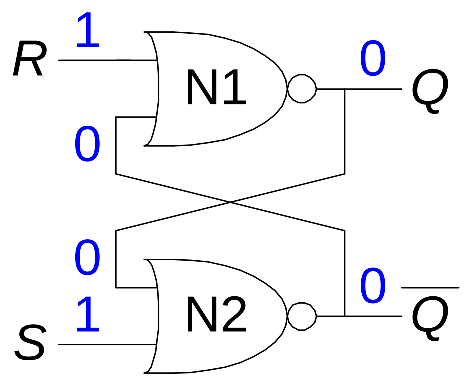


$Q_{prev} = 1$



– ***S* = 1, *R* = 1:**

then $Q = 0$, $\overline{Q} = 0$



ELSEVIER

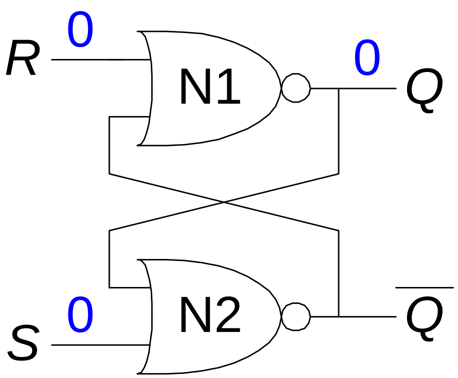
SR Latch Analysis

– ***S* = 0, *R* = 0:**

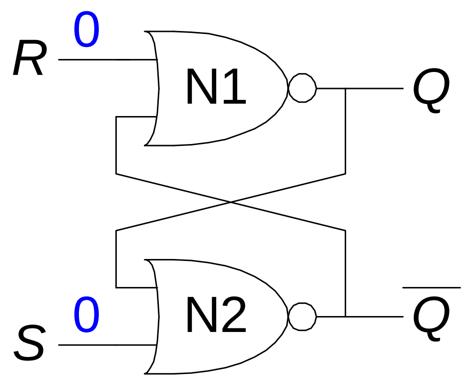
then $Q = Q_{prev}$

Memory!

$Q_{prev} = 0$



$Q_{prev} = 1$

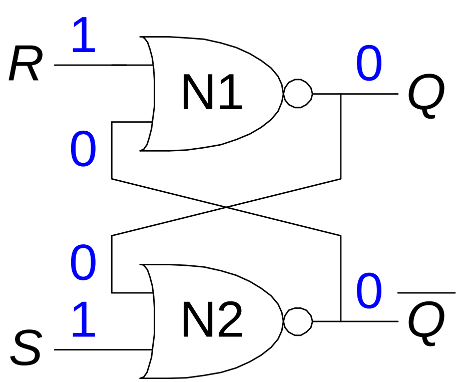


– ***S* = 1, *R* = 1:**

then $Q = 0, \overline{Q} = 0$

Invalid State

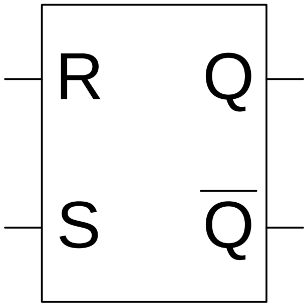
$\overline{Q} \neq \text{NOT } Q$



SR Latch Symbol

- SR stands for Set/Reset Latch
  - Stores one bit of state (*Q*)
- Control what value is being stored with *S*, *R* inputs
  - **Set:** Make the output 1
    (S = 1, R = 0, Q = **1**)
  - **Reset:** Make the output 0
    (S = 0, R = 1, Q = **0**)

SR Latch
Symbol

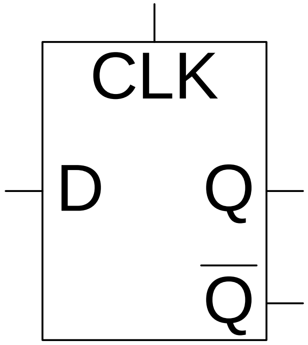


D Latch

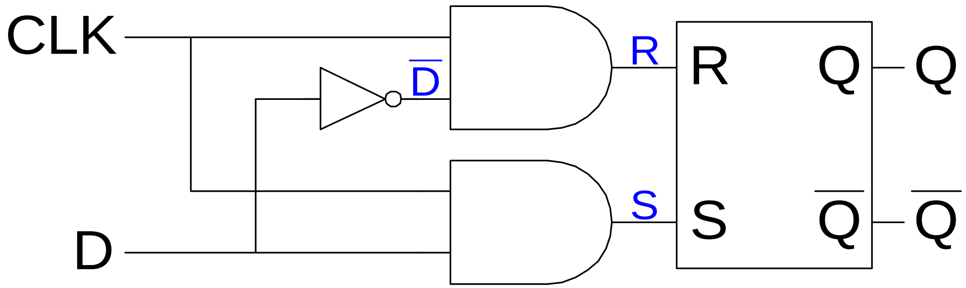
- Two inputs: *CLK*, *D*
  - ***CLK*****:** controls *when* the output changes
  - ***D*** (the data input): controls *what* the output changes to
- Function
  - When ***CLK*** **= 1**,
    D passes through to *Q* (*transparent*)
  - When ***CLK*** **= 0**,
    Q holds its previous value (*opaque*)
- Avoids invalid case when

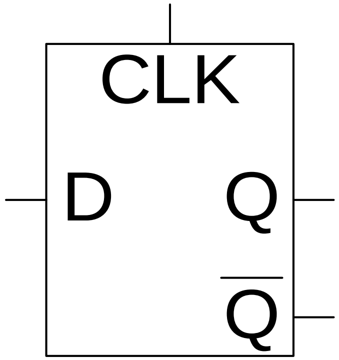
$Q \neq \text{NOT } \overline{Q}$

D Latch Symbol



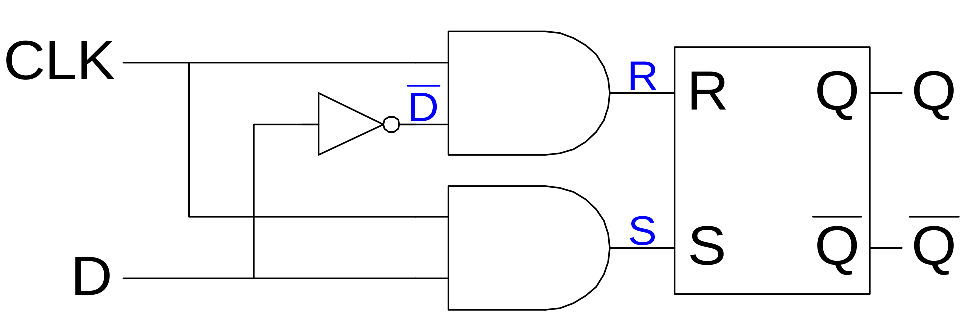
D Latch Internal Circuit

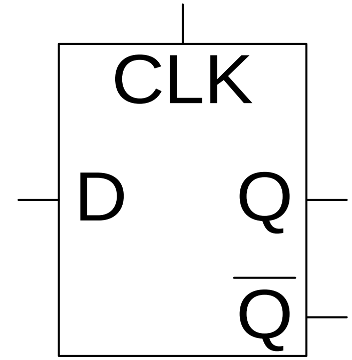




CLK	*D*	$\overline{D}$	*S*	*R*	*Q*	$\overline{Q}$
0	X					
1	0					
1	1					

D Latch Internal Circuit



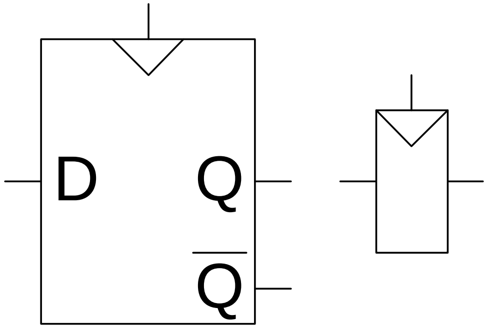


CLK	*D*	$\overline{D}$	*S*	*R*	*Q*	$\overline{Q}$
0	X	$\overline{X}$	0	0	Q_{prev}	$\overline{Q}_{prev}$
1	0	1	0	1	0	1
1	1	0	1	0	1	0

D Flip-Flop

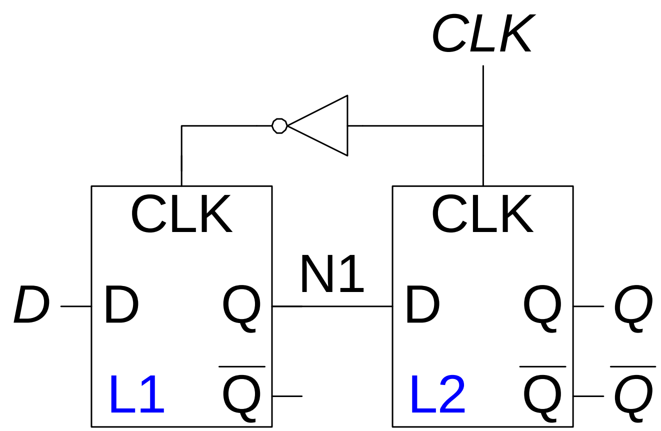
- **Inputs:** *CLK*, *D*
- **Function**
  - Samples *D* on rising edge of *CLK*
    - When *CLK* rises from 0 to 1, *D* passes through to *Q*
    - Otherwise, *Q* holds its previous value
  - *Q* changes only on rising edge of *CLK*
- Called *edge-triggered*
- Activated on the clock edge

D Flip-Flop Symbols

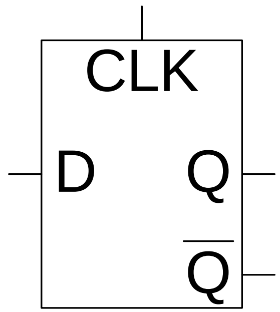


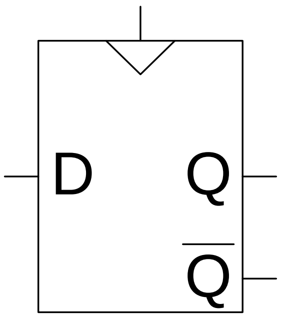
D Flip-Flop Internal Circuit

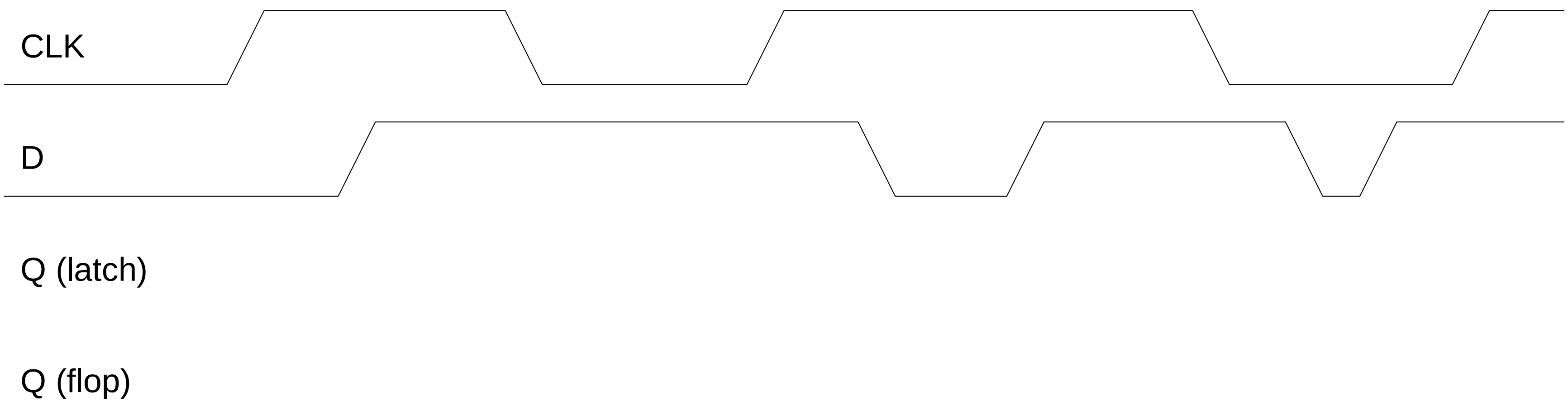
- Two back-to-back latches (L1 and L2) controlled by complementary clocks
- When ***CLK* = 0**
  - L1 is transparent
  - L2 is opaque
  - *D* passes through to N1
- When ***CLK* = 1**
  - L2 is transparent
  - L1 is opaque
  - N1 passes through to *Q*
- Thus, on the edge of the clock (when ***CLK* rises from 0 → 1**)
  - *D* passes through to *Q*



D Latch vs. D Flip-Flop

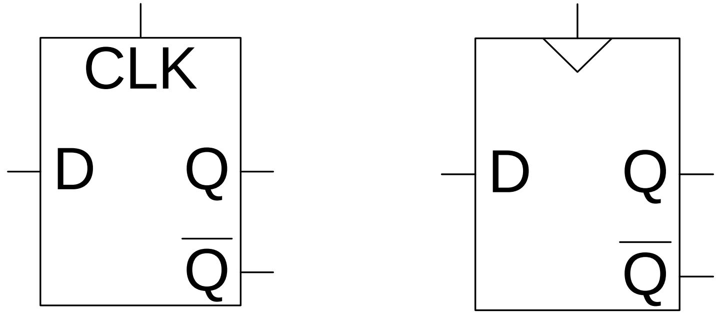


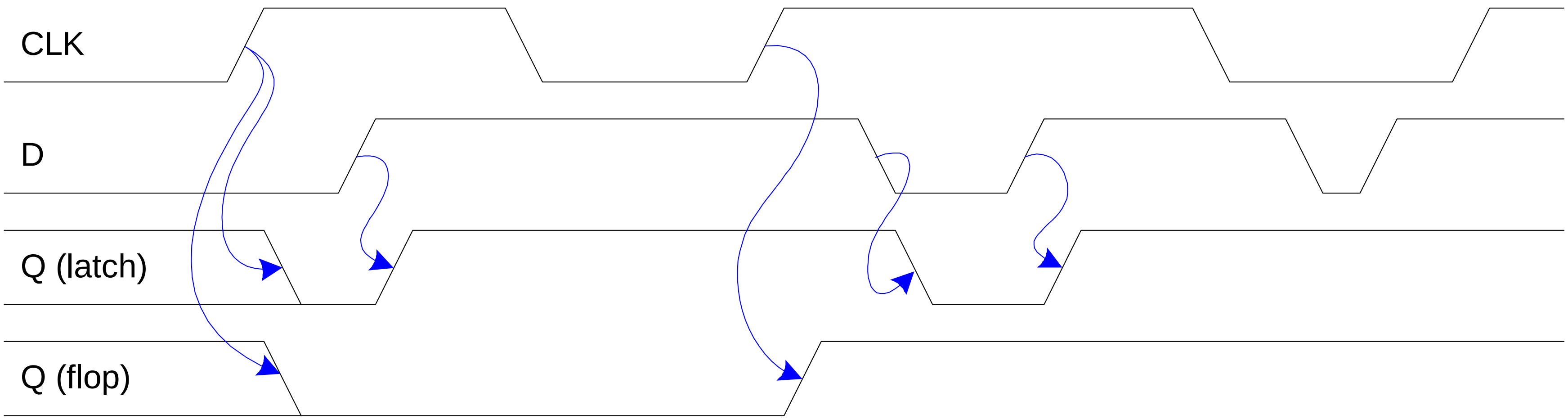




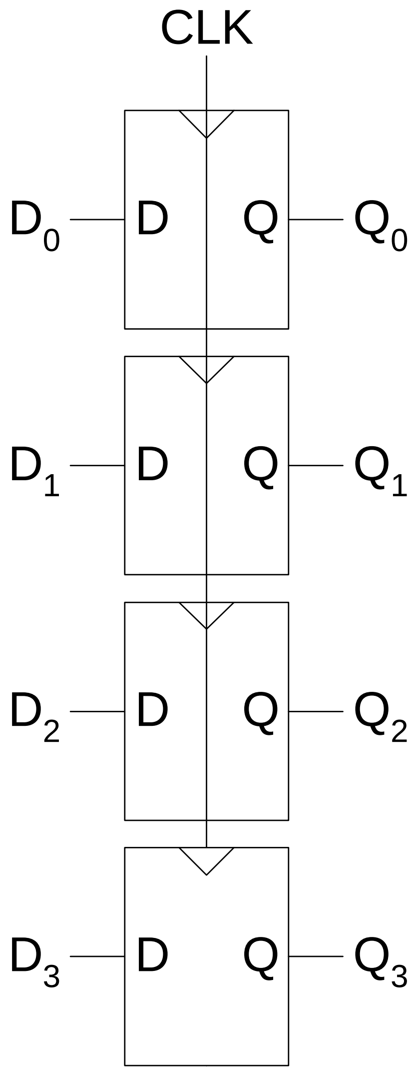
ELSEVIER

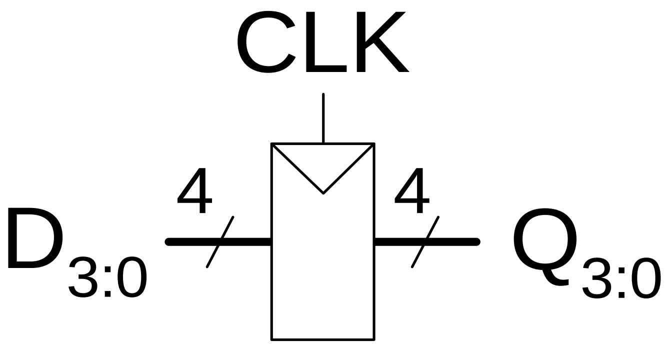
D Latch vs. D Flip-Flop





Registers

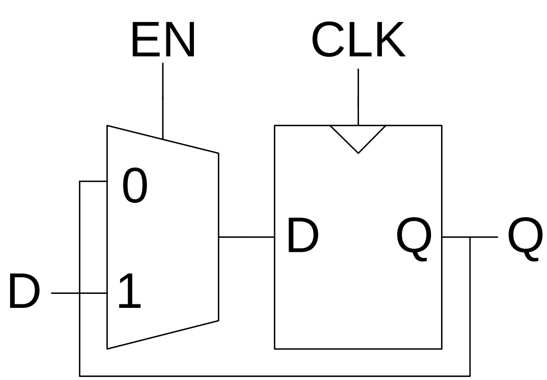




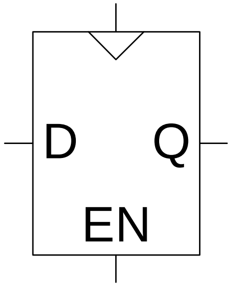
Enabled Flip-Flops

- **Inputs:** *CLK*, *D*, *EN*
  - The enable input (*EN*) controls when new data (*D*) is stored
- **Function**
  - ***EN* = 1:** *D* passes through to *Q* on the clock edge
  - ***EN* = 0:** the flip-flop retains its previous state

Internal Circuit



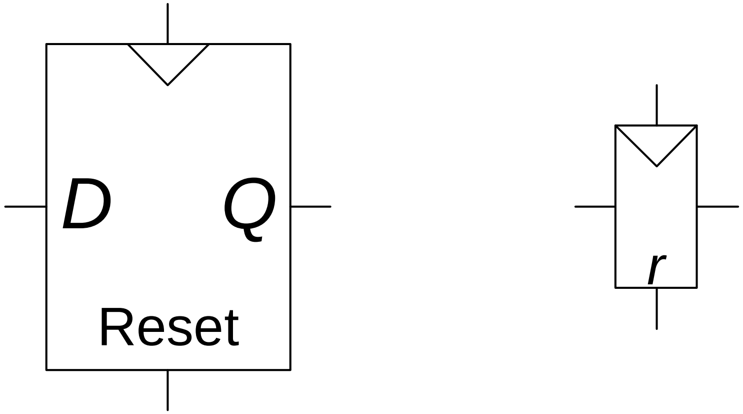
Symbol



Resettable Flip-Flops

- **Inputs:** *CLK*, *D*, *Reset*
- **Function:**
  - ***Reset* = 1:** *Q* is forced to 0
  - ***Reset* = 0:** flip-flop behaves as ordinary D flip-flop

Symbols

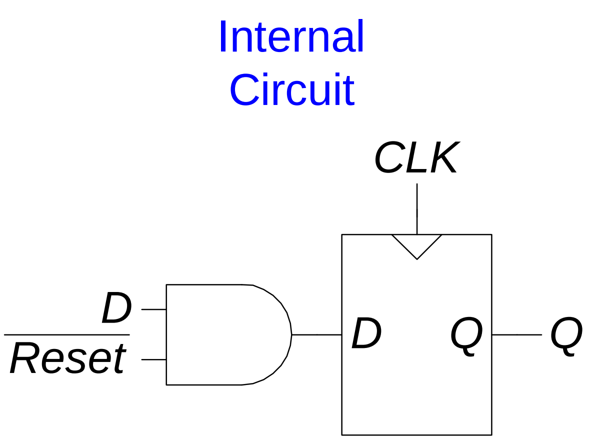


Resettable Flip-Flops

- Two types:
  - **Synchronous:** resets at the clock edge only
  - **Asynchronous:** resets immediately when *Reset* = 1
- Asynchronously resettable flip-flop requires changing the internal circuitry of the flip-flop
- Synchronously resettable flip-flop?

Resettable Flip-Flops

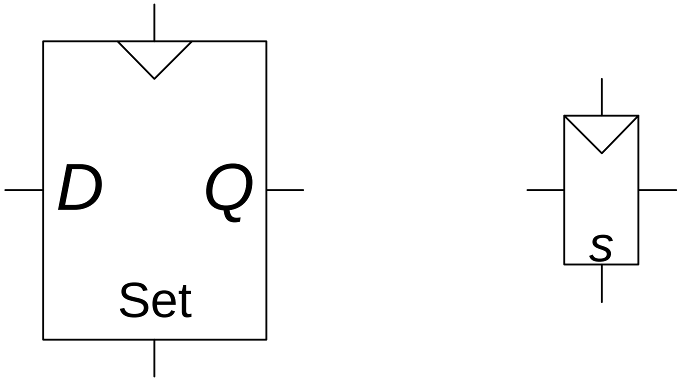
- Two types:
  - **Synchronous:** resets at the clock edge only
  - **Asynchronous:** resets immediately when *Reset* = 1
- Asynchronously resettable flip-flop requires changing the internal circuitry of the flip-flop
- Synchronously resettable flip-flop?



Settable Flip-Flops

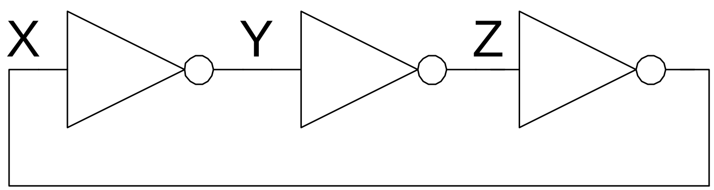
- **Inputs:** *CLK*, *D*, *Set*
- **Function:**
  - ***Set* = 1:** *Q* is set to 1
  - ***Set* = 0:** the flip-flop behaves as ordinary D flip-flop

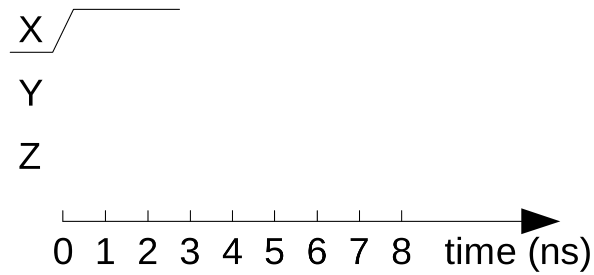
Symbols



Sequential Logic

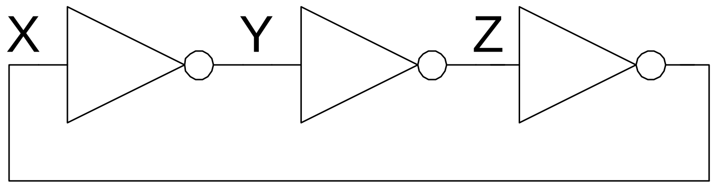
- Sequential circuits: all circuits that aren't combinational
- A problematic circuit:

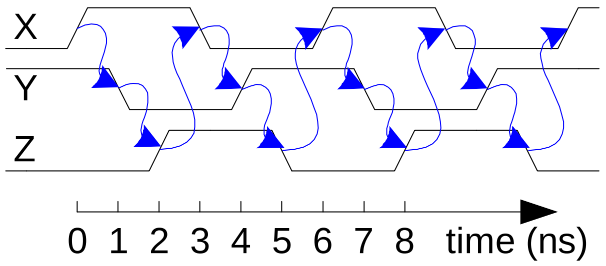




Sequential Logic

- Sequential circuits: all circuits that aren't combinational
- A problematic circuit:





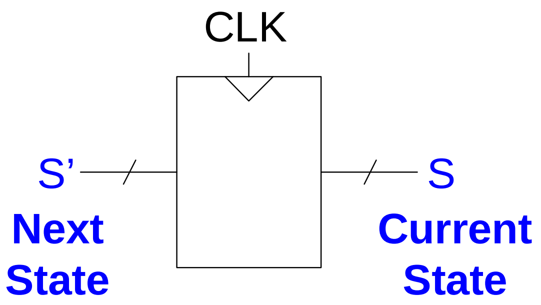
- No inputs and 1-3 outputs
- Astable circuit, oscillates
- Period depends on inverter delay
- It has a ***cyclic path***: output fed back to input

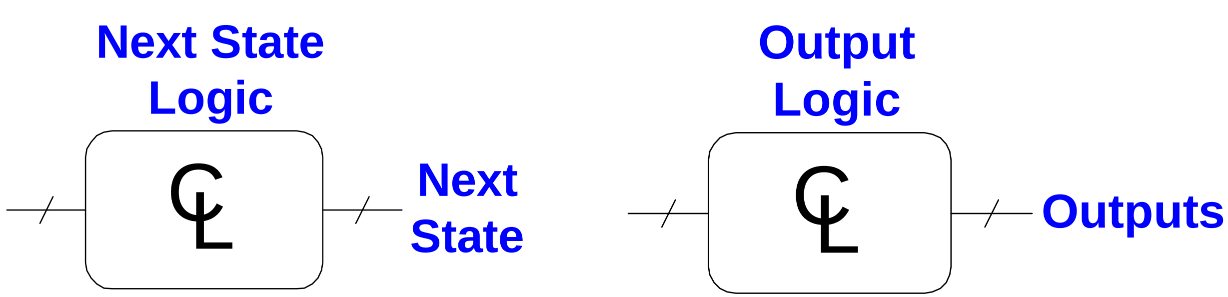
Synchronous Sequential Logic Design

- Breaks cyclic paths by **inserting registers**
- Registers contain **state** of the system
- State changes at clock edge: system **synchronized** to the clock
- **Rules** of synchronous sequential circuit composition:
  - Every circuit element is either a register or a combinational circuit
  - At least one circuit element is a register
  - All registers receive the same clock signal
  - Every cyclic path contains at least one register
- Two common synchronous sequential circuits
  - Finite State Machines (FSMs)
  - Pipelines

Finite State Machine (FSM)

- Consists of:
  - **State register**
    - Stores current state
    - Loads next state at clock edge
  - **Combinational logic**
    - Computes the next state
    - Computes the outputs

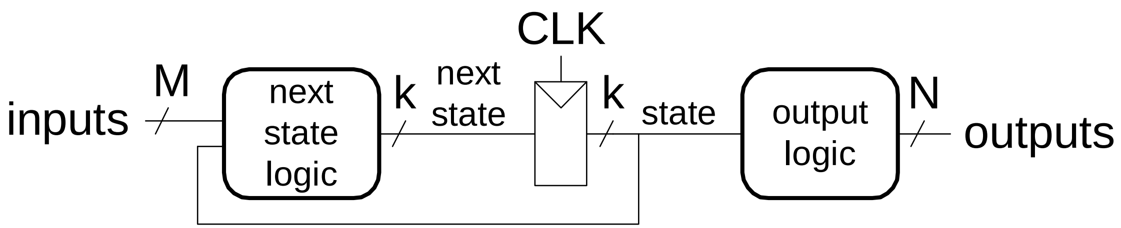




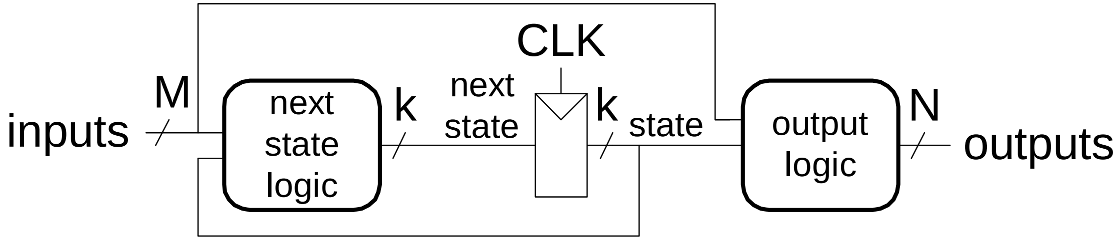
Finite State Machines

- Next state determined by current state and inputs
- Two types of finite state machines differ in output logic:
  - **Moore FSM:** outputs depend only on current state
  - **Mealy FSM:** outputs depend on current state *and* inputs

Moore FSM

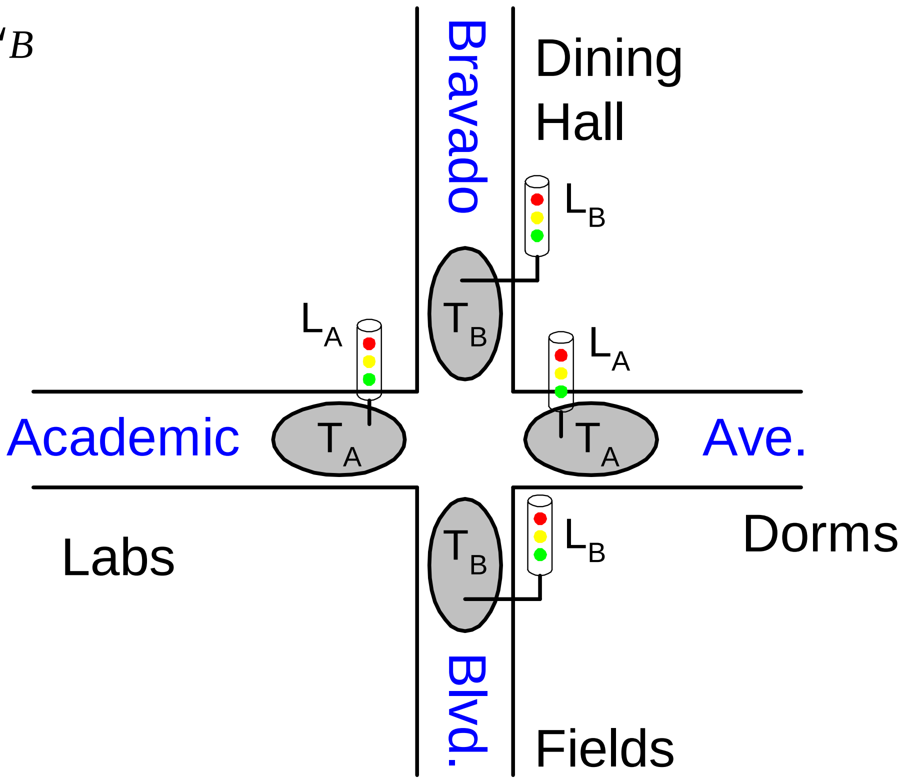


Mealy FSM



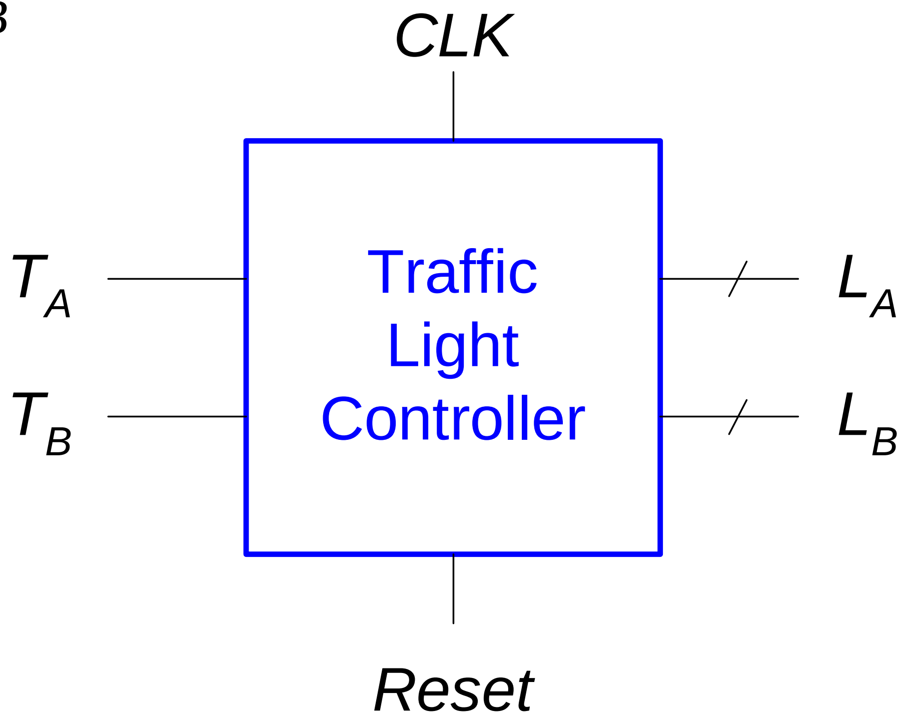
FSM Example

- Traffic light controller
  - Traffic sensors: T_A, T_B (TRUE when there's traffic)
  - Lights: L_A, L_B



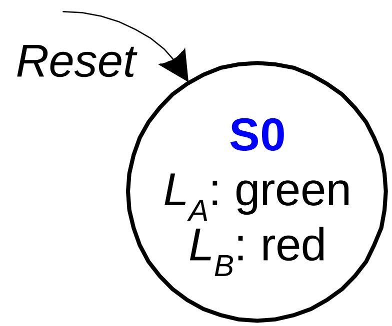
FSM Black Box

- Inputs: CLK, $Reset$, T_A, T_B
- Outputs: L_A, L_B



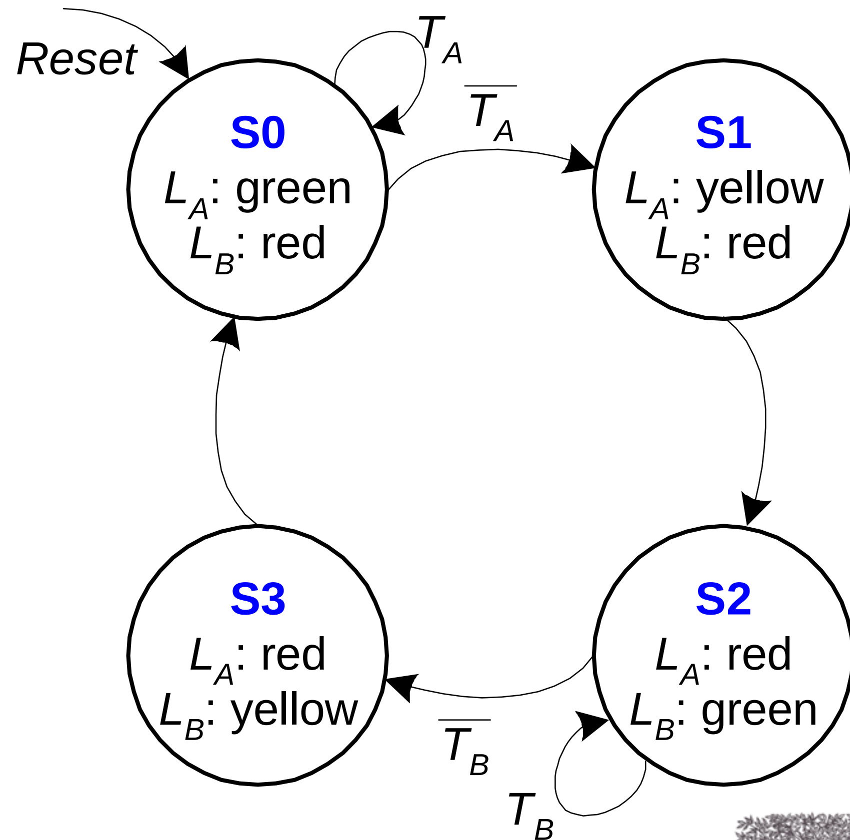
FSM State Transition

- **Moore FSM:** outputs labeled in each state
- **States:** Circles
- **Transitions:** Arcs



FSM State Transition

- **Moore FSM:** outputs labeled in each state
- **States:** Circles
- **Transitions:** Arcs



FSM State Transition Table

Current State S	Inputs T_A	T_B	Next State S'
S0	0	X	
S0	1	X	
S1	X	X	
S2	X	0	
S2	X	1	
S3	X	X	

FSM State Transition Table

Current State S	Inputs T_A	T_B	Next State S'
S0	0	X	S1
S0	1	X	S0
S1	X	X	S2
S2	X	0	S3
S2	X	1	S2
S3	X	X	S0

FSM Encoded State Transition Table

Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X		
0	0	1	X		
0	1	X	X		
1	0	X	0		
1	0	X	1		
1	1	X	X		

State	Encoding
S0	00
S1	01
S2	10
S3	11

FSM Encoded State

Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

State	Encoding
S0	00
S1	01
S2	10
S3	11

$$S'_1 = S_1 \oplus S_0$$

$$S'_0 = \overline{S_1}\,\overline{S_0}\,\overline{T_A} + S_1\overline{S_0}\,\overline{T_B}$$

ELSEVIER

FSM Output Table

Current State		Outputs			
S_1	S_0	L_{A1}	L_{A0}	L_{B1}	L_{B0}
0	0				
0	1				
1	0				
1	1				

Output	Encoding
green	00
yellow	01
red	10

FSM Output Table

Current State		Outputs			
S_1	S_0	L_{A1}	L_{A0}	L_{B1}	L_{B0}
0	0	0	0	1	0
0	1	0	1	1	0
1	0	1	0	0	0
1	1	1	0	0	1

Output	Encoding
green	00
yellow	01
red	10

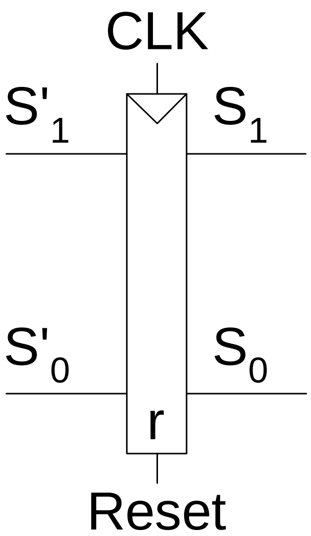
$$L_{A1} = S_1$$

$$L_{A0} = \overline{S_1} S_0$$

$$L_{B1} = S_1$$

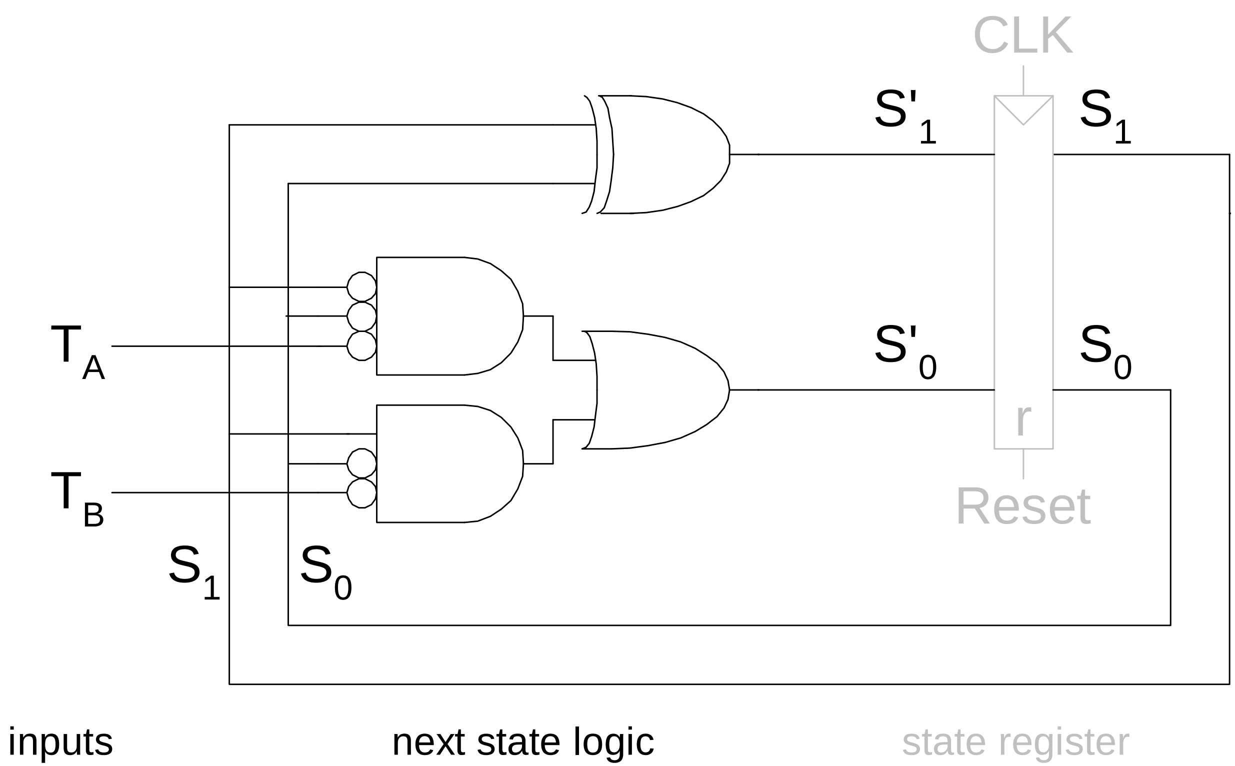
$$L_{B0} = S_1 S_0$$

FSM Schematic: State Register

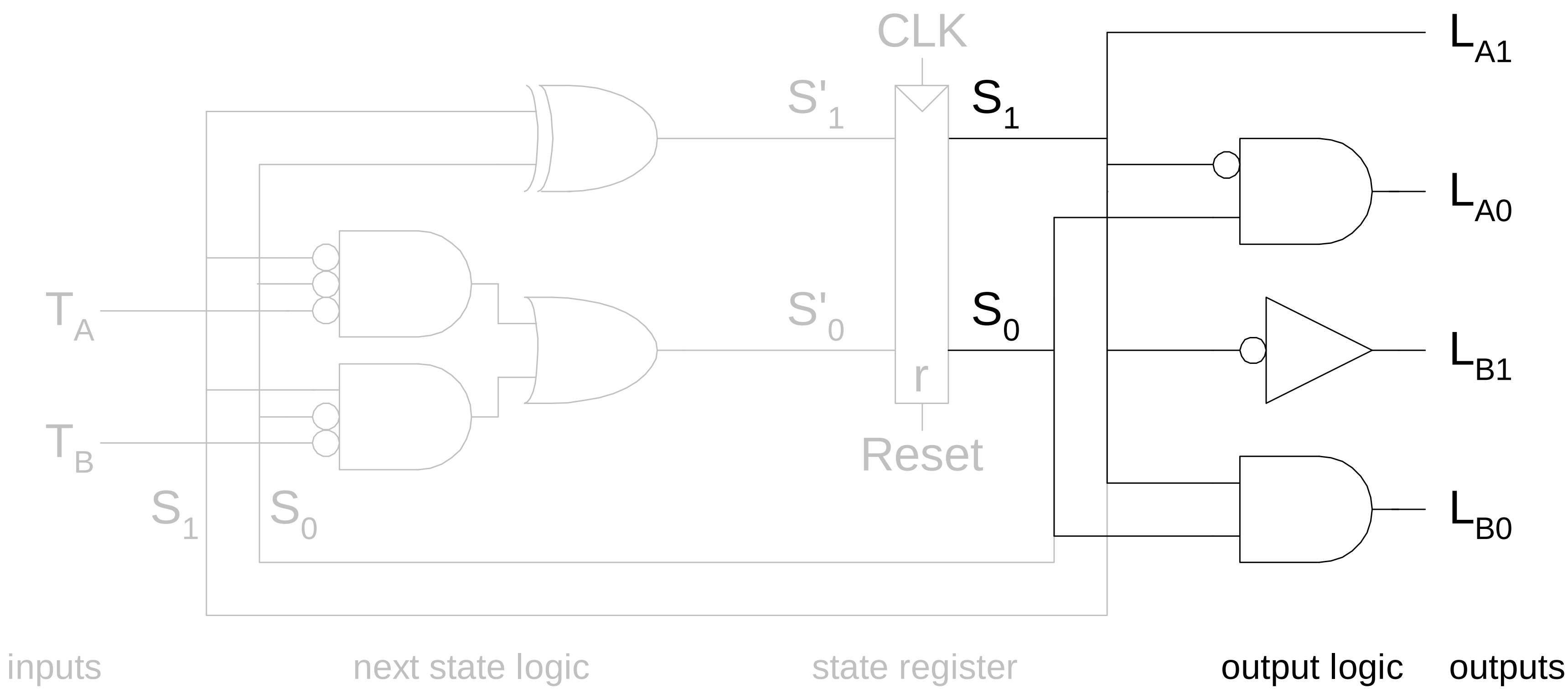


state register

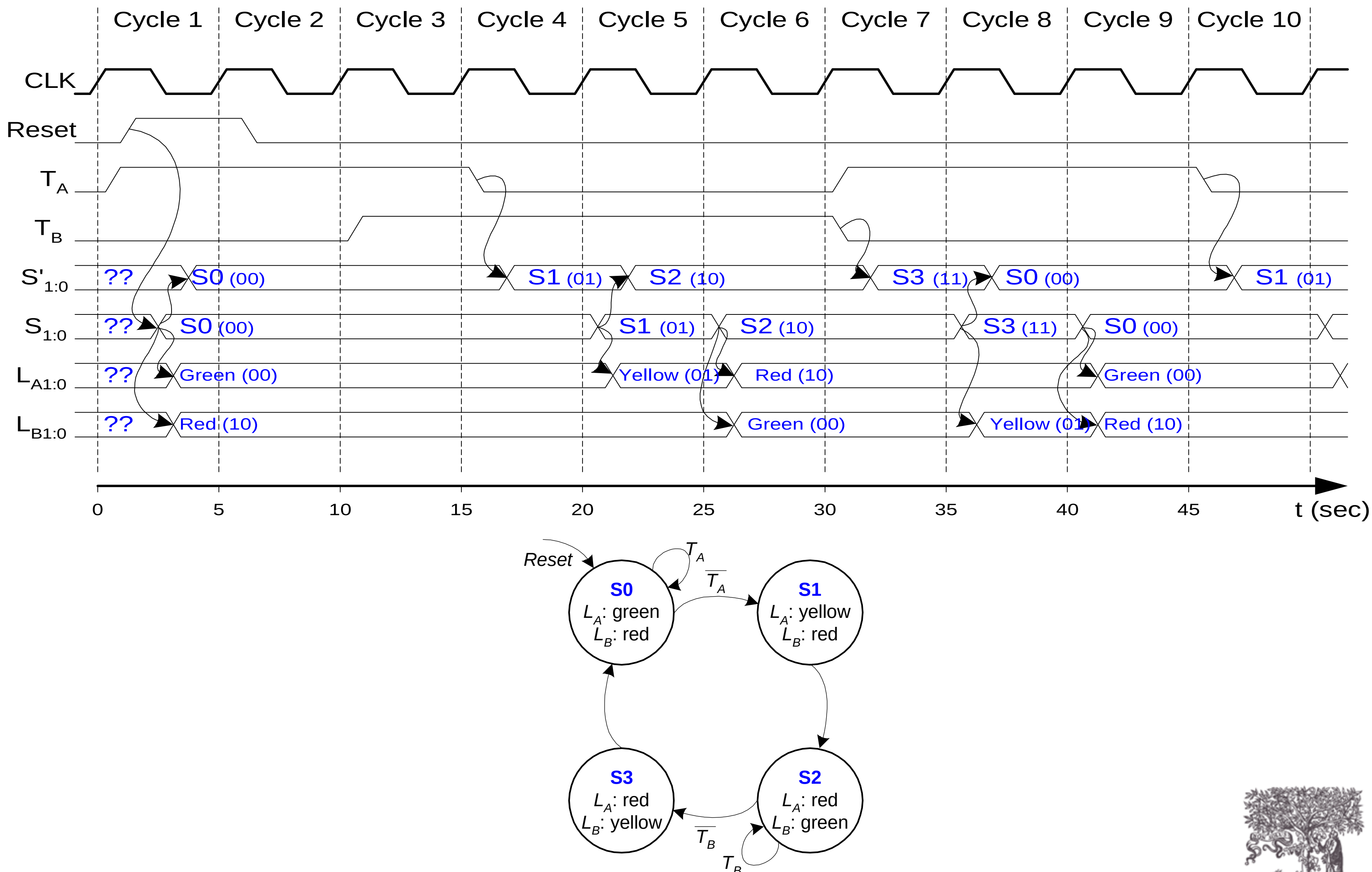
FSM Schematic: Next State



FSM Schematic: Output



FSM Timing Diagram



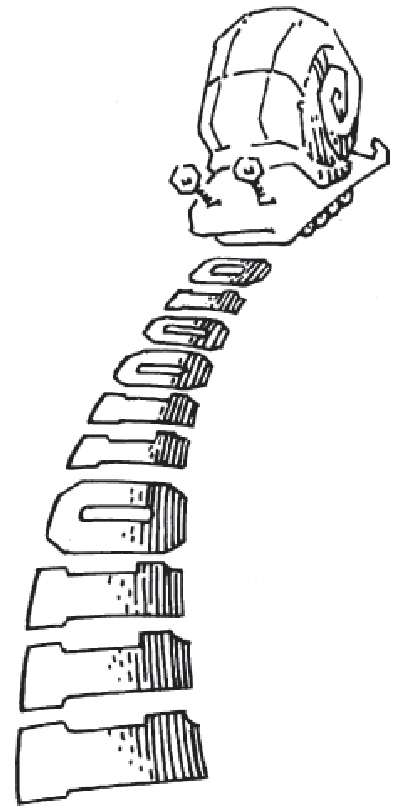
ELSEVIER

FSM State Encoding

- **Binary** encoding:
  - i.e., for four states, 00, 01, 10, 11
- **One-hot** encoding
  - One state bit per state
  - Only one state bit HIGH at once
  - i.e., for 4 states, 0001, 0010, 0100, 1000
  - Requires more flip-flops
  - Often next state and output logic is simpler

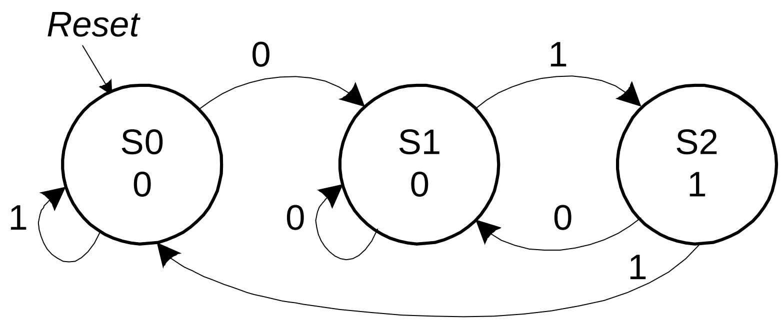
Moore vs. Mealy FSM

- Alyssa P. Hacker has a snail that crawls down a paper tape with 1's and 0's on it. The snail smiles whenever the last two digits it has crawled over are 01. Design Moore and Mealy FSMs of the snail's brain.

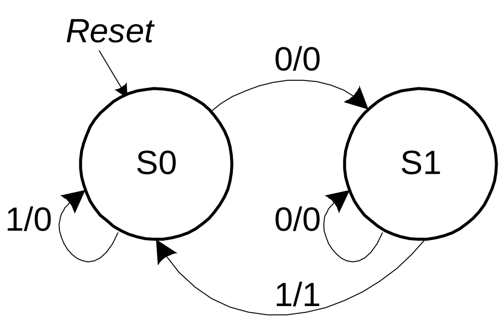


State Transition Diagrams

Moore FSM



Mealy FSM



Mealy FSM: arcs indicate input/output

Moore FSM State Transition Table

Current State		Inputs	Next State	
S_1	S_0	A	S'_1	S'_0
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		

State	Encoding
S0	00
S1	01
S2	10

ELSEVIER

Moore FSM State Transition

Current State		Inputs	Next State	
S_1	S_0	A	S'_1	S'_0
0	0	0	0	1
0	0	1	0	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0

State	Encoding
S0	00
S1	01
S2	10

$$S_1' = S_0 A$$

$$S_0' = \overline{A}$$

Moore FSM Output Table

Current State		Output
S_1	S_0	Y
0	0	
0	1	
1	0	

Moore FSM Output Table

Current State		Output
S_1	S_0	Y
0	0	0
0	1	0
1	0	1

$$Y = S_1$$

Mealy FSM State Transition & Output Table

Current State	Input	Next State	Output
S_0	A	S'_0	Y
0	0		
0	1		
1	0		
1	1		

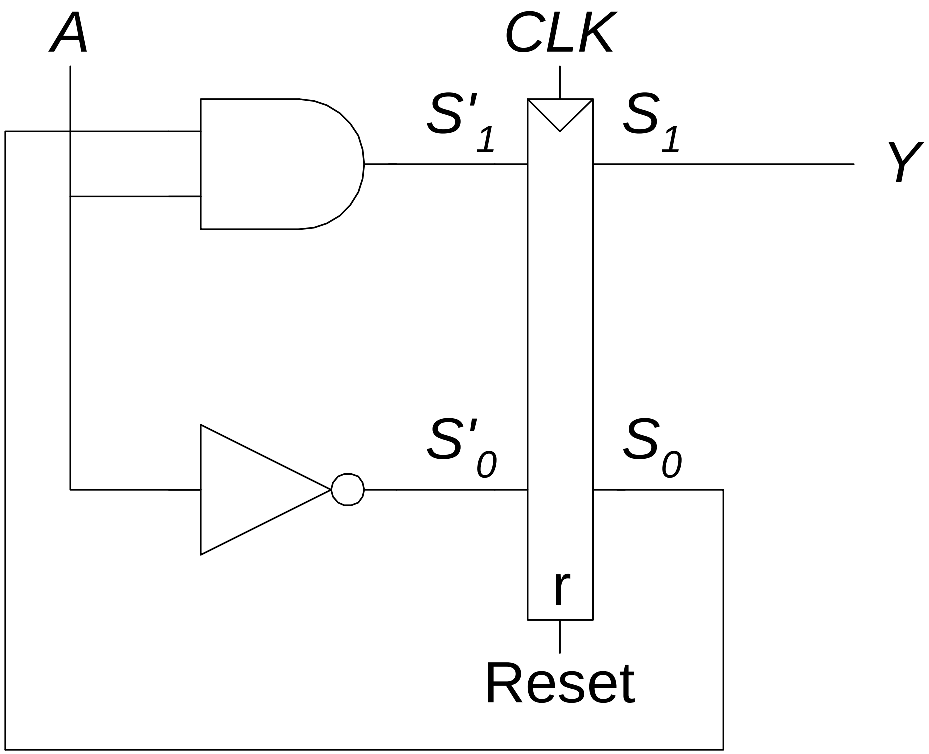
State	Encoding
S0	00
S1	01

Mealy FSM State Transition & Output Table

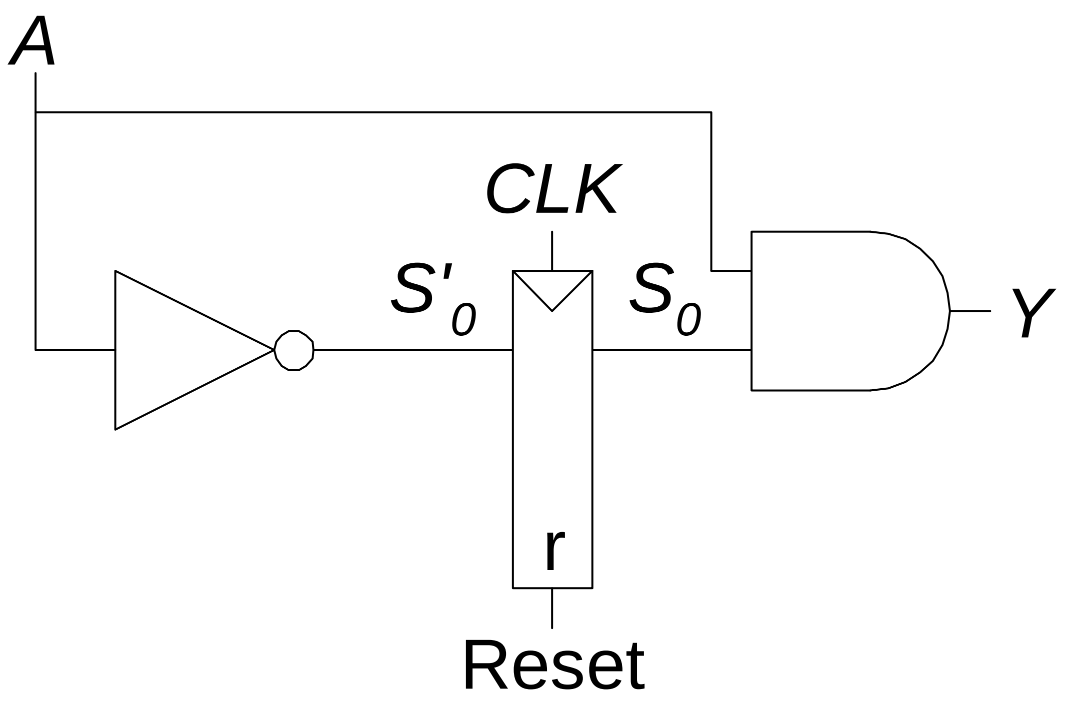
Current State	Input	Next State	Output
S_0	A	S'_0	Y
0	0	1	0
0	1	0	0
1	0	1	0
1	1	0	1

State	Encoding
S0	00
S1	01

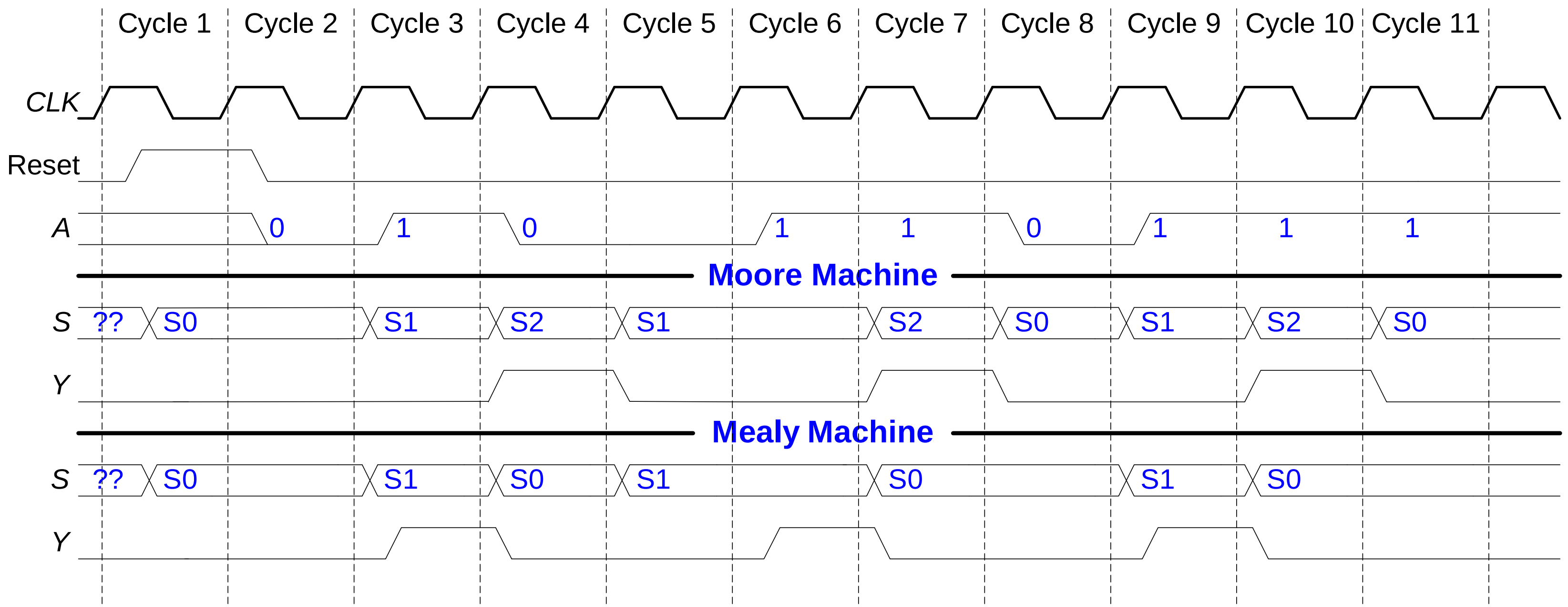
Moore FSM Schematic



Mealy FSM Schematic



Moore & Mealy Timing



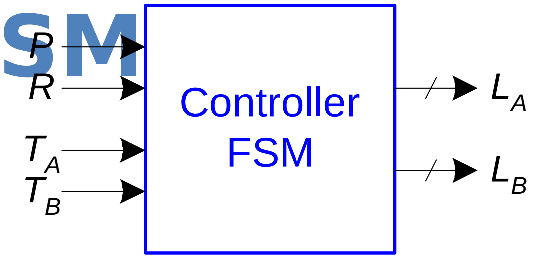
ENOUGH FOR TODAY!

Factoring State Machines

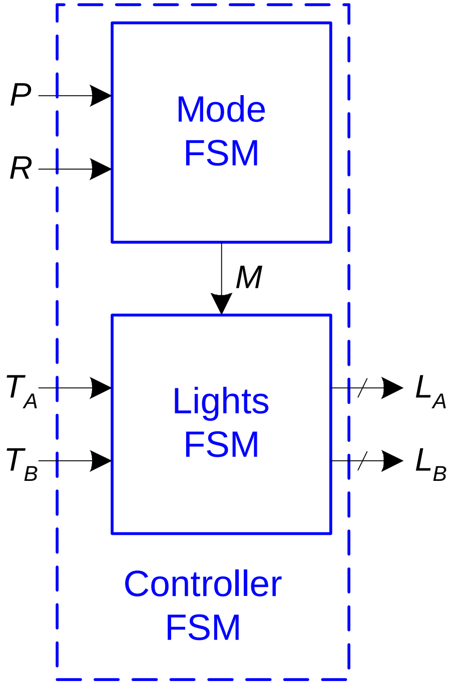
- Break complex FSMs into smaller interacting FSMs
- Example: Modify traffic light controller to have Parade Mode.
  - Two more inputs: *P*, *R*
  - When ***P* = 1**, enter Parade Mode & Bravado Blvd light stays green
  - When ***R* = 1**, leave Parade Mode

Parade FSM

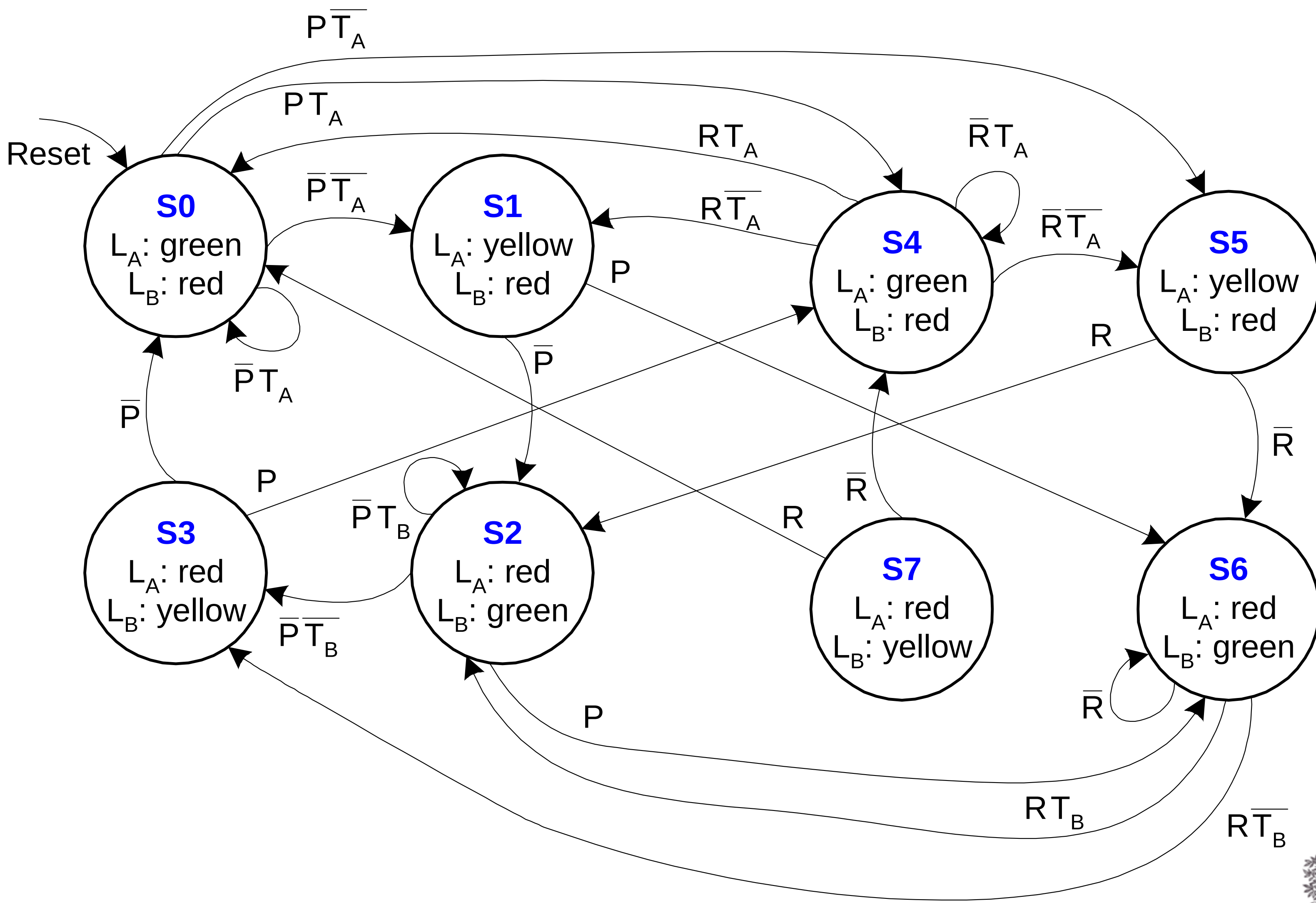
Unfactored FSM



Factored FSM

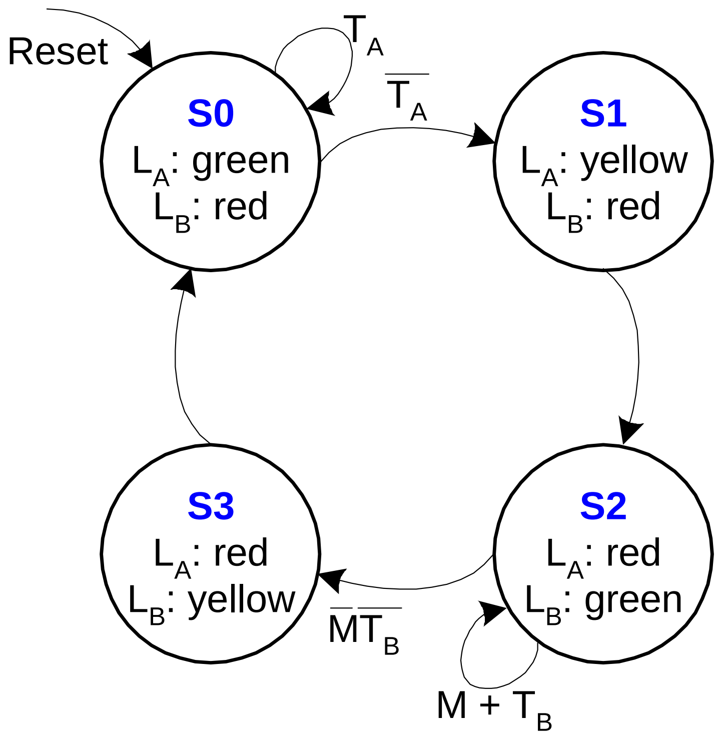


Unfactored FSM

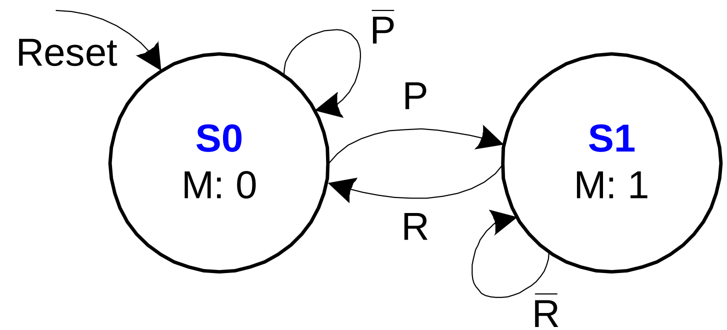


ELSEVIER

Factored FSM



Lights FSM



Mode FSM

FSM Design Procedure

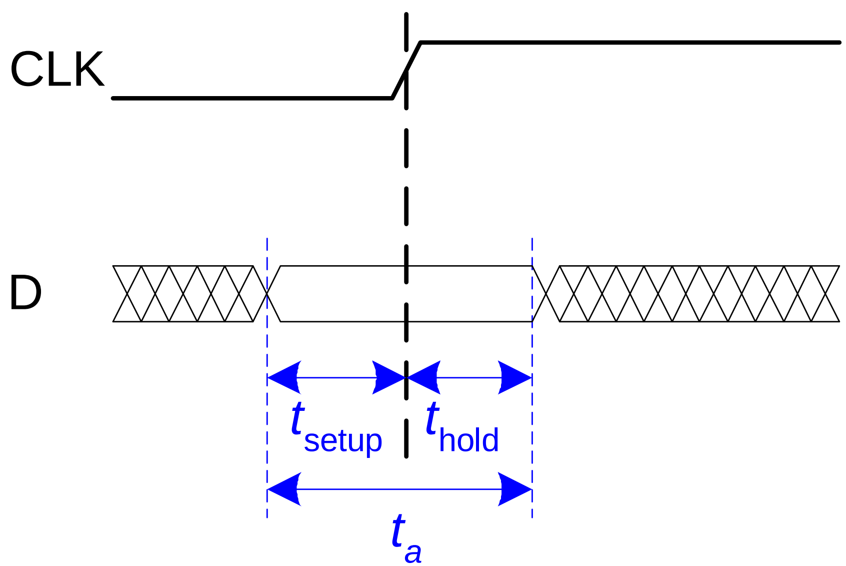
1. Identify inputs and outputs
2. Sketch state transition diagram
3. Write state transition table
4. Select state encodings
5. For Moore machine:
    1. Rewrite state transition table with state encodings
    2. Write output table
6. For a Mealy machine:
    1. Rewrite combined state transition and output table with state encodings
7. Write Boolean equations for next state and output logic
8. Sketch the circuit schematic

Timing

- Flip-flop samples *D* at clock edge
- *D* must be stable when sampled
- Similar to a photograph, *D* must be stable around clock edge
- If not, metastability can occur

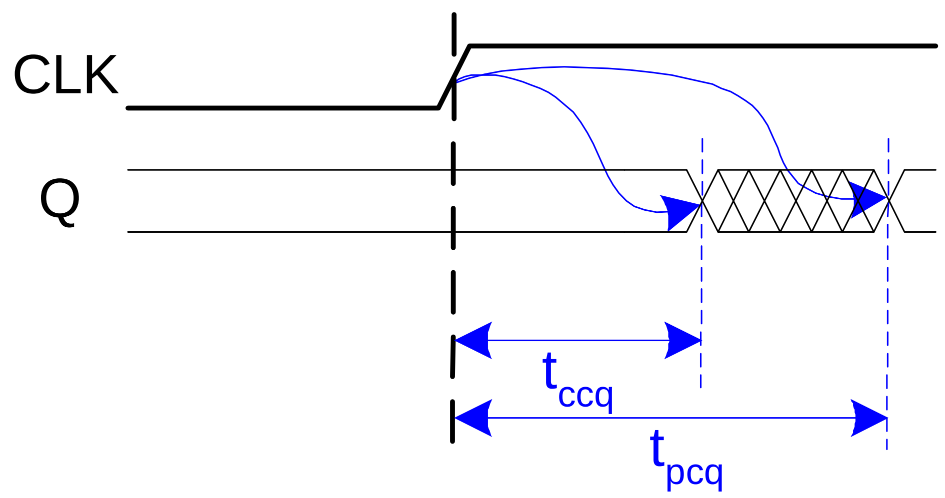
Input Timing Constraints

- **Setup time:** t_{setup} = time *before* clock edge data must be stable (i.e. not changing)
- **Hold time:** t_{hold} = time *after* clock edge data must be stable
- **Aperture time:** t_a = time *around* clock edge data must be stable ($t_a = t_{setup} + t_{hold}$)



Output Timing Constraints

- **Propagation delay:** t_{pcq} = time after clock edge that the output Q is guaranteed to be stable (i.e., to stop changing)
- **Contamination delay:** t_{ccq} = time after clock edge that Q might be unstable (i.e., start changing)

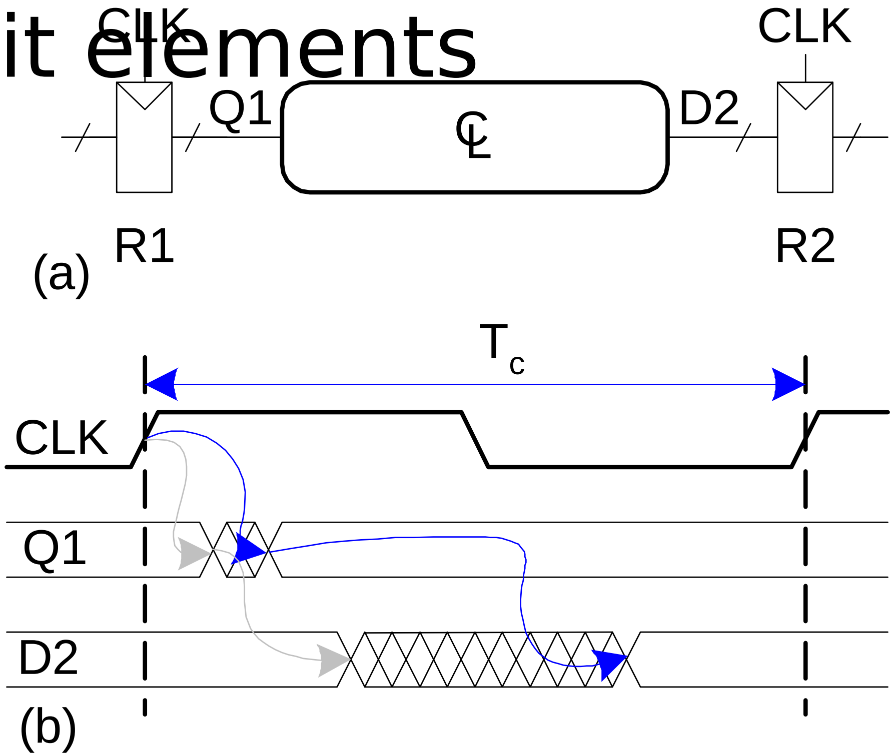


Dynamic Discipline

- Synchronous sequential circuit inputs must be stable during aperture (setup and hold) time around clock edge
- Specifically, inputs must be stable
  - at least t_{setup} before the clock edge
  - at least until t_{hold} after the clock edge

Dynamic Discipline

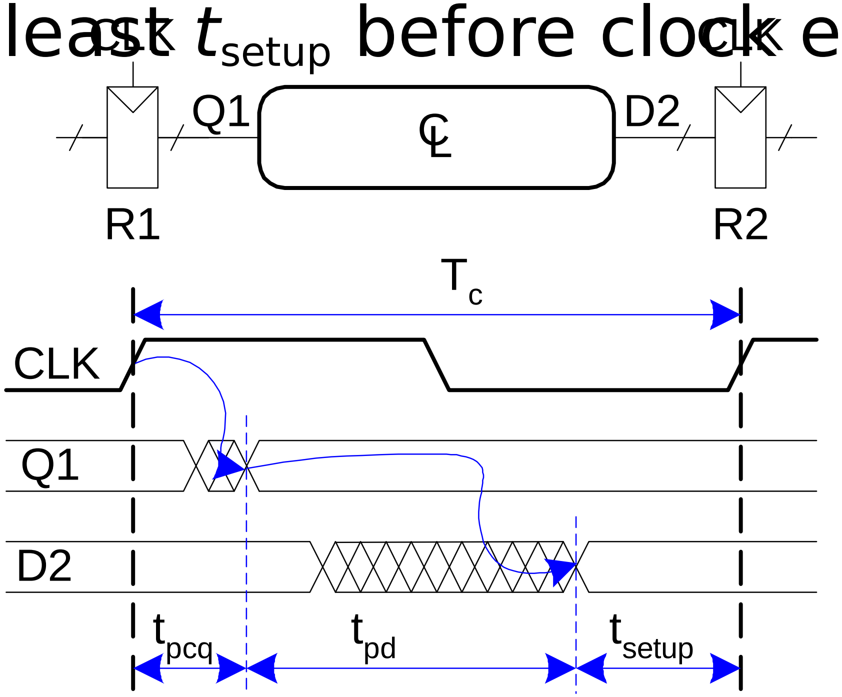
- The delay between registers has a **minimum** and **maximum** delay, dependent on the delays of the circuit elements



ELSEVIER

Setup Time Constraint

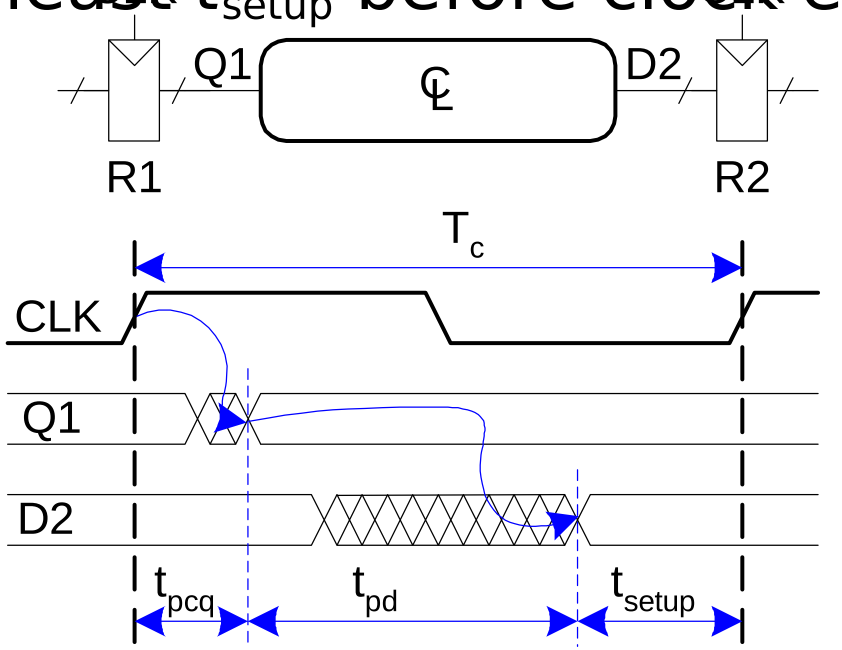
- Depends on the **maximum** delay from register R1 through combinational logic to R2
- The input to register R2 must be stable at least t_{setup} before clock edge



$T_c \geq$

Setup Time Constraint

- Depends on the **maximum** delay from register R1 through combinational logic to R2
- The input to register R2 must be stable at least t_{setup} before clock edge



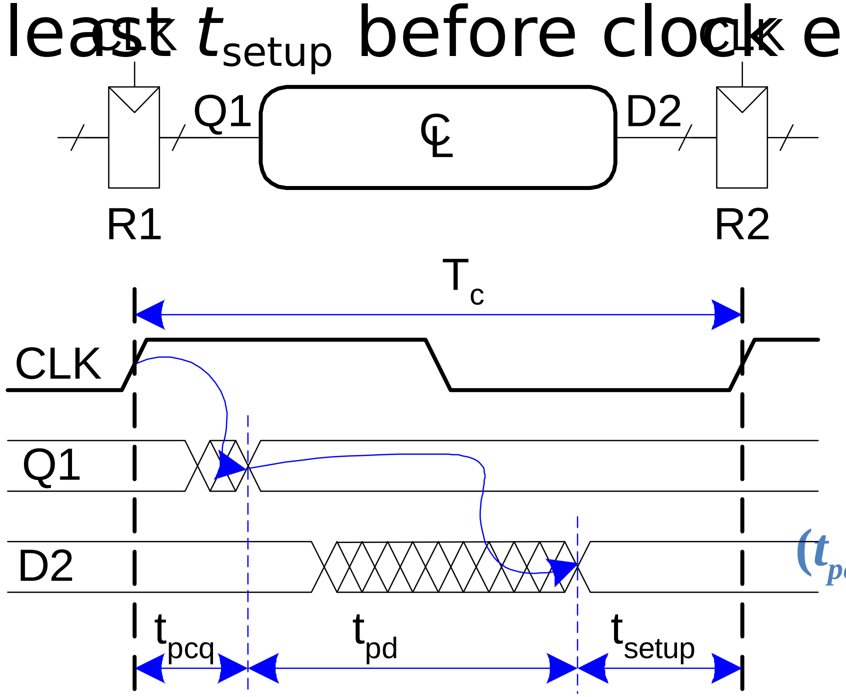
$$T_c \geq t_{pcq} + t_{pd} + t_{setup}$$

$$t_{pd} \leq$$

ELSEVIER

Setup Time Constraint

- Depends on the **maximum** delay from register R1 through combinational logic to R2
- The input to register R2 must be stable at least t_{setup} before clock edge

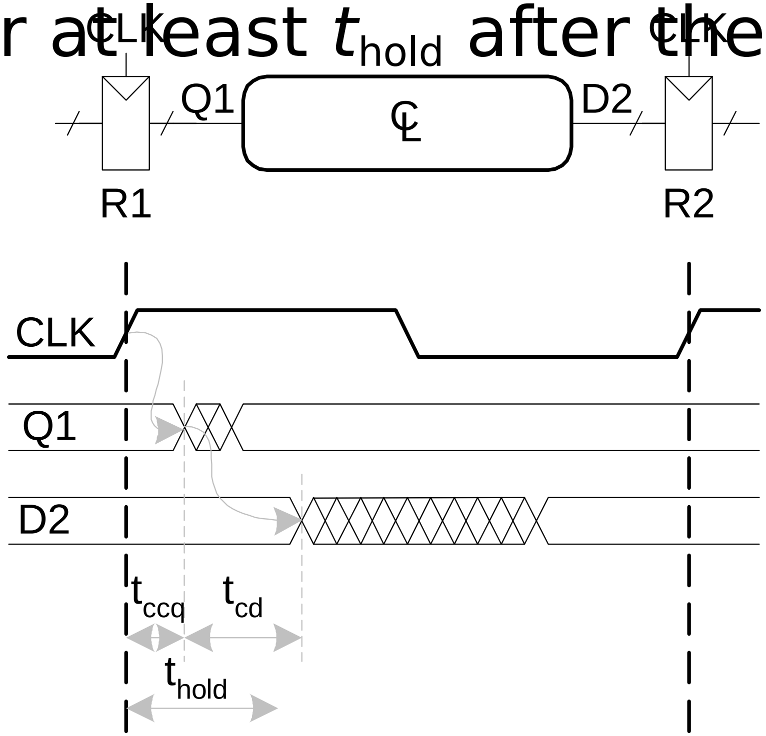


$$T_c \geq t_{pcq} + t_{pd} + t_{setup}$$
$$t_{pd} \leq T_c - (t_{pcq} + t_{setup})$$

$(t_{pcq} + t_{setup})$: *sequencing overhead*

Hold Time Constraint

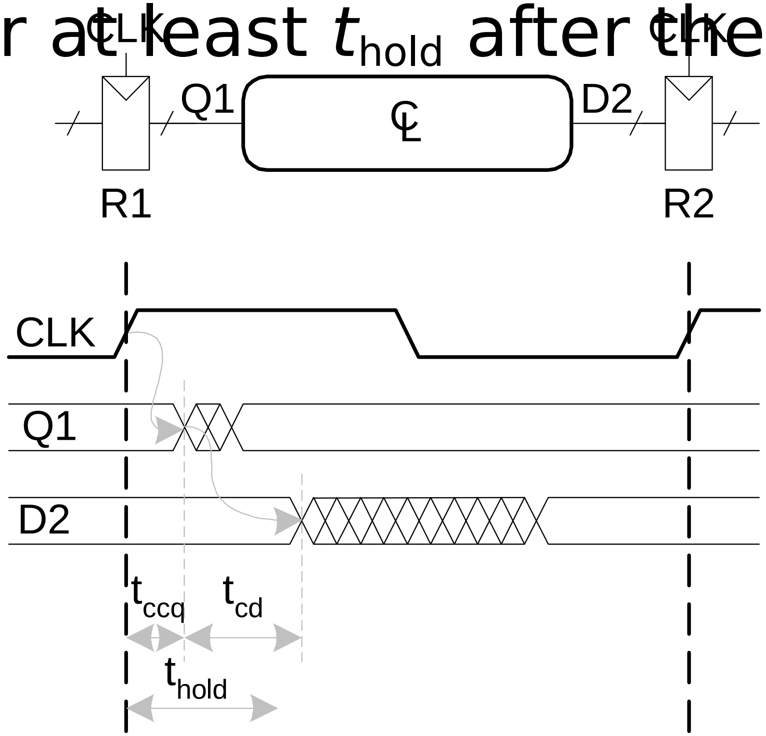
- Depends on the **minimum** delay from register R1 through the combinational logic to R2
- The input to register R2 must be stable for at least t_{hold} after the clock edge



$t_{hold} <$

Hold Time Constraint

- Depends on the **minimum** delay from register R1 through the combinational logic to R2
- The input to register R2 must be stable for at least t_{hold} after the clock edge

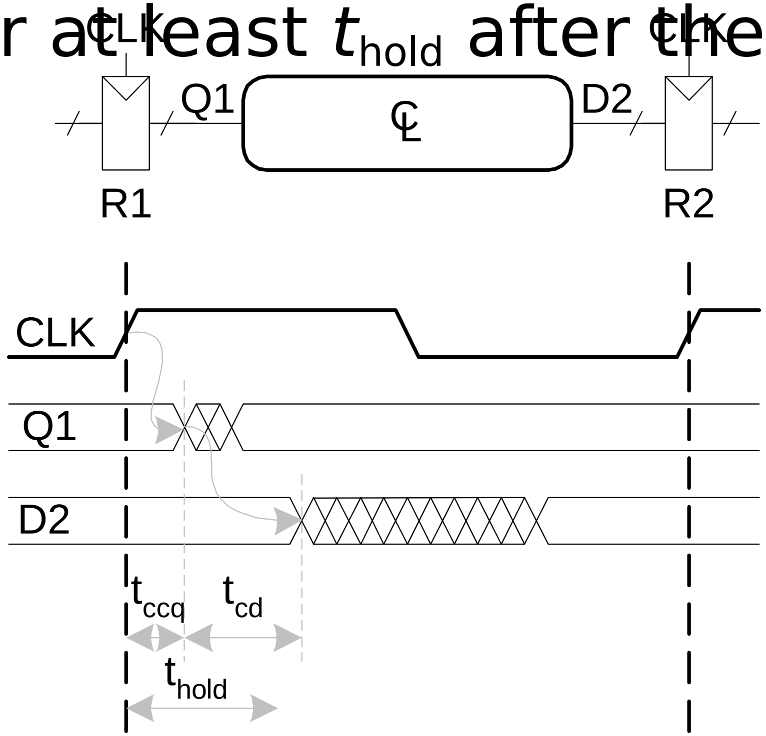


$$t_{hold} < t_{ccq} + t_{cd}$$

$$t_{cd} >$$

Hold Time Constraint

- Depends on the **minimum** delay from register R1 through the combinational logic to R2
- The input to register R2 must be stable for at least t_{hold} after the clock edge

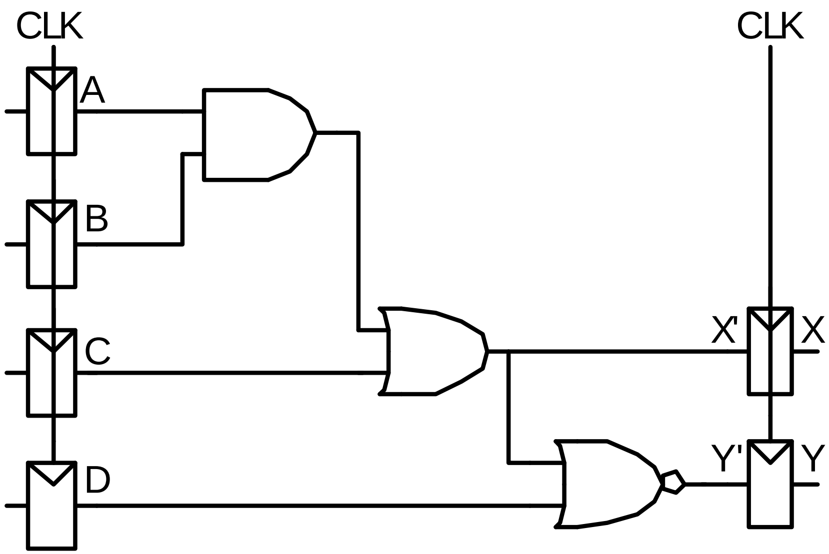


$$t_{hold} < t_{ccq} + t_{cd}$$

$$t_{cd} > t_{hold} - t_{ccq}$$

ELSEVIER

Timing Analysis



Timing Characteristics

t_{ccq} = 30 ps

t_{pcq} = 50 ps

t_{setup} = 60 ps

t_{hold} = 70 ps

per gate

t_{pd} = 35 ps

t_{cd} = 25 ps

t_{pd} =

t_{cd} =

Setup time constraint:

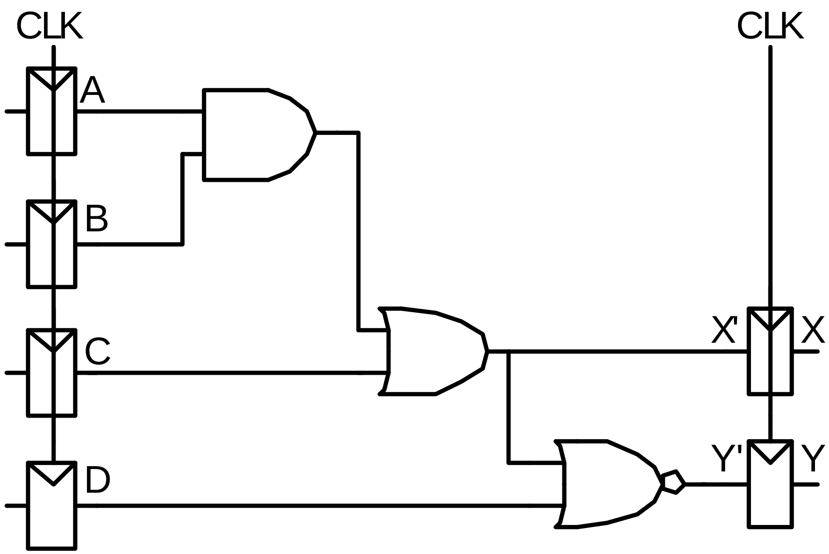
$T_c \geq$

f_c =

Hold time constraint:

$t_{ccq} + t_{cd} > t_{hold}$?

Timing Analysis



Timing Characteristics

t_{ccq} = 30 ps

t_{pcq} = 50 ps

t_{setup} = 60 ps

t_{hold} = 70 ps

per gate

t_{pd} = 35 ps

t_{cd} = 25 ps

t_{pd} = 3 x 35 ps = 105 ps

t_{cd} = 25 ps

Setup time constraint:

$T_c \geq (50 + 105 + 60)$ ps = 215 ps

$f_c = 1/T_c$ = 4.65 GHz

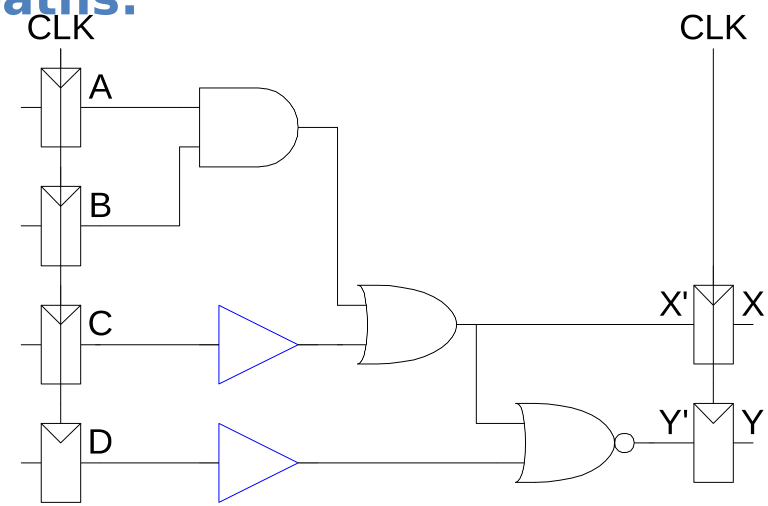
Hold time constraint:

$t_{ccq} + t_{cd} > t_{hold}$?

(30 + 25) ps > 70 ps ? **No!**

Timing Analysis

Add buffers to the short paths:



$t_{pd} =$

$t_{cd} =$

Setup time constraint:

$T_c \geq$

$f_c =$

Timing Characteristics

t_{ccq} = 30 ps

t_{pcq} = 50 ps

t_{setup} = 60 ps

t_{hold} = 70 ps

per gate

t_{pd} = 35 ps

t_{cd} = 25 ps

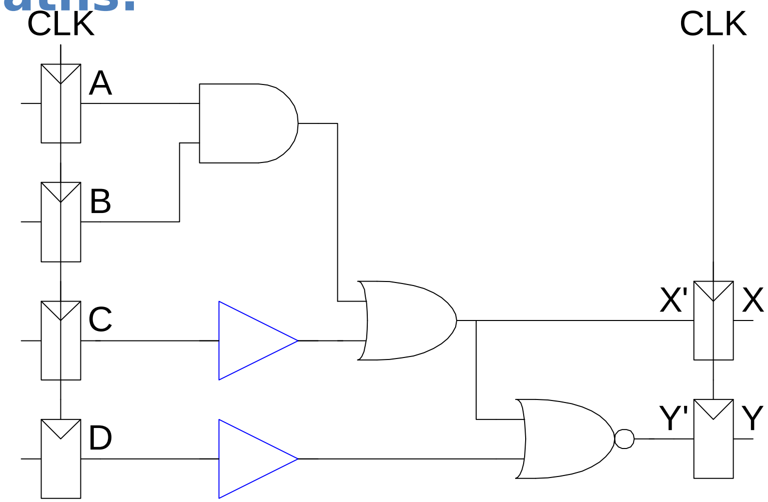
Hold time constraint:

$t_{ccq} + t_{cd} > t_{hold}$?

ELSEVIER

Timing Analysis

Add buffers to the short paths:



t_{pd} = 3 x 35 ps = 105 ps

t_{cd} = 2 x 25 ps = 50 ps

Timing Characteristics

t_{ccq} = 30 ps

t_{pcq} = 50 ps

t_{setup} = 60 ps

t_{hold} = 70 ps

per gate

t_{pd} = 35 ps

t_{cd} = 25 ps

Setup time constraint:

$T_c \geq$ (50 + 105 + 60) ps = 215 ps

$f_c = 1/T_c$ = 4.65 GHz

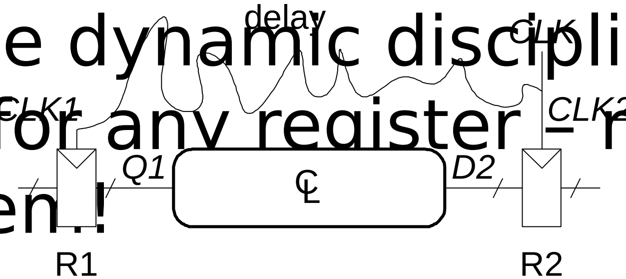
Hold time constraint:

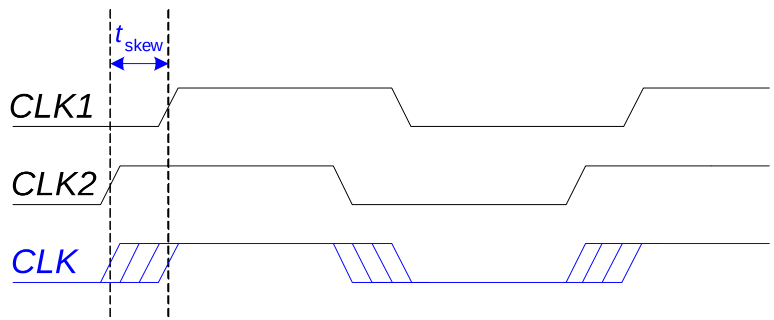
$t_{ccq} + t_{cd} > t_{hold}$?

(30 + 50) ps > 70 ps ? **Yes!**

Clock Skew

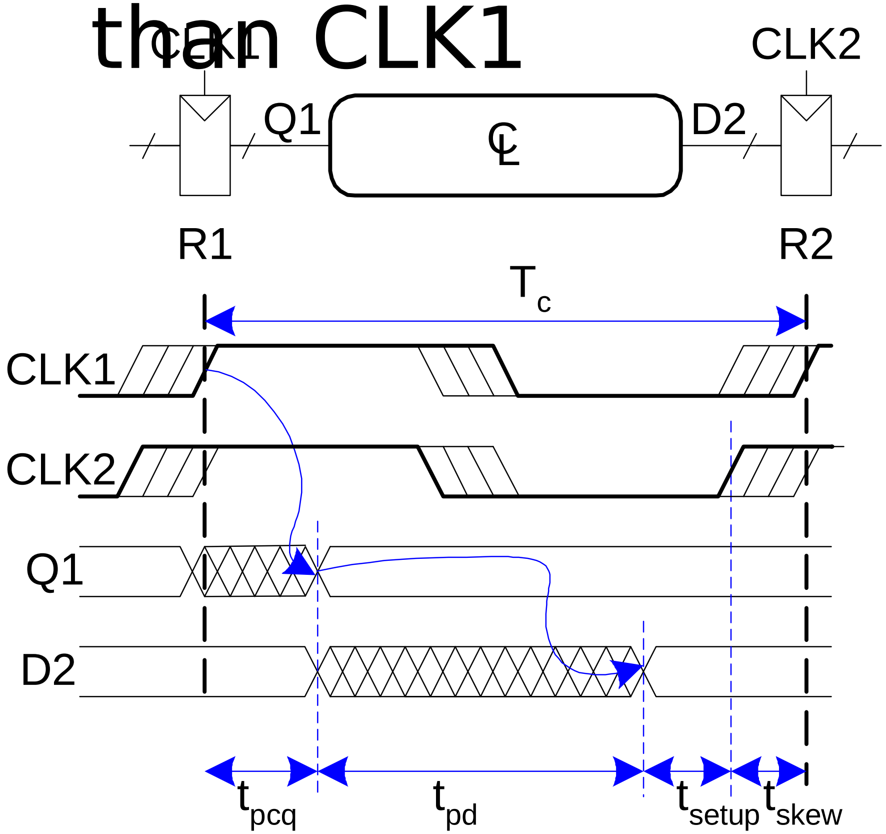
- The clock doesn’t arrive at all registers at same time
- **Skew:** difference between two clock edges
- Perform **worst case analysis** to guarantee dynamic discipline is not violated for any register – many registers in a system!





Setup Time Constraint with Skew

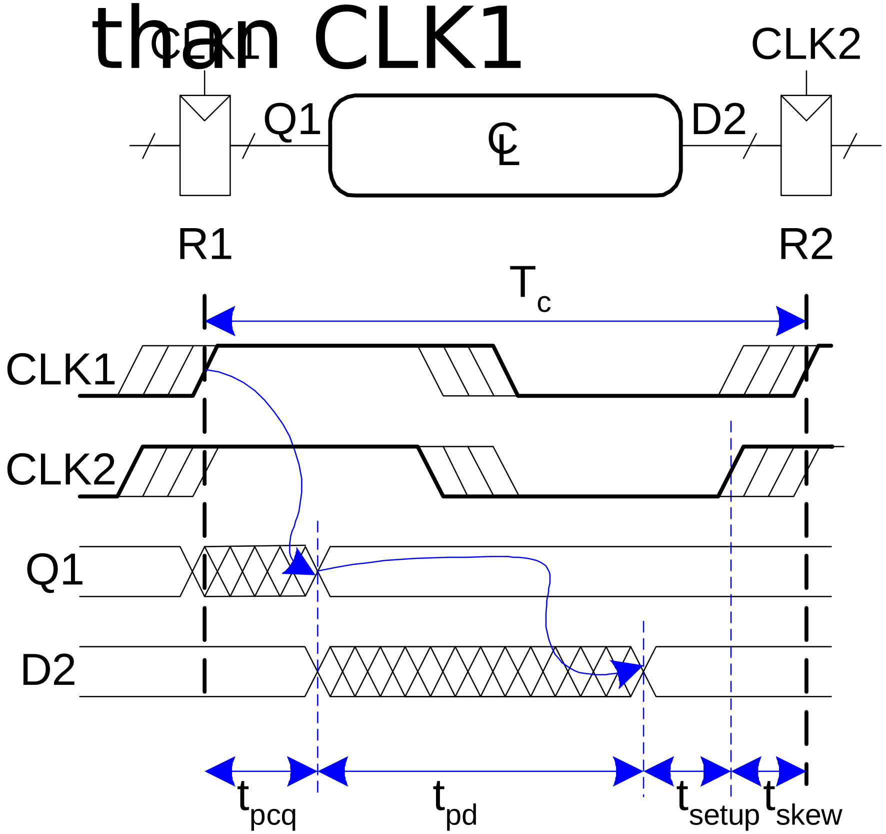
- In the worst case, CLK2 is earlier than CLK1



$T_c \geq$

Setup Time Constraint with

- In the worst case, CLK2 is earlier than CLK1

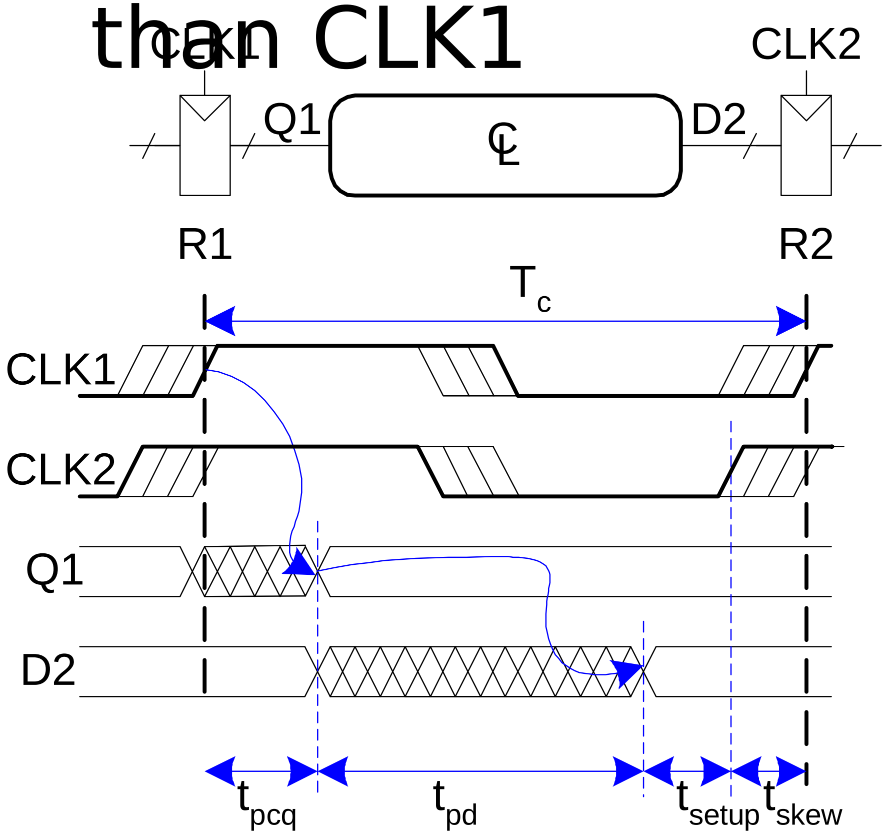


$$T_c \geq t_{pcq} + t_{pd} + t_{setup} + t_{skew}$$

$$t_{pd} \leq$$

Setup Time Constraint with Skew

- In the worst case, CLK2 is earlier than CLK1

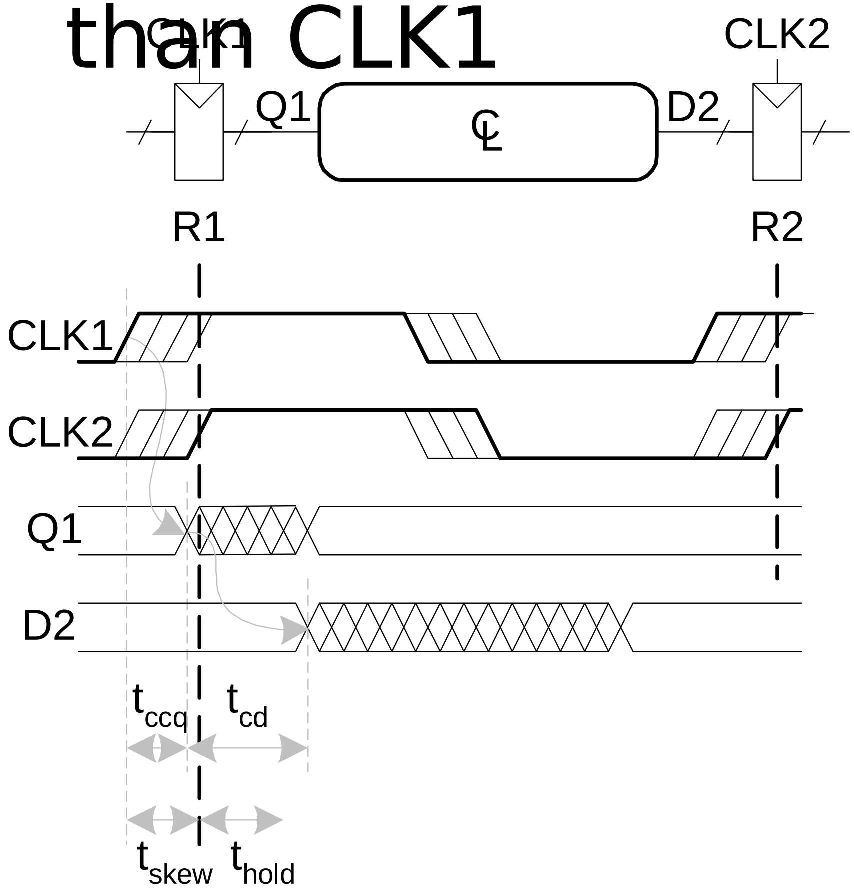


$$T_c \geq t_{pcq} + t_{pd} + t_{setup} + t_{skew}$$

$$t_{pd} \leq T_c - (t_{pcq} + t_{setup} + t_{skew})$$

Hold Time Constraint with Skew

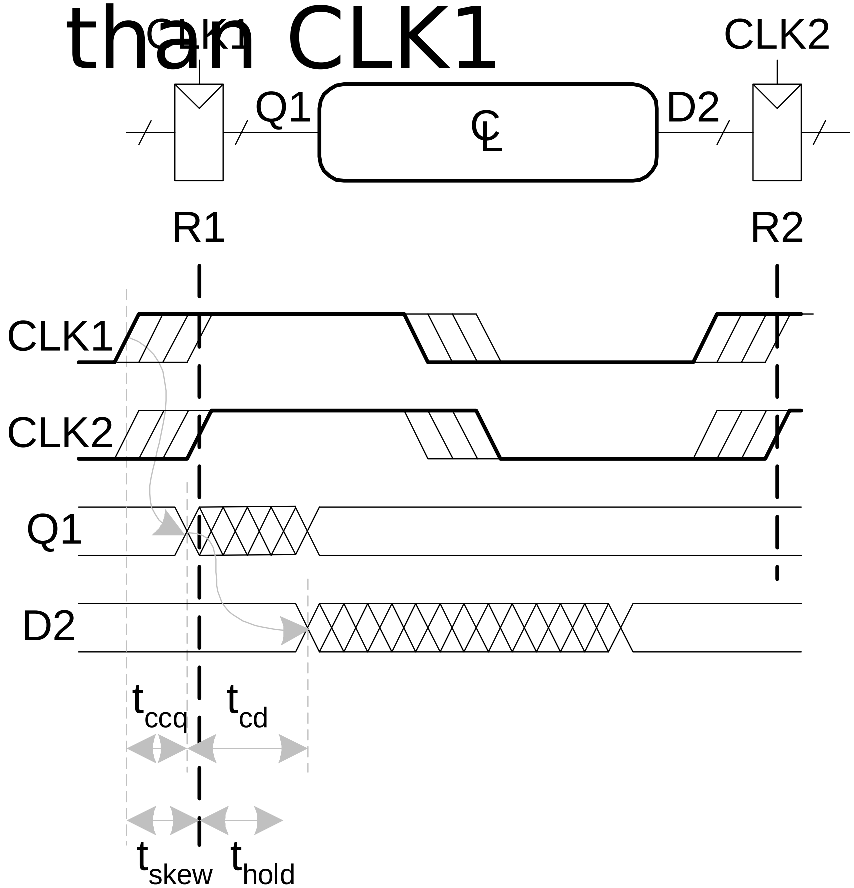
- In the worst case, CLK2 is later than CLK1



$t_{ccq} + t_{cd} >$

Hold Time Constraint with

- In the worst case, CLK2 is later than CLK1

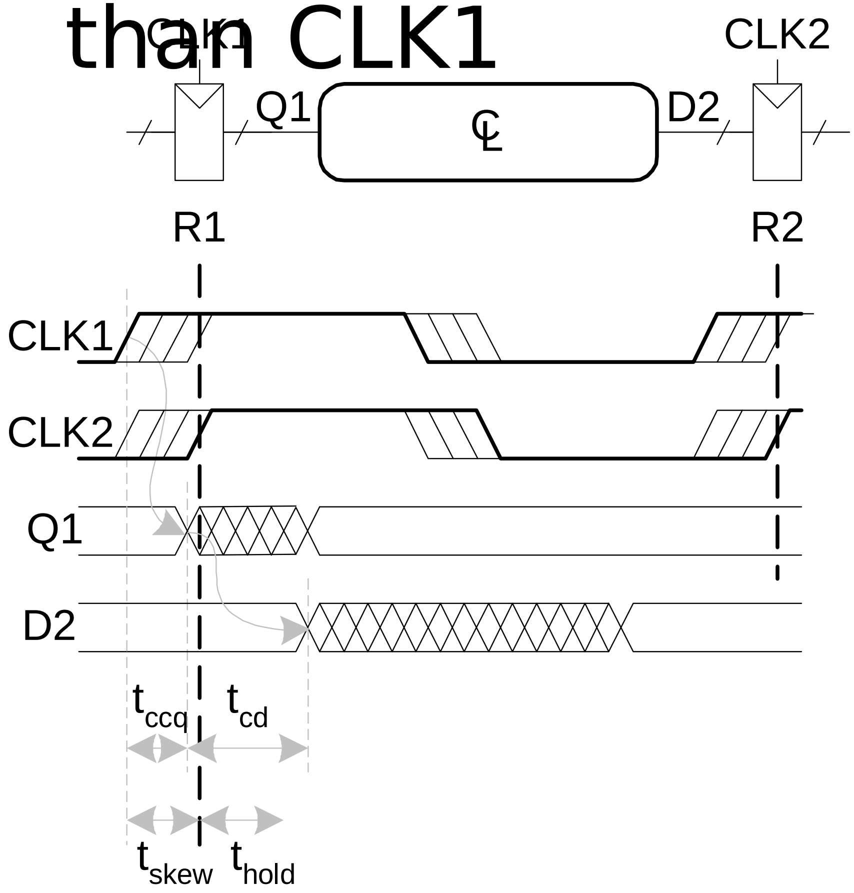


$$t_{ccq} + t_{cd} > t_{hold} + t_{skew}$$

$$t_{cd} >$$

Hold Time Constraint with Skew

- In the worst case, CLK2 is later than CLK1

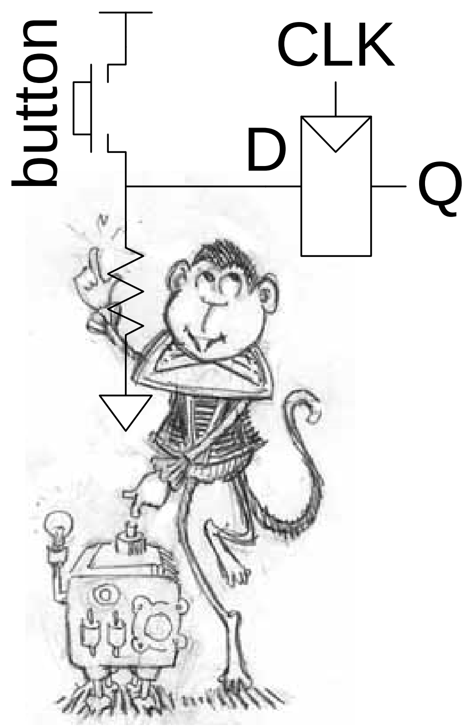


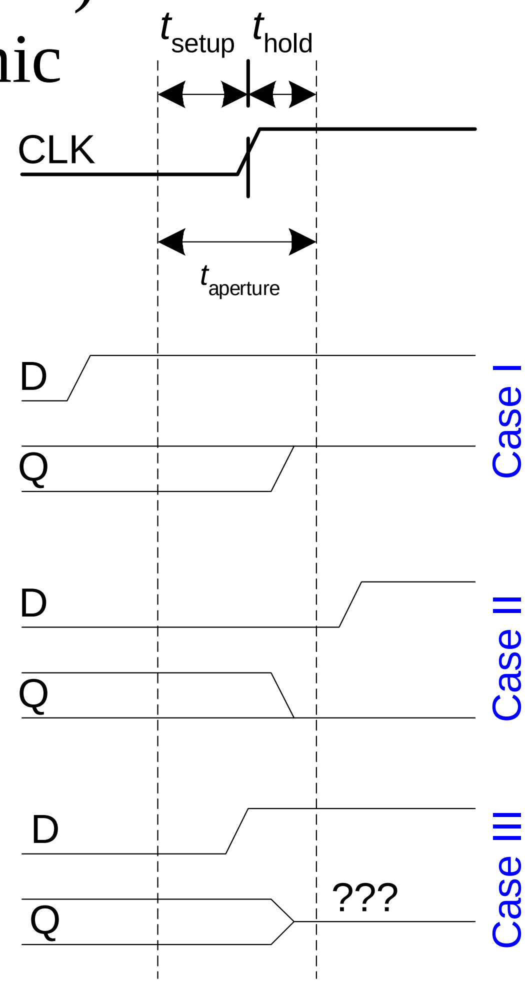
$$t_{ccq} + t_{cd} > t_{hold} + t_{skew}$$

$$t_{cd} > t_{hold} + t_{skew} - t_{ccq}$$

Violating the Dynamic

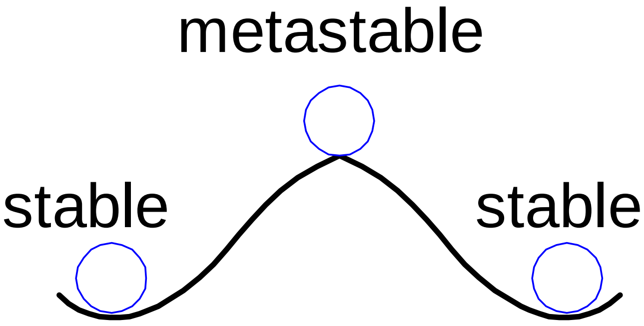
- Asynchronous (for example, user) inputs might violate the dynamic discipline





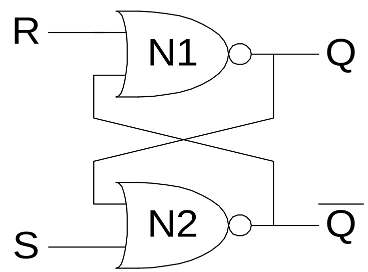
Metastability

- **Bistable devices:** two stable states, and a metastable state between them
- **Flip-flop:** two stable states (1 and 0) and one metastable state
- If flip-flop lands in metastable state, could stay there for an undetermined amount of time



Flip-Flop Internals

- Flip-flop has **feedback**: if *Q* is somewhere between 1 and 0, cross-coupled gates drive output to either rail (1 or 0)



- **Metastable signal:** if it hasn't resolved to 1 or 0
- If flip-flop input changes at random time, **probability that output *Q* is metastable** after waiting some time, *t*:

$$\mathbf{P}(t_{\text{res}} > t) = (T_0/T_c)\, e^{-t/\tau}$$

t_{res} : time to resolve to 1 or 0

T_0, τ : properties of the circuit

Metastability

- **Intuitively:**

  T_0/T_c: probability input changes at a bad time (during aperture)

  $$P(t_{res} > t) = (T_0/T_c)\, e^{-t/\tau}$$

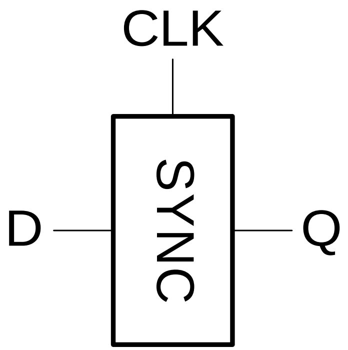
  τ: time constant for how fast flip-flop moves away from metastability

  $$P(t_{res} > t) = (T_0/T_c)\, e^{-t/\tau}$$

- In short, if flip-flop samples metastable input, if you wait long enough (t), the output will have resolved to 1 or 0 with high probability.

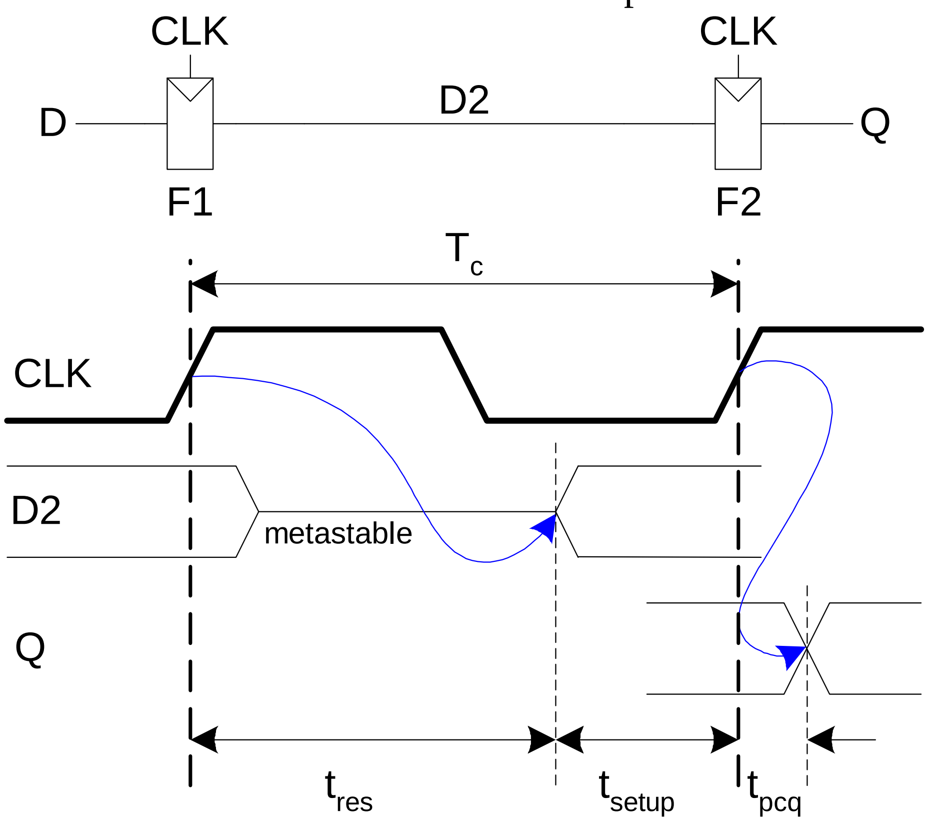
Synchronizers

- **Asynchronous inputs are inevitable** (user interfaces, systems with different clocks interacting, etc.)
- **Synchronizer goal:** make the probability of failure (the output *Q* still being metastable) low
- Synchronizer cannot make the probability of failure 0



Synchronizer Internals

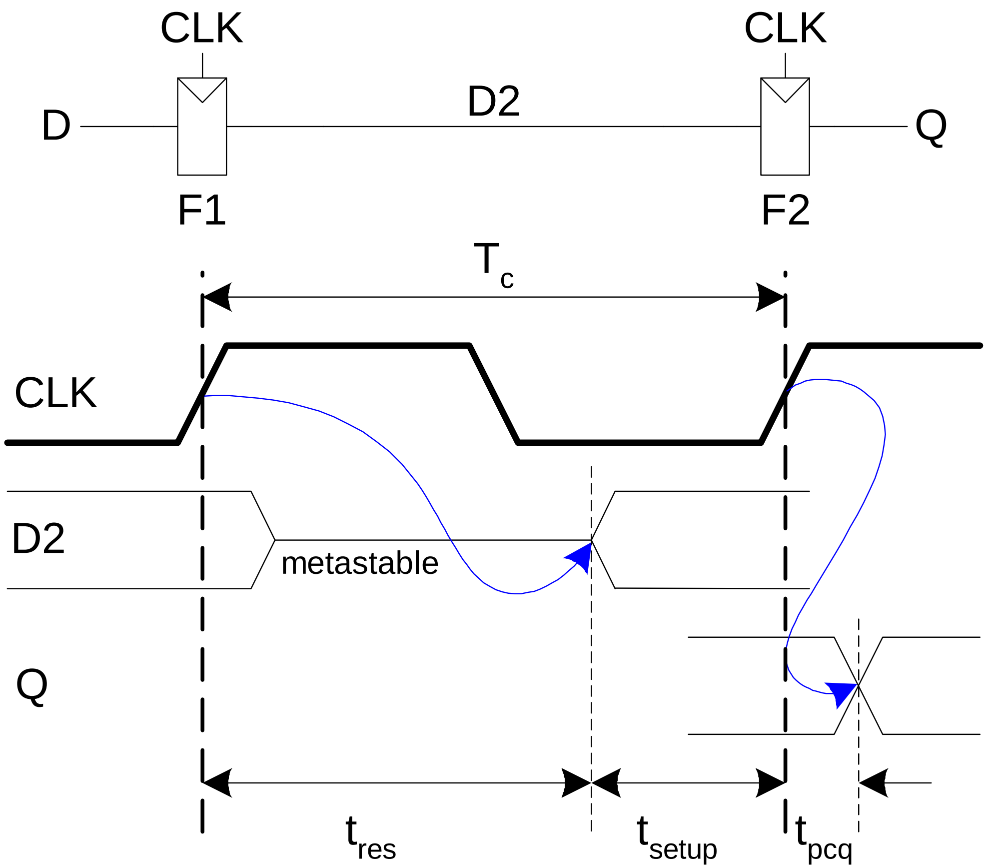
- Synchronizer: built with two back-to-back flip-flops
- Suppose D is transitioning when sampled by F1
- Internal signal D2 has (T_c - t_{setup}) time to resolve to 1 or 0



Synchronizer Probability of Failure

For each sample, probability of failure is:

$$P(\text{failure}) = (T_0/T_c)\, e^{-(T_c - t_{setup})/\tau}$$



Synchronizer Mean Time Between Failures

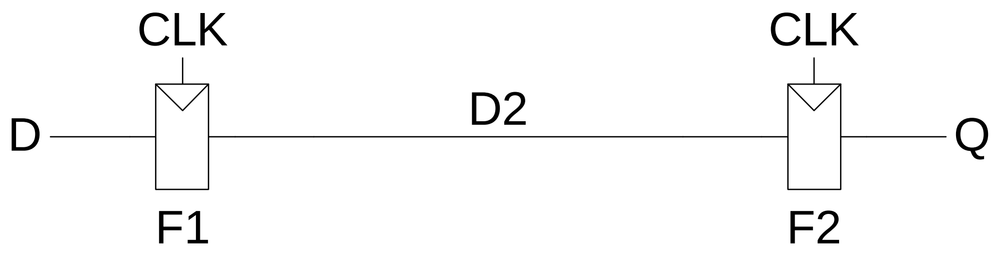
- If asynchronous input changes once per second, probability of failure per second is *P*(failure).
- If input changes *N* times per second, probability of failure per second is:

$$P(\text{failure})/\text{second} = (NT_0/T_c)\, e^{-(T_c - t_{setup})/\tau}$$

- Synchronizer fails, on average, 1/[*P*(failure)/second]
- Called ***mean time between failures***, MTBF:

$$\text{MTBF} = 1/[P(\text{failure})/\text{second}] = (T_c/NT_0)\, e^{(T_c - t_{setup})/\tau}$$

Example Synchronizer

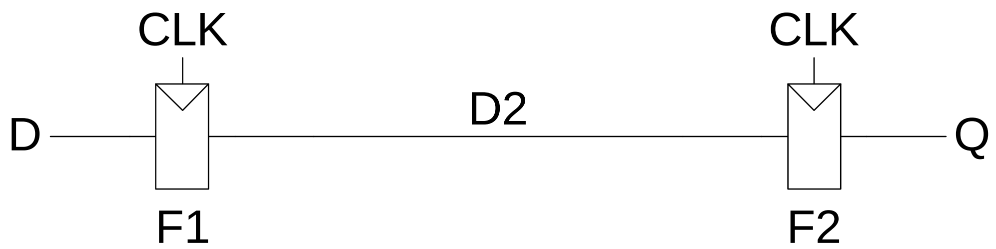


- Suppose: T_c = 1/500 MHz = 2 ns τ = 200 ps

  T_0 = 150 ps t_{setup} = 100 ps

  N = 10 events per second
- What is the probability of failure? MTBF?

Example Synchronizer



- Suppose: T_c = 1/500 MHz = 2 ns τ = 200 ps
  T_0 = 150 ps t_{setup} = 100 ps
  N = 10 events per second
- What is the probability of failure? MTBF?

$$P(\text{failure}) = (150\text{ ps}/2\text{ ns})\, e^{-(1.9\text{ ns})/200\text{ ps}}$$
$$= \mathbf{5.6 \times 10^{-6}}$$
$$P(\text{failure})/\text{second} = 10 \times (5.6 \times 10^{-6})$$
$$= 5.6 \times 10^{-5}\ /\ \text{second}$$
$$\text{MTBF} = 1/[\text{P(failure)/second}] \approx \mathbf{5\ hours}$$

Parallelism

- **Two types of parallelism:**
  - **Spatial parallelism**
    - duplicate hardware performs multiple tasks at once
  - **Temporal parallelism**
    - task is broken into multiple stages
    - also called pipelining
    - for example, an assembly line

Parallelism Definitions

- **Token:** Group of inputs processed to produce group of outputs
- **Latency:** Time for one token to pass from start to end
- **Throughput:** Number of tokens produced per unit time

Parallelism increases throughput

EQUENTIAL LOGIC DESI

Parallelism Example

- Ben Bitdiddle bakes cookies to celebrate traffic light controller installation
- 5 minutes to roll cookies
- 15 minutes to bake
- What is the latency and throughput without parallelism?

Parallelism Example

- Ben Bitdiddle bakes cookies to celebrate traffic light controller installation
- 5 minutes to roll cookies
- 15 minutes to bake
- What is the latency and throughput without parallelism?

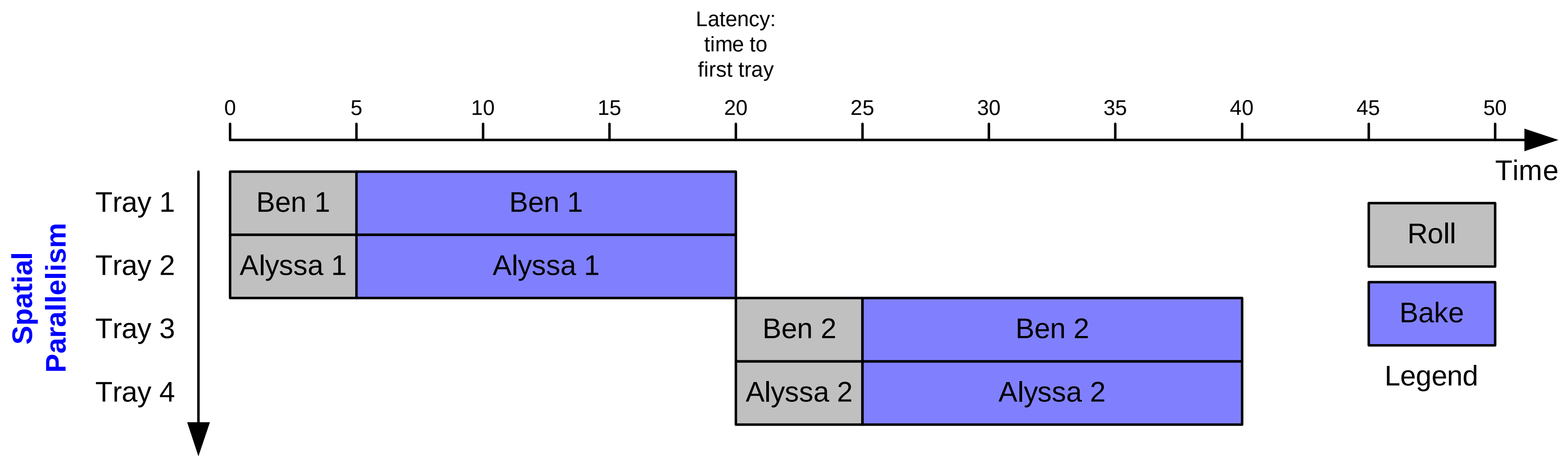
Latency = 5 + 15 = 20 minutes = **1/3 hour**

Throughput = 1 tray/ 1/3 hour = **3 trays/hour**

Parallelism Example

- What is the latency and throughput if Ben uses parallelism?
  - **Spatial parallelism:** Ben asks Allysa P. Hacker to help, using her own oven
  - **Temporal parallelism:**
    - two stages: rolling and baking
    - He uses two trays
    - While first batch is baking, he rolls the second batch, etc.

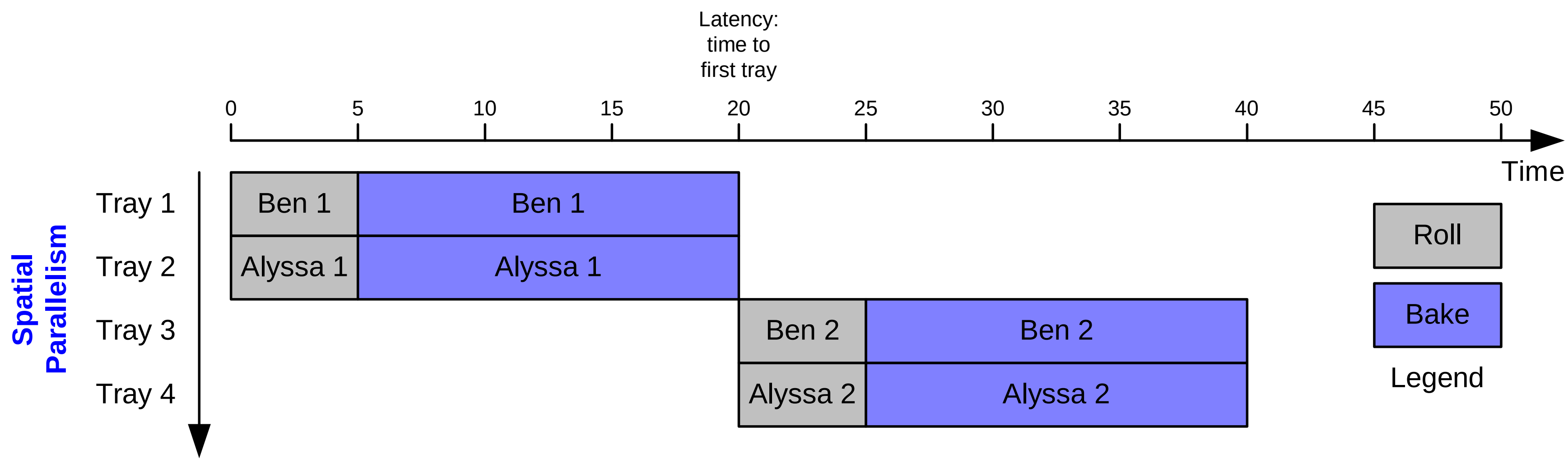
Spatial Parallelism



Latency = ?

Throughput = ?

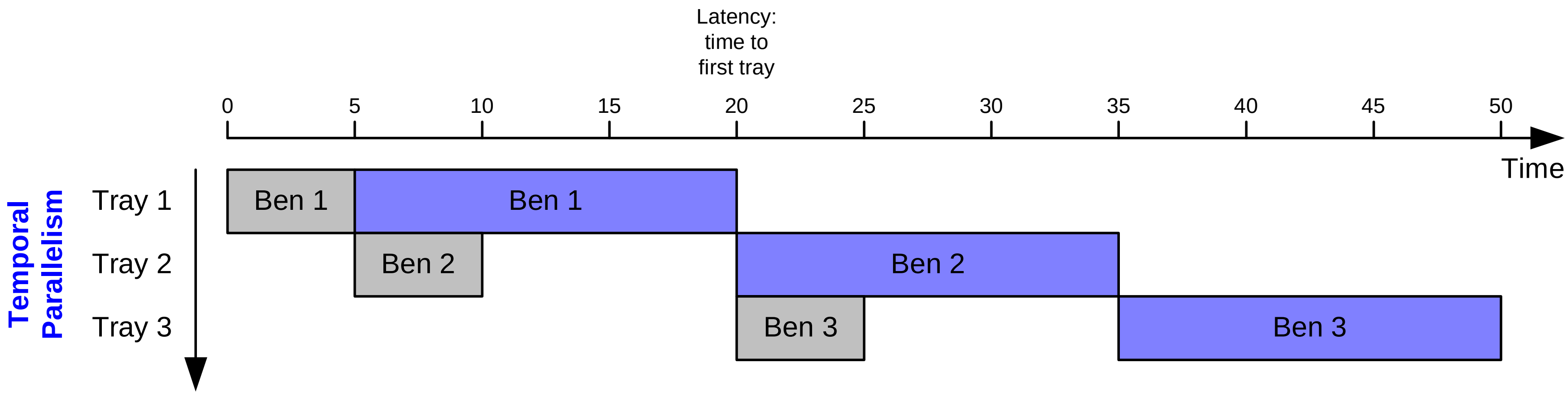
Spatial Parallelism



Latency = 5 + 15 = 20 minutes = **1/3 hour**

Throughput = 2 trays/ 1/3 hour = **6 trays/hour**

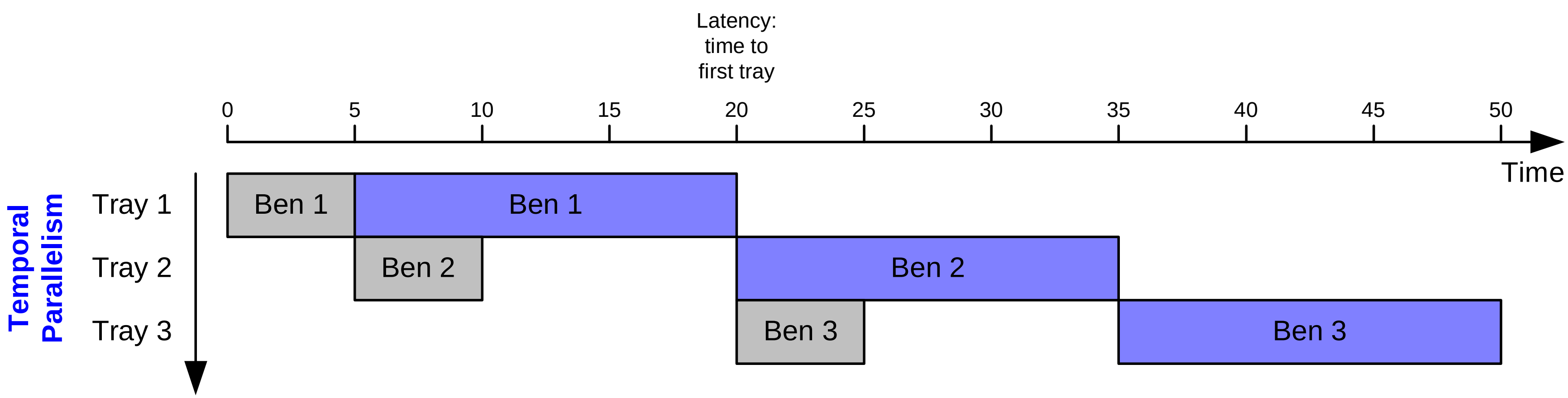
Temporal Parallelism



Latency = ?

Throughput = ?

Temporal Parallelism



Latency = 5 + 15 = 20 minutes = **1/3 hour**

Throughput = 1 trays/ 1/4 hour = **4 trays/hour**

Using both techniques, the throughput would be **8 trays/hour**